Statistics for Data Analysis-Lec 4

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Hypothesis Testing

Hypothesis Testing

- Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- The null hypothesis, denoted by H_0, is a tentative assumption about a population parameter
- The alternative hypothesis, denoted by Ha, is the opposite of what is stated in the null hypothesis
- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H0 and Ha.

Developing Null and Alternative Hypotheses

- It is not always obvious how the null and alternative hypotheses should be formulated
- Care must be taken to structure the hypotheses appropriately so that the test conclusion provides the information the researcher wants
- The context of the situation is very important in determining how the hypotheses should be stated
- In some cases it is easier to identify the alternative hypothesis first. In order cases the null is easier
- Correct hypothesis formulation will take practice

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Many applications of hypothesis testing involve and attempt to gather evidence in support of a research hypothesis
- In such cases, it is often best to begin with the alternative hypothesis and make it the conclusion that the researcher hopes to support
- The conclusion that the research hypothesis is true is made if the sample data provide sufficient evidence to show that the null hypothesis can be rejected

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Example: A new manufacturing method is believed to be better than the current method.
- Alternative Hypothesis:
    - The new manufacturing method is better
- Null Hypothesis:
    - The new methods is no better than the old method

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Example: A new bonus plan, that is developed in and attempt to increase sales
- Alternative Hypothesis:
    - The new bonus plan increase sales
- Null Hypothesis:
    - The new bonus plan does not increase sales

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Example: A new drug is developed with the goal of lowering Cholesterol-level more than the existing drug
- Alternative Hypothesis:
  - The new drug lowers Cholesterol-level more than the existing drug
- Null Hypothesis:
  - The new drug does not lower Cholesterol-level more than the existing drug

Developing Null and Alternative Hypotheses

- Null Hypothesis as and assumption to be challenged
- We might begin with a belief or assumption that a statement about the value of a population parameter is true
- **Example**: The label on a milk bottle states that it contains 1000 ml
- Null Hypothesis:
  - The label is correct. $\mu \geq 1000$ ml
- Alternative Hypothesis:
  - The label is incorrect. $\mu < 1000$ ml

Null and Alternative Hypotheses about a Population Mean μ

- The equality part of the hypotheses always appears in the null hypothesis
- In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean)

One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed
$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu = \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$	$H_a: \mu \neq \mu_0$

Null and Alternative Hypotheses

- A major hospital in Chennai provides one of the most comprehensive emergency medical services in the world
- Operating in a multiple hospital system with approximately 10 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 8 minutes or less
- The director of medical services wants to formulate a hypothesis test that coils use a sample of emergency response times to determine whether o not the *service goal of 8 minutes or less is being archived.*

Null and Alternative Hypotheses

The emergency service is meeting the response goal; no follow-up action is necessary.

$$H_0: \mu \leq 8$$

The emergency service is not meeting the response goal; appropriate follow-up action is necessary.

$$H_a: \mu > 8$$

Where: μ = mean response time for the population of medical emergency requests

Type I Error

- Because hypothesis tests are based on sample data, we must allow for the possibility of errors
- A Type I error is rejecting H_0 when it is true
- The probability of making a Type I error when the null hypothesis is called the level of significance
- Applications of hypothesis testing that only control the Type I error are often called significance tests

Type II Error

- A Type II error is accepting H_0 when it is false.

- It is difficult to control for the probability of making a Type II error.

- Statisticians avoid the risk of making a Type II error by using "do not reject H_0" and not "accept H_0".

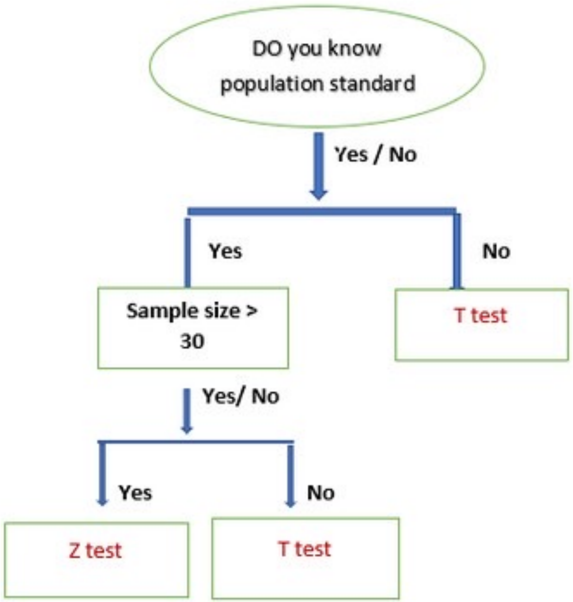
Type I and Type II Errors

	Population Condition	
Conclusion	**H0 True** $(\mu \leq 8)$	**H0 False** $(\mu > 8)$
Accept H0 (Conclude $\mu \leq 8$)	**Correct Decision**	**Type II Error**
Reject H0 (Conclude $\mu > 8$)	**Type I Error**	**Correct Decision**

Three Approached for Hypothesis Testing

- Z test = Average value
- t- test = = Average value
- Chi-square = categorical data
- ANNOVA = Analysis of variance

When to use z-test vs t-test



Problem definition 1:

The average height of all players in the academy is 168 cm with a population standard deviation of 39 . New coach believes the mean to be different . He measured height of all 36 players and found average to be 169.5.

