Instituto Superior Técnico, University of Lisbon

Integrated Master in Aerospace Engineering

Circuit Theory and Electronics Fundamentals

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Second Laboratory Report

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing a AC voltage source v_s, seven resistors, a voltage-controlled current source I_b, a capacitor C and a current-controlled voltage source V_d. The components of this circuit are distribuited by 4 elementary meshes and 8 nodes, as seen in Figure 1.

In order to analyse the circuit, the following data were obtained by running the supplied Python script:

Units for the values: V, A, F, Ohm and S

Values:

R_1	1.001471e+03
R_2	2.007807e+03
R_3	3.112697e+03
R_4	4.106096e+03
R_5	3.026707e+03
R_6	2.012925e+03
R_7	1.029052e+03
V_s	5.248421e+00
C	1.040860e-06
K_b	7.214136e-03
K_d	8.014551e+03

Table 1: Data Table

The voltage source is defined by:

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{1}$$

, where

$$u(t) = \begin{cases} 0 & \text{t<0} \\ 1 & \text{t} \geq 0 \end{cases} \tag{2}$$

In Section 2, a theoretical analysis of the circuit, performed on Octave, is presented. In Section 3, the circuit is analysed by simulation, using NGSpice, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

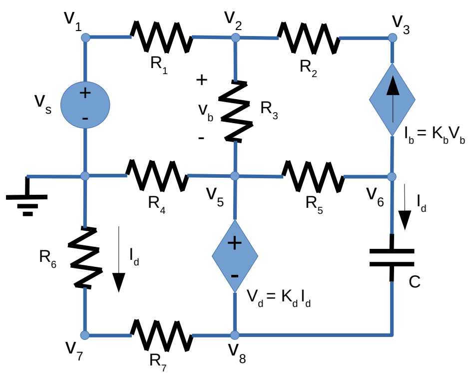

Figure 1: Circuit which will be analysed during this laboratory assignment.

2 Theoretical Analysis

In this section, the RC circuit shown in Figure 1 is analysed theoretically. We will begin by analyzing the circuit by applying the nodal method to determine the voltages in all nodes and currents in all branches for t<0.

In order to do this laboratory, we will not only use the node method, but also apply the Thévenin/Norton Theorem as well as what was lectured about capacitors.

A RC circuit is composed by resistors and capacitors and it may driven by voltage or current sources wich will produce different responses. A capacitor is an electrical component that behaves according to the following differential equations:

$$q(t) = C \cdot v(t) \Leftrightarrow \frac{d \cdot v(t)}{dt} = C \cdot \frac{d \cdot q(t)}{dt} \Leftrightarrow i(t) = C \cdot \frac{d \cdot v(t)}{dt} \tag{3}$$

Thereafter, the current of a capacitor is not proportional to the voltage in its terminals, but rather to the voltage variation rate.

2.1 Voltages in All Nodes and Currents in All Branches

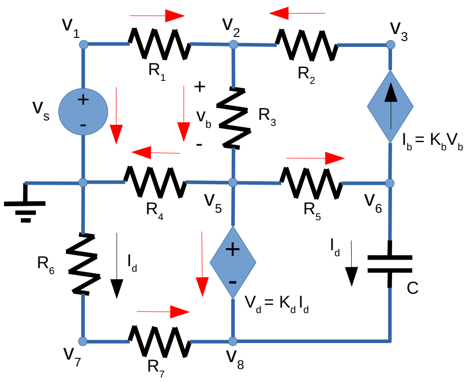

Figure 2: Circuit with each current direction arbitrarily assigned, to assist in the analysis of the circuit by the Nodal Method.

For t<0, -t>0. Because of that u(t)=0 and u(-t)=1. So:

$$v_S(t<0) = V_s \tag{4}$$

That means that the voltage source drives constant voltage V_s, in other words, the voltage doesn't varies in time. Consequently, the current of the capacitor is null:

$$I_c = 0 \tag{5}$$

To determine the nodal voltages, we use the nodal method.
We first start by calculating the values of the conductances of the various resistors:

$$G_i = 1/R_i \tag{6}$$

Then we determine the KCL equations in nodes not connected to voltage sources and another additional equations for nodes related by voltage sources.

