Homework #1 Key

0.1

a

$n - 100 = \Theta(n - 200)$

b

$n^{1/2} = O(n^{2/3})$

c

$100n - \log n = \Theta(n + (\log n)^2)$

d

$n \log n = \Theta(10n \log 10n)$

e

$\log 2n = \Theta(\log 3n)$ since $\frac{\log 2n}{\log 3n} = \frac{\log n + \log 2}{\log n + \log 3}$

f

$10 \log n = \Theta(\log n^2)$ since $\frac{10 \log n}{\log n^2} = \frac{10 \log n}{2 \log n} = 5$

m

$n2^n = O(3^n)$

0.2

If $c \neq 1$, the "closed form" formula for a geometric series is $g(n) = \frac{1-c^{n+1}}{1-c} = \frac{c^{n+1}-1}{c-1}$.

a

Since if $c < 1$, $1 > 1 - c^{n+1} > 1 - c$, then $\frac{1}{1-c} > g(n) > 1$.

b

If $c = 1$, $g(n) = 1 + 1 + 1 + \cdots + 1 = n + 1$.

c

Since if $c > 1$, $c^{n+1} > c^{n+1} - 1 > c^n$, then $\frac{c}{c-1}c^n > g(n) > \frac{1}{c-1}c^n$.

0.3 Fabonacci

a

$fabexp(n)$
  if $n = 0$ or $n = 1$ or $n = 2$ return 1
  return $fabexp(n-1) + fabexp(n-2) * fabexp(n-3)$
  Adds and Mults: Both $O(3^n)$

b

$fablin(n)$
  if $n = 0$ or $n = 1$ or $n = 2$ return 1
  Create Array $F[0...n]$
  $F[0] = F[1] = F[2] = 1$
  for $i = 3$ to n $F[i] = F[i-1] + F[i-2] * F[i-3]$
  return $F[n]$
  Adds and Mults: Both $n - 2$.

You could also figure that we need $3 * (n-2)$ subtracts to deal with the subscripts, but if you got the adds and multiplies correct that is sufficient.