A. State Null and Alternate hypothesis

B. At a 95% CI is there enough evidence to accept alternate hypothesis.

Solution:

$\mu = 168cm$, $\sigma = 39$, n= 36 , $\overline{x} = 169.5$

Null Hypothesis: H0: $\boldsymbol{\mu = 168}$

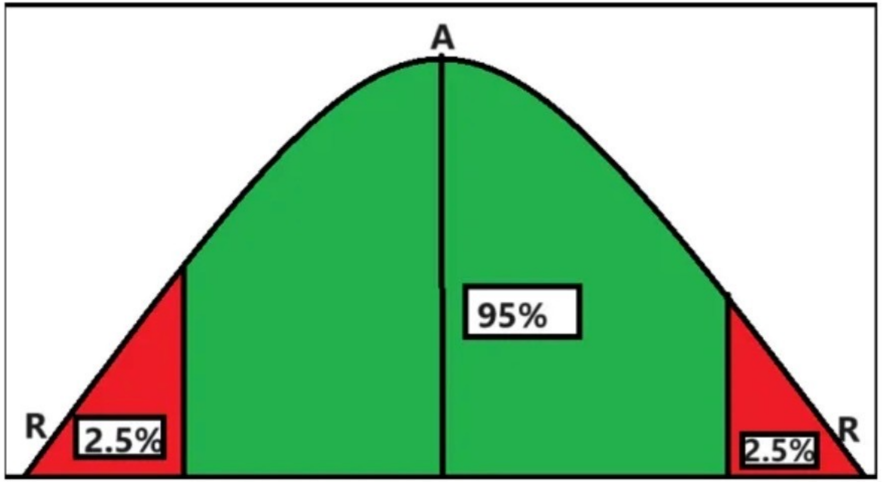
Alternate Hypothesis: H1: $\boldsymbol{\mu \neq 168}$

Considering above example, it will be two tailed test

CI = 0.95 => 95%

α = 1- CI= 1-0.95 =0.05

As it is 2 tailed it would look like one below



Here Above A reffers to Accepted Area and R refers to Rejected Area and considering that we have 2 tails 5% is divided in to 2.5% and 2.5%

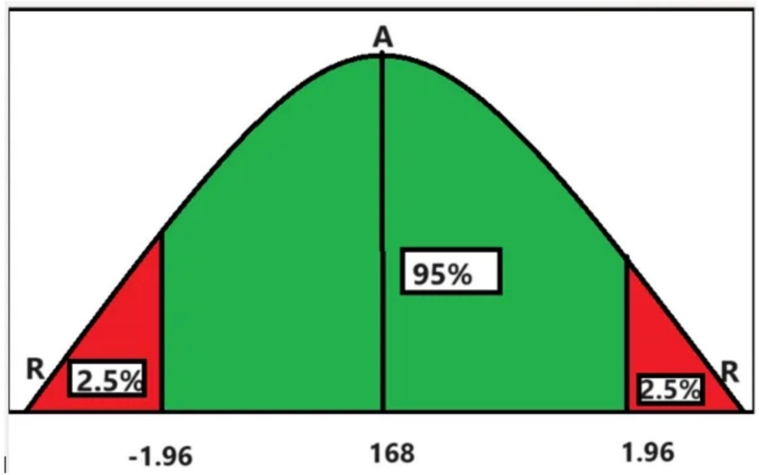
Decision Boundary: Now we are calculating decision boundary so we need to find either side as another will inverse of it .

Calculting for right hand curve:

1-0.025= 0.9750 (considering right hand curve so 100%-2.5%)

Refer Z-Table to find value of 0.9750 (Ztable)

Value is 1.96 and inverse will be -1.96



If Z score value falls between -1.96 to 1.96 then we fail to reject Null Hypothesis.

Z Score:

Z Score:

$$Z = \frac{(\overline{X} - \mu)}{(\sigma / \sqrt{n})} \qquad \mathrm{SE} = \frac{\sigma}{\sqrt{n}}$$

SE = standard error of the sample
σ = sample standard deviation
n = number of samples

$=(169.5 - 169)/(\frac{39}{\sqrt{36}}) \quad =2.31$

Conclusion:

If Z score is greater than 1.96 and lesser then -1.96 then we will reject the null hypothesis.

2.31>1.96 we reject the null hypothesis:

Hence new coach was right.

Problem definition 2

In the population, the average IQ is 100. A team of scientists wants to test a new medication to see if it has either a positive or negative effect on intelligence, or no effect at all. A sample of 30 participants who have taken the medication has a mean of 140 with a standard deviation of 20. Did the medication affect intelligence? Use alpha = 0.05.

1. Define Null and Alternative Hypotheses

$$H_0; \mu = 100$$

$$H_1; \mu \neq 100$$

Figure 1.

2. State Alpha

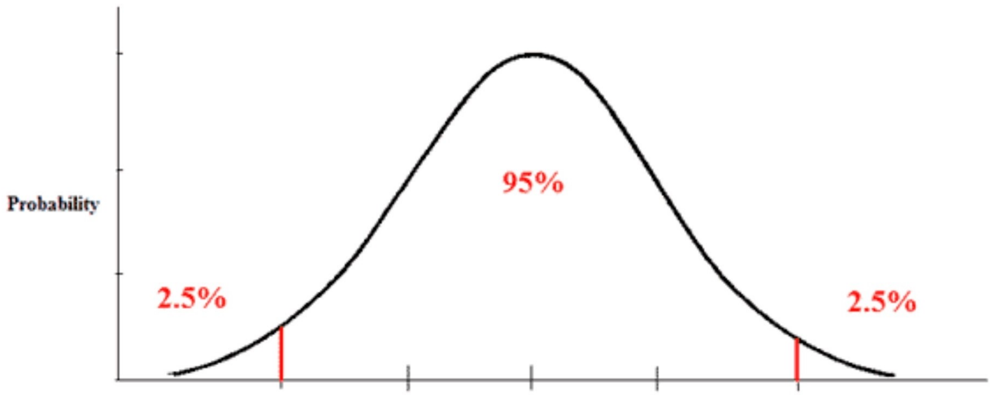
Alpha = 0.05

3. Calculate Degrees of Freedom

df = n - 1 = 30 - 1 = 29

4. State Decision Rule

Using an alpha of 0.05 with a two-tailed test with 29 degrees of freedom, we would expect our distribution to look something like this:



Use the t-table to look up a two-tailed test with 29 degrees of freedom and an alpha of 0.05. We find a critical value of 2.0452. Thus, our decision rule for this two-tailed test is:

If t is less than -2.0452, or greater than 2.0452, reject the null hypothesis.

