Instituto Superior Técnico, University of Lisbon

Integrated Master in Aerospace Engineering

Circuit Theory and Electronics Fundamentals

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Fourth Laboratory Report

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Contents

1 Introduction

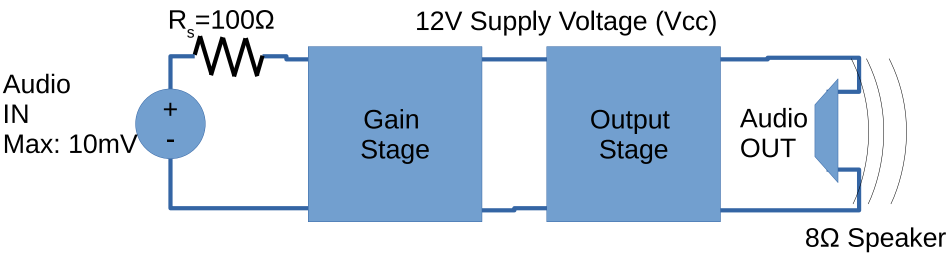


Figure 1: Audio Amplifier Circuit

The objective of this laboratory assignment is to simulate an Audio Amplifier Circuit as shown in Figure **??**.

This way, we should choose the architecture of the Gain and Output amplifier stages, however, we must consider the cost of the components in the circuit. Its diagram is shown in Figure **??**.

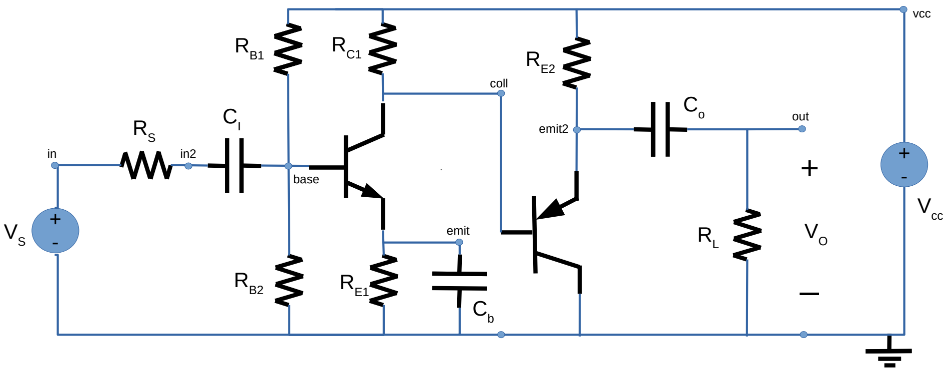


Figure 2: Audio Amplifier Circuit Diagram

In this laboratory, we use two different modles of Phillips BJT's Transistors: BC547A, a NPN transistor used in Gain Stage, and BC557A, a PNP Transistor used in Output Stage. The values of the components are exhibited in the table below.

Name	Value
R_S	1.000000e+02 Ohm
R_{B1}	8.000000e+04 Ohm
R_{B2}	2.000000e+04 Ohm
R_{C1}	9.400000e+02 Ohm
R_{E1}	7.750000e+02 Ohm
R_{E2}	2.335000e+03 Ohm
R_L	8.000000e+00 Ohm
C_I	6.900000e-04 F
C_b	4.180000e-03 F
C_O	2.250000e-03 F

Table 1: Components Values

In Section **??**, a theoretical analysis of the circuit, performed on Octave, is presented. In Section **??**, the circuit is analysed by simulation, using NGSpice, and the results are compared to the theoretical results obtained in Section **??**. The conclusions of this study are outlined in Section **??**.

2 Theoretical Analysis

In this section, the circuit shown in Figure **??** is analysed theoretically. We will begin by analyzing the Gain Stage circuit and, after that, the Output Stage circuit. Thus, we will start by computing the Operating Point using the theoretical DC model studied and comparing it to Ngspice's OP.

Then, we will compute the gain and input and output impedances separately for the 2 stages.

