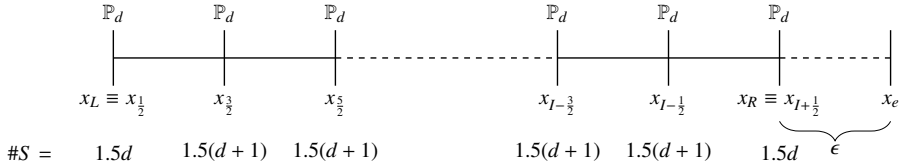
In this tests are considered 4 types of combinations:

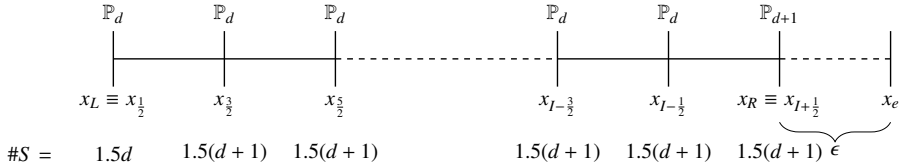
- $c1$

- $c2$



- $c3$

- $c4$



$\epsilon = 0$ – uniform mesh

Table 1: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0+$ (c1; uniform mesh; $\epsilon = 0$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	6.48E+02	✓	8.76E−04	—	5.66E−04	—	7.61E−04	—
$\mathbb{P}_1$	40	2.59E+03	✓	2.14E−04	2.03	1.42E−04	2.00	1.92E−04	1.99
	80	1.04E+04	✓	5.28E−05	2.02	3.54E−05	2.00	4.82E−05	1.99
	20	8.25E+02	✓	7.53E−05	—	1.40E−04	—	3.33E−04	—
$\mathbb{P}_2$	40	3.30E+03	✓	9.55E−06	2.98	3.61E−05	1.95	8.64E−05	1.95
	80	1.32E+04	✓	1.20E−06	2.99	9.19E−06	1.97	2.20E−05	1.97
	20	1.02E+03	✓	4.41E−06	—	6.53E−07	—	7.67E−07	—
$\mathbb{P}_3$	40	4.08E+03	✓	2.78E−07	3.99	4.07E−08	4.01	5.30E−08	3.86
	80	1.63E+04	✓	1.75E−08	3.99	2.53E−09	4.01	3.47E−09	3.93
	20	1.18E+03	✓	2.00E−07	—	2.39E−08	—	5.60E−08	—
$\mathbb{P}_4$	40	4.74E+03	✓	6.36E−09	4.98	2.07E−09	3.53	5.53E−09	3.34
	80	1.89E+04	✓	2.01E−10	4.99	1.59E−10	3.70	4.07E−10	3.77
	20	1.35E+03	✓	8.78E−09	—	1.80E−09	—	2.09E−09	—
$\mathbb{P}_5$	40	5.42E+03	✓	1.40E−10	5.97	2.70E−11	6.06	3.07E−11	6.09
	80	2.17E+04	✓	2.24E−12	5.96	1.53E−13	7.47	8.86E−13	5.11

Table 2: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (c2; uniform mesh; $\epsilon = 0$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	5.52E+03	×	9.62E−03	—	8.66E−01	—	5.81E+00	—
$\mathbb{P}_1$	40	1.84E+07	×	2.45E−03	1.97	3.45E+02	↑	4.60E+03	↑
	80	2.03E+14	×	6.20E−04	1.98	4.68E+08	↑	1.25E+10	↑
	20	6.10E+02	✓	1.89E−04	—	3.70E−04	—	8.59E−04	—
$\mathbb{P}_2$	40	2.44E+03	✓	2.48E−05	2.93	1.00E−04	1.89	2.36E−04	1.87
	80	9.75E+03	✓	3.18E−06	2.96	2.59E−05	1.95	6.16E−05	1.94
	20	6.74E+02	✓	1.08E−05	—	1.75E−06	—	2.67E−06	—
$\mathbb{P}_3$	40	2.70E+03	✓	6.80E−07	3.99	1.25E−07	3.81	1.88E−07	3.83
	80	1.08E+04	✓	4.27E−08	3.99	1.00E−08	3.64	1.70E−08	3.46
	20	7.97E+02	✓	9.17E−07	—	4.81E−07	—	1.14E−06	—
$\mathbb{P}_4$	40	3.19E+03	✓	2.98E−08	4.95	4.13E−08	3.54	9.76E−08	3.55
	80	1.28E+04	✓	9.51E−10	4.97	2.91E−09	3.82	6.92E−09	3.82
	20	8.36E+02	✓	5.19E−08	—	1.13E−08	—	2.40E−08	—
$\mathbb{P}_5$	40	3.34E+03	✓	8.21E−10	5.98	3.77E−10	4.91	8.49E−10	4.82
	80	1.34E+04	✓	1.30E−11	5.99	1.22E−11	4.95	2.82E−11	4.91

