Problem 5

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Problem statement

The Experimental Cuisine problem

We are feeling experimental and want to create a new dish. There are various ingredients we can choose from and we'd like to use as many of them as possible, but some ingredients don't go well with others. If there are n possible ingredients (numbered 1 to n, we write down the $n \times n$ matrix D giving the *discord* between any pair of ingredients. This *discord* is a real number between 0.0 and 1.0, where 0.0 means "they go together perfectly" and 1.0 means "they don't go together". For example, if $D[2,3] = 0.1$ and $D[1,5] = 1.0$, then ingredients 2 and 3 go together pretty well whereas 1 and 5 clash badly.

Notice that D is necessarely symmetric and that the diagonal entries are always 0.0. Any set of ingredients always incurs a penalty which is the sum of all discors values between pairs of ingredients. For instance the set of ingredients $\{1,2,3\}$ incurs a penalty of $D[1,2] + D[1,3] + D[2,3]$. We want the penalty to be small.

Exercise

Given n ingredients, and the discord $n \times n$ matrix D and some number p, compute the maximum number of ingredients we can choose with penaly $\leq p$

Show that if Experimental Cuisine is solvable in polinomial time, then is so 3SAT.

Answer

A way to prove that if Experimental Cuisine is solvable in polinomial time, then so is 3sat is by reducing in polynomial time 3sat to Experimental Cuisine. If a reduction was found, we'd have:

3sat $\leq_T^P$ Experimental Cuisine

Which means that Experimental Cuisine is harder or as hard to solve as 3sat. Once we've proven this to be true, there is no doubt that if we assume that Experimental Cuisine is solvable in polynomial time, given that 3sat

$\leq_T^P$ EXPERIMENTAL CUISINE, we have that 3SAT must be solvable in plynomial time, because it is easier or as easy to solve as EXPERIMENTAL CUISINE, given that we find a reduction.

One way to prove this is by reducing 3SAT to EXPERIMENTAL CUISINE. Once this is done, we can safely assume that if EXPERIMENTAL CUISINE is solvable in polynomial time, so is 3SAT.

We will proceed with this reduction as follows:

1. By reducing 3SAT to INDEPENDENT SET
2. By reducing INDEPENDENT SET to EXPERIMENTAL CUISINE

The reduction from 3SAT to INDEPENDENT SET is proven here[1].

Now we need to reduce INDEPENDENT SET to EXPERIMENTAL CUISINE.

Given an instance of INDEPENDENT SET, G, where G is a graph, construct the following:

Set D to be an incidence matrix of G, where $a_{ij} = 0$ if the graph's vertices do not share a common edge, and $a_{ij} = 1$ if they are, and assign to the maximum discordance d the value of 0.

If we feed this values to an algoritm that solves EXPERIMENTAL CUISINE, the result that it yields is a subset of verticies V such that V is an INDEPENDENT SET of G.

Proving that this reduction is correct Given a graph $G = (V, E)$ and a vertex set $I \subseteq V$ that is the independent set of G with highest cardinality, it is easy to see that, when we transform this configuration using the steps described above, it is a valid solution to EXPERIMENTAL CUISINE, and adding one more vertex to is not. This means it is the maximum subset that satisfies the problem.

References

[1] Proving a problem is np-complete. `http://www.cs.cornell.edu/courses/cs482/2005su/handouts/NPComplete.pdf`, 2005.