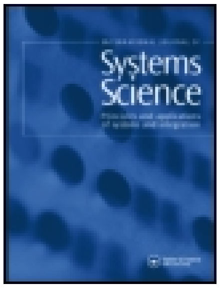

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Method of steady-state simulation of a gas network

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Method of steady-state simulation of a gas network

A. J. OSIADACZ†

A method is described for the steady-state simulation of an arbitrary gas network. Basic equations for a steady-state analysis are given. For controllable elements of the network, such as sources, compressors, regulators, and valves, the equation linking the inlet pressure, outlet pressure, and the flow through each unit is formulated. The set of equations for the steady-state simulation of a gas network and numerical methods for their solution are also described. Two examples are given to illustrate the properties of the method.

1. Introduction

The simulation of gas networks makes use of models of the gas flow in pipes as well as mathematical models of other elements such as sources, valves, regulators, and compressors. The simulation allows us to predict the behaviour of gas network systems under different conditions. Prediction by simulation can be used to guide decisions regarding the design and operation of the real system. At the stage of designing a network, simulation helps to select a structure for the network, as well as the geometric parameters of the pipes in the case of given parameters of gas supply and demands. The control of the network also requires simulation in order to obtain information about the pressures and flow rates at a given point of the network. In this paper a steady-state simulation method for a network has been described.

2. Basic equations for steady-state analysis

2.1. *Kirchhoff's first law*

Kirchhoff's first law states that the algebraic sum of the flows at any node is zero. This means that the load at any node is equal to the sum of the branch flows into and out of the node. We can write Kirchhoff's first law in the following matrix form:

$$\mathbf{A}_1\mathbf{Q} = \mathbf{L} \tag{1}$$

where $\mathbf{A}_1$ is the reduced branch-node incidence matrix

$$\mathbf{A}_1 = [a_{ij}]_{(n-n_1)\times m}$$

$$a_{ij} = \begin{cases} +1, & \text{if the flow in branch } j \text{ enters node } i \\ -1, & \text{if the flow in branch } j \text{ leaves node } i \\ \;\;0, & \text{if branch } j \text{ is not incident to node } i \end{cases}$$

n is the number of nodes, n_1 is the number of discriminated nodes (sources), m is the number of branches,

$$\mathbf{Q} = [Q_1 \quad Q_2 \quad \cdots \quad Q_m]^{\mathrm{T}}$$

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is the branch flow vector,

$$\mathbf{L} = [L_1 \quad L_2 \quad \dots \quad L_{n-n_1}]^T$$

is the node load vector.

Taking into account units, i.e. sources, compressors (compressor stations), regulators, and valves, and assuming the flow through each unit to be a positive demand at the inlet node and a negative demand at the outlet, (1) can be rewritten as

$$\mathbf{AQ} - \mathbf{Kf} = \mathbf{L} \tag{2}$$

where $\mathbf{A}$ is the branch-node incidence matrix

$$\dim \mathbf{A} = n \times m$$

$$\mathbf{K} = [k_{ij}]_{n \times u}$$

$$k_{ij} = \begin{cases} +1, & \text{if the } j\text{th unit has its inlet at node } i \\ -1, & \text{if the } j\text{th unit has its outlet at node } i \\ 0, & \text{otherwise} \end{cases}$$

u is the number of units,

$$\mathbf{f} = [f_1 \quad f_2 \quad \dots \quad f_u]^T$$

f_i is the flow through the ith unit.

2.2. *Kirchhoff's second law*

Kirchhoff's second law states that the pressure drop around any closed loop is zero. We can write Kirchhoff's second law in the following matrix form:

$$\mathbf{B}\Delta\mathbf{P} = \mathbf{0} \tag{3}$$

where $\mathbf{B}$ is the branch-loop incidence matrix

$$\mathbf{B} = [b_{ij}]_{k \times m}$$

$$b_{ij} = \begin{cases} +1, & \text{if branch } j \text{ is in loop } i \text{ and their directions are the same} \\ -1, & \text{if branch } j \text{ is in loop } i \text{ and their directions are opposite} \\ 0, & \text{if branch } j \text{ is not in loop } i \end{cases}$$

k is the number of loops.

$$\Delta\mathbf{P} = [\Delta P_1 \quad \Delta P_2 \quad \dots \quad \Delta P_m]^T$$

is the pressure drop vector.