$\epsilon = \frac{h^2}{2}$ – uniform mesh

Table 3: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0+$ (c1; uniform mesh; $\epsilon = \frac{h^2}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	6.48E+02	✓	9.92E−04	—	1.19E−03	—	2.55E−03	—
$\mathbb{P}_1$	40	2.59E+03	✓	2.28E−04	2.12	2.97E−04	2.00	6.47E−04	1.98
	80	1.04E+04	✓	5.46E−05	2.06	7.42E−05	2.00	1.63E−04	1.99
	20	8.27E+02	✓	7.05E−05	—	9.70E−05	—	2.50E−04	—
$\mathbb{P}_2$	40	3.30E+03	✓	9.24E−06	2.93	3.08E−05	1.65	7.59E−05	1.72
	80	1.32E+04	✓	1.18E−06	2.97	8.53E−06	1.85	2.07E−05	1.88
	20	1.02E+03	✓	4.25E−06	—	1.41E−06	—	2.19E−06	—
$\mathbb{P}_3$	40	4.08E+03	✓	2.73E−07	3.96	8.88E−08	3.99	1.40E−07	3.97
	80	1.63E+04	✓	1.73E−08	3.98	5.58E−09	3.99	8.85E−09	3.99
	20	1.19E+03	✓	1.95E−07	—	2.28E−08	—	4.92E−08	—
$\mathbb{P}_4$	40	4.74E+03	✓	6.27E−09	4.96	1.92E−09	3.57	5.18E−09	3.25
	80	1.90E+04	✓	1.99E−10	4.98	1.53E−10	3.65	3.94E−10	3.72
	20	1.36E+03	✓	8.57E−09	—	1.07E−09	—	4.89E−09	—
$\mathbb{P}_5$	40	5.42E+03	✓	1.38E−10	5.96	1.84E−11	5.86	8.81E−11	5.79
	80	2.17E+04	✓	2.22E−12	5.95	5.11E−13	5.17	1.87E−12	5.56

Table 4: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (c2; uniform mesh; $\epsilon = \frac{h^2}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	4.68E+03	×	9.52E−03	—	8.66E−01	—	5.82E+00	—
$\mathbb{P}_1$	40	1.56E+07	×	2.41E−03	1.98	3.45E+02	↑	4.60E+03	↑
	80	1.71E+14	×	6.06E−04	1.99	4.68E+08	↑	1.25E+10	↑
	20	5.25E+02	✓	3.51E−04	—	4.89E−04	—	1.20E−03	—
$\mathbb{P}_2$	40	2.09E+03	✓	4.56E−05	2.94	1.41E−04	1.79	3.41E−04	1.81
	80	8.36E+03	✓	5.81E−06	2.97	3.77E−05	1.91	9.04E−05	1.91
	20	5.99E+02	✓	1.52E−05	—	6.12E−06	—	1.12E−05	—
$\mathbb{P}_3$	40	2.39E+03	✓	9.77E−07	3.96	4.35E−07	3.82	8.10E−07	3.79
	80	9.56E+03	✓	6.20E−08	3.98	3.14E−08	3.79	6.03E−08	3.75
	20	7.52E+02	✓	1.16E−06	—	8.60E−07	—	2.02E−06	—
$\mathbb{P}_4$	40	3.00E+03	✓	3.84E−08	4.92	6.80E−08	3.66	1.59E−07	3.66
	80	1.20E+04	✓	1.24E−09	4.96	4.65E−09	3.87	1.10E−08	3.86
	20	7.99E+02	✓	5.47E−08	—	2.72E−08	—	5.72E−08	—
$\mathbb{P}_5$	40	3.19E+03	✓	8.76E−10	5.96	7.88E−10	5.11	1.77E−09	5.01
	80	1.27E+04	✓	1.39E−11	5.98	2.35E−11	5.07	5.45E−11	5.03

