Econometrics II - Problem Set 1

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Question 1

Item a. and b.

Below are the regression tables for each model:

	AR(1)	AR(2)	MA(1)	MA(2)
Table 1:				
ar1	0.287	0.251		
	(0.091)	(0.094)		
	p = 0.002	p = 0.008		
ar2		0.122		
		(0.095)		
		p = 0.199		
ma1			0.238	0.222
			(0.086)	(0.094)
			p = 0.006	p = 0.019
ma2				0.159
				(0.103)
				p = 0.126
intercept	4.538	4.527	4.539	4.535
	(0.538)	(0.605)	(0.480)	(0.529)
	p = 0.000	p = 0.000	p = 0.000	p = 0.000
AIC	691.99	692.349	693.979	693.718
BIC	700.352	703.499	702.341	704.868
Observations	120	120	120	120
Log Likelihood	-342.995	-342.174	-343.989	-342.859
σ^2	17.778	17.533	18.080	17.738

Note: *p<0.1; **p<0.05; ***p<0.01

	ARMA(1,1)	ARMA(2,1)	ARMA(1,2)	ARMA(2,2)
ar1	0.740	1.104	0.944	1.290
	(0.284)	(0.142)	(0.070)	(0.483)
	p = 0.010	p = 0.000	p = 0.000	p = 0.008
ar2		−0.149		−0.324
		(0.110)		(0.453)
		p = 0.178		p = 0.475
ma1	−0.516	−0.870	−0.719	−1.059
	(0.375)	(0.102)	(0.119)	(0.495)
	p = 0.169	p = 0.000	p = 0.000	p = 0.033
ma2			−0.119	0.156
			(0.097)	(0.410)
			p = 0.220	p = 0.704
intercept	4.491	4.259	4.260	4.268
	(0.706)	(1.070)	(1.072)	(1.064)
	p = 0.000	p = 0.0001	p = 0.0001	p = 0.0001
AIC	691.99	692.349	693.979	693.718
BIC	700.352	703.499	702.341	704.868
Observations	120	120	120	120
Log Likelihood	−341.905	−341.274	−341.364	−341.212
σ^2	17.450	17.250	17.275	17.233

Note: *p<0.1; **p<0.05; ***p<0.01

Item c.

Below are the forecasts for each model:

Prediction of PIB - Model MA(1)

Prediction of PIB - Model MA(2)

Prediction of PIB - Model AR(1)

Prediction of PIB - Model AR(2)

Prediction of PIB - Model ARMA(1,1)

Prediction of PIB - Model ARMA(2,1)

Prediction of PIB - Model ARMA(1,2)