Kirchhoff's second law can also be expressed in the form

$$-\mathbf{A}^T\mathbf{P} = \Delta\mathbf{P} \ (\mathbf{AB}^T = \mathbf{BA}^T = \mathbf{0}) \tag{4}$$

where

$$\mathbf{P} = [P_1 \quad P_2 \quad \dots \quad P_n]^T$$

is the pressure node vector.

(For high and medium pressure $P_i = P_i^2$, $\Delta P = P_i^2 - P_j^2$.)

2.3. *Pipe flow equations*

The flow equation is the following:

$$\Delta \mathbf{P} = \boldsymbol{\phi}(\mathbf{Q}) \tag{5}$$

where $\boldsymbol{\phi}: \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a mapping whose components are

$$\phi_i = K_i |Q_i|^{m_i - 1} Q_i, \quad i = 1, 2, \ldots, m$$

and m_i is the flow exponent.

The coefficient K_i takes values according to the approximating equations for the pipe flow. Depending on the pressure level which characterizes the gas networks, the following approximating equations are commonly used (Osiadacz 1987).

(*a*) Low pressure networks operating between 0–0·75 bar gauge

$$\Delta P = p_1 - p_2 = KQ^2 \quad \text{(Lacey's equation)} \tag{6}$$

where $K = 11{\cdot}7 \times 10^3\ (LD^{-5})$, with L: length (m), D: diameter (m), P (bars), Q ($\mathrm{m^3\,h^{-1}}$).

(*b*) Medium pressure networks operating between 0·75–7 bar gauge:

$$\Delta P = P_1 - P_2 = KQ^{1{\cdot}848} \quad \text{(Polyflo equation)} \tag{7}$$

where $K = 27{\cdot}24\ (LE^{-2}D^{-4{\cdot}848})$, with E: efficiency factor (0·9), L (m), D (mm), P ($\mathrm{bar^2}$), Q ($\mathrm{m^3\,h^{-1}}$).

(*c*) High pressure networks operating above 7·0 bar gauge:

$$\Delta P = P_1 - P_2 = KQ^{1{\cdot}854} \quad \text{(Panhandle 'A' equation)} \tag{8}$$

where $K = 18{\cdot}43\ (LE^{-2}\ D^{-4{\cdot}854}$, E (0·9), L (m), D (mm), P ($\mathrm{bar^2}$), Q ($\mathrm{m^3\,h^{-1}}$).

3. Modelling of the units

The controllable elements of the network are sources, compressors, regulators, and valves. For the purposes of a steady-state simulation we assume that there is an equation linking the inlet and outlet pressures (for high and medium pressure—squared pressures) and the flow through the unit. This equation is of the form (see Goldwater *et al.* 1976)

$$c_1 P^* + c_2 P + c_3 f = d \tag{9}$$

where P^* is the inlet node pressure (squared pressure); P is the outlet node pressure (squared pressure); f is the flow through the unit; c_1, c_2, c_3, d are coefficients.

According to the assumptions that are made for the simulation run, proper values are assigned to the coefficients of (9) specifically as follows.

(*a*) The inlet pressure (squared pressure) is kept constant at P^*_{set}. Then

$$c_1 = 1, \quad c_2 = c_3 = 0, \quad d = P^*_{\mathrm{set}}$$

(*b*) The outlet pressure (squared pressure) is kept constant at P_{set}. Then

$$c_1 = c_3 = 0, \quad c_2 = 1, \quad d = P_{\mathrm{set}}$$

(*c*) The pressures ratio (the pressures squared ratio) $\varepsilon = P^*/P$ is kept constant at $\varepsilon_{\mathrm{set}}$. Then

$$c_1 = \varepsilon_{\mathrm{set}}, \quad c_2 = -1, \quad c_3 = d = 0$$

(*d*) The flow through the unit is kept constant at f_{set}. Then

$$c_1 = c_2 = 0, \quad c_3 = 1, \quad d = f_{set}$$

For the whole network the unit equations can be written in matrix form as

$$\mathbf{C}_1 \mathbf{P}^* + \mathbf{C}_2 \mathbf{P} + \mathbf{C}_3 \mathbf{f} = \mathbf{d} \tag{10}$$