Table 5: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (c3; uniform mesh; $\epsilon = \frac{h^2}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	6.49E+02	✓	8.71E−04	—	5.23E−04	—	6.77E−04	—
$\mathbb{P}_1$	40	2.59E+03	✓	2.14E−04	2.03	1.36E−04	1.94	1.81E−04	1.90
	80	1.04E+04	✓	5.28E−05	2.02	3.47E−05	1.97	4.69E−05	1.95
	20	8.27E+02	✓	7.52E−05	—	1.49E−04	—	3.52E−04	—
$\mathbb{P}_2$	40	3.30E+03	✓	9.54E−06	2.98	3.75E−05	1.99	8.90E−05	1.98
	80	1.32E+04	✓	1.20E−06	2.99	9.36E−06	2.00	2.23E−05	2.00
	20	1.02E+03	✓	4.40E−06	—	7.35E−07	—	3.10E−06	—
$\mathbb{P}_3$	40	4.08E+03	✓	2.78E−07	3.98	4.87E−08	3.92	2.09E−07	3.89
	80	1.63E+04	✓	1.75E−08	3.99	3.13E−09	3.96	1.35E−08	3.95
	20	1.19E+03	✓	2.00E−07	—	7.86E−08	—	1.70E−07	—
$\mathbb{P}_4$	40	4.74E+03	✓	6.36E−09	4.97	9.83E−10	6.32	1.96E−09	6.43
	80	1.90E+04	✓	2.01E−10	4.99	7.92E−11	3.63	2.36E−10	3.05
	20	1.36E+03	✓	8.77E−09	—	3.75E−09	—	1.12E−08	—
$\mathbb{P}_5$	40	5.42E+03	✓	1.40E−10	5.97	6.42E−11	5.87	1.92E−10	5.87
	80	2.17E+04	✓	2.24E−12	5.96	1.32E−12	5.61	3.56E−12	5.75

Table 6: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (c4; uniform mesh; $\epsilon = \frac{h^2}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	4.66E+03	×	9.36E−03	—	8.64E−01	—	5.74E+00	—
$\mathbb{P}_1$	40	1.55E+07	×	2.39E−03	1.97	3.58E+02	↑	4.54E+03	↑
	80	1.72E+14	×	6.03E−04	1.98	5.26E+08	↑	1.23E+10	↑
	20	5.25E+02	✓	3.44E−04	—	5.59E−04	—	1.33E−03	—
$\mathbb{P}_2$	40	2.09E+03	✓	4.51E−05	2.93	1.50E−04	1.89	3.58E−04	1.89
	80	8.36E+03	✓	5.78E−06	2.96	3.89E−05	1.95	9.28E−05	1.95
	20	5.99E+02	✓	1.57E−05	—	1.78E−06	—	5.84E−06	—
$\mathbb{P}_3$	40	2.39E+03	✓	9.93E−07	3.98	1.07E−07	4.06	3.58E−07	4.03
	80	9.56E+03	✓	6.25E−08	3.99	8.90E−09	3.58	1.60E−08	4.49
	20	7.52E+02	✓	1.18E−06	—	6.57E−07	—	1.61E−06	—
$\mathbb{P}_4$	40	3.00E+03	✓	3.88E−08	4.93	6.04E−08	3.44	1.45E−07	3.48
	80	1.20E+04	✓	1.24E−09	4.96	4.40E−09	3.78	1.05E−08	3.79
	20	7.99E+02	✓	5.62E−08	—	8.48E−09	—	2.40E−08	—
$\mathbb{P}_5$	40	3.19E+03	✓	8.90E−10	5.98	4.67E−10	4.18	1.17E−09	4.36
	80	1.27E+04	✓	1.40E−11	5.99	1.82E−11	4.68	4.43E−11	4.72

