Universidade de Aveiro
Departamento de Electrónica,
Telecomunicações e Informática

Introduction to Computer Graphics
main concepts and methods - II

(Wikipedia)

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University of Aveiro, 2022

Topics

- Computer Graphics main tasks
- Geometric Primitives
- Geometric transformations
- 2D and 3D visualization
- Projections

CG Main Tasks

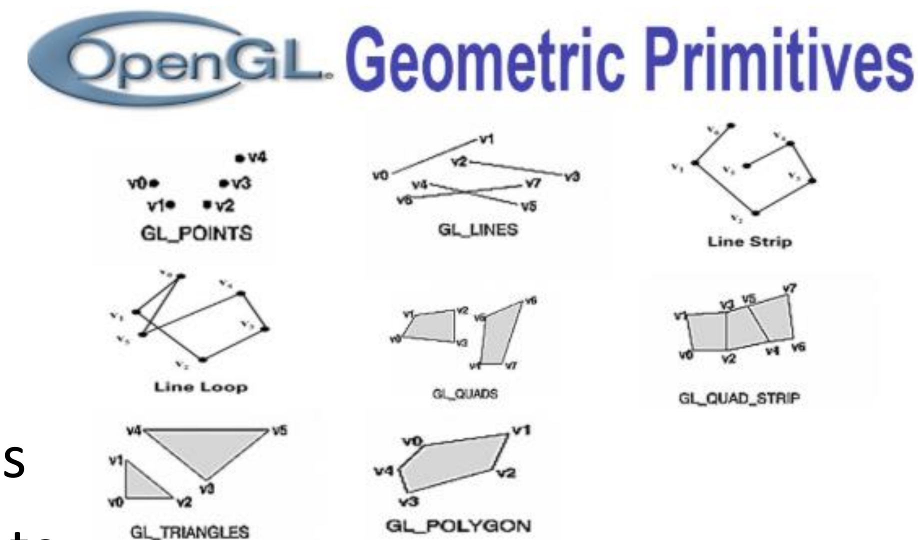
- Modeling
  - Construct individual models / objects
  - Assemble them into a 2D or 3D scene (using **transformations**)
- Rendering
  - Generate final images:
  - How is the scene illuminated?
  - What are the materials of the objects?
  - Where is the observer? How is he/she **looking at the scene**?
- Animation
  - Static vs. dynamic scenes
  - Movement and / or deformation

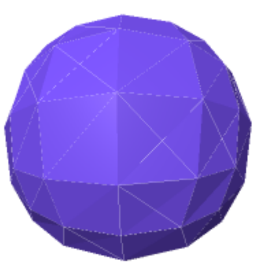
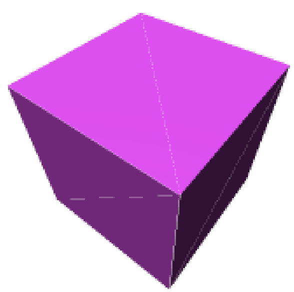
Geometric Primitives

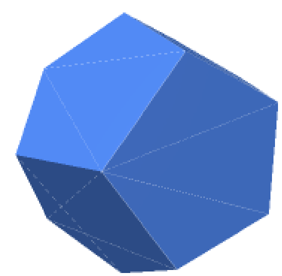
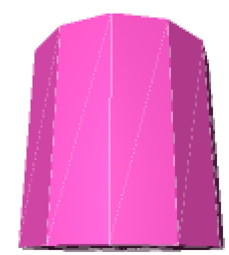
- Simple primitives
  - Points
  - Line segments
  - Polygons

- Geometric primitives
  - Parametric curves / surfaces
  - Cubes, spheres, cylinders, etc.

Examples:



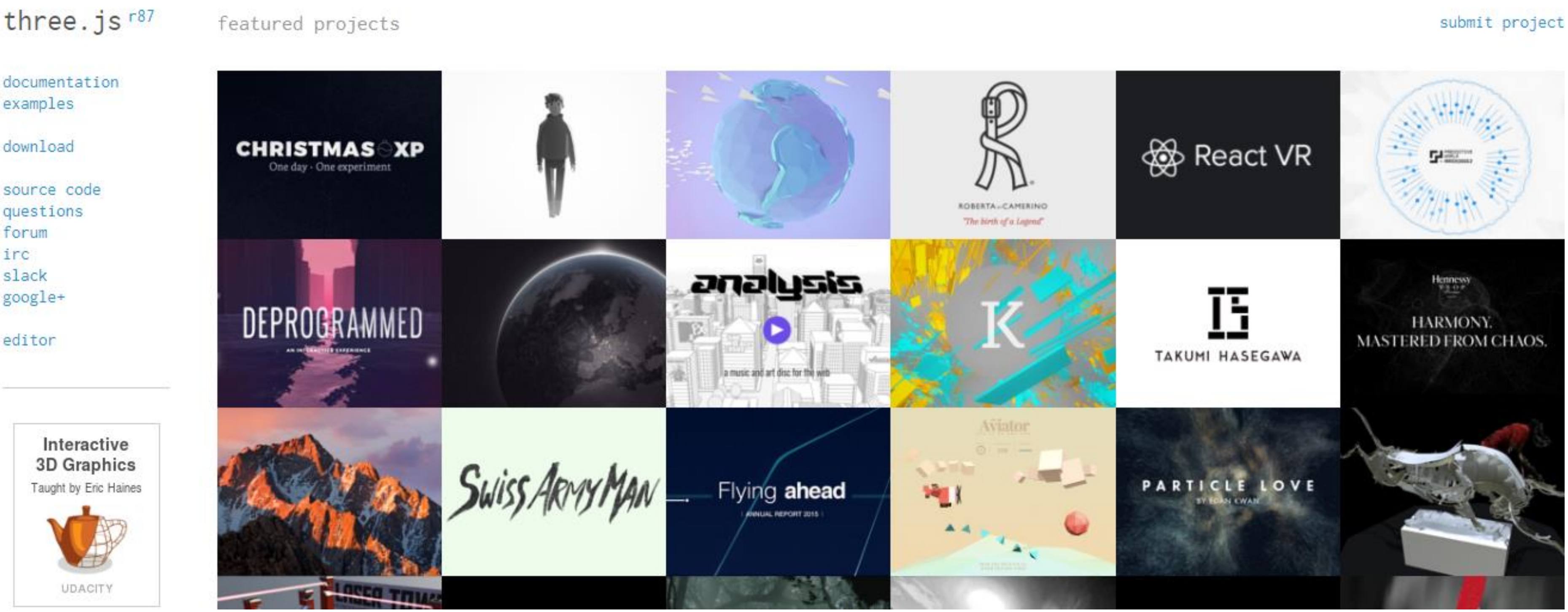
https://threejs.org/manual/#en/primitives

Computer Graphics APIs

- Create 2D / 3D scenes from simple primitives
- OpenGL and variants …
  - Rendering
  - No modeling or interaction facilities
- Direct 3D – Microsoft
- VTK
  - 3D CG + Image processing + Visualization
- Three.js
- Vulkan …

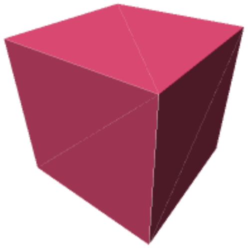
Three.js

- Cross-browser JavaScript library/API used to create and display animated 3D computer graphics in a web browser.
- Uses WebGL



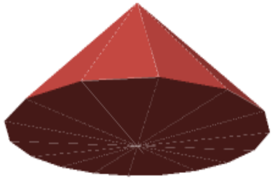
https://threejs.org/

Geometric Primitives – three.js examples

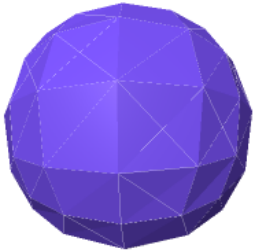


```
const width = 8; // ui: width
const height = 8; // ui: height
const depth = 8; // ui: depth
const geometry = new THREE.BoxGeometry(width, height, depth)
```

```
const radius = 6;  // ui: radius
const height = 8;  // ui: height
const radialSegments = 16;  // ui: radialSegments
const geometry = new THREE.ConeGeometry(radius, height, radialSegments);
```

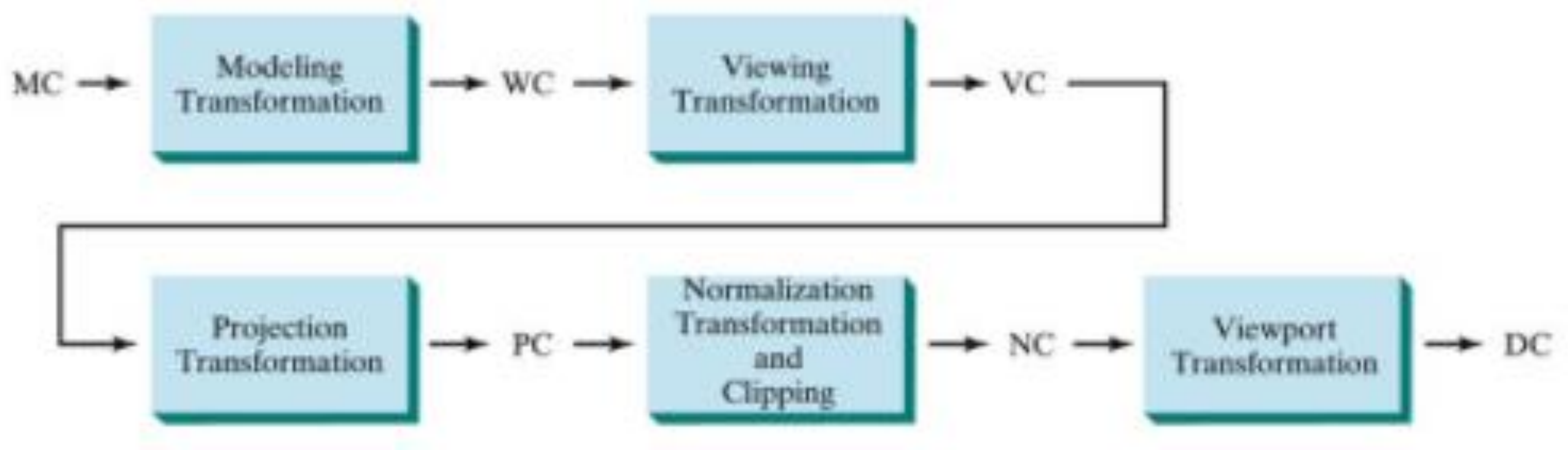


```
const radius = 7;  // ui: radius
const widthSegments = 12;  // ui: widthSegments
const heightSegments = 8;  // ui: heightSegments
const geometry = new THREE.SphereGeometry(radius, widthSegments,
heightSegments);
```



https://threejs.org/manual/#en/primitives

3D visualization pipeline (coordinate transformations)



(Hearn & Baker, 2004)

MC - Modelling coordinates

WC – World coordinates

VC – Viewing coordinates

PC – Projection coordinates

NC – Normalized coordinates

DC – Display coordinates

2D and 3D visualization pipeline

- We will start by 2D transformations and viewing a 2D scene and then generalize to 3D

- Main operations represented as point transformations
  - Basic transformation matrices
  - Homogeneous coordinates
  - Matrix multiplication and composed transformations

Basic 2D Transformations

$p = (x, y)$ → *original point*

$p' = (x', y')$ → *transformed point*

$$\boldsymbol{P} = \begin{bmatrix} x \\ y \end{bmatrix}$$

- Basic transformations:
  - Translation
  - Scaling
  - Rotation

$$\boldsymbol{P'} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Vector notation

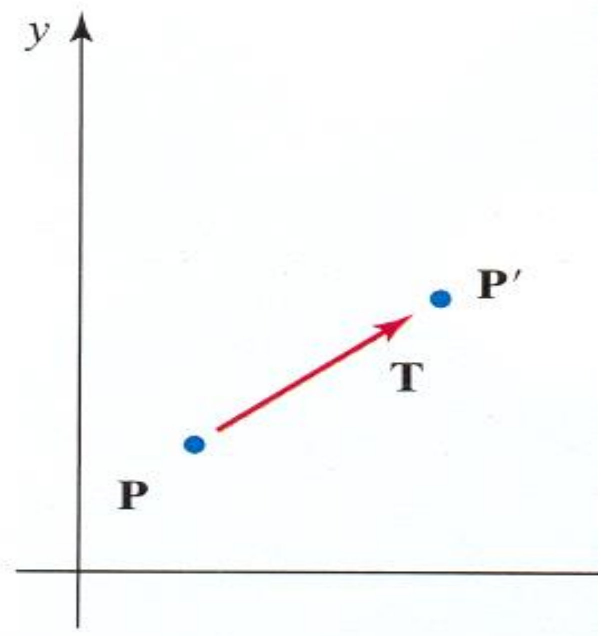
Complex transformations may be expressed as a composition of these

Translation

- It is necessary to specify translations in x and y

$$x' = x + t_x \quad y' = y + t_y$$

$$\boldsymbol{P} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \boldsymbol{P'} = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad \boldsymbol{T} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



$$\boldsymbol{P'} = \boldsymbol{P} + \boldsymbol{T}$$

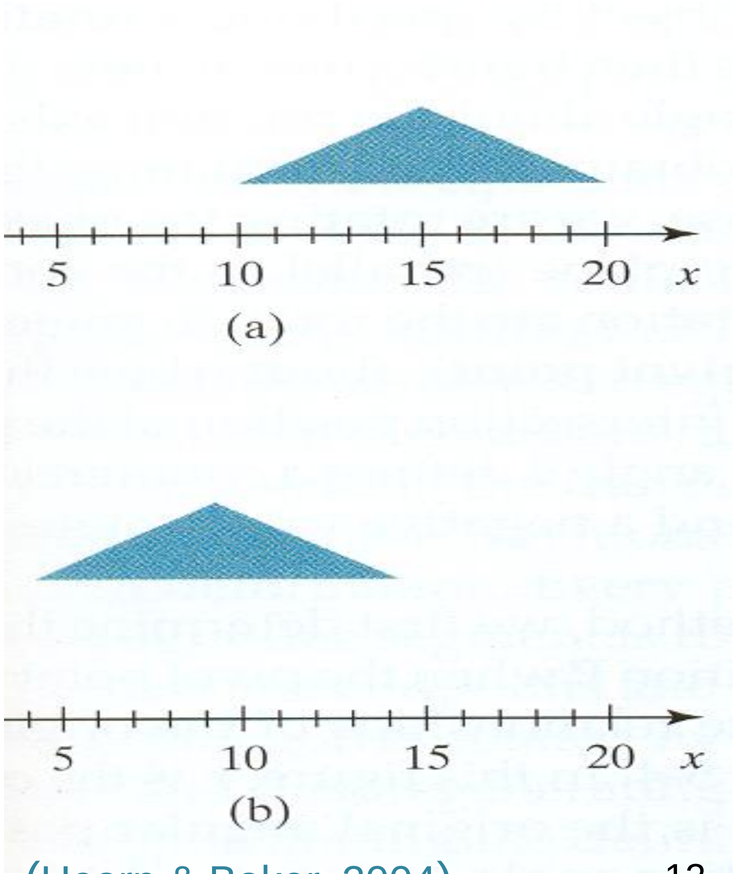
transformation matrix

Translation

- It is a rigid body transformation (it does not deform the object)