where

$$\dim \mathbf{C}_1 = u \times (n - u)$$

$\mathbf{P}^*$ is a vector of non-unit outlet node pressures,

$$\dim \mathbf{P}^* = (n - u) \times 1$$

$$\dim \mathbf{C}_2 = u \times u$$

$\mathbf{P}$ is a vector of unit outlet pressures,

$$\dim \mathbf{P} = u \times 1$$

$$\dim \mathbf{C}_3 = u \times u$$

$\mathbf{f}$ is a vector of unit flows,

$$\dim \mathbf{f} = u \times 1$$

$$\dim \mathbf{d} = u \times 1$$

The *ij*th element of $\mathbf{C}_1$ can be non-zero if the *j*th element of $\mathbf{P}^*$ corresponds to the inlet node of the machine *i*. This element is non-zero only if the pressure P_j^* is kept constant during the simulation. Matrices $\mathbf{C}_2$ and $\mathbf{C}_3$ are diagonal. The values of their diagonal elements depend on the structure of the network which is to be simulated.

4. Method of steady-state simulation with units

If we substitute (5) in (4) we have

$$-\mathbf{A}^{\mathrm{T}} \mathbf{P} - \phi(\mathbf{Q}) = \mathbf{0} \tag{11}$$

We can linearize the flow equation (4) in the following way

$$\phi(\mathbf{Q}) = \Lambda \mathbf{Q} \tag{12}$$

where

$$\Lambda = \operatorname{diag} \{K_i |Q_i|^{m_i - 1}\}, \quad i = 1, 2, \ldots, m$$

Now, (11) becomes

$$-\mathbf{A}^{\mathrm{T}} \mathbf{P} - \Lambda \mathbf{Q} = \mathbf{0} \tag{13}$$

Eliminating $\mathbf{Q}$ from (2) and (13) we obtain

$$\mathbf{G}\mathbf{P} = -\mathbf{L} - \mathbf{K}\mathbf{f} \tag{14}$$

where

$$\mathbf{G} = \mathbf{A} \Lambda^{-1} \mathbf{A}^{\mathrm{T}}$$

Since Λ^{-1} is diagonal, matrix $\mathbf{G}$ is symmetric. Matrix $\mathbf{G}$ is sparse. It has the same sparsity characteristics as the nodal Jacobi matrix (Fincham 1971, Osiadacz 1987). The number of non-zero elements of row i of $\mathbf{G}$ is equal to the number of nodes which are incident to node i plus one.

Suppose there are u units, their outlet pressures (squared pressures) are $P_1, P_2, \ldots, P_u$ and pressures (squared pressures) at the other nodes are $P_1^*, P_2, \ldots, P_v^*$ —$v = n - u$; note that each unit has two nodes except sources—(14) can be written as (see Goldwater *et al.* 1976)

$$\left[\begin{array}{c|c|c} \mathbf{G}_N & \hat{\mathbf{G}} & \mathbf{K}^I \\ \hline \hat{\mathbf{G}}^{\mathrm{T}} & \mathbf{G}^* & \mathbf{K}^0 \end{array}\right]\left[\begin{array}{c} \mathbf{P}^* \\ \hline \mathbf{P} \\ \hline \mathbf{f} \end{array}\right] = -\left[\begin{array}{c} \mathbf{L}^I \\ \hline \mathbf{L}^0 \end{array}\right] \tag{15}$$

Here, the rows and columns have been permuted from the original matrix $\mathbf{G}$, where $\mathbf{G}_N$ represents connections within the pipes network only, $\hat{\mathbf{G}}$ represents connections between the units and the network, and $\mathbf{G}^*$ represents pipe connections between units. We note that $\mathbf{K}^I$ is the inlet part of the matrix $\mathbf{K}$

$$\dim \mathbf{K}^I = (v + u)$$

$\mathbf{K}^0$ is the outlet part of the matrix $\mathbf{K}$

$$\dim \mathbf{K}^0 = (u \times u)$$

$\mathbf{P}^*$ is the vector of non-outlet pressures (for high and medium pressure $P_i^* = P_i^2$)

$$\dim \mathbf{P}^* = v \times 1$$

$\mathbf{P}$ is the vector of outlet pressures (for high and medium pressure $P_i = p_i^2$)

$$\dim \mathbf{P} = u \times 1$$

$\mathbf{L}^I$ is the vector of demands not at outlet nodes, and $\mathbf{L}^0$ is the vector of demands at outlet nodes.

