Universidade Federal de Alagoas - UFAL
Centro de Tecnologia - CTEC
Curso de Engenharia Civil

Mecânica dos Sólidos 3 - ECIV051D (2020.2)

Exercícios:
Deslocamentos em vigas isostáticas usando o método da superposição
Encontro Assíncrono

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Maceió/AL, 23/07/2021

Exemplo 9-6
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

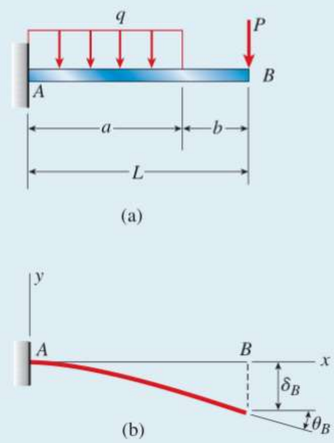
Example 9-6

A cantilever beam AB supports a uniform load of intensity q acting over part of the span and a concentrated load P acting at the free end (Fig. 9-18a).

Determine the deflection δ_B and angle of rotation θ_B at end B of the beam (Fig. 9-18b). (*Note:* The beam has length L and constant flexural rigidity EI.)

Fig. 9-18

Example 9-6: Cantilever beam with a uniform load and a concentrated load



restart :

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 2 e 4 da Tabela G-1.

$$\delta_{B2} := (q, a, L, EI) \rightarrow \frac{q \cdot a^3}{24 \cdot EI} \cdot (4 \cdot L - a) :$$

$$\theta_{B2} := (q, a, EI) \rightarrow \frac{q \cdot a^3}{6 \cdot EI} :$$

$$\delta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^3}{3 \cdot EI} :$$

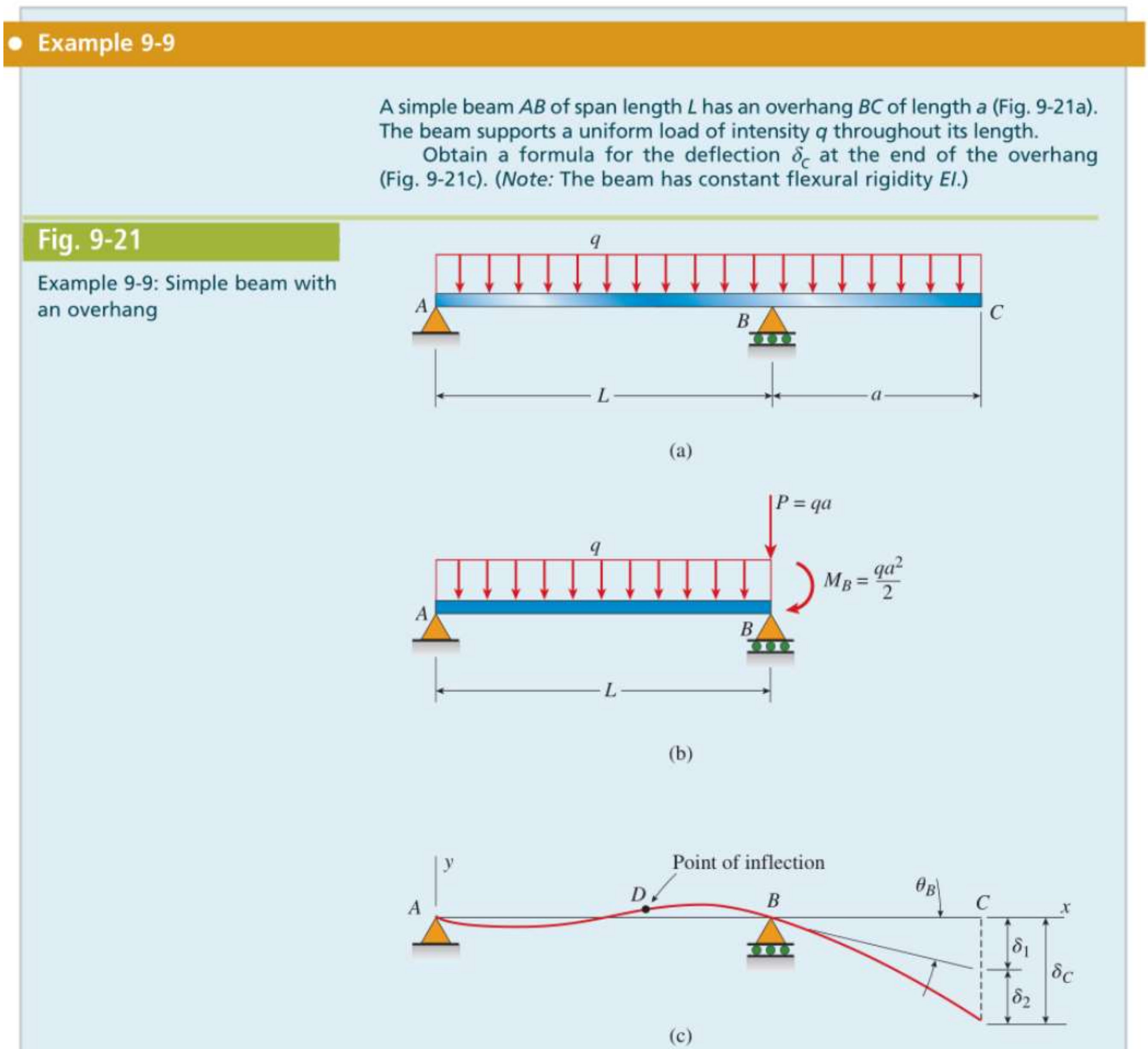
$$\theta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^2}{2 \cdot EI} :$$

