In this tests we consider:

- $\psi(x) = x^4$
- $\psi_{\mathrm{l}} = 0$
- $\psi_{\mathrm{r}} = 1$
- $\psi_{\mathrm{ll}} = 0$
- $\psi_{\mathrm{rr}} = 4$
- $g(x) = -24$

Table 1: Numerical results of PRO1 scheme.

		$\omega = 1 \vert 1, 1$		$\omega = 1 \vert 3, 1$		$\omega = 1 \vert 3, 3$		$\omega = 1 \vert 3, 10$	
	I	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$
$\mathbb{P}_3(4)$	20	3.33E−03	—	3.33E−03	—	3.33E−03	—	3.33E−03	—
	40	4.31E−04	2.95	4.31E−04	2.95	4.31E−04	2.95	4.31E−04	2.95
	80	5.46E−05	2.98	5.46E−05	2.98	5.46E−05	2.98	5.46E−05	2.98
	160	6.86E−06	2.99	6.86E−06	2.99	6.86E−06	2.99	6.86E−06	2.99
	320	8.59E−07	3.00	8.59E−07	3.00	8.59E−07	3.00	8.59E−07	3.00
	640	9.20E−08	3.22	9.20E−08	3.22	9.20E−08	3.22	9.20E−08	3.22
$\mathbb{P}_5(6)$	20	4.78E−14	—	4.78E−14	—	4.78E−14	—	4.78E−14	—
	40	1.24E−13	↑	1.24E−13	↑	1.24E−13	↑	1.24E−13	↑
	80	2.80E−12	↑	2.80E−12	↑	2.80E−12	↑	2.80E−12	↑
	160	9.42E−11	↑	9.42E−11	↑	9.42E−11	↑	9.42E−11	↑
	320	1.10E−09	↑	1.10E−09	↑	1.10E−09	↑	1.10E−09	↑
	640	7.63E−09	↑	7.63E−09	↑	7.63E−09	↑	7.63E−09	↑

In this tests we consider:

- $\psi(x) = \exp(x)$
- $\psi_{\mathrm{l}} = 1$
- $\psi_{\mathrm{r}} = e$
- $\psi_{\mathrm{ll}} = 1$
- $\psi_{\mathrm{rr}} = e$
- $g(x) = -\exp(x)$

Table 2: Numerical results of PRO1 scheme.

		$\omega = 1\|1,1$		$\omega = 1\|3,1$		$\omega = 1\|3,3$		$\omega = 1\|3,10$	
	I	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$
$\mathbb{P}_3(4)$	20	2.60E−04	—	2.60E−04	—	2.60E−04	—	2.60E−04	—
	40	3.35E−05	2.95	3.35E−05	2.95	3.35E−05	2.95	3.35E−05	2.95
	80	4.14E−06	3.02	4.14E−06	3.02	4.14E−06	3.02	4.14E−06	3.02
	160	4.90E−07	3.08	4.90E−07	3.08	4.90E−07	3.08	4.90E−07	3.08
	320	5.33E−08	3.20	5.33E−08	3.20	5.33E−08	3.20	5.33E−08	3.20
	640	1.16E−08	2.20	1.16E−08	2.20	1.16E−08	2.20	1.16E−08	2.20
$\mathbb{P}_5(6)$	20	1.78E−07	—	1.78E−07	—	1.78E−07	—	1.78E−07	—
	40	5.36E−09	5.05	5.36E−09	5.05	5.36E−09	5.05	5.36E−09	5.05
	80	1.43E−10	5.23	1.43E−10	5.23	1.43E−10	5.23	1.43E−10	5.23
	160	9.21E−11	0.64	9.21E−11	0.64	9.21E−11	0.64	9.21E−11	0.64
	320	6.88E−10	↑	6.88E−10	↑	6.88E−10	↑	6.88E−10	↑
	640	5.60E−09	↑	5.60E−09	↑	5.60E−09	↑	5.60E−09	↑

In this tests we consider:

- $\psi(x) = \sin(\pi x)$
- $\psi_{\mathrm{l}} = 0$
- $\psi_{\mathrm{ll}} = \pi$
- $\psi_{\mathrm{r}} = 0$
- $\psi_{\mathrm{rr}} = -\pi$
- $g(x) = -\pi^4 \sin(\pi x)$

Table 3: Numerical results of PRO1 scheme.

