Econometrics II – Short Research Project – Ricardo Semião

One facet of theoretical econometric research that doesn't get much attention, is the derivation of metrics with the purpose of extracting, from complex models, interesting to interpret and easy to digest results.

There are lots of topics that could benefit from this exploration, but in I'll focus on the realm of VAR models with regime switching. This literature has been very useful to allow the use of bigger time windows in the low-data world of macroeconometrics – without including bias from breaks –, and to explain otherwise unknown phenomenons in economic literature[1]. Another interesting topic of interest is the differences between the regimes and the effect of regime changes, but I argue that there could be more tools for this. My research goal is purposing metrics to fill this space, and study their statistical properties, and possible applications.

There is an added value on creating metrics that are universal to all regime switching models. So, the first step of the project is to draw a parallel between each option, for which I'll now present a sketch of. In simplistic ways, all the models partition a process in R regimes, and each regime r is "activated" by a indicator function of a *regime variable* at some lag d, $I^r : x_{t-d} \mapsto \{0, 1\}$. Then, the DGP of the vector Y_t can be written as:

$$Y_t = I^1(x_{t-d}) \left[\varphi_0^1 + \sum_{l=1}^{p_1} \varphi_l^1 Y_{t-l} \right] + \cdots + I^R(x_{t-d}) \left[\varphi_0^R + \sum_{l=1}^{p_R} \varphi_l^R Y_{t-l} \right] + \varepsilon_t,$$

where each r has a matrix $\Phi^r \coloneqq \begin{bmatrix} \varphi_0^r & \dots & \varphi_{p_r}^r \end{bmatrix}$ of coefficients. The difference between each model lies in the functional form of I^r: (i) Structural Break VAR – $x_{t-d} = t - d$, and I^r returns 1 if $t-d$ is in the time-period associated with r; (ii) Threshold VAR – I^r returns 1 if x_{t-d} is in the deterministic threshold associated with r; (iii) Markov Switching VAR – I^r returns 1 if x_{t-d} is in the stochastic Markov state associated with r; (iv) Smooth Transition VAR – continuum of states, modeled by $R = 2$ with $I^1(x_{t-d}) = 1$, $I^2(x_{t-d}) = x_{t-d}$.

The main object of interest is the comparison between some pair $P = (\Phi^{r_1}, \Phi^{r_2})$. Possible metrics are:

1. The "elementary" option is $\Delta_{r_1}^{r_2} P \coloneqq \Phi^{r_1} - \Phi^{r_2}$. With a normalized Y_t, it can be summarized by matrix norms, possibly calculated in row/column-wise fashion, to get the "Δ contribution" on/of each series i.
2. From Φ^r, one can find ACFs of lag l, $\rho_i(l; \Phi^r)$. Then, $\Delta_{r_1}^{r_2} \rho_i(l; \Phi^r)$ can quantify changes in system dynamics. I'll also consider the accumulated alternative: $\sum_{l=1}^{H} \Delta_{r_1}^{r_2} \rho_i(l; \Phi^r)$. Can be applied for CCFs too.
3. From Φ^r, one can find IRFs of horizons h (from i, to j). Then, $\Delta_{r_1}^{r_2} IRF_{ij}(h; \Phi^r)$, can quantify changes in shock responses. I'll also consider the accumulated, discounted, fashion: $\sum_{h=1}^{H} \beta^h \Delta_{r_1}^{r_2} IRF_{ij}(h; \Phi^r)$.

The goal is to carefully define and study the properties of all of the above. Each one supports confidence intervals, calculated analytically from the SEs, or by bootstrapping for the IRFs. While not my focus, each can also be used in tests, a la Andrews[2]/Zeileis[3][4] for 1, a la Box-Pierce[5] for 2, and with cum/sup-distances for 3. Another secondary goal is studying conditions for causal interpretation, using a potential outcomes framework[6].

To test drive the tools, I'll use simulated data with varying intensities of breaks in the DGP, while also applying them to the different models. Additionally, I'll see if the metrics correctly order the importance of real world historical breaks, like the Real Plan versus the lesser New Economic Matrix in Brazil, using development indicators from the World Bank. Lastly, the alongside creation of a R package would be useful.

References

[1] Tommaso Ferraresi, Andrea Roventini, and Giorgio Fagiolo. "Fiscal Policies and Credit Regimes: A TVAR Approach". In: *Journal of Applied Econometrics* 30.7 (2015), pp. 1047–1072. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/jae.2420.

[2] Donald W. K. Andrews. "Tests for Parameter Instability and Structural Change With Unknown Change Point". In: *Econometrica* 61.4 (1993), pp. 821–856. URL: http://www.jstor.org/stable/2951764.

[3] Achim Zeileis. "A Unified Approach to Structural Change Tests Based on ML Scores, F Statistics, and OLS Residuals". In: *Econometric Reviews* 24.4 (2005), pp. 445–466. URL: https://doi.org/10.1080/07474930500406053.

[4] Achim Zeileis, Ajay Shah, and Ila Patnaik. "Testing, monitoring, and dating structural changes in exchange rate regimes". In: *Computational Statistics & Data Analysis* 54.6 (2010), pp. 1696–1706. URL: https://www.sciencedirect.com/science/article/pii/S0167947309004435.

[5] G. E. P. Box and David A. Pierce. "Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models". In: *Journal of the American Statistical Association* 65.332 (1970), pp. 1509–1526. URL: https://www.tandfonline.com/doi/abs/10.1080/01621459.1970.10481180.

[6] Ashesh Rambachan and Neil Shephard. "When do common time series estimands have nonparametric causal meaning?" In: (2021). URL: https://scholar.harvard.edu/sites/scholar.harvard.edu/files/shephard/files/causalmodelformacro20211012.pdf.