

Note: The results seems a little confused and wrong. Analyse later!

## 1 Results

Table 1: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (only d).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	2.91E+02	×	1.22E-02	—	2.49E-03	—	6.56E-03	—
	30	6.52E+02	×	5.46E-03	1.97	1.04E-03	2.16	2.92E-03	2.00
	40	1.16E+03	×	3.09E-03	1.98	5.68E-04	2.09	1.64E-03	1.99
$\mathbb{P}_2$	20	2.70E+05	×	6.19E-04	—	3.24E-01	—	4.20E+00	—
	30	1.83E+08	×	1.85E-04	2.97	4.21E+01	↑	8.17E+02	↑
	40	1.47E+11	×	7.85E-05	2.98	1.05E+04	↑	2.72E+05	↑
$\mathbb{P}_3$	20	1.00E+03	×	5.63E-05	—	2.15E-04	—	4.47E-04	—
	30	1.46E+04	×	1.13E-05	3.95	7.17E-04	↑	1.47E-03	↑
	40	1.08E+05	×	3.63E-06	3.96	1.51E-03	↑	3.08E-03	↑
$\mathbb{P}_4$	20	1.46E+03	×	2.63E-06	—	1.32E-06	—	2.14E-06	—
	30	2.85E+03	×	3.55E-07	4.94	9.00E-08	6.63	1.73E-07	6.21
	40	5.07E+03	×	8.54E-08	4.95	2.35E-08	4.67	4.78E-08	4.46
$\mathbb{P}_5$	20	1.53E+03	×	2.08E-07	—	7.31E-08	—	9.44E-08	—
	30	3.39E+03	×	1.87E-08	5.94	1.31E-08	4.24	1.98E-08	3.85
	40	6.38E+03	×	3.38E-09	5.95	2.22E-09	6.17	2.95E-09	6.62

Table 2: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	2.91E+02	×	1.22E-02	—	2.49E-03	—	6.56E-03	—
	30	6.52E+02	×	5.46E-03	1.97	1.04E-03	2.16	2.92E-03	2.00
	40	1.16E+03	×	3.09E-03	1.98	5.68E-04	2.09	1.64E-03	1.99
$\mathbb{P}_2$	20	2.70E+05	×	6.19E-04	—	3.24E-01	—	4.20E+00	—
	30	1.83E+08	×	1.85E-04	2.97	4.21E+01	↑	8.17E+02	↑
	40	1.47E+11	×	7.85E-05	2.98	1.05E+04	↑	2.72E+05	↑
$\mathbb{P}_3$	20	1.00E+03	×	5.63E-05	—	2.15E-04	—	4.47E-04	—
	30	1.46E+04	×	1.13E-05	3.95	7.17E-04	↑	1.47E-03	↑
	40	1.08E+05	×	3.63E-06	3.96	1.51E-03	↑	3.08E-03	↑
$\mathbb{P}_4$	20	1.46E+03	×	2.63E-06	—	1.32E-06	—	2.14E-06	—
	30	2.85E+03	×	3.55E-07	4.94	9.00E-08	6.63	1.73E-07	6.21
	40	5.07E+03	×	8.54E-08	4.95	2.35E-08	4.67	4.78E-08	4.46
$\mathbb{P}_5$	20	1.53E+03	×	2.08E-07	—	7.31E-08	—	9.44E-08	—
	30	3.39E+03	×	1.87E-08	5.94	1.31E-08	4.24	1.98E-08	3.85
	40	6.38E+03	×	3.38E-09	5.95	2.22E-09	6.17	2.95E-09	6.62

