In this test we will consider:

- $\psi(x) = \exp(x)$
- $\psi_l = 1;$
- $\psi_{ll} = 1;$
- $\psi_r = 1;$
- $\psi_{rr} = 1;$
- $g(x) = -\exp(x).$

Table 1: Test of 01_01 with d and d+1 ($\exp(x)$ — constant mesh).

| | $I$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | $\omega = 1\|1$ | | | | $\omega = 1\|3$ | | |
| $\mathbb{P}_3$ | 50 | 1.09E−05 | — | 1.72E−05 | — | 8.41E−06 | — | 1.36E−05 | — |
| | 100 | 1.34E−06 | 3.02 | 2.10E−06 | 3.03 | 1.02E−06 | 3.04 | 1.65E−06 | 3.04 |
| | 150 | 3.84E−07 | 3.08 | 5.99E−07 | 3.09 | 2.92E−07 | 3.10 | 4.68E−07 | 3.11 |
| | 200 | 1.57E−07 | 3.12 | 2.43E−07 | 3.14 | 1.17E−07 | 3.16 | 1.88E−07 | 3.16 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 2.46E−06 | — | 3.97E−06 | — | 1.55E−06 | — | 2.39E−06 | — |
| | 100 | 2.55E−07 | 3.27 | 4.01E−07 | 3.31 | 1.57E−07 | 3.31 | 2.20E−07 | 3.44 |
| | 150 | 6.01E−08 | 3.56 | 9.68E−08 | 3.51 | 3.33E−08 | 3.82 | 4.71E−08 | 3.80 |
| | 200 | 1.91E−08 | 3.99 | 3.38E−08 | 3.65 | 8.23E−09 | 4.86 | 1.53E−08 | 3.91 |
| $\mathbb{P}_4$ | 50 | 3.85E−07 | — | 7.68E−07 | — | 3.08E−07 | — | 6.06E−07 | — |
| | 100 | 1.11E−07 | 1.79 | 2.10E−07 | 1.87 | 9.29E−08 | 1.73 | 1.76E−07 | 1.78 |
| | 150 | 5.07E−08 | 1.93 | 9.53E−08 | 1.95 | 4.28E−08 | 1.91 | 8.06E−08 | 1.93 |
| | 200 | 2.88E−08 | 1.97 | 5.42E−08 | 1.96 | 2.44E−08 | 1.95 | 4.59E−08 | 1.95 |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 4.20E−07 | — | 8.21E−07 | — | 3.66E−07 | — | 6.95E−07 | — |
| | 100 | 1.14E−07 | 1.88 | 2.14E−07 | 1.94 | 9.70E−08 | 1.92 | 1.82E−07 | 1.93 |
| | 150 | 5.10E−08 | 1.98 | 9.59E−08 | 1.99 | 4.36E−08 | 1.97 | 8.19E−08 | 1.98 |
| | 200 | 2.89E−08 | 1.97 | 5.44E−08 | 1.97 | 2.46E−08 | 2.00 | 4.61E−08 | 2.00 |
| $\mathbb{P}_5$ | 50 | 1.51E−09 | — | 2.16E−09 | — | 9.32E−10 | — | 1.46E−09 | — |
| | 100 | 3.51E−11 | 5.42 | 5.35E−11 | 5.34 | 2.82E−11 | 5.05 | 4.55E−11 | 5.00 |
| | 150 | 1.03E−12 | 8.69 | 2.29E−12 | 7.77 | 4.31E−12 | 4.64 | 8.16E−12 | 4.24 |
| | 200 | 7.66E−12 | ↑ | 1.72E−11 | ↑ | 1.27E−11 | ↑ | 2.52E−11 | ↑ |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 7.22E−10 | — | 9.92E−10 | — | 6.77E−10 | — | 1.02E−09 | — |
| | 100 | 2.13E−11 | 5.09 | 3.29E−11 | 4.92 | 2.12E−11 | 4.99 | 3.32E−11 | 4.94 |
| | 150 | 1.51E−12 | 6.52 | 3.26E−12 | 5.70 | 5.91E−12 | 3.16 | 1.34E−11 | 2.24 |
| | 200 | 3.63E−12 | ↑ | 7.61E−12 | ↑ | 1.02E−11 | ↑ | 1.79E−11 | ↑ |

