

## Instituto Superior Técnico, University of Lisbon

### Integrated Master in Aerospace Engineering

Circuit Theory and Electronics Fundamentals

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Fourth Laboratory Report

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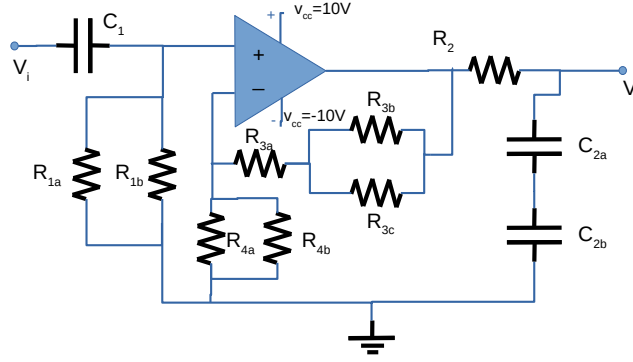
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## 1 Introduction

The objective of this laboratory assignment is to simulate a Band Pass Filter (BPF) using OPAMP as shown in Figure 1. We should have a central frequency of 1KHz and a 40 dB gain at the central frequency.

This way, we should choose its the architecture, considering the limitations imposed as well as the cost of the components in the circuit. Its diagram is shown in Figure 1.



**Figure 1:** Band Pass Filter Circuit

This circuit is composed by 6 resistors, 3 capacitors and a 741 OPAMP. The values of the components are exhibited in the table below.

Name	Value
$R_{1a}$	1.000000e+03 Ohm
$R_{1b}$	1.000000e+04 Ohm
$R_1$	9.090909e+02 Ohm
$R_2$	1.000000e+03 Ohm
$R_{3a}$	1.000000e+05 Ohm
$R_{3b}$	1.000000e+05 Ohm
$R_{3c}$	1.000000e+05 Ohm
$R_3$	1.500000e+05 Ohm
$R_{4a}$	1.000000e+03 Ohm
$R_{4b}$	1.000000e+04 Ohm
$R_4$	9.090909e+02 Ohm
$C_1$	2.200000e-07 F
$C_{2a}$	2.200000e-07 Ohm
$C_{2b}$	2.200000e-07 Ohm
$C_2$	1.100000e-07 F

**Table 1:** Components Values

Notice that some components have been simplified by its equivalent ( $R_1$ ,  $R_3$ ,  $R_4$  and  $C_2$ ).

In Section 2, a theoretical analysis of the circuit, performed on Octave, is presented. In Section 3, the circuit is analysed by simulation, using NGSpice, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

## 2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically. It is important to notice that the purpose is to build a band pass filter circuit using an OPAMP. This way, the considered circuit has a high pass filter, a signal amplifier (OPAMP) and a low pass filter in series, where the capacitor  $C_1$  and the resistor  $R_1$  act as a high pass filter, while the capacitor  $C_2$  and the resistor  $R_2$  function as a low pass filter.

Thus, We will begin by computing the gain, input and output impedances at the central frequency. Then, we will compute the frequency response  $V_o(f)/V_i(f)$ , using the incremental

circuit, solving the circuit for a frequency vector in log scale with 10 points per decade, from 10Hz to 100MHz.

## 2.1 Gain, Input and Output Impedances at central frequency

In this subsection, we will compute the gain, input and output impedances at the central frequency.

First, we will calculate low and high cutoff frequencies,  $w_L$  and  $w_H$ , respectively, as well as central frequency,  $w_O$ , which are given by:

$$w_L = \frac{1}{R_1 \cdot C_1} \quad (1)$$

$$w_H = \frac{1}{R_2 \cdot C_2} \quad (2)$$

$$w_O = \frac{1}{\sqrt{w_L \cdot w_H}} \quad (3)$$

Their values are presented in the table below:

Name	Value
$w_L$	5.000000e+03 rad/s
$w_H$	9.090909e+03 rad/s
$w_O$	6.741999e+03 rad/s
$f_O$	1.073022e+03 Hz

**Table 2:** Frequencies

As we can see, our central frequency is very close to 1KHz, as requested.