Usando o método da superposição, tem-se

$$\delta_B := \delta_{B2}(q, a, L, EI) + \delta_{B4}(P, L, EI) = \frac{1}{24} \frac{q\, a^3\, (4\, L - a)}{EI} + \frac{1}{3} \frac{P\, L^3}{EI}$$

$$\theta_B := \theta_{B2}(q, a, EI) + \theta_{B4}(P, L, EI) = \frac{1}{6} \frac{q\, a^3}{EI} + \frac{1}{2} \frac{P\, L^2}{EI}$$

Exemplo 9-9
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.


Example 9-9

A simple beam AB of span length L has an overhang BC of length a (Fig. 9-21a). The beam supports a uniform load of intensity q throughout its length.

Obtain a formula for the deflection δ_C at the end of the overhang (Fig. 9-21c). (*Note:* The beam has constant flexural rigidity EI.)

Fig. 9-21

Example 9-9: Simple beam with an overhang

restart :

1) Considerando o trecho biapoiado AB com o binário da carga $P = q \cdot a$
Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 1 e 7 da Tabela G-2.

$$\theta_{B1} := (q, L, EI) \rightarrow \frac{q \cdot L^3}{24 \cdot EI} :$$

$$\theta_{A7} := (M_0, L, EI) \rightarrow \frac{M_0 \cdot L}{3 \cdot EI} :$$

$$\theta_B := -\theta_{B1}(q, L, EI) + \theta_{A7}\left(\frac{q \cdot a^2}{2}, L, EI\right) = -\frac{1}{24} \frac{q L^3}{EI} + \frac{1}{6} \frac{L q a^2}{EI}$$

$$\delta_1 := \theta_B \cdot a = \left(-\frac{1}{24} \frac{q L^3}{EI} + \frac{1}{6} \frac{L q a^2}{EI}\right) a$$

2) Considerando o trecho engastado e livre BC

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 1 da Tabela G-1.

$$\delta_{B1} := (q, L, EI) \rightarrow \frac{q \cdot L^4}{8 \cdot EI} :$$

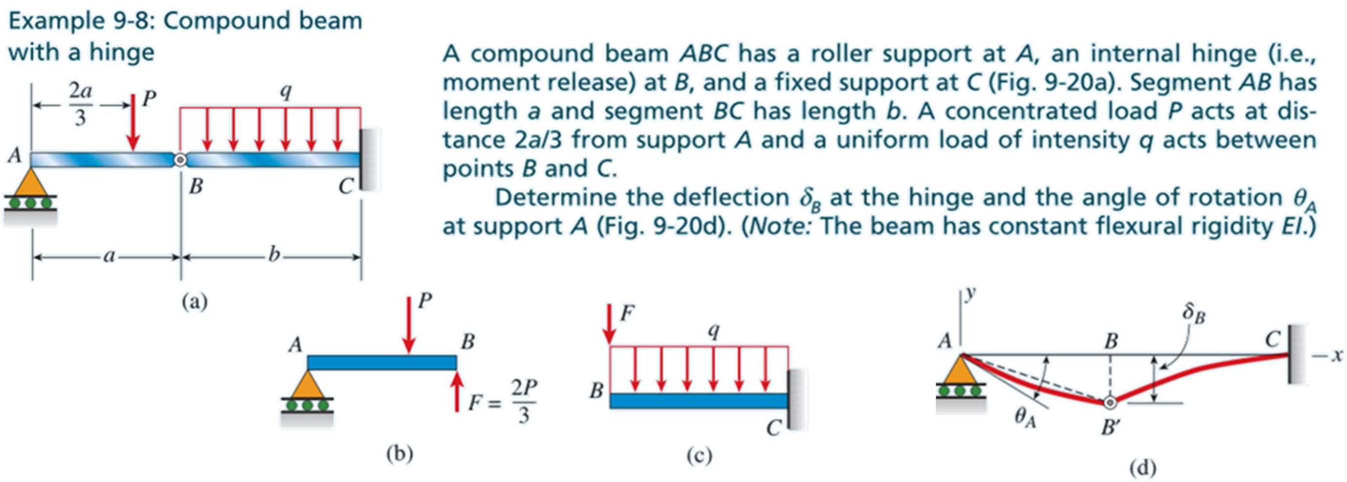
$$\delta_2 := \delta_{B1}(q, a, EI) = \frac{1}{8} \frac{q\, a^4}{EI}$$