Finally, we will compute the frequency response Vo(f)/Vi(f).

2.1 Gain Stage

In this subsection, we will analyze the Gain Stage circuit. Its function is to ensure a high input voltage so that the input signal is not degraded or distorted throughout the circuit. It also has a high gain associated, being responsible for amplifying the signal.

In order to make the analysis task easier, we used Thévenin's equivalent of bias circuit. Its diagram is represented in Figure **??** as well as the NPN BC547A model used in this assignment is shown in table **??**.

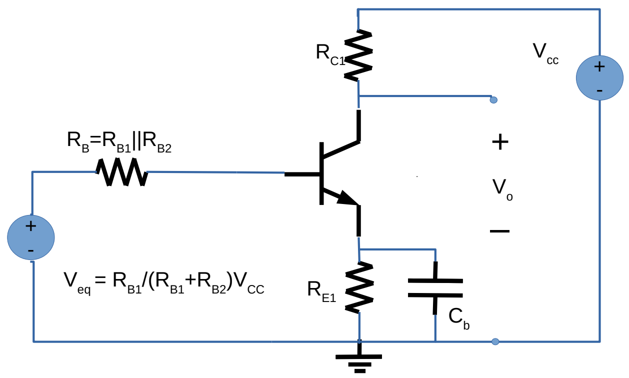


Figure 3: Gain Stage Circuit

Name	**Value**
V_T	0.025000 V
β	178.700000
V_A	69.700000 V
V_{BEON}	0.700000 V

Table 2: BC547A model

In gain stage circuit, it is important to mention that capacitor C_I is a coupling capacitor, acting as a DC Block, and C_b is a bypass capacitor.

2.1.1 Operating Point

Considering the equations of the lecture 17 and the theoretical DC model studied , we compute the OP. The table **??** presents the results obtained.

Name	**Value**
R_B	1.600000e+04 Ohm
V_{eq}	2.400000e+00 V
I_{B1}	1.094885e-05 A
I_{C1}	1.956559e-03 A
I_{E1}	1.967508e-03 A
V_{E1}	1.524818e+00 V
V_{O1}	1.016083e+01 V
V_{CE}	8.636016e+00 V

Table 3: OP - Gain Stage

2.1.2 Gain and Input and Output Impedances

Then, we will compute the gain and input and output impedances. In order to do that, we will use the incremental circuit, whose diagram is represented in Figure **??**.

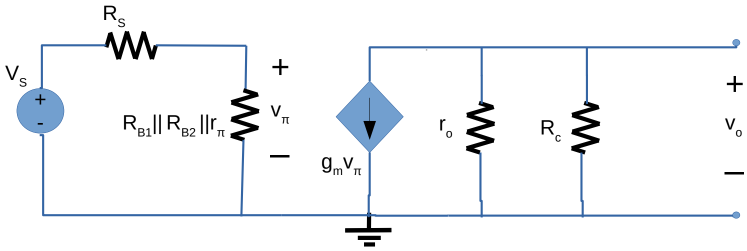


Figure 4: Incremental Gain Stage Circuit

The incremental parameters are given by:

$$g_m = \frac{I_{C1}}{V_T} \tag{1}$$

$$r_\pi = \frac{\beta}{g_m} \tag{2}$$

$$r_o = \frac{V_A}{I_{C1}} \tag{3}$$

Starting by calculating the gain, after analyzing the circuit, we obtain the following equation:

$$A_v = \frac{v_o}{v_{in}} = -g_m(r_o||R_C)\frac{R_B||r_\pi}{R_B||r_\pi + R_{in}} \tag{4}$$

Notice that the capacitor C_b was used to bypass R_E. Otherwise, the gain will be lower and the lower cutoff frequency too high. This way, CE is an open-circuit for low frequency (DC) and a short-circuit fot higher frequencies (AC).

Table **??** presents the results.

