Instituto Superior Técnico, University of Lisbon

Integrated Master in Aerospace Engineering

Circuit Theory and Electronics Fundamentals

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First Laboratory Report

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing a DC voltage source V_a, seven resistors, a voltage-controlled current source I_b and a current-controlled voltage source V_c. The components of this circuit are distribuited by 4 elementary meshes and 8 nodes, as seen in Figure 1.

In order to analyse the circuit, the following data were obtained by running the supplied Python script:

Units for the values: V, mA, kOhm and mS *Values:*
R1 = 1.00147062639
R2 = 2.0078068512
R3 = 3.11269704405
R4 = 4.10609573471
R5 = 3.02670672634

R6 = 2.01292455078
R7 = 1.02905244808
Va = 5.24842063411
Id = 1.04086013403
Kb = 7.21413591579
Kc = 8.01455113996

In Section 2, a theoretical analysis of the circuit, performed on Octave using the mesh and node methods, is presented. In Section 3, the circuit is analysed by simulation, using NGSpice, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

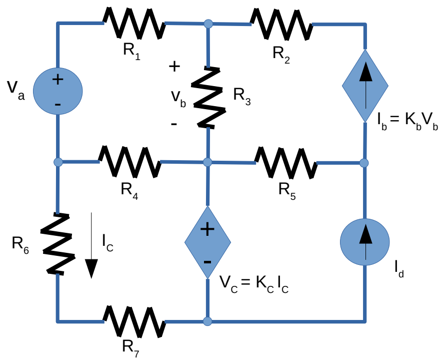

Figure 1: Circuit which will be analysed during this laboratory assignment.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically. We will begin by analyzing the circuit by applying the Mesh Method and, after that, we will analyze it again using the Nodal Method . In order to this, we will use Kirchhoff's circuit laws (KVL and KCL) together with Ohm's Law to obtain the theoretical results.

Note that we defined one of the nodes as GND, which is the central node of the circuit, having then randomly assigned numbers to the remaining ones. This will help us to later simulate the circuit and perform its analysis.

2.1 Mesh Method

— A mesh is a loop that contains no other loops. — The Mesh Method consists of assigning a current with a certain direction to each loop in a circuit, as shown in Figure 2, and then evaluating the circuit based on the new currents defined.

In this way, the KVL equations are applied to loops which do not have current sources or whose current is not known.

Equations obtained to:

Mesh A

$$-V_a + R_1 \cdot I_A + R_3 \cdot (I_A + I_B) + R_4 \cdot (I_A + I_C) = 0 \tag{1}$$

Mesh B

$$I_B = I_b = K_b \cdot V_b = K_b \cdot R_3 \cdot (I_A + I_B) \tag{2}$$

Mesh C

$$I_C = I_c \tag{3}$$

$$R_4 \cdot (I_C + I_A) + R_6 \cdot I_C + R_7 \cdot I_C - K_c \cdot I_C = 0 \tag{4}$$

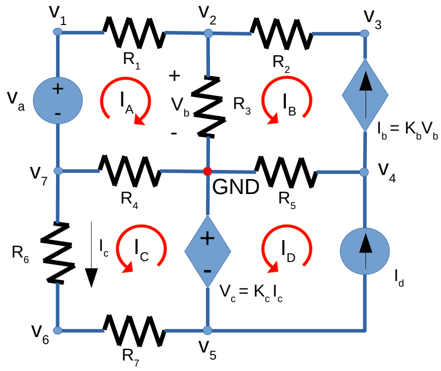

Figure 2: Circuit with each mesh current clockwise or counterclockwise direction assigned, to assist in the analysis of the circuit by the Mesh Method.

Mesh D

$$I_D = I_d \tag{5}$$

Then, the equations are solved in order to obtain the currents of each mesh.

