Universidade Federal de Alagoas - UFAL
Centro de Tecnologia - CTEC
Curso de Engenharia Civil

Mecânica dos Sólidos 3 - ECIV051D (2020.2)

Exercícios:
Deslocamentos em vigas isostáticas usando o método da superposição

Encontro Assíncrono

Monitores:
Hugo Vinícius F. Azevedo
Milton Mateus G. Santos
Ricardo A. Fernandes

Professor/Supervisor:
Adeildo S. Ramos Jr.

Maceió/AL, 23/07/2021

Exemplo 9-6
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

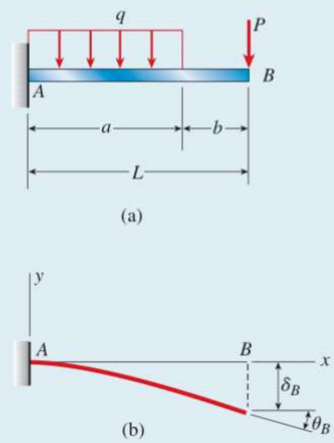
Example 9-6

A cantilever beam AB supports a uniform load of intensity q acting over part of the span and a concentrated load P acting at the free end (Fig. 9-18a).

Determine the deflection δ_B and angle of rotation θ_B at end B of the beam (Fig. 9-18b). (*Note:* The beam has length L and constant flexural rigidity EI.)

Fig. 9-18

Example 9-6: Cantilever beam with a uniform load and a concentrated load



restart :

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 2 e 4 da Tabela G-1.

$$\delta_{B2} := (q, a, L, EI) \rightarrow \frac{q \cdot a^3}{24 \cdot EI} \cdot (4 \cdot L - a) :$$

$$\theta_{B2} := (q, a, EI) \rightarrow \frac{q \cdot a^3}{6 \cdot EI} :$$

$$\delta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^3}{3 \cdot EI} :$$

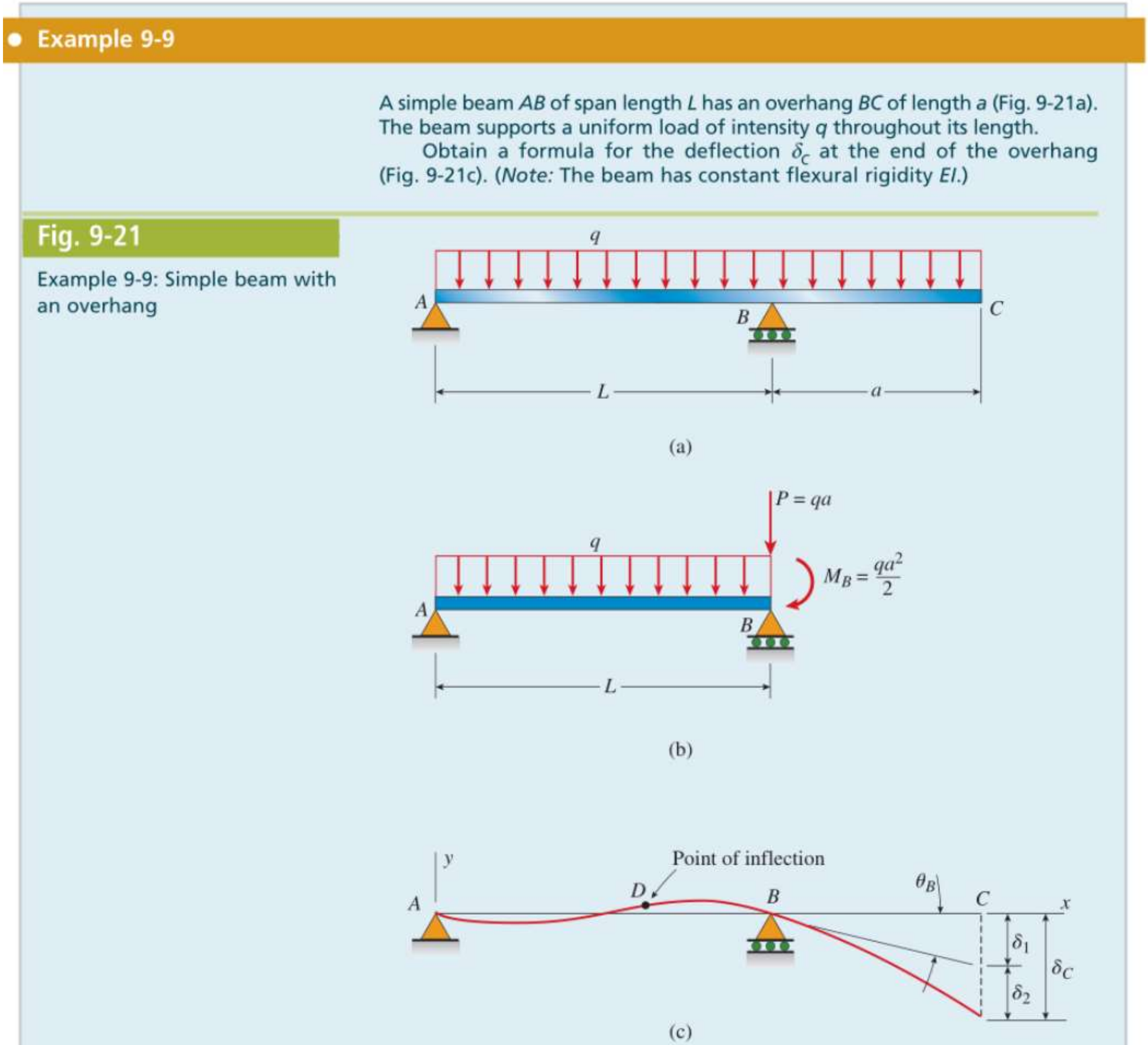
$$\theta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^2}{2 \cdot EI} :$$