Name	Value
A_{V1}	3.668323e+01 dB

Table 4: Gain - Gain Stage

In order to obtain the impedances, we used the following equations:

$$Z_{I1} = R_B1||R_B2||r_\pi \tag{5}$$

$$Z_{O1} = R_C||R_o \tag{6}$$

The results are presented in Table **??**

Name	Value
Z_{I1}	1.998186e+03 Ohm
Z_{O1}	9.158340e+02 Ohm

Table 5: Input and Output Impedances - Gain Stage

Notice that, in this section, we have made the approximation $R_E1 \approx 0$, because it is assumed the capacitors are shor-circuited, i.e. high frequency analysis.

Finally, we should pay attention to the values of the output impedance of gain stage. Its values are high when compared with the load. That's why we need the Output Stage.

2.2 Output Stage

In this subsection, we will analyze the Output Stage circuit, wich presents a lower output impedance. Its diagram is represented in Figure **??** as well as the PNP BC547A model used in this assignment is shown in table **??**.

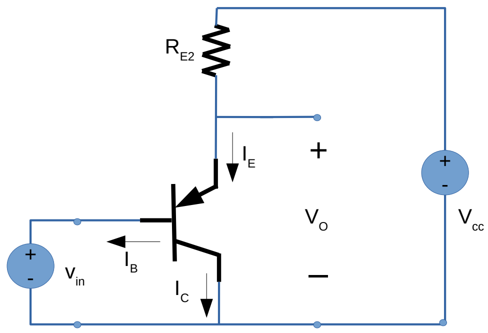


Figure 5: Output Stage Circuit

Name	Value
β	227.300000
V_{AFP}	37.200000 V
V_{BEON}	0.700000 V

Table 6: BC557A model

2.2.1 Operating Point

Considering the equations of the lecture 17, we compute the OP. The table **??** presents the results obtained.

Name	Value
V_{I2}	1.016083e+01 V
I_{E2}	4.878652e-04 A
I_{C2}	4.857283e-04 A
V_{O2}	1.086083e+01 V

Table 7: Operating Point - Output Stage

It is important to notice that output current I_E is much stronger than in Gain Stage and that part of this current will feed the Load.

2.2.2 Gain and Input and Output Impedances

Then, we will compute the gain and input and output impedances. In order to do that, we will use the incremental circuit, whose diagram is represented in Figure **??**.

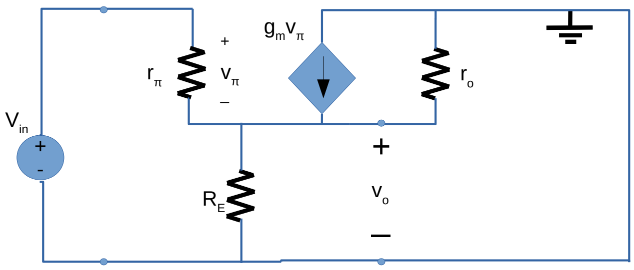


Figure 6: Output Stage Circuit

The incremental parameter is given by:

$$g_m = \frac{I_{C2}}{V_T} \tag{7}$$

Starting by calculating the gain, after analyzing the circuit, we obtain the following equation:

$$A_v = \frac{v_o}{v_{in}} = \frac{g_m}{g_\pi + g_E + g_o + g_m} \tag{8}$$

, where g_π, g_E and g_o are the admittances of the respective resistors.

Table **??** presents the results and, as predicted, we obtained almost unitary gain.

Name	**Value**
A_{V2}	9.736018e-01 dB

Table 8: Gain - Output Stage

In order to obtain the impedances, we used the following equations:

$$Z_{I2} = \frac{g_\pi + g_E + g_o + g_m}{g_\pi(g_\pi + g_E + g_o)} \tag{9}$$

$$Z_{O2} = \frac{1}{g_\pi + g_E + g_o + g_m} \tag{10}$$

The results are presented in Table **??**

Name	**Value**
Z_{I2}	4.431714e+05 Ohm
Z_{O2}	5.011041e+01 Ohm

Table 9: Input and Output Impedances - Output Stage

To connect the two stages with without significant signal loss, it is important to ensure that the input impedance of the output impedance is greater than the output impedance of the gain stage.L ooking at Table **??** (Z_{O1}) and Table **??** (Z_{I2}), we can conclude that this goal was achieved.

