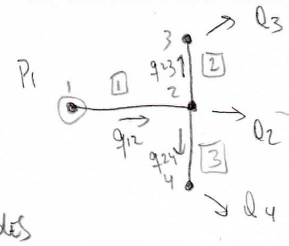
STEADY-STATE SIMULATION OF A GAS NETWORK



BASIC EQUATIONS

m: number of nodes

m_1: number of discriminated nodes (sources)

m: number of branches

→ KIRCHHOFF'S FIRST LAW

Σ flow at any node $= 0$

$$\underline{A}_1 \, \underline{Q} = \underline{L}$$

NODE LOAD VECTOR $\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_{m-m_1} \end{bmatrix}$

$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{bmatrix}$ BRANCH FLOW VECTOR

Reduced branch-node incidence matrix

$$\underline{A}_1 = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{(m-m_1)\times m} \quad \underline{Q} = \begin{bmatrix} q_{12} \\ q_{23} \\ q_{24} \end{bmatrix}_{m\times 1} \quad \underline{L} = \begin{bmatrix} Q_2 \\ Q_3 \\ Q_4 \end{bmatrix}_{(m-m_1)\times 1}$$

$$\underline{A}_1 = [a_{ij}]_{(m-m_1)\times m}$$

$$a_{ij} = \begin{cases} +1, & \text{flow in branch } j \text{ enters node } i \\ -1, & \text{" " " " leaves " "} \\ 0, & \text{branch } j \text{ is not incident to node } i \end{cases}$$

u: number of units

$$\underline{K} = [K_{ij}]_{m\times u} \qquad K_{ij} = \begin{cases} +1, & j^{th} \text{ unit has inlet at } i \\ -1, & j^{th} \text{ " " outlet " "} \\ 0, & \text{otherwise} \end{cases}$$

Taking into account UNITS

- SOURCES
- COMPRESSORS
- REGULATORS
- VALVES

and assuming the flow through each unit to be a positive demand at the inlet node and neg. at the outlet

flow through unit

$$\underline{f} = [f_1 \; f_2 \; \dots \; f_u]^T$$

branch-node incidence matrix

$$\Rightarrow \quad \underset{m\times m}{\underline{A}}\,\underset{m\times 1}{\underline{Q}} - \underset{m\times u}{\underline{K}}\,\underset{m\times 1}{\underline{f}} = \underset{m\times 1^*}{\underline{L}}$$

→ KIRCHHOFF'S SECOND LAW

Δp around any closed loop $= 0$

$$\underline{B}\,\underline{\Delta P} = \underline{0}$$

k: number of loops

pressure drop vector $\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_m \end{bmatrix}$

BRANCH-LOOP INCIDENCE MATRIX

$$\underline{B} = [b_{ij}]_{k\times m}$$

$$b_{ij} = \begin{cases} +1, & \text{branch } j \text{ is in loop } i \text{ (same directions)} \\ -1, & \text{" " " " " " (opposite ")} \\ 0, & \text{branch } j \text{ is not in loop } i \end{cases}$$

$$\underline{A} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{m\times m} \qquad \underline{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}_{m\times 1}$$

$$\underline{\Delta P} = -\underline{A}^T \underline{P} = \begin{bmatrix} P_1 - P_2 \\ P_2 - P_3 \\ P_2 - P_4 \end{bmatrix} \quad ✓$$

$$\Rightarrow \underline{A}\,\underline{B}^T = \underline{B}\,\underline{A}^T = \underline{0}$$

For high and medium pressure

$$P_i = p_i^2, \qquad \Delta P = p_i^2 - p_j^2$$

→ **Pipe flow equations**

flow exponent

$$\Delta \underline{P} = \phi(\underline{Q})$$

mapping $\phi : \mathbb{R}^m \to \mathbb{R}^m \Rightarrow \phi_i = K_i |Q_i|^{m_i - 1} Q_i \quad , \quad i = 1, 2, \ldots, m$

Approximations

a) low pressure networks (0 – 0,75 bar gauge)

$\Delta P = p_1 - p_2 = K Q^2$ [Lacey's equation] → $K = 11.7 \times 10^3 \; L D^{-5}$ [m] [m]

Q [m³/h], P [bars]

b) medium pressure (0.75 – 7 bar gauge)

$\Delta P = P_1 - P_2 = K Q^{1.848}$ [Polyflo's eq] → $K = 27{,}24 \; (L E^{-2} D^{-4.848})$ [m] [0.9] [mm]

Q [m³/h], P [bar²]

c) high pressure (≥7 bar gauge)

$\Delta P = P_1 - P_2 = K Q^{1.854}$ [Panhandle A] → $K = 18.43 \; (L E^{-2} D^{-4.854})$ [m] (0.9) [mm]

Q [m³/h], P [bar²]

Modelling of the units

We assume that there is an equation linking the inlet and outlet pressures (for high and medium pressure – squared pressures) and the flow through the unit.