Table 2: Test of 01_01 with d and d+1 ($\exp(x)$ — non-constant mesh).

| | $I$ | $\omega = 1|1$ | | | | $\omega = 1|3$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
| $\mathbb{P}_3$ | 50 | 1.14E−05 | — | 1.79E−05 | — | 7.04E−06 | — | 1.05E−05 | — |
| | 100 | 1.30E−06 | 3.13 | 2.02E−06 | 3.15 | 6.72E−07 | 3.39 | 9.69E−07 | 3.44 |
| | 150 | 3.35E−07 | 3.35 | 5.18E−07 | 3.35 | 1.30E−07 | 4.05 | 2.12E−07 | 3.75 |
| | 200 | 1.18E−07 | 3.62 | 1.88E−07 | 3.53 | 2.65E−08 | 5.53 | 6.98E−08 | 3.85 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 1.95E−06 | — | 3.29E−06 | — | 1.14E−06 | — | 2.34E−06 | — |
| | 100 | 7.65E−08 | 4.67 | 2.33E−07 | 3.82 | 1.40E−07 | 3.03 | 3.01E−07 | 2.96 |
| | 150 | 5.94E−08 | 0.62 | 1.34E−07 | 1.37 | 1.03E−07 | 0.75 | 2.21E−07 | 0.76 |
| | 200 | 4.77E−08 | 0.77 | 1.03E−07 | 0.91 | 7.27E−08 | 1.22 | 1.50E−07 | 1.35 |
| $\mathbb{P}_4$ | 50 | 7.17E−08 | — | 1.61E−07 | — | 7.74E−08 | — | 1.31E−07 | — |
| | 100 | 3.09E−08 | 1.22 | 6.13E−08 | 1.39 | 2.80E−08 | 1.46 | 5.46E−08 | 1.26 |
| | 150 | 1.64E−08 | 1.55 | 3.16E−08 | 1.64 | 1.44E−08 | 1.65 | 2.75E−08 | 1.70 |
| | 200 | 1.01E−08 | 1.68 | 1.96E−08 | 1.66 | 8.52E−09 | 1.82 | 1.61E−08 | 1.84 |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 1.50E−07 | — | 3.32E−07 | — | 7.32E−08 | — | 1.47E−07 | — |
| | 100 | 3.93E−08 | 1.94 | 7.45E−08 | 2.16 | 3.03E−08 | 1.27 | 5.84E−08 | 1.33 |
| | 150 | 1.69E−08 | 2.08 | 3.24E−08 | 2.05 | 1.49E−08 | 1.75 | 2.83E−08 | 1.78 |
| | 200 | 1.46E−08 | 0.51 | 2.67E−08 | 0.68 | 8.69E−09 | 1.87 | 1.64E−08 | 1.89 |
| $\mathbb{P}_5$ | 50 | 2.80E−09 | — | 4.63E−09 | — | 2.25E−09 | — | 3.76E−09 | — |
| | 100 | 1.17E−10 | 4.59 | 2.01E−10 | 4.53 | 1.07E−10 | 4.40 | 1.87E−10 | 4.33 |
| | 150 | 1.88E−11 | 4.50 | 3.31E−11 | 4.44 | 1.94E−11 | 4.20 | 3.49E−11 | 4.14 |
| | 200 | 4.60E−12 | 4.89 | 8.42E−12 | 4.76 | 7.54E−12 | 3.29 | 1.35E−11 | 3.29 |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 2.33E−09 | — | 3.90E−09 | — | 2.37E−09 | — | 4.00E−09 | — |
| | 100 | 9.65E−11 | 4.59 | 1.68E−10 | 4.53 | 1.08E−10 | 4.46 | 1.89E−10 | 4.40 |
| | 150 | 1.44E−11 | 4.69 | 2.57E−11 | 4.64 | 1.74E−11 | 4.49 | 3.10E−11 | 4.47 |
| | 200 | 5.38E−12 | 3.42 | 9.70E−12 | 3.39 | 6.21E−12 | 3.58 | 1.08E−11 | 3.66 |

In this test we will consider:

- $\psi(x) = -\exp(x) + \mathbb{P}_3$

- $\psi_l = 0$;

- $\psi_{ll} = 0$;

- $\psi_r = 0$;

- $\psi_{rr} = 0$;

- $g(x) = \exp(x)$.