5. Calculate Test Statistic

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$\bar{x} = 140$

$\mu = 100$

$s = 20$

$n = 30$

$$t = \frac{140 - 100}{20 / \sqrt{30}} = \frac{40}{3.65} = 10.96$$

6. State Results

t = 10.96

Result: Reject the null hypothesis.

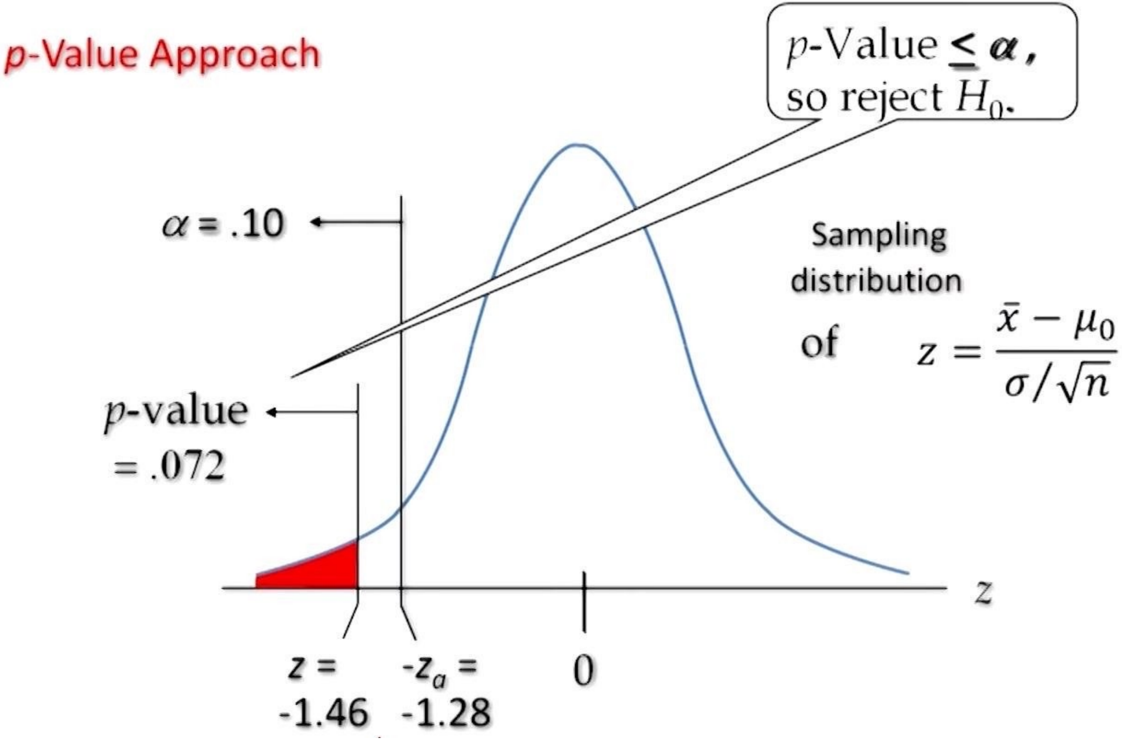
7. State Conclusion

Medication significantly affected intelligence, $t = 10.96$, $p < 0.05$.

p-Value Approach to One-Tailed Hypothesis Testing

- The p-value is the probability, computed using the test statistic, that measures the support (or lack of support) provided by the sample for the null hypothesis
- If the p-value is less than or equal to the level of significance α, the value of the test statistic is in the rejection region
- Reject H_0 if the p-value $\leq \alpha$

Lower-Tailed Test About a Population Mean: σ Known



p-Value Approach

Importing library

```
In [2]: from scipy import stats
```

```
In [3]: stats.norm.cdf(1.96)
Out[3]: 0.9750021048517795
```

Finding P Value

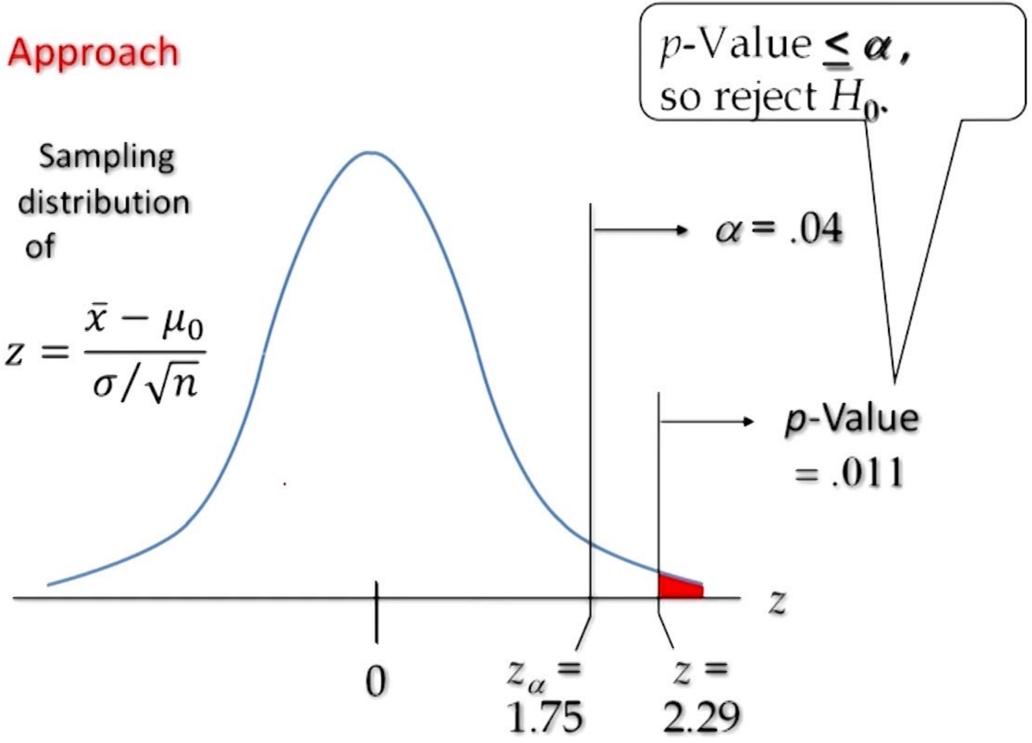
```
In [3]: stats.norm.cdf(-1.46)
Out[3]: 0.0721450369658937 8
```