Now we can determine the gain at central frequency, which is given by the following equation:

$$T_{w_O} = \frac{R_1 \cdot C_1 \cdot jw_O}{1 + R_1 \cdot C_1 \cdot jw_O} \cdot \left(1 + \frac{R_3}{R_4}\right) \cdot \frac{1}{1 + R_2 \cdot C_2 \cdot jw_O} \quad (4)$$

As well as  $Z_{in}$  and  $Z_{out}$ , at the central frequency, are determined by:

$$Z_{in} = R_1 + \frac{1}{jw_O \cdot C_1} \quad (5)$$

$$Z_{out} = R_2 \cdot \frac{\frac{1}{jw_O \cdot C_2}}{R_2 + \frac{1}{jw_O \cdot C_2}} \quad (6)$$

The results are shown in tables 3 and 4:

Name	Value
$Z_{in}$	9.090909e+02 + j-6.741999e+02 Ohm
$Z_{out}$	6.451613e+02 + j-4.784644e+02 Ohm
$ Z_{in} $	1.131809e+03 Ohm
$ Z_{out} $	8.032193e+02 Ohm

**Table 3:** Impedances at central frequency

Name	Value
$AV_{HP}$	-1.903317e+00 dB
$AV_{OPAMP}$	4.440216e+01 dB
$AV_{LP}$	-1.903317e+00 dB
$AV$	4.059553e+01 dB

**Table 4:** Gain at central frequency

,where  $AV_{HP}$ ,  $AV_{OPAMP}$ ,  $AV_{LP}$ ,  $AV$  correspond to the high pass, OPAMP, low pass and final gain.

It is important to remember that the input impedance should be higher as possible to have a minimum input signal degradation, and output impedance should be close to null. This target became hard to achieve because of the limitations imposed on the circuit components. That's why our output impedance presents a too high value.

As also requested, we can observe that the circuit has a gain close to 40 dB.

Table 5 presents the same gain results, but considering, this time, a central frequency of 1KHz.

Name	Value
$AV_{HP}$	-2.130546e+00 dB
$AV_{OPAMP}$	4.440216e+01 dB
$AV_{LP}$	-1.695830e+00 dB
$AV$	4.057579e+01 dB

**Table 5:** Gain at 1KHz

As we can notice, these results are very similar to the ones in the previous table.

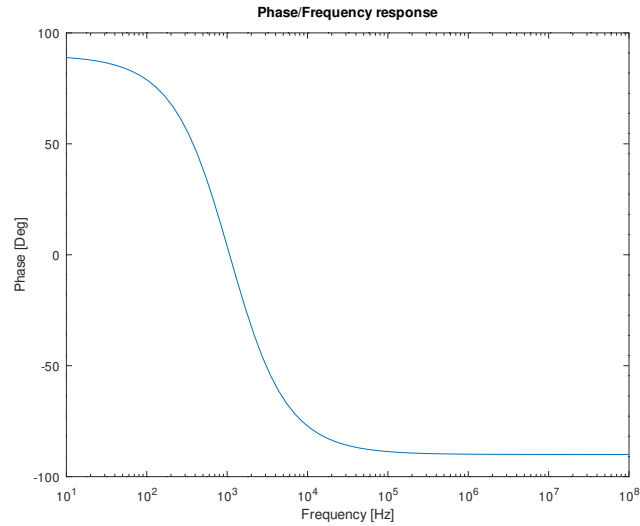
## 2.2 Frequency response $V_o(f)/V_i(f)$

Now, considering the transfer function  $T(s)$ , it is given by:

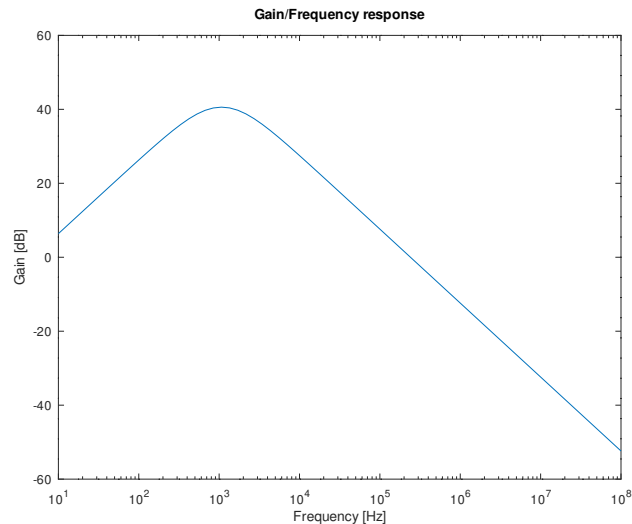
$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = 1 + \frac{Z_{in}(s)}{Z_{out}(s)} = \frac{R_1 \cdot C_1 \cdot s}{1 + R_1 \cdot C_1 \cdot s} \cdot \left(1 + \frac{R_3}{R_4}\right) \cdot \frac{1}{1 + R_2 \cdot C_2 \cdot s} \quad (7)$$

, where  $\frac{R_1 \cdot C_1 \cdot s}{1 + R_1 \cdot C_1 \cdot s}$ ,  $1 + \frac{R_3}{R_4}$  and  $\frac{1}{1 + R_2 \cdot C_2 \cdot s}$  represent the gain at high pass filter, OP-AMP and low pass filter, respectively.

