Universidade Federal de Alagoas - UFAL
Centro de Tecnologia - CTEC
Curso de Engenharia Civil

Mecânica dos Sólidos 3 - ECIV051D (2020.2)

Memorial de Cálculo da Lista de Exercícios:
Teorema de Castigliano e Instabilidade em Colunas

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Questão 1

restart :

Fazendo uso da simetria do problema

$$R_A := \frac{q \cdot L}{2} + \frac{P}{2} :$$

$$M_{AC} := \frac{q \cdot L \cdot x}{2} - \frac{q \cdot x^2}{2} + \frac{P \cdot x}{2} :$$

$$\delta_C := 2 \cdot \left(int\left(\frac{M_{AC}}{EI} \cdot diff\left(M_{AC}, P\right), x = 0 .. \frac{L}{4} \right) + int\left(\frac{M_{AC}}{2 \cdot EI} \cdot diff\left(M_{AC}, P\right), x = \frac{L}{4} .. \frac{L}{2} \right) \right) :$$

$$subs\left(P = 0, \delta_C\right) = \frac{31\ q\ L^4}{4096\ EI}$$

Questão 2

restart :

$$assign\left(solve\left(\left\{ R_A + R_B - P = 0, R_B \cdot L - P \cdot (L + a) = 0 \right\}, \left\{ R_A, R_B \right\} \right) \right) :$$

$$M_{AB}, M_{BC} := R_A \cdot x, -P \cdot x :$$

$$U_B := \frac{{R_B}^2}{2 \cdot k} :$$

$$\delta_C := diff\left(U_B, P\right) + int\left(\frac{M_{AB}}{EI} \cdot diff\left(M_{AB}, P\right), x = 0 .. L \right) + int\left(\frac{M_{BC}}{EI} \cdot diff\left(M_{BC}, P\right), x = 0 .. a \right) :$$

$$\delta_C = \frac{P\ (L + a)^2}{L^2\ k} + \frac{P\ a^2\ L}{3\ EI} + \frac{P\ a^3}{3\ EI}$$

Questão 3

restart :

$$R_B, R_C := \frac{3 \cdot q \cdot L}{4} - \frac{P}{4}, \frac{3 \cdot q \cdot L}{4} + \frac{5 \cdot P}{4} :$$

$$M_{AB} := -q \cdot x^2 :$$

$$M_{BC} := -q \cdot \frac{L}{4} \cdot \left(\frac{L}{8} + x \right) - \frac{q \cdot x^2}{2} + R_B \cdot x :$$

$$M_{CD} := -\frac{q \cdot x^2}{2} - P \cdot x :$$

$$\delta_{D,\ AB} := int\left(\frac{M_{AB}}{EI} \cdot diff\left(M_{AB}, P\right), x = 0 .. \frac{L}{4} \right) :$$

$$\delta_{D,\ BC} := int\left(\frac{M_{BC}}{EI} \cdot diff\left(M_{BC}, P\right), x = 0 .. L \right) :$$

$$\delta_{D,\ CD} := int\left(\frac{M_{CD}}{EI} \cdot diff\left(M_{CD}, P\right), x = 0 .. \frac{L}{4} \right) :$$

$$subs\left(P = 0, \delta_{D,\ AB} + \delta_{D,\ BC} + \delta_{D,\ CD}\right) = -\frac{37\ q\ L^4}{6144\ EI}$$

Questão 4

restart :

$$A, E := 4.5,\ convert\left(200.,'units','GPa',\ '1000\cdot\frac{lbf}{inch^2}'\right) = 4.5,\ 29007.54755$$

AB, BC, CD, DE, EF, AF, AE, CE, BE

$$N := P\cdot\left\langle \frac{2}{3}, \frac{2}{3}, 0, 0, 0, 0, -\frac{5}{6}, -\frac{5}{6}, 1\right\rangle :$$

$$L := \langle 96, 96, 72, 96, 96, 72, 120, 120, 72\rangle :$$

$$subs\left(P=5,\ add\left(N\cdot\sim diff\left(N, P\right)\cdot\sim\frac{L}{E\cdot A}\right)\right) = 0.0124105631247000$$

AB, BC, CD, DE, EF, AF, AE, CE, BE

$$N := \left\langle \frac{2\cdot P+10}{3}, \frac{2\cdot P+10}{3}, 0, 0, 0, 0, -\frac{5\cdot P+25}{6}, -\frac{5\cdot P+25}{6}, 5\right\rangle :$$

$$subs\left(P=0,\ add\left(N\cdot\sim diff\left(N, P\right)\cdot\sim\frac{L}{E\cdot A}\right)\right) = 0.00965266020810000$$