Taking into account (10), (15) can be rewritten in the form

$$\left[\begin{array}{c|c|c} \mathbf{G}_N & \hat{\mathbf{G}} & \mathbf{K}^1 \\ \hline \hat{\mathbf{G}}^{\mathrm{T}} & \mathbf{G} & \mathbf{K}^0 \\ \hline \mathbf{C}_1 & \mathbf{C}_2 & \mathbf{C}_3 \end{array}\right]\left[\begin{array}{c} \mathbf{P}^* \\ \hline \mathbf{P} \\ \hline \mathbf{f} \end{array}\right] = \left[\begin{array}{c} -\mathbf{L}^1 \\ \hline -\mathbf{L}^0 \\ \hline \mathbf{d} \end{array}\right] \tag{16}$$

Matrix $\mathbf{G}_N$ has the following properties.

(*a*) It is symmetric. This is true since $\mathbf{G}_N$ is obtained by moving down the rows of $\mathbf{G}$ corresponding to outlet nodes and moving right the corresponding columns.

(*b*) It is positive definite.

Now, it is possible to decompose system (16) by eliminating $\mathbf{P}^*$. Using Cholesky's decomposition method we may write

$$\mathbf{G}_N = \mathbf{L}\mathbf{L}^{\mathrm{T}} = \mathbf{L}\mathbf{U}$$

and

$$\mathbf{U}\mathbf{P}^* = -\mathbf{L}^{-1}\mathbf{L}^I - \mathbf{L}^{-1}\hat{\mathbf{G}}\mathbf{P} - \mathbf{L}^{-1}\mathbf{K}^I\mathbf{f} \tag{17}$$

Since $\mathbf{P}^*$ is expressed in terms of $\mathbf{P}$ and $\mathbf{f}$, we can write (16) in the form

$$\left[\begin{array}{c|c} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \hline \mathbf{D}_{21} & \mathbf{D}_{22} \end{array}\right]\left[\begin{array}{c} \mathbf{P} \\ \hline \mathbf{f} \end{array}\right] = \left[\begin{array}{c} \mathbf{R}_1 \\ \hline \mathbf{R}_2 \end{array}\right] \tag{18}$$

where

$$\mathbf{D}_{11} = \mathbf{G}^* - \hat{\mathbf{G}}^T\mathbf{U}^{-1}\mathbf{L}^{-1}\hat{\mathbf{G}}$$
$$\mathbf{D}_{12} = \mathbf{K}^0 - \hat{\mathbf{G}}^T\mathbf{U}^{-1}\mathbf{L}^{-1}\mathbf{K}^I$$
$$\mathbf{D}_{21} = \mathbf{C}_2 - \mathbf{C}_1\mathbf{U}^{-1}\mathbf{L}^{-1}\hat{\mathbf{G}}$$
$$\mathbf{D}_{22} = \mathbf{C}_3 - \mathbf{C}_1\mathbf{U}^{-1}\mathbf{L}^{-1}\mathbf{K}^I$$
$$\mathbf{R}_1 = -\mathbf{L}^0 + \hat{\mathbf{G}}^T\mathbf{U}^{-1}\mathbf{L}^{-1}\mathbf{L}^I$$
$$\mathbf{R}_2 = \mathbf{d} + \mathbf{C}_1\mathbf{U}^{-1}\mathbf{L}^{-1}\mathbf{L}^I$$

Equation (18) is solved for $\mathbf{P}$ and $\mathbf{f}$. Crout's method with iterative refinement of the solution is used in order to solve this system and $\mathbf{P}^*$ is obtained from (17) using backward substitution. Since the pipe flow equation is linearized, the above set of equations is solved iteratively until convergence is obtained for the values of $\mathbf{P}^*$, $\mathbf{P}$ and $\mathbf{f}$.

5. Results of investigations

Two examples are now given which illustrate the properties of the above simulation method.