Table 3: Test of 01_01 with d and d+1 ($\exp(x) + \mathbb{P}_3$ — constant mesh).

| | $I$ | $\omega = 1\|1$ | | | | $\omega = 1\|3$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
| $\mathbb{P}_3$ | 50 | 1.09E−05 | — | 1.72E−05 | — | 8.41E−06 | — | 1.36E−05 | — |
| | 100 | 1.34E−06 | 3.02 | 2.10E−06 | 3.03 | 1.02E−06 | 3.04 | 1.65E−06 | 3.04 |
| | 150 | 3.84E−07 | 3.08 | 5.99E−07 | 3.09 | 2.92E−07 | 3.10 | 4.68E−07 | 3.11 |
| | 200 | 1.56E−07 | 3.12 | 2.43E−07 | 3.14 | 1.17E−07 | 3.16 | 1.88E−07 | 3.16 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 2.46E−06 | — | 3.97E−06 | — | 1.55E−06 | — | 2.39E−06 | — |
| | 100 | 2.55E−07 | 3.27 | 4.01E−07 | 3.31 | 1.57E−07 | 3.31 | 2.20E−07 | 3.44 |
| | 150 | 6.01E−08 | 3.56 | 9.68E−08 | 3.51 | 3.33E−08 | 3.82 | 4.71E−08 | 3.80 |
| | 200 | 1.91E−08 | 3.99 | 3.38E−08 | 3.65 | 8.23E−09 | 4.86 | 1.53E−08 | 3.91 |
| $\mathbb{P}_4$ | 50 | 3.85E−07 | — | 7.68E−07 | — | 3.08E−07 | — | 6.06E−07 | — |
| | 100 | 1.11E−07 | 1.79 | 2.10E−07 | 1.87 | 9.29E−08 | 1.73 | 1.76E−07 | 1.78 |
| | 150 | 5.07E−08 | 1.93 | 9.53E−08 | 1.95 | 4.28E−08 | 1.91 | 8.06E−08 | 1.93 |
| | 200 | 2.88E−08 | 1.97 | 5.42E−08 | 1.96 | 2.44E−08 | 1.96 | 4.58E−08 | 1.96 |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 4.20E−07 | — | 8.21E−07 | — | 3.66E−07 | — | 6.95E−07 | — |
| | 100 | 1.14E−07 | 1.88 | 2.14E−07 | 1.94 | 9.70E−08 | 1.92 | 1.82E−07 | 1.93 |
| | 150 | 5.10E−08 | 1.98 | 9.59E−08 | 1.99 | 4.36E−08 | 1.97 | 8.19E−08 | 1.98 |
| | 200 | 2.89E−08 | 1.97 | 5.44E−08 | 1.97 | 2.46E−08 | 1.99 | 4.62E−08 | 1.99 |
| $\mathbb{P}_5$ | 50 | 1.51E−09 | — | 2.16E−09 | — | 9.32E−10 | — | 1.46E−09 | — |
| | 100 | 3.58E−11 | 5.40 | 5.45E−11 | 5.31 | 2.95E−11 | 4.98 | 4.73E−11 | 4.94 |
| | 150 | 5.37E−12 | 4.68 | 8.57E−12 | 4.56 | 3.40E−12 | 5.34 | 5.39E−12 | 5.36 |
| | 200 | 4.18E−12 | 0.87 | 7.61E−12 | 0.41 | 1.79E−12 | 2.22 | 3.01E−12 | 2.03 |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 7.22E−10 | — | 9.92E−10 | — | 6.77E−10 | — | 1.02E−09 | — |
| | 100 | 1.88E−11 | 5.27 | 2.85E−11 | 5.12 | 2.08E−11 | 5.02 | 3.24E−11 | 4.98 |
| | 150 | 2.91E−12 | 4.60 | 4.68E−12 | 4.45 | 3.15E−12 | 4.66 | 5.10E−12 | 4.56 |
| | 200 | 2.57E−12 | 0.43 | 4.65E−12 | 0.02 | 6.22E−13 | 5.64 | 1.04E−12 | 5.52 |