1+2) Usando o método da superposição e assumindo deflexão nula em C, tem-se

$$\delta_C := \delta_1 + \delta_2 = \left(-\frac{1}{24} \frac{q\, L^3}{EI} + \frac{1}{6} \frac{L\, q\, a^2}{EI} \right) a + \frac{1}{8} \frac{q\, a^4}{EI}$$

$$simplify\left(\delta_C - \frac{q \cdot a}{24 \cdot EI} \cdot \left(L \cdot \left(4 \cdot a^2 - L^2 \right) + 3 \cdot a^3 \right) \right) = 0$$

Exemplo 9-8
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.


Example 9-8: Compound beam with a hinge

A compound beam *ABC* has a roller support at *A*, an internal hinge (i.e., moment release) at *B*, and a fixed support at *C* (Fig. 9-20a). Segment *AB* has length *a* and segment *BC* has length *b*. A concentrated load *P* acts at distance $2a/3$ from support *A* and a uniform load of intensity *q* acts between points *B* and *C*.

Determine the deflection δ_B at the hinge and the angle of rotation θ_A at support *A* (Fig. 9-20d). (*Note:* The beam has constant flexural rigidity *EI*.)

restart :

$$F := solve\left(-P\cdot\frac{2\cdot a}{3} + F\cdot a = 0, F\right) = \frac{2}{3}\,P$$

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 1 e 4 da Tabela G-1.

$$\delta_{B1} := (q, L, EI) \rightarrow \frac{q\cdot L^4}{8\cdot EI} :$$

$$\delta_{B4} := (P, L, EI) \rightarrow \frac{P\cdot L^3}{3\cdot EI} :$$

Usando o método da superposição, determina-se a deflexão em B

$$\delta_B := \delta_{B1}(q, b, EI) + \delta_{B4}(F, b, EI) = \frac{1}{8}\,\frac{q\,b^4}{EI} + \frac{2}{9}\,\frac{P\,b^3}{EI}$$

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 5 da Tabela G-2.

$$\theta_{A5} := (P, a, b, L, EI) \rightarrow \frac{P\cdot a\cdot b\cdot (L + b)}{6\cdot L\cdot EI} :$$

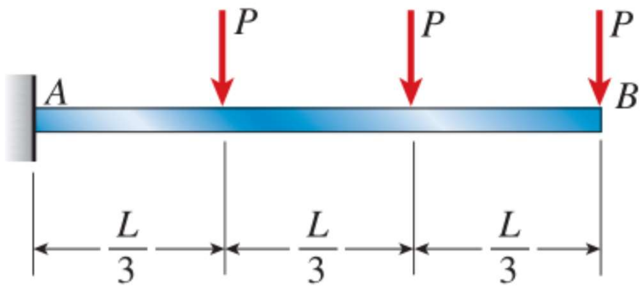
Usando o método da superposição, determina-se a rotação em A

$$\theta_{A,\,2} := \theta_{A5}\left(P, \frac{2\cdot a}{3}, \frac{a}{3}, a, EI\right) = \frac{4}{81}\,\frac{a^2\,P}{EI}$$

$$\theta_A := expand\left(\frac{\delta_B}{a} + \theta_{A,\,2}\right) = \frac{1}{8}\,\frac{q\,b^4}{a\,EI} + \frac{2}{9}\,\frac{P\,b^3}{a\,EI} + \frac{4}{81}\,\frac{a^2\,P}{EI}$$

Problema 9.5-1
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-1 A cantilever beam AB carries three equally spaced concentrated loads, as shown in the figure. Obtain formulas for the angle of rotation θ_B and deflection δ_B at the free end of the beam.