Table 3: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 1 cell with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	4.58E+02	×	6.16E-03	—	1.19E-03	—	2.27E-03	—
	30	1.03E+03	×	2.68E-03	2.05	5.25E-04	2.03	9.44E-04	2.16
	40	1.83E+03	×	1.49E-03	2.04	2.93E-04	2.02	5.29E-04	2.01
$\mathbb{P}_2$	20	3.00E+05	×	3.61E-04	—	3.63E-01	—	4.74E+00	—
	30	1.81E+08	×	1.04E-04	3.07	4.18E+01	↑	8.18E+02	↑
	40	1.33E+11	×	4.32E-05	3.05	9.61E+03	↑	2.51E+05	↑
$\mathbb{P}_3$	20	1.63E+03	×	3.41E-05	—	1.05E-04	—	2.21E-04	—
	30	1.26E+04	×	6.67E-06	4.02	1.55E-04	↑	3.18E-04	↑
	40	1.37E+05	×	2.10E-06	4.02	4.83E-04	↑	9.85E-04	↑
$\mathbb{P}_4$	20	1.31E+03	×	1.66E-06	—	3.47E-07	—	7.85E-07	—
	30	2.86E+03	×	2.15E-07	5.04	9.33E-08	3.24	1.76E-07	3.68
	40	5.06E+03	×	5.06E-08	5.03	1.58E-08	6.18	4.01E-08	5.15
$\mathbb{P}_5$	20	2.03E+03	×	1.25E-07	—	3.56E-08	—	6.54E-08	—
	30	4.51E+03	×	1.07E-08	6.05	3.81E-09	5.51	7.79E-09	5.25
	40	8.05E+03	×	1.90E-09	6.03	5.09E-10	7.00	1.17E-09	6.60

Table 4: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 2 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	5.47E+02	×	5.59E-03	—	1.72E-03	—	6.82E-03	—
	30	1.23E+03	×	2.51E-03	1.98	7.28E-04	2.12	3.18E-03	1.88
	40	2.18E+03	×	1.42E-03	1.98	3.98E-04	2.10	1.83E-03	1.92
$\mathbb{P}_2$	20	3.19E+05	×	3.24E-04	—	3.76E-01	—	5.06E+00	—
	30	1.95E+08	×	9.61E-05	2.99	4.40E+01	↑	8.90E+02	↑
	40	1.46E+11	×	4.06E-05	2.99	1.03E+04	↑	2.76E+05	↑
$\mathbb{P}_3$	20	2.10E+03	×	2.27E-05	—	4.77E-05	—	1.01E-04	—
	30	1.19E+04	×	4.27E-06	4.12	3.81E-05	0.55	7.94E-05	0.60
	40	1.50E+05	×	1.32E-06	4.09	1.35E-04	↑	2.75E-04	↑
$\mathbb{P}_4$	20	1.30E+03	×	1.27E-06	—	1.64E-07	—	3.55E-07	—
	30	2.90E+03	×	1.59E-07	5.12	8.05E-08	1.76	1.59E-07	1.98
	40	5.13E+03	×	3.68E-08	5.09	1.03E-08	7.16	2.26E-08	6.79
$\mathbb{P}_5$	20	2.13E+03	×	9.71E-08	—	2.78E-08	—	3.70E-08	—
	30	4.73E+03	×	8.10E-09	6.13	1.67E-09	6.94	3.60E-09	5.75
	40	8.43E+03	×	1.40E-09	6.09	3.06E-10	5.90	7.60E-10	5.41

Table 5: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 3 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	5.57E+02	×	5.86E-03	—	2.94E-03	—	2.43E-02	—
	30	1.25E+03	×	2.59E-03	2.01	1.10E-03	2.43	1.12E-02	1.90
	40	2.21E+03	×	1.45E-03	2.01	5.57E-04	2.36	6.47E-03	1.92
$\mathbb{P}_2$	20	5.61E+04	×	3.69E-04	—	1.02E-01	—	5.56E-01	—
	30	3.24E+07	×	1.05E-04	3.09	1.13E+01	↑	9.24E+01	↑
	40	2.34E+10	×	4.37E-05	3.07	2.54E+03	↑	2.77E+04	↑
$\mathbb{P}_3$	20	2.48E+03	×	2.20E-05	—	1.41E-05	—	2.93E-05	—
	30	1.12E+04	×	4.17E-06	4.10	2.27E-06	4.50	4.38E-06	4.69
	40	1.53E+05	×	1.29E-06	4.07	1.40E-05	↑	2.94E-05	↑
$\mathbb{P}_4$	20	1.31E+03	×	9.93E-07	—	2.60E-07	—	5.32E-07	—
	30	2.92E+03	×	1.21E-07	5.20	5.57E-08	3.80	1.34E-07	3.40
	40	5.16E+03	×	2.75E-08	5.15	9.51E-09	6.14	2.14E-08	6.37
$\mathbb{P}_5$	20	2.16E+03	×	8.33E-08	—	2.79E-08	—	3.90E-08	—
	30	4.77E+03	×	6.80E-09	6.18	2.34E-09	6.11	7.86E-09	3.95
	40	8.52E+03	×	1.16E-09	6.13	3.32E-10	6.79	4.91E-10	9.64