- To apply a translation to a line segment we need only to transform the end points

- To apply a translation to a polygon we need only to transform the vertices



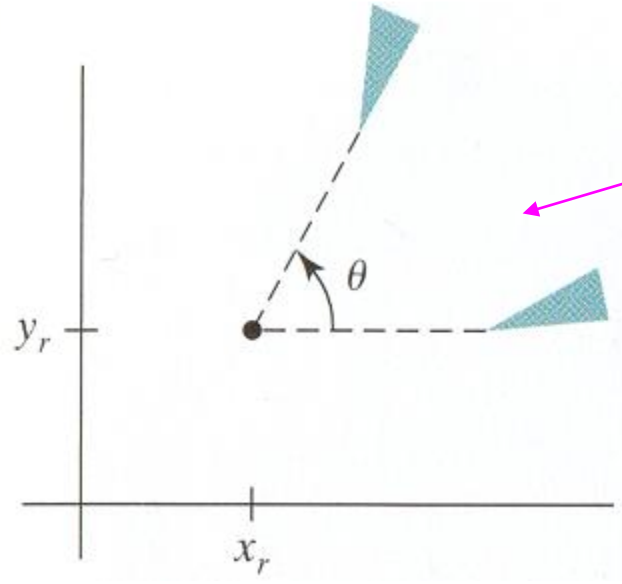
(Hearn & Baker, 2004)

Rotation

- To apply a 2D rotation we need to specify:

  - a point (center of rotation) (x_r, y_r)

  - a rotation angle θ (the convention is: positive -> counter-clockwise)



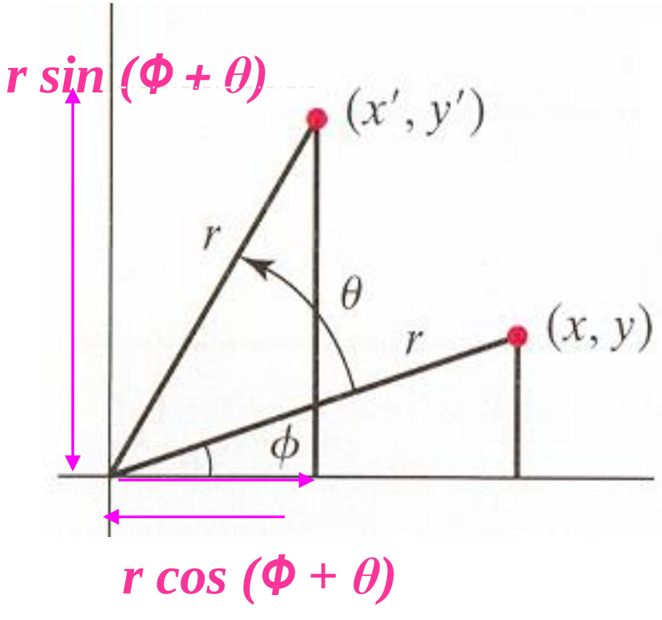
Positive rotation

Rotation around the origin

- The simplest case:

$$x' = r\cos(\Phi + \Theta) = r\cos\Phi\cos\Theta - r\sin\Phi\sin\Theta$$

$$y' = r\sin(\Phi + \Theta) = r\cos\Phi\sin\Theta + r\sin\Phi\cos\Theta$$



Polar coordenates of the original point:

$$x = r\cos\Phi$$
$$y = r\sin\Phi$$

Replacing:

$$x' = x\cos\Theta - y\sin\Theta$$
$$y' = x\sin\Theta + y\cos\Theta$$

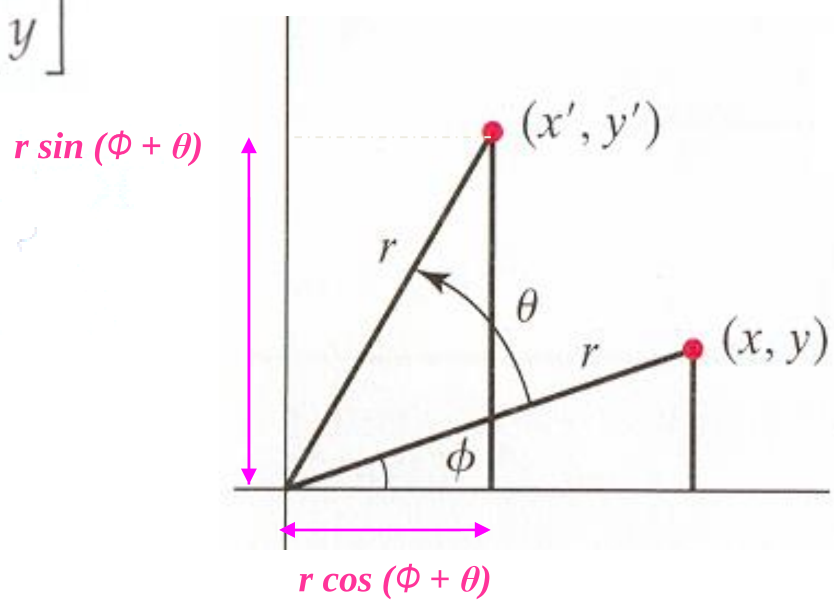
2D Rotation in matrix notation

$$x' = r\cos(\Phi + \Theta) = r\cos\Phi\cos\Theta - r\sin\Phi\sin\Theta$$

$$y' = r\sin(\Phi + \Theta) = r\cos\Phi\sin\Theta + r\sin\Phi\cos\Theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$



Reminder:

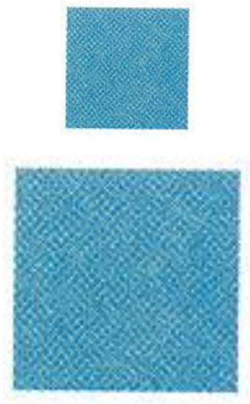
cos(α+β) = cos α cos β − sin α sin β

cos (α−β) = cos α cos β + sin α sin β

Scaling

- Modifies the size of an object; we need to specify scaling factors: s_x and s_y

$$x' = x \cdot s_x$$
$$y' = y \cdot s_y$$



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Trasformation matrix

$$P' = S \cdot P$$

Transforming a square into a larger square applying a scaling s_x=2, s_y=2

(Hearn & Baker, 2004)

2D Transformations (composed)

- Matrix representation
  - Homogeneous coordinates !!
  - Concatenation = Matrix products

- Complex transformations ?
  - Decompose into a sequence of basic transformations

Homogeneous coordinates

- Most applications involve sequences of transformations

- For instance:

  - visualization transformations involve a sequence of translations and rotations to render an image of a scene

  - animations may imply that an object is rotated and translated between two consecutive frames

- **Homogeneous coordinates provide an efficient way to represent and apply sequences of transformations**

- It is possible to combine in a matrix the multiplying and additive terms if we use 3x3 matrices

- **All transformations may be represented by multiplying matrices in homogenous coordinates**

- Each point is now represented by 3 coordinates

$$(x, y) \rightarrow (x_h, y_h, h), \qquad h \neq 0$$

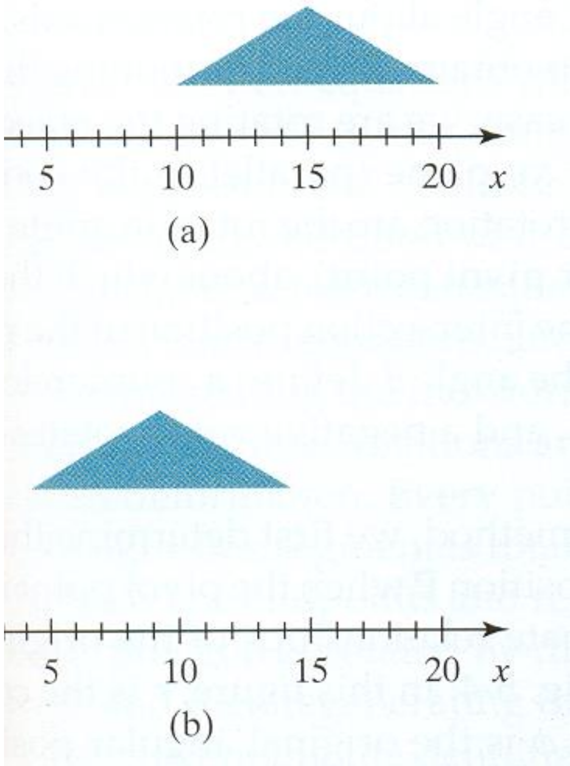
$$x = x_h / h \qquad y = y_h / h$$

$$(x.h, y.h, h)$$

2D Translation
(in homogeneous coordinates)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{T}(t_x, t_y) \cdot \mathbf{P}$$



(Hearn & Baker, 2004)

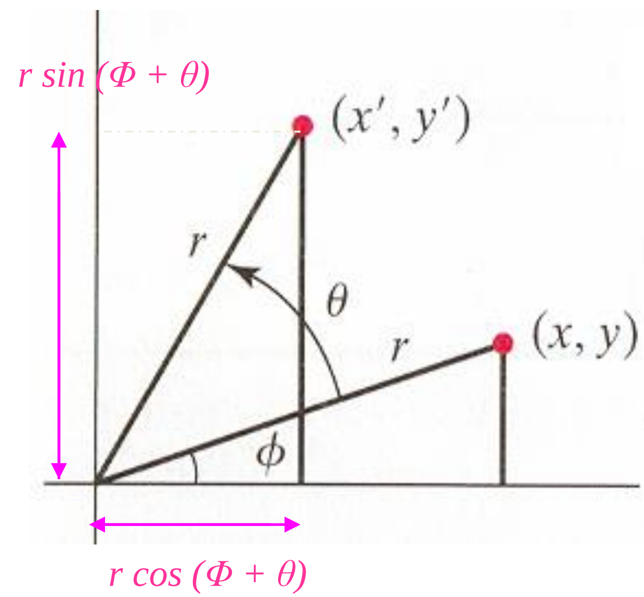
2D Rotation
(in homogeneous coordinates)

$$x' = r\cos(\Phi + \Theta) = r\cos\Phi\cos\Theta - r\sin\Phi\sin\Theta$$

$$y' = r\sin(\Phi + \Theta) = r\cos\Phi\sin\Theta + r\sin\Phi\cos\Theta$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

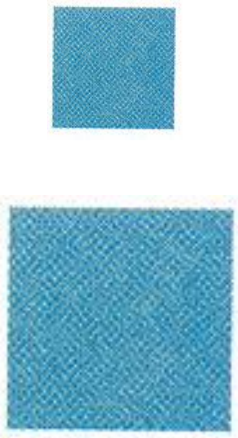
$$\mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$



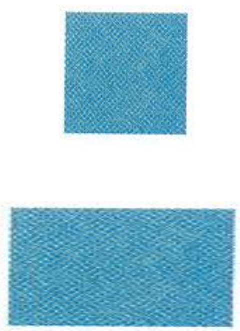
2D Scaling
(in homogeneous coordinates)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{S}(s_x, s_y) \cdot \mathbf{P}$$



$(Sx = Sy)$



$(Sx \neq Sy)$

(Hearn & Baker, 2004)

Concatenation of two translations

$$\begin{aligned}\mathbf{P}' &= \mathbf{T}(t_{2x}, t_{2y}) \cdot \{\mathbf{T}(t_{1x}, t_{1y}) \cdot \mathbf{P}\} \\ &= \{\mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y})\} \cdot \mathbf{P}\end{aligned}$$

$$\begin{bmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

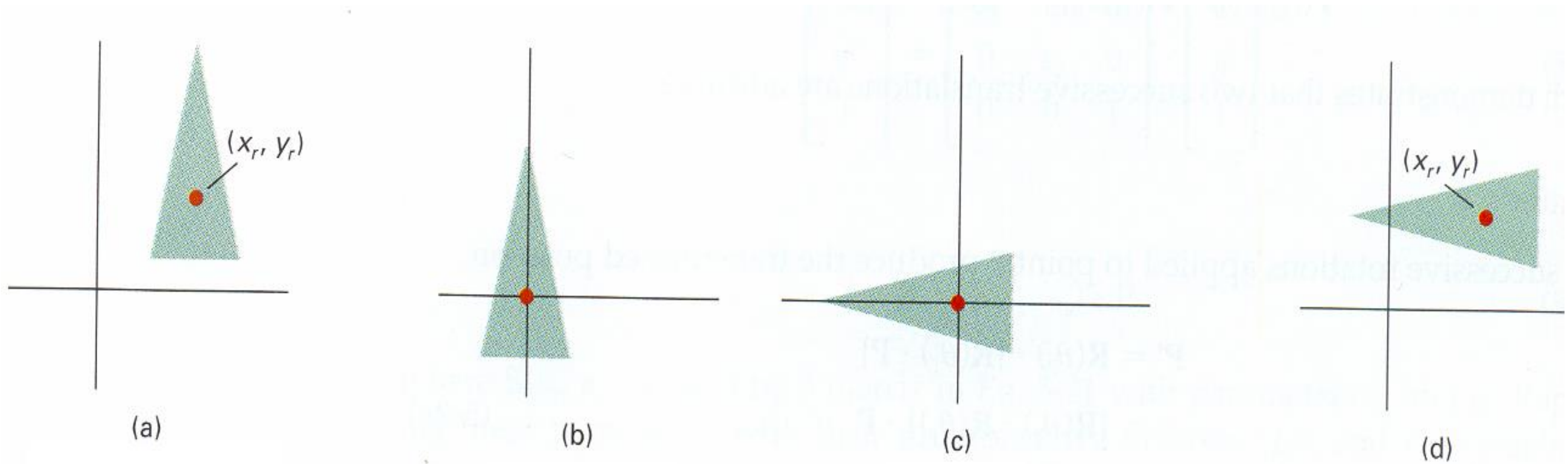
$$\mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y}) = \mathbf{T}(t_{1x} + t_{2x}, t_{1y} + t_{2y})$$