Matricial Equation obtained:

$$\begin{bmatrix} R_1 + R_3 + R_4 & R_3 & R_4 & 0 \\ K_b \cdot R_3 & K_b \cdot R_3 - 1 & 0 & 0 \\ R_4 & 0 & R_4 + R_6 + R_7 - K_c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ I_d \end{bmatrix} \tag{6}$$

The solution of this matricial equation is determined by Octave:

Mesh Current	**Ampere[A]**
I_A	2.015673e-03
I_B	-2.109620e-03
I_C	-1.157872e-03
I_D	1.040860e-03

Table 1: Mesh Current Values

Knowing these currents, it is possible to determine any nodal voltages and branch currents, using Ohm's Law.

$$I_C = I_c \tag{7}$$

$$V_c = K_c \cdot I_c \tag{8}$$

$$V_b = R_3 \cdot (I_A + I_B) \tag{9}$$

$$I_b = K_b \cdot V_b \tag{10}$$

$$I_{R1} = I_A \tag{11}$$

$$I_{R2} = I_B \tag{12}$$

$$I_{R3} = I_A + I_B \tag{13}$$

$$I_{R4} = I_A + I_C \tag{14}$$

$$I_{R5} = I_B - I_D \tag{15}$$

$$I_{R6} = I_C \tag{16}$$

$$I_{R7} = I_C \tag{17}$$

Name	Value [A or V]
V_b	-2.924287e-01
I_b	-2.109620e-03
V_c	-9.279827e-06
I_c	-1.157872e-03
I_{R1}	2.015673e-03
I_{R2}	-2.109620e-03
I_{R3}	-9.394704e-05
I_{R4}	8.578007e-04
I_{R5}	-3.150480e-03
I_{R6}	-1.157872e-03
I_{R7}	-1.157872e-03

Table 2: Voltages and currents of some circuit components

2.2 Nodal Method

— Nodes connect components in a circuit — The Nodal Method consists of choosing a reference node against which all other voltages are measured. Through the KCL equations and other additional equations, it is possible to determine the voltage at each node.

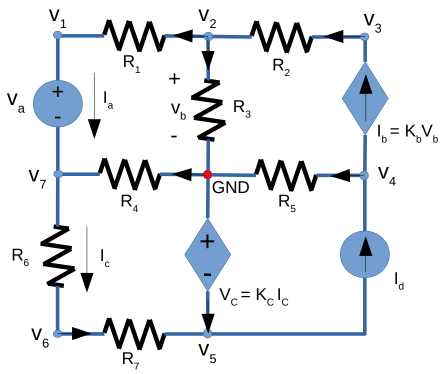

Figure 3: Circuit with each current direction arbitrarily assigned, to assist in the analysis of the circuit by the Nodal Method.

It was assigned potencial 0 to the central node in order to proceed with the analysis. Because of that, we can write:

$$V_b = V_2 - 0 = V_2 \tag{18}$$

$$I_b = K_b \cdot V_b = K_b \cdot V_2 \tag{19}$$

We first start by calculating the values of the conductances of the various resistors:

$$G_i = 1/R_i \tag{20}$$

Then we determine the equations obtained by applying KCL in nodes not connected to voltage sources:

Node number 2

$$-(V_2-V_1)\cdot G_1 - V_b\cdot G_3 + (V_3-V_2)\cdot G_2 = 0 \Leftrightarrow V_1\cdot G_1 + V_2\cdot(-G_1-G_2-G_3) + V_3\cdot G_2 = 0 \quad (21)$$

Node number 3

$$I_b - (V_3-V_2)\cdot G_2 = 0 \Leftrightarrow V_2(G_2+K_b) - G_2\cdot V_3 = 0 \quad (22)$$

Node number 4

$$-(V_4-0)\cdot G_5 + I_d - K_b\cdot V_2 = 0 \Leftrightarrow -K_b\cdot V_2 - V_4\cdot G_5 = -I_d \quad (23)$$

Node number 6

$$V_5\cdot G_7 + V_6\cdot(-G_6-G_7) + V_7\cdot G_6 = 0 \quad (24)$$

However, we need additional equations for nodes related to voltage sources:
Node number 1

$$(V_2-V_1)\cdot G_1 - I_a = 0 \Leftrightarrow I_a = (V_2-V_1)\cdot G_1 \quad (25)$$

Node number 7

$$V_1\cdot G_1 - G_1\cdot V_2 - G_6\cdot V_6 + V_7\cdot(G_4+G_6) = 0 \quad (26)$$