Table 4: Test of 01_01 with d and d+1 ($\exp(x) + \mathbb{P}_3$ — non-constant mesh).

|  | $I$ | $\omega = 1|1$ | | | | $\omega = 1|3$ | | | |
|---|---|---|---|---|---|---|---|---|---|
|  |  | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
| $\mathbb{P}_3$ | 50 | 1.14E−05 | — | 1.79E−05 | — | 7.04E−06 | — | 1.05E−05 | — |
|  | 100 | 1.30E−06 | 3.13 | 2.02E−06 | 3.15 | 6.72E−07 | 3.39 | 9.69E−07 | 3.44 |
|  | 150 | 3.35E−07 | 3.35 | 5.18E−07 | 3.35 | 1.30E−07 | 4.05 | 2.12E−07 | 3.75 |
|  | 200 | 1.18E−07 | 3.62 | 1.88E−07 | 3.53 | 2.65E−08 | 5.53 | 6.98E−08 | 3.85 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 1.95E−06 | — | 3.29E−06 | — | 1.14E−06 | — | 2.34E−06 | — |
|  | 100 | 7.65E−08 | 4.67 | 2.33E−07 | 3.82 | 1.40E−07 | 3.03 | 3.01E−07 | 2.96 |
|  | 150 | 5.94E−08 | 0.62 | 1.34E−07 | 1.37 | 1.03E−07 | 0.75 | 2.21E−07 | 0.76 |
|  | 200 | 4.77E−08 | 0.77 | 1.03E−07 | 0.91 | 7.27E−08 | 1.22 | 1.50E−07 | 1.35 |
| $\mathbb{P}_4$ | 50 | 7.17E−08 | — | 1.61E−07 | — | 7.74E−08 | — | 1.31E−07 | — |
|  | 100 | 3.09E−08 | 1.22 | 6.13E−08 | 1.39 | 2.80E−08 | 1.46 | 5.46E−08 | 1.26 |
|  | 150 | 1.64E−08 | 1.55 | 3.16E−08 | 1.64 | 1.44E−08 | 1.65 | 2.75E−08 | 1.70 |
|  | 200 | 1.01E−08 | 1.68 | 1.96E−08 | 1.66 | 8.52E−09 | 1.82 | 1.61E−08 | 1.85 |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 1.50E−07 | — | 3.32E−07 | — | 7.32E−08 | — | 1.47E−07 | — |
|  | 100 | 3.93E−08 | 1.94 | 7.45E−08 | 2.16 | 3.03E−08 | 1.27 | 5.84E−08 | 1.33 |
|  | 150 | 1.69E−08 | 2.08 | 3.24E−08 | 2.05 | 1.49E−08 | 1.75 | 2.83E−08 | 1.78 |
|  | 200 | 1.46E−08 | 0.51 | 2.67E−08 | 0.68 | 8.69E−09 | 1.87 | 1.64E−08 | 1.89 |
| $\mathbb{P}_5$ | 50 | 2.80E−09 | — | 4.63E−09 | — | 2.25E−09 | — | 3.76E−09 | — |
|  | 100 | 1.17E−10 | 4.58 | 2.02E−10 | 4.52 | 1.07E−10 | 4.40 | 1.87E−10 | 4.33 |
|  | 150 | 1.87E−11 | 4.53 | 3.28E−11 | 4.48 | 1.90E−11 | 4.25 | 3.39E−11 | 4.20 |
|  | 200 | 6.09E−12 | 3.89 | 1.08E−11 | 3.85 | 4.71E−12 | 4.85 | 8.46E−12 | 4.83 |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 2.33E−09 | — | 3.90E−09 | — | 2.37E−09 | — | 4.00E−09 | — |
|  | 100 | 9.59E−11 | 4.60 | 1.67E−10 | 4.54 | 1.07E−10 | 4.47 | 1.89E−10 | 4.41 |
|  | 150 | 1.61E−11 | 4.39 | 2.89E−11 | 4.33 | 1.84E−11 | 4.35 | 3.29E−11 | 4.31 |
|  | 200 | 4.05E−12 | 4.81 | 7.24E−12 | 4.81 | 6.05E−12 | 3.86 | 1.10E−11 | 3.82 |

In this test we will consider:

- $\psi(x) = \sin(x)$
- $\psi_{\mathrm{l}} = 0$;
- $\psi_{\mathrm{ll}} = 1$;
- $\psi_{\mathrm{r}} = \sin(1)$;
- $\psi_{\mathrm{rr}} = \cos(1)$;
- $g(x) = -\sin(x)$.