		$\omega = 1\|1,1$		$\omega = 1\|3,1$		$\omega = 1\|3,3$		$\omega = 1\|3,10$	
	I	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$
$\mathbb{P}_3(4)$	20	5.37E−03	—	4.42E−03	—	4.42E−03	—	4.42E−03	—
	40	7.55E−04	2.83	6.90E−04	2.68	6.90E−04	2.68	6.90E−04	2.68
	80	1.51E−04	2.32	1.47E−04	2.24	1.47E−04	2.24	1.47E−04	2.24
	160	3.53E−05	2.09	3.50E−05	2.07	3.50E−05	2.07	3.50E−05	2.07
	320	8.67E−06	2.02	8.65E−06	2.02	8.65E−06	2.02	8.65E−06	2.02
	640	2.14E−06	2.02	2.15E−06	2.01	2.16E−06	2.00	2.15E−06	2.01
$\mathbb{P}_5(6)$	20	2.68E−05	—	2.24E−05	—	2.24E−05	—	2.24E−05	—
	40	3.73E−07	6.17	4.59E−07	5.61	4.59E−07	5.61	4.59E−07	5.61
	80	5.88E−08	2.66	5.41E−08	3.08	5.42E−08	3.08	5.41E−08	3.08
	160	4.11E−09	3.84	3.93E−09	3.78	3.55E−09	3.93	3.75E−09	3.85
	320	4.63E−10	3.15	1.62E−09	1.28	2.64E−09	0.43	3.06E−10	3.61
	640	4.31E−08	↑	3.45E−09	↑	8.41E−09	↑	1.32E−08	↑

In this tests we consider:

- $\psi(x) = \sin(2\pi x)$
- $\psi_{\mathrm{l}} = 0$
- $\psi_{\mathrm{ll}} = 2\pi$
- $\psi_{\mathrm{r}} = 0$
- $\psi_{\mathrm{rr}} = 2\pi$
- $g(x) = -16\pi^4 \sin(2\pi x)$

Table 4: Numerical results of PRO1 scheme.

		$\omega = 1\|1,1$		$\omega = 1\|3,1$		$\omega = 1\|3,3$		$\omega = 1\|3,10$	
	I	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$
$\mathbb{P}_3(4)$	20	6.05E−02	—	4.96E−02	—	4.96E−02	—	4.96E−02	—
	40	6.90E−03	3.13	6.47E−03	2.94	6.47E−03	2.94	6.47E−03	2.94
	80	1.24E−03	2.47	1.23E−03	2.40	1.23E−03	2.40	1.23E−03	2.40
	160	2.82E−04	2.14	2.82E−04	2.12	2.82E−04	2.12	2.82E−04	2.12
	320	6.89E−05	2.03	6.88E−05	2.03	6.88E−05	2.03	6.88E−05	2.03
	640	1.71E−05	2.01	1.71E−05	2.01	1.71E−05	2.01	1.71E−05	2.01
$\mathbb{P}_5(6)$	20	3.65E−03	—	1.85E−03	—	1.85E−03	—	1.85E−03	—
	40	1.55E−05	7.88	1.95E−05	6.57	1.95E−05	6.57	1.95E−05	6.57
	80	1.54E−06	3.34	1.76E−06	3.47	1.76E−06	3.47	1.76E−06	3.47
	160	1.29E−07	3.57	1.20E−07	3.87	1.20E−07	3.87	1.20E−07	3.87
	320	8.64E−09	3.90	8.64E−09	3.80	8.93E−09	3.75	8.95E−09	3.75
	640	1.80E−08	↑	1.65E−08	↑	3.84E−08	↑	1.46E−08	↑

In this tests we consider:

- $\psi(x) = \sin(6\pi x)\exp(x)$
- $\psi_{\mathrm{l}} = 0$
- $\psi_{\mathrm{ll}} = 6\pi$
- $\psi_{\mathrm{r}} = 0$
- $\psi_{\mathrm{rr}} = 6\mathrm{e}\pi$
- $g(x) = \exp(x)\left(24\pi(36\pi^2 - 1)\cos(6\pi x) - (1296\pi^4 - 216\pi^2 + 1)\sin(6\pi x)\right)$

Table 5: Numerical results of PRO1 scheme.