Concatenation of two scaling transformations

$$\begin{bmatrix} s_{2x} & 0 & 0 \\ 0 & s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{1x} & 0 & 0 \\ 0 & s_{1y} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{1x} \cdot s_{2x} & 0 & 0 \\ 0 & s_{1y} \cdot s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{S}(s_{2x}, s_{2y}) \cdot \mathbf{S}(s_{1x}, s_{1y}) = \mathbf{S}(s_{1x} \cdot s_{2x}, \quad s_{1y} \cdot s_{2y})$$

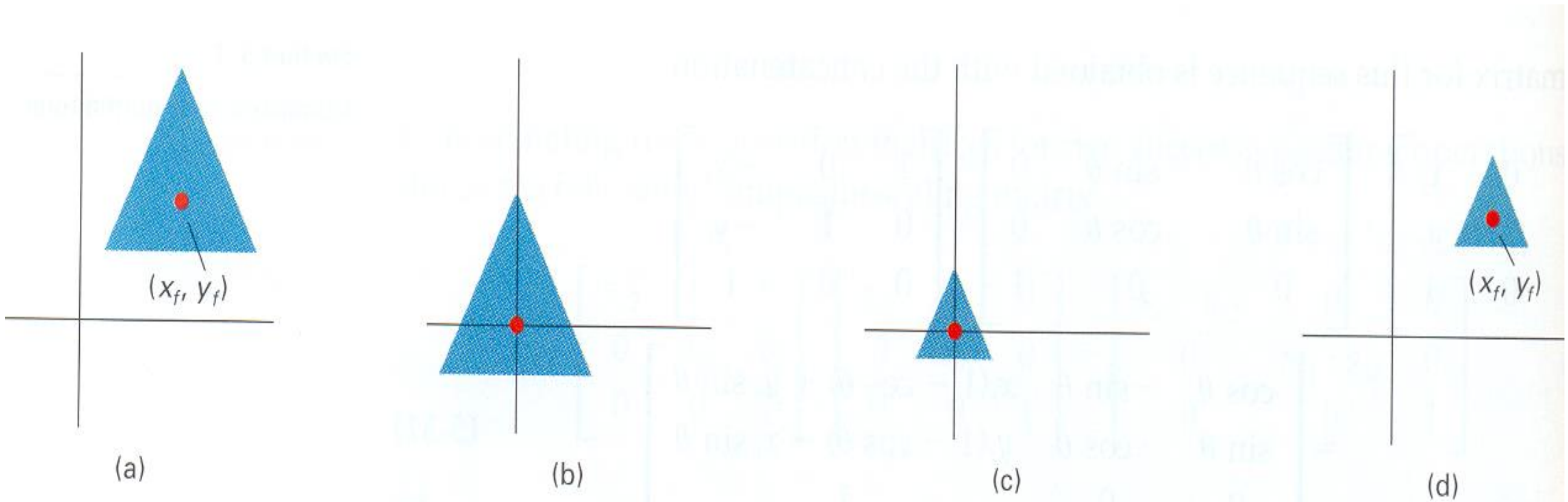
Arbitrary Rotation
(around any point)



(Hearn & Baker, 2004)

Translation + (to the origin)	Rotation (around the origin)	+ Inverse Translation (to the initial position)

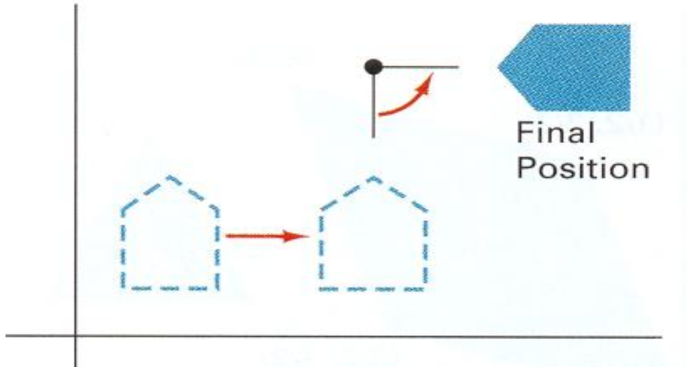
Arbitrary Scaling

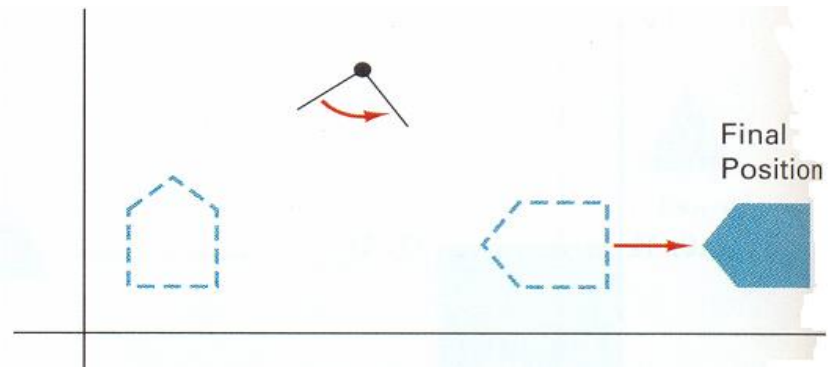


(Hearn & Baker, 2004)

Translation + Scaling + Inverse Translation

Order is important !





a) Translation + rotation

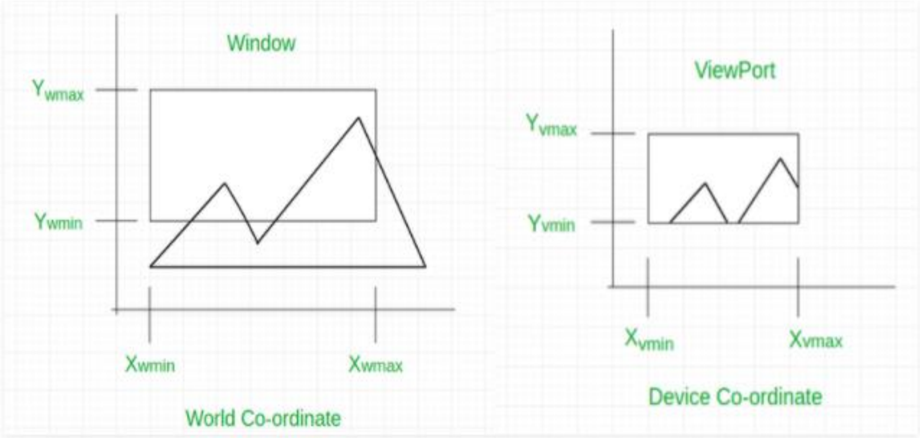
b) Rotation + translation

(Hearn & Baker, 2004)

Results may be different if transformations are applied in a different order!

The case of viewing 2D scenes

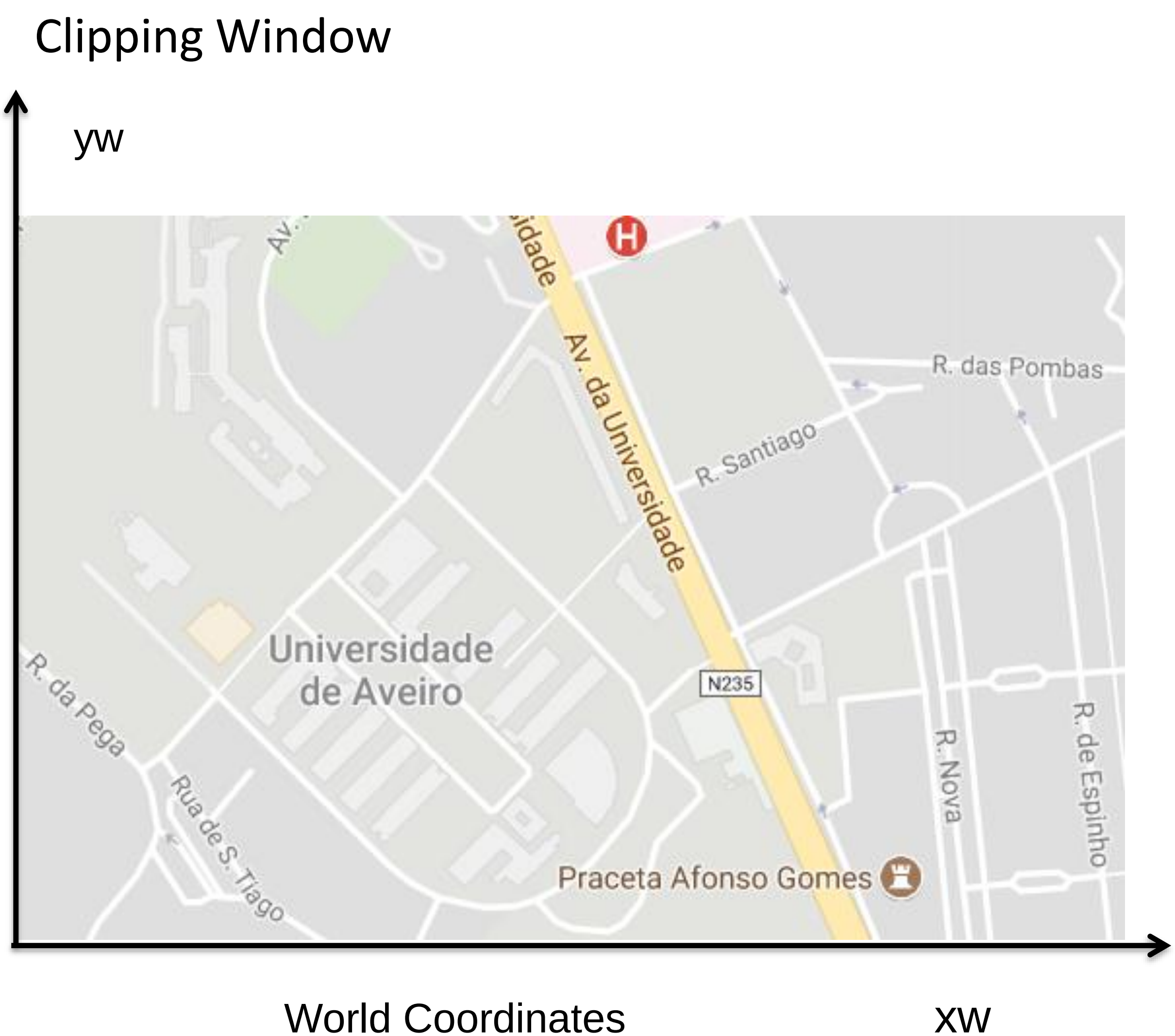
- Define a 2D scene in the world coordinate system
- Select a clipping window in the XOY plane
  - The window contents will be displayed
- Select a viewport in the display
- The viewport displays the contents of the clipping window

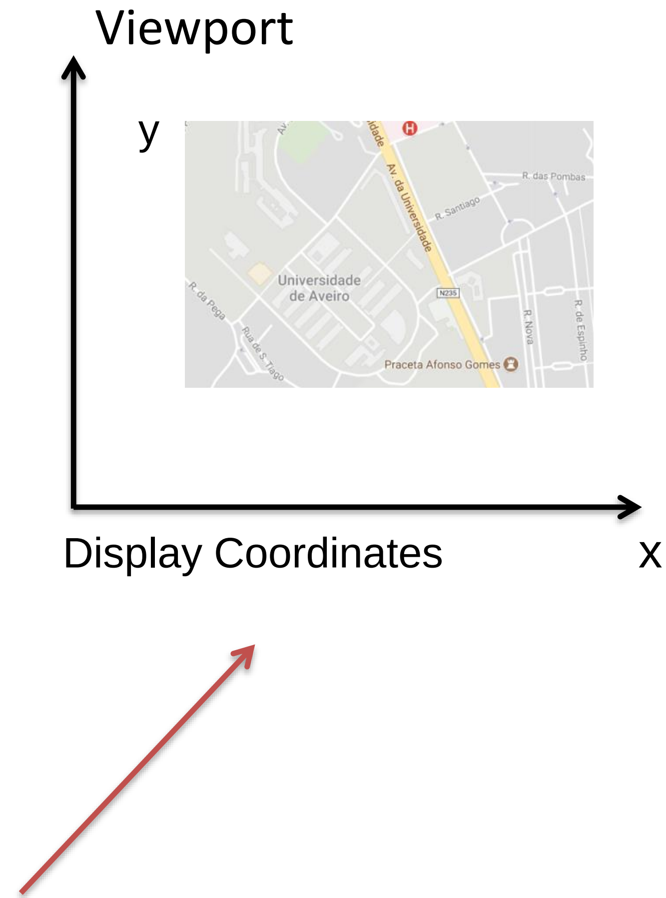


Note: Clipping window and viewport are the traditional terms in CG

https://www.geeksforgeeks.org/window-to-viewport-transformation-in-computer-graphics-with-implementation/