After doing that we can now obtain the matricial equation wich will allow us to determine de nodal voltages:

$$\begin{bmatrix} 1 & -0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2+K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & -G_3 & 0 & G_3+G_4+G_5 & -G_5 & -G_7 & G_7 \\ 0 & -K_b & 0 & G_5+K_b & -G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & -G_7 \\ 0 & 0 & 0 & 1 & 0 & G_6 \cdot K_d & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

The solution of this matricial equation is determined by Octave:

Node	Voltage[V]
V_1	5.248421e+00
V_2	5.033187e+00
V_3	4.581561e+00
V_5	5.064366e+00
V_6	5.745178e+00
V_7	-2.050083e+00
V_8	-3.098131e+00

Table 2: Nodal Voltages Values

Knowing these voltages, it is also possible to determine branch currents, using Ohm's Law and Kirchoff's Laws. Note that:

$$I_S = I_1 \tag{8}$$

$$I_{Vd} = -I_7 \tag{9}$$

$$I_b = K_b \cdot (V_2 - V_5) \tag{10}$$

Name	Current[A]
I_{R1}	2.149179e-04
I_{R2}	-2.249348e-04
I_{R3}	-1.001695e-05
I_{R4}	1.233378e-03
I_{R5}	-2.249348e-04
I_{R6}	1.018460e-03
I_{R7}	1.018460e-03
I_s	-2.149179e-04
I_b	-2.249348e-04
I_{Vd}	-1.018460e-03
I_c	0.000000e+00

Table 3: Currents of circuit components

2.2 Equivalent Resistance, R_{eq}

The objective of this task is to calculate R_{eq} as seen in terminals of the capacitor. That can be useful to study the circuit and the variation of its nodal values for t = 0. In order to solve it, we used the concept of the Thévenin and Norton Theorems.

These theorems state that all linear circuits of sources and resistors can be substituted by a simpler circuit with an equivalent resistor and a voltage or current source, respectively, from a chosen point of view.

In that way, we first replaced the capcitor by an independent voltage source V_x and, then, we put the independent voltage source V_s to 0V. It is important to notice that the depedent voltage source cannot be put equal to 0V and the dependent current source cannot be removed from the circuit.

The analysed circuit is represented on figure 3.

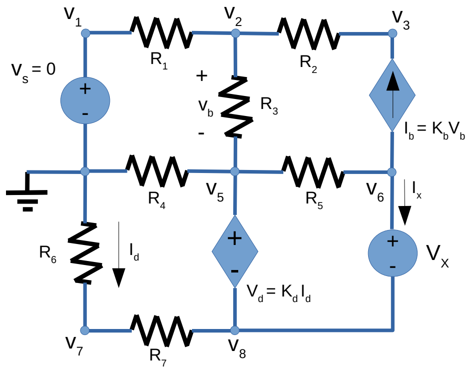

Figure 3: Circuit Analysed

Although the voltage in the nodes is not necessarily continuous, the difference pottencial in the capacitor, wich is given by the difference between nodes 6 and 8, is. Therefore, we considered:

$$V_x = V_6 - V_8 \tag{11}$$

, where V_6 and V_8 are the voltages in nodes 6 and 8 obtained in the previous subsection.

Then, we run the node analysis to determine the current I_x in order to determine the R_{eq} by using Ohm's Law:

$$R_{eq} = \frac{V_x}{I_x} \tag{12}$$

This way, we can also determine the time constante:

$$\tau = R_{eq} \cdot C \tag{13}$$

The matrix equation used in the octave tool is the following:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2 + K_b & -G_2 & -K_b & 0 & 0 & 0 \\ -G_1 & G_1 & 0 & G_4 & 0 & G_6 & 0 \\ 0 & 0 & 0 & 1 & 0 & G_6 \cdot K_d & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -G_6 - G_7 & G_7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \end{bmatrix} \tag{14}$$