$$C_1 P^* + C_2 P + C_3 f = d$$

P^*: inlet node pressure; P: outlet node pressure; f: flow through the unit

C_1, C_2, C_3, d are coefficients

Proper values:

a) Inlet pressure kept constant at P^*_{SET}

$C_1 = 1 \quad C_2 = C_3 = 0 \quad d = P^*_{SET}$

b) Outlet pressure kept constant at P_{SET}

$C_1 = C_3 = 0 \quad C_2 = 1 \quad d = P_{SET}$

c) Pressures ratio $\varepsilon = P^*/P$ kept constant at ε_{SET}

$C_1 = \varepsilon_{SET} \quad C_2 = -1 \quad C_3 = d = 0$

d) Flow through unit kept constant at f_{SET}

$C_1 = C_2 = 0 \quad C_3 = 1 \quad d = f_{SET}$

non-unit outlet node pressures; unit outlet pressures; unit flows

$$\underset{u \times (m-u)}{\underline{C_1}} \; \underset{(m-u) \times 1}{\underline{P^*}} + \underset{u \times u}{\underline{C_2}} \; \underset{u \times 1}{\underline{P}} + \underset{u \times u}{\underline{C_3}} \; \underset{u \times 1}{\underline{f}} = \underset{u \times 1}{\underline{d}}$$

$C_{1\,ij} \neq 0$ if P_j^* is the inlet node of unit i and if P_j^* is kept constant during simulation

$\underline{C_2}$ and $\underline{C_3}$ are diagonal

depends on the structure of the network

Method of Steady-State Simulation with Units

$$\Delta \underline{P} = \underline{\phi}(\underline{Q}) \Rightarrow -\underline{A}^T \underline{P} - \underline{\phi}(\underline{Q}) = \underline{0}$$

Linearizing flow equation: $\underline{\phi}(\underline{Q}) = \underline{\Lambda}\,\underline{Q}$ $\Rightarrow -\underline{A}^T\underline{P} - \underline{\Lambda}\,\underline{Q} = \underline{0}$

$$\underline{\Lambda} = \operatorname{diag}\left\{ K_i |Q_i|^{m_j - 1} \right\}$$

$$-\underline{A}^T \underline{P} - \underline{\Lambda}\,\underline{Q} = \underline{0}$$

$$-\underline{A}\,\underline{\Lambda}^{-1}\underline{A}^T \underline{P} - \underline{A}\,\underline{Q} = \underline{0} \xrightarrow{\underline{A}\underline{Q} - \underline{K}\underline{f} = \underline{L}} \underbrace{(\underline{A}\,\underline{\Lambda}^{-1}\underline{A}^T)}_{\underline{G}\ \text{(symmetric)}} \underline{P} = -\underline{L} - \underline{K}\underline{f}$$

sparse

* The number of non-zero elements of row i of $\underline{G}$ is equal to the number of nodes which are incident to node i $(+1)$.

* Each unit has two nodes except sources

$$\underline{G}\,\underline{P} = -\underline{L} - \underline{K}\underline{f} \Rightarrow \begin{bmatrix} G_N & \hat{G} & K^I \\ \hat{G}^T & G^* & K^O \end{bmatrix} \begin{bmatrix} P^* \\ P \\ f \end{bmatrix} = -\begin{bmatrix} L^I \\ L^O \end{bmatrix}$$

$\underline{G}_N$: connections within the pipes network

$\hat{\underline{G}}$: connections between units and the network

$\underline{G}^*$: pipe connections between units

$\underline{K}^I, \underline{K}^O$: inlet and outlet parts of $\underline{K}$

$\underline{P}^*$: non-outlet pressures

$\underline{P}$: outlet pressures

$\underline{L}^I, \underline{L}^O$: demands not at outlet and at outlet nodes

$$\rightsquigarrow \begin{bmatrix} \underline{G}_N & \hat{\underline{G}} & \underline{K}^I \\ \hat{\underline{G}}^T & \underline{G}^* & \underline{K}^O \\ \underline{C}_1 & \underline{C}_2 & \underline{C}_3 \end{bmatrix} \begin{bmatrix} \underline{P}^* \\ \underline{P} \\ \underline{f} \end{bmatrix} = \begin{bmatrix} -\underline{L}^I \\ -\underline{L}^O \\ \underline{d} \end{bmatrix}$$

symmetric p.d.

Cholesky's decomposition

$$\underline{G}_N = \underline{L}\,\underline{L}^T = \underline{L}\,\underline{U}$$

$$\underline{U}\,\underline{P}^* = -\underline{L}^{-1}\underline{L}^I - \underline{L}^{-1}\hat{\underline{G}}\underline{P} - \underline{L}^{-1}\underline{K}^I\underline{f}$$

$$\Rightarrow \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} P \\ f \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

$$D_{11} = G^* - \hat{G}^T U^{-1} L^{-1} \hat{G}$$

$$D_{12} = K^O - \hat{G}^T U^{-1} L^{-1} K^I$$

$$D_{21} = C_2 - C_1 U^{-1} L^{-1} \hat{G}$$

$$D_{22} = C_3 - C_1 U^{-1} L^{-1} K^I$$

$$R_1 = -L^O + \hat{G}^T U^{-1} L^{-1} L^I$$

$$R_2 = d + C_1 U^{-1} L^{-1} L^I$$

* Solve for $\underline{P}$ and $\underline{f}$

Crout's method iterative until convergence