Usando o método da superposição, tem-se

$$\delta_B := \delta_{B2}(q, a, L, EI) + \delta_{B4}(P, L, EI) = \frac{1}{24} \frac{q\, a^3\, (4\, L - a)}{EI} + \frac{1}{3} \frac{P\, L^3}{EI}$$

$$\theta_B := \theta_{B2}(q, a, EI) + \theta_{B4}(P, L, EI) = \frac{1}{6} \frac{q\, a^3}{EI} + \frac{1}{2} \frac{P\, L^2}{EI}$$

Exemplo 9-9
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.



Example 9-9

A simple beam AB of span length L has an overhang BC of length a (Fig. 9-21a). The beam supports a uniform load of intensity q throughout its length.

Obtain a formula for the deflection δ_C at the end of the overhang (Fig. 9-21c). (*Note:* The beam has constant flexural rigidity EI.)

Fig. 9-21

Example 9-9: Simple beam with an overhang

restart :

1) Considerando o trecho biapoiado AB com o binário da carga $P = q \cdot a$
Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 1 e 7 da Tabela G-2.

$$\theta_{B1} := (q, L, EI) \rightarrow \frac{q \cdot L^3}{24 \cdot EI} :$$

$$\theta_{A7} := (M_0, L, EI) \rightarrow \frac{M_0 \cdot L}{3 \cdot EI} :$$

$$\theta_B := -\theta_{B1}(q, L, EI) + \theta_{A7}\left(\frac{q \cdot a^2}{2}, L, EI\right) = -\frac{1}{24}\,\frac{q\,L^3}{EI} + \frac{1}{6}\,\frac{L\,q\,a^2}{EI}$$

$$\delta_1 := \theta_B \cdot a = \left(-\frac{1}{24}\,\frac{q\,L^3}{EI} + \frac{1}{6}\,\frac{L\,q\,a^2}{EI}\right) a$$

2) Considerando o trecho engastado e livre BC

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 1 da Tabela G-1.