Questão 5

(A)

restart :

$$\Sigma M_A := -\left(\beta\cdot\theta\cdot a\right)\cdot a + -\beta_R\cdot\theta + R_C\cdot L :$$

$$R_C := solve\left(\Sigma M_A = 0, R_C\right) = \frac{\theta\left(a^2\beta+\beta_R\right)}{L}$$

$$\Sigma M_B := R_C\cdot\left(L-a\right) - P\cdot\left(\theta\cdot a\right) :$$

$$collect\left(\Sigma M_B = 0, \theta\right) = \left(\frac{\left(a^2\beta+\beta_R\right)\left(L-a\right)}{L} - P\,a\right)\theta = 0$$

$$P_{cr} := solve\left(diff\left(lhs\left(\%\right), \theta\right) = 0, P\right) = \frac{\left(a^2\beta+\beta_R\right)\left(L-a\right)}{L\,a}$$

(B)

restart :

$$\Sigma M_A := -\left(\beta\cdot\theta\cdot\frac{L}{2}\right)\cdot\frac{L}{2} + -\beta_R\cdot\theta + R_C\cdot L :$$

$$R_C := solve\left(\Sigma M_A = 0, R_C\right) = \frac{1}{4}\ \frac{\theta\left(L^2\beta+4\,\beta_R\right)}{L}$$

$$\Sigma M_B := R_C\cdot\frac{L}{2} - P\cdot\left(\frac{\theta\cdot L}{2}\right) + \beta_R\cdot\left(2\cdot\theta\right) :$$

$$collect\left(\Sigma M_B = 0, \theta\right) = \left(\frac{1}{8}\,\beta\,L^2 + \frac{5}{2}\,\beta_R - \frac{1}{2}\,P\,L\right)\theta = 0$$

$$P_{cr} := solve\left(diff\left(lhs\left(\%\right), \theta\right) = 0, P\right) = \frac{1}{4}\ \frac{L^2\beta+20\,\beta_R}{L}$$

Questão 6

restart :

$I_1, I_2 := \frac{b \cdot h^3}{12}, \frac{h \cdot b^3}{12}$:

$P_1, P_2 := \frac{\pi^2 \cdot E \cdot I_1}{L^2}, \frac{\pi^2 \cdot E \cdot I_2}{\left(\frac{L}{2}\right)^2}$:

$solve(P_1 = P_2, h) = 0, 2\ b, -2\ b$

h, b > 0 => h = 2b

Questão 7

restart :

$L, E, I_1, I_2 := 4, 200 \cdot 10^9, 2140 \cdot (10^{-2})^4, 117 \cdot (10^{-2})^4$:

$P_{cr1}, P_{cr2} := \frac{\pi^2 \cdot E \cdot I_1}{4. \cdot L^2}, \frac{2.046 \cdot \pi^2 \cdot E \cdot I_2}{L^2} = 6.600297945\ 10^5, 2.953257052\ 10^5$

$P_{cr} := \min(P_{cr1}, P_{cr2}) = 2.953257052\ 10^5$

Questão 8

restart :

$E := 200. \cdot 10^9$:

$L, d_2, t := 7., 100. \cdot 10^{-3}, 6. \cdot 10^{-3}$:

$\theta_1, \theta_2 := 40. \cdot \frac{\pi}{180}, 55. \cdot \frac{\pi}{180}$:

$d_1 := d_2 - 2 \cdot t$:

$mI := \frac{\pi}{64} \cdot (d_2^4 - d_1^4) = 0.000001964990806$

$L_{AB}, L_{BC} := \frac{L \cdot \sin(\theta_2)}{\sin(\pi - \theta_1 - \theta_2)}, \frac{L \cdot \sin(\theta_1)}{\sin(\pi - \theta_I - \theta_2)} = 5.755967506, 4.516700679$

$P_{crAB}, P_{crBC} := \frac{\pi^2 \cdot E \cdot mI}{L_{AB}^2}, \frac{\pi^2 \cdot E \cdot mI}{L_{BC}^2} = 1.170721507\ 10^5, 1.901286840\ 10^5$

Resolvendo equilíbrio em B

$solve(\{N_{AB} \cdot \cos(\theta_1) - N_{BC} \cdot \cos(\theta_2) = 0, N_{AB} \cdot \sin(\theta_1) + N_{BC} \cdot \sin(\theta_2) - W = 0\}, \{N_{AB}, N_{BC}\})$:

assign(%) :

$N_{AB}, N_{BC} = 0.5757674049\ W, 0.7689706083\ W$

$W_{crAB} := solve(N_{AB} = P_{crAB}, W) = 2.033323695\ 10^5$

$W_{crBC} := solve(N_{BC} = P_{crBC}, W) = 2.472509117\ 10^5$

$W_{cr} := \min(W_{crAB}, W_{crBC}) = 2.033323695\ 10^5$