PROB. 9.5-1

restart :

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 4 e 5 da Tabela G-1.

$$\delta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^3}{3 \cdot EI} :$$

$$\theta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^2}{2 \cdot EI} :$$

$$\delta_{B5} := (P, a, L, EI) \rightarrow \frac{P \cdot a^2}{6 \cdot EI} \cdot (3 \cdot L - a) :$$

$$\theta_{B5} := (P, a, EI) \rightarrow \frac{P \cdot a^2}{2 \cdot EI} :$$

Usando o método da superposição, tem-se

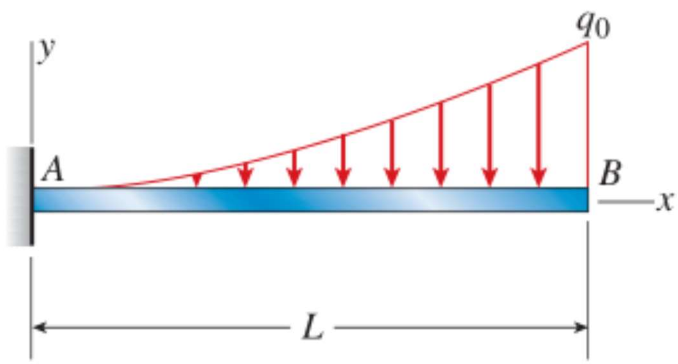
$$\theta_B := \theta_{B5}\left(P, \frac{L}{3}, EI\right) + \theta_{B5}\left(P, \frac{2 \cdot L}{3}, EI\right) + \theta_{B4}(P, L, EI) = \frac{7}{9} \frac{P L^2}{EI}$$

$$\delta_B := \delta_{B5}\left(P, \frac{L}{3}, L, EI\right) + \delta_{B5}\left(P, \frac{2 \cdot L}{3}, L, EI\right) + \delta_{B4}(P, L, EI) = \frac{5}{9} \frac{P L^3}{EI}$$

Problema 9.5-13

Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-13 Determine the angle of rotation θ_B and deflection δ_B at the free end of a cantilever beam AB supporting a parabolic load defined by the equation $q(x) = q_0 x^2/L^2$ (see figure).



PROB. 9.5-13

restart :

$$q := x \rightarrow q_0 \cdot \frac{x^2}{L^2} :$$

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 5 da Tabela G-1.

$$\delta_{B5} := (P, a, L, EI) \rightarrow \frac{P \cdot a^2}{6 \cdot EI} \cdot (3 \cdot L - a) :$$

$$\theta_{B5} := (P, a, EI) \rightarrow \frac{P \cdot a^2}{2 \cdot EI} :$$

Usando o método da superposição e integrando para $0 \leq x \leq L$, tem-se

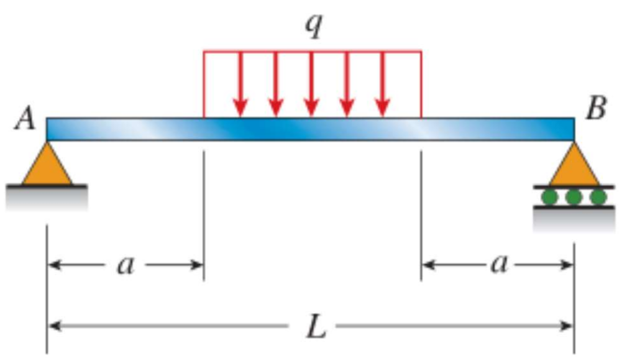
$$\theta_B := \int_0^L \theta_{B5}(q(x), x, EI) \, dx = \frac{1}{10} \frac{q_0 L^3}{EI}$$

$$\delta_B := \int_0^L \delta_{B5}(q(x), x, L, EI) \, dx = \frac{13}{180} \frac{q_0 L^4}{EI}$$

Problema 9.5-14
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-14 A simple beam AB supports a uniform load of intensity q acting over the middle region of the span (see figure).

Determine the angle of rotation θ_A at the left-hand support and the deflection δ_{max} at the midpoint.