World -> display

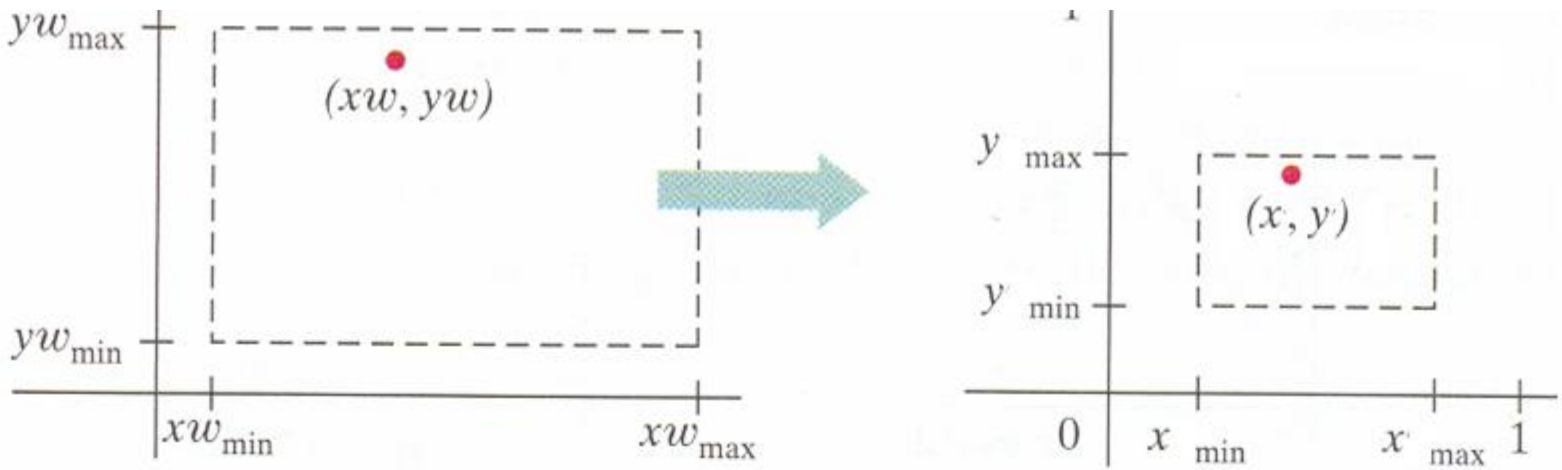




Coordinate mapping

Clipping Window

Viewport



World Coordinates

Screen coordinates

(Hearn & Baker, 2004)

Objects inside the clipping window are mapped to the viewport (the area on the screen where they will be displayed).

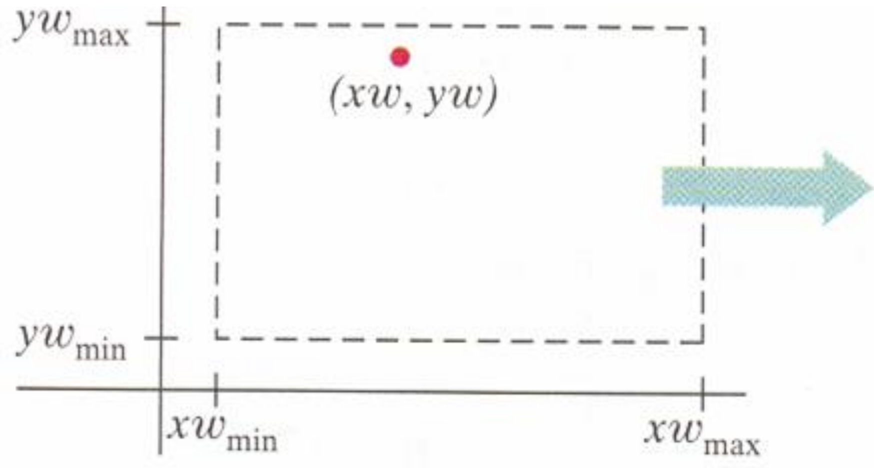
Home work:

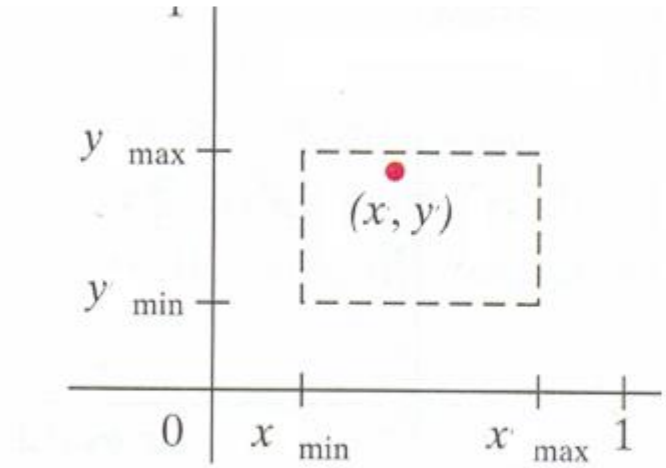
Compute (x,y) given (xw, yw)

Coordinate mapping

Clipping Window

Viewport





World Coordinates

Screen coordinates

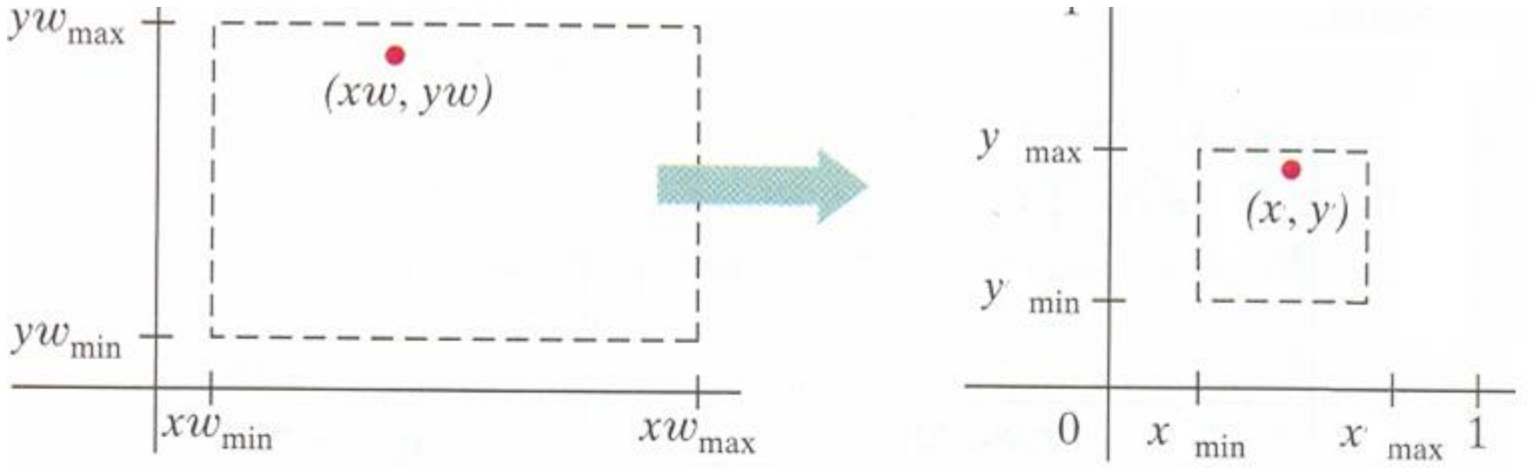
(x,y) given (xw, yw) :

$$\frac{xw - xwmin}{xwmax - xwmin} = \frac{x - xmin}{xmax - xmin}$$

$$\frac{yw - ywmin}{ywmax - ywmin} = \frac{y - ymin}{ymax - ymin}$$

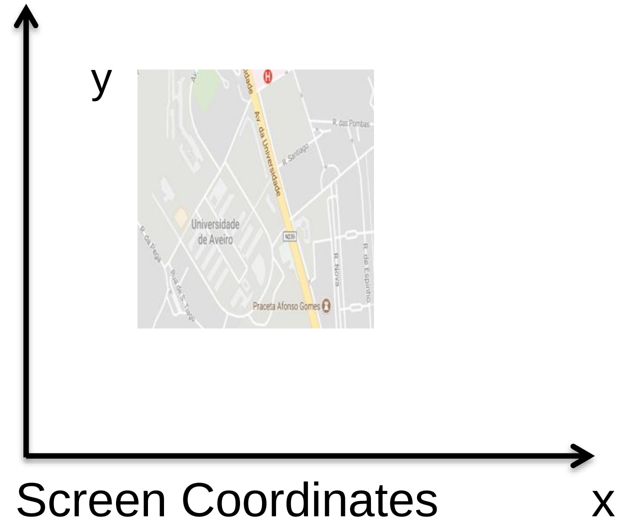
Coordinate mapping

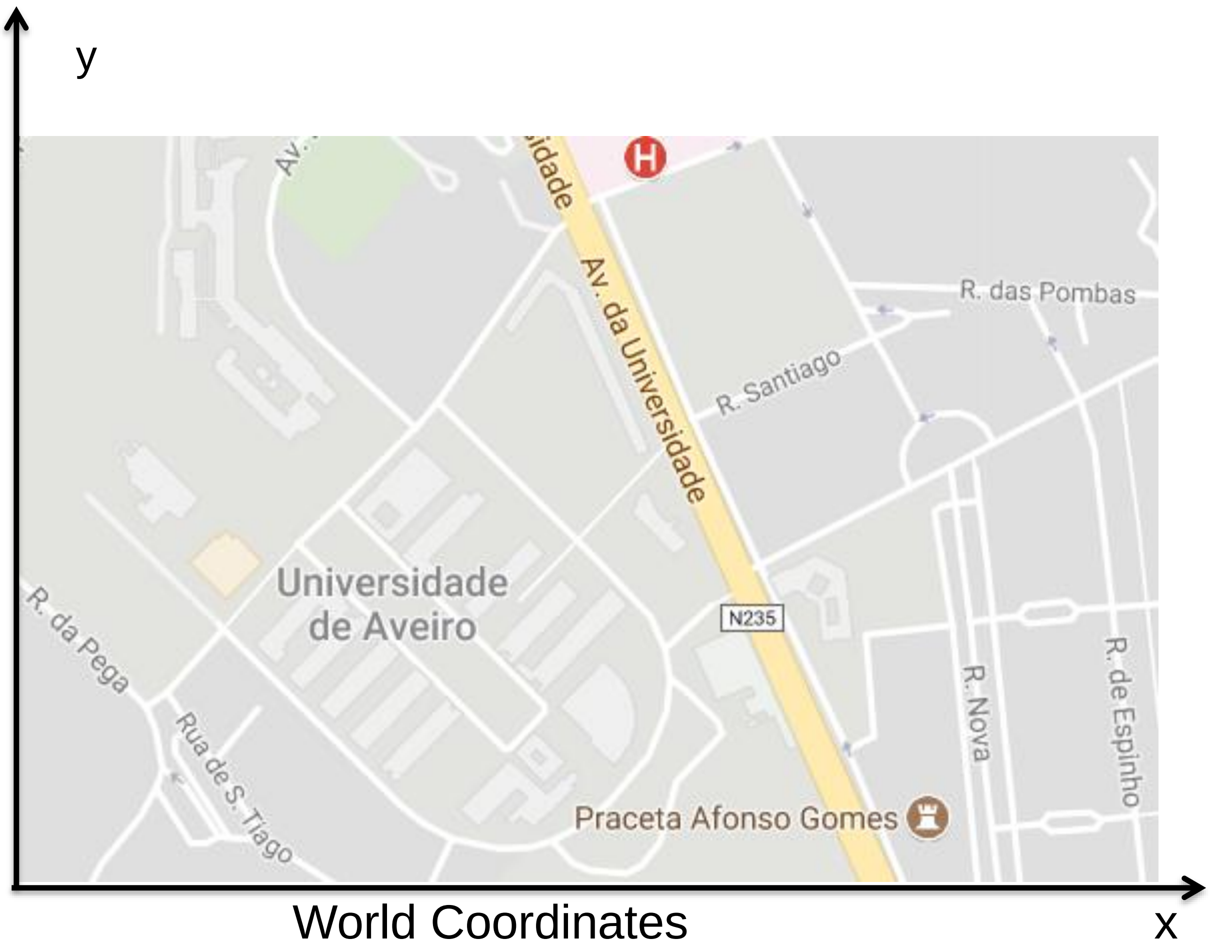
If the **aspect ratio** is not the same in both situations the result is distortion



(Hearn & Baker, 2004)

World -> screen





The **aspect ratio** is not the same in both situations: distortion!