Finding Z Value

```
In [5]: stats.norm.ppf(0.1)
Out[5]: -1.2815515655446004
```

Upper-Tailed Test About a Population Mean: σ Known

p-Value Approach



p-Value Approach

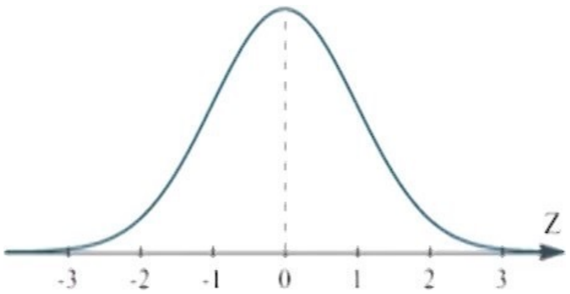
```
In [4]: 1-stats.norm.cdf(1.75)

Out[4]: 0.040059156863817114
```

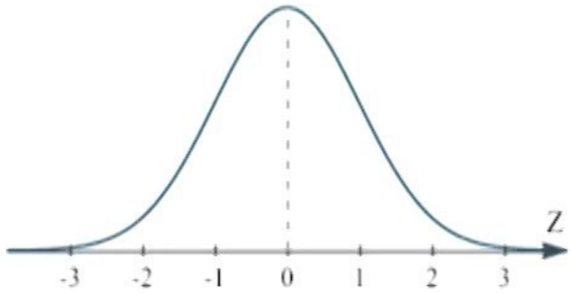
```
In [5]: 1-stats.norm.cdf(2.29)

Out[5]: 0.011010658324411393
```

Critical Value Approach to One-Tailed Hypothesis Testing

- The test statistic z has a standard normal probability distribution.
- We can use the standard normal probability distribution table to find the z-value with and area of α in the lower (or upper) tail of the distribution.

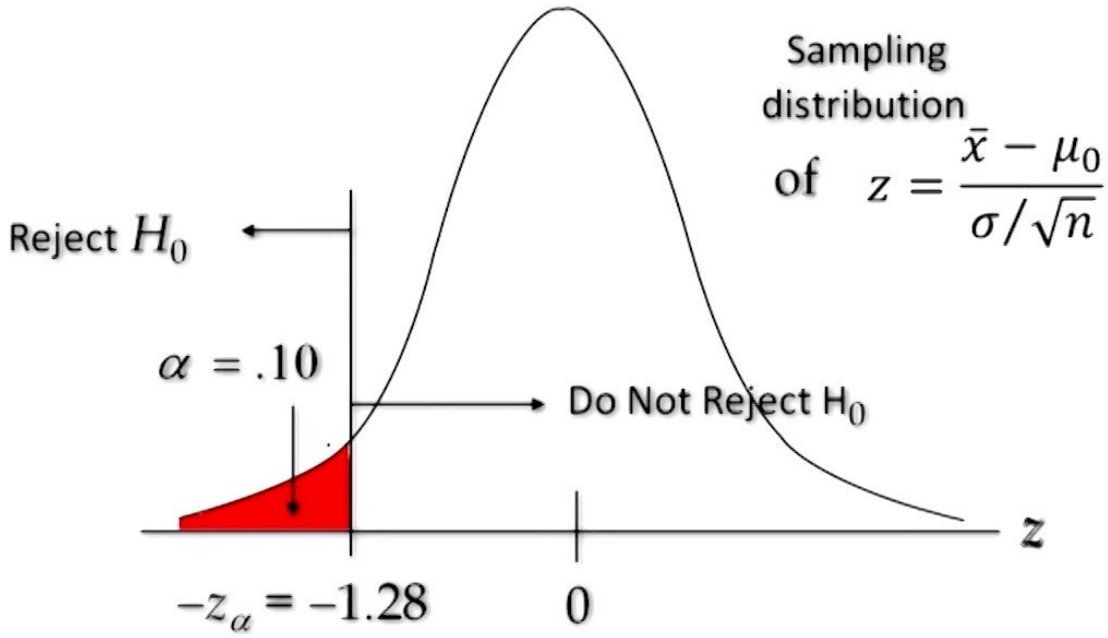
- The value of the test statistic that established the boundary of the rejection region is called the critical value for the test.
- The rejection rule is:
  - Lower tail: Reject H_0 if $z \le -z_\alpha$
  - Upper tail: Reject H_0 if $z \ge z_\alpha$