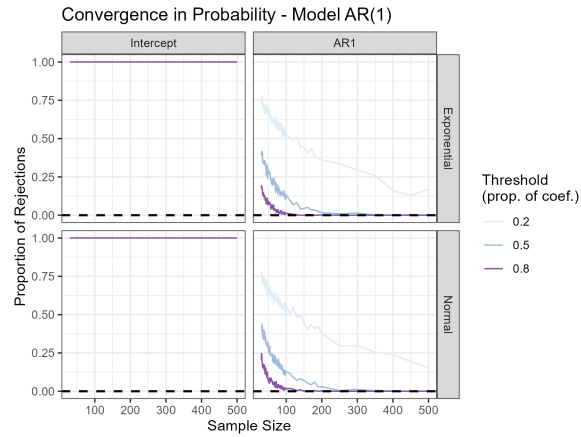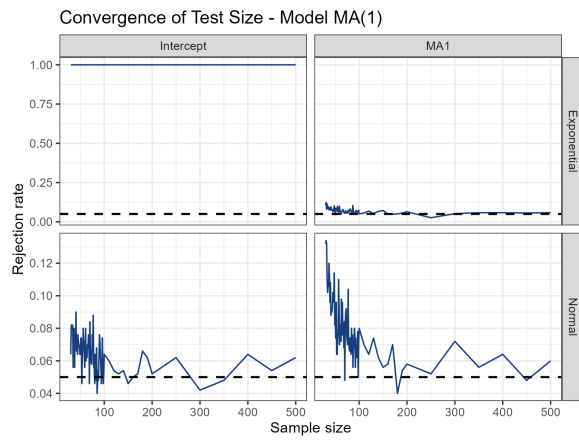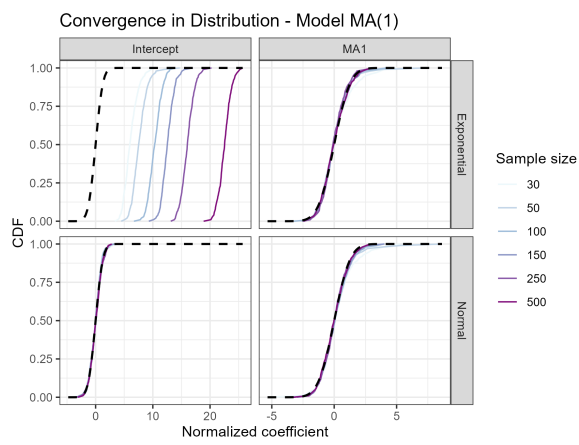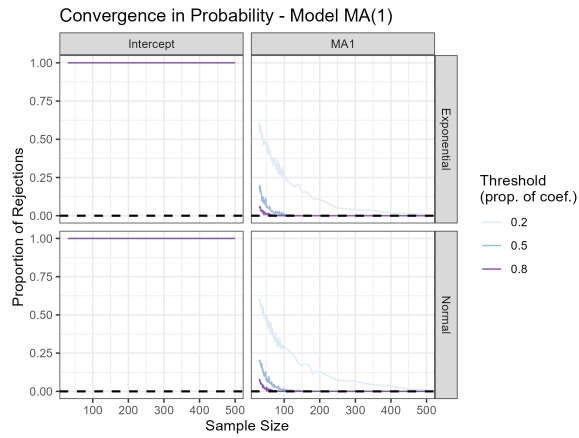Prediction of PIB - Model ARMA(2,2)
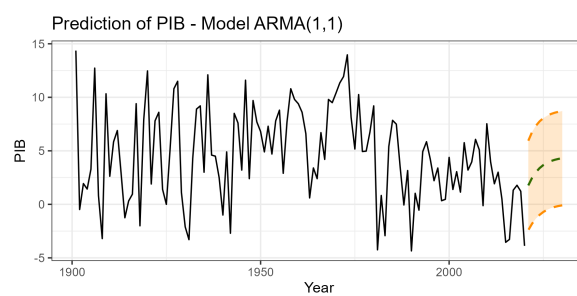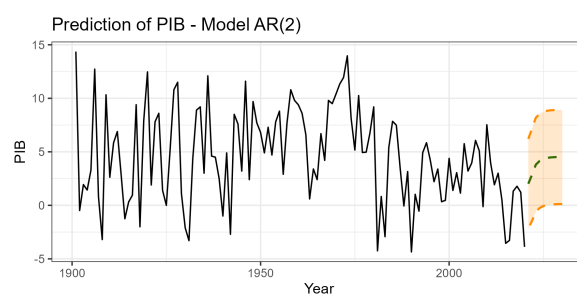