This way, the gain frequency response is plotted in the figure below.



**Figure 2: Phase Frequency Response**



**Figure 3: Gain Frequency Response**

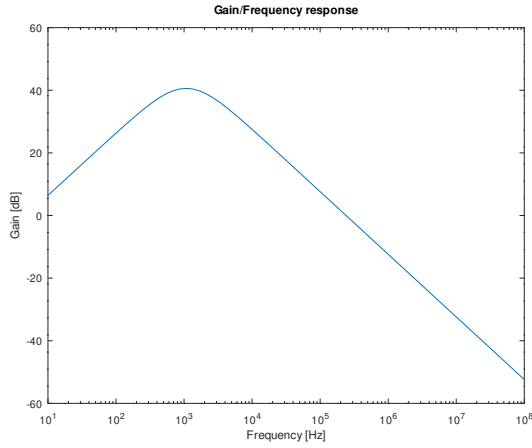
As we can see there is a maximum for the frequencies in the central frequency, which is near to 1KHz, otherwise low and high frequencies have a low gain. That was expected because both low and high frequencies were blocked by high and low pass filters.

### 3 Simulation Analysis and Comparison with Theoretical Values

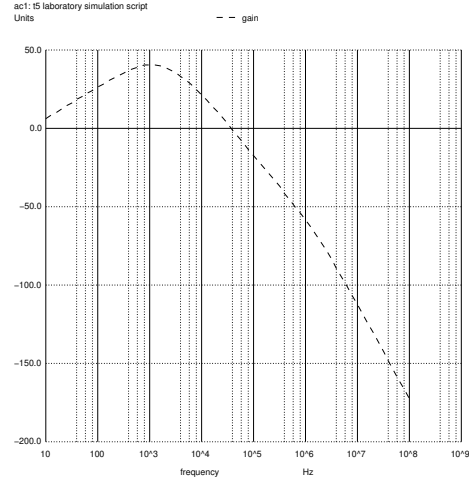
#### 3.1 Frequency Analysis

##### 3.1.1 Magnitude Response

Figure 4 shows the frequency magnitude response of the circuit, for the voltage gain, which is expressed in dB. Comparing to the theoretical graphic, which is on the right, we notice that the curves are very similar. We only notice the differences by taking a closer look to both curves, and see that the maximum voltage gain is not the same for the theoretical and simulated results. Both results approach very well the reference value, 40dB, and the differences between these two results might rely on the fact that NGSpice uses a real OPAMP model.



