Escola de Economia de São Paulo - Fundação Getulio Vargas

Course: Econometria 2

Instructor: Vitor Possebom

Problem Set 6 - Total = 140 points

Question 1 (Comparing Estimators of a Dynamic Panel Data Model - 100 points)

We want to compare different estimators for the coefficients in a Dynamic Panel Data Model. Specifically, our model is given by

$$Y_{i,t} = \alpha \cdot Y_{i,t-1} + \eta_i + \nu_{i,t},$$

where

- $i \in \{1, 2, \ldots, N\}$ *with* $N = 10,000$,
- $t \in \{2, 3, \ldots, T\}$ *with* $T = 3$,
- $\alpha = 0.2$,
- $Y_{i,1} \sim$ *i.i.d. Cauchy* $(location = 0, scale = 1)$,
- $\eta_i \sim$ *i.i.d.* $Unif\,(0, 5)$,
- $\nu_{i,t} \sim$ *i.i.d.* $N\,(0, 1)$, *and*
- $(Y_{i,1}, \eta_i, \nu_{i,t})$ *are jointly independent.*

This comparison will be done with a Monte Carlo exercise with seed number 220206 *and* $M = 10,000$ *repetitions.*

1. *For each repetition, use OLS to estimate* α *in our model. Report the average bias:* $Bias_{OLS} = \sum_{m=1}^{M} (\hat{\alpha}_{OLS,m} - \alpha)$. *Have in mind that standard results for omitted variable bias indicate that, at least in large samples, the OLS estimator is biased upwards. (20 points)*

2. *For each repetition, use the Within-Groups estimator to estimate α in our model. Report the average bias: $Bias_{WG} = \sum_{m=1}^{M} (\hat{\alpha}_{WG,m} - \alpha)$. Have in mind that standard results for omitted variable bias indicate that, at least in large samples, the Within-Groups estimator is biased downwards. (20 points)*

3. *If the Within-Groups estimator is larger than the OLS estimator, we may suspect that our model is misspecified. So, in real-world applications, it is always useful to compare those two inconsistent estimators as a sanity check. To illustrate this point, compare $\hat{\alpha}_{WG,m}$ against $\hat{\alpha}_{OLS,m}$ and report the share of repetitions where $\hat{\alpha}_{WG,m} < \hat{\alpha}_{OLS,m}$. (20 points)*

4. *For each repetition, use the First-Differences estimator to estimate α in our model. Report the average bias: $Bias_{FD} = \sum_{m=1}^{M} (\hat{\alpha}_{FD,m} - \alpha)$. Have in mind that, in this case, the bias will be negative. Moreover, when $T = 3$, the First-Differences and the Within-Groups Estimators are numerically identical. (20 points)*

5. *For each repetition, use the Anderson-Hsiao estimator to estimate α in our model. Report the average bias: $Bias_{AH} = \sum_{m=1}^{M} (\hat{\alpha}_{AH,m} - \alpha)$. Have in mind that, in this case, the bias should be close to zero. (20 points)*

Question 2 (Near Unit Root Properties - 40 points)

We want to understand what happens with the Anderson-Hsiao estimator when our model is close to a unit root process. Specifically, our model is given by

$$Y_{i,t} = \alpha \cdot Y_{i,t-1} + \eta_i + \nu_{i,t},$$

where

- $i \in \{1, 2, \ldots, N\}$ *with* $N = 10,000$,
- $t \in \{2, 3, \ldots, T\}$ *with* $T = 3$,
- $\alpha = 0.9999$,

- $Y_{i,1} \sim$ *i.i.d. Cauchy* $(location = 0, scale = 1)$,
- $\eta_i \sim$ *i.i.d. Unif* $(0, 5)$,
- $\nu_{i,t} \sim$ *i.i.d.* $N(0, 1)$, *and*
- $(Y_{i,1}, \eta_i, \nu_{i,t})$ *are jointly independent.*

This comparison will be done with a Monte Carlo exercise with seed number 220206 *and* $M = 10,000$ *repetitions. For each repetition, use the Anderson-Hsiao estimator to estimate* α *in our model.*

1. *Report the observed deciles of the first-stage F-statistic. Have in mind that the threshold for a strong instrument is* $F = 10$ *according to the standard practice or* $F = 104.7$ *according to Lee, McCrary, Moreira and Porter (2020, arxiv.org/abs/2010.05058). Generally, when our* $AR(1)$ *model is close to being a unit root process, the Anderson-Hsiao instrument will be weak. (20 points)*
2. *Report the average bias:* $Bias_{AH} = \sum_{m=1}^{M} (\hat{\alpha}_{AH,m} - \alpha)$. *Due to a weak instrument, this bias will be larger than the one in Question 1.6. (20 points)*