PROB. 9.5-14

restart :

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 5 da Tabela G-2.

$$\theta_{A5} := (P, x, b, L, EI) \rightarrow \frac{P \cdot x \cdot b \cdot (L + b)}{6 \cdot L \cdot EI} :$$

$$\delta_{C5} := (P, x, L, EI) \rightarrow \frac{P \cdot x \cdot (3 \cdot L^2 - 4 \cdot x^2)}{48 \cdot EI} :$$

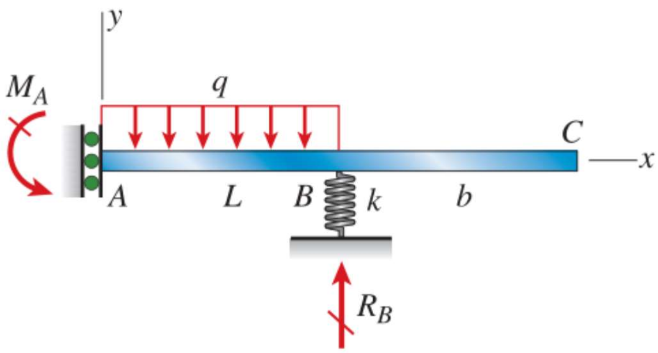
Usando o método da superposição e integrando para $a \leq x \leq L - a$, tem-se

$$\theta_A := simplify\left(\int_a^{L-a} \theta_{A5}(q, x, L - x, L, EI)\, dx\right) = \frac{1}{24} \frac{q\,(L^3 - 6\,L\,a^2 + 4\,a^3)}{EI}$$

$$\delta_{max} := simplify\left(2 \cdot \int_a^{\frac{L}{2}} \delta_{C5}(q, x, L, EI)\, dx\right) = \frac{1}{384} \frac{q\,(5\,L^4 - 24\,L^2\,a^2 + 16\,a^4)}{EI}$$

Problema 9.5-17
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-17 An overhanging beam ABC with flexural rigidity $EI = 45\ \mathrm{N \cdot m^2}$ is supported by a guided support at A and by a spring of stiffness k at point B (see figure). Span AB has length $L = 0.75$ m and carries a uniform load. The overhang BC has length $b = 375$ mm. For what stiffness k of the spring will the uniform load produce no deflection at the free end C?



PROB. 9.5-17

restart :
Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 1 da Tabela G-2.

$$\theta_{B1} := (q, L, EI) \rightarrow \frac{q \cdot L^3}{24 \cdot EI} :$$

1) Considerando um apoio simples em B
Trecho AB: metade de uma viga biapoiada com carregamento uniforme (cortante nula em A)

$$\delta_{C1} := \theta_{B1}(q, 2 \cdot L, EI) \cdot b = \frac{1}{3} \frac{q L^3 b}{EI}$$

2) Considerado o efeito do apoio elástico

$$R_B := q \cdot L :$$

$$\delta_B := solve\left(k \cdot \delta_B = R_B, \delta_B\right) = \frac{q L}{k}$$

1+2) Usando o método da superposição e assumindo deflexão nula em C, tem-se

$$\delta_C := -\delta_{C1} + \delta_B = -\frac{1}{3} \frac{q L^3 b}{EI} + \frac{q L}{k}$$

$$k := solve\left(\delta_C = 0, k\right) = \frac{3 EI}{L^2 b}$$

*** Verificação: Resolvendo pela equação diferencial da elástica**

$restart:$

Reações de apoio

$$assign\left(solve\left(\left\{R_B - q\cdot L = 0, M_A + (q\cdot L)\cdot\frac{L}{2} = 0\right\}, \{R_B, M_A\}\right)\right):$$

$$R_B, M_A = q\,L, -\frac{q\,L^2}{2}$$

Momento fletor

$$M_{AB} := solve\left(M + M_A + (q\cdot x)\cdot\frac{x}{2} = 0, M\right):$$

Deflexão

$$bc := \mathrm{D}[1](v)(0) = 0, v(L) = -\frac{R_B}{k}:$$

$$v := unapply\left(simplify\left(rhs\left(dsolve\left(\left\{diff(v(x), x\$2) = \frac{M_{AB}}{EI}, bc\right\}, v(x)\right)\right)\right), x\right):$$

