# Macro III - Problem Set 2

August 2024

**Deadline:** Monday, Sept 2

## 1 RBC model with capital adjustment costs

For the following problems, consider an economy in which there is a continuum of infinitely lived households of measure 1 that choose how much to consume $c_t \in \mathbb{R}_+$, and invest $i_t \in \mathbb{R}_+$, in each period $t \in \mathbb{N}_0$. They earn income from profits of the firm $\omega_t \in \mathbb{R}_+$, and from renting the capital stock, taking prices $r_t \in \mathbb{R}_+$ as given. As an extension to the basic RBC model, we have a convex cost of adjusting capital.

$$\max_{(c_t, i_t)_t} \quad \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1.1}$$

$$\text{s.t.} \quad c_t + i_t \leq r_t k_t + \omega_t \tag{1.2}$$

$$k_{t+1} = i_t + (1-\delta)k_t - \frac{\phi}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t, \tag{1.3}$$

where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$.

For the supply side, there is a representative competitive firm that produces according to the technology

$$Y_t = A_t K_t^{\alpha}, \tag{1.4}$$

renting capital at the given rate $r_t$.

### 1.1 Deterministic equilibrium (5 pts)

For now, assume that $\forall t \in \mathbb{N}_0,\ A_t = A$.

**(a)** Define the decentralized competitive equilibrium for this economy and write down the system of equations that describe the equilibrium of the model. Be precise about the problem of each agent, and market clearing conditions. Make sure that you have the same number of endogenous variables as equations. What about the social's planner problem? Does it yield the same result? (1 pt)

**(b)** Find the steady state value of all the endogenous variables. (0.25 pts)

Consider now that $\beta = 0.98, \sigma = 2, A = 1, \alpha = 1/3, \delta = 0.05, \phi = 5$.

**(c)** Discuss the challenges of implementing the shooting algorithm (bisection) to solve for the path of capital in this model, compared to the simple RBC model we've seen in class. Build a shooting algorithm that manages to surpass these challenges. Solve for the path of the endogenous variables for $T$ (large) periods, assuming that the initial capital stock is half of the steady-state value. Compare the path of the endogenous variables you obtained to the one when there's no capital adjustment cost ($\phi = 0$). Discuss. (0.75 pts)

**(d)** Do the same thing you did in the previous item, but instead of using a shooting algorithm, solve for the root of the system of $T$ simultaneous equations. (0.5 pts + 0.25 extra for those that build their own function to solve the system)

**(e)** Write down the household's problem recursively. Define the recursive competitive equilibrium. (0.5 pt)

**(f)** Solve the model using value function iteration. Discretize the capital space as you wish, but justify your choice. Plot the policy function for consumption and next period capital. (1.5 pts)

**(g)** Using your constructed policy function of the previous item, simulate the path for all the endogenous variables, assuming that you're initially at half of the steady state value of the capital stock. How does this path compare to the ones obtained previously? Did they yield the same result? Which one was faster? (0.5 pts)

## 1.2 Stochastic equilibrium (5 pts)

**(h)** Describe, in your own words and great detail, the general procedure for approximating an AR(1) process

$$y_t = \mu(1-\rho) + \rho y_{t_1} + \epsilon_t, \tag{1.5}$$

where $\epsilon_t \sim N\left(0, \sigma^2\right)$, with a Markov chain using the Tauchen method. Make sure to clearly explain each step in the process and how they contribute to the approximation. (0.25 pts)

**(i)** Implement the Tauchen method numerically, and simulate $T = 5000$ realizations from a Markov chain approximation of the AR(1) process

$$y_t = 0.9y_{t-1} + \epsilon_t,$$

where $\epsilon_t \sim N(0, 0.05)$. Use $r = 3$, $N = 7$. Calculate the empirical mean and variance of the process. Is it consistent with the theoretical mean and variance? Repeat this exercise with $\rho = 0.98$. (0.5 pt)

**(j)** Now implement the Rouwenhorst method to approximate the same process of the previous item using $N = 7$. Simulate 5000 realizations. Which method does a better job of approximating the true mean and variance for $\rho = 0.9$? And for $\rho = 0.98$? Ps: you will use the Rouwenhorst method a lot throughout this course, your code should be reusable. (0.75 pt)

Returning to the model, assume that productivity follows an AR(1) process

$$\ln A_t = \rho \ln A_{t-1} + \epsilon_t, \tag{1.6}$$

with $\rho = 0.95$, and $\epsilon_t \sim N(0, 0.01)$.

**(k)** Re-write the recursive competitive equilibrium, considering that now we have uncertainty. Write also the social's planner problem in the recursive formulation. (0.5 pts)

**(l)** Solve the model using value function iteration. You should discretize the shock space using the Rouwenhorst method with $N = 11$. Report the number of value function iterations and the time it took to find the optimal value function and plot the policy functions. (2 pts)

**(m)** Using your constructed policy functions, simulate the path for all the endogenous variables starting at half of the steady-state level of capital. Simulate the path for a long period, and report the first and second moment of each endogenous variable. (0.5 pts)

**(n)** Simulate 1000 times the path of capital and consumption, again starting at half of the steady-state level of capital. Compute the average path, and build a 90% confidence interval. Plot them. (0.5 pts)