Table 6: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 4 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	1.77E+03	×	5.72E-03	—	3.15E-02	—	3.32E-01	—
	30	1.63E+03	×	2.54E-03	2.00	8.60E-03	3.21	1.14E-01	2.63
	40	2.25E+03	×	1.43E-03	2.00	3.93E-03	2.72	6.02E-02	2.23
$\mathbb{P}_2$	20	3.64E+04	×	3.63E-04	—	7.24E-02	—	3.55E-01	—
	30	2.35E+07	×	1.04E-04	3.08	8.96E+00	↑	6.58E+01	↑
	40	1.82E+10	×	4.32E-05	3.06	2.16E+03	↑	2.12E+04	↑
$\mathbb{P}_3$	20	2.01E+03	×	2.26E-05	—	4.38E-05	—	8.83E-05	—
	30	1.08E+04	×	4.25E-06	4.11	6.04E-05	↑	1.22E-04	↑
	40	1.38E+05	×	1.31E-06	4.08	2.28E-04	↑	4.67E-04	↑
$\mathbb{P}_4$	20	1.30E+03	×	9.89E-07	—	3.00E-07	—	5.77E-07	—
	30	2.94E+03	×	1.20E-07	5.19	5.30E-08	4.28	1.46E-07	3.39
	40	5.15E+03	×	2.74E-08	5.15	9.85E-09	5.85	2.38E-08	6.31
$\mathbb{P}_5$	20	2.13E+03	×	8.10E-08	—	2.99E-08	—	4.23E-08	—
	30	4.74E+03	×	6.58E-09	6.19	9.61E-10	8.48	6.63E-09	4.57
	40	8.41E+03	×	1.12E-09	6.14	3.73E-10	3.29	5.40E-10	8.72

Table 7: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 5 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	5.55E+02	×	5.58E-03	—	6.78E-03	—	7.55E-02	—
	30	1.24E+03	×	2.50E-03	1.98	2.15E-03	2.83	3.25E-02	2.08
	40	2.21E+03	×	1.41E-03	1.98	9.93E-04	2.68	1.83E-02	1.99
$\mathbb{P}_2$	20	7.08E+04	×	3.57E-04	—	1.36E-01	—	6.63E-01	—
	30	1.60E+08	×	1.03E-04	3.07	5.88E+01	↑	4.30E+02	↑
	40	2.97E+10	×	4.28E-05	3.05	3.40E+03	↑	3.32E+04	↑
$\mathbb{P}_3$	20	2.83E+03	×	2.23E-05	—	1.30E-04	—	2.73E-04	—
	30	8.25E+03	×	4.22E-06	4.11	5.89E-05	1.96	1.22E-04	1.99
	40	1.24E+05	×	1.31E-06	4.08	2.63E-04	↑	5.37E-04	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.89E-07	—	2.19E-07	—	4.72E-07	—
	30	3.40E+03	×	1.20E-07	5.19	1.66E-07	0.69	2.93E-07	1.17
	40	5.11E+03	×	2.74E-08	5.15	1.02E-08	9.68	2.43E-08	8.65
$\mathbb{P}_5$	20	2.13E+03	×	8.13E-08	—	9.60E-09	—	3.48E-08	—
	30	4.73E+03	×	6.60E-09	6.19	2.55E-09	3.27	7.47E-09	3.80
	40	8.40E+03	×	1.13E-09	6.14	2.10E-10	8.69	1.10E-09	6.65

Table 8: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 6 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	5.70E+02	×	5.45E-03	—	6.69E-03	—	7.15E-02	—
	30	1.25E+03	×	2.46E-03	1.96	2.49E-03	2.44	3.61E-02	1.68
	40	2.21E+03	×	1.40E-03	1.97	1.22E-03	2.48	2.15E-02	1.80
$\mathbb{P}_2$	20	7.21E+03	×	3.51E-04	—	1.63E-02	—	6.01E-02	—
	30	1.75E+07	×	1.02E-04	3.06	7.51E+00	↑	4.16E+01	↑
	40	1.44E+10	×	4.25E-05	3.04	1.93E+03	↑	1.43E+04	↑
$\mathbb{P}_3$	20	3.38E+04	×	2.21E-05	—	1.71E-04	—	3.58E-04	—
	30	7.48E+03	×	4.19E-06	4.10	2.77E-06	10.17	4.47E-06	10.81
	40	1.36E+05	×	1.30E-06	4.07	2.00E-05	↑	4.16E-05	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.95E-07	—	1.58E-07	—	3.92E-07	—
	30	3.24E+03	×	1.21E-07	5.20	2.07E-07	↑	3.50E-07	0.28
	40	5.11E+03	×	2.75E-08	5.15	9.07E-09	10.88	1.92E-08	10.10
$\mathbb{P}_5$	20	2.12E+03	×	8.10E-08	—	2.80E-08	—	5.56E-08	—
	30	4.73E+03	×	6.58E-09	6.19	1.22E-09	7.73	4.03E-09	6.47
	40	8.38E+03	×	1.12E-09	6.14	3.03E-10	4.83	1.17E-09	4.31