$\epsilon = \frac{h}{2}$ – uniform mesh

Table 7: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0+$ (c1; uniform mesh; $\epsilon = \frac{h}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	6.48E+02	✓	4.26E−03	—	3.38E−02	—	6.63E−02	—
$\mathbb{P}_1$	40	2.59E+03	✓	1.06E−03	2.01	1.70E−02	1.00	3.36E−02	0.98
	80	1.04E+04	✓	2.65E−04	2.00	8.49E−03	1.00	1.69E−02	0.99
	20	8.75E+02	✓	1.15E−04	—	1.12E−03	—	2.13E−03	—
$\mathbb{P}_2$	40	3.40E+03	✓	1.44E−05	3.00	2.81E−04	2.00	5.40E−04	1.98
	80	1.34E+04	✓	1.80E−06	3.00	7.03E−05	2.00	1.36E−04	1.99
	20	1.13E+03	✓	4.73E−06	—	3.86E−05	—	7.34E−05	—
$\mathbb{P}_3$	40	4.29E+03	✓	2.95E−07	4.00	4.82E−06	3.00	9.40E−06	2.96
	80	1.67E+04	✓	1.84E−08	4.00	6.02E−07	3.00	1.19E−06	2.98
	20	1.50E+03	×	1.91E−07	—	6.55E−07	—	1.26E−06	—
$\mathbb{P}_4$	40	5.40E+03	×	5.98E−09	5.00	4.12E−08	3.99	8.14E−08	3.95
	80	2.03E+04	×	1.87E−10	5.00	2.57E−09	4.00	5.15E−09	3.98
	20	2.47E+03	×	7.90E−09	—	2.97E−08	—	6.22E−08	—
$\mathbb{P}_5$	40	7.96E+03	×	1.24E−10	6.00	1.10E−09	4.75	2.25E−09	4.79
	80	2.72E+04	×	1.96E−12	5.98	3.61E−11	4.93	7.28E−11	4.95

Table 8: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (c2; uniform mesh; $\epsilon = \frac{h}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	4.68E+03	×	1.28E−02	—	8.69E−01	—	5.88E+00	—
$\mathbb{P}_1$	40	1.56E+07	×	3.24E−03	1.98	3.45E+02	↑	4.60E+03	↑
	80	1.73E+14	×	8.16E−04	1.99	4.68E+08	↑	1.25E+10	↑
	20	5.53E+02	✓	4.92E−04	—	1.16E−03	—	2.08E−03	—
$\mathbb{P}_2$	40	2.15E+03	✓	6.40E−05	2.94	2.80E−04	2.05	5.01E−04	2.05
	80	8.48E+03	✓	8.16E−06	2.97	6.89E−05	2.02	1.23E−04	2.03
	20	6.38E+02	✓	2.19E−05	—	1.23E−04	—	2.32E−04	—
$\mathbb{P}_3$	40	2.47E+03	✓	1.39E−06	3.98	1.57E−05	2.96	3.06E−05	2.93
	80	9.72E+03	✓	8.80E−08	3.99	1.99E−06	2.98	3.92E−06	2.96
	20	8.37E+02	✓	1.27E−06	—	4.79E−06	—	9.36E−06	—
$\mathbb{P}_4$	40	3.18E+03	✓	4.11E−08	4.94	3.25E−07	3.88	6.54E−07	3.84
	80	1.24E+04	✓	1.31E−09	4.97	2.10E−08	3.95	4.31E−08	3.92
	20	8.96E+02	✓	6.32E−08	—	4.13E−07	—	7.97E−07	—
$\mathbb{P}_5$	40	3.39E+03	✓	9.99E−10	5.98	1.38E−08	4.90	2.73E−08	4.87
	80	1.32E+04	✓	1.57E−11	5.99	4.48E−10	4.95	8.93E−10	4.93

