5. Obtenció del criteri d'en Fisher

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10 d'octubre de 2017

Usant les defnicions vistes a classe per les matrius de dispersió (*scatter*) intra-classes S_W i inter-classes S_B , demostreu que el criteri d'en Fisher es pot escriure com:

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

1. **Demostreu primer que** $s_1^2 + s_2^2 = w^T S_W w$

   Repasamos primero las definiciones:

$$m_i = \frac{1}{N_i} \sum_{j \in C_i} x_j$$

$$\mu_i = w^T \cdot m_i$$

$$s_i^2 = \sum_{i \in C_i} (w^T \cdot x_i - \mu_i)^2$$

$$S_W = \sum_{i \in C_1} (x_i - m_1)(x_i - m_1)^T + \sum_{i \in C_2} (x_i - m_2)(x_i - m_2)^T$$

$$S_B = (m_2 - m_1)(m_2 - m_1)^T$$

   Por lo tanto hay que demostrar que:

$$\sum_{i \in C_1} (w^T \cdot x_i - \mu_1)^2 + \sum_{i \in C_2} (w^T \cdot x_i - \mu_2)^2 = \\ = w^T \cdot \Big[\sum_{i \in C_1} (x_i - m_1)(x_i - m_1)^T + \sum_{i \in C_2} (x_i - m_2)(x_i - m_2)^T \Big] \cdot w$$

$$\sum_{i \in C_1} (w^T \cdot x_i - w^T \cdot m_1)^2 + \sum_{i \in C_2} (w^T \cdot x_i - w^T \cdot m_2)^2 = \\ = w^T \cdot \Big[\sum_{i \in C_1} (x_i - m_1)(x_i - m_1)^T + \sum_{i \in C_2} (x_i - m_2)(x_i - m_2)^T \Big] \cdot w$$

$$
\begin{aligned}
&\sum_{i \in C_1} \Big(w^T (x_i - m_1) \Big)^2 + \sum_{i \in C_2} \Big(w^T (x_i - m_2) \Big)^2 = \\
&\quad = w^T \cdot \Big[\sum_{i \in C_1} (x_i - m_1)(x_i - m_1)^T + \sum_{i \in C_2} (x_i - m_2)(x_i - m_2)^T \Big] \cdot w
\end{aligned}
$$

2. **Demostreu que** $(\mu_1 - \mu_2)^2 = w^T S_B w$