Cálculo da deflexão em C

$$v(L) = -\frac{q\,L}{k}$$

$$\mathrm{D}[1](v)(L) = \frac{L^3\,q}{3\,EI}$$

$$v_C := v(L) + \mathrm{D}[1](v)(L)\cdot b:$$

Cálculo de k assumindo deflexão nula em C

$$solve(v_C = 0, k) = \frac{3\,EI}{L^2\,b}$$

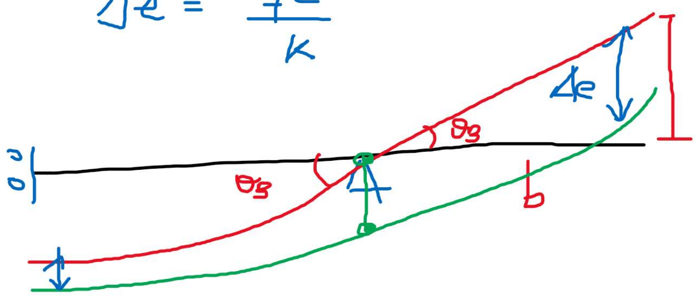
Esquema ilustrativo

$$K \Delta e = R_B = qL$$

$$\Delta e = \frac{qL}{K}$$

$$b \cdot \theta_B - \frac{qL}{K} = 0$$

$$K$$



$$\frac{\sin \theta_B}{\cos \theta_B} = \tan \theta_B = \frac{\delta_c}{b}$$

$$\theta_B \ll 1 \Longrightarrow \sin \theta_B \simeq \theta_B$$

$$\cos \theta_B \approx 1$$

$$\boxed{\delta_c = b \cdot \theta_B}$$

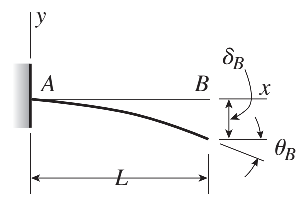
$$- \Delta e$$

APPENDIX G

Deflections and Slopes of Beams

Table G-1

Deflections and Slopes of Cantilever Beams



v = deflection in the y direction (positive upward)

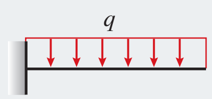
$v' = dv/dx$ = slope of the deflection curve

$\delta_B = -v(L)$ = deflection at end B of the beam (positive downward)

$\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise)

EI = constant

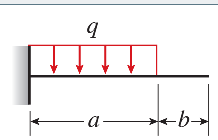
1



$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$$

2



$$v = -\frac{qx^2}{24EI}(6a^2 - 4ax + x^2) \qquad (0 \le x \le a)$$

$$v' = -\frac{qx}{6EI}(3a^2 - 3ax + x^2) \qquad (0 \le x \le a)$$

$$v = -\frac{qa^3}{24EI}(4x - a) \qquad v' = -\frac{qa^3}{6EI} \qquad (a \le x \le L)$$

$$\text{At } x = a\text{:} \quad v = -\frac{qa^4}{8EI} \qquad v' = -\frac{qa^3}{6EI}$$

$$\delta_B = \frac{qa^3}{24EI}(4L - a) \qquad \theta_B = \frac{qa^3}{6EI}$$

(*Continued*)

Table G-1 (Continued)

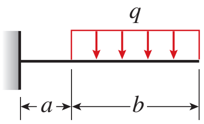
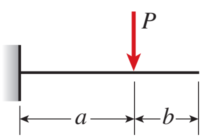
3		$v = -\frac{qbx^2}{12EI}(3L + 3a - 2x) \quad (0 \le x \le a)$ $v' = -\frac{qbx}{2EI}(L + a - x) \quad (0 \le x \le a)$ $v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \quad (a \le x \le L)$ $v' = -\frac{q}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3) \quad (a \le x \le L)$ At $x = a$: $v = -\frac{qa^2b}{12EI}(3L + a) \quad v' = -\frac{qabL}{2EI}$ $\delta_B = \frac{q}{24EI}(3L^4 - 4a^3L + a^4) \quad \theta_B = \frac{q}{6EI}(L^3 - a^3)$
4		$v = -\frac{Px^2}{6EI}(3L - x) \quad v' = -\frac{Px}{2EI}(2L - x)$ $\delta_B = \frac{PL^3}{3EI} \quad \theta_B = \frac{PL^2}{2EI}$
5		$v = -\frac{Px^2}{6EI}(3a - x) \quad v' = -\frac{Px}{2EI}(2a - x) \quad (0 \le x \le a)$ $v = -\frac{Pa^2}{6EI}(3x - a) \quad v' = -\frac{Pa^2}{2EI} \quad (a \le x \le L)$ At $x = a$: $v = -\frac{Pa^3}{3EI} \quad v' = -\frac{Pa^2}{2EI}$ $\delta_B = \frac{Pa^2}{6EI}(3L - a) \quad \theta_B = \frac{Pa^2}{2EI}$
6		$v = -\frac{M_0x^2}{2EI} \quad v' = -\frac{M_0x}{EI}$ $\delta_B = \frac{M_0L^2}{2EI} \quad \theta_B = \frac{M_0L}{EI}$

Table G-1 (Continued)

