Homework #15 Key

1 Matrix multiplication

$M_1 : 20 \times 5, M_2 : 5 \times 10, M_3 : 10 \times 12, M_4 : 12 \times 6, M_5 : 6 \times 25$

The first thing we do is calculate the cost of multiplying any two adjacent matrices.

M_1M_2	M_1
M_2	$20 \cdot 5 \cdot 10 = 1000$

M_2M_3	M_2
M_3	$5 \cdot 10 \cdot 12 = 600$

M_3M_4	M_3
M_4	$10 \cdot 12 \cdot 6 = 720$

M_4M_5	M_4
M_5	$12 \cdot 6 \cdot 25 = 1800$

We now calculate the minimum cost of multiplying any three consecutive matrices. For example, to calculate the cost of $M_1 \times M_2 \times M_3$, we consider the cost of $M_1 \times (M_2 \times M_3)$ and $(M_1 \times M_2) \times M_3$. The total cost includes the cost of $M_2 \times M_3$ and $M_1 \times M_2$ respectively.

$M_1M_2M_3$	M_1	M_1M_2
M_2M_3	$20 \cdot 5 \cdot 12 + 600 = 1800$	
M_3		$20 \cdot 10 \cdot 12 + 1000 = 3400$

$M_2M_3M_4$	M_2	M_2M_3
M_3M_4	$5 \cdot 10 \cdot 6 + 720 = 1020$	
M_4		$5 \cdot 12 \cdot 6 + 600 = 960$

$M_3M_4M_5$	M_3	M_3M_4
M_4M_5	$10 \cdot 12 \cdot 25 + 1800 = 4800$	
M_5		$10 \cdot 6 \cdot 25 + 720 = 2220$

We do the same thing for any four consecutive matrices, referring to the tables above as necessary. When determining how much to add from a table above, we choose the minimum. In this example, to obtain $M_1M_2M_3M_4$, it is cheapest to calculate $M_2M_3M_4$ and then multiply M_1 by that result.

$M_1M_2M_3M_4$	M_1	M_1M_2	$M_1M_2M_3$
$M_2M_3M_4$	$20 \cdot 5 \cdot 6 + 960 = 1560$		
M_3M_4		$20 \cdot 10 \cdot 6 + 1000 + 720 = 2920$	
M_4			$20 \cdot 12 \cdot 6 + 1800 = 3240$

$M_2M_3M_4M_5$	M_2	M_2M_3	$M_2M_3M_4$
$M_3M_4M_5$	$5 \cdot 10 \cdot 25 + 2220 = 3470$		
M_4M_5		$5 \cdot 12 \cdot 25 + 600 + 1800 = 3900$	
M_5			$5 \cdot 6 \cdot 25 + 960 = 1710$

Finally, we can calculate the minimum cost to find the product of all the matrices.

$M_1M_2M_3M_4M_5$	M_1	M_1M_2	$M_1M_2M_3$	$M_1M_2M_3M_4$
$M_2M_3M_4M_5$	$20 \cdot 5 \cdot 25$ $+1710 = 4210$			
$M_3M_4M_5$		$20 \cdot 10 \cdot 25$ $+1000 + 2220 = 8220$		
M_4M_5			$20 \cdot 12 \cdot 25$ $+1800 + 1800 = 9600$	
M_5				$20 \cdot 6 \cdot 25$ $+1560 = 4560$

2 All paths

The Initial table of 1-hop distances from the graph, Table$(i, j, 0)$. We only need to consider the upper triangle, because the graph is undirected, so distances are symmetric.

	a	b	c	d	e
a	0	3	5	-	9
b		0	-	-	-
c			0	2	4
d				0	1
e					0

Table$(i, j, 1)$, being the shortest distances considering hopping through vertex a:

	a	b	c	d	e
a	0	3	5	-	9
b		0	8	-	12
c			0	2	4
d				0	1
e					0

Table$(i, j, 2)$, being the shortest distances considering hopping through vertex b:

	a	b	c	d	e
a	0	3	5	-	9
b		0	8	-	12
c			0	2	4
d				0	1
e					0

Table$(i, j, 3)$, being the shortest distances considering hopping through vertex c:

	a	b	c	d	e
a	0	3	5	7	9
b		0	8	10	12
c			0	2	4
d				0	1
e					0

Table$(i, j, 4)$, being the shortest distances considering hopping through vertex d:

	a	b	c	d	e
a	0	3	5	7	8
b		0	8	10	11
c			0	2	3
d				0	1
e					0

Table$(i, j, 5)$, being the shortest distances considering hopping through vertex e:

	a	b	c	d	e
a	0	3	5	7	8
b		0	8	10	11
c			0	2	3
d				0	1
e					0