Equation (25)+(26)

$$V_1\cdot G_1 + V_2\cdot(-G_1) + V6\cdot(-G_6) + V_7\cdot(G_4+G_6) = 0 \quad (27)$$

Another additional Equations:

$$V_a = V_1 - V_7 \quad (28)$$

$$0 - V_5 = K_c\cdot G_6\cdot(V_7-V_6) \Leftrightarrow V_5 + V_6\cdot K_c\cdot(-G_6) + V_7\cdot K_c\cdot G_6 = 0 \quad (29)$$

Matricial equation obtained using the nodal method:

$$\begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & -G_6 & G_4+G_6 \\ G_1 & -G_1-G_2-G_3 & G_2 & 0 & 0 & 0 & 0 \\ 0 & G_2+K_b & -G_2 & 0 & 0 & 0 & 0 \\ 0 & K_b & 0 & G_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_7 & -G_6-G_7 & G_6 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -G_6\cdot K_c & G_6\cdot K_c \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_d \\ 0 \\ V_a \\ 0 \end{bmatrix} \quad (30)$$

The solution of this matricial equation is determined by Octave:

Node	**Voltage[V]**
V_1	1.726209e+00
V_2	-2.924287e-01
V_3	-4.528138e+00
V_4	9.535580e+00
V_5	9.279827e-06
V_6	-1.191502e+00
V_7	-3.522212e+00

Table 3: Nodal Voltages Values

Knowing these voltages, it is also possible to determine any components' voltages, by subtracting the voltages of each node where the component is connected, and branch currents, using Ohm's Law.

$$I_c = (V_7 - V_6) \cdot G_6 \tag{31}$$

$$V_c = K_c \cdot I_c \tag{32}$$

$$V_b = V_2 \tag{33}$$

$$I_b = K_b \cdot V_b \tag{34}$$

Name	Value [A or V]
V_b	-2.924287e-01
I_b	-2.109620e-03
V_c	-9.279827e-06
I_c	-1.157872e-03
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I_{R3}	-9.394704e-05
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I_{R5}	3.150480e-03
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I_{R7}	-1.157872e-03

Table 4: Voltages and currents of some circuit components

3 Simulation Analysis

3.1 Operating Point Analysis

Table 5 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, we notice that the simulation results are accurate, except for the last decimal places, as a consequence of the cientific notation and the number of significative algharisms used by each program to present the results. Despite that, we realise that the values with more significant algharisms (used in NGSpice) match correctly the rounded values (used in Octave).

In the table, we can see an nineth node (node 8), which has the same voltage value as node 6. This happens because, in NGSPice, when we want to simulate circuits with current-dependent sources, we must add a 0V voltage source in series to a component to sense the current flowing through it. Therefore, an aditional node appears in the simulated circuit.

Note that we can not perform aditional simulation analysis, namely transient and frequency ones, with phase and magnitude responses and input impedance, because the circuit does not have any electrical component which output is a function of time.

4 Conclusion

In this laboratory assignment the objective of analysing a circuit with resistors, voltage sources and current sources by applying the Nodal and Mesh methods together with Kirchhoff's Circuit Laws and Ohm's Law has been achieved. The theoreticall analysis was performed with the help of the Octave math tool and the circuit simulation using the Ngspice tool. The simulation results matched the theoretical results perfectly. This happens mainly because the method used to obtain the theoretical results is the same method Ngspice uses to emulate the described circuit. Another reason why the results match perfectly is that the circuit was composed by simple

Name	Value [A or V]
@gb[i]	-2.10962e-03
@id[current]	1.040860e-03
@r1[i]	-2.01567e-03
@r2[i]	-2.10962e-03
@r3[i]	-9.39470e-05
@r4[i]	8.578007e-04
@r5[i]	3.150480e-03
@r6[i]	-1.15787e-03
@r7[i]	-1.15787e-03
v1	1.726209e+00
v2	-2.92429e-01
v3	-4.52814e+00
v4	9.535580e+00
v5	9.279827e-06
v6	-1.19150e+00
v7	-3.52221e+00
v8	-1.19150e+00

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

components.For more complex components, the theoretical and simulation models may differ but it is not the case in this assignment.