Problem 16

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Problem statement

The problem

We have to prove that board games like chess or go in nxn boards are in EXP.

The solution

This is easy. We only have to give an exponential algorithm that solves such a game.

The algorithm is as follows. We construct a tree of possible states that can be produced from the initial states by moving pieces around. For every configuration in the board, there are (at most) two nodes in the graph. One for the case that it is white's turn to play, and one for the case that it is black's turn.

Let us consider that in each game, there are c kinds of pieces. For example, in chess we have $c = 6$ (king,queen,rook,bishop,knight,pawn). That means that in every square of the board can have none or one of 2*c pieces (every piece can be either black or white). Therefore, the number of possible configurations on a $n * n$ board is $(2 * c + 1)^{n^2}$.

Clearly, this is a directed tree with $(2 * c + 1)^{n^2}$ nodes. Also, there are back edges, although these will not trouble us a lot, as shown later. While creating this tree, there some configurations in which either black or white wins. These are the leaves of the tree.

The creation of this tree takes exponential time. Why? Let us describe the constuction process with in detail.

Each state is in the form $s_i =< board_i, white\_plays_i >$. The first variable gives us the board configuration and the second is a boolean that is true if it is white's turn to play and false otherwise.

We begin with an initial state s_0, given as input to the problem. Then, we start exploring in a BFS manner. Firstly, we create the $p(n)$ states that can be generated by this state, where $p(n)$ is a polynomial of size n. We add the newly created states to a queue, and then we repeat the process for each one of the states in the queue, but without creating duplicate states. In this manner, we shall create no more than $2(2c + 1)^{n^2}$ nodes, and since for each node we try a p(n) of moves, the creation of tree is complete after $O((2c + 1)^{n^2} p(n))$ steps.

This tree will help us check if one of the two players has a winning strategy. This concludes the proof.