$$\delta_{B1} := (q, L, EI) \rightarrow \frac{q \cdot L^4}{8 \cdot EI} :$$

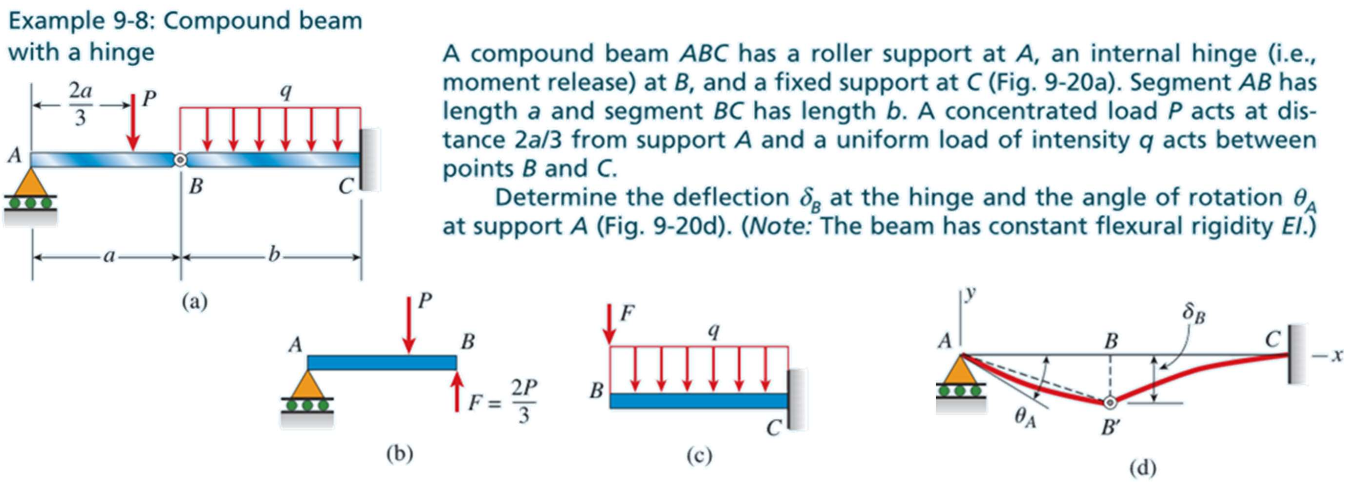
$$\delta_2 := \delta_{B1}(q, a, EI) = \frac{1}{8} \frac{q\, a^4}{EI}$$

1+2) Usando o método da superposição e assumindo deflexão nula em C, tem-se

$$\delta_C := \delta_1 + \delta_2 = \left(-\frac{1}{24} \frac{q\, L^3}{EI} + \frac{1}{6} \frac{L\, q\, a^2}{EI} \right) a + \frac{1}{8} \frac{q\, a^4}{EI}$$

$$simplify\left(\delta_C - \frac{q \cdot a}{24 \cdot EI} \cdot \left(L \cdot \left(4 \cdot a^2 - L^2 \right) + 3 \cdot a^3 \right) \right) = 0$$

Exemplo 9-8
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.


Example 9-8: Compound beam with a hinge

A compound beam *ABC* has a roller support at *A*, an internal hinge (i.e., moment release) at *B*, and a fixed support at *C* (Fig. 9-20a). Segment *AB* has length *a* and segment *BC* has length *b*. A concentrated load *P* acts at distance 2*a*/3 from support *A* and a uniform load of intensity *q* acts between points *B* and *C*.

Determine the deflection δ_B at the hinge and the angle of rotation θ_A at support *A* (Fig. 9-20d). (*Note:* The beam has constant flexural rigidity *EI*.)