2.3 Final Outputs

For the final output, it is important to mention that was used another coupling capacitor between the Output stage and the load. Final Gain and Output Impedance are given, respectively, by:

$$AV = (gB + gm2/gpi2 * gB)/(gB + ge2 + go2 + gm2/gpi2 * gB) * AV1 \tag{11}$$

$$Z_O = 1/(go2 + gm2/gpi2 * gB + ge2 + gB) \tag{12}$$

So, these outputs are in Table **??**.

Name	Value
A_V	3.647396e+01 dB
Z_I	1.998186e+03 Ohm
Z_O	5.393985e+01 Ohm

Table 10: Output Values

This way, the theoretical value of the lower cut-off frequency finally is given by:

$$LowerCOfreq = 1/R_{eqi}/C_i + 1/R_{eqb}/C_b + 1/R_{eqo}/C_o \tag{13}$$

, where R_{eqi}, R_{eqb} and R_{eqo} are the equivalent resistances seen by each capacitor. This equation is an aproximation and represents the time constants method. Its result is presented in Table **??**.

Name	Value
Higher CO freq	1.000000e+05 Hz
Lower CO freq	4.156647e+00 Hz
Bandwidth	9.999584e+04 Hz

Table 11: Cut-Off Frequencies

Finaly, in Figure **??**, is presented the the frequency response Vo(f)/Vi(f), for which we used the incremental circuit:

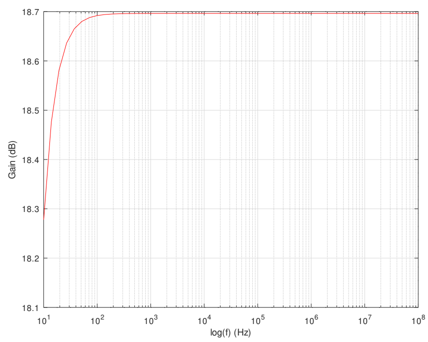


Figure 7: Frequency response Vo(f)/Vi(f)

3 Simulation Analysis

Tables **??** and **??** shows the voltages required to confirm that the BJTs are operating Forawrd Active Region (F.A.R), by comparing V_{CE} and V_{BE} for the NPN transistor and V_EC and V_EB for the PNP. Analysing the results, we can confirmar that, in fact, the BJTs do operate in F.A.R.