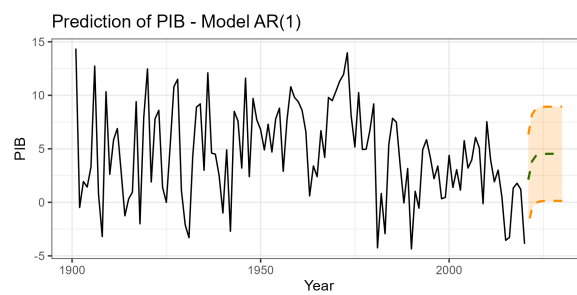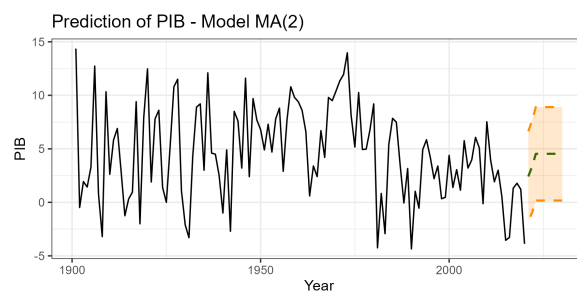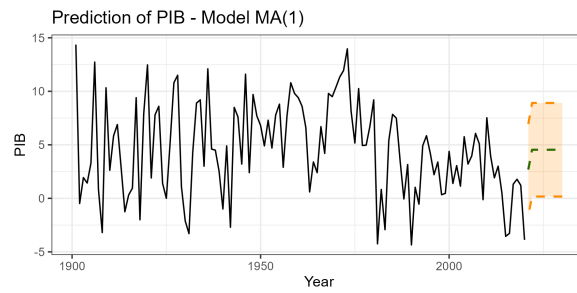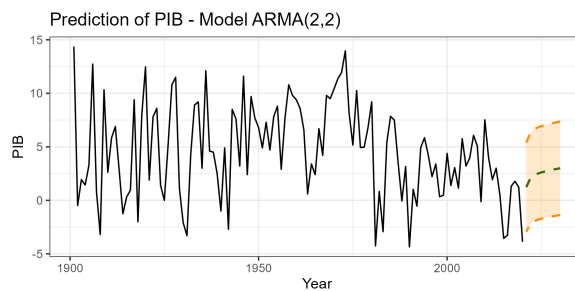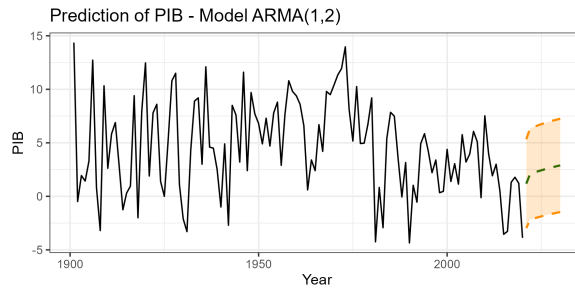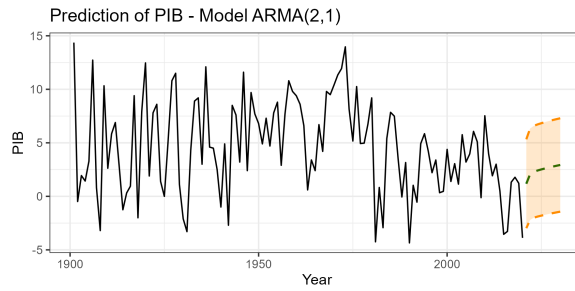
Item d.

