Universidade Federal de Alagoas - UFAL
Centro de Tecnologia - CTEC
Curso de Engenharia Civil

Mecânica dos Sólidos 3 - ECIV051D (2020.2)

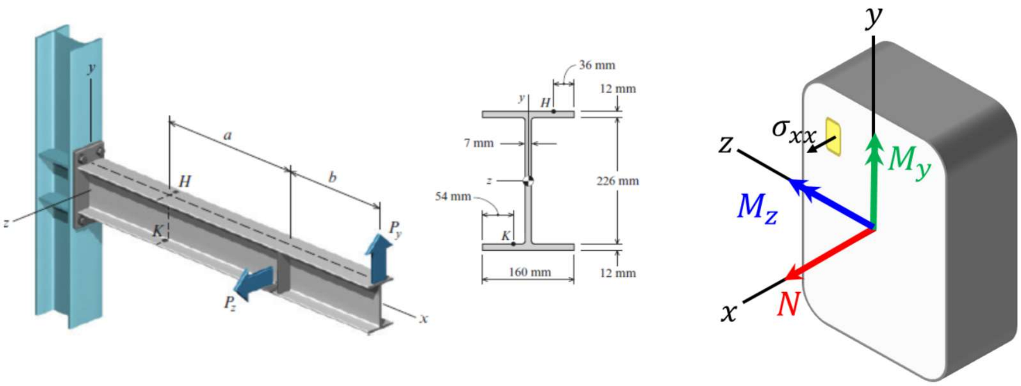
Exercícios:
Flexão oblíqua e composta
Encontro Assíncrono

Monitores:
Hugo Vinícius F. Azevedo
Milton Mateus G. Santos
Ricardo A. Fernandes

Professor/Supervisor:
Adeildo S. Ramos Jr.

Maceió/AL, 08/09/2021

Exemplo 2



Determinar as tensões normais nos pontos H, K e as tensões normais máximas na mesma seção transversal da viga mostrada.
Dados: $a = 1{,}15\ m; b = 0{,}85\ m; P_y = 13\ kN; P_z = 6\ kN$

restart :
Dados no SI
$a, b := 1.15, 0.85$:
$P_y, P_z := 13.\cdot 10^3, 6.\cdot 10^3$:

Propriedades geométricas

$A := convert(226.\cdot 7 + 2\cdot 160\cdot 12, 'units', 'mm^2', 'm^2') = 0.005422000000$

$I_z := \int y^2 \, \mathrm{d}A$

$I_z := \frac{7\cdot 226^3}{12.} + 2\cdot\left(\frac{160\cdot 12^3}{12} + (160\cdot 12)\cdot\left(\frac{226}{2} + \frac{12}{2}\right)^2\right)$:

$I_z := convert(I_z, 'units', 'mm^4', 'm^4') = 0.00006115783933$

$I_y := \int z^2 \, \mathrm{d}A$

$I_y := \frac{226\cdot 7^3}{12.} + 2\cdot\left(\frac{12\cdot 160^3}{12}\right)$:

$I_y := convert(I_y, 'units', 'mm^4', 'm^4') = 0.000008198459833$

$I_{yz} := \int y\cdot z \, \mathrm{d}A$

$I_{yz} := 0 + 0 + (160\cdot 12)\cdot\left(\frac{226}{2} + \frac{12}{2}\right)\cdot(0 - 0) + 0 + (160\cdot 12)\cdot\left(-\frac{226}{2} - \frac{12}{2}\right)\cdot(0 - 0) = 0$

Esforço normal e momentos fletores

$N := 0.$:
$M_z := P_y\cdot(a + b) = 26000.00$
$M_y := -P_z\cdot a = -6900.00$

$\sigma_x := (y, z) \rightarrow \frac{N}{A} - \frac{M_z\cdot I_y + M_y\cdot I_{yz}}{I_z\cdot I_y - I_{yz}^2}\cdot y + \frac{M_y\cdot I_z + M_z\cdot I_{yz}}{I_z\cdot I_y - I_{yz}^2}\cdot z$:

