Macro III: Problem Set 4

Tiago Cavalcanti

September 2024

Deadline: Friday, 4 Oct 2024

1. **Aiyagari Model.** Time is discrete and indexed by $t = 0, 1, 2...$ Let $\beta \in (0, 1)$ be the subjective discount factor, $c_t \geq 0$ be consumption at period t. Agents are *ex-ante* identical and have the following preferences:

   Preferences:
$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} \right) \right],$$
   where $\sigma > 0$. Expectations are taken over an idiosyncratic shock, z_t, on labor productivity, where
$$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_{t+1}, \ \rho \in [0, 1].$$
   Variable ϵ_{t+1} is an iid shock with zero mean and variance σ_ϵ^2. Markets are incomplete as in Huggett (1993) and Aiyagari (1994). There are no state contingent assets and agents trade a risk-free bond, a_{t+1}, which pays interest rate r_t at period t. In order to avoid a Ponzi game assume that $a \geq -\phi$ with $\phi > 0$.

   Technology: There is no aggregate uncertainty and the technology is represented by $Y_t = K_t^\alpha N_t^{1-\alpha}$. Let I_t be investment at period t. Capital evolves according to:
$$K_{t+1} = (1 - \delta) K_t + I_t.$$
   Let $\delta = 0.08$, $\beta = 0.96$, $\alpha = 0.4$ and $\sigma = 2$. Start with $\phi = 0$.

   (a) Use a finite approximation for the autoregressive process
$$\ln(z') = \rho \ln(z) + \epsilon.$$
   where ϵ' is normal iid with zero mean and variance σ_ϵ^2. Use a 7 state Markov process spanning 3 standard deviations of the log wage. Let ρ be equal to 0.98 and assume that $\sigma_z^2 = \frac{\sigma_\epsilon^2}{1-\rho^2} = 0.621$. Simulate this shock and report results.

(b) State the households' problem.

(c) State the representative firm's problem.

(d) Define the recursive competitive equilibrium for this economy.

(e) For $r = 2\%$ (notice that w can also be found from the firm's problem), write down a code with VFI to solve the households problem. Find the policy functions for a' and c. Plot these functions and comment your results. How long does your code take? Suppose $\beta = 0.975$, how long does it take?

(f) For $r = 2\%$ (notice that w can also be found from the firm's problem), write down a code with EGM to solve the households problem. Find the policy functions for a' and c. Plot these functions and comment your results. Compare the two solutions. How long does your code take? Suppose $\beta = 0.975$, how long does it take?

(g) Solve out for the equilibrium allocations (r and w are determined in equilibrium) and compute statistics for this economy (stationary equilibrium). Report basic statistics about this economy, such as: investment rate, the capital-to-output ratio, cumulative distribution of income (e.g., bottom 1%, 5%, 10%, 50%, top 1%, top 5%, top 10%), cumulative distribution of wealth (e.g., bottom 1%, 5%, 10%, 50%, top 1%, top 5%, top 10%), Gini of income, Gini of Wealth.

(h) Keeping all other parameters the same, could you find a value for β such that the equilibrium interest rate is equal to 4% per year? Report this value and explain how you found it.

(i) Keeping all other parameters the same, could you find a value for σ_z^2 such that the p_{90}/p_{10} of wealth is equal to 7? Report this value and explain how you found it.

(j) Keeping all other parameters the same, could you find joint value for β and σ_z^2 such that the equilibrium interest rate is equal to 4% per year and the p_{90}/p_{10} of wealth is equal to 7? Report these values and explain how you found them. If possible use the simulated methods of moments explained in the lectures.

(k) Could you find a value for ϕ such that the debt over income is equal to 10%? Report this value and explain how you found it.

(l) Could you find a value for $(\beta, \sigma_z^2, \phi)$ such that the equilibrium interest rate is equal to 4%, the p_{90}/p_{10} of wealth is equal to 7, and the debt over income is equal to 10%? Report the values and explain how you found them. If possible use the simulated methods of moments explained in the lectures.

2. **Hopenhayn model.** On paying a fixed operating cost $\kappa > 0$, an incumbent firm that hires n workers produces flow output $y = zn^{\alpha}$ with $0 < \alpha < 1$ where $z > 0$ is a firm-level productivity level. The productivity of an incumbent firm evolves according to an AR(1) in logs

$$\ln(z_{t+1}) = (1-\rho)\ln(\bar{z}) + \rho\ln(z_t) + \sigma\epsilon_{t+1}, \ \rho \in (0,1), \ \sigma > 0$$

where $\epsilon_{t+1} \sim N(0,1)$. Firms discount flow profits according to a constant discount factor $0 < \beta < 1$. There is an unlimited number of potential entrants. On paying a sunk entry cost $\kappa_e > 0$, an entrant receives an initial productivity draw $z_0 > 0$ and then starts operating the next period as an incumbent firm. For simplicity, assume that initial productivity z_0 is drawn from the stationary productivity distribution implied by the AR(1) above.

Individual firms take the price p of their output as given. Industry-wide demand is given by the $D(p) = \bar{D}/p$ for some constant $\bar{D} > 0$. Let labor be the numeraire, so that the wage is $w = 1$. Let $\pi(z)$ and $v(z)$ denote respectively the profit function and value function of a firm with productivity z. Let v_e denote the corresponding expected value of an entering firm. Let $\mu(z)$ denote the (stationary) distribution of firms and let m denote the associated measure of entering firms.

(a) Derive an expression for the profit function.

(b) Set the parameter values $\alpha = 2/3$, $\beta = 0.8$, $\kappa = 20$, $\kappa_e = 40$, $\ln(\bar{z}) = 1.4$, $\sigma = 0.20$, $\rho = 0.9$ and $\bar{D} = 100$. Discretize the AR(1) process to a Markov chain on 33 nodes. Solve the model on this grid of productivity levels. Calculate the equilibrium price p^* and measure of entrants m^* . Let z^* denote the cutoff level of productivity below which a firm exits. Calculate the equilibrium z^* . Plot the stationary distribution of firms and the implied distribution of employment across firms. Explain how these compare to the stationary distribution of productivity levels implies by the AR(1).

(c) Now suppose the demand curve shifts, with $\bar{D}$ increasing to 120. How does this change the equilibrium price and measure of entrants? How does this change the stationary distributions of firms and employment? Give intuition for your results.

(d) Now, as in Hopenhayn and Rogerson (1993), introduce a firing cost such that firms have to pay $g(n, n_{-1}) = \tau(n_{-1} - n)$ if $n < n_{-1}$ with $\tau > 0$, where n_{-1} corresponds to employment of the firm in the end of the previous period. In addition, $g(n, n_{-1}) = 0$ if $n \geq n_{-1}$. Repeat (a), (b) and (c) with different values for τ (e.g., $\tau = 0.1$, $\tau = 0.2$) and study the implications for aggregate output.