Universidade Federal de Alagoas - UFAL
Centro de Tecnologia - CTEC
Curso de Engenharia Civil

Mecânica dos Sólidos 3 - ECIV051D (2020.2)

Exercícios:
Deslocamentos em vigas hiperestáticas usando a equação diferencial da elástica

Encontro Assíncrono

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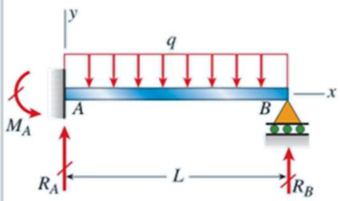
Exemplo 10-1
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

Example 10-1



A propped cantilever beam *AB* of length *L* supports a uniform load of intensity *q* (Fig. 10-6). Analyze this beam by solving the second-order differential equation of the deflection curve (the bending-moment equation). Determine the reactions, shear forces, bending moments, slopes, and deflections of the beam.

restart :

Reações de apoio (em função de R_B)

$$assign\left(solve\left(\left\{R_A + R_B - q\cdot L = 0,\ M_A - (q\cdot L)\cdot\frac{L}{2} + R_B\cdot L = 0\right\},\ \{R_A, M_A\}\right)\right) :$$

$$R_A, M_A = q\,L - R_B,\ \frac{1}{2}\,q\,L^2 - R_B\,L$$

Esforço cortante

$$V := unapply\left(solve\left(-V + R_A - q\cdot x = 0,\ V\right)\right) :$$

$$V(x) = q\,L - q\,x - R_B$$

Momento fletor

$$M := unapply\left(solve\left(M + M_A - R_A\cdot x + (q\cdot x)\cdot\frac{x}{2} = 0,\ M\right), x\right) :$$

$$M(x) = -\frac{1}{2}\,q\,L^2 + R_B\,L + L\,q\,x - R_B\,x - \frac{1}{2}\,q\,x^2$$

Equação da curva de deflexão

$$bc := v(0) = 0,\ \mathrm{D}[1](v)(0) = 0,\ v(L) = 0 :$$

$$assign\left(dsolve\left(\left\{diff(v(x), x\$2) = \frac{M(x)}{EI},\ bc\right\},\ \{v(x), R_B\}\right)\right) :$$

$$v := unapply(v(x), x) :$$

Respostas: reações, esforço cortante, momento fletor, deflexão e inclinação

$$R_A, R_B, M_A = \frac{5\,q\,L}{8},\ \frac{3\,q\,L}{8},\ \frac{q\,L^2}{8}$$

$$V(x), M(x) = \frac{5}{8}\,q\,L - q\,x,\ -\frac{1}{8}\,q\,L^2 + \frac{5}{8}\,L\,q\,x - \frac{1}{2}\,q\,x^2$$

$$v(x), \mathrm{D}[1](v)(x) = \frac{-\frac{3}{2}\,L^2\,q\,x^2 + \frac{5}{2}\,L\,q\,x^3 - q\,x^4}{24\,EI},\ \frac{-3\,L^2\,q\,x + \frac{15}{2}\,L\,q\,x^2 - 4\,q\,x^3}{24\,EI}$$

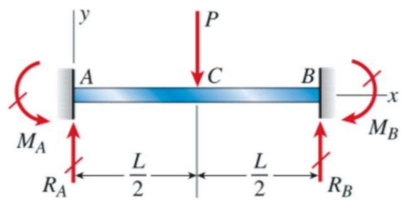
Exemplo 10-2
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

Example 10-2

The fixed-end beam *ACB* shown in Fig. 10-9 supports a concentrated load *P* at the midpoint. Analyze this beam by solving the fourth-order differential equation of the deflection curve (the load equation). Determine the reactions, shear forces, bending moments, slopes, and deflections of the beam.