Lower-Tailed Test About a Population Mean: σ

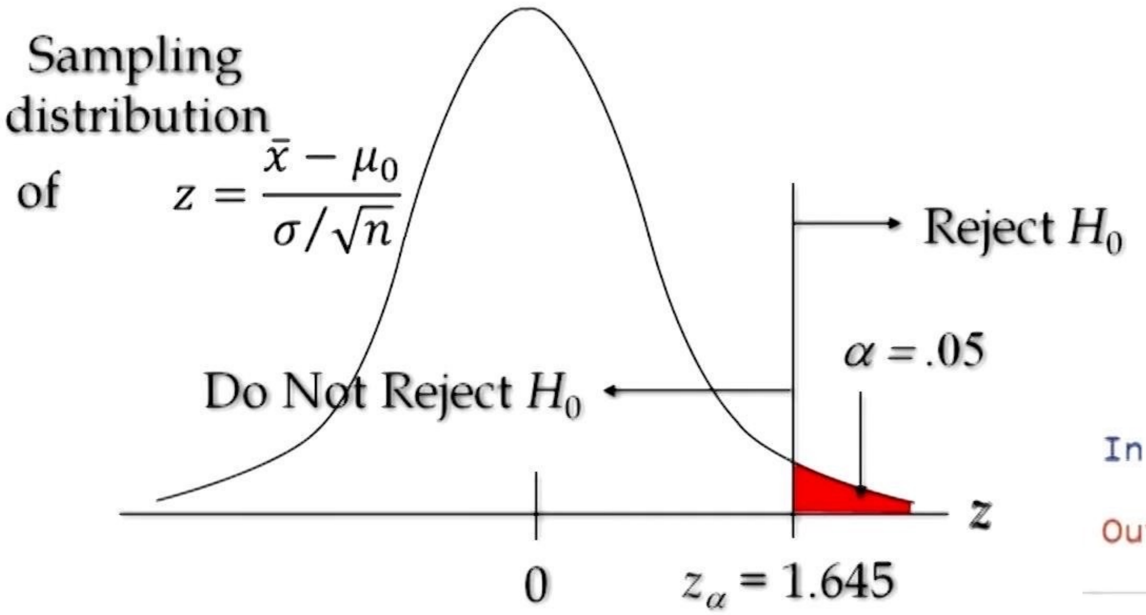
Critical Value Approach




```
In [6]: stats.norm.ppf(0.1)
Out[6]: -1.2815515655446004
```

Upper-Tailed Test About a Population Mean: σ

Critical Value Approach




```
In [7]: stats.norm.ppf(0.95)
Out[7]: 1.6448536269514722
```

Steps of Hypothesis Testing - P value approach

Step 1: Develop the null and alternative hypotheses.

Step 2: Specify the level of significance α.

Step 3: Collect the sample data and compute the test statistic.

P-Value Approach

Step 4: Use the value of the test statistic to compute the p-value.

Step 5: Reject H_0 if p-value ≤ α.

Steps of Hypothesis Testing

Critical Value Approach

Step 4: Use the level of significance α to determine the critical value and the rejection rule.

Step 5: Use the value of the test statistic and the rejection rule to determine whether to reject H_0.

One-Tailed Tests about a population Mean: σ Known

- **Example**: The mean response times for a random sample of 30 Pizza Deliveries is 32 minutes
- The population standard deviation is believed to be 10 minutes.
- The pizza delivery services director wants to perform a hypothesis test, with α = 0.05 level of significance, to determine whether the service goal of 30 minutes or less is begin achieved.

Given Values

- Sample
- Sample mean = 32 min
- Sample size = 30

- Population
- $\alpha = 0.05$
- Population mean = 30 min

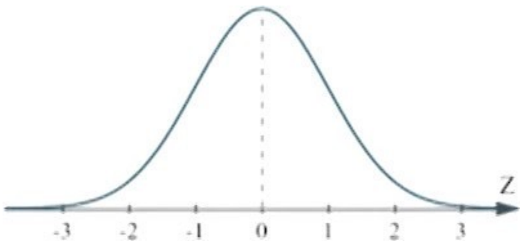
One-Tailed Tests about a Population Mean: α Known

1. Develop the hypotheses.
2. Specify the level of significance.
3. Compute the value of the test statistic.

$$H_0: \mu \leq 30$$
$$H_a: \mu > 30$$

$$\alpha = .05$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{32 - 30}{10 / \sqrt{30}} = 1.09$$




```
In [8]: 1-stats.norm.cdf(1.09)
Out[8]: 0.13785657203203556
```