Table 5: Test of 01_01 with d and d+1 ($\sin(x)$ — constant mesh).

| | $I$ | $\omega = 1\vert1$ | | | | $\omega = 1\vert3$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
| $\mathbb{P}_3$ | 50 | 2.73E−06 | — | 4.84E−06 | — | 2.30E−06 | — | 4.05E−06 | — |
| | 100 | 3.57E−07 | 2.94 | 6.30E−07 | 2.94 | 3.03E−07 | 2.92 | 5.34E−07 | 2.92 |
| | 150 | 1.10E−07 | 2.91 | 1.93E−07 | 2.91 | 9.42E−08 | 2.89 | 1.65E−07 | 2.89 |
| | 200 | 4.80E−08 | 2.87 | 8.44E−08 | 2.88 | 4.15E−08 | 2.85 | 7.28E−08 | 2.86 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 7.68E−07 | — | 1.39E−06 | — | 4.76E−07 | — | 8.59E−07 | — |
| | 100 | 1.08E−07 | 2.83 | 1.91E−07 | 2.86 | 7.47E−08 | 2.67 | 1.33E−07 | 2.69 |
| | 150 | 3.57E−08 | 2.73 | 6.29E−08 | 2.74 | 2.63E−08 | 2.57 | 4.68E−08 | 2.57 |
| | 200 | 1.67E−08 | 2.64 | 2.94E−08 | 2.64 | 1.28E−08 | 2.50 | 2.29E−08 | 2.49 |
| $\mathbb{P}_4$ | 50 | 1.11E−07 | — | 2.30E−07 | — | 1.03E−07 | — | 2.05E−07 | — |
| | 100 | 3.12E−08 | 1.83 | 5.98E−08 | 1.94 | 2.71E−08 | 1.93 | 5.17E−08 | 1.99 |
| | 150 | 1.42E−08 | 1.94 | 2.70E−08 | 1.96 | 1.22E−08 | 1.97 | 2.31E−08 | 1.98 |
| | 200 | 8.08E−09 | 1.97 | 1.53E−08 | 1.97 | 6.92E−09 | 1.97 | 1.31E−08 | 1.98 |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 1.19E−07 | — | 2.44E−07 | — | 9.83E−08 | — | 1.94E−07 | — |
| | 100 | 3.20E−08 | 1.90 | 6.09E−08 | 2.00 | 2.69E−08 | 1.87 | 5.13E−08 | 1.92 |
| | 150 | 1.43E−08 | 1.98 | 2.71E−08 | 1.99 | 1.22E−08 | 1.96 | 2.31E−08 | 1.97 |
| | 200 | 8.12E−09 | 1.97 | 1.54E−08 | 1.96 | 6.91E−09 | 1.97 | 1.31E−08 | 1.98 |
| $\mathbb{P}_5$ | 50 | 3.37E−10 | — | 5.16E−10 | — | 2.20E−10 | — | 3.75E−10 | — |
| | 100 | 6.59E−12 | 5.68 | 1.09E−11 | 5.56 | 6.92E−12 | 4.99 | 1.17E−11 | 5.01 |
| | 150 | 1.13E−12 | 4.35 | 1.90E−12 | 4.31 | 1.08E−12 | 4.59 | 1.90E−12 | 4.47 |
| | 200 | 6.04E−13 | 2.18 | 1.45E−12 | 0.94 | 4.82E−12 | ↑ | 1.18E−11 | ↑ |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 1.49E−10 | — | 2.39E−10 | — | 1.41E−10 | — | 2.37E−10 | — |
| | 100 | 2.88E−12 | 5.69 | 5.23E−12 | 5.52 | 3.53E−12 | 5.32 | 6.26E−12 | 5.24 |
| | 150 | 4.39E−13 | 4.64 | 8.58E−13 | 4.46 | 1.17E−12 | 2.73 | 2.50E−12 | 2.27 |
| | 200 | 3.73E−13 | 0.57 | 5.76E−13 | 1.38 | 1.14E−12 | 0.09 | 2.11E−12 | 0.59 |

