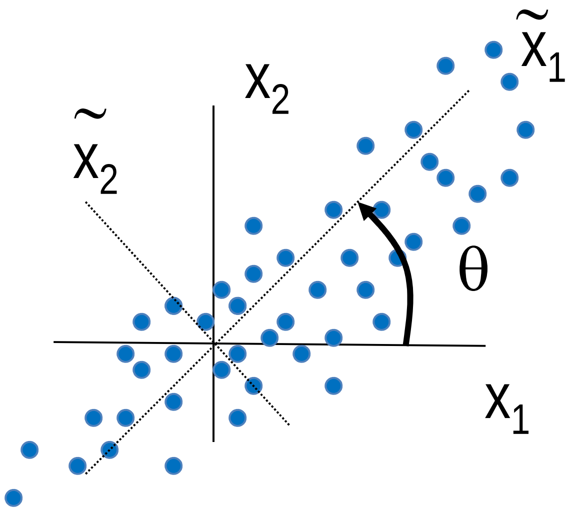
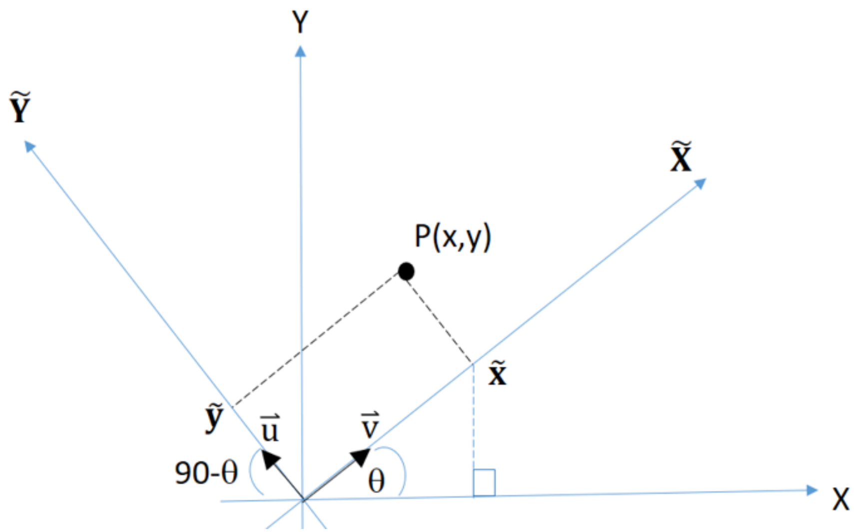
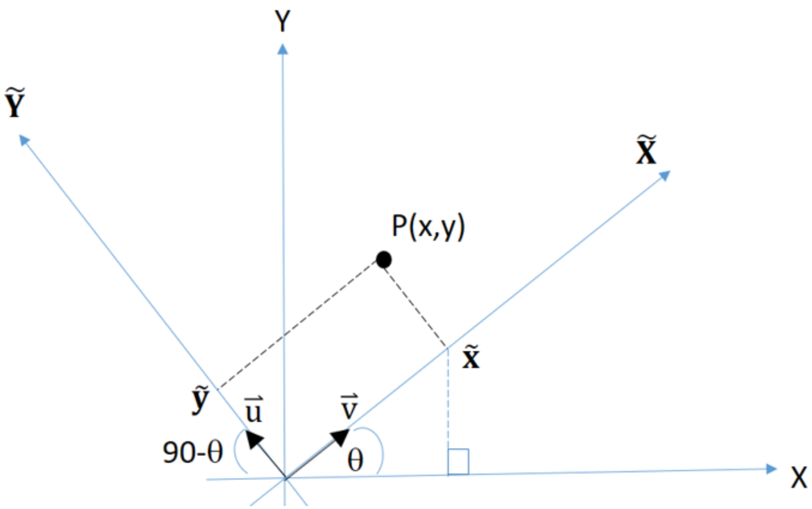
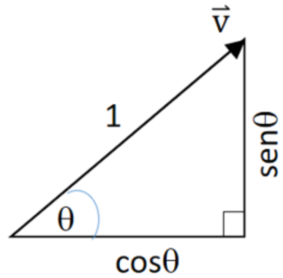
1 Análisis del modelo de rotación de ejes