Voltages	V
t4 laboratory simulation results	
AC Analysis Sun May 23 23:21:26 2021	
———————————————————————	
Index frequency vce	
———————————————————————	
0 1.000000e-01 -1.78971e+00, -2.50922e+00	
1 1.258925e-01 -1.85837e+00, -3.03593e+00	
2 1.584893e-01 -1.95300e+00, -3.71922e+00	
3 1.995262e-01 -2.09327e+00, -4.59247e+00	
4 2.511886e-01 -2.30792e+00, -5.69537e+00	
5 3.162278e-01 -2.63970e+00, -7.07306e+00	
6 3.981072e-01 -3.15178e+00, -8.77289e+00	
7 5.011872e-01 -3.93526e+00, -1.08372e+01	
8 6.309573e-01 -5.11603e+00, -1.32899e+01	
9 7.943282e-01 -6.85760e+00, -1.61147e+01	
10 1.000000e+00 -9.35316e+00, -1.92245e+01	
11 1.258925e+00 -1.27966e+01, -2.24252e+01	
12 1.584893e+00 -1.73193e+01, -2.53877e+01	
13 1.995262e+00 -2.28898e+01, -2.76659e+01	
14 2.511886e+00 -2.92185e+01, -2.88026e+01	
15 3.162278e+00 -3.57616e+01, -2.85175e+01	
16 3.981072e+00 -4.18827e+01, -2.68578e+01	
17 5.011872e+00 -4.70877e+01, -2.41810e+01	
18 6.309573e+00 -5.11625e+01, -2.09805e+01	
19 7.943282e+00 -5.41495e+01, -1.76986e+01	
20 1.000000e+01 -5.62348e+01, -1.46317e+01	
21 1.258925e+01 -5.76414e+01, -1.19295e+01	
22 1.584893e+01 -5.85682e+01, -9.63669e+00	
23 1.995262e+01 -5.91695e+01, -7.73744e+00	
24 2.511886e+01 -5.95558e+01, -6.18817e+00	
25 3.162278e+01 -5.98022e+01, -4.93662e+00	
26 3.981072e+01 -5.99589e+01, -3.93180e+00	
27 5.011872e+01 -6.00581e+01, -3.12819e+00	
28 6.309573e+01 -6.01210e+01, -2.48704e+00	
29 7.943282e+01 -6.01607e+01, -1.97626e+00	
30 1.000000e+02 -6.01858e+01, -1.56968e+00	
31 1.258925e+02 -6.02016e+01, -1.24617e+00	
32 1.584893e+02 -6.02116e+01, -9.88757e-01	
33 1.995262e+02 -6.02179e+01, -7.83870e-01	
34 2.511886e+02 -6.02219e+01, -6.20662e-01	
35 3.162278e+02 -6.02244e+01, -4.90473e-01	
36 3.981072e+02 -6.02260e+01, -3.86386e-01	
37 5.011872e+02 -6.02270e+01, -3.02868e-01	
38 6.309573e+02 -6.02276e+01, -2.35475e-01	
39 7.943282e+02 -6.02280e+01, -1.80619e-01	
40 1.000000e+03 -6.02283e+01, -1.35381e-01	
41 1.258925e+03 -6.02284e+01, -9.73525e-02	
42 1.584893e+03 -6.02285e+01, -6.45077e-02	
43 1.995262e+03 -6.02286e+01, -3.50981e-02	
44 2.511886e+03 -6.02286e+01, -7.55745e-03	
45 3.162278e+03 -6.02286e+01, 1.958071e-02	
46 3.981072e+03 -6.02286e+01, 4.776161e-02	
47 5.011872e+03 -6.02285e+01, 7.848592e-02	
48 6.309573e+03 -6.02284e+01, 1.133898e-01	
49 [illegible]	