3D Transformations
(in homogeneous coordinates)

- Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\boldsymbol{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

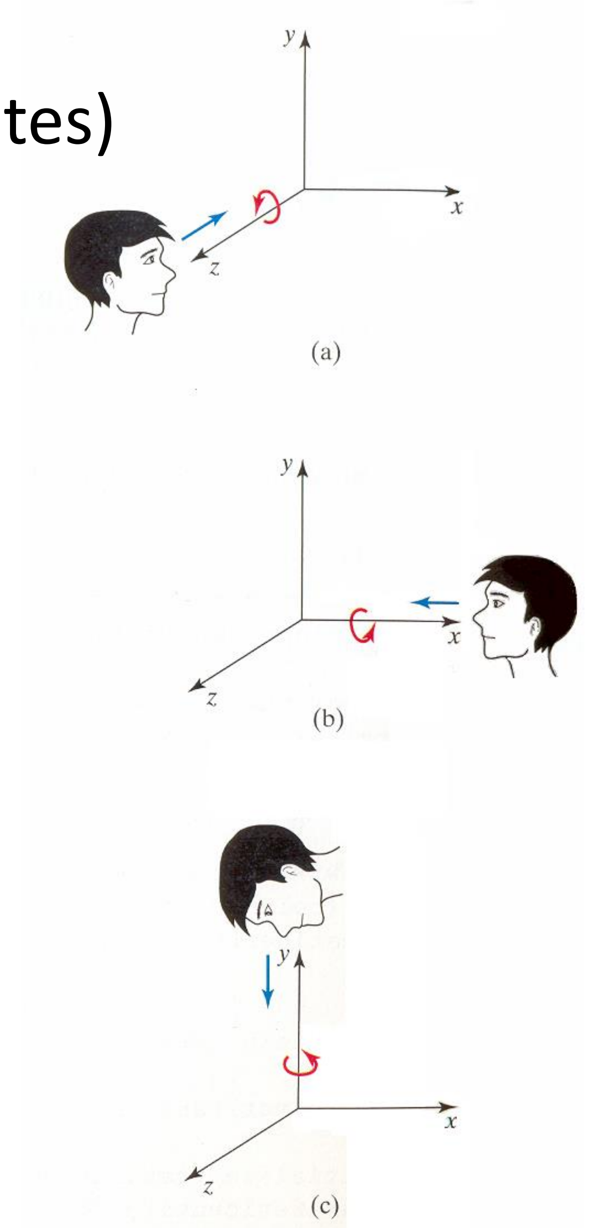
- Scaling

$$\mathbf{S} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{P'} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

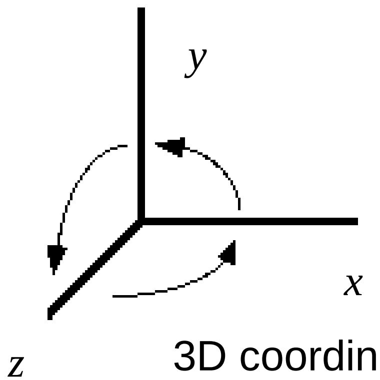
A 3D point in vector notation

3D Rotation
(in homogeneous coordinates)



- Rotation around each one of the coordinate axis

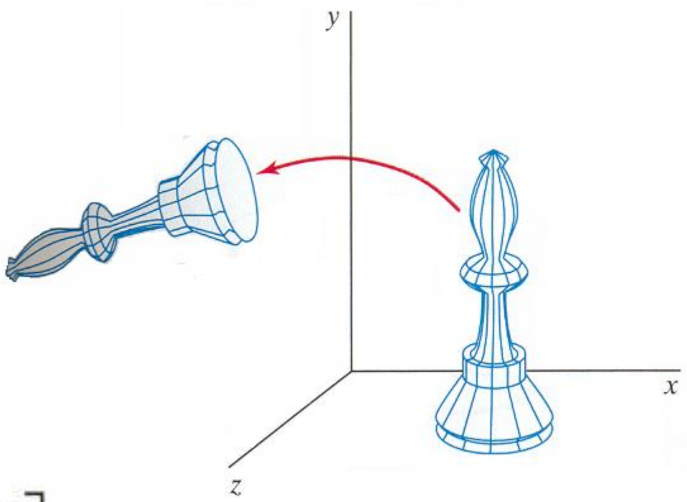
- Positive rotations are CCW (counter clock wise)!!



3D coordinate system in CG

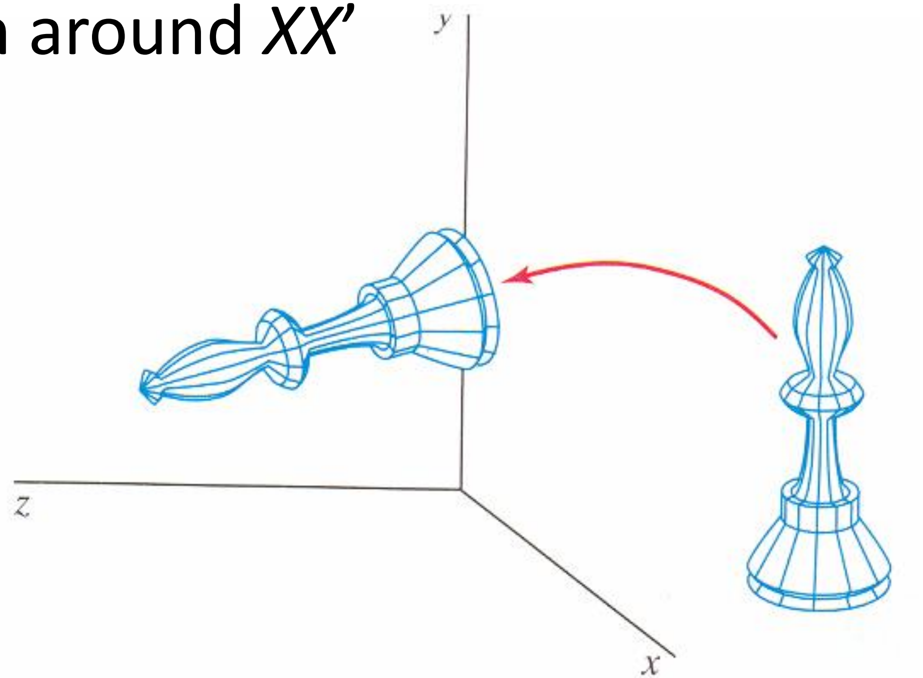
(Hearn & Baker, 2004)

Rotation around ZZ'



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

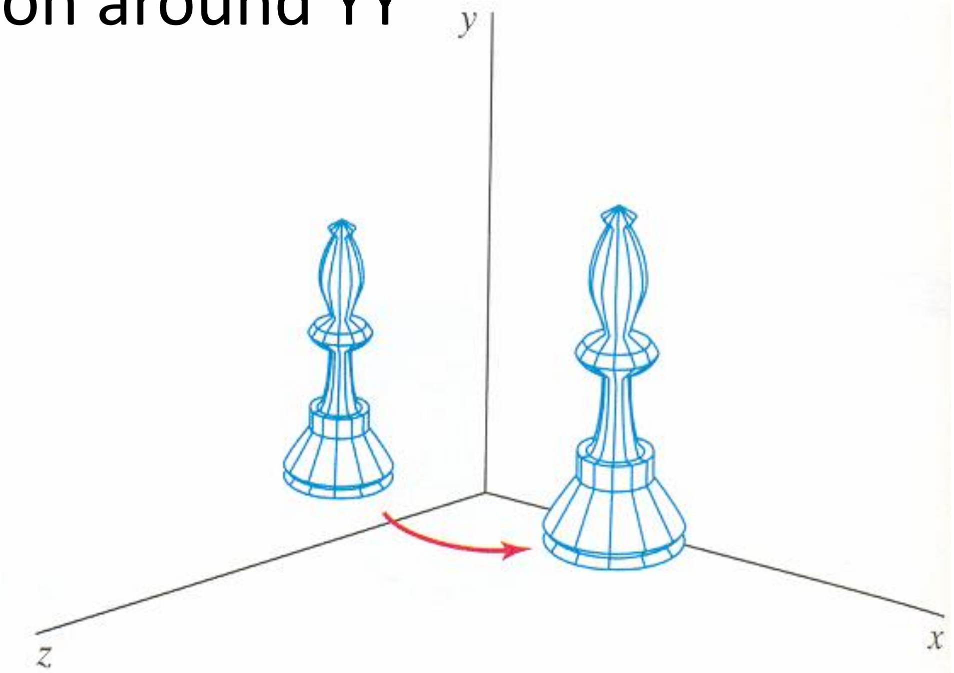
Rotation around *XX'*



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(Hearn & Baker, 2004)

Rotation around YY’



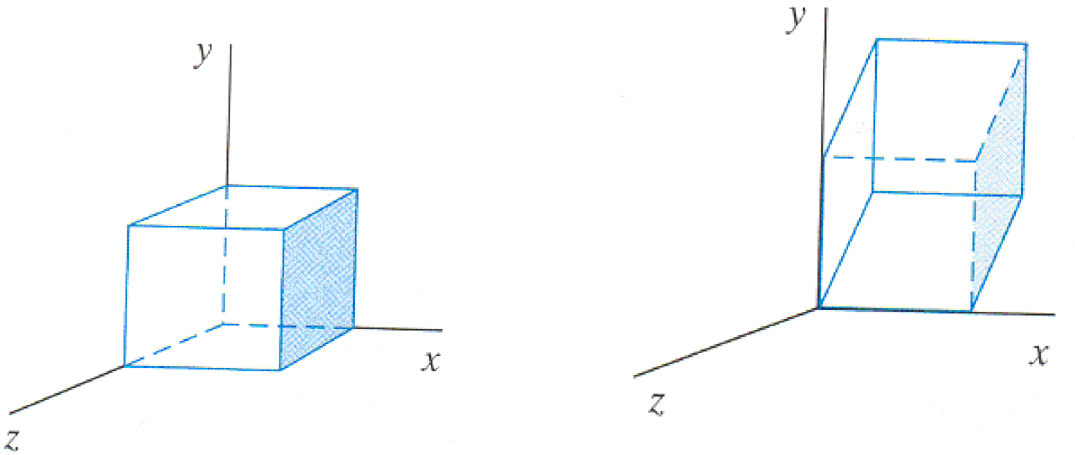
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(Hearn & Baker, 2004)

Other useful 3D Transformations

- Shears
- Reflections

Shear in ZZ'



Transformations in three.js

Matrix4

A class representing a 4x4 matrix.

The most common use of a 4x4 matrix in 3D computer graphics is as a Transformation Matrix. For an introduction to transformation matrices as used in WebGL, check out this tutorial.

This allows a Vector3 representing a point in 3D space to undergo transformations such as translation, rotation, shear, scale, reflection, orthogonal or perspective projection and so on, by being multiplied by the matrix. This is known as *applying* the matrix to the vector.

https://threejs.org/docs/#api/math/Matrix4

.makeTranslation (x, y, z)

x - the amount to translate in the X axis.
y - the amount to translate in the Y axis.
z - the amount to translate in the Z axis.

Sets this matrix as a translation transform:

```
1, 0, 0, x,
0, 1, 0, y,
0, 0, 1, z,
0, 0, 0, 1
```

.makeScale (x, y, z)

x - the amount to scale in the X axis.
y - the amount to scale in the Y axis.
z - the amount to scale in the Z axis.

Sets this matrix as scale transform:

```
x, 0, 0, 0,
0, y, 0, 0,
0, 0, z, 0,
0, 0, 0, 1
```

.makeRotationX (theta)

theta — Rotation angle in radians.

Sets this matrix as a rotational transformation around the X axis by theta (θ) radians. The resulting matrix will be:

```
1 0      0       0
0 cos(θ) -sin(θ) 0
0 sin(θ) cos(θ)  0
0 0      0       1
```

.makeRotationY (theta)

theta — Rotation angle in radians.

Sets this matrix as a rotational transformation around the Y axis by theta (θ) radians. The resulting matrix will be:

```
cos(θ)  0 sin(θ) 0
0       1 0      0
-sin(θ) 0 cos(θ) 0
0       0 0      1
```

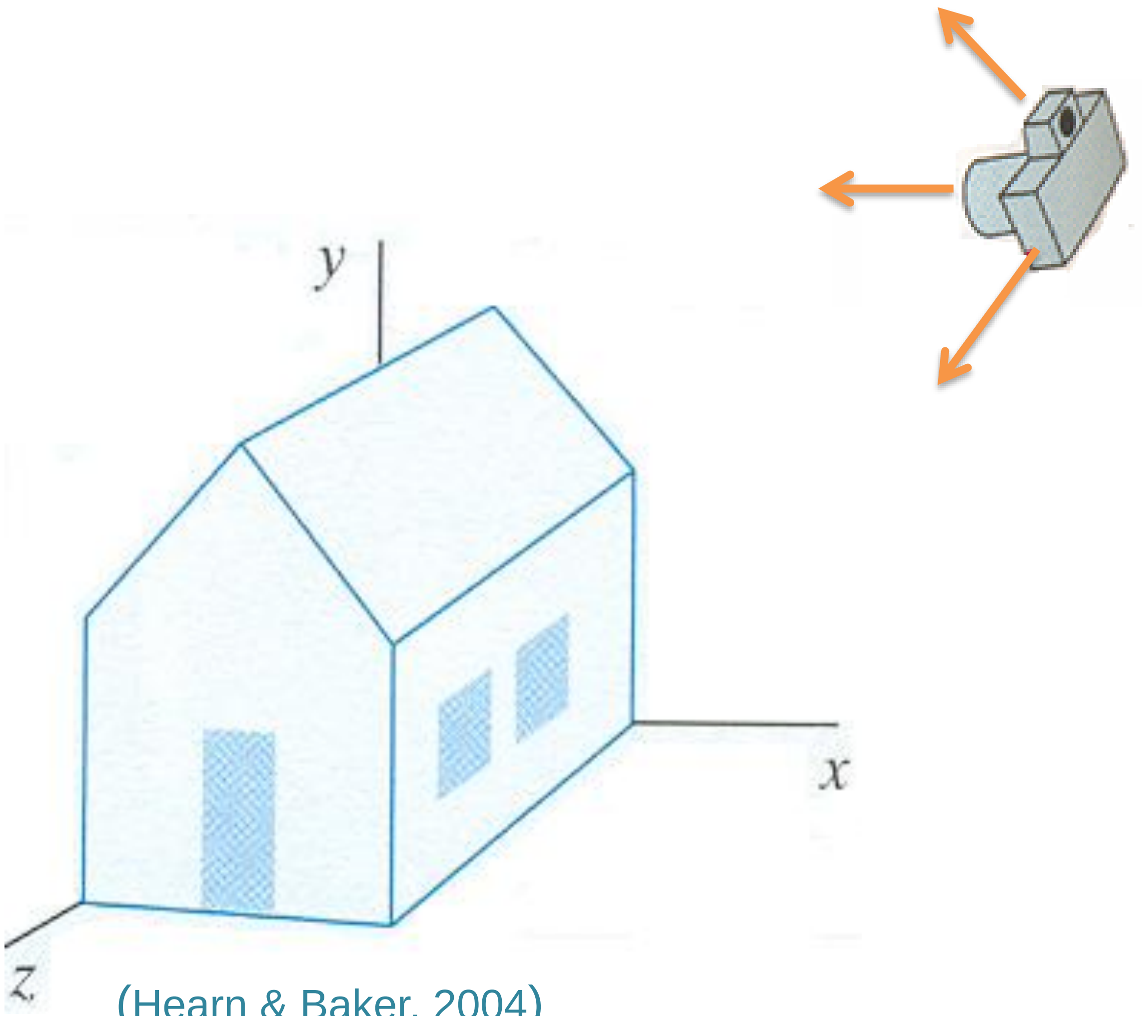
.makeRotationZ (theta)

theta — Rotation angle in radians.