Si x_1 y x_2 son variables aleatorias y no varían independientemente

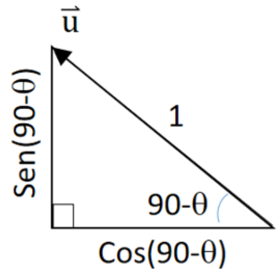








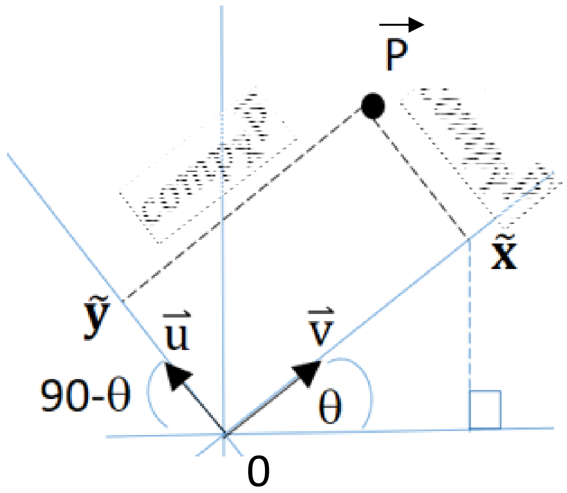
Dado que $\|\vec{v}\|=1$, entonces, $\vec{v} = (\cos\theta, \mathrm{sen}\theta)$



Dado que $\|\vec{u}\|=1$, entonces,

$$\vec{u} = (\cos(90-\theta), \mathrm{sen}(90-\theta)) \Rightarrow \vec{u} = (-\mathrm{sen}\theta, \cos\theta)$$

Calculamos las coordenadas de **P** en $\tilde{X}\tilde{Y}$:



$$P = (comp_{\tilde{X}}\vec{P},\ comp_{\tilde{Y}}\vec{P})$$

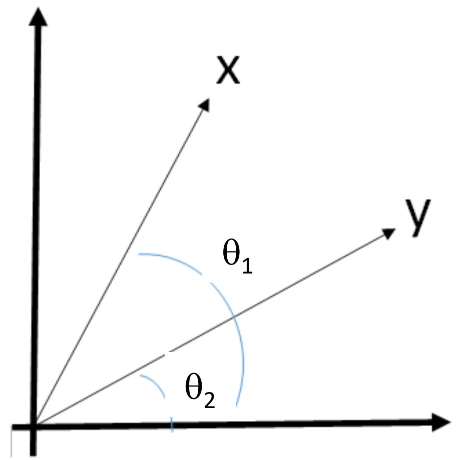
$$\text{Comp}_{\vec{v}}\vec{p} = \frac{\vec{P}'\vec{v}}{\|\vec{v}\|} = \vec{P}'\vec{v} = (x, y)\begin{pmatrix}\cos\theta \\ \text{sen}\theta\end{pmatrix} = x\cos\theta + y\,\text{sen}\theta$$

$$\text{Comp}_{\vec{u}}\vec{p} = \frac{\vec{P}'\vec{u}}{\|\vec{u}\|} = \vec{P}'\vec{u} = (x, y)\begin{pmatrix}-\text{sen}\theta \\ \cos\theta\end{pmatrix} = -x\,\text{sen}\theta + y\cos\theta$$

Se verifica que: $\left\|\overrightarrow{\mathbf{P}\tilde{x}\tilde{y}}\right\| = \left\|\overrightarrow{\mathbf{P}xy}\right\|$

