Lexical Analysis

Masters in Informatics and Computing Engineering
(MIEIC), 3rd Year

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Formal Languages

- Natural Languages
  - Ambiguous
    - Problem in the language processing
    - Context dependence allows shorter messages
- Formal (artificial) Languages
  - Obey to rules specified rigorously using appropriate formalisms
  - Rules guarantee that the languages are not ambiguous

Definition of Formal Languages

- Necessity to define precisely a language
- Definition of the languages structured in layers
  - Start by the set of the symbols of the language (the alphabet, Σ)
  - **Lexical structure** – identifies "words" of the language (each word is a sequence of symbols)
  - **Syntactic structure** – identifies "sentences" in the language (each sentence is a sequence of words)
  - **Semantic** – meaning of the program (specifies the results that should be output for the inputs)

Formal Specification of Languages

- Regular expressions (generative method)
  - There exist cases not possible to describe using regular expressions
- Finite Automata (method by recognition)
  - Non-Deterministic (NFAs)
  - Deterministic (DFAs)
  - Implement any regular expression

Specification of Lexemas Using Regular Expressions (REs)

- Given an alphabet Σ = set of symbols
- Regular Expressions are built with:
  - ε - emprty string
  - Any symbol from alphabet Σ
  - $r_1 r_2$ – RE r_1 followed by RE r_2 : concatenation (sometimes we use '.')
  - $r_1 \mid r_2$ – RE r_1 or RE r_2 (OR)
  - r* - Kleene start: ε | r | rr | …
  - Parentesis to indicate precedences
    - Priority: *, ., |

Regular Expressions (REs)

- Generation of the strings of the language represented by an RE:
  - Rewrite the RE until we have a sequence of alphabet symbols (string)
  - Different application of the rules can conduct to different results

General Rules

1) $r_1 | r_2 \rightarrow r_1$
2) $r_1 | r_2 \rightarrow r_2$
3) $r^* \rightarrow rr^*$
4) $r^* \rightarrow \varepsilon$

Example 1

(0 | 1)*”.”(0 | 1)*
(0 | 1)(0 | 1)*”.”(0 | 1)*
1(0 | 1)*”.”(0 | 1)*
1”.”(0 | 1)*
1”.”(0 | 1)(0 | 1)*
1”.”(0 | 1)
1”.”0

Example 2

(0 | 1)*”.”(0 | 1)*
(0 | 1)(0 | 1)*”.”(0 | 1)*
0(0 | 1)*”.”(0 | 1)*
0”.”(0 | 1)*
0”.”(0 | 1)(0 | 1)*
0”.”(0 | 1)
0”.”1

Language Generated by a Regular Expression

- Set of all the strings generated by the regular expression is a language of regular expressions
- In general, a language can be infinite
- A String of the language is known as token

Regular Languages

- $\Sigma = \{0, 1, \text{"."}\}$
  - (0 | 1)*"."(0 | 1)* - binary numbers with integer and fractional part (representing real numbers)
- $\Sigma = \{0\}$
  - (00)* - sequences of 0's with even length
- $\Sigma = \{0, 1\}$
  - (1*01*01*)* - strings in the alphabet {0,1} with an even number of 0's
- $\Sigma = \{a, b, c, 0, 1, 2\}$
  - (a | b | c)(a | b | c | 0 | 1 | 2)* - alphanumeric identifiers
- $\Sigma = \{0, 1, 2\}$
  - (0|1|2)* - ternary numbers

Regular Expressions

- Other constructs:
  - r^+ - one or more occorrunces of r: r | rr | rrr ...
    - Equivalent to: $r.r^*$
  - $r^?$ – zero or one occurrence of r: (r | ε)
  - [ ] – symbol classes:
    - [ac] is the same as: (a | c)
    - [a-c] is the same as: (a | b | c)
    - [a-c0-2] is the same as: (a | b | c | 0 | 1 | 2)

Regular Expressions

- Exercises
  - Specify the language of the integers
  - Specify the language of the identifiers (a letter followed by zero or more letters/numbers)
  - Enumerate algebraic properties of regular expressions
  - Give examples of languages that cannot be specified by regular expressions

