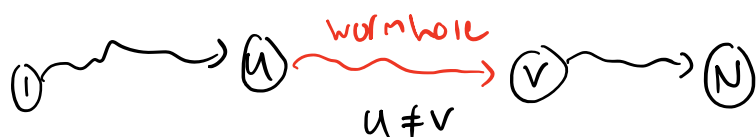


## Cosmic Commute

- Problem Summary: given an undirected graph with random portals what is the expected shortest path between nodes  $(1, N)$

- Slow Solution: Try all possible paths if you reach a wormhole location try taking vs. ignoring it you take it average over all possible destinations distances to the end. Use backtracking to find all paths this is exponential complexity

- Observation: If we take a wormhole at node  $u$  then we don't care about the actual path we know it will look something like this



- We know fixing  $u$  we must travel  $\text{dist}(1, u) + \text{dist}(v, N)$ . But  $v$  is not known and is random

- But this means our expected distance is the average across all  $v$ s

- So if we take wormhole at location  $u$  our expected distance is

$$\frac{1}{K-1} \left[ \sum_{\substack{i=1 \\ K_i \neq u}}^K \text{dist}(1, u) + \text{dist}(K_i, N) \right] = \frac{1}{K-1} \left[ \text{dist}(1, u) \cdot (K-1) + \left[ \sum_{i=1}^K \text{dist}(K_i, N) \right] - \text{dist}(u, N) \right]$$

- Now we just need to efficiently compute the distances

$\text{dist}(1, x)$ : BFS from node 1 and store results

$\text{dist}(x, N)$ : BFS from  $N$

- Last case is not taking any teleporters which is just  $\text{dist}(1, N)$

- Now we have all expected probabilities we can just take the minimum to compare  $\frac{a}{b} < \frac{c}{d} \Rightarrow a \cdot d < b \cdot c$

- Finally to simplify use  $\text{gcd}(a, b) = g$   $\frac{a}{b} = \frac{(a/g)}{(b/g)}$