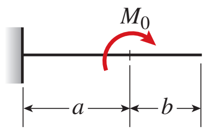
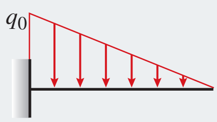
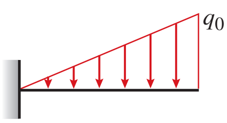
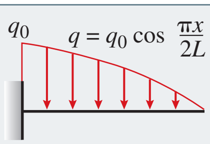
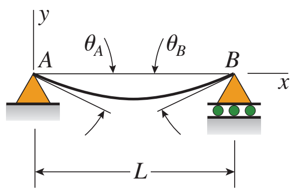
7		$v = -\frac{M_0 x^2}{2EI} \qquad v' = -\frac{M_0 x}{EI} \qquad (0 \le x \le a)$ $v = -\frac{M_0 a}{2EI}(2x - a) \qquad v' = -\frac{M_0 a}{EI} \qquad (a \le x \le L)$ At $x = a$: $\quad v = -\frac{M_0 a^2}{2EI} \qquad v' = -\frac{M_0 a}{EI}$ $\delta_B = \frac{M_0 a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0 a}{EI}$
8		$v = -\frac{q_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$ $v' = -\frac{q_0 x}{24LEI}(4L^3 - 6L^2 x + 4Lx^2 - x^3)$ $\delta_B = \frac{q_0 L^4}{30EI} \qquad \theta_B = \frac{q_0 L^3}{24EI}$
9		$v = -\frac{q_0 x^2}{120LEI}(20L^3 - 10L^2 x + x^3)$ $v' = -\frac{q_0 x}{24LEI}(8L^3 - 6L^2 x + x^3)$ $\delta_B = \frac{11 q_0 L^4}{120EI} \qquad \theta_B = \frac{q_0 L^3}{8EI}$
10		$v = -\frac{q_0 L}{3\pi^4 EI}\left(48L^3 \cos \frac{\pi x}{2L} - 48L^3 + 3\pi^3 Lx^2 - \pi^3 x^3\right)$ $v' = -\frac{q_0 L}{\pi^3 EI}\left(2\pi^2 Lx - \pi^2 x^2 - 8L^2 \sin \frac{\pi x}{2L}\right)$ $\delta_B = \frac{2q_0 L^4}{3\pi^4 EI}(\pi^3 - 24) \qquad \theta_B = \frac{q_0 L^3}{\pi^3 EI}(\pi^2 - 8)$

Table G-2

Deflections and Slopes of Simple Beams



v = deflection in the y direction (positive upward)

$v' = dv/dx$ = slope of the deflection curve

$\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward)

x_1 = distance from support A to point of maximum deflection

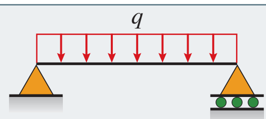
$\delta_{\max} = -v_{\max}$ = maximum deflection (positive downward)

$\theta_A = -v'(0)$ = angle of rotation at left-hand end of the beam (positive clockwise)

$\theta_B = v'(L)$ = angle of rotation at right-hand end of the beam (positive counterclockwise)

EI = constant

1

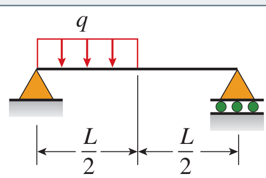


$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{\max} = \frac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \frac{qL^3}{24EI}$$

2



$$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

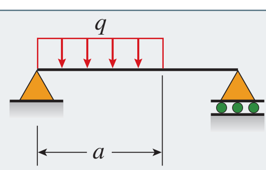
$$v' = -\frac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$v' = -\frac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$\delta_C = \frac{5qL^4}{768EI} \qquad \theta_A = \frac{3qL^3}{128EI} \qquad \theta_B = \frac{7qL^3}{384EI}$$

3



$$v = -\frac{qx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad (0 \le x \le a)$$

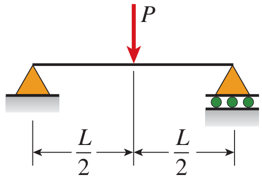
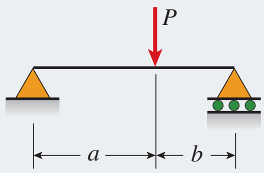
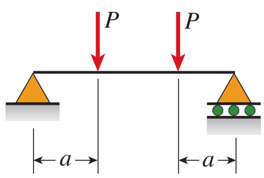
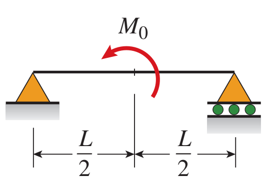
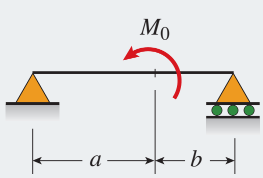
$$v' = -\frac{q}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad (0 \le x \le a)$$

$$v = -\frac{qa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad (a \le x \le L)$$

$$v' = -\frac{qa^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad (a \le x \le L)$$