One-Tailed Tests about a Population Mean: α Known

p-Value Approach

4. Compute the p-value,

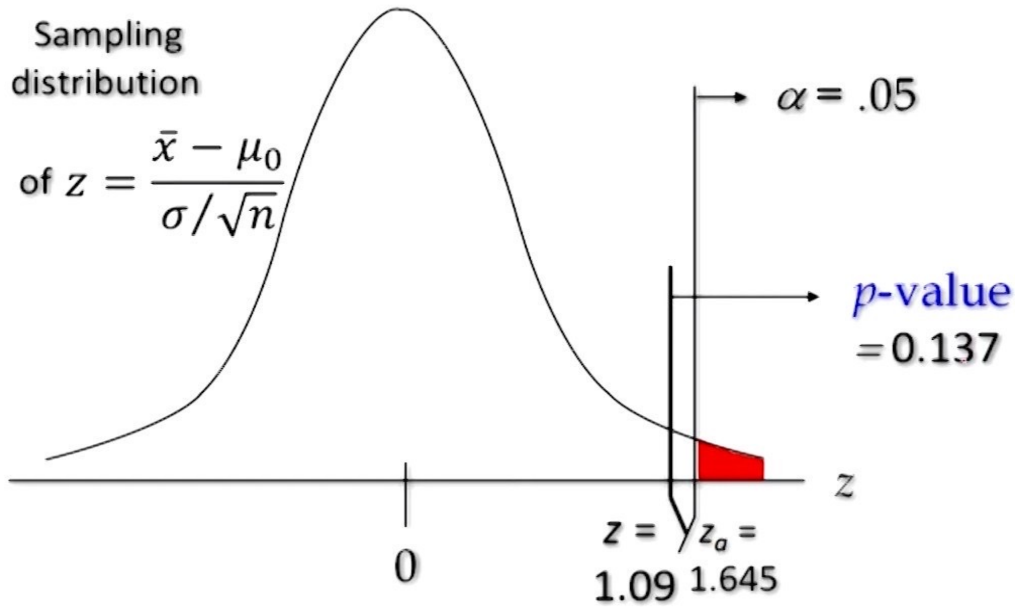
For $z = 1.09$, p-value == 0.137

5. Determine whether to reject H0.

- Because p-value = $0.137 > \alpha = .05$, we do not reject H_0.
- There are not sufficient statistical evidence to infer that Pizza delivery services is not meeting the response goal of 30 minutes.

One-Tailed Tests about a Population Mean: σ Known

p –Value Approach



Critical Value Approach

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Compute the p-value using the following three steps:

Critical value Approach

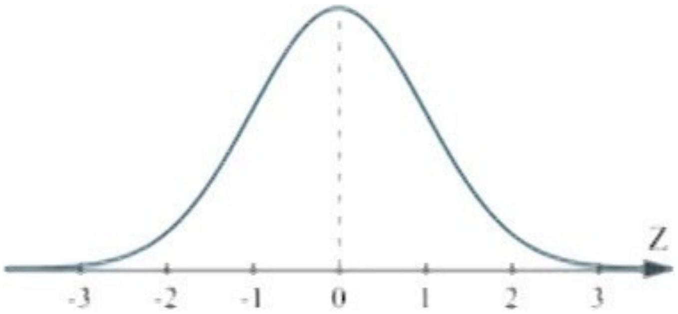
4. Determine the critical value and rejection rule.

For $\alpha = .05$, $z_{.05} = 1.645$

Reject H_0 if $z \geq 1.645$

5. Determine whether to reject H_0.

- Because $1.645 \geq 1.05$, we do not reject H_0.



Compute the p-value using the following three steps:

- Compute the value of the test statistic z.
- If z is in the upper tail ($z > 0$), find the area under the standard normal curve to the right of z.
- If z is in the lower tail ($z < 0$), find the area under the standard normal curve to the left of z.
- Double the tail area obtained in step 2 to obtain the p -value.
- The rejection rule:
  - Reject H_0 of the p-value $\leq \alpha$.

Critical Value Approach to Two-Tailed Hypothesis Testing

- The critical values will occur in both the lower and upper tails of the standard normal curve.
- Use the standard normal probability distribution table to find $z_{\alpha/2}$ (the z-value with and area of $\alpha/2$ in the upper tail of the distribution).

  - The rejection rule is:

Reject H_0 if $z \le -z_{\alpha/2}$ or $z \ge z_{\alpha/2}$.

Two-Tailed Tests about a Population Mean: σ Known

- **Example**: Milk Carton
- Assume that a sample of 30 milk carton provides a sample mean of 505 ml.
- The population standard deviation is believed to be 10 ml.
- Perform a hypothesis test, at the 0.03 level of significance, population mean 500 ml and help determine whether the filling process should continue operating and corrected.

Given Values

- Sample
- Sample mean = 505 ml
- Sample size = 30

- Population
- Population mean = 500 ml
- Standard deviation = 10 ml
- Significance level 0.03

p-value approach

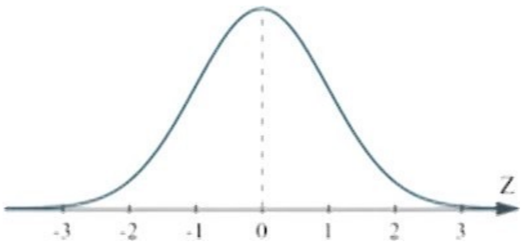
One-Tailed Tests about a Population Mean: σ Known

1. Determine the hypotheses.
2. Specify the level of significance.
3. Compute the value of the test statistic.

$$H_0: \mu = 500$$
$$H_a: \mu \neq 500$$

$$\alpha = .03$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{505 - 500}{10 / \sqrt{30}} = 2.74$$




```
In [9]: 1-stats.norm.cdf(2.74)
Out[9]: 0.0030719592186504444
```

```
In [10]: (1-stats.norm.cdf(2.74))*2
Out[10]: 0.006143918437300888
```