$restart:$

$$F := solve\left(-P\cdot\frac{2\cdot a}{3}+F\cdot a=0, F\right) = \frac{2}{3}\,P$$

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 1 e 4 da Tabela G-1.

$$\delta_{B1} := (q, L, EI) \rightarrow \frac{q\cdot L^4}{8\cdot EI}:$$

$$\delta_{B4} := (P, L, EI) \rightarrow \frac{P\cdot L^3}{3\cdot EI}:$$

Usando o método da superposição, determina-se a deflexão em B

$$\delta_B := \delta_{B1}(q, b, EI) + \delta_{B4}(F, b, EI) = \frac{1}{8}\,\frac{q\,b^4}{EI}+\frac{2}{9}\,\frac{P\,b^3}{EI}$$

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 5 da Tabela G-2.

$$\theta_{A5} := (P, a, b, L, EI) \rightarrow \frac{P\cdot a\cdot b\cdot(L+b)}{6\cdot L\cdot EI}:$$

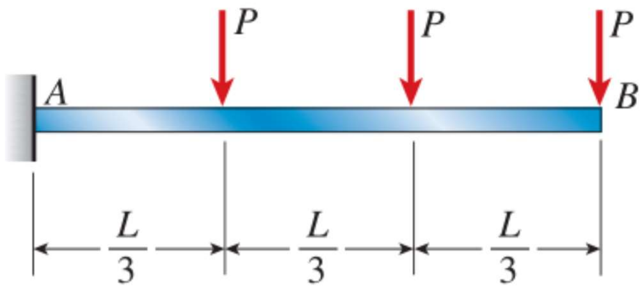
Usando o método da superposição, determina-se a rotação em A

$$\theta_{A,2} := \theta_{A5}\left(P, \frac{2\cdot a}{3}, \frac{a}{3}, a, EI\right) = \frac{4}{81}\,\frac{a^2\,P}{EI}$$

$$\theta_A := expand\left(\frac{\delta_B}{a}+\theta_{A,2}\right) = \frac{1}{8}\,\frac{q\,b^4}{a\,EI}+\frac{2}{9}\,\frac{P\,b^3}{a\,EI}+\frac{4}{81}\,\frac{a^2\,P}{EI}$$

Problema 9.5-1
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-1 A cantilever beam AB carries three equally spaced concentrated loads, as shown in the figure. Obtain formulas for the angle of rotation θ_B and deflection δ_B at the free end of the beam.



PROB. 9.5-1

restart :

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso dos casos 4 e 5 da Tabela G-1.

$$\delta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^3}{3 \cdot EI} :$$

$$\theta_{B4} := (P, L, EI) \rightarrow \frac{P \cdot L^2}{2 \cdot EI} :$$

$$\delta_{B5} := (P, a, L, EI) \rightarrow \frac{P \cdot a^2}{6 \cdot EI} \cdot (3 \cdot L - a) :$$

$$\theta_{B5} := (P, a, EI) \rightarrow \frac{P \cdot a^2}{2 \cdot EI} :$$

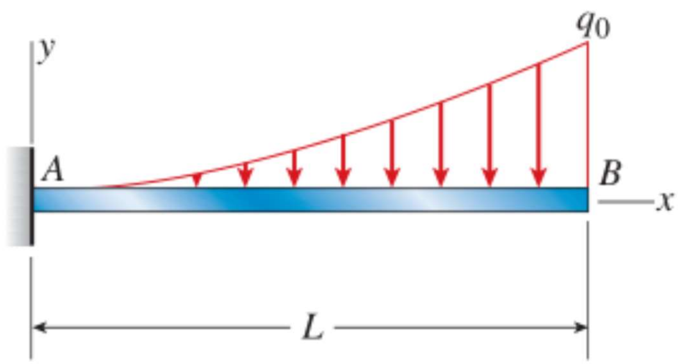
Usando o método da superposição, tem-se

$$\theta_B := \theta_{B5}\left(P, \frac{L}{3}, EI\right) + \theta_{B5}\left(P, \frac{2 \cdot L}{3}, EI\right) + \theta_{B4}(P, L, EI) = \frac{7}{9} \frac{P L^2}{EI}$$

$$\delta_B := \delta_{B5}\left(P, \frac{L}{3}, L, EI\right) + \delta_{B5}\left(P, \frac{2 \cdot L}{3}, L, EI\right) + \delta_{B4}(P, L, EI) = \frac{5}{9} \frac{P L^3}{EI}$$

Problema 9.5-13
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-13 Determine the angle of rotation θ_B and deflection δ_B at the free end of a cantilever beam AB supporting a parabolic load defined by the equation $q(x) = q_0 x^2/L^2$ (see figure).