And the results obtained are expressed in this table:

Name	Value [A or V]
I_{R1}	0.000000e+00
I_{R2}	0.000000e+00
I_{R3}	0.000000e+00
I_{R4}	0.000000e+00
I_{R5}	-2.921760e-03
I_{R6}	0.000000e+00
I_{R7}	-0.000000e+00
I_s	-0.000000e+00
I_b	0.000000e+00
I_{Vd}	0.000000e+00
V_1	0.000000e+00
V_2	0.000000e+00
V_3	0.000000e+00
V_5	0.000000e+00
V_6	8.843309e+00
V_7	-0.000000e+00
V_8	0.000000e+00

Table 4: Voltages and currents of circuit components

V_x	8.843309e+00
I_x	2.921760e-03
R_{eq}	3.026707e+03
tau	3.150378e-03

Table 5: Calculated Values

2.3 Natural Solution, t $\geq$ 0

This tasks' objective is to compute the natural solution of the voltage in node 6. Using the results previously obtained in task 2, we obtained the value of the voltage in this node when t= 0 seconds and v_{6n}(t=0s)=V_x , since the result obtain for v8 is zero in the previous task.

The natural solution as the following format:

$$v_n(t) = A \cdot \exp(\frac{-t}{RC}) \tag{15}$$

In this particular case, A= v_6(t=0) =V_x, giving us the equation below:

$$v_6n(t) = V_x \cdot \exp(\frac{-t}{RC}) \tag{16}$$

The following plot was obtained using the octave tool and it shows the graph obtained for the natural solution of the voltage in node 6 and it is as expected a negative exponetial:

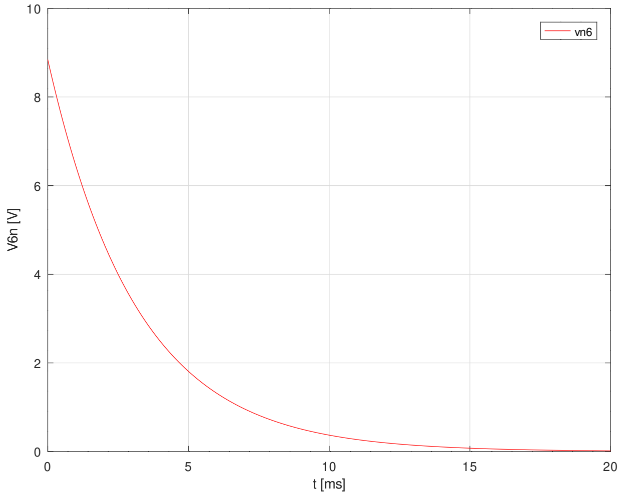

Figure 4: Natural response.

2.4 Forced Solution, t $\geq$ 0

In order to compute the forced solution for the voltage in node 6,it was utilized the same time interval considered in the previous task.

But to calculate the forced solution, we need to used the regular circuit and to do that we assumed a phasor Vs and substituited the capacitor C for its impedance, Zc.

The phasor used was:

$$V_s = \exp(-i \cdot \frac{\pi}{2}) \tag{17}$$

And the formula to obtain Z_c is:

$$Z_c = \frac{1}{i \cdot C \cdot \omega} \tag{18}$$

With all of that taken into consideration, the matrix equation used to calculate the forced solution in this time interval is the following:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & G_1 & 0 & G_4 & 0 & G_6 & 0 \\ G_1 & -G_1 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & G_2+Kb & -G_2 & -K_b & 0 & 0 & 0; \\ 0 & -Kb & 0 & Kb+G_5 & -G_5-\frac{1}{Z_c} & 0 & \frac{1}{Z_c} \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & 0 & 0 & 1 & 0 & G_6 \cdot K_d & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \end{bmatrix} \tag{19}$$

The result obtained can be seen in the plot below:

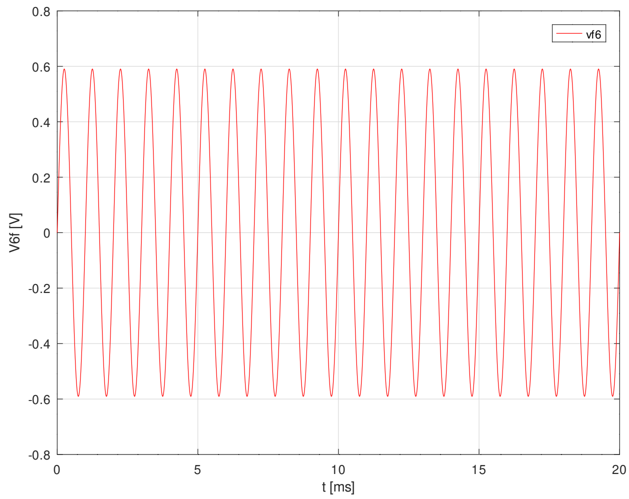

Figure 5: Forced response.

The phasor amplitude values and phases were also computed in this task and can be consulted in the Table (X):

Name	Value [V]
V_1	1.000000e+00
V_2	9.589907e-01
V_3	8.729409e-01
V_5	9.649315e-01
V_6	5.902978e-01
V_7	3.906094e-01
V_8	5.902978e-01

Table 6: Amplitude

2.5 Final Total Solution

The total solution for the node 6 for a frequency,f = 1000 Hz was calculated by superimposing the forced and the natural solution for t $\geq$ 0. For t $<$0 , the voltage in node 6 is the one calculated in the first task. This gives us the following function for v6:

$$v_6t = \begin{cases} V_6 & \text{t} < 0 \\ v_6n + v_6A * cos(\omega * t - v_6Ph) & \end{cases} \tag{20}$$

The total solution was plotted together with v_s and this is the result obtained:

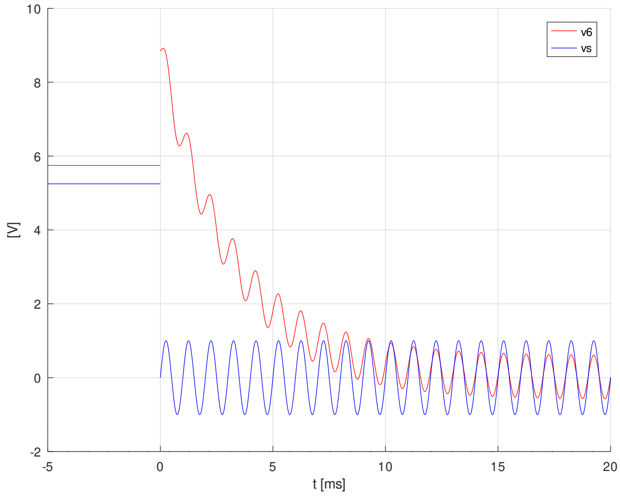

Figure 6: Final response.

It is worth noticing that $v6t$ is not continuous and tends to diminish.

2.6 Frequency Responses

The last assignment is to study how the amplitude ,in decibels, and the phase, in degrees, of v6, vs and vc= v6 - v8 vary with different frequencies. In order to achieve this objective, we plotted this fuctions together twice, in one of them we study the variance of the amplitude (Figure X) and in the other the variance of the phase (Figure Y). To obtain the intervals we used the logspace in the range of 0,1 Hz to 1 MHz.

Plot studying the amplitude variance:

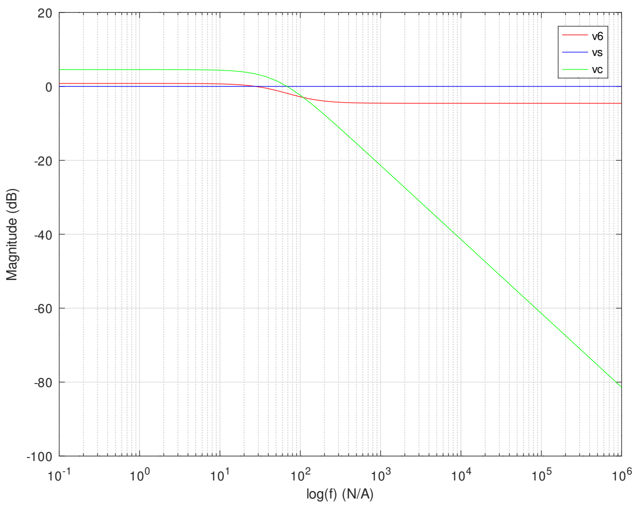

Figure 7: Magnitude response.