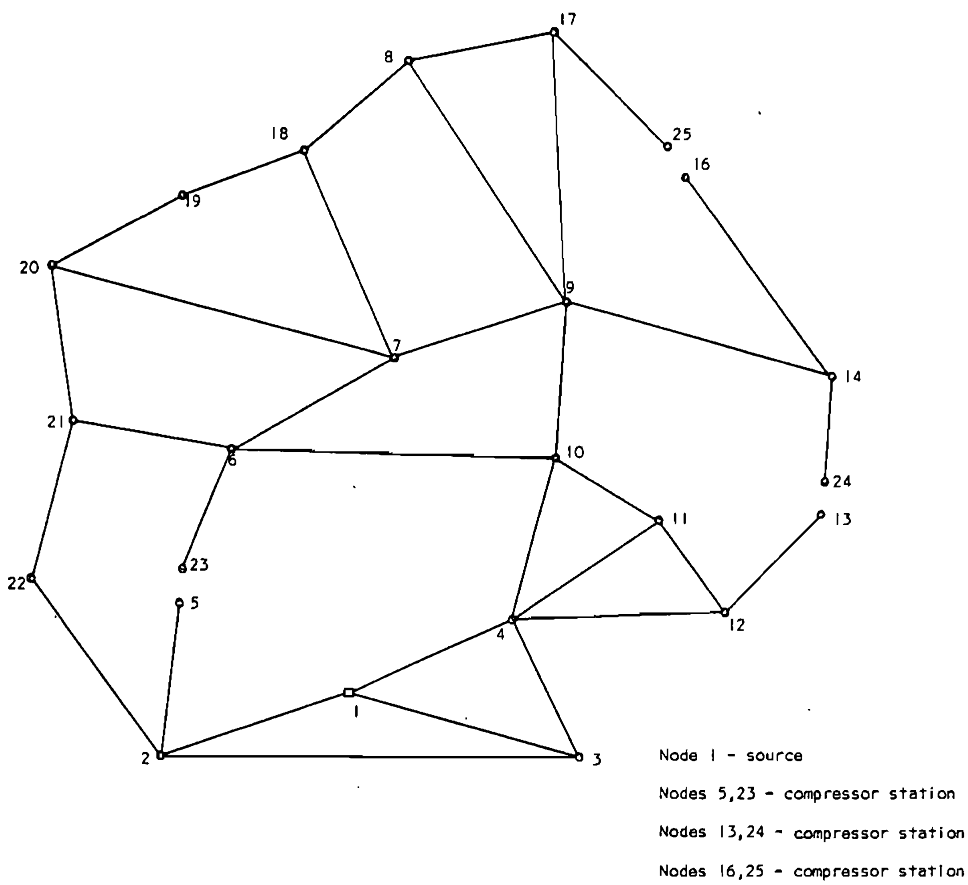

Figure 1. Graph of the gas network.

Example 1

The structure of the network is shown in Fig. 1. Table 1 shows the pipe data and Table 2 gives the node data. Panhandle's 'A' equation has been used assuming $E = 0{\cdot}9$. The end of calculations depended on achieving the following inequality

$$\|\Delta \mathbf{Q}\|_2 \leqslant 0{\cdot}4 \times 10^2 \text{ m}^3 \text{ h}^{-1} \tag{19}$$

where $\Delta \mathbf{Q} = \mathbf{Q}^{k+1} - \mathbf{Q}^k$ (k is the number of iterations).

Condition (19) was achieved after 13 iterations and the result obtained was

$$\|\Delta \mathbf{Q}\|_2 = 0{\cdot}359 \times 10^2 \text{ m}^3 \text{ h}^{-1}$$

The results of the simulation are shown in Tables 3 and 4.

Example 2

The structure of the network is shown in Fig. 2. Table 5 gives the pipe data and Table 6 shows the node data.

Pipe	Sending node	Receiving node	Diameter (mm)	Length (m)
1	1	2	0·7E + 03	0·24E + 05
2	1	3	0·7E + 03	0·25E + 05
3	1	4	0·7E + 03	0·20E + 05
4	4	3	0·7E + 03	0·30E + 05
5	3	2	0·7E + 03	0·40E + 05
6	2	22	0·7E + 03	0·45E + 05
7	2	5	0·6E + 03	0·70E + 05
8	5	6	0·6E + 03	0·60E + 05
9	22	21	0·7E + 03	0·52E + 05
10	6	21	0·6E + 03	0·30E + 05
11	4	10	0·7E + 03	0·40E + 05
12	4	11	0·6E + 03	0·35E + 05
13	4	12	0·7E + 03	0·55E + 05
14	12	13	0·6E + 03	0·70E + 05
15	12	11	0·7E + 03	0·30E + 05
16	11	10	0·6E + 03	0·50E + 05
17	13	14	0·6E + 03	0·60E + 05
18	10	14	0·6E + 03	0·10E + 05
19	14	15	0·7E + 03	0·80E + 05
20	10	9	0·6E + 03	0·75E + 05
21	15	9	0·7E + 03	0·80E + 05
22	15	16	0·6E + 03	0·75E + 05
23	16	17	0·7E + 03	0·80E + 05
24	10	6	0·6E + 03	0·40E + 05
25	9	17	0·7E + 03	0·65E + 05
26	9	8	0·6E + 03	0·40E + 05
27	8	17	0·6E + 03	0·55E + 05
28	9	7	0·6E + 03	0·45E + 05
29	8	18	0·7E + 03	0·30E + 05
30	7	18	0·6E + 03	0·42E + 05
31	6	7	0·7E + 03	0·20E + 05
32	18	19	0·6E + 03	0·30E + 05
33	7	20	0·7E + 03	0·40E + 05
34	20	19	0·6E + 03	0·32E + 05
35	20	21	0·7E + 03	0·45E + 05

