

3. (20 pts) For each one of the loops below, give a Big-Oh analysis of the running time. Show work/reasoning.

a) `sum = 0;`
 `for (i = 0; i < n; ++i)`
 `++sum;`

$O(n)$
 for loop runs n times in total from 0 to $n-1$

b) `sum = 0;`
 `for (i = 0; i < n; ++i)`
 `for (j = 0; j < n; j++)`
 `++sum;`

$O(n^2)$
 outer loop runs total n times, for each outer loop n times of run inner loop runs n times also.
 $Total = n \times n = n^2$

c) `sum = 0;`
 `for (i = 0; i < n; ++i)`
 `for (j = 0; j < n*n; ++j)`
 `++sum;`

$O(n^3)$
 outer loop need n times to run, for each outer loop, inner loop need n^2 times to run.
 $Total = n \times n^2 = n^3$

d) `sum = 0;`
 `for (i = 0; i < n; ++i)`
 `for (j = 0; j < i; ++j)`
 `++sum;`

$O(n^2)$
 outer loops runs n times, inner times runs up to i times. When $i = n-1$, inner loop runs $n-1$ times.
 $Total = \frac{n \times (n-1)}{2} = \frac{n^2 - n}{2} = n^2$

e) `sum = 0;`
 `for (i = 0; i < n; ++i)`
 `for (j = 0; j < i * i; ++j)`
 `for (k = 0; k < j; ++k)`
 `++sum;`

$O(n^4)$
 outer loop runs n times and for each mid loop runs, then inner loop runs up to i^2 times
 $Total = n \times \frac{(n)(n+1)(2n+1)}{6} = n^4$

f) `sum = 0;`
 `for (i = 1; i < n; ++i)`
 `for (j = 1; j < i * i; ++j)`
 `if (j % i == 0)`
 `for (k = 0; k < j; ++k)`
 `++sum;`

$O(n^4)$
 outer loop runs in $n-1$ times, and for each outer loop mid loop runs up to i^2 times, then inner loop runs for each even j .
 $Total = \frac{n(n+1)(2n+1)}{6} \times \frac{n}{2} = n^4$