Table 6: Test of 01_01 with d and d+1 ($\sin(x)$ — non-constant mesh).

| | $I$ | $\omega = 1|1$ | | | | $\omega = 1|3$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
| $\mathbb{P}_3$ | 50 | 1.99E−06 | — | 3.18E−06 | — | 1.66E−06 | — | 2.69E−06 | — |
| | 100 | 2.87E−07 | 2.79 | 4.71E−07 | 2.75 | 2.57E−07 | 2.69 | 4.26E−07 | 2.66 |
| | 150 | 1.00E−07 | 2.59 | 1.68E−07 | 2.54 | 9.33E−08 | 2.50 | 1.58E−07 | 2.45 |
| | 200 | 4.93E−08 | 2.47 | 8.39E−08 | 2.41 | 4.69E−08 | 2.39 | 8.06E−08 | 2.34 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 8.43E−07 | — | 1.44E−06 | — | 7.38E−07 | — | 1.31E−06 | — |
| | 100 | 1.59E−07 | 2.41 | 2.82E−07 | 2.35 | 1.50E−07 | 2.30 | 2.73E−07 | 2.26 |
| | 150 | 6.44E−08 | 2.23 | 1.16E−07 | 2.19 | 6.26E−08 | 2.16 | 1.15E−07 | 2.13 |
| | 200 | 3.46E−08 | 2.16 | 6.30E−08 | 2.13 | 3.41E−08 | 2.11 | 6.32E−08 | 2.08 |
| $\mathbb{P}_4$ | 50 | 3.03E−07 | — | 6.50E−07 | — | 2.79E−07 | — | 5.61E−07 | — |
| | 100 | 1.08E−07 | 1.48 | 2.08E−07 | 1.65 | 9.37E−08 | 1.57 | 1.79E−07 | 1.65 |
| | 150 | 5.15E−08 | 1.83 | 9.74E−08 | 1.87 | 4.41E−08 | 1.86 | 8.33E−08 | 1.88 |
| | 200 | 2.99E−08 | 1.89 | 5.66E−08 | 1.88 | 2.54E−08 | 1.93 | 4.77E−08 | 1.94 |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 4.29E−07 | — | 8.48E−07 | — | 3.23E−07 | — | 6.31E−07 | — |
| | 100 | 1.18E−07 | 1.86 | 2.22E−07 | 1.93 | 9.73E−08 | 1.73 | 1.84E−07 | 1.78 |
| | 150 | 5.22E−08 | 2.01 | 9.85E−08 | 2.01 | 4.49E−08 | 1.91 | 8.45E−08 | 1.92 |
| | 200 | 3.41E−08 | 1.47 | 6.30E−08 | 1.55 | 2.56E−08 | 1.95 | 4.81E−08 | 1.96 |
| $\mathbb{P}_5$ | 50 | 2.20E−10 | — | 3.18E−10 | — | 1.69E−10 | — | 2.60E−10 | — |
| | 100 | 3.96E−12 | 5.79 | 6.72E−12 | 5.57 | 4.98E−12 | 5.08 | 9.47E−12 | 4.78 |
| | 150 | 7.33E−13 | 4.16 | 1.72E−12 | 3.36 | 2.48E−12 | 1.72 | 4.76E−12 | 1.70 |
| | 200 | 5.79E−12 | ↑ | 1.10E−11 | ↑ | 5.93E−12 | ↑ | 1.29E−11 | ↑ |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 1.55E−10 | — | 2.23E−10 | — | 1.60E−10 | — | 2.45E−10 | — |
| | 100 | 2.00E−12 | 6.28 | 4.15E−12 | 5.75 | 2.26E−12 | 6.15 | 4.67E−12 | 5.71 |
| | 150 | 3.96E−13 | 4.00 | 9.29E−13 | 3.69 | 5.67E−12 | ↑ | 1.13E−11 | ↑ |
| | 200 | 4.07E−12 | ↑ | 7.61E−12 | ↑ | 3.66E−12 | 1.52 | 7.71E−12 | 1.33 |

In this test we will consider:

- $\psi(x) = x^4$

- $\psi_l = 0$;

- $\psi_{ll} = 0$;

- $\psi_r = 1$;

- $\psi_{rr} = 4$;

- $g(x) = -24$.

