Latex Template

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Contents

1 Statement

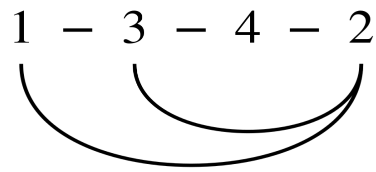
We've been given the following problem to talk about.

MINIMUM CUT LINEAR ARRANGEMENT
Given a graph $G = (V, E)$, compute a one-to-one function $f : V \to [1..|V|]$ so that the maximum number of cut edges in any integer point is minimised, i.e.

$$\max_{i \in 1..|V|} |\{\{u, v\} \in E : f(u) \leq i < f(v)\}|$$

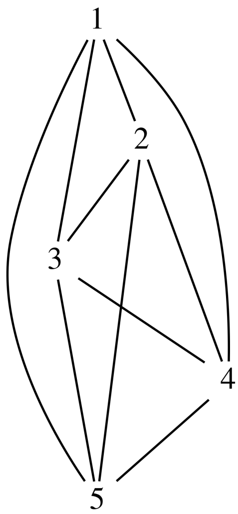
2 Example Instances

In order to visualize the problem and attack it using more intuition, we must consider that the cuts at integer points can be seen easily in the following manner. Given a graph G and an ordering function f, we draw the nodes in an horizontal line, in the order given by f. What we are trying to minimize is the size maximum cut among the S_i's , where $S_i = \{v \in V : f(v) \leq i\}$, for $i = 1, 2, \ldots, n-1$. But on the graph drawing mentioned above, the size of every S_i is the number of edges existing in the space between the vertical lines passing by nodes i and i+1. To give an example, in the following linear arrangement we easily deduce that the minimum S_i size is 3:



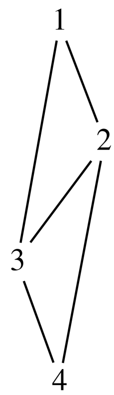
TODO: draw another example

The problem instances are Graphs G. As an example, we take $G = K_5$:

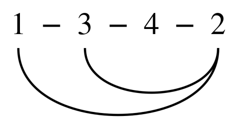


Here, every node is connected to every one of the rest. Therefore, every node is identical to every one else. In this case, every assignment of numbers $1 \ldots n$ to the nodes yields the same graph. Therefore to solve this instance, we consider a random labeling of our nodes, and we compute the cuts at all integer points between 1 and n.

Another instance, could be this graph.



Here, the assignment matters. In order to attack the problem in a more intuitive manner, it would be more suitable to have visualize the problem as follows: Given graph G, create a drawing of G such that the nodes are placed on a straight horizontal line. The objective is to minimize the maximum number of edges that exist in the space between two consecutive nodes. For example, the graph below is a visualization of the assignment $f(1) = 1, f(2) = 4, f(3) = 2, f(4) = 3$, which gives us $max(S_i) = 3$.



By the way, the above assignment gives us the best possible results, w.r.t. the requirements of the problems. That is, there is no assingment that gives us a lower maximum cutwidth. **explain better here**

3 Decisional Problem

Given a graph $G = (V, E)$ and $k \in N$, say if there exists one ordering of the vertexs so that the maximum number of cut edges in any integer point is less than or equal k, i.e.

$$\exists f : V \to [1..|V|] | \max_{i \in 1..|V|} | \{\{u, v\} \in E : f(u) \leq i < f(v)\} | \leq k$$

4 Parametrized Problems

Using this problem, and combining it with a computable function $\kappa : \Sigma^* \to N$ we can formulate a Parametrized version of this problem.

TODO: Add Natural numbers symbol

An example:

Input	Graph G and tw = treewidth(G)
Paremeters	tw
Question	What is the Min-Cut Linear Arrangement of G?

Given that we find an expression Monadic Second Order Logic, the above can be solved in $O(n * f(tw))$ [1].

5 A $(log^2(n))$ Approximation

There exists an algorithm that runs in subexponential time and achieves an $(log^2(n))$ approximation of the min.cut linear arrangement problem.

Source: [2]

TODO: examine explain how the time is subexponential TODO: explain the proof below in a better way (look at Vazirani-Approximation algorithms)

Problem 21.27 (Minimum b-balanced cut)

6 Various results

In this section we will present a number of results regarding our problem.

The problem for genera1 graphs is NP-complete [4], [5], even for planar graphs [8] For fixed cost k, the problem can be solved in $O(n^{k-1})$ time [9]. For trees, better results have been found.

The restriction of the problem to trees has been open for some time. In 1982 Lengauer described an approximation algorithm for trees in [6] that produces a layout with cutwidth at most twice the optimal. He also determined exactly the cutwidth of complete k-ary trees. M. Chung et al. present in [7] an algorithm that solves the MINCUT problem on trees in time $O(n(logn)^{d-2})$ where d is the maximum degree of the tree. Thus, their algorithm works in polynomial time for bounded degree trees, but exponential time in general. Yanakakis in 1983, presents an algorithm that finds a min-cut linear arrangement of a tree in O(n log n) time [3].

References

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