PROB. 9.5-13

restart :

$q := x \rightarrow q_0 \cdot \frac{x^2}{L^2}$:

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 5 da Tabela G-1.

$\delta_{B5} := (P, a, L, EI) \rightarrow \frac{P \cdot a^2}{6 \cdot EI} \cdot (3 \cdot L - a)$:

$\theta_{B5} := (P, a, EI) \rightarrow \frac{P \cdot a^2}{2 \cdot EI}$:

Usando o método da superposição e integrando para $0 \leq x \leq L$, tem-se

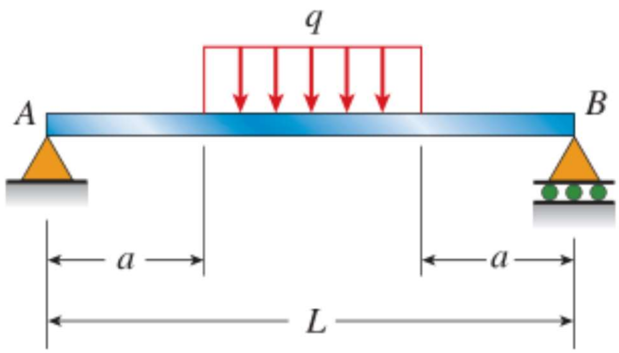
$$\theta_B := \int_0^L \theta_{B5}(q(x), x, EI)\, dx = \frac{1}{10} \frac{q_0 L^3}{EI}$$

$$\delta_B := \int_0^L \delta_{B5}(q(x), x, L, EI)\, dx = \frac{13}{180} \frac{q_0 L^4}{EI}$$

Problema 9.5-14
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-14 A simple beam AB supports a uniform load of intensity q acting over the middle region of the span (see figure).

Determine the angle of rotation θ_A at the left-hand support and the deflection δ_{max} at the midpoint.



PROB. 9.5-14

restart :

Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 5 da Tabela G-2.

$$\theta_{A5} := (P, x, b, L, EI) \rightarrow \frac{P \cdot x \cdot b \cdot (L+b)}{6 \cdot L \cdot EI} :$$

$$\delta_{C5} := (P, x, L, EI) \rightarrow \frac{P \cdot x \cdot (3 \cdot L^2 - 4 \cdot x^2)}{48 \cdot EI} :$$

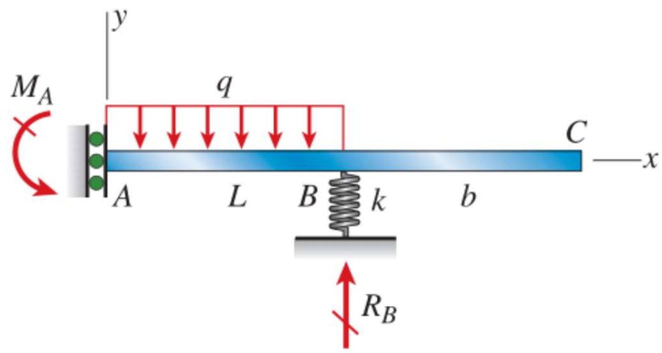
Usando o método da superposição e integrando para $a \leq x \leq L - a$, tem-se

$$\theta_A := simplify\left(\int_a^{L-a} \theta_{A5}(q, x, L-x, L, EI)\, dx\right) = \frac{1}{24} \frac{q\,(L^3 - 6\,L\,a^2 + 4\,a^3)}{EI}$$

$$\delta_{max} := simplify\left(2 \cdot \int_a^{\frac{L}{2}} \delta_{C5}(q, x, L, EI)\, dx\right) = \frac{1}{384} \frac{q\,(5\,L^4 - 24\,L^2\,a^2 + 16\,a^4)}{EI}$$

Problema 9.5-17
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

9.5-17 An overhanging beam ABC with flexural rigidity $EI = 45\ \mathrm{N \cdot m^2}$ is supported by a guided support at A and by a spring of stiffness k at point B (see figure). Span AB has length $L = 0.75$ m and carries a uniform load. The overhang BC has length $b = 375$ mm. For what stiffness k of the spring will the uniform load produce no deflection at the free end C?



