Mecânica dos Sólidos 3 - AB2.2

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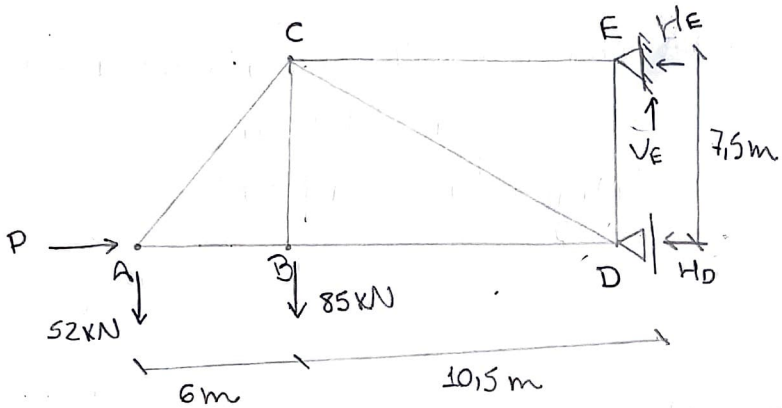
1.



$A = 1600\ mm^2 = 0,0016\ m^2$

$E = 200\ GPa = 2 \cdot 10^8\ KPa$

* Reações de apoio

$\Sigma F_x = 0$

$-H_D - H_E + P = 0 \rightarrow \boxed{H_E = -H_D + P}$

$\Sigma F_y = 0$

$-P - 85 + V_E = 0 \rightarrow V_E = 52 + 85$

$\boxed{V_E = 137\ KN}$

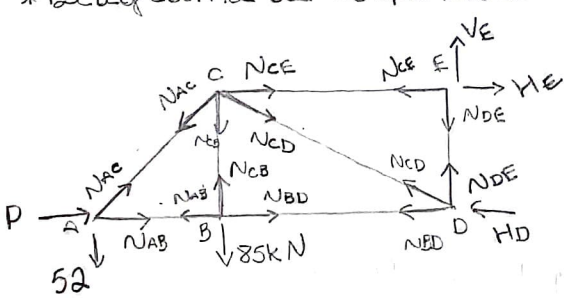
$\Sigma M_E = 0$

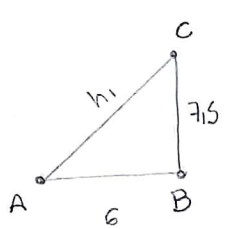
$52 \cdot 16,5 + P \cdot 7,5 + 85 \cdot 10,5 - H_D \cdot 7,5 = 0 \rightarrow H_D = \frac{858 + 7,5P + 892,5}{7,5}$

$H_D = \frac{1750,5 + 7,5P}{7,5}$

$\boxed{H_D = 233,4 + P}$

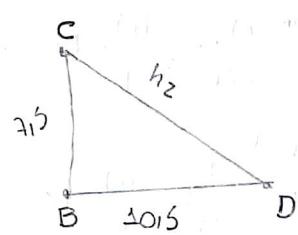
* Diagrama de corpo livre (Método dos Nós)





$h_1 = \sqrt{6^2 + 7,5^2}$

$h_1 = 9,6$



$h_2 = \sqrt{7,5^2 + 10,5^2}$

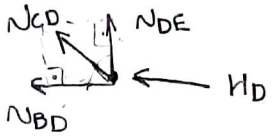
$h_2 = 12,9$

*Nó E

$\Sigma F_y = 0 \rightarrow V_E - N_{DE} = 0 \rightarrow 52 + 85 - N_{DE} = 0 \rightarrow N_{DE} = 52 + 85 \rightarrow \boxed{N_{DE} = 137\ KN}$

$\Sigma F_x = 0 \rightarrow H_E - N_{CE} = 0 \rightarrow -233,4 - N_{CE} = 0 \rightarrow \boxed{N_{CE} = -233,4\ KN}$