Sets this matrix as a rotational transformation around the Z axis by theta (θ) radians. The resulting matrix will be:

```
cos(θ) -sin(θ) 0 0
sin(θ) cos(θ)  0 0
0      0       1 0
0      0       0 1
```

3D Viewing



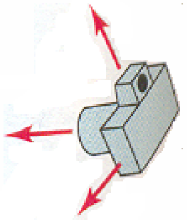
(Hearn & Baker, 2004)

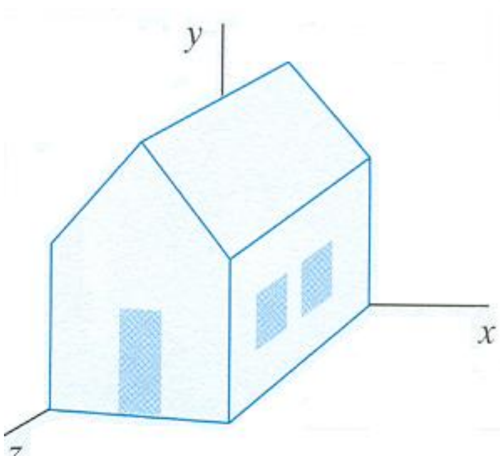
3D Viewing

- Where is the observer / the camera ?
  - Position ?
  - Close to the 3D scene ?
  - Far away ?

- How is the camera/observer looking at the scene ?
  - Orientation ?

- How to represent as a 2D image ?
  - Projection ?



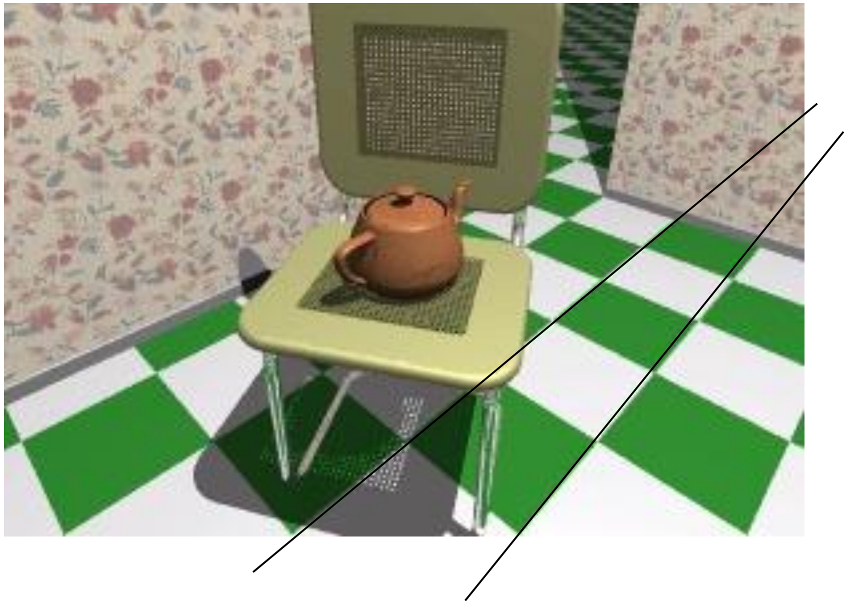


3D visualization pipeline

- Instantiate models of the scene
  - Position, orientation, size
- Establish viewing parameters
  - Camera position and orientation
- Compute illumination and shade polygons
- Perform clipping
- Project into 2D
- Rasterize

Projection (from 3D to 2D)

Obtaining a 2D representation of a 3D scene may be done in different ways …



Parallel Projection

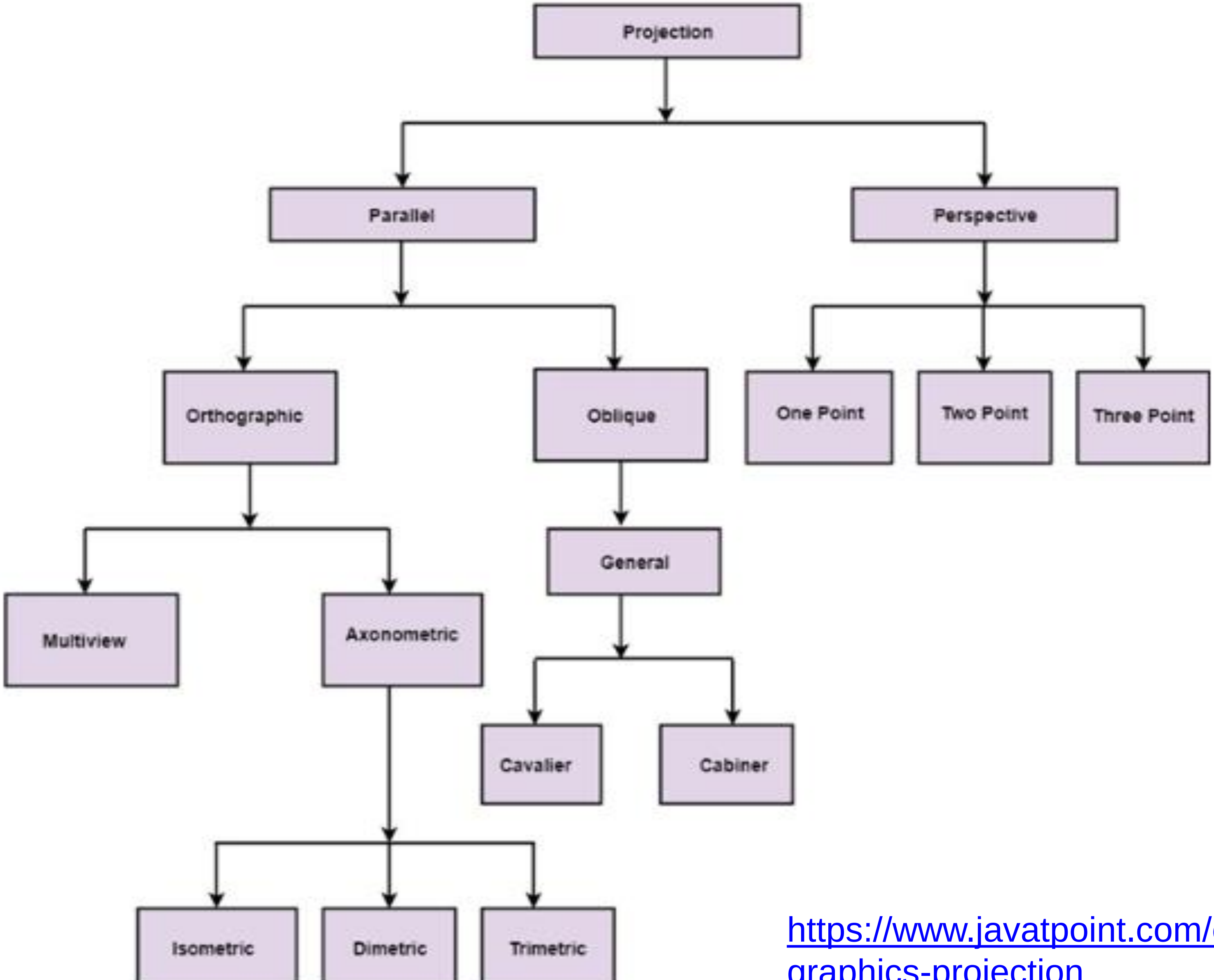
(allows measures)

Perspective Projection

(more realistic images)

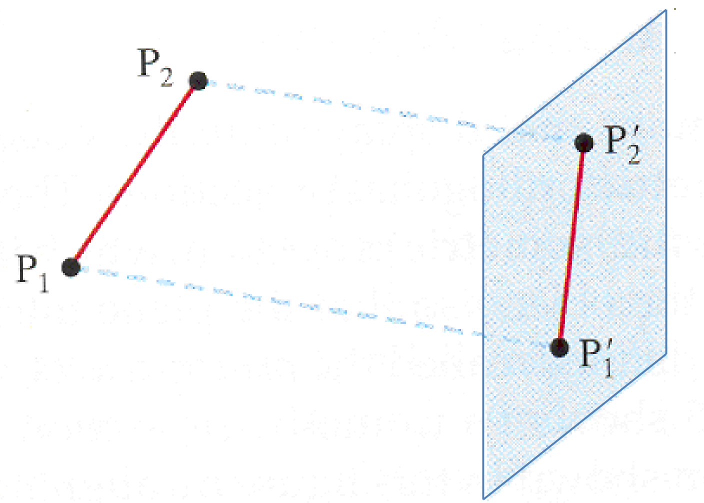
https://www.britannica.com/science/projection-geometry

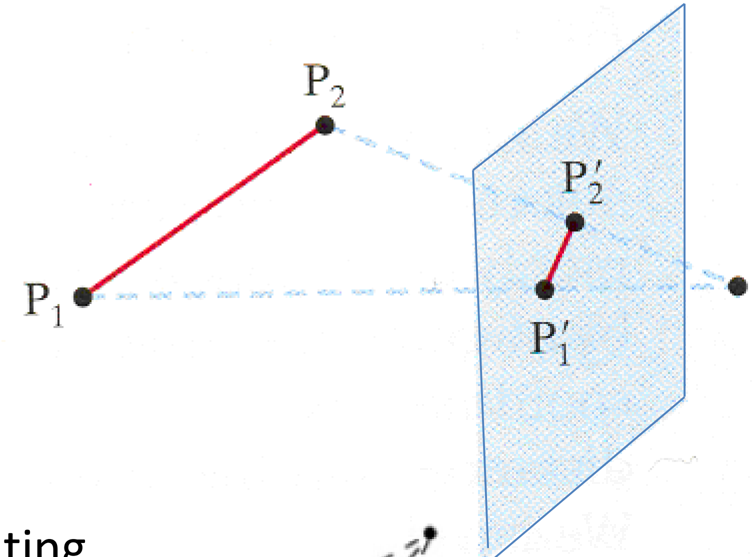
Projections



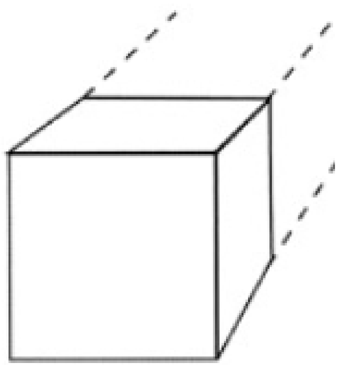
https://www.javatpoint.com/computer-graphics-projection

Projections

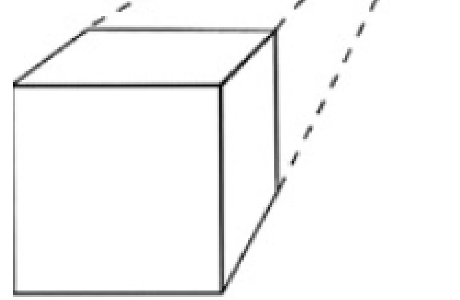




Examples of resulting representation on the viewing plane



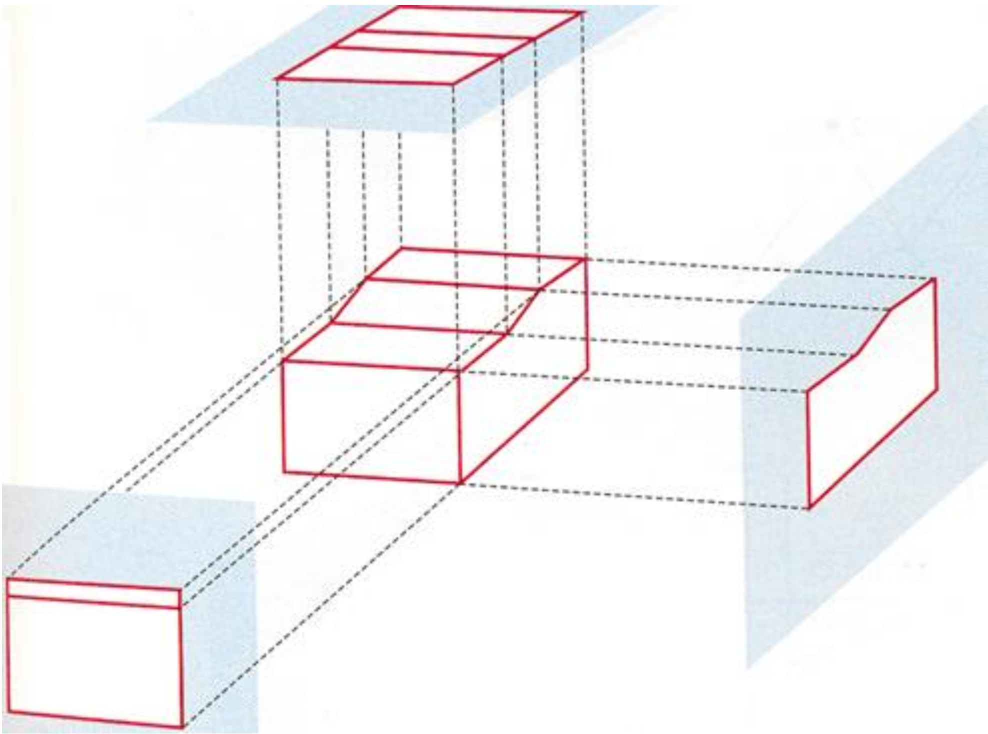
Parallel Projection



Perspective Projection

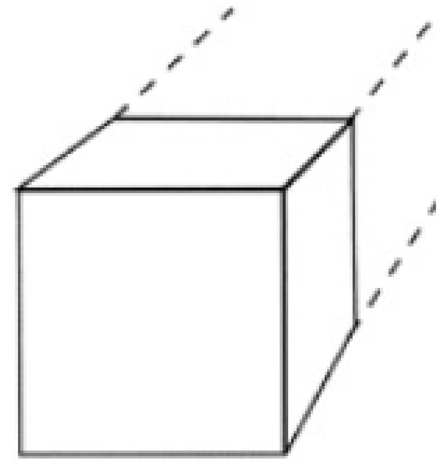
(Hearn & Baker, 2004)