Fig. 10-9

Example 10-2: Fixed-end beam with a concentrated load at the midpoint



restart :

Reações de apoio (em função de R_B e M_B)

$$assign\left(solve\left(\left\{R_A + R_B - P = 0, M_A - M_B - P\cdot\frac{L}{2} + R_B\cdot L = 0\right\}, \{R_A, M_A\}\right)\right):$$

$$R_A, M_A = -R_B + P, \frac{1}{2}\,P\,L - R_B\,L + M_B$$

Esforço cortante

$$V_{AC} := solve(-V + R_A = 0, V):$$
$$V_{CB} := solve(V + R_B = 0, V):$$
$$V_{AC}, V_{CB} = -R_B + P, -R_B$$

Momento fletor

$$M_{AC} := unapply(solve(M + M_A - R_A\cdot x = 0, M), x):$$
$$M_{CB} := unapply(solve(-M - M_B + R_B\cdot(L - x) = 0, M), x):$$
$$M_{AC}(x), M_{CB}(x) = x\,P - x\,R_B - \frac{1}{2}\,P\,L + R_B\,L - M_B, R_B\,L - x\,R_B - M_B$$

Equação da curva de deflexão

$$edo_{AC} := diff\,(v_{AC}(x), x\$2) = \frac{M_{AC}(x)}{EI}:$$
$$edo_{CB} := diff\,(v_{CB}(x), x\$2) = \frac{M_{CB}(x)}{EI}:$$
$$bc_v := v_{AC}(0) = 0,\ v_{AC}\left(\frac{L}{2}\right) = v_{CB}\left(\frac{L}{2}\right),\ v_{CB}(L) = 0:$$
$$bc_\theta := \mathrm{D}[1](v_{AC})(0) = 0,\ \mathrm{D}[1](v_{AC})\left(\frac{L}{2}\right) = \mathrm{D}[1](v_{CB})\left(\frac{L}{2}\right), \mathrm{D}[1](v_{CB})(L) = 0:$$
$$assign(dsolve(\{edo_{AC}, edo_{CB}, bc_v, bc_\theta\}, \{v_{AC}(x), v_{CB}(x), R_B, M_B\})):$$
$$v_{AC} := unapply(v_{AC}(x), x) : v_{CB} := unapply(v_{CB}(x), x):$$

Respostas: reações, esforço cortante, momento fletor, deflexão e inclinação

$R_A, R_B, M_A, M_B = \frac{P}{2}, \frac{P}{2}, \frac{PL}{8}, \frac{PL}{8}$

$V_{AC}, V_{CB} = \frac{P}{2}, -\frac{P}{2}$

$M_{AC}(x), M_{CB}(x) = \frac{1}{2}\,x\,P - \frac{1}{8}\,P\,L, \frac{3}{8}\,P\,L - \frac{1}{2}\,x\,P$

$$v_{AC}(x), v_{CB}(x) = \frac{-\frac{3}{4}\,L\,P\,x^2 + P\,x^3}{12\,EI}, \frac{\frac{1}{8}\,L^3\,P - \frac{3}{4}\,L^2\,P\,x + \frac{9}{8}\,L\,P\,x^2 - \frac{1}{2}\,P\,x^3}{6\,EI}$$

$$\mathrm{D}[1](v_{AC})(x), \mathrm{D}[1](v_{CB})(x) = \frac{-\frac{3}{2}\,L\,P\,x + 3\,P\,x^2}{12\,EI}, \frac{-\frac{3}{4}\,L^2\,P + \frac{9}{4}\,L\,P\,x - \frac{3}{2}\,P\,x^2}{6\,EI}$$

$$simplify\left(v_{AC}(x) - \left(-\frac{P\cdot x^2}{48\cdot EI}\cdot(3\cdot L - 4\cdot x)\right)\right) = 0$$

$$simplify\left(\mathrm{D}[1](v_{AC})(x) - \left(-\frac{P\cdot x}{8\cdot EI}\cdot(L - 2\cdot x)\right)\right) = 0$$

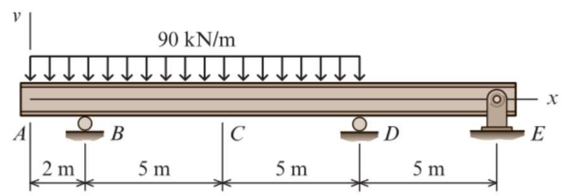
Exemplo 11.4
Referência: Philpot (2017). Mechanics of Materials: An integrated Learning System, 4th Edition. Wiley.