Table 7: Test of 01_01 with d and d+1 ($x^4$ — constant mesh).

| | $I$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | $\omega = 1\|1$ | | | | $\omega = 1\|3$ | | |
| $\mathbb{P}_3$ | 50 | 1.47E−04 | — | 2.22E−04 | — | 1.08E−04 | — | 1.65E−04 | — |
| | 100 | 1.86E−05 | 2.98 | 2.80E−05 | 2.99 | 1.36E−05 | 2.98 | 2.07E−05 | 2.99 |
| | 150 | 5.54E−06 | 2.99 | 8.33E−06 | 2.99 | 4.06E−06 | 2.99 | 6.15E−06 | 3.00 |
| | 200 | 2.34E−06 | 2.99 | 3.52E−06 | 3.00 | 1.72E−06 | 2.99 | 2.60E−06 | 3.00 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 3.71E−05 | — | 5.71E−05 | — | 2.92E−05 | — | 4.67E−05 | — |
| | 100 | 4.56E−06 | 3.02 | 6.92E−06 | 3.04 | 3.81E−06 | 2.94 | 5.97E−06 | 2.97 |
| | 150 | 1.34E−06 | 3.03 | 2.02E−06 | 3.04 | 1.15E−06 | 2.97 | 1.78E−06 | 2.98 |
| | 200 | 5.61E−07 | 3.02 | 8.46E−07 | 3.03 | 4.87E−07 | 2.98 | 7.54E−07 | 2.99 |
| $\mathbb{P}_4$ | 50 | 2.53E−14 | — | 7.51E−14 | — | 9.54E−14 | — | 2.50E−13 | — |
| | 100 | 3.36E−13 | ↑ | 8.20E−13 | ↑ | 1.37E−11 | ↑ | 2.54E−11 | ↑ |
| | 150 | 2.57E−12 | ↑ | 5.02E−12 | ↑ | 7.51E−12 | 1.49 | 1.85E−11 | 0.78 |
| | 200 | 5.60E−12 | ↑ | 1.16E−11 | ↑ | 3.18E−10 | ↑ | 6.12E−10 | ↑ |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 9.08E−14 | — | 1.83E−13 | — | 4.22E−13 | — | 8.47E−13 | — |
| | 100 | 2.42E−12 | ↑ | 4.59E−12 | ↑ | 1.56E−11 | ↑ | 3.01E−11 | ↑ |
| | 150 | 4.54E−12 | ↑ | 8.92E−12 | ↑ | 5.89E−12 | 2.41 | 1.48E−11 | 1.74 |
| | 200 | 1.50E−11 | ↑ | 2.85E−11 | ↑ | 4.64E−11 | ↑ | 1.01E−10 | ↑ |
| $\mathbb{P}_5$ | 50 | 3.22E−13 | — | 5.87E−13 | — | 5.88E−13 | — | 1.13E−12 | — |
| | 100 | 1.38E−12 | ↑ | 2.58E−12 | ↑ | 8.40E−12 | ↑ | 1.55E−11 | ↑ |
| | 150 | 2.86E−12 | ↑ | 7.08E−12 | ↑ | 8.22E−12 | 0.05 | 1.91E−11 | ↑ |
| | 200 | 9.51E−12 | ↑ | 1.67E−11 | ↑ | 2.18E−11 | ↑ | 4.34E−11 | ↑ |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 4.43E−13 | — | 8.42E−13 | — | 1.45E−13 | — | 3.63E−13 | — |
| | 100 | 1.48E−12 | ↑ | 2.93E−12 | ↑ | 2.75E−13 | ↑ | 5.41E−13 | ↑ |
| | 150 | 5.15E−12 | ↑ | 1.13E−11 | ↑ | 7.38E−12 | ↑ | 1.84E−11 | ↑ |
| | 200 | 2.05E−12 | 3.20 | 4.53E−12 | 3.18 | 1.98E−11 | ↑ | 3.85E−11 | ↑ |

Table 8: Test of 01_01 with d and d+1 ($x^4$ — non-constant mesh).