2

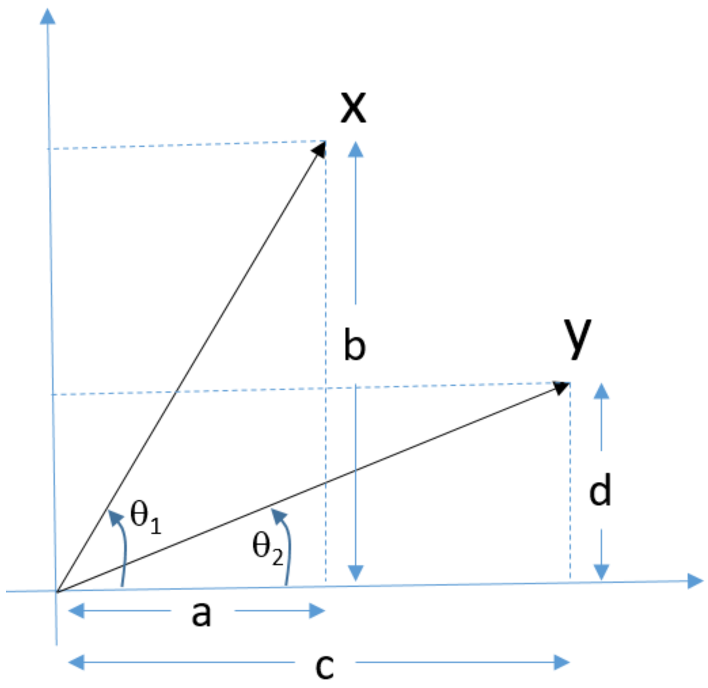
A partir de los datos proporcionados en el gráfico determine la correlación entre ***x*** e ***y***.



$$x = \begin{pmatrix} a \\ b \end{pmatrix} \qquad y = \begin{pmatrix} c \\ d \end{pmatrix}$$

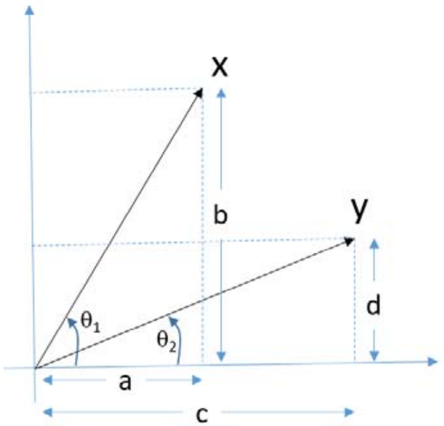
Por definición: $\mathbf{r}_{xy} = \cos\theta$

$$\mathbf{r}_{xy} = \cos(\theta_1 - \theta_2)$$



$$\mathbf{r}_{xy} = \text{Cos}\,(\theta_1 - \theta_2) \quad \Rightarrow \quad \mathbf{r}_{xy} = \text{Cos}\,\theta_1\,\text{Cos}\,\theta_2 + \text{Sen}\,\theta_1\,\text{Sen}\,\theta_2 \qquad \text{(i)}$$

Del gráfico:



$$\left.\begin{aligned} \text{Cos}\,\theta_1 &= \frac{a}{\|x\|} & \text{Cos}\,\theta_2 &= \frac{c}{\|y\|} \\ \text{Sen}\,\theta_1 &= \frac{b}{\|x\|} & \text{Sen}\,\theta_2 &= \frac{d}{\|y\|} \end{aligned}\right\} \text{ en (i)}$$

$$\Rightarrow \quad \mathbf{r}_{xy} = \frac{a}{\|x\|}\frac{c}{\|y\|} + \frac{b}{\|x\|}\frac{d}{\|y\|}$$

$$\Rightarrow \quad \mathbf{r}_{xy} = \frac{ac + bd}{\|x\|\,\|y\|} \quad \Rightarrow \quad \mathbf{r}_{xy} = \frac{x'y}{\|x\|\,\|y\|}$$

3

Muestre mediante geometría muestral que la media y la matriz varianza covarianza muestral son independientes.