Voltages	V
t4 laboratory simulation results	
AC Analysis Sun May 23 23:21:26 2021	
————————————————————————	
Index frequency vec	
————————————————————————	
0 1.000000e-01 -1.34608e-01, 1.190096e-01	
1 1.258925e-01 -2.16533e-01, 1.619388e-01	
2 1.584893e-01 -3.44185e-01, 2.284126e-01	
3 1.995262e-01 -5.40995e-01, 3.361049e-01	
4 2.511886e-01 -8.39487e-01, 5.174617e-01	
5 3.162278e-01 -1.28056e+00, 8.310501e-01	
6 3.981072e-01 -1.90571e+00, 1.378709e+00	
7 5.011872e-01 -2.73342e+00, 2.326156e+00	
8 6.309573e-01 -3.70881e+00, 3.914666e+00	
9 7.943282e-01 -4.62436e+00, 6.434068e+00	
10 1.000000e+00 -5.04401e+00, 1.011697e+01	
11 1.258925e+00 -4.31348e+00, 1.494751e+01	
12 1.584893e+00 -1.74480e+00, 2.047377e+01	
13 1.995262e+00 3.047502e+00, 2.579466e+01	
14 2.511886e+00 9.848241e+00, 2.983046e+01	
15 3.162278e+00 1.782267e+01, 3.177467e+01	
16 3.981072e+00 2.584599e+01, 3.143708e+01	
17 5.011872e+00 3.296602e+01, 2.923871e+01	
18 6.309573e+00 3.868359e+01, 2.590991e+01	
19 7.943282e+00 4.293980e+01, 2.215542e+01	
20 1.000000e+01 4.593928e+01, 1.847559e+01	
21 1.258925e+01 4.797427e+01, 1.514675e+01	
22 1.584893e+01 4.932005e+01, 1.227840e+01	
23 1.995262e+01 5.019518e+01, 9.880387e+00	
24 2.511886e+01 5.075807e+01, 7.913159e+00	
25 3.162278e+01 5.111759e+01, 6.318420e+00	
26 3.981072e+01 5.134620e+01, 5.035281e+00	
27 5.011872e+01 5.149116e+01, 4.007705e+00	
28 6.309573e+01 5.158290e+01, 3.187210e+00	
29 7.943282e+01 5.164090e+01, 2.533253e+00	
30 1.000000e+02 5.167754e+01, 2.012598e+00	
31 1.258925e+02 5.170067e+01, 1.598316e+00	
32 1.584893e+02 5.171528e+01, 1.268747e+00	
33 1.995262e+02 5.172450e+01, 1.006541e+00	
34 2.511886e+02 5.173031e+01, 7.978358e-01	
35 3.162278e+02 5.173399e+01, 6.315670e-01	
36 3.981072e+02 5.173630e+01, 4.989059e-01	
37 5.011872e+02 5.173776e+01, 3.928006e-01	
38 6.309573e+02 5.173868e+01, 3.076070e-01	
39 7.943282e+02 5.173926e+01, 2.387913e-01	
40 1.000000e+03 5.173963e+01, 1.826906e-01	
41 1.258925e+03 5.173986e+01, 1.363180e-01	
42 1.584893e+03 5.174000e+01, 9.720440e-02	
43 1.995262e+03 5.174008e+01, 6.326710e-02	
44 2.511886e+03 5.174012e+01, 3.269892e-02	
45 3.162278e+03 5.174014e+01, 3.872029e-03	
46 3.981072e+03 5.174012e+01, -2.47487e-02	
47 5.011872e+03 5.174008e+01, -5.46873e-02	
48 6.309573e+03 5.174000e+01, -8.75380e-02	
[illegible]	

Regarding the simulation results, we present them below in Table **??**. Note that, in the previous section, we presented this results to compare to the theoretical ones.

