In this tests we will consider this two attempts:

Figure 1: Old attempt.

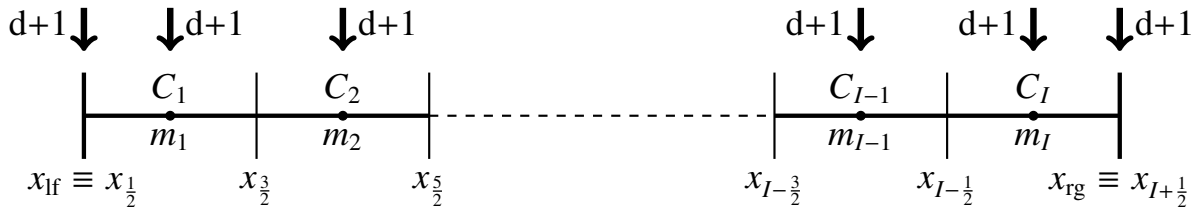

Figure 2: New attempt.

In this tests we are also using a pre-conditioner.
In this test we will consider:

- $\psi(x) = \exp(x)$
- $\psi_{\mathrm{l}} = 1$;
- $\psi_{\mathrm{ll}} = 1$;
- $\psi_{\mathrm{r}} = 1$;
- $\psi_{\mathrm{rr}} = 1$;
- $g(x) = -\exp(x)$.

Table 1: Test of 01_01 with d and d+1 ($\exp(x)$ — constant mesh).

		$\omega = 1\|1$				$\omega = 1\|3$			
	I	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$
$\mathbb{P}_3$	50	1.09E−05	—	1.72E−05	—	8.41E−06	—	1.36E−05	—
	100	1.33E−06	3.03	2.08E−06	3.04	1.05E−06	3.00	1.69E−06	3.01
	150	3.88E−07	3.04	6.05E−07	3.05	3.28E−07	2.87	5.27E−07	2.87
	200	1.90E−07	2.49	2.95E−07	2.49	1.66E−07	2.36	2.67E−07	2.37
$\mathbb{P}_3/\mathbb{P}_4$ (old)	50	2.46E−06	—	3.97E−06	—	1.55E−06	—	2.39E−06	—
	100	2.74E−07	3.16	4.25E−07	3.22	1.87E−07	3.05	2.69E−07	3.15
	150	1.02E−07	2.45	1.58E−07	2.45	7.43E−08	2.28	1.13E−07	2.14
	200	5.40E−08	2.19	8.39E−08	2.20	4.69E−08	1.60	7.50E−08	1.43
$\mathbb{P}_3/\mathbb{P}_4$ (new)	50	3.38E−06	—	6.75E−06	—	1.31E−06	—	2.11E−06	—
	100	1.71E−06	0.98	2.87E−06	1.23	1.43E−07	3.19	2.23E−07	3.24
	150	2.63E−06	↑	4.91E−06	↑	3.60E−07	↑	7.58E−07	↑
	200	7.86E−06	↑	1.34E−05	↑	4.71E−07	↑	1.02E−06	↑
$\mathbb{P}_4$	50	3.85E−07	—	7.68E−07	—	3.08E−07	—	6.06E−07	—