Table 9: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (c3; uniform mesh; $\epsilon = \frac{h}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	E_1	O_1	E_∞	O_∞
	20	6.87E+02	✓	9.15E−04	—	7.72E−04	—	1.70E−03	—
$\mathbb{P}_1$	40	2.67E+03	✓	2.19E−04	2.07	1.94E−04	1.99	4.35E−04	1.97
	80	1.05E+04	✓	5.34E−05	2.03	4.87E−05	2.00	1.10E−04	1.99
	20	9.08E+02	✓	7.24E−05	—	3.45E−04	—	7.34E−04	—
$\mathbb{P}_2$	40	3.47E+03	✓	9.36E−06	2.95	9.55E−05	1.85	2.04E−04	1.85
	80	1.35E+04	✓	1.19E−06	2.98	2.50E−05	1.93	5.35E−05	1.93
	20	1.24E+03	×	4.31E−06	—	3.26E−05	—	6.54E−05	—
$\mathbb{P}_3$	40	4.55E+03	×	2.75E−07	3.97	4.43E−06	2.88	8.87E−06	2.88
	80	1.73E+04	×	1.74E−08	3.98	5.77E−07	2.94	1.15E−06	2.94
	20	1.89E+03	×	1.96E−07	—	2.60E−06	—	5.08E−06	—
$\mathbb{P}_4$	40	6.31E+03	×	6.30E−09	4.96	1.73E−07	3.91	3.41E−07	3.90
	80	2.23E+04	×	2.00E−10	4.98	1.11E−08	3.95	2.21E−08	3.95
	20	3.62E+03	×	8.64E−09	—	1.83E−07	—	3.62E−07	—
$\mathbb{P}_5$	40	1.10E+04	×	1.38E−10	5.96	6.22E−09	4.88	1.24E−08	4.87
	80	3.48E+04	×	2.23E−12	5.96	2.02E−10	4.94	4.03E−10	4.94

Table 10: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (c4; uniform mesh; $\epsilon = \frac{h}{2}$).

	I	cond(A)	$A^{-1} \geq 0$	$\mathrm{E}_{c,1}$	$\mathrm{O}_{c,1}$	E_1	O_1	E_∞	O_∞
	20	4.66E+03	×	9.50E−03	—	9.81E−01	—	4.93E+00	—
$\mathbb{P}_1$	40	1.95E+07	×	2.41E−03	1.98	8.65E+02	↑	4.50E+03	↑
	80	4.59E+14	×	6.06E−04	1.99	4.06E+09	↑	1.62E+10	↑
	20	5.60E+02	✓	3.52E−04	—	6.37E−04	—	1.48E−03	—
$\mathbb{P}_2$	40	2.16E+03	✓	4.57E−05	2.95	1.88E−04	1.76	4.31E−04	1.77
	80	8.51E+03	✓	5.82E−06	2.97	5.07E−05	1.89	1.16E−04	1.89
	20	6.56E+02	✓	1.53E−05	—	2.74E−05	—	6.02E−05	—
$\mathbb{P}_3$	40	2.51E+03	✓	9.80E−07	3.97	4.19E−06	2.71	8.77E−06	2.78
	80	9.80E+03	✓	6.21E−08	3.98	5.77E−07	2.86	1.18E−06	2.90
	20	8.76E+02	✓	1.16E−06	—	2.55E−06	—	4.95E−06	—
$\mathbb{P}_4$	40	3.26E+03	✓	3.83E−08	4.92	1.76E−07	3.86	3.34E−07	3.89
	80	1.25E+04	✓	1.24E−09	4.95	1.17E−08	3.91	2.19E−08	3.93
	20	9.97E+02	✓	5.51E−08	—	2.15E−07	—	4.29E−07	—
$\mathbb{P}_5$	40	3.61E+03	✓	8.80E−10	5.97	7.80E−09	4.79	1.54E−08	4.80
	80	1.36E+04	✓	1.40E−11	5.98	2.61E−10	4.90	5.15E−10	4.90