Instituto Superior Técnico, University of Lisbon

Integrated Master in Aerospace Engineering

Circuit Theory and Electronics Fundamentals

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Second Laboratory Report

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a AC voltage source v_s, seven resistors, a voltage-controlled current source I_b, a capacitor C and a current-controlled voltage source V_d. The components of this circuit are distribuited by 4 elementary meshes and 8 nodes, as seen in Figure 1.

In order to analyse the circuit, the following data were obtained by running the supplied Python script:

Units for the values: V, mA, uC, kOhm and mS
Values:
R1 = 1.00147062639
R2 = 2.0078068512

R3 = 3.11269704405

R4 = 4.10609573471

R5 = 3.02670672634

R6 = 2.01292455078

R7 = 1.02905244808

Vs = 5.24842063411

C = 1.04086013403

Kb = 7.21413591579

Kd = 8.01455113996

In Section 2, a theoretical analysis of the circuit, performed on Octave, is presented. In Section 3, the circuit is analysed by simulation, using NGSpice, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

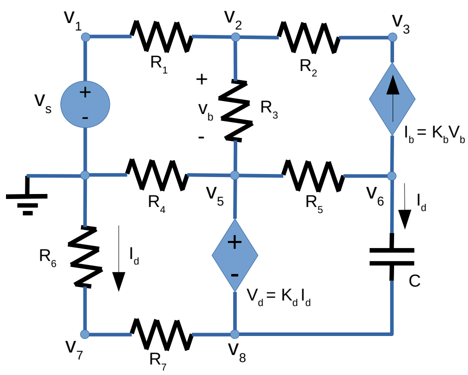

Figure 1: Circuit which will be analysed during this laboratory assignment.

2 Theoretical Analysis

In this section, the RC circuit shown in Figure 1 is analysed theoretically. We will begin by analyzing the circuit by applying the nodal method to determine the voltages in all nodes and currents in all branches for t<0.

In order to do this laboratory, we will not only use the node method, but also apply the Thévenin/Norton Theorem as well as what was lectured about capacitors. A RC circuit is composed by resistors and capacitors and it may driven by voltage or current sources wich will produce different responses. A capacitor is an electrical component that behaves according to the following differential equations:

$$q(t) = C \cdot v(t) \Leftrightarrow \frac{d \cdot v(t)}{dt} = C \cdot \frac{d \cdot q(t)}{dt} \Leftrightarrow i(t) = C \cdot \frac{d \cdot v(t)}{dt} \tag{1}$$

Thereafter, the current of a capacitor is not proportional to the voltage in its terminals, but rather to the voltage variation rate.

2.1 Voltages in Nodes and Currents in Branches

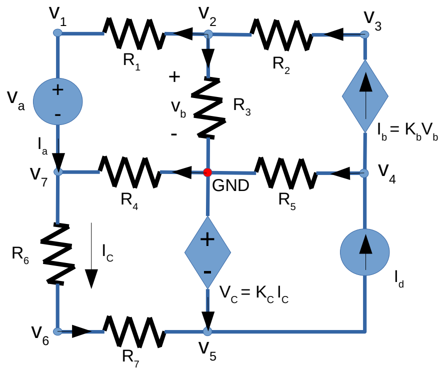

Figure 2: Circuit with each current direction arbitrarily assigned, to assist in the analysis of the circuit by the Nodal Method.

For t<0, -t>0. Because of that u(t)=0 and u(-t)=1. So:

$$V_S(t<0) = V_s \tag{2}$$

That means that the voltage source drives constant voltage Vs, in other words, the voltage doesn't varies in time. Consequently, the current of the capacitor is null:

$$I_c = 0 \tag{3}$$

To determine the nodal voltages, we use the nodal method.

We first start by calculating the values of the conductances of the various resistors:

$$G_i = 1/R_i \tag{4}$$

Then we determine the KCL equations in nodes not connected to voltage sources and another additional equations for nodes related by voltage sources.

After doing that we can now obtain the matricial equation wich will allow us to determine de nodal voltages:

$$\begin{bmatrix} 1 & -0 & 0 & 0 & 0 & 0 & 0+0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2+K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & -G_3 & 0 & G_3+G_4+G_5 & -G_5 & -G_7 & G_7 \\ 0 & -K_b & 0 & G_5+K_b & -G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & -G_7 \\ 0 & 0 & 0 & 1 & 0 & G_6 \cdot K_d & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

The solution of this matricial equation is determined by Octave:

Node	Voltage[V]
V_1	5.248421e+00
V_2	5.033187e+00
V_3	4.581561e+00
V_5	5.064366e+00
V_6	5.745178e+00
V_7	-2.050083e+00
V_8	-3.098131e+00