Table 1. Pipe data.

Node	Load ($m^3 h^{-1}$)	Pressure (bar)
1	0	0·4E + 02 (source)
2	0·9E + 05	—
3	0·29E + 05	—
4	0·75E + 05	—
5	0	—
6	0·55E + 05	—
7	0·85E + 05	—
8	0·28E + 05	—
9	0·9E + 05	—
10	0·41E + 05	—
11	0·39E + 05	—
12	0·2E + 05	—
13	0	—
14	0·8E + 05	—
15	0·45E + 05	—
16	0	—
17	0·12E + 05	—
18	0·42E + 05	—
19	0·18E + 05	—
20	0·35E + 05	—
21	0·29E + 05	—
22	0·71E + 05	—
23	0	0·4E + 02 (discharge pressure)
24	0	0·4E + 02 (discharge pressure)
25	0	0·4E + 02 (discharge pressure)

Table 2. Node data.

Node	p (bar)
1	0·4E + 02
2	0·38517E + 02
3	0·38829E + 02
4	0·38419E + 02
5	0·35519E + 02
6	0·37546E + 02
7	0·37316E + 02
8	0·37301E + 02
9	0·37254E + 02
10	0·37670E + 02
11	0·37704E + 02
12	0·37630E + 02
13	0·34328E + 02
14	0·37368E + 02
15	0·36373E + 02
16	0·31492E + 02
17	0·37852E + 02
18	0·37286E + 02
19	0·37287E + 02
20	0·37321E + 02
21	0·37509E + 02
22	0·37760E + 02
23	0·4E + 02
24	0·4E + 02
25	0·4E + 02

Pipe	Q ($m^3 h^{-1}$)
1	0·29422E + 06
2	0·25390E + 06
3	0·33577E + 06
4	−0·12925E + 06
5	0·95646E + 05
6	0·14364E + 06
7	0·15625E + 06
8	0·15625E + 06
9	0·72644E + 05
10	0·23371E + 05
11	0·15202E + 06
12	0·10643E + 06
13	0·13158E + 06
14	0·16209E + 06
15	−0·50503E + 05
16	0·16933E + 05
17	0·16209E + 06
18	0·37655E + 05
19	0·11975E + 06
20	0·52188E + 05
21	−0·11207E + 06
22	0·18682E + 06
23	0·18682E + 06
24	0·38117E + 05
25	−0·10286E + 06
26	−0·22544E + 05
27	−0·71960E + 05
28	−0·24471E + 05
29	0·21415E + 05
30	0·17151E + 05
31	0·11600E + 06
32	−0·34190E + 04
33	−0·10620E + 05
34	0·21410E + 05
35	−0·67018E + 05

Table 4. Results of simulation.

Similar to the first example Panhandle's 'A' equation has been used assuming $E = 0{\cdot}9$. The end of the calculations depended on achieving the following inequality

$$\|\Delta \mathbf{Q}\|_2 \leqslant 0{\cdot}1 \text{ m}^3 \text{ h}^{-1} \tag{20}$$

Case (*a*)

$$\frac{p_5}{p_4} = 1{\cdot}8 \quad \text{and} \quad \frac{p_7}{p_6} = 1{\cdot}4$$

Condition (20) was achieved after 10 iterations and the result obtained was

$$\|\Delta \mathbf{Q}\|_2 = 0{\cdot}828 \times 10^{-1} \text{ m}^3 \text{ h}^{-1}$$

The results of the simulation are shown in Tables 7 and 8.