* Nó D



$$\Sigma F_Y = N_{DE} + N_{CD} \cdot \left(\frac{7,5}{12,9}\right) = 0 \rightarrow 137 + 0,581\, N_{CD} = 0 \rightarrow \boxed{N_{CD} = -235,7\ KN}$$

$$\Sigma F_X = 0 \rightarrow -N_{BD} - H_D - N_{CD}\left(\frac{10,5}{12,9}\right) = 0 \rightarrow N_{BD} = -233,4 - P + 235,7 \cdot 0,814 = 0$$

$$N_{BD} = -233,4 - P + 191,85$$

$$\boxed{N_{BD} = -41,6 - P}\ KN$$

* Nó B

$$\Sigma F_Y = 0 \rightarrow N_{CB} - 85 = 0 \rightarrow \boxed{N_{CB} = 85\ KN}$$

$$\Sigma F_X = 0 \rightarrow -N_{AB} + N_{BD} = 0 \rightarrow N_{AB} = N_{BD} \rightarrow \boxed{N_{AB} = (-41,6 - P)\ KN}$$

* Nó A

$$\Sigma F_Y = 0 \rightarrow -52 + N_{AC}\left(\frac{7,5}{9,6}\right) = 0 \rightarrow \boxed{N_{AC} = 66,59\ KN}$$

* Teorema de Castigliano ($P = 0$)

$$\Delta_{DE} = \int_0^{7,5} \frac{N_{DE}}{EA} \cdot \frac{dN}{dP}\, dx = 0\ m$$

$$\Delta_{CE} = \int_0^{10,5} \frac{N_{CE}}{EA} \cdot \frac{dN}{dP}\, dx = 0\ m$$

$$\Delta_{CD} = \int_0^{12,9} \frac{N_{CD}}{EA} \cdot \frac{dN}{dP}\, dx = 0\ m$$

$$\Delta_{BD} = \int_0^{10,5} \frac{N_{BD}}{EA} \cdot \frac{dN}{dP}\, dx = \int_0^{10,5} (-41,6 - P) \cdot (-1)\, dx = 1,365 \cdot 10^{-6}\ m$$

$$\Delta_{CB} = \int_0^{7,5} \frac{N_{CB}}{EA} \cdot \frac{dN}{dP}\, dx = 0\ m$$

$$\Delta_{AB} = \int_0^{6} \frac{N_{AB}}{EA} \cdot \frac{dN}{dP}\, dx = \int_0^{6} (-41,6 - P) \cdot (-1)\, dx = 7,8 \cdot 10^{-7}\ m$$

$$\Delta_{AC} = \int_0^{9,6} \frac{N_{AC}}{EA} \cdot \frac{dN}{dP}\, dx = 0$$

Sendo assim, o deslocamento horizontal em A:

$$\Delta_A = \Delta_{DE} + \Delta_{CE} + \Delta_{CD} + \Delta_{BD} + \Delta_{CD} + \Delta_{BD} + \Delta_{CB} + \Delta_{AB} + \Delta_{AC}$$

$$\Delta_A = 2{,}145 \cdot 10^{-3}\ m \text{ (esquerda para direita)}$$

2.

$\sigma_{adm} = 250\ MPa$

$E = 200\ GPa$

$I_z = 128 \cdot 10^6\ mm^4$

$I_y = 18,4 \cdot 10^6\ mm^4$

$r_z = 130\ mm$

$n_f = 2$

Utilizando a tabela para uma configuração com um engaste e uma extremidade livre → $K = 2$

* Carga Crítica (plano XZ)

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_y}{(KL)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 (18,4 \cdot 10^6 \cdot 10^{-12})}{(2 \cdot 0,7)^2}$$

$$\boxed{P_{cr} = 18530,6\ KN}$$

* Carga Crítica (plano XY)

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I_z}{(KL)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9 \cdot (128 \cdot 10^6 \cdot 10^{-12})}{(2 \cdot 9)^2}$$

$$\boxed{P_{cr} = 779,821\ KN}$$

Então, a carga crítica adotada será a menor ($P_{cr} = 779,821\ KN$)

* Tensão

$$\sigma_{cr} = \frac{\pi^2 \cdot E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \cdot 200 \cdot 10^9}{\left(\frac{2 \cdot 9}{130 \cdot 10^{-3}}\right)^2} = 102,9\ MPa$$

Como $\sigma_{cr} < \sigma_{adm}$, os calculos são validados.

* Utilizando os critérios de estabilidade

$$\sigma \leq \frac{\sigma_{cr}}{n}$$

$$\frac{P}{A} \leq \frac{F_{cr}}{A \cdot n}$$

Sendo assim,

$$P \leq \frac{Pcr}{n}$$

$$P \leq \frac{779,821}{2}$$

$$P \leq 389,91\,KN$$

Então,

a carga admissível é $389,91\,KN$