Since the transfer function is calculated via a quocient of the output and the input, the result expected and obtained for vs is zero. It is also noticeable that vc rapidly decreases as the frenquency shoots upwards.

Regarding v6, it is noticeable that for higher frequencies that the functions is constant, this is due to the fact that for this kind of frequencies the capacitator acts as a shunt, meaning that only the resistors (that do not vary with the frequency) influence the variance, making it constant. For lower frequencies, it is observable that there is a slightly decrease. This is expected in a RC circuit and it is due to the impedance of the capacitor.

Plot studying the phase variance:

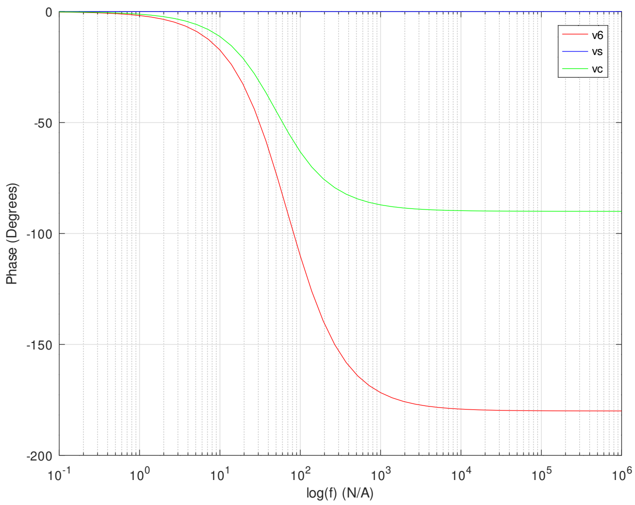

Figure 8: Phase response.

It is worth noticing that, as expected , the phase variance of vs is null. This is also due to the fact that the transfer function is calculated using a quocient between the output and the input. For v6 and vc, it is observable that with the frequency increment the phase achieve lower values and are all negative, as expected.

3 Simulation Analysis

3.1 Operating Point Analysis

Table 7 shows the simulated operating point results for the circuit under analysis for t¡0, which means that we will consider the voltage source v_s as a DC voltage source equal to V_s, given in the Python script. Notice that the values on the right were obtained by Octave, therefore they are the theoretical values, which we already mentioned previously but, to facilitate the comparison between the simulation and theoretical values, we mentioned them again.

Compared to the theoretical analysis results, we notice that both simulation results are accurate, except for the last decimal places, as a consequence of the cientific notation and the number of significative algharisms used by each program to present the results. Despite that, we realise that the values with more significant algharisms match correctly the rounded values.

Table 8 shows the simulated operating point results for the circuit under analysis for t=0, where we will consider the voltage source v_s=0 and replace the capacitor for a voltage source V_x=V_6-V_8, where V_6 and V_8 are the voltages in nodes 6 and 8, respectively. This step will allow us to verify the theoretical results obtained for the node voltages and the current I_x, which are accurate, as we can see. Again, we show the theoretical values obtained for this step, to make it easier to compare between simulation and theoretical values.