Name	V or dB
t4 laboratory simulation results	
AC Analysis Sun May 23 23:21:26 2021	
———————————————————————	
Index frequency vec	
———————————————————————	
0 1.000000e-01 -1.34608e-01, 1.190096e-01	
1 1.258925e-01 -2.16533e-01, 1.619388e-01	
2 1.584893e-01 -3.44185e-01, 2.284126e-01	
3 1.995262e-01 -5.40995e-01, 3.361049e-01	
4 2.511886e-01 -8.39487e-01, 5.174617e-01	
5 3.162278e-01 -1.28056e+00, 8.310501e-01	
6 3.981072e-01 -1.90571e+00, 1.378709e+00	
7 5.011872e-01 -2.73342e+00, 2.326156e+00	
8 6.309573e-01 -3.70881e+00, 3.914666e+00	
9 7.943282e-01 -4.62436e+00, 6.434068e+00	
10 1.000000e+00 -5.04401e+00, 1.011697e+01	
11 1.258925e+00 -4.31348e+00, 1.494751e+01	
12 1.584893e+00 -1.74480e+00, 2.047377e+01	
13 1.995262e+00 3.047502e+00, 2.579466e+01	
14 2.511886e+00 9.848241e+00, 2.983046e+01	
15 3.162278e+00 1.782267e+01, 3.177467e+01	
16 3.981072e+00 2.584599e+01, 3.143708e+01	
17 5.011872e+00 3.296602e+01, 2.923871e+01	
18 6.309573e+00 3.868359e+01, 2.590991e+01	
19 7.943282e+00 4.293980e+01, 2.215542e+01	
20 1.000000e+01 4.593928e+01, 1.847559e+01	
21 1.258925e+01 4.797427e+01, 1.514675e+01	
22 1.584893e+01 4.932005e+01, 1.227840e+01	
23 1.995262e+01 5.019518e+01, 9.880387e+00	
24 2.511886e+01 5.075807e+01, 7.913159e+00	
25 3.162278e+01 5.111759e+01, 6.318420e+00	
26 3.981072e+01 5.134620e+01, 5.035281e+00	
27 5.011872e+01 5.149116e+01, 4.007705e+00	
28 6.309573e+01 5.158290e+01, 3.187210e+00	
29 7.943282e+01 5.164090e+01, 2.533253e+00	
30 1.000000e+02 5.167754e+01, 2.012598e+00	
31 1.258925e+02 5.170067e+01, 1.598316e+00	
32 1.584893e+02 5.171528e+01, 1.268747e+00	
33 1.995262e+02 5.172450e+01, 1.006541e+00	
34 2.511886e+02 5.173031e+01, 7.978358e-01	
35 3.162278e+02 5.173399e+01, 6.315670e-01	
36 3.981072e+02 5.173630e+01, 4.989059e-01	
37 5.011872e+02 5.173776e+01, 3.928006e-01	
38 6.309573e+02 5.173868e+01, 3.076070e-01	
39 7.943282e+02 5.173926e+01, 2.387913e-01	
40 1.000000e+03 5.173963e+01, 1.826906e-01	
41 1.258925e+03 5.173986e+01, 1.363180e-01	
42 1.584893e+03 5.174000e+01, 9.720440e-02	
43 1.995262e+03 5.174008e+01, 6.326710e-02	
44 2.511886e+03 5.174012e+01, 3.269892e-02	
45 3.162278e+03 5.174014e+01, 3.872029e-03	
46 3.981072e+03 5.174012e+01, -2.47487e-02	
47 5.011872e+03 5.174008e+01, -5.46873e-02	
48 6.309573e+03 5.174000e+01, -8.75380e-02	
[illegible]	

It is important, in order to guarantee a high compatibility with AUDIO IN and speakers, that we obtain a very high input impedance (Z_I) and a very low output impedance (Z_O). Analysing our results, we notice that despite having a small input impedance (in result of a compromise we had to make to obtain a higher merit figure), the output impedance is very low, as desired.

3.1 Coupling Capacitors

In order to analyse this circuit, we need to understand the coupling capacitors influence. In this BJT amplifier circuit, there are two couplin capacitors, C_{in} and C_O - because their functions are similar, we will focus only on capacitor C_{in}. In the graphics below, we present the frequency response of the circuit, but with C_{in} values drastically differents.

(a) Low C_{in}=1nF

Images/couplingbig-eps-converted-t

(b) High C_{in}=1F

Figure 8: C_{in} influence on the frequency response of the circuit.

As we can notice, the change in the capacitance of the coupling capacitor does not influence the value of the higher cut-off frequency. However, the increase of that value leads to a larger bandwidth, which is desired.

This happenas because, when $\omega \to 0$, $Z(C_{in}) \to \inf$ and, therefore, this capacitor prevents the transistor from entering on the saturation or the cut-off regions, by blocking the DC component of the AUDIO IN source. This helps mantaining the OP of the transistor, so that it can operate at lower frequencies, as C_{in} increases.

3.2 Bypass Capacitor

Next, we must analyse the influence of the Bypass Capacitor C_b on the circuit. In the graphics below, we present the frequency response of the circuit, but with C_b values drastically differents.

(a) Low C_E=1nF

(b) High C_E=1F

Figure 9: C_E influence on the circuit.

By placing the bypass capacitor in parallel with R_E, this resistor becomes short for medium and high frequencies and because the amplifier's first stage gain is inversely dependent on this resistance, the bypass capacitor helps maximizing the gain for medium and high frequencies, which is desired.

3.3 Resistor R_C

Finally, we must understand the effect on the circuit of changing R_C. In the graphics below, we present the frequency response of the circuit, but with R_C values drastically differents.