Mostrar:

$$\underset{(px1)}{\bar{\mathbf{X}}} = \begin{pmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \bar{X}_p \end{pmatrix} \quad \textit{es independientes a} \quad S = \begin{bmatrix} \frac{s_{11}}{n} & \cdots & \frac{s_{1p}}{n} \\ \vdots & \ddots & \vdots \\ \frac{s_{p1}}{n} & \cdots & \frac{s_{pp}}{n} \end{bmatrix}$$

Veamos:

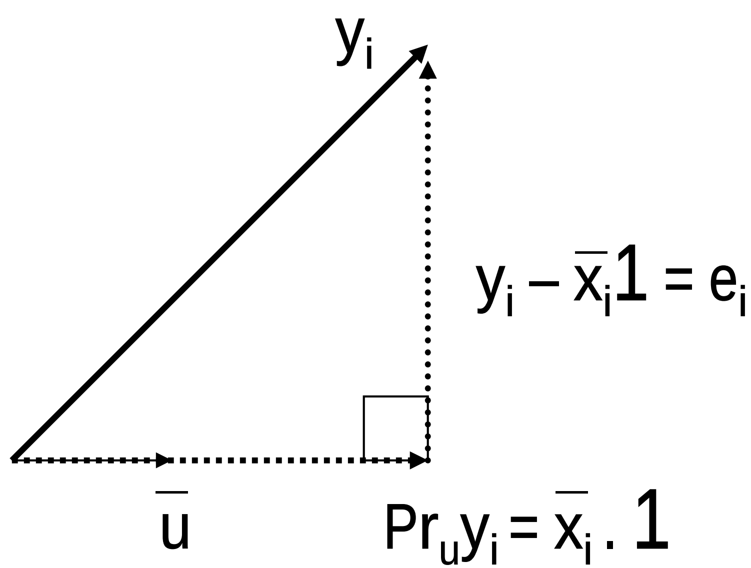
$$S = \begin{bmatrix} \frac{(\mathrm{x}_1 - \bar{\mathrm{x}}_1 \mathbf{1}_\mathrm{n})'(\mathrm{x}_1 - \bar{\mathrm{x}}_1 \mathbf{1}_\mathrm{n})}{n} & \cdots & \frac{(\mathrm{x}_1 - \bar{\mathrm{x}}_1 \mathbf{1}_\mathrm{n})'(\mathrm{x}_\mathrm{p} - \bar{\mathrm{x}}_\mathrm{p} \mathbf{1}_\mathrm{n})}{n} \\ \vdots & \ddots & \vdots \\ \frac{(\mathrm{x}_\mathrm{p} - \bar{\mathrm{x}}_\mathrm{p} \mathbf{1}_\mathrm{n})'(\mathrm{x}_1 - \bar{\mathrm{x}}_1 \mathbf{1}_\mathrm{n})}{n} & \cdots & \frac{(\mathrm{x}_\mathrm{p} - \bar{\mathrm{x}}_\mathrm{p} \mathbf{1}_\mathrm{n})'(\mathrm{x}_\mathrm{p} - \bar{\mathrm{x}}_\mathrm{p} \mathbf{1}_\mathrm{n})}{n} \end{bmatrix}$$

$$nS=\begin{bmatrix}(\mathrm{x}_1-\bar{\mathrm{x}}_1\mathbf{1}_n)'(\mathrm{x}_1-\bar{\mathrm{x}}_1\mathbf{1}_n) & \cdots & (\mathrm{x}_1-\bar{\mathrm{x}}_1\mathbf{1}_n)'(\mathrm{x}_p-\bar{\mathrm{x}}_p\mathbf{1}_n)\\ \vdots & \ddots & \vdots\\ (\mathrm{x}_p-\bar{\mathrm{x}}_p\mathbf{1}_n)'(\mathrm{x}_1-\bar{\mathrm{x}}_1\mathbf{1}_n) & \ldots & (\mathrm{x}_p-\bar{\mathrm{x}}_p\mathbf{1}_n)'(\mathrm{x}_p-\bar{\mathrm{x}}_p\mathbf{1}_n)\end{bmatrix}$$

$$nS=\begin{bmatrix}(\mathrm{x}_1-\bar{\mathrm{x}}_1\mathbf{1}_n)'\\ (\mathrm{x}_2-\bar{\mathrm{x}}_2\mathbf{1}_n)'\\ \vdots\\ (\mathrm{x}_p-\bar{\mathrm{x}}_p\mathbf{1}_n)'\end{bmatrix}\begin{bmatrix}(\mathrm{x}_1-\bar{\mathrm{x}}_1\mathbf{1}_n) & (\mathrm{x}_2-\bar{\mathrm{x}}_2\mathbf{1}_n) & \cdots & (\mathrm{x}_p-\bar{\mathrm{x}}_p\mathbf{1}_n)\end{bmatrix}$$