Furthermore, we can see that for t¡0, V_6 will have a significant different voltage value than for t=0; however, the voltage in the capacitor C, which is given by V_6-V_8 remains the same, despite the different values for V_6 and V_8. Therefore, we witness a discontinuity in the node voltages, that does not happen for the voltage in the capacitor - well, this happens because

Name	Value [A or V]
@c[i]	0.000000e+00
@gb[i]	-2.24935e-04
@r1[i]	2.149178e-04
@r2[i]	-2.24935e-04
@r3[i]	-1.00169e-05
@r4[i]	1.233378e-03
@r5[i]	-2.24935e-04
@r6[i]	1.018460e-03
@r7[i]	1.018460e-03
v1	5.248421e+00
v2	5.033187e+00
v3	4.581562e+00
v4	-2.05008e+00
v5	5.064367e+00
v6	5.745178e+00
v7	-2.05008e+00
v8	-3.09813e+00

Node	Voltage[V]
V_1	5.248421e+00
V_2	5.033187e+00
V_3	4.581561e+00
V_5	5.064366e+00
V_6	5.745178e+00
V_7	-2.050083e+00
V_8	-3.098131e+00

Name	Value [A or V]
I_{R1}	2.149179e-04
I_{R2}	-2.249348e-04
I_{R3}	-1.001695e-05
I_{R4}	1.233378e-03
I_{R5}	-2.249348e-04
I_{R6}	1.018460e-03
I_{R7}	1.018460e-03
I_s	-2.149179e-04
I_b	-2.249348e-04
I_{Vd}	-1.018460e-03
I_c	0.000000e+00

Table 7: Operating point for t¡0. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt. The values on the right are the theoretical values for the currents and voltages in the circuit.

only the voltages in capacitors and inductors remain continuous, therefore the voltages in the resistors will have 'jumps' in their values to mantain the continuity previously mentioned.

In both tables, we can see an nineth node (node 8), which has the same voltage value as node 6. This happens because, in NGSPice, when we want to simulate circuits with current-dependent sources, we must add a 0V voltage source in series to a component to sense the current flowing through it. Therefore, an aditional node appears in the simulated circuit.

3.2 Transient Analysis

Figure **??** shows the simulated transient analysis results for the natural response of the circuit under analysis, using the boundary conditions for V_6 and V_8 obtained previously in the second operating point analysis performed.

Name	Value [A or V]
@gb[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.92176e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v1	0.000000e+00
v2	0.000000e+00
v3	0.000000e+00
v4	0.000000e+00
v5	0.000000e+00
v6	8.843309e+00
v7	0.000000e+00
v8	0.000000e+00

Name	Value [A or V]
I_{R1}	0.000000e+00
I_{R2}	0.000000e+00
I_{R3}	0.000000e+00
I_{R4}	0.000000e+00
I_{R5}	-2.921760e-03
I_{R6}	0.000000e+00
I_{R7}	-0.000000e+00
I_s	-0.000000e+00
I_b	0.000000e+00
I_{Vd}	0.000000e+00
V_1	0.000000e+00
V_2	0.000000e+00
V_3	0.000000e+00
V_5	0.000000e+00
V_6	8.843309e+00
V_7	-0.000000e+00
V_8	0.000000e+00

Name	Value [A or V]
V_x	8.843309e+00
I_x	2.921760e-03
R_{eq}	3.026707e+03
tau	3.150378e-03

Table 8: Operating point for t=0, where the capacitor C was replaced by a voltage source V_x=V_6-V_8, with V_6 and V_8 obtained previously. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt. The values on the right are the theoretical values for the currents and voltages in the circuit.

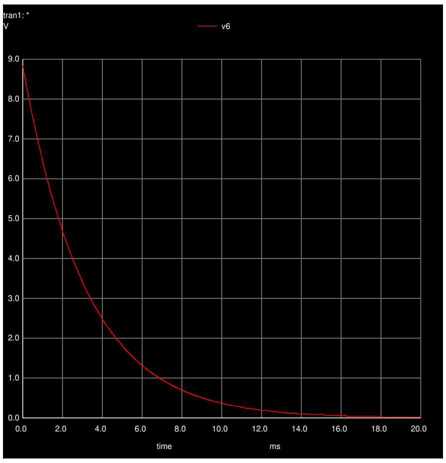

Figure 9: Transient output voltage for node 6, with boundary conditions V_6 and V_8 obtained previously.

