Computational Methods in Economics

Sao Paulo School of Economics - FGV

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Problem Set 3

Due on March 25th, 2025 at 23h59

Please remember:

- *Late responses are only accepted within the first 24 hours and with a 20% penalty*
- *You are allowed to complete this problem in any programming language.*
- *You should submit the answers* ***along with a fully reproducible code****.*
- *Remember our good coding practices: code with documentation, folder structure, relative paths only, use of git, defensive programming, etc.*

Question 1. Assume X is a random variable with a normal distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$.

a) We are interested in computing $E[X]$ using numerical integration. What is the value of $E[X]$ using Gauss-Hermite quadrature with $n = 3$ nodes? And with $n = 5$ nodes? And with $n = 10$ nodes?

b) What wold be the estimated value of $E[X]$ using Monte Carlo integration with $n = 10^2$ simulations? And with $n = 10^4$ simulations? And with $n = 10^6$ simulations?

c) Now we are interested in computing $E[\max(1, X)]$. What is the analytical result? Hint: use the truncated normal distribution.

d) Using numerical methods, how do your answers from a)-b) change when considering the expectation of $\max(1, X)$ instead of X?

Question 2. Suppose utility is given by

$$u = \max\{x, y\},$$

where X and Y are random variables, each following a normal distribution with mean $\mu = 0$, variance $\sigma^2 = 1$ and independent.

1. Using Gauss-Hermite quadrature, compute the expected value of u.
2. Using Monte Carlo integration, compute the expected value of u.

Question 3. Using the trapezoid rule, compute the integral of the following functions. Compute for $n = 3, 5, 10, 15, 20$ nodes.

a) $\int_0^1 x\ dx$

b) $\int_0^1 xsin(x)\ dx$

c) $\int_0^1 \sqrt{1-x^2}\ dx$

Question 4. Using finite differences methods, compute the derivative of the following functions. Compute for centered differences with 2 and 4 points. Do it for hsteps of $0.001, 0.005, 0.01, 0.05$.

a) $f(x) = x^2$ at $x = 5$

b) $f(x) = log(x)$ at $x = 10$

c) $f(x) = x * sin(x)$ at $x = 1$

d) How do your results compare with the analytical solution?