Parallel Projections



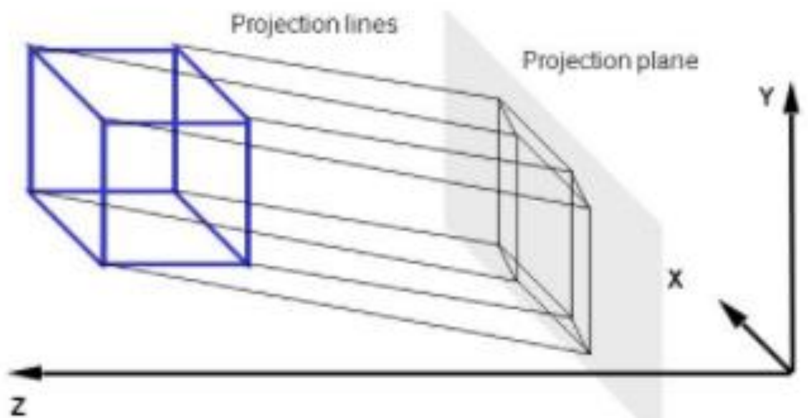
Orthographic/ Multiview projection

(Hearn & Baker, 2004)



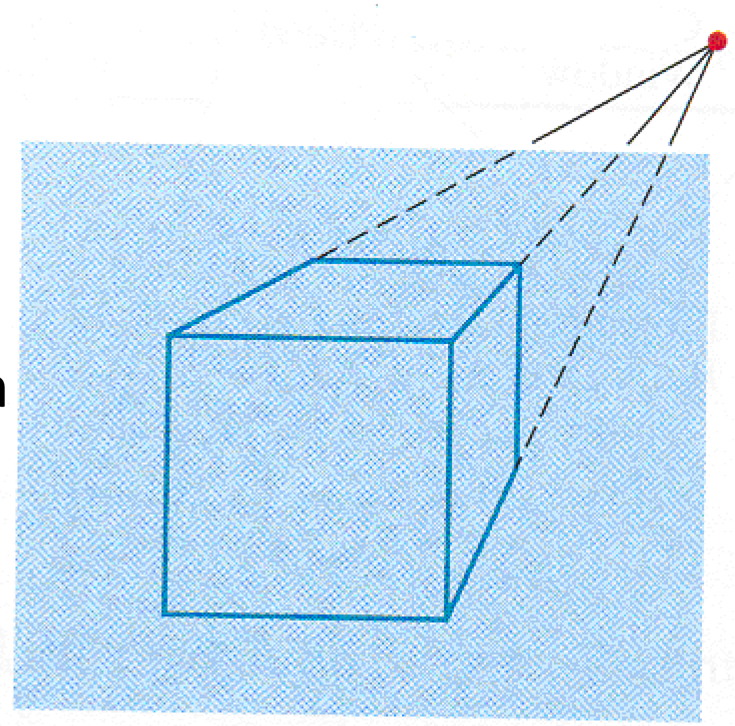
Orthographic / Axonometric projection

Oblique projection

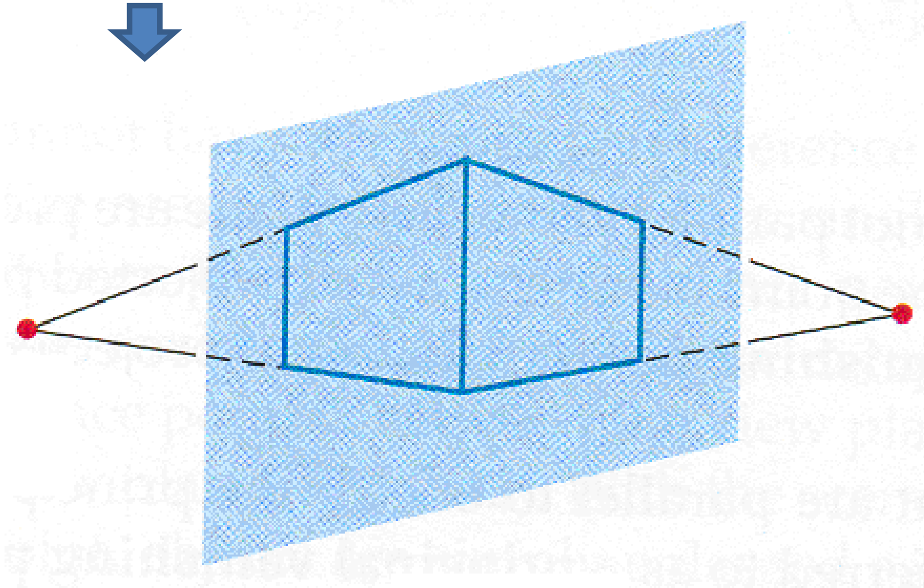


Perspective Projections

One vanishing point perspective projection



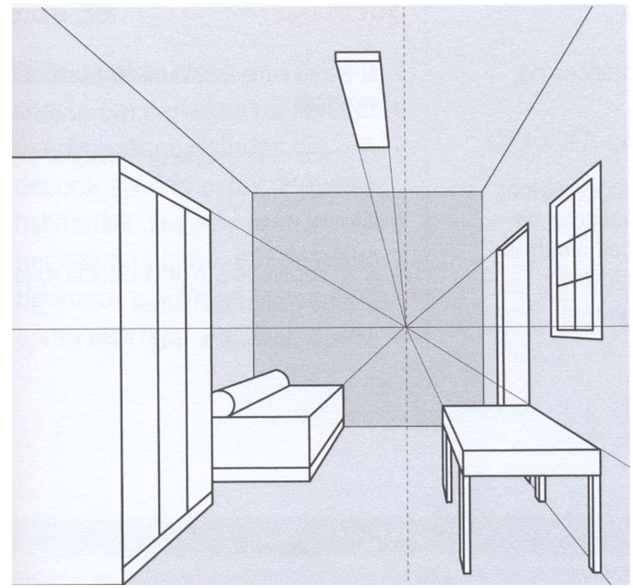
Two vanishing points perspective projection



(Hearn & Baker, 2004)

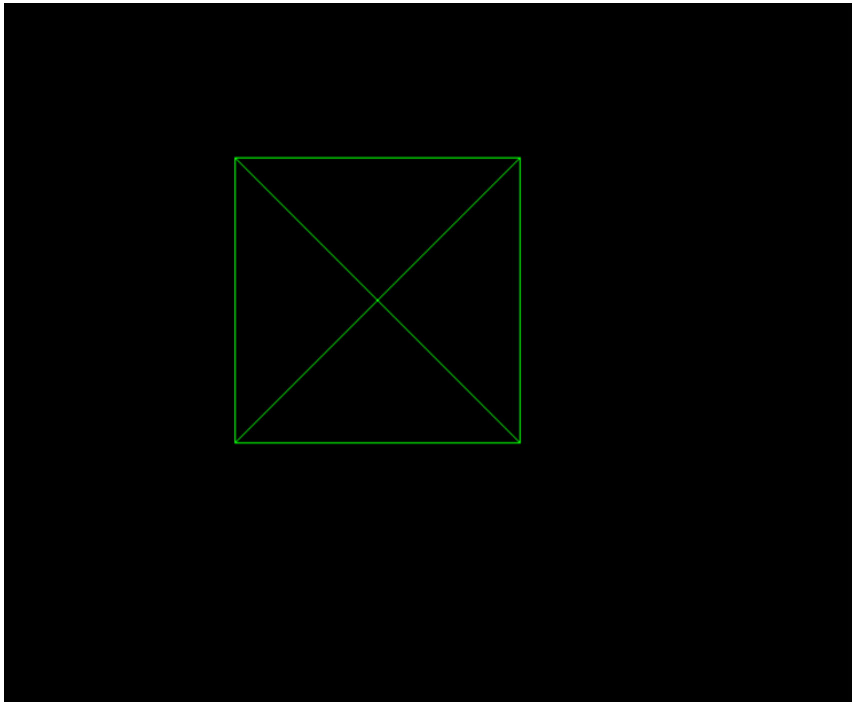
Perspective Projections

Foreshortening indicates a perspective projection

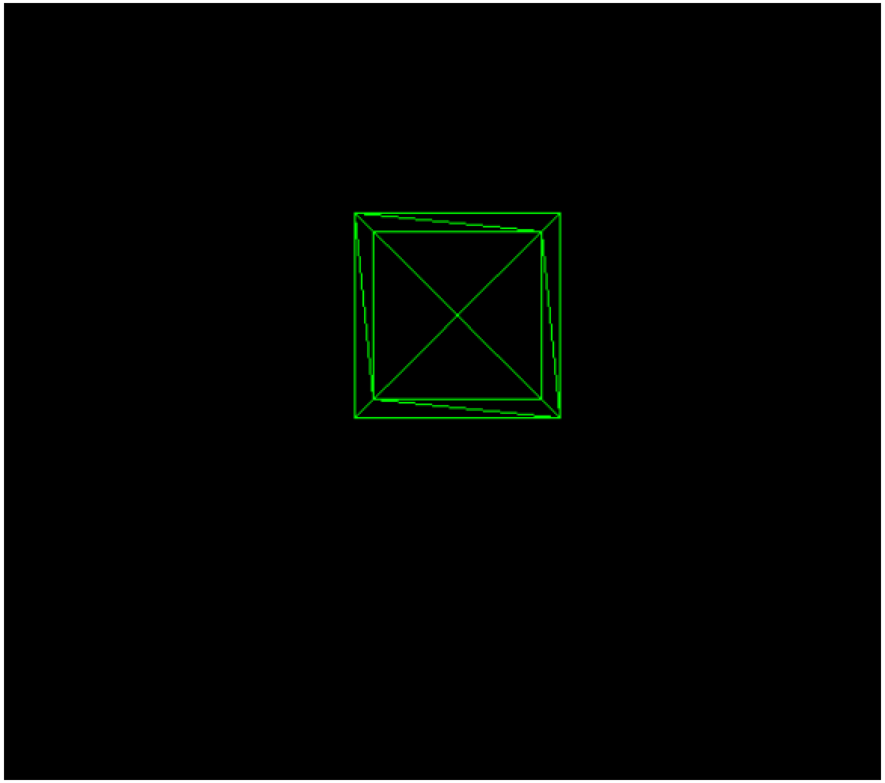


Approximate representation as a scene is seen by the eye

Object's dimensions along the line of sight appear shorter than its dimensions across the line of sight
https://en.wikipedia.org/wiki/Perspective_(graphical)



Orthographic *vs* perspective camera



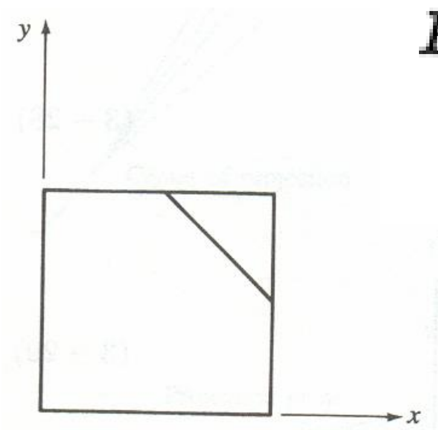
How to represent ?

- Projection matrices
- Homogeneous coordinates
- Concatenation through matrix multiplication
- Don’t worry !
- Graphics APIs implement usual projections !

How to apply Projections?

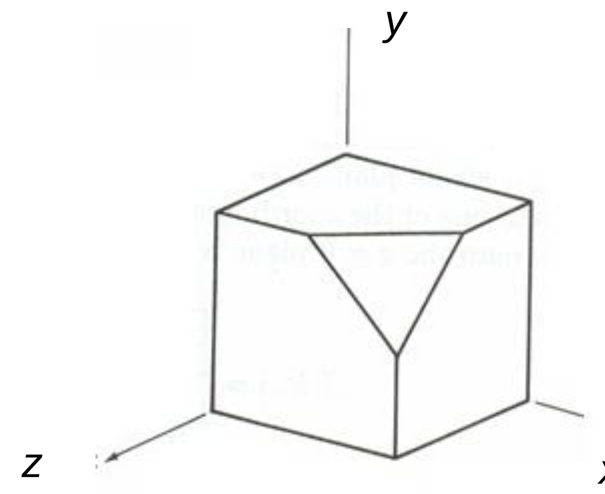
- Also by matrix multiplication

Example: Matrix of the orthographic projection on the *xy* plane in homogeneous coordinates:



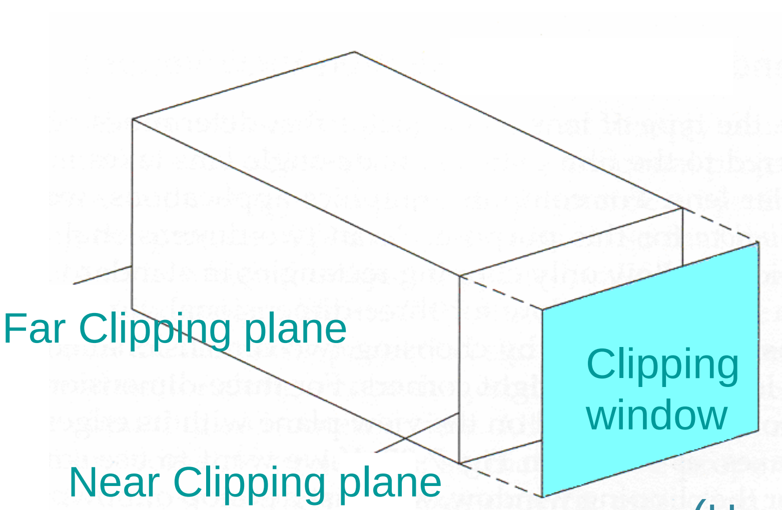
$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

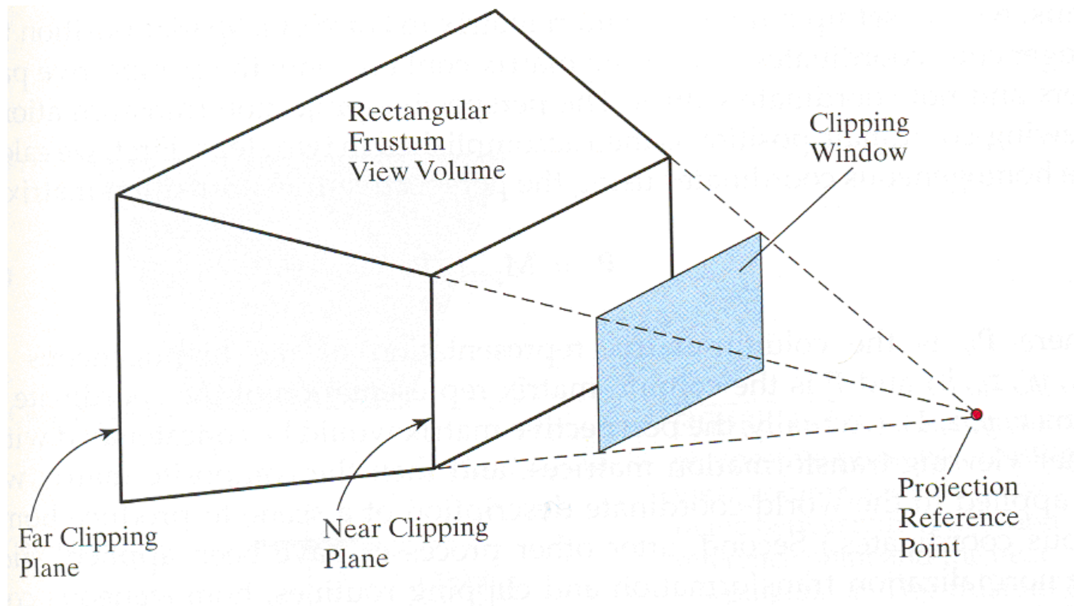
zz coordinates are discarded



How to limit what is observed and represented ?