(a) Low R_C=10Ω

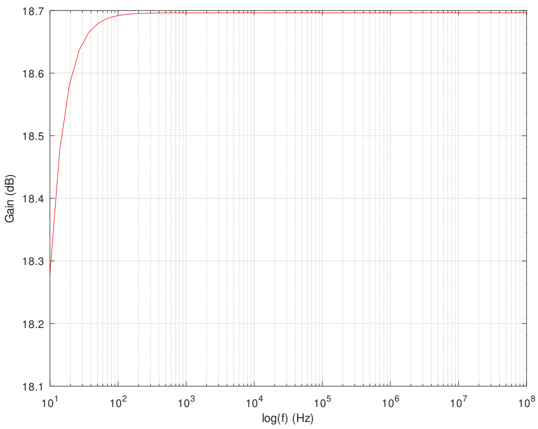
(b) High R_C=10$k\Omega$

Figure 10: R_C influence (on the Gain)

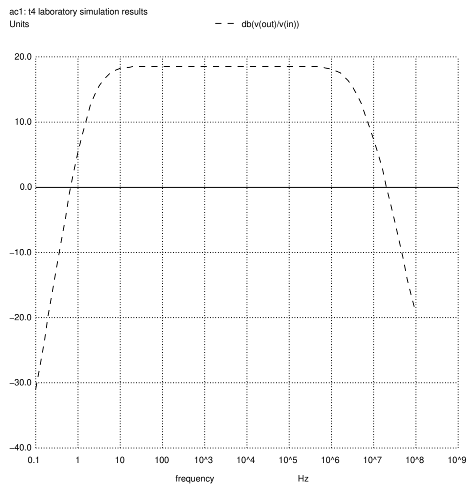
As we can notice, when we increase R_C, the gain increases and the passaband is antecipated. Note that for high values of resistance, a bizarre graphic is described.

3.4 Gain.

We are now ready to make a global comparison of the two approaches, with the chosen values for the constants. Below, we present both the theoretical and simulation graphs of the gain:



(a) Theoretical Gain



(b) Simulation Gain

Figure 11: Gain

In this case, the comparison of the shape can only be done on the left side, since we don't have the theoretical higher cut-off frequency.

Comparing both figures, we can conclude that the overall shape of the graphs is similar, noting that the theoretical curve can be thought of as assymptotical approach. Furthermore, we can see that the Gain and the lower cut-off frequency are quite similar - they may differ slightly because not only the theoretical result is an approximation, as said previously, but the transistors used in the NGSPice were real ones.

3.5 Merit

To end this section, we outline below the 4 values that influence the merit figure and the respective value of the merit.

Name	Values
lowcutoff	3.202582e+00
cost	7.224358e+03
gain	1.857653e+01
bandwidth	3.038871e+06
merit	2.439930e+03

Table 15: Values for the calculation of the Merit.

As we can see, we obtained a very high merit value. However, that value was obtained at the cost of degrading the quality of the circuit (for example, the low input impedance.)

4 Conclusion

In this laboratory assignment the objective of building and analysing an Audio Amplifier Circuit, made of a gain stage and an output stage, has been achieved. The theoretical analysis was performed with the help of the Octave math tool and the circuit simulation using the Ngspice tool. For both analysis, we determined the gain and input and output voltages of the circuit, as well as, lower cut-off frequency. We also plotted the frequency responce Vo(f)/Vi(f). At the end, we calculate the merit of our work.

This way, in theoretical analysis, we explained why the two stages could be connected without significant signal loss. And, in simulation analysis, we also explaneid the purpose of the coupling capacitors and their effect on the bandwidth, the purpose of the bypass capacitor and its effect on the gain and the effect of resistor RC on the gain.

As previously mentioned, the simulation results had slight differences from the theoretical ones. However, we designed an acceptable Audio Amplifier Circuit - furthermore, we obtained a decent figure of merit, despite the cost being too high.