(a) Theoretical Phase Response

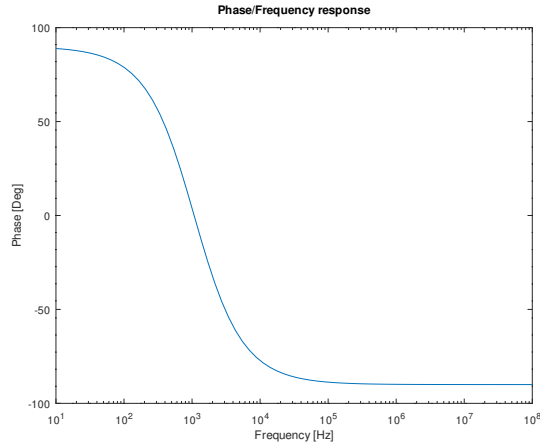


(b) Simulation Phase Response

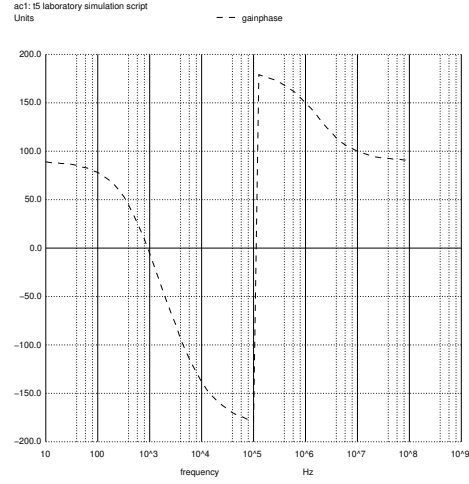
**Figure 4: Gain Response**

### 3.1.2 Phase Response

Figure 5 shows the frequency phase response of the circuit, for the voltage gain, which is expressed in degrees. Comparing to the theoretical graphic, which is on the right, we might consider that the curves are very different, since we see a sudden jump of phase, in the simulated graphic. However, if we take a closer look, we notice that the simulated graphic shows a jump of +180 degrees in the phase in order to fit the scale. If we ignore that necessary jump, we conclude that the theoretical curve only has 2 poles, while the simulated one has 4. That can be explained by the fact that NGSPice simulates a real OPAMP model, while Octave performs a theoretical analysis of the circuit, with an idealized OPAMP model.



(a) Theoretical Phase Response



(b) Simulation Phase Response

**Figure 5: Phase Response**

### 3.2 Simulation Results

Table 6 shows the simulated results, using NGSPICE, for the input and output impedances, at the central frequency. Remind that the values on the right are the theoretical ones obtained by Octave, which are reproduced in this section to help compare them to the simulated results. Comparing both the values, we conclude that the results are very close, some differences resulting on the fact that, on NGSpice, we used an approximate value of the central frequency to calculate the impedances and because in this simulating tool, it considers the components real (the OPAMP, most importantly).

Name	Ohm
zin	1.161878e+03

Name	Ohm
zout	8.265986e+02

Name	Value
$Z_{in}$	9.090909e+02 + j-6.741999e+02 Ohm
$Z_{out}$	6.451613e+02 + j-4.784644e+02 Ohm
$ Z_{in} $	1.131809e+03 Ohm
$ Z_{out} $	8.032193e+02 Ohm

**Table 6:** Comparison of Input and Output Impedances (Left - Simulation Values, Right - Theoretical Values)

Finally, Table 7 shows the values obtained for the gain and central frequency of the circuit under analysis. The table shows also the merit result and the values used for the calculation of the merit figure, which are the cost, the central frequency deviation (which should be ideally 1kHz) and the linearly voltage gain deviation (which should be ideally 100).

Name	Value
$AV_{HP}$	-1.903317e+00 dB
$AV_{OPAMP}$	4.440216e+01 dB
$AV_{LP}$	-1.903317e+00 dB
$AV$	4.059553e+01 dB

Name	Value
$w_L$	5.000000e+03 rad/s
$w_H$	9.090909e+03 rad/s
$w_O$	6.741999e+03 rad/s
$f_O$	1.073022e+03 Hz

Name	Value
Gain(dB)	40.4785
Gain	105.663
Central Frequency(Hz)	1021.94
Gain deviation	5.66302
Central frequency deviation(Hz)	21.938
Cost(MU)	13646.7
Merit	2.6549E-06

**Table 7:** Comparison of Gain (Left - Theoretical Values, Right - Simulation Values)

Comparing to the theoretical results, we can see that both results approach closely the reference values. However, we can notice some slight differences on the gain and the central frequency results, which can be explained by the fact that NGSpice simulates a real OPAMP model, while Octave performs a theoretical analysis of the circuit.

Lastly, as one can notice, we achieved a very low merit value. However, if we analyse the restrictions imposed on the uses of certain components and the results for the gain and the central frequency, which are quite close to the reference values, we conclude that, despite the low merit value, we achieved a very efficient circuit whose quality would degrade significantly if we changed randomly the values of the components to achieve the highest merit figure possibly.

## 4 Conclusion

In this laboratory assignment the objective of simulating a Band Pass Filter (BPF) using OPAMP has been achieved. The theoretical analysis was performed with the help of the Octave math tool and the circuit simulation using the Ngspice tool. For both analysis, we determined the gain, the input and the output impedances of the circuit using a central frequency, as well as, the gain and phase frequency response for an array of frequency values that range from 10 Hz to 100MHz, the results were also plotted for this values. At the end, we calculate the merit of our work.

This way, in theoretical analysis, we explained and compute the results obtained using an ideal OPAMP with the help of the math tool previously mentioned and the formulas that we learned in the theoretical lectures. And, in simulation analysis, we compare both the simulation and the theoretical results, explaining why the results are similar or differ.

As previously mentioned, the simulation results had slight differences from the theoretical ones. However, we designed an acceptable Band Pass Filter - furthermore, we believe that the objective of this assignment was achieved with great success.