Table 9: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 7 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	7.55E+02	×	5.33E−03	—	1.38E−02	—	1.70E−01	—
	30	1.26E+03	×	2.42E−03	1.95	4.28E−03	2.88	7.69E−02	1.96
	40	2.21E+03	×	1.38E−03	1.96	1.91E−03	2.80	4.45E−02	1.90
$\mathbb{P}_2$	20	9.34E+03	×	3.46E−04	—	2.04E−02	—	9.93E−02	—
	30	1.09E+07	×	1.01E−04	3.05	4.55E+00	↑	3.35E+01	↑
	40	2.84E+09	×	4.21E−05	3.03	3.68E+02	↑	3.61E+03	↑
$\mathbb{P}_3$	20	1.74E+03	×	2.19E−05	—	2.78E−05	—	5.69E−05	—
	30	7.76E+03	×	4.16E−06	4.10	4.01E−05	↑	8.07E−05	↑
	40	1.02E+05	×	1.29E−06	4.07	1.63E−04	↑	3.34E−04	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.91E−07	—	2.27E−07	—	4.61E−07	—
	30	3.27E+03	×	1.21E−07	5.20	1.55E−07	0.95	3.10E−07	0.97
	40	5.11E+03	×	2.74E−08	5.15	9.39E−09	9.74	2.00E−08	9.53
$\mathbb{P}_5$	20	2.13E+03	×	8.07E−08	—	2.78E−08	—	4.07E−08	—
	30	4.73E+03	×	6.56E−09	6.19	1.18E−09	7.81	1.11E−08	3.21
	40	8.40E+03	×	1.12E−09	6.14	3.09E−10	4.64	4.89E−10	10.84

Table 10: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 8 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	2.03E+03	×	5.22E−03	—	2.65E−02	—	3.35E−01	—
	30	2.86E+03	×	2.38E−03	1.94	1.02E−02	2.36	1.91E−01	1.39
	40	3.51E+03	×	1.36E−03	1.95	4.78E−03	2.62	1.18E−01	1.66
$\mathbb{P}_2$	20	1.84E+04	×	3.41E−04	—	4.02E−02	—	1.84E−01	—
	30	2.79E+06	×	9.96E−05	3.04	1.17E+00	↑	8.03E+00	↑
	40	1.40E+09	×	4.17E−05	3.02	1.84E+02	↑	1.68E+03	↑
$\mathbb{P}_3$	20	1.29E+03	×	2.18E−05	—	4.17E−05	—	8.85E−05	—
	30	5.44E+03	×	4.14E−06	4.10	3.97E−05	0.13	8.23E−05	0.18
	40	6.23E+04	×	1.28E−06	4.06	1.46E−04	↑	2.98E−04	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.88E−07	—	2.09E−07	—	4.24E−07	—
	30	4.80E+03	×	1.20E−07	5.19	3.15E−07	↑	5.52E−07	↑
	40	5.11E+03	×	2.73E−08	5.15	9.96E−09	12.01	2.26E−08	11.11
$\mathbb{P}_5$	20	2.12E+03	×	8.04E−08	—	1.67E−08	—	5.37E−08	—
	30	4.72E+03	×	6.54E−09	6.19	2.35E−09	4.83	8.33E−09	4.60
	40	8.39E+03	×	1.12E−09	6.14	1.55E−10	9.46	1.44E−09	6.11