	100	1.03E−07	1.90	1.98E−07	1.96	8.86E−08	1.80	1.70E−07	1.84
	150	4.74E−08	1.92	9.04E−08	1.93	3.77E−08	2.11	7.29E−08	2.08
	200	2.38E−08	2.39	4.65E−08	2.31	1.51E−08	3.19	2.84E−08	3.27
$\mathbb{P}_4/\mathbb{P}_5$ (old)	50	4.20E−07	—	8.21E−07	—	3.67E−07	—	6.96E−07	—
	100	1.08E−07	1.96	2.06E−07	2.00	9.34E−08	1.97	1.77E−07	1.98
	150	4.77E−08	2.02	9.08E−08	2.02	3.65E−08	2.32	7.10E−08	2.25
	200	2.33E−08	2.49	4.56E−08	2.39	2.22E−08	1.74	4.25E−08	1.79
$\mathbb{P}_4/\mathbb{P}_5$ (new)	50	4.32E−07	—	8.31E−07	—	3.75E−07	—	7.09E−07	—
	100	1.13E−07	1.94	2.13E−07	1.96	9.76E−08	1.94	1.84E−07	1.95
	150	5.37E−08	1.83	1.06E−07	1.73	4.64E−08	1.83	8.63E−08	1.86
	200	1.02E−06	↑	2.29E−06	↑	2.48E−08	2.18	4.72E−08	2.10
$\mathbb{P}_5$	50	9.64E−10	—	1.45E−09	—	7.96E−10	—	1.23E−09	—
	100	3.33E−09	↑	5.14E−09	↑	1.28E−09	↑	1.98E−09	↑
	150	3.01E−10	5.93	4.50E−10	6.01	5.82E−11	7.62	9.38E−11	7.53
	200	1.61E−10	2.17	3.17E−10	1.22	9.78E−11	↑	1.82E−10	↑
$\mathbb{P}_5/\mathbb{P}_6$ (old)	50	6.95E−10	—	1.04E−09	—	5.88E−10	—	8.74E−10	—
	100	1.77E−09	↑	2.72E−09	↑	8.18E−10	↑	1.25E−09	↑
	150	1.40E−10	6.26	2.09E−10	6.34	2.92E−11	8.22	5.80E−11	7.58
	200	9.70E−11	1.27	1.84E−10	0.43	7.09E−11	↑	1.34E−10	↑
$\mathbb{P}_5/\mathbb{P}_6$ (new)	50	7.87E−08	—	1.16E−07	—	1.06E−09	—	1.85E−09	—
	100	1.27E−08	2.63	2.56E−08	2.18	5.19E−09	↑	8.64E−09	↑
	150	7.86E−07	↑	1.23E−06	↑	5.41E−09	↑	8.38E−09	0.07
	200	3.21E−06	↑	5.09E−06	↑	8.81E−09	↑	1.32E−08	↑