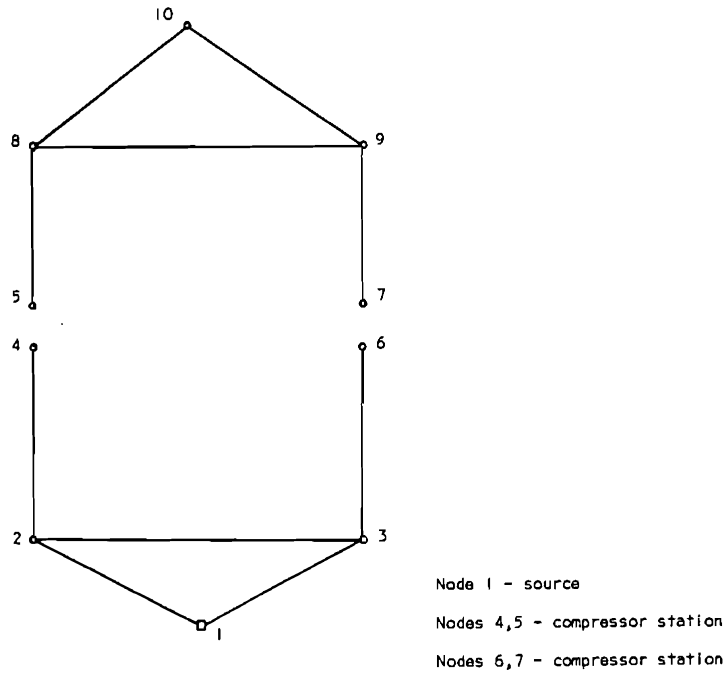

Figure 2.

Pipe	Sending node	Receiving node	Diameter (mm)	Length (m)
1	1	2	0·7E + 03	0·7E + 05
2	1	3	0·7E + 03	0·6E + 05
3	2	3	0·7E + 03	0·9E + 05
4	2	4	0·6E + 03	0·5E + 05
5	3	6	0·6E + 03	0·45E + 05
6	5	8	0·6E + 03	0·7E + 05
7	7	9	0·6E + 03	0·8E + 05
8	8	9	0·5E + 03	0·7E + 05
9	8	10	0·5E + 03	0·45E + 05
10	9	10	0·5E + 03	0·75E + 05

Table 5. Pipe data.

Node	Load ($m^3 h^{-1}$)	Pressure (bar)
1	0	0·5E + 02
2	0·2E + 05	—
3	0·2E + 05	—
4	0	—
5	0	—
6	0	—
7	0	—
8	0·15E + 05	—
9	0·30E + 06	—
10	0·45E + 05	—

Table 6. Node data.

Node	p (bar)
1	0·5E + 02
2	0·48079E + 02
3	0·48742E + 02
4	0·42244E + 02
5	0·76040E + 02
6	0·48482E + 02
7	0·67875E + 02
8	0·71023E + 02
9	0·67545E + 02
10	0·69105E + 02

Table 7. Results of simulation.

Pipe	Q ($m^3 h^{-1}$)
1	0·21410E + 06
2	0·18589E + 06
3	−0·10455E + 06
4	0·29866E + 06
5	0·61335E + 06
6	0·29866E + 06
7	0·61335E + 06
8	0·14727E + 06
9	0·13639E + 06
10	−0·91391E + 06

Table 8. Results of simulation.

Case (*b*)

$$\frac{p_5}{p_4} = 1{\cdot}5 \quad \text{and} \quad p_6 = 45 \text{ bar} \quad \text{(suction pressure)}$$

Condition (20) was achieved after 15 iterations and the result obtained was

$$\|\Delta \mathbf{Q}\|_2 = 0{\cdot}955 \times 10^{-1} \text{ m}^3 \text{ h}^{-1}$$

The results of the simulation are shown in Tables 9 and 10.

Node	p (bar)
1	0·5E + 02
2	0·48516E + 02
3	0·48371E + 02
4	0·47494E + 02
5	0·71241E + 02
6	0·45E + 02
7	0·73739E + 02
8	0·70270E + 02
9	0·69842E + 02
10	0·69881E + 02

Table 9. Results of simulation.

Pipe	Q ($m^3 h^{-1}$)
1	0·18675E + 06
2	0·21325E + 06
3	0·46133E + 05
4	0·12061E + 06
5	0·23938E + 06
6	0·12061E + 06
7	0·23938E + 06
8	0·47898E + 05
9	0·57716E + 05
10	−0·12717E + 05

Table 10. Results of simulation.

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