Table 11: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 9 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	1.54E+04	×	5.11E-03	—	2.83E-01	—	3.65E+00	—
	30	3.06E+04	×	2.34E-03	1.92	1.22E-01	2.08	2.34E+00	1.09
	40	4.24E+04	×	1.34E-03	1.93	6.10E-02	2.41	1.56E+00	1.43
$\mathbb{P}_2$	20	1.50E+03	×	3.37E-04	—	3.26E-03	—	1.51E-02	—
	30	1.82E+06	×	9.85E-05	3.03	7.46E-01	↑	5.18E+00	↑
	40	3.72E+09	×	4.14E-05	3.02	4.75E+02	↑	4.40E+03	↑
$\mathbb{P}_3$	20	2.01E+03	×	2.16E-05	—	1.15E-05	—	2.41E-05	—
	30	4.01E+03	×	4.11E-06	4.09	1.43E-05	↑	2.81E-05	↑
	40	5.59E+04	×	1.28E-06	4.06	7.42E-05	↑	1.52E-04	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.85E-07	—	1.66E-07	—	3.65E-07	—
	30	3.85E+03	×	1.20E-07	5.19	3.09E-07	↑	5.15E-07	↑
	40	5.11E+03	×	2.73E-08	5.15	8.74E-09	12.39	1.77E-08	11.72
$\mathbb{P}_5$	20	2.12E+03	×	8.02E-08	—	3.02E-08	—	8.46E-08	—
	30	4.72E+03	×	6.52E-09	6.19	1.84E-09	6.90	6.35E-09	6.39
	40	8.38E+03	×	1.12E-09	6.14	2.78E-10	6.57	1.64E-09	4.71

Table 12: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 10 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	1.23E+04	×	5.00E-03	—	1.47E-01	—	1.89E+00	—
	30	1.48E+04	×	2.31E-03	1.91	5.05E-02	2.64	9.67E-01	1.66
	40	1.76E+04	×	1.33E-03	1.92	2.30E-02	2.72	5.85E-01	1.75
$\mathbb{P}_2$	20	7.00E+02	×	3.32E-04	—	6.54E-04	—	2.15E-03	—
	30	2.48E+05	×	9.75E-05	3.02	1.16E-01	↑	6.63E-01	↑
	40	3.02E+08	×	4.10E-05	3.01	4.41E+01	↑	3.35E+02	↑
$\mathbb{P}_3$	20	1.19E+04	×	2.14E-05	—	1.59E-04	—	3.44E-04	—
	30	3.90E+03	×	4.08E-06	4.09	7.27E-07	13.29	2.09E-06	12.59
	40	6.04E+04	×	1.27E-06	4.06	3.62E-06	↑	8.10E-06	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.82E-07	—	2.11E-07	—	3.99E-07	—
	30	4.05E+03	×	1.20E-07	5.19	2.68E-07	↑	4.55E-07	↑
	40	5.10E+03	×	2.72E-08	5.15	9.07E-09	11.77	1.82E-08	11.20
$\mathbb{P}_5$	20	2.12E+03	×	7.99E-08	—	3.04E-08	—	4.67E-08	—
	30	4.72E+03	×	6.50E-09	6.19	1.40E-09	7.59	1.55E-08	2.72
	40	8.41E+03	×	1.11E-09	6.13	3.77E-10	4.56	6.04E-10	11.28

Table 13: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 11 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	1.10E+04	×	4.90E-03	—	2.21E-01	—	2.86E+00	—
	30	1.48E+04	×	2.27E-03	1.90	5.87E-02	3.27	1.13E+00	2.28
	40	1.81E+04	×	1.31E-03	1.91	2.55E-02	2.90	6.56E-01	1.90
$\mathbb{P}_2$	20	6.99E+02	×	3.28E-04	—	3.98E-04	—	1.97E-03	—
	30	1.95E+05	×	9.65E-05	3.02	8.45E-02	↑	6.08E-01	↑
	40	3.65E+08	×	4.07E-05	3.00	4.92E+01	↑	4.72E+02	↑
$\mathbb{P}_3$	20	1.20E+03	×	2.13E-05	—	2.60E-05	—	5.42E-05	—
	30	3.86E+03	×	4.06E-06	4.08	1.62E-05	1.17	3.40E-05	1.15
	40	4.07E+04	×	1.26E-06	4.05	6.34E-05	↑	1.29E-04	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.79E-07	—	1.77E-07	—	3.43E-07	—
	30	2.97E+03	×	1.19E-07	5.19	5.63E-08	2.82	1.09E-07	2.84
	40	5.13E+03	×	2.71E-08	5.14	1.08E-08	5.73	2.29E-08	5.41
$\mathbb{P}_5$	20	2.11E+03	×	7.96E-08	—	2.05E-08	—	1.39E-07	—
	30	4.72E+03	×	6.49E-09	6.18	2.52E-09	5.17	1.11E-08	6.23
	40	8.41E+03	×	1.11E-09	6.13	2.88E-10	7.53	2.88E-09	4.70

