1 $\frac{h}{2}$

Table 1: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (d).

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	3.67E+02	3.96E−02	—	8.19E−02	—
$\mathbb{P}_1$	30	8.28E+02	2.69E−02	0.96	5.56E−02	0.96
	40	1.47E+03	2.03E−02	0.97	4.21E−02	0.97
	20	5.58E+02	1.19E−03	—	2.15E−03	—
$\mathbb{P}_2$	30	1.23E+03	5.19E−04	2.05	9.37E−04	2.05
	40	2.17E+03	2.90E−04	2.03	5.21E−04	2.04
	20	6.38E+02	1.23E−04	—	2.34E−04	—
$\mathbb{P}_3$	30	1.41E+03	3.72E−05	2.96	7.17E−05	2.92
	40	2.47E+03	1.58E−05	2.97	3.08E−05	2.94
	20	8.35E+02	4.86E−06	—	9.50E−06	—
$\mathbb{P}_4$	30	1.82E+03	1.02E−06	3.86	2.02E−06	3.82
	40	3.17E+03	3.29E−07	3.91	6.63E−07	3.88
	20	8.96E+02	4.14E−07	—	7.98E−07	—
$\mathbb{P}_5$	30	1.94E+03	5.71E−08	4.89	1.12E−07	4.85
	40	3.39E+03	1.39E−08	4.92	2.73E−08	4.89

Table 2: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ $(d+1)$.

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	5.58E+02	1.19E−03	—	2.15E−03	—
$\mathbb{P}_1$	30	1.23E+03	5.19E−04	2.05	9.37E−04	2.05
	40	2.17E+03	2.90E−04	2.03	5.21E−04	2.04
	20	6.38E+02	1.23E−04	—	2.34E−04	—
$\mathbb{P}_2$	30	1.41E+03	3.72E−05	2.96	7.17E−05	2.92
	40	2.47E+03	1.58E−05	2.97	3.08E−05	2.94
	20	8.35E+02	4.86E−06	—	9.50E−06	—
$\mathbb{P}_3$	30	1.82E+03	1.02E−06	3.86	2.02E−06	3.82
	40	3.17E+03	3.29E−07	3.91	6.63E−07	3.88
	20	8.96E+02	4.14E−07	—	7.98E−07	—
$\mathbb{P}_4$	30	1.94E+03	5.71E−08	4.89	1.12E−07	4.85
	40	3.39E+03	1.39E−08	4.92	2.73E−08	4.89
	20	1.22E+03	1.08E−09	—	2.34E−09	—
$\mathbb{P}_5$	30	2.55E+03	5.20E−11	7.48	1.10E−10	7.53
	40	4.37E+03	5.66E−12	7.71	1.08E−11	8.08

Table 3: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ $(75\%d,\ 25\%d+1)$.

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	3.89E+02	6.48E−03	—	1.26E−02	—
$\mathbb{P}_1$	30	8.62E+02	4.43E−03	0.94	8.50E−03	0.97
	40	1.52E+03	3.32E−03	1.01	6.13E−03	1.14
	20	5.64E+02	4.34E−04	—	8.49E−04	—
$\mathbb{P}_2$	30	1.24E+03	1.95E−04	1.97	3.79E−04	1.99
	40	2.18E+03	1.14E−04	1.87	2.12E−04	2.02
	20	6.59E+02	5.70E−06	—	9.45E−06	—
$\mathbb{P}_3$	30	1.45E+03	1.27E−06	3.71	2.19E−06	3.61
	40	2.53E+03	3.98E−07	4.03	6.96E−07	3.98
	20	8.57E+02	2.05E−06	—	4.20E−06	—
$\mathbb{P}_4$	30	1.84E+03	7.03E−08	8.32	1.68E−07	7.93
	40	3.19E+03	6.45E−08	0.30	1.24E−07	1.05
	20	1.00E+03	4.19E−08	—	8.04E−08	—
$\mathbb{P}_5$	30	2.12E+03	8.43E−10	9.64	1.86E−09	9.29
	40	3.67E+03	3.71E−10	2.86	7.40E−10	3.20

Table 4: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ $(5, d+1)$.

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	3.90E+02	6.13E−03	—	1.14E−02	—
$\mathbb{P}_1$	30	8.62E+02	4.54E−03	0.74	9.05E−03	0.56
	40	1.52E+03	3.55E−03	0.86	7.38E−03	0.71
	20	5.63E+02	3.28E−04	—	6.41E−04	—
$\mathbb{P}_2$	30	1.24E+03	1.88E−04	1.38	3.84E−04	1.26
	40	2.18E+03	1.17E−04	1.65	2.47E−04	1.54
	20	6.65E+02	6.38E−06	—	1.09E−05	—
$\mathbb{P}_3$	30	1.45E+03	1.27E−06	3.97	2.19E−06	3.95
	40	2.53E+03	4.05E−07	3.98	7.00E−07	3.96
	20	8.54E+02	1.42E−06	—	2.86E−06	—
$\mathbb{P}_4$	30	1.85E+03	3.09E−07	3.77	6.37E−07	3.71
	40	3.21E+03	1.01E−07	3.88	2.13E−07	3.81
	20	1.01E+03	3.05E−09	—	8.11E−09	—
$\mathbb{P}_5$	30	2.13E+03	4.71E−10	4.61	1.30E−09	4.52
	40	3.64E+03	1.13E−10	4.95	3.30E−10	4.76

2 h^2

Table 5: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ (d).