EXAMPLE 11.4

For the statically indeterminate beam shown, use discontinuity functions to determine

(a) the force reactions at B, D, and E.
(b) the deflection of the beam at A.
(c) the deflection of the beam at C.

Assume a constant value of $EI = 120{,}000\ \text{kN}\cdot\text{m}^2$ for the beam.



restart :

Dados no SI

$L_{AB}, L_{BC}, L_{CD}, L_{DE} := 2, 5, 5, 5 :$

$q, EI := 90\cdot 10^3, 120000\cdot 10^3 :$

$L := L_{AB} + L_{BC} + L_{CD} + L_{DE} = 17$

Reações de apoio (em função de R_E)

$sfv := R_B + R_D + R_E - q\cdot(L_{AB} + L_{BC} + L_{CD}) = 0 :$

$$smE := -R_B\cdot(L_{BC} + L_{CD} + L_{DE}) - R_D\cdot L_{DE} + q\cdot(L_{AB} + L_{BC} + L_{CD})\cdot\left(L_{DE} + \frac{L_{AB} + L_{BC} + L_{CD}}{2}\right) = 0 :$$

$assign(solve(\{sfv, smE\}, \{R_B, R_D\})) :$

$$R_B, R_D = 648000 + \frac{R_E}{2}, 432000 - \frac{3\,R_E}{2}$$

Momento fletor

$$M_{AB} := unapply\left(solve\left(M + q\cdot\frac{x^2}{2} = 0, M\right), x\right) :$$

$$M_{BD} := unapply\left(solve\left(M + q\cdot\frac{x^2}{2} - R_B\cdot(x - L_{AB}) = 0, M\right), x\right) :$$

$$M_{DE} := unapply(solve(-M + R_E\cdot(L_{AB} + L_{BC} + L_{CD} + L_{DE} - x) = 0, M), x) :$$

$$M := unapply(piecewise(x \geq 0 \textbf{ and } x \leq L_{AB}, M_{AB}(x), x > L_{AB} \textbf{ and } x \leq L - L_{DE}, M_{BD}(x), x > L - L_{DE} \textbf{ and } x \leq L, M_{DE}(x), 0), x) :$$

$$M(x) = \begin{cases} -45000\,x^2 & 0 \leq x \leq 2 \\ -45000\,x^2 + 648000\,x - 1296000 + \frac{1}{2}\,R_E\,x - R_E & 2 < x \leq 12 \\ -R_E\,(-17 + x) & 12 < x \leq 17 \\ 0 & otherwise \end{cases}$$

Equação da curva de deflexão

$bc := v(L_{AB}) = 0,\ v(L - L_{DE}) = 0,\ v(L) = 0 :$

$assign\left(dsolve\left(\left\{diff(v(x), x\$2) = \frac{M(x)}{EI}, bc\right\}, \{v(x), R_E\}\right)\right) :$

$v := unapply(v(x), x) :$

$v(x) =$

$$-\frac{47\,x}{3000} + \frac{191}{6000} + \left(\begin{cases} 0 & x \le 0 \\ -\frac{x^4}{32000} & x \le 2 \\ \frac{193}{240000}x^3 - \frac{193}{40000}x^2 - \frac{1}{32000}x^4 + \frac{193}{20000}x - \frac{193}{30000} & x \le 12 \\ -\frac{391}{40000}x^2 + \frac{23}{120000}x^3 + \frac{3541}{20000}x - \frac{26761}{30000} & x \le 17 \\ \frac{87\,x}{8000} + \frac{397}{8000} & 17 < x \end{cases}\right)$$