Table 14: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 12 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	1.72E+04	×	4.81E-03	—	1.79E-01	—	2.31E+00	—
	30	2.47E+04	×	2.24E-03	1.89	8.12E-02	1.95	1.57E+00	0.96
	40	3.08E+04	×	1.29E-03	1.90	3.91E-02	2.54	1.00E+00	1.55
$\mathbb{P}_2$	20	7.00E+02	×	3.24E-04	—	4.24E-04	—	1.80E-03	—
	30	2.27E+05	×	9.56E-05	3.01	1.00E-01	↑	6.89E-01	↑
	40	1.38E+09	×	4.03E-05	3.00	1.90E+02	↑	1.74E+03	↑
$\mathbb{P}_3$	20	1.16E+03	×	2.11E-05	—	4.76E-06	—	9.26E-06	—
	30	3.07E+03	×	4.03E-06	4.08	1.03E-05	↑	2.02E-05	↑
	40	2.59E+04	×	1.26E-06	4.05	4.26E-05	↑	8.76E-05	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.76E-07	—	1.11E-07	—	2.84E-07	—
	30	2.92E+03	×	1.19E-07	5.19	5.93E-08	1.54	1.10E-07	2.34
	40	5.14E+03	×	2.71E-08	5.14	8.94E-09	6.57	1.76E-08	6.35
$\mathbb{P}_5$	20	2.11E+03	×	7.94E-08	—	3.09E-08	—	1.09E-07	—
	30	4.71E+03	×	6.47E-09	6.18	1.46E-09	7.53	9.09E-09	6.12
	40	8.38E+03	×	1.11E-09	6.13	3.62E-10	4.84	2.20E-09	4.94

Table 15: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 13 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	3.70E+04	×	4.72E-03	—	8.89E-01	—	1.15E+01	—
	30	5.96E+04	×	2.20E-03	1.88	2.39E-01	3.24	4.63E+00	2.24
	40	7.61E+04	×	1.28E-03	1.89	1.06E-01	2.81	2.75E+00	1.81
$\mathbb{P}_2$	20	6.99E+02	×	3.20E-04	—	1.99E-04	—	8.46E-04	—
	30	5.26E+04	×	9.46E-05	3.01	2.37E-02	↑	1.57E-01	↑
	40	4.30E+07	×	4.00E-05	2.99	6.03E+00	↑	5.32E+01	↑
$\mathbb{P}_3$	20	8.26E+03	×	2.10E-05	—	4.00E-04	—	8.73E-04	—
	30	2.79E+03	×	4.01E-06	4.08	4.66E-06	10.98	1.03E-05	10.95
	40	2.47E+04	×	1.25E-06	4.05	2.47E-05	↑	4.97E-05	↑
$\mathbb{P}_4$	20	1.29E+03	×	9.73E-07	—	1.87E-07	—	3.27E-07	—
	30	2.93E+03	×	1.19E-07	5.19	4.96E-08	3.28	9.51E-08	3.05
	40	5.14E+03	×	2.70E-08	5.14	1.08E-08	5.30	2.08E-08	5.29
$\mathbb{P}_5$	20	2.11E+03	×	7.92E-08	—	3.23E-08	—	8.84E-08	—
	30	4.71E+03	×	6.45E-09	6.18	2.23E-09	6.60	2.32E-08	3.30
	40	8.39E+03	×	1.11E-09	6.13	3.33E-10	6.60	1.16E-09	10.42

