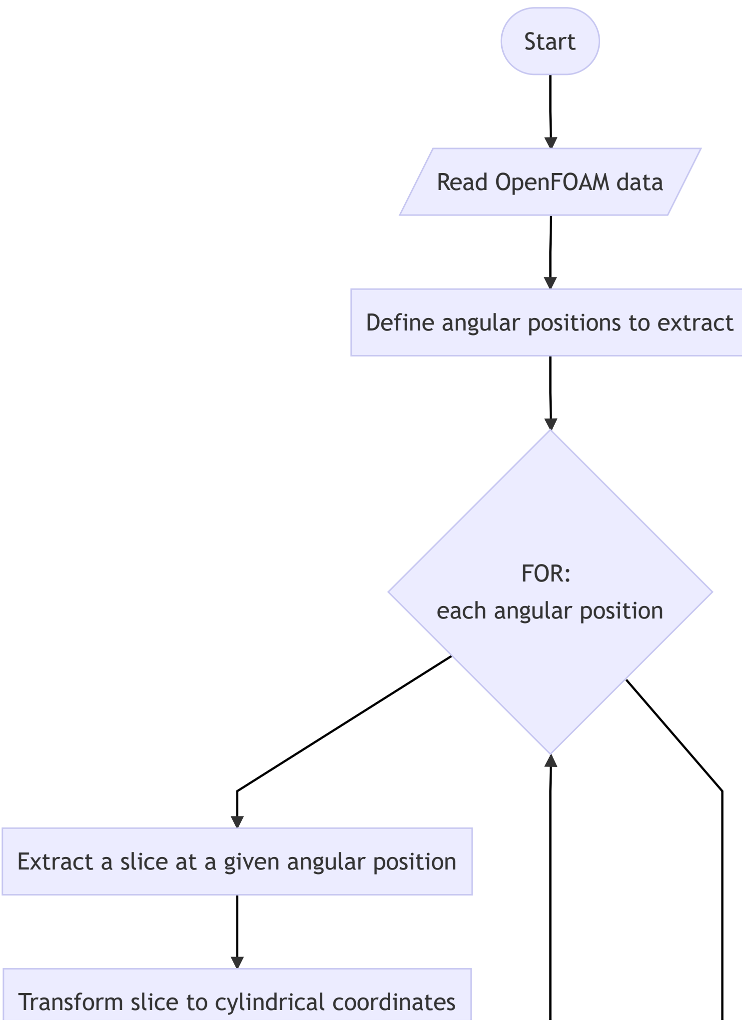
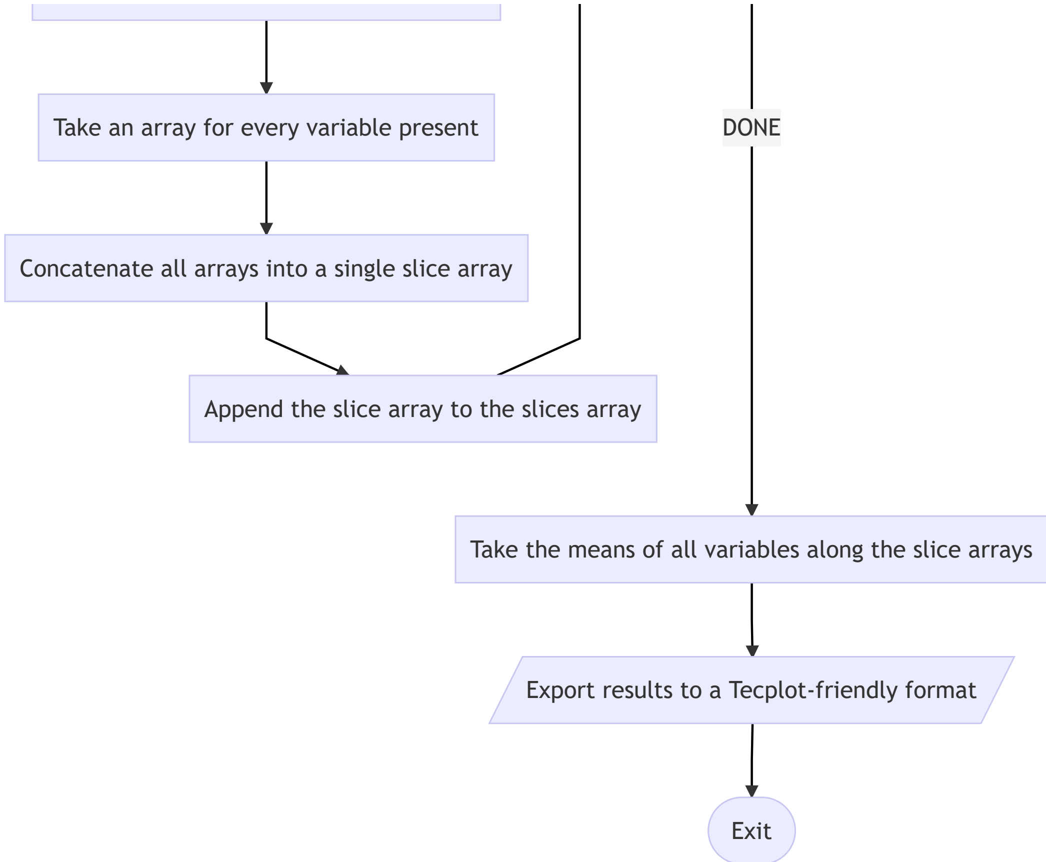
Azimuthal Average of OpenFOAM results

Conceptual Design





Cylindrical transformation for vectorial and tensorial quantities

Vectors

$$\vec{u} : \text{Vector in cartesian coordinates } (u_x, u_y, u_z)$$
$$\vec{v} : \text{Vector in cylindrical coordinates } (v_r, v_t, v_z)$$