Two-Tailed Tests about a Population Mean: σ Known

p-Value Approach

4. Compute the p-value,

For $z = 2.74$, p-value $= 2(1 - .9969) = .0061$

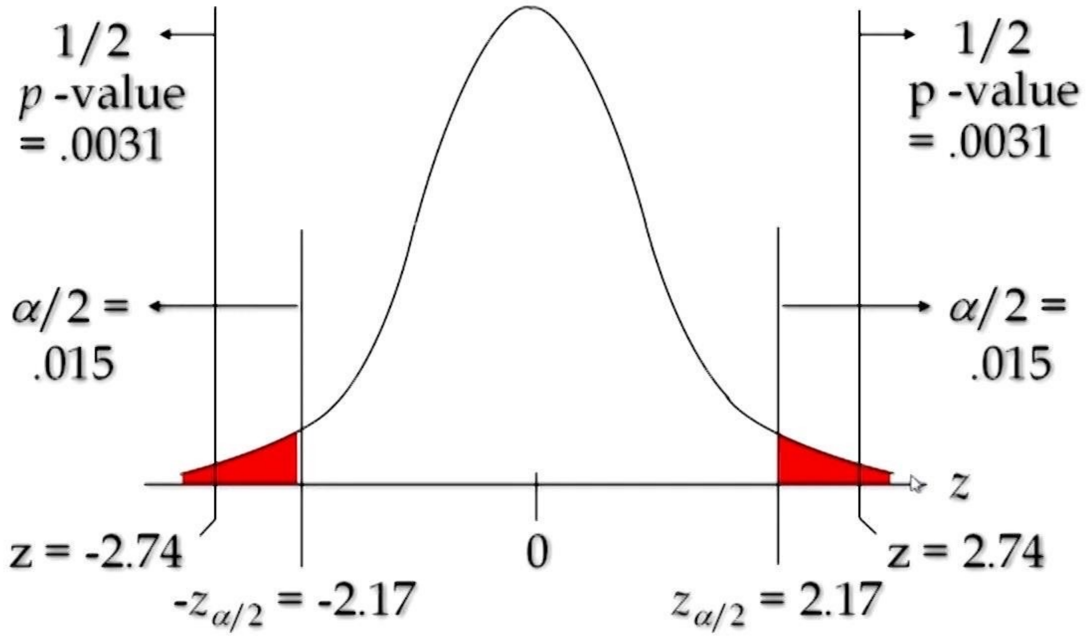
5. Determine whether to reject H0.

- Because p-value $= .0062 < \alpha = .03$, we reject H_0.

There are no sufficient statistical evidence to infer that the null hypothesis is true (i.e. the mean filling quantity is not 500 ml)

Two-Tailed Tests about a Population Mean: σ Known

p-Value Approach



Critical Value Approach

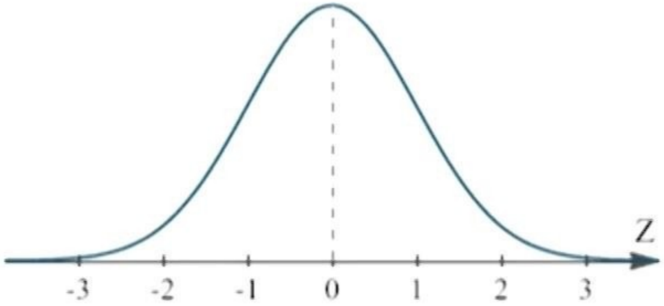
Two-Tailed Tests about a Population Mean: σ Known

Critical value Approach

4. Determine the critical value and rejection rule.

For $\alpha/2 = .03/2 = .015$, z.0.15 = 2.

Reject H_0 if $z < -2.17$ or $z > 2.17$



5. Determine whether to reject H_0.

- Because $2.74 > 2.17$, we reject H_0.

There is sufficient statistical evidence to infer that the null hypothesis is not true

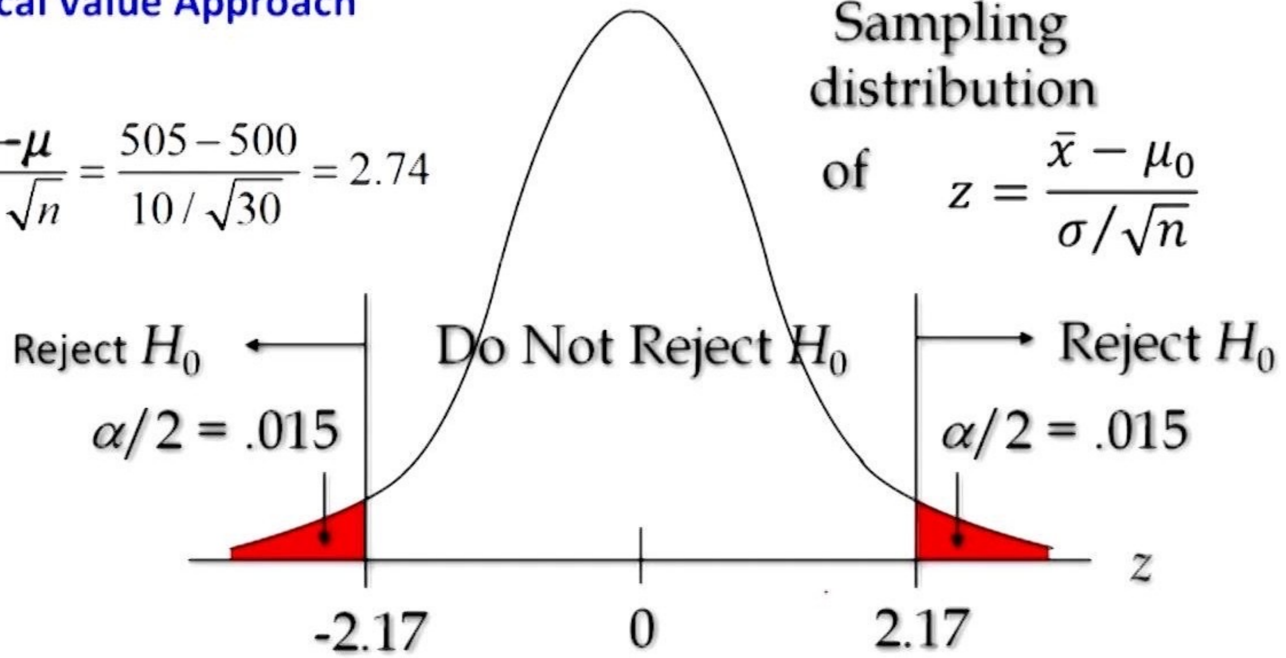
```
In [12]: stats.norm.ppf(0.015)

Out[12]: -2.1700903775845606
```

Two-Tailed Tests about a Population Mean: σ Known

Critical Value Approach

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{505 - 500}{10 / \sqrt{30}} = 2.74$$



Confidence Interval Approach

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Confidence Interval Approach to Two-Tailed Tests about a Population Mean