Respostas: Forças de reação, Deflexões em A e C

$R_B, R_D, R_E = 579000,\ 639000,\ -138000$

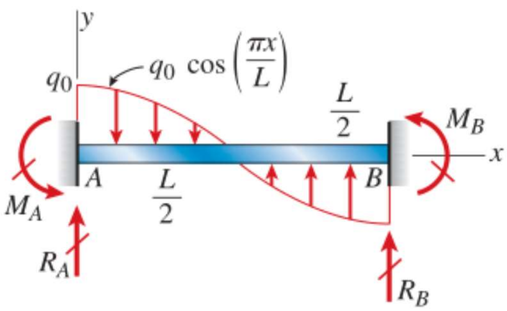
$\delta_A, \delta_C := -v(0.),\ -v(evalf(L_{AB} + L_{BC})) = -0.03183333333,\ 0.05234375000$

Problema 10.3-9
Referência: Gere & Goodno (2013). Mechanics of Materials, 8th Edition. Cengage Learning.

10.3-8 A fixed-end beam of length L is loaded by a distributed load in the form of a cosine curve with maximum intensity q_0 at A.

(a) Use the fourth-order differential equation of the deflection curve to solve for reactions at A and B and also the equation of the deflection curve.

(b) Repeat part (a) using the distributed load $q_0 \sin(\pi x/L)$.



PROB. 10.3-8

restart :

Equação diferencial de 4a ordem

$bc := v(0) = 0, v(L) = 0, \mathrm{D}[1](v)(0) = 0, \mathrm{D}[1](v)(L) = 0$:

$edo := diff(v(x), x\$4) = -\dfrac{q(x)}{EI}$:

A) usando a carga cossenoidal

$q := x \rightarrow q_0 \cdot \cos\left(\dfrac{\pi \cdot x}{L}\right)$:

$v := unapply(rhs(dsolve(\{edo, bc\}, v(x))), x)$:

$$v(x) = -\frac{6 L^2 q_0 x^2}{EI \pi^4} - \frac{q_0 L^4 \cos\left(\frac{\pi x}{L}\right)}{EI \pi^4} + \frac{4 L q_0 x^3}{EI \pi^4} + \frac{q_0 L^4}{EI \pi^4}$$

$V := unapply(EI \cdot diff(v(x), x\$3), x)$:

$R_A, R_B := V(0), -V(L)$:

$$R_A, R_B = \frac{24 L q_0}{\pi^4}, -\frac{24 L q_0}{\pi^4}$$

$M := unapply(EI \cdot diff(v(x), x\$2), x)$:

$assign\big(solve\big(\{M(0) + M_A = 0, -M(L) + M_B\}, \{M_A, M_B\}\big)\big)$:

$$M_A, M_B = -\frac{L^2 q_0 (\pi^2 - 12)}{\pi^4}, -\frac{L^2 q_0 (\pi^2 - 12)}{\pi^4}$$

B) usando a carga senoidal

$unassign('v', 'V', 'M', 'R_A', 'R_B', 'M_A', 'M_B')$:

$q := x \rightarrow q_0 \cdot \sin\left(\frac{\pi \cdot x}{L}\right)$:

$v := unapply(rhs(dsolve(\{edo, bc\}, v(x))), x)$:

$$v(x) = -\frac{L^2 q_0 x^2}{EI \pi^3} - \frac{q_0 L^4 \sin\left(\frac{\pi x}{L}\right)}{EI \pi^4} + \frac{q_0 L^3 x}{EI \pi^3}$$

$V := unapply(EI \cdot diff(v(x), x\$3), x)$:

$R_A, R_B := V(0), -V(L)$:

$$R_A, R_B = \frac{q_0 L}{\pi}, \frac{q_0 L}{\pi}$$

$M := unapply(EI \cdot diff(v(x), x\$2), x)$:

$assign(solve(\{M(0) + M_A = 0, -M(L) + M_B\}, \{M_A, M_B\}))$:

$$M_A, M_B = \frac{2 L^2 q_0}{\pi^3}, -\frac{2 L^2 q_0}{\pi^3}$$