Table 16: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 14 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	1.18E+05	×	4.63E-03	—	7.72E-01	—	9.99E+00	—
	30	2.82E+05	×	2.17E-03	1.87	8.99E-01	↑	1.75E+01	↑
	40	3.86E+05	×	1.26E-03	1.88	4.78E-01	2.19	1.24E+01	1.19
$\mathbb{P}_2$	20	6.99E+02	×	3.17E-04	—	9.29E-05	—	2.86E-04	—
	30	1.45E+04	×	9.37E-05	3.00	6.89E-03	↑	4.27E-02	↑
	40	1.18E+07	×	3.97E-05	2.99	1.75E+00	↑	1.44E+01	↑
$\mathbb{P}_3$	20	3.78E+03	×	2.08E-05	—	1.34E-04	—	2.72E-04	—
	30	2.79E+03	×	3.99E-06	4.08	2.22E-06	10.12	5.02E-06	9.85
	40	2.55E+04	×	1.25E-06	4.04	1.03E-05	↑	2.04E-05	↑
$\mathbb{P}_4$	20	1.30E+03	×	9.71E-07	—	1.44E-07	—	2.81E-07	—
	30	2.91E+03	×	1.18E-07	5.19	4.03E-08	3.15	7.90E-08	3.12
	40	5.21E+03	×	2.70E-08	5.14	1.37E-08	3.74	2.58E-08	3.89
$\mathbb{P}_5$	20	2.10E+03	×	7.90E-08	—	1.53E-08	—	1.27E-07	—
	30	4.71E+03	×	6.44E-09	6.18	2.65E-09	4.32	1.51E-08	5.25
	40	8.41E+03	×	1.10E-09	6.13	3.46E-10	7.07	4.87E-09	3.94



Table 17: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 15 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	9.20E+05	×	4.55E-03	—	3.16E+01	—	4.10E+02	—
	30	6.58E+05	×	2.14E-03	1.86	2.70E+00	6.07	5.25E+01	5.07
	40	7.64E+05	×	1.25E-03	1.87	1.07E+00	3.23	2.76E+01	2.23
$\mathbb{P}_2$	20	6.98E+02	×	3.13E-04	—	8.66E-05	—	1.67E-04	—
	30	8.66E+03	×	9.29E-05	3.00	3.93E-03	↑	2.63E-02	↑
	40	8.13E+06	×	3.94E-05	2.98	1.14E+00	↑	1.03E+01	↑
$\mathbb{P}_3$	20	1.12E+03	×	2.07E-05	—	9.01E-06	—	1.48E-05	—
	30	2.74E+03	×	3.96E-06	4.08	4.61E-06	1.66	8.75E-06	1.30
	40	1.61E+04	×	1.24E-06	4.04	1.92E-05	↑	3.96E-05	↑
$\mathbb{P}_4$	20	1.31E+03	×	9.68E-07	—	7.19E-08	—	2.02E-07	—
	30	2.89E+03	×	1.18E-07	5.19	3.93E-08	1.49	7.47E-08	2.45
	40	5.73E+03	×	2.69E-08	5.14	1.98E-08	2.37	3.21E-08	2.93
$\mathbb{P}_5$	20	2.09E+03	×	7.88E-08	—	3.40E-08	—	1.66E-07	—
	30	4.71E+03	×	6.42E-09	6.18	1.74E-09	7.34	1.59E-08	5.79
	40	8.38E+03	×	1.10E-09	6.13	4.09E-10	5.03	3.11E-09	5.67

Table 18: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 16 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	2.10E+06	×	4.47E-03	—	2.54E+01	—	3.29E+02	—
	30	4.32E+05	×	2.11E-03	1.85	1.34E+00	7.25	2.61E+01	6.25
	40	5.46E+05	×	1.23E-03	1.87	6.65E-01	2.44	1.72E+01	1.44
$\mathbb{P}_2$	20	7.00E+02	×	3.10E-04	—	9.47E-05	—	1.89E-04	—
	30	7.87E+03	×	9.20E-05	2.99	3.46E-03	↑	2.44E-02	↑
	40	7.72E+06	×	3.91E-05	2.98	1.06E+00	↑	9.98E+00	↑
$\mathbb{P}_3$	20	1.12E+03	×	2.06E-05	—	1.25E-05	—	2.16E-05	—
	30	2.60E+03	×	3.94E-06	4.07	3.58E-06	3.09	7.68E-06	2.55
	40	1.11E+04	×	1.23E-06	4.04	1.48E-05	↑	2.96E-05	↑
$\mathbb{P}_4$	20	1.31E+03	×	9.68E-07	—	1.42E-07	—	2.66E-07	—
	30	2.89E+03	×	1.18E-07	5.19	3.23E-08	3.65	6.45E-08	3.49
	40	5.49E+03	×	2.69E-08	5.14	2.79E-08	0.51	4.20E-08	1.49
$\mathbb{P}_5$	20	2.09E+03	×	7.49E-08	—	2.04E-08	—	5.80E-08	—
	30	4.71E+03	×	6.41E-09	6.06	2.91E-09	4.80	3.01E-08	1.62
	40	8.38E+03	×	1.10E-09	6.13	3.34E-10	7.53	2.05E-09	9.34