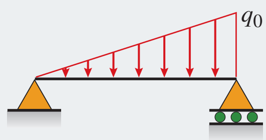
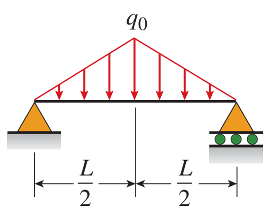
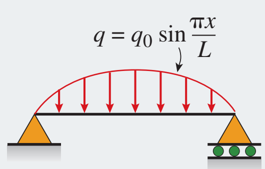
$$\theta_A = \frac{qa^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{qa^2}{24LEI}(2L^2 - a^2)$$

Table G-2 (Continued)

4		$v = -\frac{Px}{48EI}(3L^2 - 4x^2)$ $v' = -\frac{P}{16EI}(L^2 - 4x^2)$ $\left(0 \le x \le \frac{L}{2}\right)$ $\delta_C = \delta_{max} = \frac{PL^3}{48EI}$ $\theta_A = \theta_B = \frac{PL^2}{16EI}$
5		$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2)$ $(0 \le x \le a)$ $\theta_A = \frac{Pab(L+b)}{6LEI}$ $\theta_B = \frac{Pab(L+a)}{6LEI}$ If $a \ge b$, $\delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI}$ If $a \le b$, $\delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$ If $a \ge b$, $x_1 = \sqrt{\frac{L^2 - b^2}{3}}$ and $\delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$
6		$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2)$ $v' = -\frac{P}{2EI}(aL - a^2 - x^2)$ $(0 \le x \le a)$ $v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2)$ $v' = -\frac{Pa}{2EI}(L - 2x)$ $(a \le x \le L - a)$ $\delta_C = \delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2)$ $\theta_A = \theta_B = \frac{Pa(L-a)}{2EI}$
7		$v = -\frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2)$ $v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$ $\delta_C = \frac{M_0 L^2}{16EI}$ $\theta_A = \frac{M_0 L}{3EI}$ $\theta_B = \frac{M_0 L}{6EI}$ $x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right)$ and $\delta_{max} = \frac{M_0 L^2}{9\sqrt{3}EI}$
8		$v = -\frac{M_0 x}{24LEI}(L^2 - 4x^2)$ $v' = -\frac{M_0}{24LEI}(L^2 - 12x^2)$ $\left(0 \le x \le \frac{L}{2}\right)$ $\delta_C = 0$ $\theta_A = \frac{M_0 L}{24EI}$ $\theta_B = -\frac{M_0 L}{24EI}$
9		$v = -\frac{M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2)$ $(0 \le x \le a)$ $v' = -\frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2)$ $(0 \le x \le a)$ At $x = a$: $v = \frac{M_0 ab}{3LEI}(2a - L)$ $v' = -\frac{M_0}{3LEI}(3aL - 3a^2 - L^2)$ $\theta_A = \frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2)$ $\theta_B = \frac{M_0}{6LEI}(3a^2 - L^2)$

(Continued)

Table G-2 (Continued)

10		$v = -\dfrac{M_0 x}{2EI}(L - x) \qquad v' = -\dfrac{M_0}{2EI}(L - 2x)$ $\delta_C = \delta_{max} = \dfrac{M_0 L^2}{8EI} \qquad \theta_A = \theta_B = \dfrac{M_0 L}{2EI}$
11		$v = -\dfrac{q_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$ $v' = -\dfrac{q_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$ $\delta_C = \dfrac{5q_0L^4}{768EI} \qquad \theta_A = \dfrac{7q_0L^3}{360EI} \qquad \theta_B = \dfrac{q_0L^3}{45EI}$ $x_1 = 0.5193L \qquad \delta_{max} = 0.00652\dfrac{q_0L^4}{EI}$
12		$v = -\dfrac{q_0 x}{960LEI}(5L^2 - 4x^2)^2 \qquad \left(0 \le x \le \dfrac{L}{2}\right)$ $v' = -\dfrac{q_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2) \qquad \left(0 \le x \le \dfrac{L}{2}\right)$ $\delta_C = \delta_{max} = \dfrac{q_0L^4}{120EI} \qquad \theta_A = \theta_B = \dfrac{5q_0L^3}{192EI}$
13		$v = -\dfrac{q_0L^4}{\pi^4 EI}\sin\dfrac{\pi x}{L} \qquad v' = -\dfrac{q_0L^3}{\pi^3 EI}\cos\dfrac{\pi x}{L}$ $\delta_C = \delta_{max} = \dfrac{q_0L^4}{\pi^4 EI} \qquad \theta_A = \theta_B = \dfrac{q_0L^3}{\pi^3 EI}$