In this test we will consider:

- $\psi(x) = -\exp(x) + \mathbb{P}_3$
- $\psi_{\mathrm{l}} = 0$;
- $\psi_{\mathrm{ll}} = 0$;
- $\psi_{\mathrm{r}} = 0$;
- $\psi_{\mathrm{rr}} = 0$;
- $g(x) = \exp(x)$.

Table 2: Test of 01_01 with d and d+1 ($\exp(x) + \mathbb{P}_3$ — constant mesh).

		$\omega = 1\|1$				$\omega = 1\|3$			
	I	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$
$\mathbb{P}_3$	50	1.09E−05	—	1.72E−05	—	8.41E−06	—	1.36E−05	—
	100	1.34E−06	3.02	2.10E−06	3.03	1.02E−06	3.04	1.65E−06	3.04
	150	3.84E−07	3.08	5.99E−07	3.09	2.92E−07	3.10	4.68E−07	3.11
	200	1.56E−07	3.12	2.43E−07	3.14	1.17E−07	3.16	1.88E−07	3.16
$\mathbb{P}_3/\mathbb{P}_4$ (old)	50	2.46E−06	—	3.97E−06	—	1.55E−06	—	2.39E−06	—
	100	2.55E−07	3.27	4.01E−07	3.31	1.57E−07	3.31	2.20E−07	3.44
	150	6.01E−08	3.56	9.68E−08	3.51	3.33E−08	3.82	4.71E−08	3.80
	200	1.91E−08	3.99	3.38E−08	3.65	8.32E−09	4.82	1.54E−08	3.88
$\mathbb{P}_3/\mathbb{P}_4$ (new)	50	3.38E−06	—	6.75E−06	—	1.31E−06	—	2.11E−06	—
	100	6.77E−07	2.32	1.22E−06	2.47	1.43E−07	3.19	2.23E−07	3.24
	150	2.40E−07	2.56	4.14E−07	2.67	3.09E−08	3.77	5.31E−08	3.54
	200	1.10E−07	2.71	1.85E−07	2.79	7.65E−09	4.86	1.84E−08	3.69
$\mathbb{P}_4$	50	3.85E−07	—	7.68E−07	—	3.08E−07	—	6.06E−07	—
	100	1.11E−07	1.79	2.10E−07	1.87	9.29E−08	1.73	1.76E−07	1.78
	150	5.07E−08	1.93	9.53E−08	1.95	4.28E−08	1.91	8.06E−08	1.93
	200	2.88E−08	1.97	5.41E−08	1.97	2.44E−08	1.96	4.58E−08	1.96
$\mathbb{P}_4/\mathbb{P}_5$ (old)	50	4.20E−07	—	8.21E−07	—	3.66E−07	—	6.95E−07	—
	100	1.14E−07	1.88	2.14E−07	1.94	9.70E−08	1.92	1.82E−07	1.93
	150	5.10E−08	1.98	9.59E−08	1.99	4.36E−08	1.97	8.19E−08	1.98
	200	2.89E−08	1.97	5.43E−08	1.98	2.46E−08	1.99	4.62E−08	1.99
$\mathbb{P}_4/\mathbb{P}_5$ (new)	50	4.32E−07	—	8.31E−07	—	3.75E−07	—	7.09E−07	—
	100	1.14E−07	1.92	2.15E−07	1.95	9.76E−08	1.94	1.83E−07	1.95
	150	5.14E−08	1.97	9.65E−08	1.98	4.37E−08	1.98	8.21E−08	1.98
	200	2.90E−08	1.99	5.44E−08	1.99	2.47E−08	1.99	4.63E−08	1.99
$\mathbb{P}_5$	50	1.51E−09	—	2.16E−09	—	9.32E−10	—	1.46E−09	—
	100	3.45E−11	5.45	5.37E−11	5.33	2.99E−11	4.96	4.79E−11	4.93
	150	4.20E−12	5.19	6.71E−12	5.13	3.72E−12	5.14	6.09E−12	5.09
	200	1.82E−12	2.92	2.88E−12	2.95	1.48E−12	3.21	2.38E−12	3.27
$\mathbb{P}_5/\mathbb{P}_6$ (old)	50	7.22E−10	—	9.93E−10	—	6.77E−10	—	1.02E−09	—
	100	2.40E−11	4.91	3.75E−11	4.73	2.18E−11	4.96	3.40E−11	4.90
	150	3.00E−12	5.13	4.82E−12	5.06	2.78E−12	5.09	4.44E−12	5.02
	200	1.23E−12	3.11	1.96E−12	3.13	1.09E−12	3.24	1.74E−12	3.25
$\mathbb{P}_5/\mathbb{P}_6$ (new)	50	1.59E−09	—	2.43E−09	—	5.20E−10	—	8.44E−10	—
	100	2.99E−11	5.74	4.61E−11	5.72	1.65E−11	4.97	2.60E−11	5.02
	150	1.92E−11	1.09	3.69E−11	0.55	2.03E−12	5.17	3.39E−12	5.03
	200	4.02E−11	↑	6.17E−11	↑	5.61E−13	4.47	9.74E−13	4.34

In this test we will consider:

- $\psi(x) = \sin(x)$
- $\psi_{\mathrm{l}} = 0$;
- $\psi_{\mathrm{ll}} = 1$;
- $\psi_{\mathrm{r}} = \sin(1)$;
- $\psi_{\mathrm{rr}} = \cos(1)$;
- $g(x) = -\sin(x)$.

Table 3: Test of 01_01 with d and d+1 ($\sin(x)$ — constant mesh).