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	3.67E+02	8.78E−03	—	2.14E−02	—
$\mathbb{P}_1$	30	8.28E+02	5.60E−03	1.11	1.36E−02	1.11
	40	1.47E+03	4.10E−03	1.08	9.98E−03	1.09
	20	5.31E+02	3.98E−04	—	1.01E−03	—
$\mathbb{P}_2$	30	1.19E+03	2.08E−04	1.60	5.13E−04	1.66
	40	2.11E+03	1.26E−04	1.75	3.07E−04	1.78
	20	6.01E+02	1.01E−05	—	1.80E−05	—
$\mathbb{P}_3$	30	1.35E+03	2.04E−06	3.94	3.72E−06	3.89
	40	2.40E+03	6.54E−07	3.96	1.20E−06	3.92
	20	7.54E+02	9.46E−07	—	2.17E−06	—
$\mathbb{P}_4$	30	1.69E+03	2.09E−07	3.72	4.83E−07	3.70
	40	3.00E+03	6.95E−08	3.83	1.61E−07	3.81
	20	8.01E+02	3.87E−08	—	7.74E−08	—
$\mathbb{P}_5$	30	1.80E+03	4.43E−09	5.35	9.37E−09	5.21
	40	3.19E+03	9.67E−10	5.29	2.11E−09	5.19

Table 6: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ $(d+1)$.

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	5.31E+02	3.98E−04	—	1.01E−03	—
$\mathbb{P}_1$	30	1.19E+03	2.08E−04	1.60	5.13E−04	1.66
	40	2.11E+03	1.26E−04	1.75	3.07E−04	1.78
	20	6.01E+02	1.01E−05	—	1.80E−05	—
$\mathbb{P}_2$	30	1.35E+03	2.04E−06	3.94	3.72E−06	3.89
	40	2.40E+03	6.54E−07	3.96	1.20E−06	3.92
	20	7.54E+02	9.46E−07	—	2.17E−06	—
$\mathbb{P}_3$	30	1.69E+03	2.09E−07	3.72	4.83E−07	3.70
	40	3.00E+03	6.95E−08	3.83	1.61E−07	3.81
	20	8.01E+02	3.87E−08	—	7.74E−08	—
$\mathbb{P}_4$	30	1.80E+03	4.43E−09	5.35	9.37E−09	5.21
	40	3.19E+03	9.67E−10	5.29	2.11E−09	5.19
	20	9.54E+02	1.57E−09	—	3.42E−09	—
$\mathbb{P}_5$	30	2.14E+03	1.40E−10	5.96	3.14E−10	5.89
	40	3.80E+03	2.49E−11	6.00	5.66E−11	5.95

Table 7: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ $(75\%d, 25\%d+1)$.

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	3.70E+02	4.97E−03	—	1.00E−02	—
$\mathbb{P}_1$	30	8.32E+02	3.73E−03	0.71	7.35E−03	0.77
	40	1.48E+03	2.91E−03	0.87	5.48E−03	1.02
	20	5.31E+02	4.88E−04	—	9.46E−04	—
$\mathbb{P}_2$	30	1.19E+03	2.27E−04	1.88	4.30E−04	1.94
	40	2.11E+03	1.29E−04	1.97	2.35E−04	2.10
	20	6.01E+02	2.27E−06	—	2.99E−06	—
$\mathbb{P}_3$	30	1.35E+03	4.41E−07	4.04	5.97E−07	3.98
	40	2.40E+03	1.38E−07	4.05	1.93E−07	3.93
	20	7.54E+02	8.19E−07	—	1.73E−06	—
$\mathbb{P}_4$	30	1.69E+03	1.67E−07	3.93	3.41E−07	4.00
	40	3.00E+03	5.78E−08	3.68	1.11E−07	3.90
	20	8.02E+02	1.71E−08	—	3.27E−08	—
$\mathbb{P}_5$	30	1.80E+03	2.09E−09	5.19	4.05E−09	5.15
	40	3.19E+03	5.38E−10	4.71	9.98E−10	4.87

Table 8: Numerical results of pure diffusion for $\phi(x) = \exp(x)$, $\kappa(x) = 1$, and $u(x) = 0$ $(5, d+1)$.

	I	$k(A)$	E_1	O_1	E_∞	O_∞
	20	3.70E+02	4.66E−03	—	8.91E−03	—
$\mathbb{P}_1$	30	8.32E+02	3.85E−03	0.47	7.87E−03	0.31
	40	1.48E+03	3.15E−03	0.70	6.67E−03	0.57
	20	5.31E+02	4.57E−04	—	8.32E−04	—
$\mathbb{P}_2$	30	1.19E+03	2.32E−04	1.67	4.57E−04	1.48
	40	2.11E+03	1.37E−04	1.82	2.83E−04	1.67
	20	6.01E+02	2.35E−06	—	3.20E−06	—
$\mathbb{P}_3$	30	1.35E+03	4.38E−07	4.15	5.80E−07	4.21
	40	2.40E+03	1.34E−07	4.11	1.75E−07	4.17
	20	7.54E+02	7.43E−07	—	1.49E−06	—
$\mathbb{P}_4$	30	1.69E+03	1.84E−07	3.45	3.87E−07	3.32
	40	3.00E+03	6.35E−08	3.69	1.38E−07	3.59
	20	8.02E+02	1.37E−08	—	2.52E−08	—
$\mathbb{P}_5$	30	1.80E+03	2.21E−09	4.51	4.44E−09	4.28
	40	3.19E+03	5.71E−10	4.70	1.20E−09	4.56