

Monitoria: Cálculo Numérico (EPET-019A)

Data: 27/04/2021

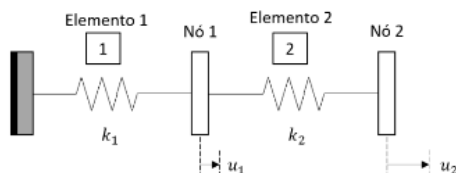
- **Monitores:**
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- **Assuntos abordados:**
 - Dúvidas sobre o trabalho da AB1 (SEL)
- **Links úteis:**
 - [Operações com matrizes em Julia](#)
 - [MATLAB-Python-Julia cheatsheet](#)

Como utilizar os valores tabelados?

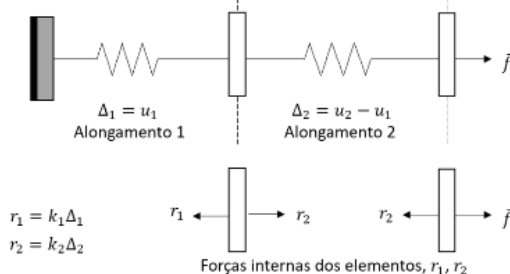
Nome do aluno ou dupla	Rigidez de cada mola dividida pelo número de molas, k_i/n	Valor da carga externa aplicada, \bar{f}
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Sistema mecânico discreto

Configuração indeformada



Configuração deformada



Para 2 graus de liberdade (2 nós livres)

Determinação dos deslocamentos

- Com base nas forças internas dos elementos (r_1, r_2) e na força externa aplicada no nó da extremidade livre (\bar{f}), faz-se o equilíbrio de forças em cada nó

$$r_1 - r_2 = 0$$

$$r_2 - \bar{f} = 0$$

- Expressando as forças internas em função dos deslocamentos, tem-se

$$r_1 - r_2 = 0 \Rightarrow k_1 u_1 - k_2 (u_2 - u_1) = 0$$

$$r_2 - \bar{f} = 0 \Rightarrow k_2 (u_2 - u_1) - \bar{f} = 0$$

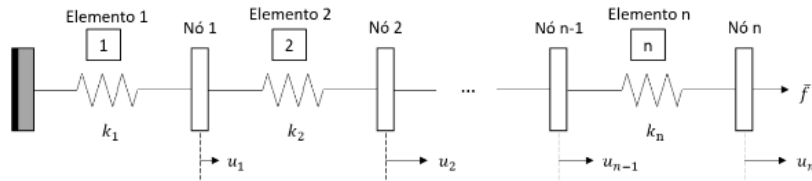
- Utilizando o formato matricial, as equações acima podem ser expressas como

$$\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{f} \end{bmatrix}$$

Sistema mecânico discreto

Para n graus de liberdade (n nós livres)

Generalizando para um sistema com n elementos em série e com carga aplicada na extremidade livre



Determinação da rigidez equivalente do sistema

$$\Delta = \frac{1}{K_{eq}} f \Rightarrow \frac{1}{K_{eq}} = \left[\frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n} \right]$$

Determinação dos deslocamentos

$$\begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \dots & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \dots & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & \dots & 0 & 0 & 0 \\ 0 & 0 & -k_4 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -k_{n-2} & 0 & 0 \\ 0 & 0 & 0 & \dots & k_{n-2} + k_{n-1} & -k_{n-1} & 0 \\ 0 & 0 & 0 & \dots & -k_{n-1} & k_{n-1} + k_n & -k_n \\ 0 & 0 & 0 & \dots & 0 & -k_n & k_n \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_{n-2} \\ u_{n-1} \\ u_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ f \end{bmatrix}$$