As a general rule,

$$
\begin{aligned}
v_r &= u_x \cos\theta + u_y \sin\theta \\
v_t &= -u_x \sin\theta + u_y \cos\theta \\
v_z &= u_z
\end{aligned}
$$

Tensors

$$
\begin{gathered}
T : \text{Tensor in cartesian coordinates} \\
\begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} \\
R : \text{Tensor in cylindrical coordinates} \\
\begin{bmatrix} R_{rr} & R_{rt} & R_{rz} \\ R_{tr} & R_{tt} & R_{tz} \\ R_{zr} & R_{zt} & R_{zz} \end{bmatrix}
\end{gathered}
$$

For the particular case in which T and R are symmetric tensors,

$$
\begin{aligned}
R_{rr} &= T_{xx} \cos^2\theta + 2T_{xy} \sin\theta\cos\theta + T_{yy} \sin^2\theta \\
R_{tt} &= T_{xx} \sin^2\theta - 2T_{xy} \sin\theta\cos\theta + T_{yy} \cos^2\theta \\
R_{zz} &= T_{zz} \\
R_{zt} &= -T_{xz} \sin\theta + T_{yz} \cos\theta \\
R_{zr} &= T_{xz} \cos\theta + T_{yz} \sin\theta \\
R_{rt} &= \left(T_{yy} - T_{xx}\right) \cos\theta \sin\theta + T_{xy} (\cos^2\theta - sin^2\theta)
\end{aligned}
$$

Rotation Matrix

Both vector and tensor transformations can be summarized into a single operation using lineal algebra.

Let,

$$
Q : \text{Rotation Matrix}
$$

Where,

$$
Q = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

The following holds:

$$
\begin{gathered}
\vec{v} = Q \cdot \vec{u} \\
R = Q \cdot T \cdot Q^T
\end{gathered}
$$

```
def get_cartesian_to_cylindrical_rotation_matrix(radial_vector):
    sin_t, cos_t = radial_vector.get_sin_t_and_cos_t()
```

```python
    return np.array(
        [[cos_t, sin_t, 0],
         [-sin_t, cos_t, 0],
         [0, 0, 1]]
    )


r"""
/*---------------------------------*\

             VECTORS

\*---------------------------------*/
"""


class CartesianVector:
    """Class for vectors in cartesian coordinates.

    Instance attributes:

    self.vector -- numpy array containing the vector
    self.x -- radial component
    self.y -- azimuthal component
    self.z -- axial component
    """

    def __init__(self, array_like):
        """Constructor for a CylindricalVector instance.

        Arguments:

        array_like -- Array like structure ordered as (x_comp, y_comp,
z_comp)
        """
        self.vector = np.array(array_like)
        self.x = self.vector[0]
        self.y = self.vector[1]
        self.z = self.vector[2]
        return

    def get_sin_t_and_cos_t(self):
        radial_magnitude = np.sqrt(self.y**2 + self.x**2)
        return self.y / radial_magnitude, self.x / radial_magnitude

    def get_magnitude(self):
        return np.sqrt(self.vector.dot(self.vector))
```

```python
    def convert_to_cylindrical(self, position_vector):
        rotation_matrix = 
get_cartesian_to_cylindrical_rotation_matrix(position_vector)

        cylindrical_vector_array = rotation_matrix.dot(self.vector)
        return CylindricalVector(cylindrical_vector_array)

    def __str__(self):
        return f"{self.vector}"


class CylindricalVector:
    """Class for vectors in cylindrical coordinates.

    Instance attributes:

    self.vector -- numpy array containing the vector
    self.r -- radial component
    self.t -- azimuthal component
    self.z -- axial component
    """

    def __init__(self, array_like):
        """Constructor for a CylindricalVector instance.

        Arguments:

        array_like -- Array like structure ordered as (r_comp, t_comp, 
z_comp)
        """
        self.vector = np.array(array_like)
        self.r = self.vector[0]
        self.t = self.vector[1]
        self.z = self.vector[2]
        return

    def __str__(self):
        return f"{self.vector}"


r"""
/*---------------------------------*\

             TENSORS

\*---------------------------------*/
"""
```

```python
def rotate_tensor(tensor, rotation_matrix):
    return rotation_matrix.dot(tensor.dot(rotation_matrix.T))


class SymmetricCartesianTensor:

    list_of_components = ["xx", "xy", "xz", "yy", "yz", "zz"]

    def __init__(self, array_like):
        for comp, array_elem in
zip(SymmetricCartesianTensor.list_of_components, array_like):
            setattr(self, comp, array_elem)
        self.tensor = np.array(
            [[self.xx, self.xy, self.xz],
             [self.xy, self.yy, self.yz],
             [self.xz, self.yz, self.zz]]
        )
        return

    def convert_to_cylindrical(self, radial_vector):
        rotation_matrix =
get_cartesian_to_cylindrical_rotation_matrix(radial_vector)
        cylindrical_tensor_array = rotate_tensor(self.tensor,
rotation_matrix)
        return
SymmetricCylindricalTensor(cylindrical_tensor_array[np.triu_indices(3)])

    def __str__(self):
        return f"{self.tensor}"


class SymmetricCylindricalTensor:
    list_of_components = ["rr", "rt", "rz", "tt", "tz", "zz"]

    def __init__(self, array_like):
        for comp, array_elem in
zip(SymmetricCylindricalTensor.list_of_components, array_like):
            setattr(self, comp, array_elem)
        self.tensor = np.array(
            [[self.rr, self.rt, self.rz],
             [self.rt, self.tt, self.tz],
             [self.rz, self.tz, self.zz]]
        )
    def __str__(self):
        return f"{self.tensor}"
```