- Clipping window on the projection plane
- View volume (frustum) in 3D



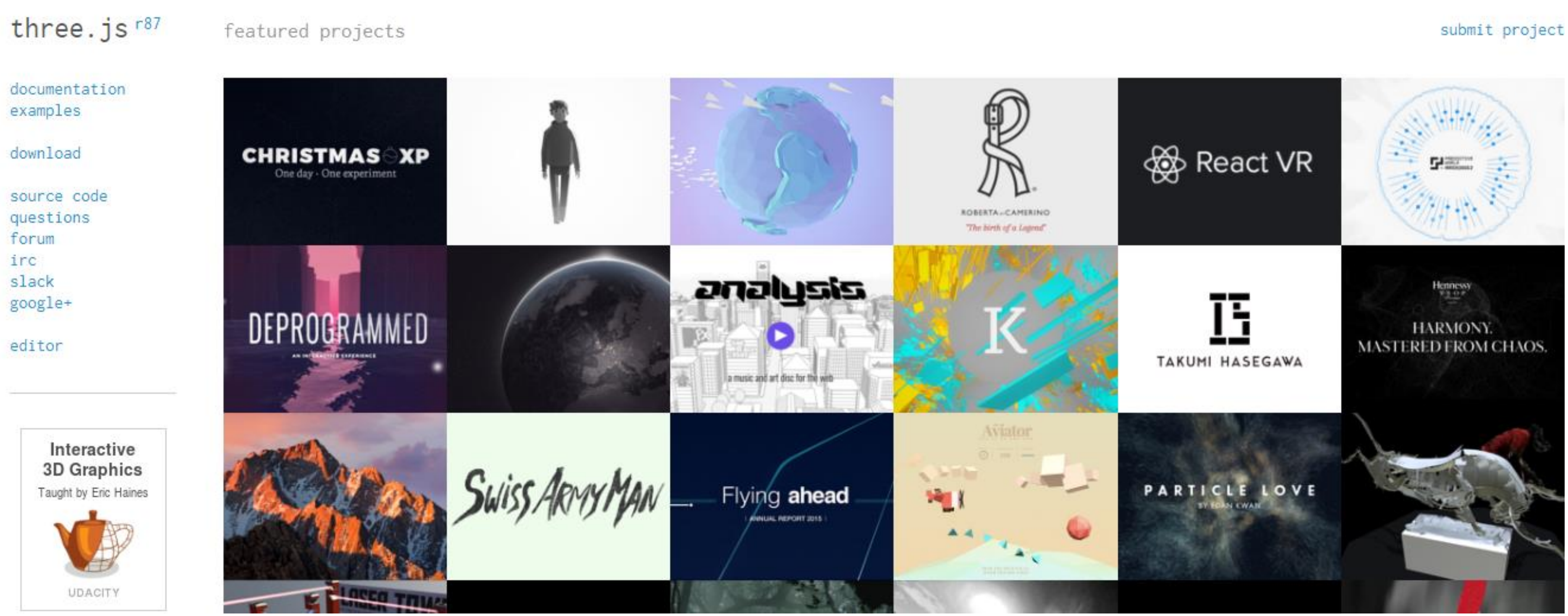


(Hearn & Baker, 2004)

Parallel projection

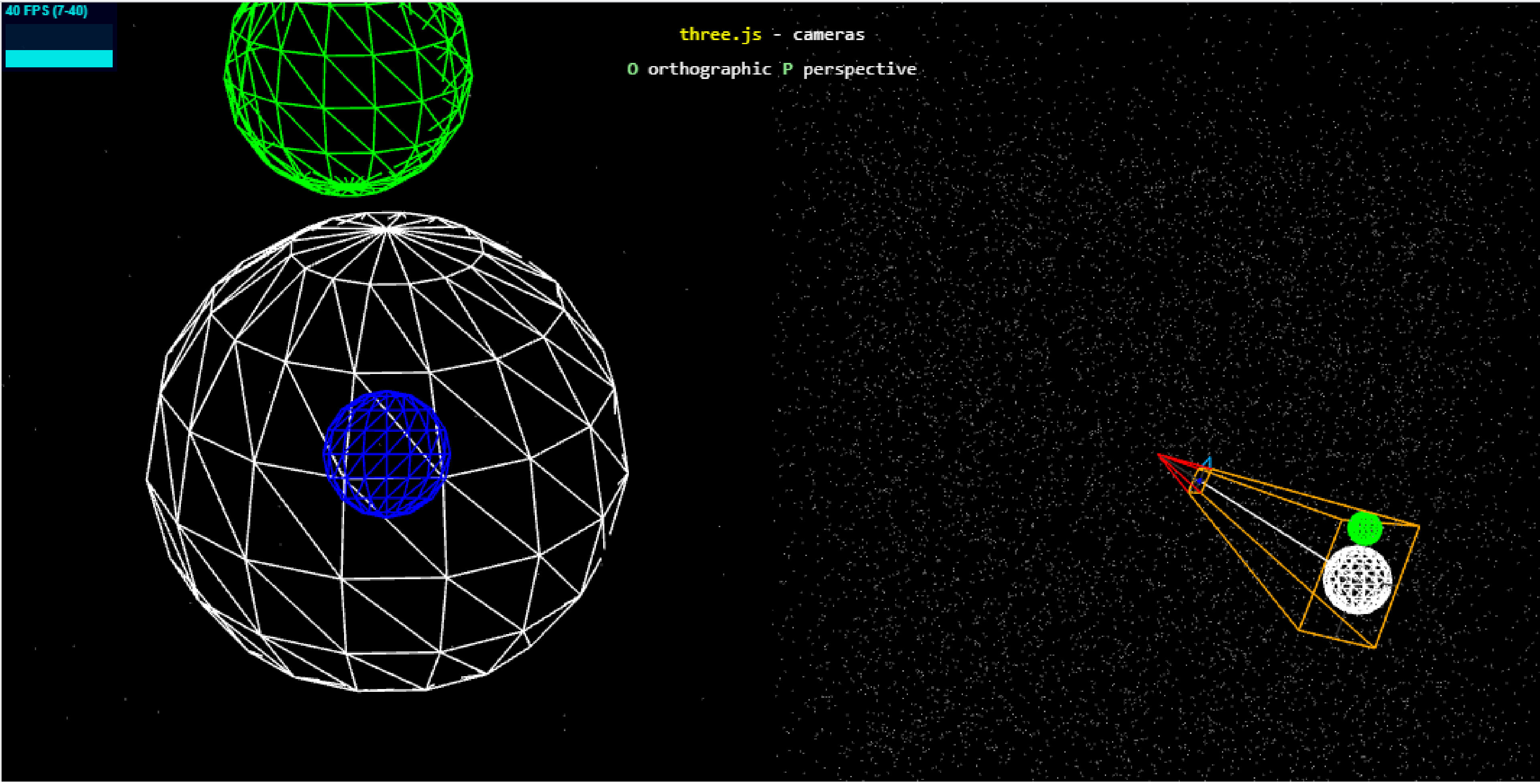
Perspective projection

Examples using Three.js



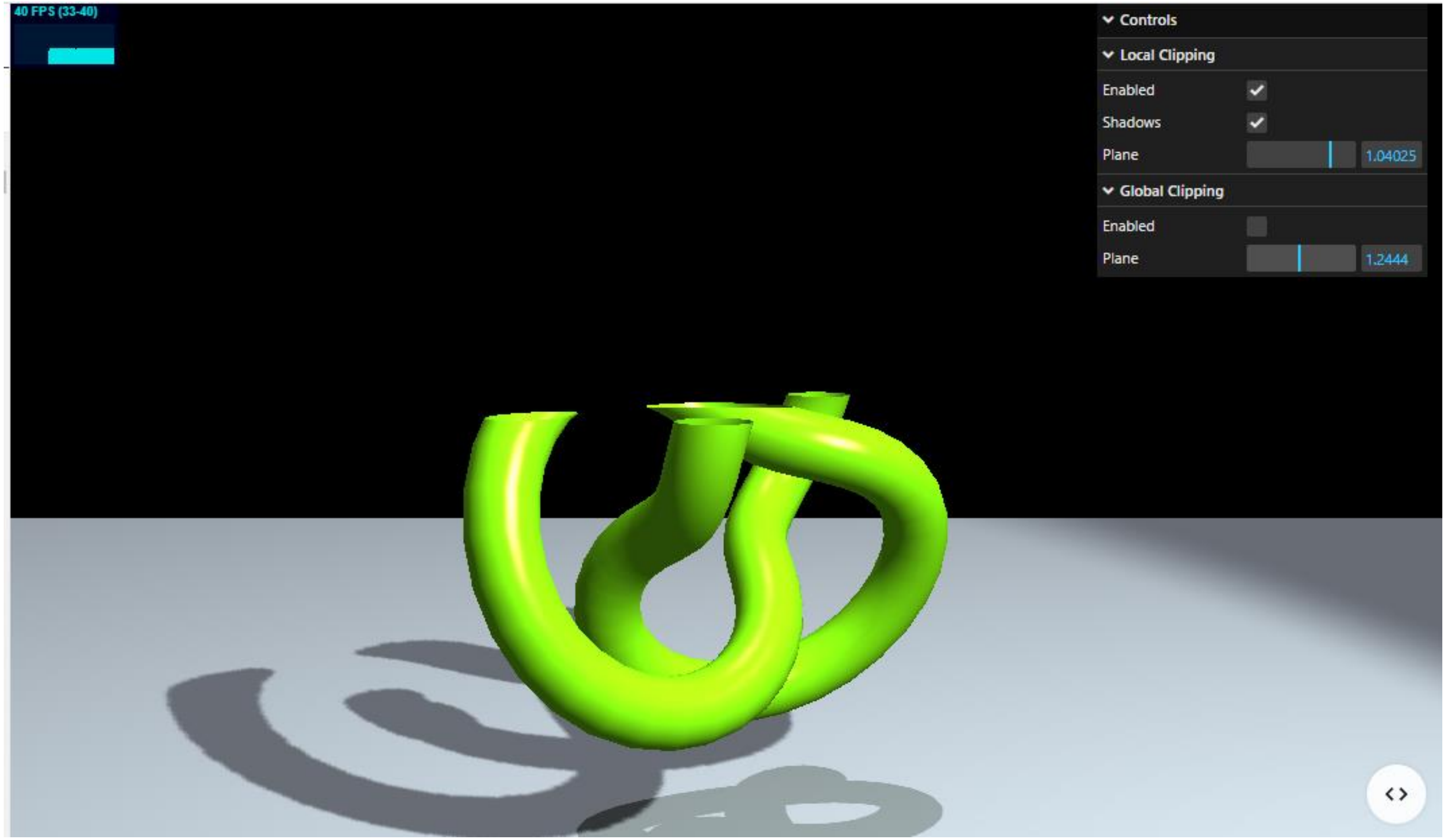
https://threejs.org/

Projections



https://threejs.org/examples/#webgl_camera

Clipping (and shadows)



https://threejs.org/examples/#webgl_clipping

Thee.js first example

1. Defining the scene, the camera and where the scene is rendered

```
var scene = new THREE.Scene();
var camera = new THREE.PerspectiveCamera( 75,
window.innerWidth / window.innerHeight, 0.1, 1000 );
var renderer = new THREE.WebGLRenderer();
renderer.setSize( window.innerWidth, window.innerHeight );
document.body.appendChild( renderer.domElement );
```

2.Creating an object and camera position

```
var geometry = new THREE.BoxGeometry(1,1,1);
var material = new THREE.MeshBasicMaterial( {
color: 0x00ff00 } );
var cube = new THREE.Mesh( geometry, material );
scene.add( cube );
camera.position.z = 5;
```

3. Scene rendering

```
function render() {
requestAnimationFrame(render);
renderer.render(scene, camera);
}
```

4. Scene animation

```
render();
cube.rotation.x += 0.1;
cube.rotation.y += 0.1;
```

Adding lights and shading

```
var material = new THREE.MeshPhongMaterial({
ambient: '#006063',
color: '#00abb1',
specular: '#a9fcff',
shininess: 100
});
```

Some reference books

- D. Hearn and M. P. Baker, *Computer Graphics with OpenGL*, 3rd Ed., Addison-Wesley, 2004
- E. Angel and D. Shreiner, *Introduction to Computer Graphics,* 6th Ed., Pearson Education, 2012
- J. Foley et al., *Introduction to Computer Graphics*, Addison-Wesley, 1993
- Hughes, J., A. Van Dam, et al., *Computer Graphics, Principles and Practice*, 3rd Ed., Addison Wesley, 2013