- Select a simple random sample from the population and use the value of the sample mean to develop the confidence interval for the population mean μ.
- If the confidence interval contains the hypothesis value 500, do not reject H_0.
- Otherwise, reject H_0.
- Actually, H_0 should be rejected if μ_0 happens to be equal to one of the end points of the confidence interval

Confidence Interval Approach to Two-Tailed Tests about a Population Mean

- The 97% confidence interval for 500 is

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 505 \pm 2.17\frac{10}{\sqrt{30}} = 505 \pm 3.9619$$
$$= \quad 501.03814\ ,508.96186$$

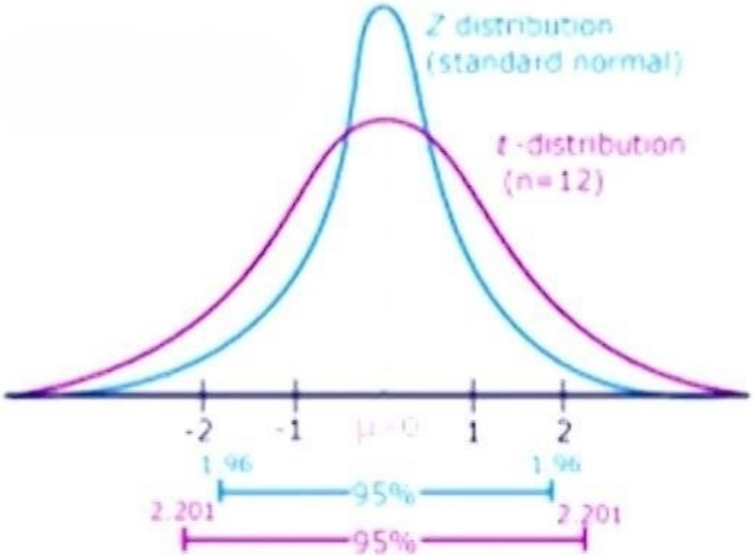
- Because the hypothesized value for the population mean, μ_0 = 500 ml, is not in this interval, the hypothesis-testing conclusion is that the null hypothesis, H_0: μ = 500, is rejected.

Tests About a Population Mean: α Unknown

- Test Statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- This test statistic has a t distribution with n - 1 degrees of freedom.



Tests About a Population Mean: α Unknown

- Rejection Rule: p-value Approach
  - Reject H0 if p-value $\leq \alpha$
- Rejection Rule: Critical Value Approach
  - H0: $\mu \geq \mu_0$ Reject H0 if $t \leq -t_\alpha$
  - H0: $\mu \leq \mu_0$ Reject H0 if $t \geq t_\alpha$
  - H_0: $\mu = \mu_0$ Reject H0 if $t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$

```
In [10]: from scipy import stats
         import numpy as np

In [11]: x=[10,12,20,21,22,24,18,15]
         stats.ttest_1samp(x,15)

Out[11]: Ttest_1sampResult(statistic=1.5623450931857947, pvalue=0.1621787560592894)
```

One-Tailed Test About a Population Mean: α Unknown

Example: Ice Cream Demand

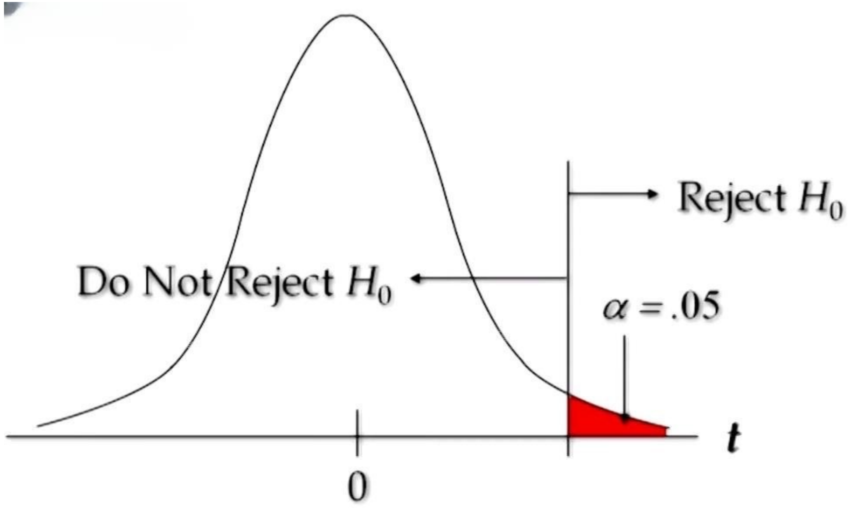
- In a ice cream parlor at IIT Roorkee, the following data represent the number of ice-creams sold in 20 days
- Test hypothesis H_0: $\mu \leq 10$
- Use $\alpha = .05$ to test the hypothesis.

Day	No. of Ice-cream Sold	Day	No. of Ice-cream Sold
1	13	11	12
2	8	12	11
3	10	13	11
4	10	14	12
5	8	15	10
6	9	16	12
7	10	17	7
8	11	18	10
9	6	19	11
10	8	20	8

```
In [8]: x=[13,8,10,10,8,9,10,11,6,8,12,11,11,12,10,12,7,10,11,8]

In [9]: stats.ttest_1samp(x,10)

Out[9]: Ttest_1sampResult(statistic=-0.35843385854878496, pvalue=0.7239703579964252)
```

```
In [10]: 0.7239703579964252/2

Out[10]: 0.3619851789982126
```

One-Tailed Test About a Population Mean: α Unknown




```
In [3]:  stats.t.cdf(-0.384,19)
Out[3]: 0.3526210256795583
```

```
[2]:  stats.t.ppf(0.05,19)
Out[2]: -1.7291328115213678
```

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