		$\omega = 1\|1$				$\omega = 1\|3$			
	I	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$
$\mathbb{P}_3$	50	2.73E−06	—	4.84E−06	—	2.30E−06	—	4.05E−06	—
	100	3.56E−07	2.94	6.25E−07	2.95	3.08E−07	2.90	5.42E−07	2.90
	150	1.11E−07	2.88	1.94E−07	2.88	1.02E−07	2.73	1.79E−07	2.74
	200	5.55E−08	2.39	9.75E−08	2.39	5.37E−08	2.22	9.43E−08	2.22
$\mathbb{P}_3/\mathbb{P}_4$ (old)	50	7.68E−07	—	1.39E−06	—	4.76E−07	—	8.59E−07	—
	100	1.11E−07	2.79	1.95E−07	2.84	8.19E−08	2.54	1.45E−07	2.57
	150	4.38E−08	2.29	7.68E−08	2.29	3.57E−08	2.05	6.29E−08	2.05
	200	2.47E−08	1.99	4.32E−08	2.00	2.36E−08	1.45	4.14E−08	1.45
$\mathbb{P}_3/\mathbb{P}_4$ (new)	50	1.05E−06	—	2.41E−06	—	4.78E−07	—	8.80E−07	—
	100	2.34E−07	2.16	4.73E−07	2.35	7.92E−08	2.59	1.42E−07	2.63
	150	3.26E−07	↑	6.63E−07	↑	1.34E−07	↑	2.70E−07	↑
	200	3.11E−06	↑	5.30E−06	↑	1.09E−07	0.73	2.28E−07	0.59
$\mathbb{P}_4$	50	1.11E−07	—	2.30E−07	—	1.03E−07	—	2.05E−07	—
	100	2.99E−08	1.89	5.84E−08	1.98	2.69E−08	1.94	5.16E−08	1.99
	150	1.34E−08	1.98	2.61E−08	1.99	1.17E−08	2.05	2.28E−08	2.02
	200	6.99E−09	2.26	1.40E−08	2.16	6.86E−09	1.86	1.36E−08	1.80
$\mathbb{P}_4/\mathbb{P}_5$ (old)	50	1.19E−07	—	2.44E−07	—	9.83E−08	—	1.94E−07	—
	100	3.14E−08	1.93	6.00E−08	2.02	2.66E−08	1.89	5.10E−08	1.93
	150	1.35E−08	2.08	2.62E−08	2.05	1.20E−08	1.96	2.29E−08	1.98
	200	6.92E−09	2.32	1.39E−08	2.20	6.53E−09	2.11	1.27E−08	2.06
$\mathbb{P}_4/\mathbb{P}_5$ (new)	50	1.22E−07	—	2.40E−07	—	1.03E−07	—	2.00E−07	—
	100	3.20E−08	1.93	6.07E−08	1.98	2.71E−08	1.93	5.15E−08	1.96
	150	1.45E−08	1.94	2.83E−08	1.88	1.20E−08	2.01	2.28E−08	2.01
	200	1.10E−07	↑	2.58E−07	↑	5.65E−09	2.63	1.10E−08	2.52
$\mathbb{P}_5$	50	2.10E−10	—	3.39E−10	—	1.86E−10	—	3.08E−10	—
	100	7.85E−10	↑	1.34E−09	↑	3.16E−10	↑	5.30E−10	↑
	150	1.19E−10	4.66	2.21E−10	4.44	5.44E−11	4.34	1.02E−10	4.06
	200	4.25E−11	3.57	8.65E−11	3.26	2.73E−11	2.40	5.00E−11	2.47
$\mathbb{P}_5/\mathbb{P}_6$ (old)	50	1.44E−10	—	2.32E−10	—	1.21E−10	—	1.94E−10	—
	100	4.21E−10	↑	7.10E−10	↑	1.99E−10	↑	3.28E−10	↑
	150	6.43E−11	4.63	1.20E−10	4.39	3.29E−11	4.44	6.36E−11	4.04
	200	2.60E−11	3.15	5.09E−11	2.97	2.33E−11	1.20	4.28E−11	1.37
$\mathbb{P}_5/\mathbb{P}_6$ (new)	50	1.85E−08	—	2.73E−08	—	6.78E−11	—	1.33E−10	—
	100	1.30E−09	3.84	3.41E−09	3.00	4.79E−10	↑	9.65E−10	↑
	150	8.62E−08	↑	1.35E−07	↑	5.00E−09	↑	7.61E−09	↑
	200	1.53E−06	↑	2.42E−06	↑	1.41E−08	↑	2.16E−08	↑

In this test we will consider:

- $\psi(x) = x^4$
- $\psi_{\mathrm{l}} = 0$;
- $\psi_{\mathrm{ll}} = 0$;
- $\psi_{\mathrm{r}} = 1$;
- $\psi_{\mathrm{rr}} = 4$;
- $g(x) = -24$.

Table 4: Test of 01_01 with d and d+1 (x^4 — constant mesh).