Por exemplo, para o sistema com 5 elementos (molas)

```
In [1]: n = 5
s, f = 1.0e+7, 2.0e+5 # Valores lidos da tabela
k = s * n # Valor da rigidez de cada mola
```

```
Out[1]: 5.0e7
```

```
In [2]: # Montando o vetor b de constante
b = [0, 0, 0, 0, f]
# ou
b = zeros(n)
b[n] = f
b
```

```
Out[2]: 5-element Vector{Float64}:
 0.0
 0.0
 0.0
 0.0
200000.0
```

```
In [3]: # Montando a matriz A dos coeficientes
k1, k2, k3, k4, k5 = k, k, k, k, k

A = zeros(n, n)
# Acumular contribuição do primeiro elemento
A[1, 1] = A[1, 1] + k1
# Acumular contribuição do segundo elemento
i, j = 1, 2
A[i, i] += k2
A[i, j] -= k2
A[j, i] -= k2
A[j, j] += k2
# Acumular contribuição do terceiro elemento
i, j = 2, 3
A[i, i] += k3
A[i, j] -= k3
A[j, i] -= k3
A[j, j] += k3
```

```
# Acumular contribuição do quarto elemento
i, j = 3, 4
A[i, i] += k4
A[i, j] -= k4
A[j, i] -= k4
A[j, j] += k4
# Acumular contribuição do quinto elemento
i, j = 4, 5
A[i, i] += k5
A[i, j] -= k5
A[j, i] -= k5
A[j, j] += k5

A
```

```
Out[3]: 5x5 Matrix{Float64}:
 1.0e8 -5.0e7  0.0    0.0    0.0
-5.0e7  1.0e8 -5.0e7  0.0    0.0
 0.0   -5.0e7  1.0e8 -5.0e7  0.0
 0.0    0.0   -5.0e7  1.0e8 -5.0e7
 0.0    0.0    0.0   -5.0e7  5.0e7
```

```
In [4]: # Resolvendo o sistema
u5 = A \ b
```

```
Out[4]: 5-element Vector{Float64}:
 0.003999999999999999
 0.007999999999999998
 0.011999999999999997
 0.015999999999999997
 0.02
```

```
In [5]: # Usando uma estrutura de repetição (Loops)
soma = 0.
for i = 1:6
    soma += i # soma = soma + i
end
soma
```

```
Out[5]: 21.0
```

Como calcular a norma de um vetor

```
In [6]: x = rand(5)
```

```
Out[6]: 5-element Vector{Float64}:
 0.2382945653567623
 0.6041705024299904
 0.7804322527233665
 0.15761489697658448
 0.12343442401983729
```

```
In [7]: nL2 = 0. # norma L2 de um vetor
for x_ in x
    nL2 += x_^2
end
nL2 = sqrt(nL2)
```

```
Out[7]: 1.0348716392664319
```

```
In [8]: using LinearAlgebra
        norm(x, 2), norm(x)
```

```
Out[8]: (1.0348716392664319, 1.0348716392664319)
```

Como calcular a norma de uma matriz

```
In [9]: ?norm
```

```
search: norm normpath normalize normalize! opnorm issubnormal UniformScaling
```

```
Out[9]: norm(A, p::Real=2)
```

For any iterable container A (including arrays of any dimension) of numbers (or any element type for which `norm` is defined), compute the p -norm (defaulting to $p=2$) as if A were a vector of the corresponding length.

The p -norm is defined as

$$\|A\|_p = \left(\sum_{i=1}^n |a_i|^p \right)^{1/p}$$

with a_i the entries of A , $|a_i|$ the norm of a_i , and n the length of A . Since the p -norm is computed using the norms of the entries of A , the p -norm of a vector of vectors is not compatible with the interpretation of it as a block vector in general if $p \neq 2$.

p can assume any numeric value (even though not all values produce a mathematically valid vector norm). In particular, `norm(A, Inf)` returns the largest value in `abs.(A)`, whereas `norm(A, -Inf)` returns the smallest. If A is a matrix and $p=2$, then this is equivalent to the Frobenius norm.

The second argument p is not necessarily a part of the interface for `norm`, i.e. a custom type may only implement `norm(A)` without second argument.

Use `opnorm` to compute the operator norm of a matrix.

Examples

```
jldoctest
julia> v = [3, -2, 6]
3-element Vector{Int64}:
 3
-2
 6
```

```
julia> norm(v)
7.0
```

```
julia> norm(v, 1)
11.0
```

```
julia> norm(v, Inf)
6.0
```

```
julia> norm([1 2 3; 4 5 6; 7 8 9])
16.881943016134134

julia> norm([1 2 3 4 5 6 7 8 9])
16.881943016134134

julia> norm(1:9)
16.881943016134134

julia> norm(hcat(v,v), 1) == norm(vcat(v,v), 1) != norm([v,v], 1)
true

julia> norm(hcat(v,v), 2) == norm(vcat(v,v), 2) == norm([v,v], 2)
true

julia> norm(hcat(v,v), Inf) == norm(vcat(v,v), Inf) != norm([v,v], Inf)
true
```

```
norm(x::Number, p::Real=2)
For numbers, return  $(|x|^p)^{1/p}$ .
```

Examples

```
jldoctest
julia> norm(2, 1)
2.0

julia> norm(-2, 1)
2.0

julia> norm(2, 2)
2.0

julia> norm(-2, 2)
2.0

julia> norm(2, Inf)
2.0

julia> norm(-2, Inf)
2.0
```

In [10]:

```
?opnorm
```

search: **opnorm**

Out[10]:

```
opnorm(A::AbstractMatrix, p::Real=2)
Compute the operator norm (or matrix norm) induced by the vector  $p$ -norm, where valid
values of  $p$  are  $1$ ,  $2$ , or  $\text{Inf}$ . (Note that for sparse matrices,  $p=2$  is currently not
implemented.) Use norm to compute the Frobenius norm.
```

When $p=1$, the operator norm is the maximum absolute column sum of A :

$$\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^m |a_{ij}|$$

with a_{ij} the entries of A , and m and n its dimensions.

