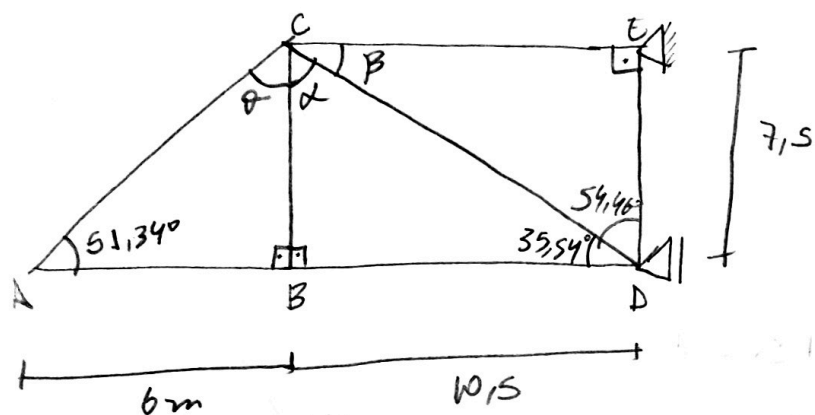


# Resoluções ABZPR

1) Em posse das cotas, calcular-se os ângulos presentes nas barras:



$$\theta = \tan^{-1} \left( \frac{6}{7,5} \right) = 38,66^\circ$$

$$\alpha = \tan^{-1} \left( \frac{10,5}{7,5} \right) = 54,96^\circ$$

$$\beta = \tan^{-1} \left( \frac{7,5}{10,5} \right) = 35,54^\circ$$

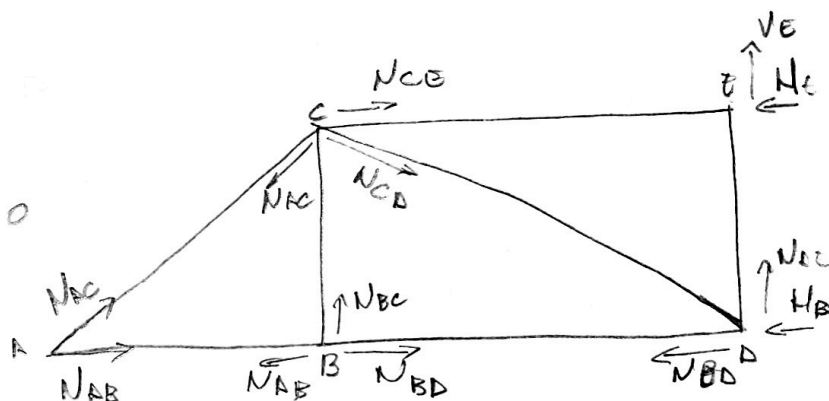
Pelo método dos nós pode-se encontrar as tensões atuantes nas barras; Logo:

Nó D:

$$\sum F_x = 0 \therefore -N_D - N_{BD} - N_{CD} \cdot \cos(35,54^\circ) = 0$$

$$\sum F_y = 0 \therefore N_{DE} + N_{CD} \cdot \sin(54,96^\circ) = 0$$

$$N_{CD} = 237,5 \cdot 0,58 = 137,00 \text{ KN}$$



Nó B:

$$\sum F_x = 0 \therefore N_{BD} = N_{AB} \Rightarrow N_{BD} = -41,623,7 \text{ KN}$$

$$\sum F_y = 0 \therefore N_{CB} - 85 = 0 \therefore N_{CB} = 85 \text{ KN}$$

Nó A:

$$\sum F_x = 0 \therefore N_{AB} + N_{AC} \cdot \cos(51,34^\circ) + P = 0 \Rightarrow N_{AB} = -41,623,7 + P$$

$$\sum F_y = 0 \therefore N_{AC} \cdot \sin(51,34^\circ) - 52 = 0 \Rightarrow N_{AC} = 66,6 \text{ KN}$$

Nó C:

$$\sum F_x = 0 \therefore N_{CE} - N_{AC} \cdot \cos(38,66^\circ) + N_{CD} \cdot \cos(35,54^\circ) = 0 \Rightarrow N_{CE} = 41,6 + 191791,43$$

$$\sum F_y = 0 \therefore -N_{AC} \cdot \sin(38,66^\circ) - N_{CB} - N_{CD} \cdot \cos(54,96^\circ) = 0$$

$$-52004,16 - 85000 = 0,58 N_{CD} \Rightarrow N_{CD} = -235,7 \text{ KN}$$

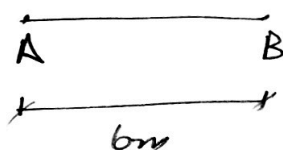
Ver B

$$\sum F_x = 0 \therefore -N_B - N_{CE} = 0 \Rightarrow \underline{N_E = -233,39 \text{ kN}}$$

$$\sum F_y = 0 \therefore V_E - N_{DE} = 0$$

calculando para os barras, tem-se que:

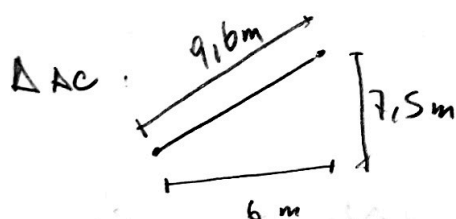
$\Delta_B$ :



$$\Delta_{AB} = \int_0^6 \frac{-41623,70 - P}{EA} (-1) dx = \int_0^6 \frac{41623,70 + P}{EA} dx$$

$$= \left( \frac{41623,70 + P}{EA} \right) x \Big|_0^6 = - \frac{41623,70 \cdot 0}{200 \cdot 10^9 \cdot 0,0016} + \frac{41623,70 \cdot 6}{200 \cdot 10^9 \cdot 0,0016} = 0,0008 \text{ m}$$

Obs: Uma vez que  $P=0!!$



$$\Delta_{AC} = \int_0^{9,6} \frac{66598}{EA} \cdot 0 dx = 0, \text{ mais } \frac{\partial N_{AC}}{\partial P} = 0$$

$$\Delta_{BD} = \int_0^{10,5} \frac{-41623,70 - P}{EA} (-1) dx = \int_0^{10,5} \frac{41623,70 + P}{EA} = \left( \frac{41623,70 + 0}{EA} \right) x \Big|_0^{10,5}$$

$$= \frac{41623,70}{200 \cdot 10^9 \cdot 0,0016} \cdot 10,5 - \frac{41623,70}{200 \cdot 10^9 \cdot 0,0016} \cdot 0 \Rightarrow \underline{\Delta_{BD} = 0,0014 \text{ m}}$$

$$\Delta_{CD} = \int_0^{12,7} \frac{-235700}{E \cdot A} \cdot 0 \cdot dx \Rightarrow \underline{\Delta_{CD} = 0} \quad \frac{\partial \Delta_{CD}}{\partial P} = 0$$

$$\Delta_{DE} = \int_0^{7,5} \frac{137005}{E \cdot A} \cdot 0 \cdot dx \Rightarrow \underline{\Delta_{DE} = 0} \quad \frac{\partial \Delta_{DE}}{\partial P} = 0$$

$$\Delta_{CE} = \int_0^{10,5} \frac{233395}{E \cdot A} \cdot 0 \cdot dx = \underline{0} \quad \frac{\partial \Delta_{CE}}{\partial P} = 0$$

$$\Delta_{BC} = \int_0^{7,5} \frac{85000}{E \cdot A} \cdot 0 \cdot dx = 0 \quad \frac{\partial \Delta_{BC}}{\partial P} = 0$$

$$\Delta_A = \sum \Delta_i \Rightarrow \Delta_A = \Delta_{AB} + \Delta_{AC} + \Delta_{BC} + \Delta_{BD} + \Delta_{CD} + \Delta_{CE} + \Delta_{DE}$$

$$\Delta_A = 0,0008 + 0,0014 + 0 + 0 + 0 + 0 + 0 = \underline{0,0022 \text{ m}}$$

Pelo fato da força  $P$  ter o sentido adotado como positivo para a direita ( $\rightarrow$ ), no ponto B, o deslocamento será de 2,2 mm para a direita.

Dividindo o problema em planos de trabalho,

No plano  $xz$ , tem-se que:

considerando  $K=2$

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_y}{(K \cdot l)_y^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 \cdot 18,4 \cdot 10^{-9}}{(2 \cdot 0,7)^2_y} = \underline{915,10 \text{ kN}}$$

No plano  $xy$ :

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_z}{(K \cdot l)_z^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 \cdot 128 \cdot 10^{-6}}{(2 \cdot 9)^2_z} = \underline{779,82 \text{ kN}}$$

Adota-se, então, o valor de  $P_{cr}$  igual a 779,82 kN.

Calculando a tensão crítica:

$$\sigma_{cr} = \frac{\pi^2 \cdot E}{\left(\frac{K \cdot l}{r}\right)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9}{\left(\frac{18}{130 \cdot 10^{-3}}\right)^2} = \underline{102,9 \text{ MPa}}$$

Índice de esbeltez:

$$\lambda_z = \frac{l_z}{r_z} = \frac{K \cdot l}{r_z} = \frac{2 \cdot 9 \cdot 10^3}{130} = \underline{138,5}$$

Como  $\sigma_{cr} < \sigma_{adm}$ , o  $P_{cr}$  adotado se confirma.

Por critério de estabilidade:

$$\sigma \leq \frac{\sigma_{cr}}{n_p} \Rightarrow \frac{P}{A} \leq \frac{P_{cr}}{A_{np}}; \quad P \leq \frac{779,82}{2} \leq \underline{389,91 \text{ kN}}$$