Table 1: Nodal Voltages Values

Knowing these voltages, it is also possible to determine branch currents, using Ohm's Law or Kirchoff's Laws. Note that:

$$I_S = I_1 \tag{6}$$

$$I_{Vd} = -I_7 \tag{7}$$

$$I_b = K_b \cdot (V_2 - V_5) \tag{8}$$

Name	Value [A or V]
I_{R1}	-2.149179e-04
I_{R2}	-2.249348e-04
I_{R3}	-1.001695e-05
I_{R4}	1.233378e-03
I_{R5}	-2.249348e-04
I_{R6}	1.018460e-03
I_{R7}	1.018460e-03
I_s	-2.149179e-04
I_b	-2.249348e-04
I_{Vd}	-1.018460e-03
I_c	0.000000e+00

Table 2: Voltages and currents of some circuit components

2.2 Equivalent Resistance

Name	Value [A or V]
V_1	0.000000e+00
V_2	0.000000e+00
V_3	0.000000e+00
V_5	0.000000e+00
V_6	8.843309e+00
V_7	-0.000000e+00
V_8	0.000000e+00

Table 3: Voltages and currents of some circuit components

Name	Value [A or V]
V_x	8.843309e+00
I_x	2.921760e-03
R_{eq}	3.026707e+03

Table 4: Voltages and currents of some circuit components

3 Simulation Analysis

3.1 Operating Point Analysis

Table 5 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, we notice that the simulation results are accurate, except for the last decimal places, as a consequence of the cientific notation and the number of significative algharisms used by each program to present the results. Despite that, we realise that the values with more significant algharisms (used in NGSpice) match correctly the rounded values (used in Octave).

Name	Value [A or V]
@c[i]	0.000000e+00
@gb[i]	-2.24935e-04
@r1[i]	2.149178e-04
@r2[i]	-2.24935e-04
@r3[i]	-1.00169e-05
@r4[i]	1.233378e-03
@r5[i]	-2.24935e-04
@r6[i]	1.018460e-03
@r7[i]	1.018460e-03
v1	5.248421e+00
v2	5.033187e+00
v3	4.581562e+00
v4	-2.05008e+00
v5	5.064367e+00
v6	5.745178e+00
v7	-2.05008e+00
v8	-3.09813e+00

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

In the table, we can see an nineth node (node 8), which has the same voltage value as node 6. This happens because, in NGSPice, when we want to simulate circuits with current-dependent sources, we must add a 0V voltage source in series to a component to sense the current flowing through it. Therefore, an aditional node appears in the simulated circuit.

Note that we can not perform aditional simulation analysis, namely transient and frequency ones, with phase and magnitude responses and input impedance, because the circuit does not have any electrical component which output is a function of time.

3.2 Point 2

3.3 Point 3

4 Conclusion

In this laboratory assignment the objective of analysing a circuit with resistors, voltage sources and current sources by applying the Nodal and Mesh methods together with Kirchhoff's Circuit Laws and Ohm's Law has been achieved. The theoreticall analysis was performed with the help of the Octave math tool and the circuit simulation using the Ngspice tool. The simulation results matched the theoretical results perfectly. This happens mainly because the method used to obtain the theoretical results is the same method Ngspice uses to emulate the described circuit. Another reason why the results match perfectly is that the circuit was composed by simple components.For more complex components, the theoretical and simulation models may differ but it is not the case in this assignment.

Name	Value [A or V]
@gb[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.92176e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v1	0.000000e+00
v2	0.000000e+00
v3	0.000000e+00
v4	0.000000e+00
v5	0.000000e+00
v6	8.843309e+00
v7	0.000000e+00
v8	0.000000e+00

Table 6: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

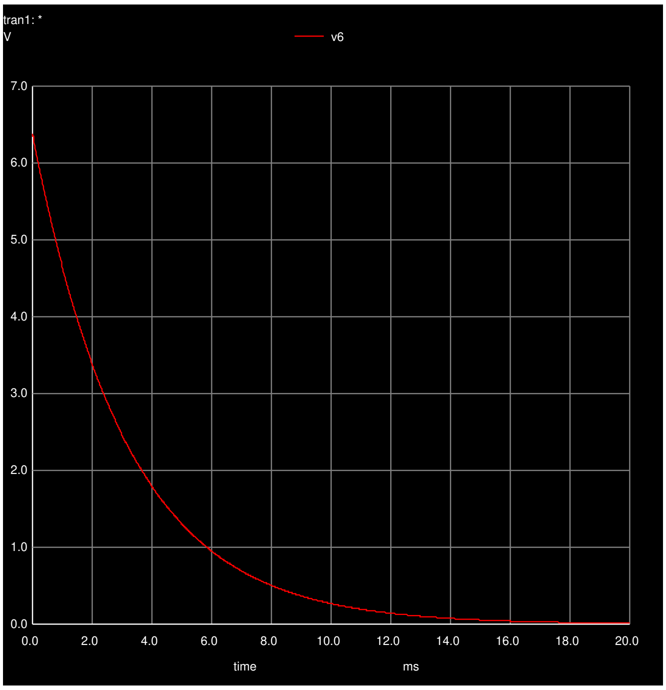

Figure 3: Transient output voltage