When $p=2$, the operator norm is the spectral norm, equal to the largest singular value of A .

When $p=\text{Inf}$, the operator norm is the maximum absolute row sum of A :

$$\|A\|_\infty = \max_{1 \leq i \leq m} \sum_{j=1}^n |a_{ij}|$$

Examples

```
jldoctest
julia> A = [1 -2 -3; 2 3 -1]
2×3 Matrix{Int64}:
 1  -2  -3
 2   3  -1
```

```
julia> opnorm(A, Inf)
6.0
```

```
julia> opnorm(A, 1)
5.0
```

```
opnorm(x::Number, p::Real=2)
```

For numbers, return $(|x|^p)^{1/p}$. This is equivalent to `norm`.

```
opnorm(A::Adjoint{<:Any,<:AbstractVector}, q::Real=2)
```

```
opnorm(A::Transpose{<:Any,<:AbstractVector}, q::Real=2)
```

For Adjoint/Transpose-wrapped vectors, return the operator q -norm of A , which is equivalent to the p -norm with value $p = q/(q-1)$. They coincide at $p = q = 2$. Use `norm` to compute the p -norm of A as a vector.

The difference in norm between a vector space and its dual arises to preserve the relationship between duality and the dot product, and the result is consistent with the operator p -norm of a $1 \times n$ matrix.

Examples

```
jldoctest
julia> v = [1; im];
```

```
julia> vc = v';
```

```
julia> opnorm(vc, 1)
1.0
```

```
julia> norm(vc, 1)
```

2.0

```
julia> norm(v, 1)
2.0
```

```
julia> opnorm(v, 2)
1.4142135623730951
```

```
julia> norm(v, 2)
1.4142135623730951
```

```
julia> norm(v, 2)
1.4142135623730951
```

```
julia> opnorm(v, Inf)
2.0
```

```
julia> norm(v, Inf)
1.0
```

```
julia> norm(v, Inf)
1.0
```

```
In [11]: A = rand(10, 10)
```

```
Out[11]: 10×10 Matrix{Float64}:
 0.0321996  0.848028      0.246259    ...  0.187193  0.115787  0.0375074
 0.764159  0.494211      0.325979    ...  0.740394  0.288967  0.717975
 0.719314  0.000133823    0.394789    ...  0.914745  0.845386  0.654728
 0.464919  0.329652      0.330919    ...  0.197058  0.724115  0.0996294
 0.51432   0.00505249    0.904945    ...  0.946565  0.57118  0.841934
 0.665503  0.478294      0.971108    ...  0.971047  0.566544  0.417113
 0.534591  0.0185831     0.807919    ...  0.413103  0.786164  0.690539
 0.376258  0.80084       0.0591936   ...  0.507808  0.299568  0.558993
 0.338805  0.137231      0.565251    ...  0.583319  0.428675  0.397012
 0.736682  0.675626      0.763323    ...  0.556122  0.324425  0.907104
```

```
In [12]: norm(A, 2)
```

```
Out[12]: 5.696711128657197
```

```
In [13]: opnorm(A, 2)
```

```
Out[13]: 5.129693176207508
```

Como medir tempo computacional

```
In [14]: A = rand(10, 10)
         b = rand(10)

         # Resolver sistema A * x = b
         @time x = A \ b
         @time x = A \ b
         @time x = A \ b
         @time x = A \ b

         t = @elapsed x = A \ b
```

```
println("t = ", t, " secs.")

t5 = @elapsed begin
    x = A \ b
    x = A \ b
    x = A \ b
    x = A \ b
    x = A \ b
end
println("t5 = ", t5, " secs.")
```

```
0.000013 seconds (3 allocations: 1.188 KiB)
0.000013 seconds (3 allocations: 1.188 KiB)
0.000013 seconds (3 allocations: 1.188 KiB)
0.000015 seconds (3 allocations: 1.188 KiB)
t = 1.13e-5 secs.
t5 = 2.18e-5 secs.
```

Como construir gráficos de barras

```
In [15]: using StatsPlots

xlabel = ["n 1", "n 2", "n 3"]
groups = ["Grupo 1", "Grupo 2", "Grupo 3"]

nlabels, ngroups = length(xlabel), length(groups)
results = rand(nlabels, ngroups)

name = repeat(xlabel, outer=ngroups)
leg = repeat(groups, inner=nlabels)
groupedbar(name, results, group=leg, xlabel="O que estou variando", ylabel="Resultados obtidos")
```

