Cosmic Commute

- Problem summary: given an undirected graph with random portals what is the expected shortest path between nodes $(1, N)$

- Slow solution: Try all possible paths if you reach a wormhole location try taking vs. ignoring if you take it average over all possible destinations distances to the end. Use back tracking to find all paths this is exponential complexity

- observation: If we take a wormhole at node u than we don't care about the actual path we know it will look something like this

$$1 \rightsquigarrow u \xrightarrow{\text{wormhole}} v \rightsquigarrow N$$

$$u \neq v$$

- We know know fixing u we must travel $dist(1, u) + dist(v, N)$. But v is not know and in fact random

- But this means our expected distance is the average across all vs

- So if we take wormhole at location u our expected distance is

$$\frac{1}{K-1}\left[\sum_{\substack{i=1 \\ K_i \neq u}}^{K} dist(1, u) + dist(K_i, N)\right] = \frac{1}{K-1}\left[dist(1, u) \cdot (K-1) + \left[\sum_{i=1}^{K} dist(K_i, N)\right] - dist(u, N)\right]$$

- Now we just need to efficiently compute the distances

$dist(1, X)$: BFS from node 1 and store results
$dist(X, N)$: BFS from N

- Last case is Not taking any teleporters which is just $dist(1, N)$

- Now we have all expected probabilities we can just take the minimum to compare $\frac{a}{b} < \frac{c}{d} \Rightarrow a \cdot d < b \cdot c$

- finally to simplify use $gcd(a, b) = g$ $\quad \frac{a}{b} = \frac{(a/g)}{(b/g)}$