		$\omega = 1\|1,1$		$\omega = 1\|3,1$		$\omega = 1\|3,3$		$\omega = 1\|3,10$	
	I	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$
$\mathbb{P}_3(4)$	20	1.91E+01	—	1.64E+01	—	1.64E+01	—	1.64E+01	—
	40	1.98E+00	3.27	1.57E+00	3.38	1.57E+00	3.38	1.57E+00	3.38
	80	1.30E−01	3.92	9.14E−02	4.11	9.14E−02	4.11	9.14E−02	4.11
	160	2.95E−02	2.14	3.33E−02	1.46	3.33E−02	1.46	3.33E−02	1.46
	320	1.04E−02	1.51	1.08E−02	1.63	1.08E−02	1.63	1.08E−02	1.63
	640	2.89E−03	1.85	2.93E−03	1.88	2.93E−03	1.88	2.93E−03	1.88
$\mathbb{P}_5(6)$	20	1.82E+00	—	1.24E+00	—	1.24E+00	—	1.24E+00	—
	40	2.02E−01	3.18	1.99E−01	2.64	1.99E−01	2.64	1.99E−01	2.64
	80	8.66E−03	4.54	7.57E−03	4.71	7.57E−03	4.71	7.57E−03	4.71
	160	3.28E−04	4.72	3.05E−04	4.63	3.05E−04	4.63	3.05E−04	4.63
	320	1.49E−05	4.46	1.52E−05	4.32	1.52E−05	4.32	1.52E−05	4.32
	640	9.82E−07	3.92	7.32E−07	4.38	8.10E−07	4.23	9.19E−07	4.05

In this tests we consider:

- $\psi(x) = -\exp(x) - (\mathrm{e}-3)x^3 - (5-2\mathrm{e})x^2 + x + 1$
- $\psi_{\mathrm{l}} = 0$
- $\psi_{\mathrm{ll}} = 0$
- $\psi_{\mathrm{r}} = 0$
- $\psi_{\mathrm{rr}} = 0$
- $g(x) = \exp(x)$

Table 6: Numerical results of PRO1 scheme.

		$\omega = 1\|1,1$		$\omega = 1\|3,1$		$\omega = 1\|3,3$		$\omega = 1\|3,10$	
	I	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$	$\mathrm{E}_{\infty,0}$	$\mathrm{O}_{\infty,0}$
$\mathbb{P}_3(4)$	20	2.60E−04	—	2.07E−04	—	2.07E−04	—	2.06E−04	—
	40	3.35E−05	2.95	2.65E−05	2.96	2.65E−05	2.96	2.65E−05	2.96
	80	4.14E−06	3.02	3.27E−06	3.02	3.27E−06	3.02	3.27E−06	3.02
	160	4.90E−07	3.08	3.82E−07	3.10	3.82E−07	3.10	3.82E−07	3.10
	320	5.40E−08	3.18	4.03E−08	3.25	4.01E−08	3.25	4.11E−08	3.22
	640	1.07E−08	2.34	7.36E−09	2.45	6.71E−09	2.58	1.09E−08	1.91
$\mathbb{P}_5(6)$	20	1.78E−07	—	1.48E−07	—	1.48E−07	—	1.48E−07	—
	40	5.36E−09	5.05	4.46E−09	5.06	4.45E−09	5.06	4.45E−09	5.06
	80	1.54E−10	5.12	1.57E−10	4.82	1.58E−10	4.82	1.40E−10	4.99
	160	3.50E−11	2.14	1.11E−10	0.51	1.69E−10	↑	4.62E−10	↑
	320	1.77E−09	↑	1.18E−09	↑	4.14E−09	↑	1.60E−10	1.53
	640	1.09E−08	↑	2.60E−08	↑	3.89E−08	↑	1.19E−08	↑