Tensões normais em H e K

$$y_H, z_H := \frac{\frac{226}{2}+12}{1000.}, \frac{-\frac{160}{2}+36}{1000.} = 0.1250000000, -0.04400000000$$

$$y_K, z_K := -\frac{\frac{226}{2}}{1000.}, \frac{\frac{160}{2}-54}{1000.} = -0.1130000000, 0.02600000000$$

$$\sigma_H := \sigma_x(y_H, z_H) = -1.610983845\ 10^7$$

$$\sigma_K := \sigma_x(y_K, z_K) = 2.615747165\ 10^7$$

-16,11 MPa (H)
+26,16 MPa (K)

Tensões normais máximas
Ao longo do eixo vertical

$$\sigma_x(y, 0) = -4.251294730\ 10^8\ y$$

$$\sigma_x\left(\frac{\frac{226}{2}+12}{1000.}, 0\right) = -5.314118412\ 10^7$$

Ao longo do topo

$$\sigma_x\left(\frac{\frac{226}{2}+12}{1000.}, z\right) = -5.314118412\ 10^7 - 8.416214925\ 10^8\ z$$

$$\sigma_x\left(\frac{\frac{226}{2}+12}{1000.}, \frac{\frac{160}{2}}{1000.}\right) = -1.204709035\ 10^8$$

$$\frac{\frac{226}{2}+12}{1000.}, \frac{\frac{160}{2}}{1000.} = 0.1250000000, 0.08000000000$$

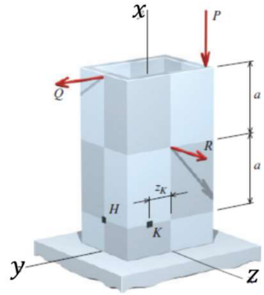
Ao longo da base

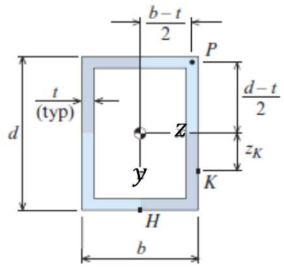
$$\sigma_x\left(\frac{-\frac{226}{2}-12}{1000.}, z\right) = 5.314118412\ 10^7 - 8.416214925\ 10^8\ z$$

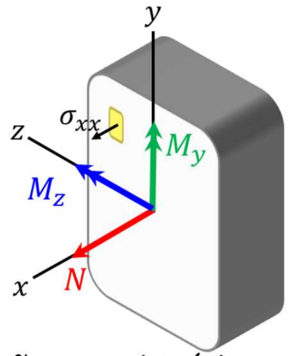
$$\sigma_x\left(\frac{-\frac{226}{2}-12}{1000.}, -\frac{\frac{160}{2}}{1000.}\right) = 1.204709035\ 10^8$$

-120,47 MPa em y=0,125 m e z=0,08 m
+120,47 MPa em y=-0,125 m e z=-0,08 m

Exemplo 3







Determinar as tensões normais nos pontos H, K e as tensões normais máximas na mesma seção transversal da coluna mostrada. Dados: $a = 125\ mm$; $b = 150\ mm$; $d = 200\ mm$; $t = 10\ mm$ $P = 13\ kN$; $Q = 60\ kN$; $R = 85\ kN$

restart :

Dados no SI

$a, b, d, t := 125.\cdot 10^{-3}, 150.\cdot 10^{-3}, 200\cdot 10^{-3}, 10.\cdot 10^{-3}$:

$P, Q, R := 13.\cdot 10^{3}, 60.\cdot 10^{3}, 85.\cdot 10^{3}$:

Propriedades geométricas

$A := b\cdot d - (b - 2\cdot t)\cdot(d - 2\cdot t) = 0.00660000000$

$$I_z := \frac{b\cdot d^3}{12} - \frac{(b-2\cdot t)\cdot(d-2\cdot t)^3}{12} = 0.00003682000000$$

$$I_y := \frac{d\cdot b^3}{12} - \frac{(d-2\cdot t)\cdot(b-2\cdot t)^3}{12} = 0.00002329500000$$

$I_{yz} := 0$:

Esforço normal e momentos fletores

$N := -P = -13000.$

with(*LinearAlgebra*) :

$$r_P := \left\langle 2\cdot a, -\frac{d-t}{2}, \frac{b-t}{2} \right\rangle :$$

$$r_Q := \left\langle 2\cdot a, \frac{d}{2.}, 0 \right\rangle :$$

$$r_R := \left\langle a, 0, \frac{b}{2} \right\rangle :$$

$M := CrossProduct\left(r_P, \langle -P, 0, 0\rangle\right) + CrossProduct\left(r_Q, \langle 0, Q, 0\rangle\right) + CrossProduct\left(r_R, \langle 0, 0, R\rangle\right)$:

$M^{\%T} = \begin{bmatrix} 0. & -11535. & 13765. \end{bmatrix}$

$M_y, M_z := M[2], M[3]$:

$M_y, M_z = -11535., 13765.$

$$\sigma_x := (y, z) \rightarrow \frac{N}{A} - \frac{M_z\cdot I_y + M_y\cdot I_{yz}}{I_z\cdot I_y - I_{yz}^2}\cdot y + \frac{M_y\cdot I_z + M_z\cdot I_{yz}}{I_z\cdot I_y - I_{yz}^2}\cdot z :$$

Tensão normal em H e K

$y_H, z_H := \frac{d}{2.}, 0 = 0.1000000000, 0$

$y_K, z_K := \frac{b}{2.}, \frac{b}{2.} = 0.07500000000, 0.07500000000$

$\sigma_H := \sigma_x(y_H, z_H) = -3.93542705713036\ 10^7$

$\sigma_K := \sigma_x(y_K, z_K) = -6.71459249816667\ 10^7$

-39,35 MPa (H)
-67,15 MPa (K)

Tensões normais máximas

$\sigma_x\left(-\frac{d}{2}, z\right) = 3.54148766313036\ 10^7 - 4.95170637475853\ 10^8\ z$

$\sigma_x\left(-\frac{d}{2} + t, z\right) = 3.16764192711733\ 10^7 - 4.95170637475853\ 10^8\ z$

$\sigma_x(0, z) = -1.96969697000000\ 10^6 - 4.95170637475853\ 10^8\ z$

$\sigma_x\left(\frac{d}{2} - t, z\right) = -3.56158132111733\ 10^7 - 4.95170637475853\ 10^8\ z$

$\sigma_x\left(\frac{d}{2}, z\right) = -3.93542705713036\ 10^7 - 4.95170637475853\ 10^8\ z$

$\sigma_x\left(y, -\frac{b}{2}\right) = 3.51681008406890\ 10^7 - 3.73845736013036\ 10^8\ y$

$\sigma_x\left(y, -\frac{b}{2} + t\right) = 3.02163944659305\ 10^7 - 3.73845736013036\ 10^8\ y$

$\sigma_x(y, 0) = -1.96969697000000\ 10^6 - 3.73845736013036\ 10^8\ y$

$\sigma_x\left(y, \frac{b}{2} - t\right) = -3.41557884059305\ 10^7 - 3.73845736013036\ 10^8\ y$

$\sigma_x\left(y, \frac{b}{2}\right) = -3.91074947806890\ 10^7 - 3.73845736013036\ 10^8\ y$

$\sigma_x\left(-\frac{d}{2}, -\frac{b}{2}\right) = 7.25526744419926\ 10^7$

$\sigma_x\left(\frac{d}{2}, \frac{b}{2}\right) = -7.64920683819926\ 10^7$

+72,55 MPa em y=-d/2 e z=-b/2
-76,49 MPa em y=d/2 e z=b/2