I would choose the ARMA(1,1) model, as it has the lowest AIC and BIC values. It also seems to be a balanced choice, given that normally it is not needed many lags to capture the dynamics of yearly GDP series. Lastly, it was the one that had the most smooth return to the historical mean, which is a desirable feature given the nature of that mean being so much bigger than the current values.

Question 2

Item a., b., and c.

Below there is, for each model, one plot for each topic.

Convergence in Probability - Model AR(1)

Convergence in Distribution - Model AR(1)

Convergence of Test Size - Model AR(1)

Convergence in Probability - Model MA(1)

Convergence in Distribution - Model MA(1)

Convergence of Test Size - Model MA(1)

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Convergence in Probability - Model ARMA(1,1)

Convergence in Distribution - Model ARMA(1,1)

Convergence of Test Size - Model ARMA(1,1)
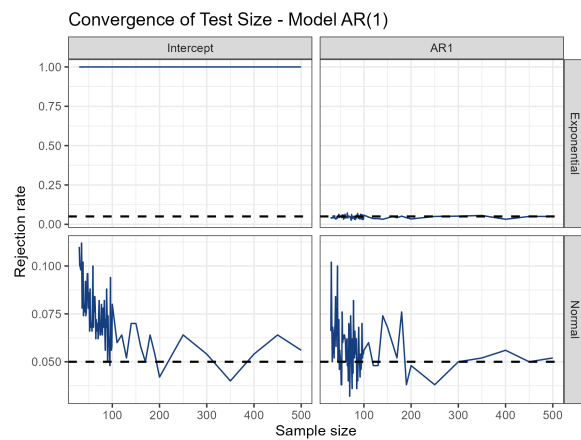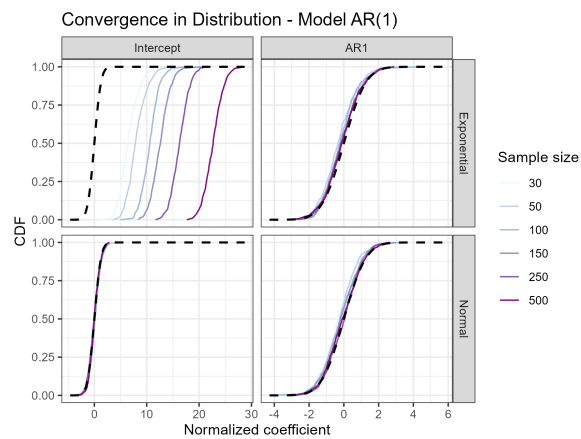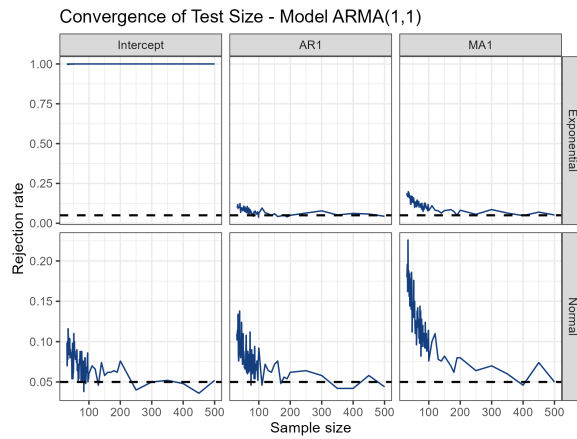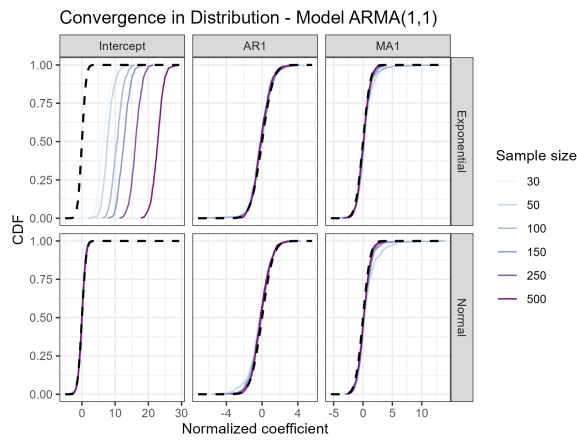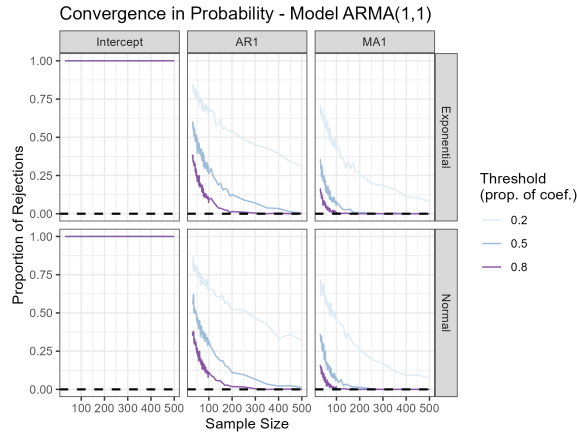
About convergence in probability: the AR and MA coefficients converge, regardless of the model and the distribution. The intercept never converges, as it is consistently different than zero, it always presents some bias, even if it is small.

About convergence in distribution: all coefficients converge, regardless of the model and the distribution. The only exception is the intercept with exponential distribution, which does not converge, and is associated with a right-skewed distribution, with a skewness increasing with the sample size.

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About convergence of test size: all tests converge to the desired level of 0.05, regardless of the model and the distribution. The only exception is the intercept with exponential distribution, which always rejects the null hypothesis, given the aforementioned bias.

Item d.

Yes, 500 periods seem to be enough to yield good approximations in the sense of convergence in probability, distribution, and test size. The only exception is the intercept with exponential distribution, which yield poor results for the intercepts, but that is more of a problem of poor model specification than of sample size.

Question 3

a) A white noise, a random process with zero mean, constant variance, and no autocorrelation by definition, is clearly weakly stationary.

b) A random walk with drift – $Y_t = Y_{t-1} + \delta + \epsilon_t$ with ϵ_t white noise –, a random process that has a stochastic trend, will have a time-varying average, thus not weakly stationary.

Question 4

If Y_t is weakly stationary, the $E[Y_t] = \mu \forall t$ and $E[(Y_t - \mu)(Y_{t-j} - \mu)] = \gamma_j \forall t, j$. Then, this is valid for the period $t + j$ too:

$$\gamma_j = E[(Y_{t+j} - \mu)(Y_{t+j-j} - \mu)] = E[(Y_{t+j} - \mu)(Y_t - \mu)] = \gamma_{-j}$$