We can notice that the graphic for the voltage V_6 is in a form of a negative exponential, which is confirmed by the expression and respective graphic obtained for the theoretical analysis, performed in Octave. Furthermore, because the theoretical analysis is performed for the interval [-5; 20]ms, we can easily notice the discontinuity for the voltage values v_s and V_6, that does not happen for the voltage in the capacitor.

Figure **??** shows the simulated transient analysis results for the circuit under analysis, where we finally consider the voltage source v_s, as a sinusoidal fuction of time. We perform this simulation to obtain the total solution (natural+forced).

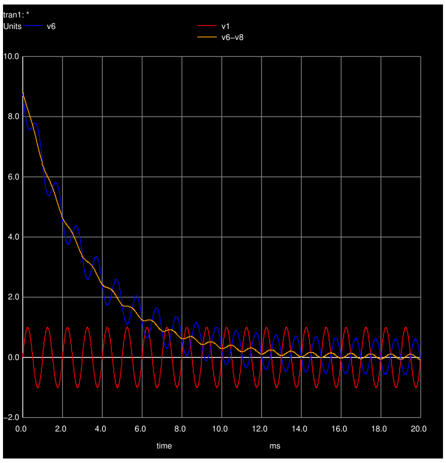

Figure 10: Transient output voltage for node 6, voltage source V_s and capacitor C

By analysing the result of the simulation, we notice that the total solution resulted of an overlap between the natural solution (in the form of a negative exponential) and the forced solution (in a form of a sinusoidal function). Comparing to the theoretical analysis, we notice again, because of the time interval used, the discontinuity of the values in t=0, that does not happen for the voltage in the capacitor. As far as the result is concerned, the form of the theoretical solution (negative exponential+sinusoidal fuction) match the one obtained by NGSpice.

3.3 Frequency Analysis

3.3.1 Magnitude Response

Figure 11 shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

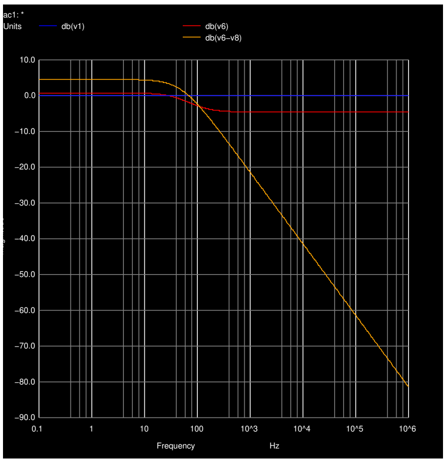

Figure 11: Magnitude response for node 6, voltage source V_s and capacitor C

3.3.2 Phase Response

Figure 12 shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

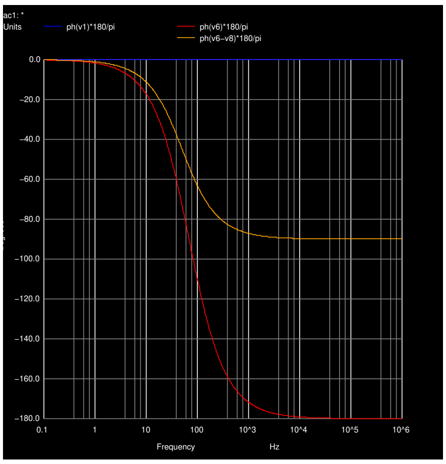

Figure 12: Phase response for node 6, voltage source V_s and capacitor C

4 Conclusion

In this laboratory assignment the objective of analysing a RC circuit by determining its natural and forced solution has been achieved. The theoretical analysis was performed with the help of the Octave math tool and the circuit simulation using the Ngspice tool. In both analysis, we had to determine the operating point results for t¡0 and t=0, and then apply the results obtained previously to plot the solutions, which were the time fuction of v_6, v_s and the voltage in the capacitor C and the respective frequency fuctions of their magnitudes and phases. The simulation results matched the theoretical results accurately - furthermore, because we use the time interval [-5;20]ms for the theoretical analysis, we can notice the discontinuities of some voltage values, which were previously explained in this report.