PROB. 9.5-17

restart :
Consultando o Anexo G (Gere & Goodno, 2013), faz-se uso do caso 1 da Tabela G-2.

$$\theta_{B1} := (q, L, EI) \rightarrow \frac{q \cdot L^3}{24 \cdot EI} :$$

1) Considerando um apoio simples em B
Trecho AB: metade de uma viga biapoiada com carregamento uniforme (cortante nula em A)

$$\delta_{C1} := \theta_{B1}(q, 2 \cdot L, EI) \cdot b = \frac{1}{3} \frac{q\, L^3\, b}{EI}$$

2) Considerado o efeito do apoio elástico
$R_B := q \cdot L :$

$$\delta_B := solve\left(k \cdot \delta_B = R_B, \delta_B\right) = \frac{q\, L}{k}$$

1+2) Usando o método da superposição e assumindo deflexão nula em C, tem-se

$$\delta_C := -\delta_{C1} + \delta_B = -\frac{1}{3} \frac{q\, L^3\, b}{EI} + \frac{q\, L}{k}$$

$$k := solve\left(\delta_C = 0, k\right) = \frac{3\, EI}{L^2\, b}$$

*** Verificação: Resolvendo pela equação diferencial da elástica**

$restart:$

Reações de apoio

$$assign\left(solve\left(\left\{R_B - q\cdot L = 0, M_A + (q\cdot L)\cdot\frac{L}{2} = 0\right\}, \{R_B, M_A\}\right)\right):$$

$$R_B, M_A = q\,L, -\frac{q\,L^2}{2}$$

Momento fletor

$$M_{AB} := solve\left(M + M_A + (q\cdot x)\cdot\frac{x}{2} = 0, M\right):$$

Deflexão

$$bc := \mathrm{D}[1](v)(0) = 0, v(L) = -\frac{R_B}{k}:$$

$$v := unapply\left(simplify\left(rhs\left(dsolve\left(\left\{diff(v(x), x\$2) = \frac{M_{AB}}{EI}, bc\right\}, v(x)\right)\right)\right), x\right):$$

Cálculo da deflexão em C

$$v(L) = -\frac{q\,L}{k}$$

$$\mathrm{D}[1](v)(L) = \frac{L^3\,q}{3\,EI}$$

$$v_C := v(L) + \mathrm{D}[1](v)(L)\cdot b:$$

Cálculo de k assumindo deflexão nula em C

$$solve(v_C = 0, k) = \frac{3\,EI}{L^2\,b}$$

Esquema ilustrativo

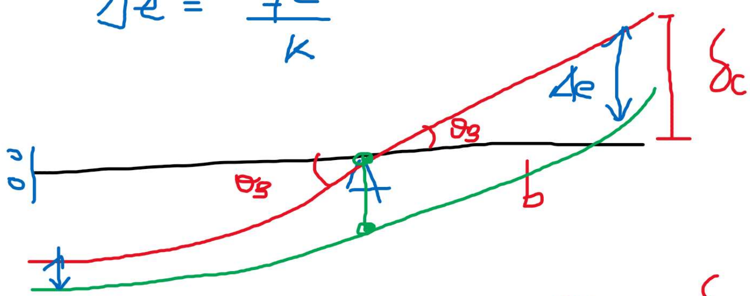
$$K\,\Delta e = R_B = qL$$

$$\Delta e = \frac{qL}{K}$$

$$b \cdot \theta_B - \frac{qL}{K} = 0$$

$\downarrow$

K



$$\frac{\sin \theta_B}{\cos \theta_B} = \tan \theta_B = \frac{\delta_c}{b}$$

$$\theta_B << 1 \implies \sin \theta_B \simeq \theta_B \qquad \cos \theta_B \simeq 1$$

$$\boxed{\delta_C = b \cdot \theta_B}$$

$$-\Delta e$$