Bastaría con analizar la independencia entre:

$$\underset{(pxn)}{\begin{bmatrix}(\mathrm{x}_1-\bar{\mathrm{x}}_1\mathbf{1}_n)'\\ (\mathrm{x}_2-\bar{\mathrm{x}}_2\mathbf{1}_n)'\\ \vdots\\ (\mathrm{x}_p-\bar{\mathrm{x}}_p\mathbf{1}_n)'\end{bmatrix}} \quad y \quad \underset{(px1)}{\bar{\mathbf{X}}} = \begin{pmatrix}\bar{X}_1\\ \bar{X}_2\\ \cdot\\ \cdot\\ \cdot\\ \bar{X}_p\end{pmatrix}$$



Sabiendo que:

$y_i = (x_{i1}, x_{i2}, \ldots, x_{1n})$ y $1 = (1, 1, \ldots, 1)_{nx1}$

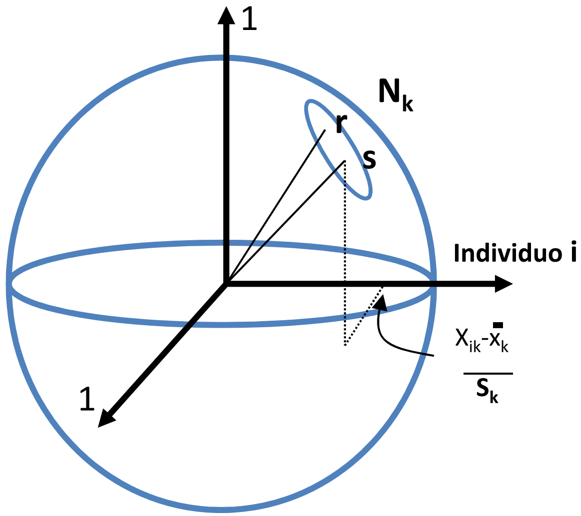
$$\Rightarrow \quad \bar{x}_i \,.\, 1 \perp y_i - \bar{x}_i 1 = e_i$$

$$\Rightarrow \quad \bar{x} \perp S$$

4

Proponga una medida de proximidad en el espacio de individuos para los puntos variables ***r***, ***s***.

$$d^2(r.s) = \frac{1}{n}\sum_k \left[\frac{x_{kr}-\bar{x}_r}{s_r} - \frac{x_{ks}-\bar{x}_s}{s_s}\right]^2$$



$$d^2(r.s) = \frac{1}{n}\sum_k \left[\left(\frac{x_{kr}-\bar{x}_r}{s_r}\right)^2 + \left(\frac{x_{ks}-\bar{x}_s}{s_s}\right)^2 - 2\left(\frac{x_{kr}-\bar{x}_r}{s_r}\right)\left(\frac{x_{ks}-\bar{x}_s}{s_s}\right)\right]$$

$$= \sum_k \frac{(x_{kr}-\bar{x}_r)^2}{n s_r^2} + \sum_k \frac{(x_{ks}-\bar{x}_s)^2}{n s_s^2} - 2\sum_k \frac{(x_{kr}-\bar{x}_r)(x_{ks}-\bar{x}_s)}{n s_r s_s}$$

$$\Rightarrow \quad = \frac{s_r^2}{s_r^2} + \frac{s_s^2}{s_s^2} - 2\frac{s_{rs}}{s_r s_s} = 2 - 2r_{rs}$$

$$\Rightarrow \quad d^2(r.s) = 2(1 - r_{rs})$$

Conclusión: $r_{rs}\uparrow$ $\begin{cases} d^2(r.s)\downarrow \\ \cos(\theta)\uparrow \quad \theta\downarrow \quad d^2(r.s)\downarrow \end{cases}$

GRACIAS!!!