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AB2-PT2 - MEC 3 - 24/09/2021

Maria Beatriz Menezes Costa Alves

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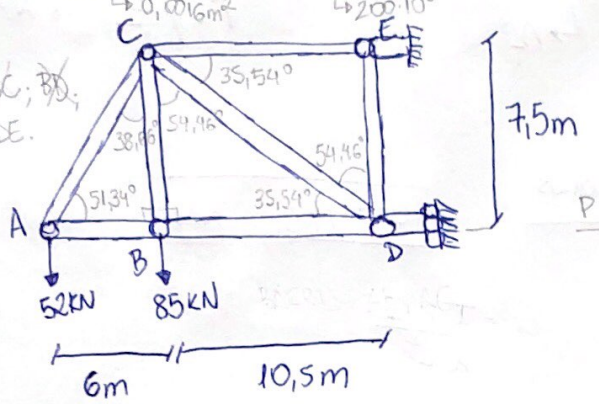
01 Castigliano; Calcular o deslocamento horizontal no ponto A.

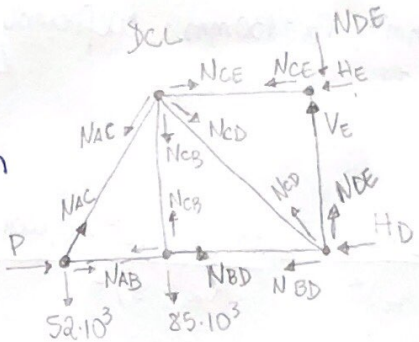
Abarras = 1600mm²; E = 200GPa.

↳ 0,0016m² ↳ 200·10⁹

BARRAS

AB; AC; BC; BD; CD; CE; DE.





~~N_{AB}, N_{AC}, N_{BC}, N_{BD}, N_{CD}, N_{CE}, N_{DE}~~

$L_{AC} = 9,6047m$; $L_{CD} = 12,9035m$.

- MÉTODO DOS NÓS:

- ΣF_{HD}: $-H_D - N_{BD} - N_{CD}\cdot\cos(35,54°) = 0$
- ΣF_{VD}: $N_{DE} + N_{CD}\cdot\cos(54,46°) = 0 \rightarrow \boxed{N_{DE} = 137,005\,kN}$
- ΣF_{HB}: $\boxed{N_{BD} = N_{AB}} \rightarrow \boxed{N_{BD} = -41623,75 - P}$
- ΣF_{VB}: $N_{CB} - 85\cdot10^3 = 0 \rightarrow \boxed{N_{CB} = 85kN}$
- ΣF_{HA}: $N_{AB} + N_{AC}\cdot\cos(51,34°) + P = 0 \rightarrow N_{AB} + 0,625N_{AC} + P = 0 \rightarrow \boxed{N_{AC} = -1,6N_{AB} - 1,6P}$
- ΣF_{VA}: $N_{AC}\cdot\text{sen}(51,34°) - 52\cdot10^3 = 0 \rightarrow \boxed{N_{AC} = 66,598\cdot kN}$

$66598 = -1,6N_{AB} - 1,6P$

$\boxed{N_{AB} = -41623,75 - P}$

- ΣF_{HC}: $N_{CE} - N_{AC}\cdot\text{sen}(38,66°) + N_{CD}\cdot\cos(35,54°) = 0$

$N_{CE} - 41603,615 - 191791,43 = 0$

$\boxed{N_{CE} = 233,395kN}$

- ΣF_{VC}: $-N_{AC}\cdot\cos(38,66°) - N_{CB} - N_{CD}\cdot\cos(54,46°) = 0 \rightarrow -52004,162 - 85000 = 0,58127\,N_{CD}$

$\boxed{N_{CD} = -235,7kN}$

- ΣF_{HE}: $-H_E - N_{CE} = 0 \rightarrow \boxed{H_E = -233,395kN}$
- ΣF_{VE}: $\boxed{V_E - N_{DE} = 0}$

Para Barras: $\Delta = \sum_{i=1}^{N} \int_0^{L_i} \frac{N_i}{E_i\cdot A_i}\cdot\frac{\partial N_i}{\partial P}\,dx$ *Lembrar que P=0, POIS P não existe!!!

- BARRA AB: $\Delta_{AB} = \int_0^6 \frac{(-41623,75 - P)\cdot(-1)}{EA}\,dx = \int_0^6 \frac{41623,75 + P}{EA}\,dx = \frac{41623,75x + \frac{Px}{2}^{0}}{EA}\Big]_0^6$

$\frac{\partial N_{AB}}{\partial P} = -1$

$= \frac{41623,75\cdot 6}{200\cdot10^9\cdot 0,0016} \rightarrow \boxed{\Delta_{AB} = 0,00098m}$

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01 continuação.