		$\omega = 1\|1$				$\omega = 1\|3$			
	I	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$	E_1	$\mathcal{O}_1$	E_∞	$\mathcal{O}_\infty$
$\mathbb{P}_3$	50	1.47E−04	—	2.22E−04	—	1.08E−04	—	1.65E−04	—
	100	1.86E−05	2.98	2.80E−05	2.99	1.36E−05	2.98	2.07E−05	2.99
	150	5.54E−06	2.99	8.33E−06	2.99	4.08E−06	2.98	6.18E−06	2.98
	200	2.34E−06	3.00	3.51E−06	3.00	1.76E−06	2.92	2.67E−06	2.92
$\mathbb{P}_3/\mathbb{P}_4$ (old)	50	3.71E−05	—	5.71E−05	—	2.92E−05	—	4.67E−05	—
	100	4.56E−06	3.02	6.92E−06	3.04	3.81E−06	2.94	5.97E−06	2.97
	150	1.34E−06	3.02	2.03E−06	3.02	1.17E−06	2.92	1.81E−06	2.94
	200	6.18E−07	2.70	9.33E−07	2.70	5.31E−07	2.74	8.15E−07	2.78
$\mathbb{P}_3/\mathbb{P}_4$ (new)	50	4.90E−05	—	8.07E−05	—	2.34E−05	—	3.71E−05	—
	100	1.01E−05	2.28	1.59E−05	2.34	3.28E−06	2.84	5.05E−06	2.87
	150	3.68E−06	2.49	5.72E−06	2.53	9.59E−07	3.03	1.65E−06	2.76
	200	4.15E−06	↑	8.40E−06	↑	3.98E−07	3.05	7.35E−07	2.81
$\mathbb{P}_4$	50	3.39E−12	—	6.47E−12	—	2.38E−12	—	4.74E−12	—
	100	7.61E−11	↑	1.36E−10	↑	2.51E−11	↑	4.14E−11	↑
	150	8.79E−11	↑	1.71E−10	↑	2.17E−11	0.36	4.09E−11	0.03
	200	2.94E−10	↑	5.78E−10	↑	7.77E−11	↑	1.15E−10	↑
$\mathbb{P}_4/\mathbb{P}_5$ (old)	50	4.97E−11	—	9.61E−11	—	2.76E−11	—	5.33E−11	—
	100	7.28E−12	2.77	1.20E−11	3.00	4.54E−12	2.60	7.62E−12	2.80
	150	1.95E−11	↑	2.99E−11	↑	3.68E−12	0.52	5.93E−12	0.62
	200	1.00E−11	2.31	2.02E−11	1.36	1.14E−11	↑	2.31E−11	↑
$\mathbb{P}_4/\mathbb{P}_5$ (new)	50	1.83E−14	—	4.88E−14	—	6.14E−11	—	1.06E−10	—
	100	8.31E−09	↑	1.72E−08	↑	1.66E−10	↑	2.51E−10	↑
	150	2.48E−08	↑	4.61E−08	↑	1.28E−09	↑	2.28E−09	↑
	200	1.18E−07	↑	3.16E−07	↑	7.23E−10	1.98	1.52E−09	1.40
$\mathbb{P}_5$	50	1.82E−12	—	3.63E−12	—	1.78E−12	—	3.20E−12	—
	100	4.34E−11	↑	8.40E−11	↑	3.49E−11	↑	7.02E−11	↑
	150	1.51E−09	↑	2.28E−09	↑	9.52E−10	↑	1.46E−09	↑
	200	1.56E−09	↑	2.52E−09	↑	1.13E−09	↑	1.82E−09	↑
$\mathbb{P}_5/\mathbb{P}_6$ (old)	50	1.56E−12	—	2.46E−12	—	1.41E−12	—	2.20E−12	—
	100	2.72E−11	↑	5.42E−11	↑	1.96E−11	↑	3.92E−11	↑
	150	9.62E−10	↑	1.44E−09	↑	7.51E−10	↑	1.14E−09	↑
	200	1.02E−09	↑	1.64E−09	↑	8.48E−10	↑	1.40E−09	↑
$\mathbb{P}_5/\mathbb{P}_6$	50	1.96E−09	—	2.89E−09	—	3.52E−11	—	6.05E−11	—
	100	1.41E−07	↑	2.10E−07	↑	1.47E−09	↑	2.92E−09	↑
	150	1.43E−07	↑	2.30E−07	↑	3.23E−09	↑	5.71E−09	↑
	200	1.80E−06	↑	2.86E−06	↑	3.21E−09	0.02	5.71E−09	↑