| | $I$ | $\omega = 1\|1$ | | | | $\omega = 1\|3$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ | $E_1$ | $\mathcal{O}_1$ | $E_\infty$ | $\mathcal{O}_\infty$ |
| $\mathbb{P}_3$ | 50 | 1.96E−04 | — | 3.38E−04 | — | 1.02E−04 | — | 1.73E−04 | — |
| | 100 | 2.03E−05 | 3.27 | 3.66E−05 | 3.20 | 5.90E−06 | 4.11 | 1.52E−05 | 3.51 |
| | 150 | 4.32E−06 | 3.82 | 9.03E−06 | 3.45 | 1.85E−06 | 2.86 | 3.41E−06 | 3.69 |
| | 200 | 1.08E−06 | 4.83 | 3.20E−06 | 3.61 | 1.50E−06 | 0.73 | 3.21E−06 | 0.21 |
| $\mathbb{P}_3/\mathbb{P}_4$ | 50 | 1.74E−05 | — | 5.22E−05 | — | 1.38E−05 | — | 3.48E−05 | — |
| | 100 | 5.05E−06 | 1.79 | 1.11E−05 | 2.23 | 7.80E−06 | 0.82 | 1.65E−05 | 1.08 |
| | 150 | 3.31E−06 | 1.04 | 6.89E−06 | 1.17 | 4.55E−06 | 1.33 | 9.22E−06 | 1.44 |
| | 200 | 2.20E−06 | 1.42 | 4.44E−06 | 1.53 | 2.90E−06 | 1.57 | 5.74E−06 | 1.65 |
| $\mathbb{P}_4$ | 50 | 9.79E−14 | — | 1.86E−13 | — | 3.31E−13 | — | 6.33E−13 | — |
| | 100 | 2.92E−13 | ↑ | 5.95E−13 | ↑ | 1.49E−12 | ↑ | 2.86E−12 | ↑ |
| | 150 | 6.92E−13 | ↑ | 1.74E−12 | ↑ | 6.43E−12 | ↑ | 1.28E−11 | ↑ |
| | 200 | 2.41E−12 | ↑ | 5.94E−12 | ↑ | 1.15E−11 | ↑ | 2.73E−11 | ↑ |
| $\mathbb{P}_4/\mathbb{P}_5$ | 50 | 5.64E−14 | — | 1.23E−13 | — | 6.20E−14 | — | 1.15E−13 | — |
| | 100 | 6.94E−13 | ↑ | 1.34E−12 | ↑ | 2.13E−12 | ↑ | 4.26E−12 | ↑ |
| | 150 | 1.60E−12 | ↑ | 3.41E−12 | ↑ | 5.21E−12 | ↑ | 1.12E−11 | ↑ |
| | 200 | 2.39E−12 | ↑ | 4.90E−12 | ↑ | 2.72E−11 | ↑ | 5.53E−11 | ↑ |
| $\mathbb{P}_5$ | 50 | 9.79E−14 | — | 1.57E−13 | — | 2.35E−13 | — | 4.91E−13 | — |
| | 100 | 2.09E−12 | ↑ | 4.13E−12 | ↑ | 4.17E−12 | ↑ | 8.43E−12 | ↑ |
| | 150 | 2.94E−12 | ↑ | 5.22E−12 | ↑ | 6.53E−12 | ↑ | 1.34E−11 | ↑ |
| | 200 | 1.15E−11 | ↑ | 2.48E−11 | ↑ | 2.09E−11 | ↑ | 4.12E−11 | ↑ |
| $\mathbb{P}_5/\mathbb{P}_6$ | 50 | 9.42E−14 | — | 1.69E−13 | — | 6.24E−13 | — | 1.21E−12 | — |
| | 100 | 2.25E−12 | ↑ | 4.30E−12 | ↑ | 3.95E−12 | ↑ | 7.44E−12 | ↑ |
| | 150 | 5.83E−13 | 3.33 | 1.60E−12 | 2.44 | 1.37E−11 | ↑ | 2.71E−11 | ↑ |
| | 200 | 2.71E−12 | ↑ | 6.11E−12 | ↑ | 1.97E−11 | ↑ | 4.26E−11 | ↑ |