- $\Delta_{AC} = \int_0^{9,6} \frac{66598}{EA} \cdot 0 \, dx \rightarrow \boxed{\Delta_{AC} = 0}$

$\frac{\partial N_{AC}}{\partial P} = 0$

- $\Delta_{BC} = \int_0^{7,5} \frac{85000}{EA} \cdot 0 \, dx \rightarrow \boxed{\Delta_{BC} = 0}$

$\frac{\partial N_{BC}}{\partial P} = 0$

- $\Delta_{BD} = \int_0^{10,5} \frac{(-41623,75 - P)}{EA} \cdot (-1) \, dx = \int_0^{10,5} \frac{41623,75 + P}{EA} \, dx = \frac{41623,75x + Px}{EA} \Big]_0^{10,5}$

$\frac{\partial N_{BD}}{\partial P} = -1$ $\Big\}$ $= \frac{41623,75 \cdot 10,5}{200 \cdot 10^9 \cdot 0,0016} \rightarrow \boxed{\Delta_{BD} = 0,0014 \, m}$

- $\Delta_{CD} = \int_0^{12,9} -\frac{235700}{EA} \cdot 0 \, dx \rightarrow \boxed{\Delta_{CD} = 0}$

$\frac{\partial N_{CD}}{\partial P} = 0$

- $\Delta_{CE} = \int_0^{10,5} \frac{233395}{EA} \cdot 0 \, dx \rightarrow \boxed{\Delta_{CE} = 0}$

$\frac{\partial N_{CE}}{\partial P} = 0$

- $\Delta_{DE} = \int_0^{7,5} \frac{137005}{EA} \cdot 0 \, dx \rightarrow \boxed{\Delta_{DE} = 0}$

$\frac{\partial N_{DE}}{\partial P} = 0$

$\Delta_A = \Delta_{AB} + \Delta_{AC} + \Delta_{BC} + \Delta_{BD} + \Delta_{CD} + \Delta_{CE} + \Delta_{DE} = 0,0008 + 0,0014$

$\boxed{\Delta_A = 0,0022 \, m}$ $\Delta_A = 2,2 \, mm$

* Como a força P em B foi considerada positiva para direita, tem-se que o deslocamento em B é 2,2mm para a direita

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02 Calcular a carga admissível. $\sigma_{adm} = 250\,MPa$; $E = 200\,GPa$ ($\hookrightarrow 200 \cdot 10^9$); $I_z = 128 \cdot 10^6\,mm^4$ ($\hookrightarrow 0,000128\,m^4$).

$I_y = 18,4 \cdot 10^6\,mm^4$ ($\hookrightarrow 1,84 \cdot 10^{-5}\,m^4$); $r_z = 130\,mm$ ($\hookrightarrow 0,13\,m$). coeficiente $= 2$ ($\hookrightarrow n_f$); $L = 9\,m$.

$r = \sqrt{\frac{I}{A}} \rightarrow A = I / r^2$

- No eixo z:

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_z}{(K \cdot L)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 \cdot 0,000128}{(2 \cdot 9)^2} \rightarrow \boxed{P_{cr} = 779,82\,kN}$$

$$\sigma_{cr} = \frac{P_{cr_z}}{A_z} \; ; \; \sigma \leq \frac{\sigma_{cr}}{n_f} \; ; \; \sigma_{cr} = \frac{\pi^2 \cdot E \cdot I}{A \cdot L_e^2}$$

$$\sigma_{cr_z} = \frac{\pi^2 \cdot E \cdot r_z^2}{(KL)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 \cdot 0,13^2}{(2 \cdot 9)^2} \rightarrow \boxed{\sigma_{cr} = 1,0296 \cdot 10^8\,Pa} \; (\hookrightarrow 102,96\,MPa)$$

$\sigma_{adm} > \sigma_{cr} \rightarrow 250\,MPa > 102,96\,MPa$ ✓

$$\sigma \leq \frac{\sigma_{cr}}{n_f} \rightarrow \frac{P}{A} \leq \frac{P_{cr}}{A \cdot n_f} \rightarrow P \leq \frac{779,82}{2} \rightarrow \boxed{P \leq 389,91\,kN}$$

- No eixo y:

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_y}{(K \cdot L)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 \cdot 1,84 \cdot 10^{-5}}{(0,7 \cdot 9)^2} \rightarrow \boxed{P_{cr} = 915,1\,kN}$$

$$\sigma_{cr_y} = \frac{\pi^2 \cdot E \cdot I_y}{A \cdot (KL)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 \cdot 1,84 \cdot 10^{-5}}{A \cdot (KL)^2} \rightarrow \sigma_{cr_y} = \frac{915100}{A}$$

$$\sigma \leq \frac{\sigma_{cr}}{n_f} \rightarrow \frac{P}{A} \leq \frac{915100}{A \cdot 2} \rightarrow \boxed{P \leq 457,55\,kN}$$

∴ Desta forma, a carga admissível para o sistema é 389,91 kN.