Table 19: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 17 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	4.98E+05	×	4.40E−03	—	2.13E+01	—	2.76E+02	—
	30	6.18E+05	×	2.08E−03	1.85	2.67E+00	5.13	5.18E+01	4.13
	40	8.11E+05	×	1.22E−03	1.86	1.14E+00	2.96	2.94E+01	1.96
$\mathbb{P}_2$	20	3.88E+03	×	3.08E−04	—	8.90E−04	—	2.47E−03	—
	30	8.96E+03	×	9.12E−05	3.00	4.45E−03	↑	2.57E−02	↑
	40	3.51E+06	×	3.88E−05	2.97	5.43E−01	↑	4.19E+00	↑
$\mathbb{P}_3$	20	1.12E+03	×	1.96E−05	—	1.21E−05	—	1.95E−05	—
	30	2.56E+03	×	3.92E−06	3.97	5.92E−07	7.43	1.97E−06	5.64
	40	1.13E+04	×	1.23E−06	4.04	5.65E−06	↑	1.20E−05	↑
$\mathbb{P}_4$	20	1.31E+03	×	9.51E−07	—	1.12E−07	—	2.26E−07	—
	30	2.89E+03	×	1.18E−07	5.16	3.52E−08	2.85	6.45E−08	3.09
	40	7.40E+03	×	2.68E−08	5.14	4.99E−08	↑	7.28E−08	↑
$\mathbb{P}_5$	20	2.09E+03	×	7.35E−08	—	1.77E−08	—	6.56E−08	—
	30	4.71E+03	×	6.39E−09	6.02	2.94E−09	4.42	2.14E−08	2.76
	40	8.41E+03	×	1.10E−09	6.13	4.94E−10	6.21	7.99E−09	3.43

Table 20: Numerical results of pure diffusion for  $\phi(x) = \exp(x)$ ,  $\kappa(x) = 1$ , and  $u(x) = 0$  (Neumann bc and 18 cells with  $d + 1$  and the other cells with  $d$ ).

	$I$	cond(A)	$A^{-1} \geq 0$	$E_{c,1}$	$O_{c,1}$	$E_1$	$O_1$	$E_\infty$	$O_\infty$
$\mathbb{P}_1$	20	6.22E+05	×	4.34E−03	—	2.98E+01	—	3.86E+02	—
	30	1.28E+06	×	2.05E−03	1.84	3.85E+00	5.05	7.48E+01	4.05
	40	1.75E+06	×	1.21E−03	1.85	2.10E+00	2.11	5.44E+01	1.11
$\mathbb{P}_2$	20	3.01E+03	×	2.13E−04	—	7.47E−04	—	1.43E−03	—
	30	2.17E+03	×	9.05E−05	2.11	1.05E−03	↑	6.49E−03	↑
	40	1.08E+06	×	3.85E−05	2.97	1.64E−01	↑	1.37E+00	↑
$\mathbb{P}_3$	20	1.13E+03	×	1.56E−05	—	1.44E−05	—	2.29E−05	—
	30	2.57E+03	×	3.90E−06	3.43	4.49E−07	8.56	1.92E−06	6.11
	40	1.09E+04	×	1.22E−06	4.03	3.94E−06	↑	8.51E−06	↑
$\mathbb{P}_4$	20	1.31E+03	×	7.77E−07	—	1.23E−07	—	2.73E−07	—
	30	2.89E+03	×	1.17E−07	4.66	3.28E−08	3.25	5.98E−08	3.75
	40	5.45E+03	×	2.68E−08	5.14	1.16E−08	3.61	2.69E−08	2.78
$\mathbb{P}_5$	20	2.10E+03	×	6.21E−08	—	1.36E−08	—	7.57E−08	—
	30	4.71E+03	×	6.38E−09	5.61	2.06E−09	4.66	2.46E−08	2.77
	40	8.39E+03	×	1.09E−09	6.13	4.72E−10	5.12	4.53E−09	5.88