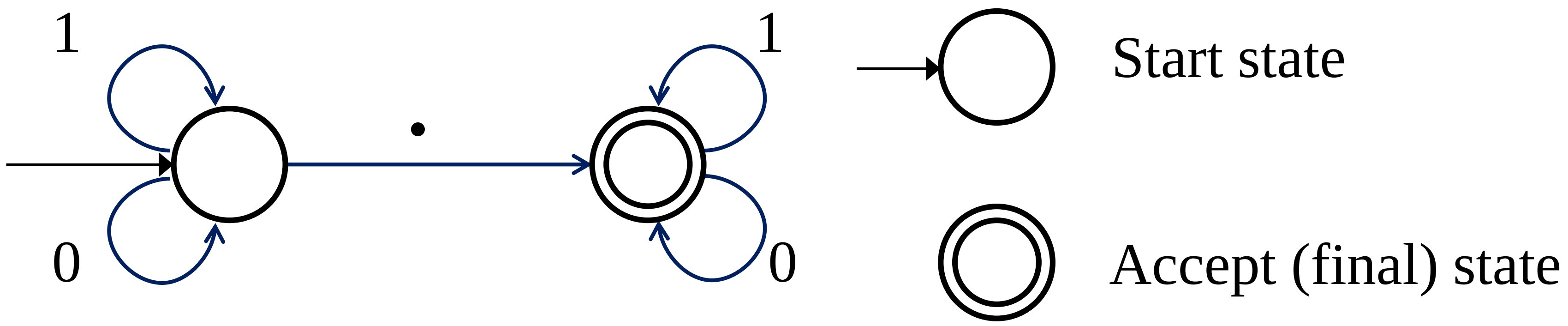
Finite Automata (FAs)

- Set of states
  - 1 start state
  - 1 or more final states (or accepting states)
- Alphabet of symbols: Σ (it can include the empty string symbol: ε)
- Transitions between states is triggered by the occurrence of a symbol of the alphabet
- Transitions are labeled with symbols

Finite Automata (FAs)

- Example:

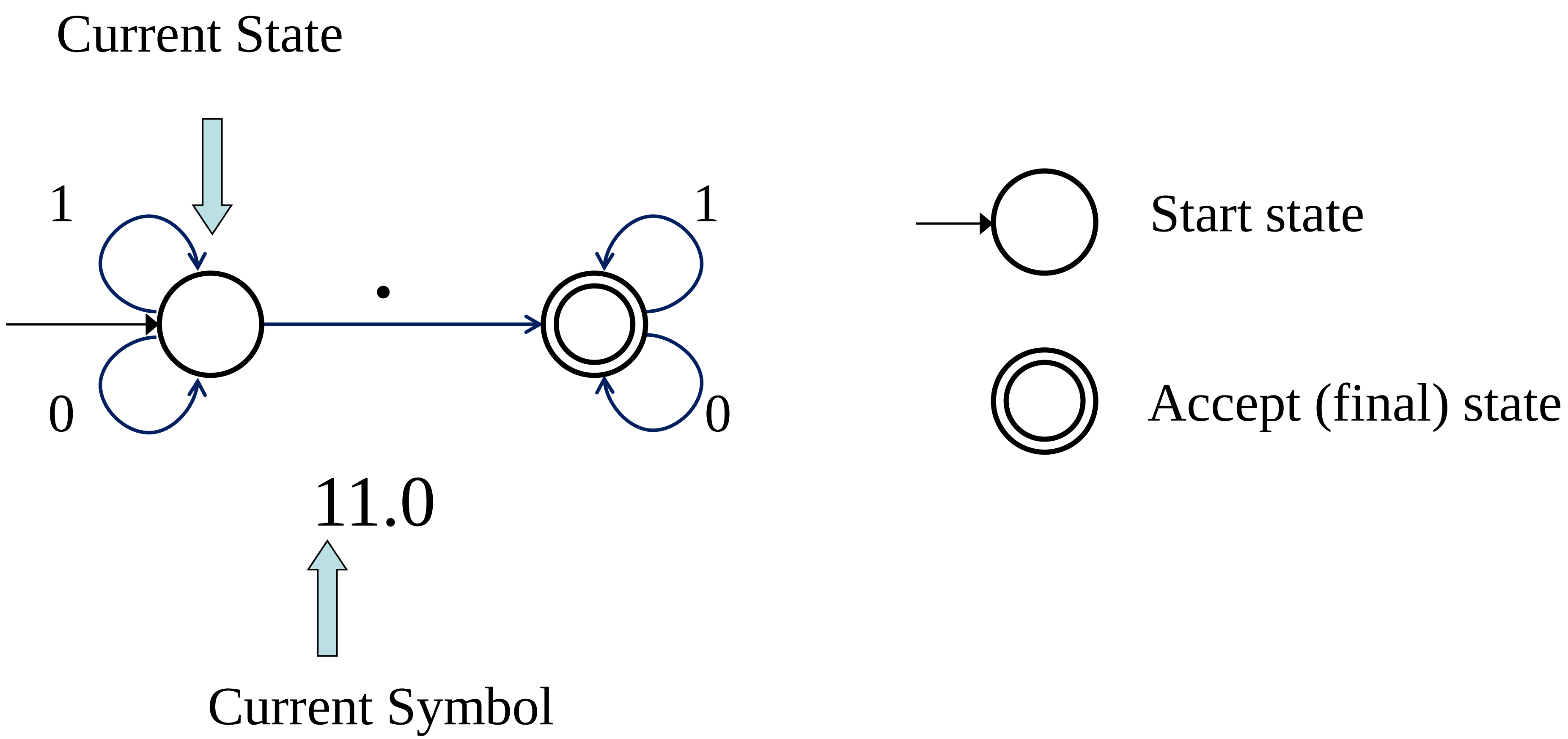
(0 | 1)*"."(0 | 1)*



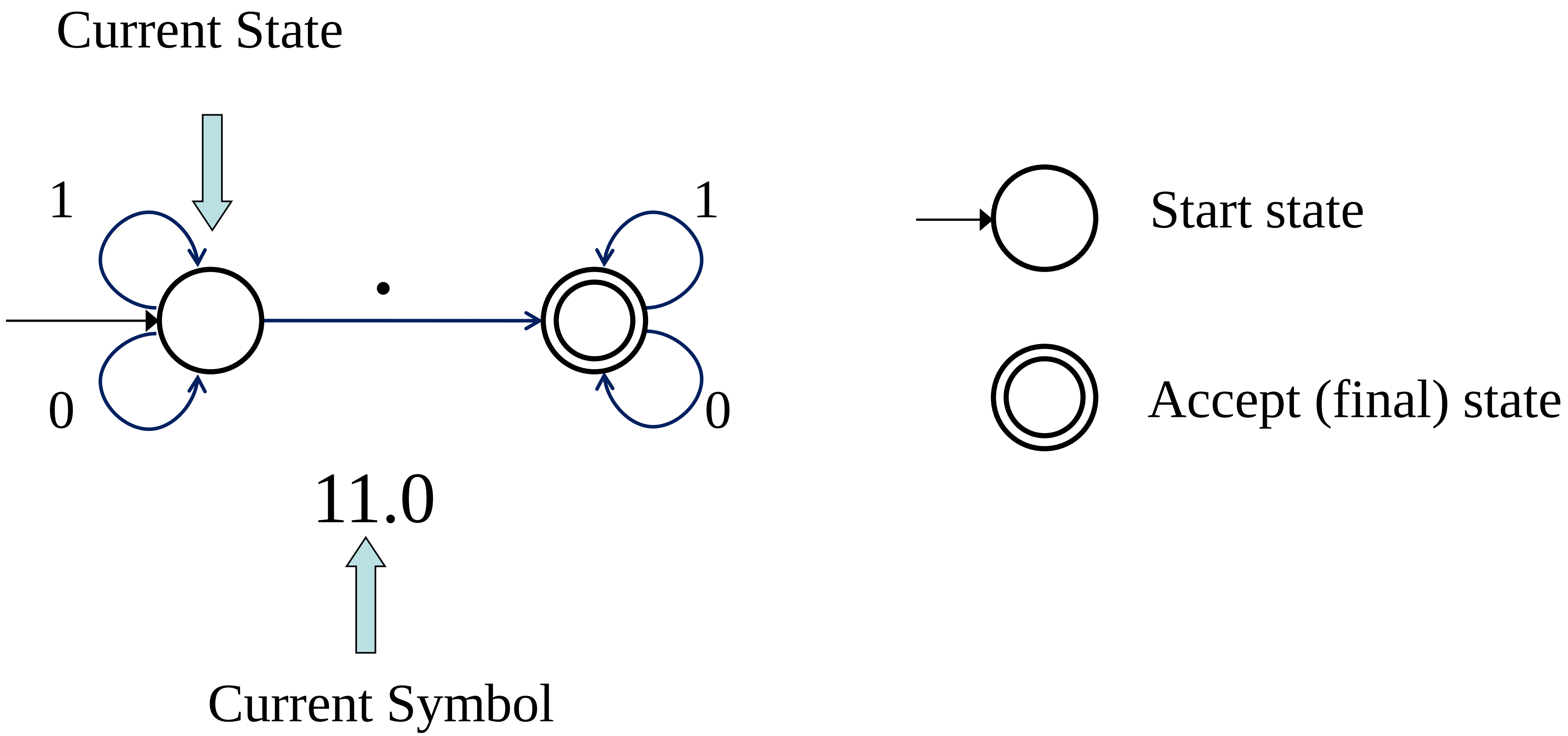
Accepting a String

- Recognition through the execution of the automaton
  - Start with the start state and with the first symbol of the string
  - Store current state and the current symbol of the string
  - In each step, match the current symbol with the transition labeled with that symbol
  - Continue until the end of the string or until the match fails
  - If the state after processing the last symbol of the string is a final state, then the string is accepted
  - The language of the automaton consists of the strings it accepts

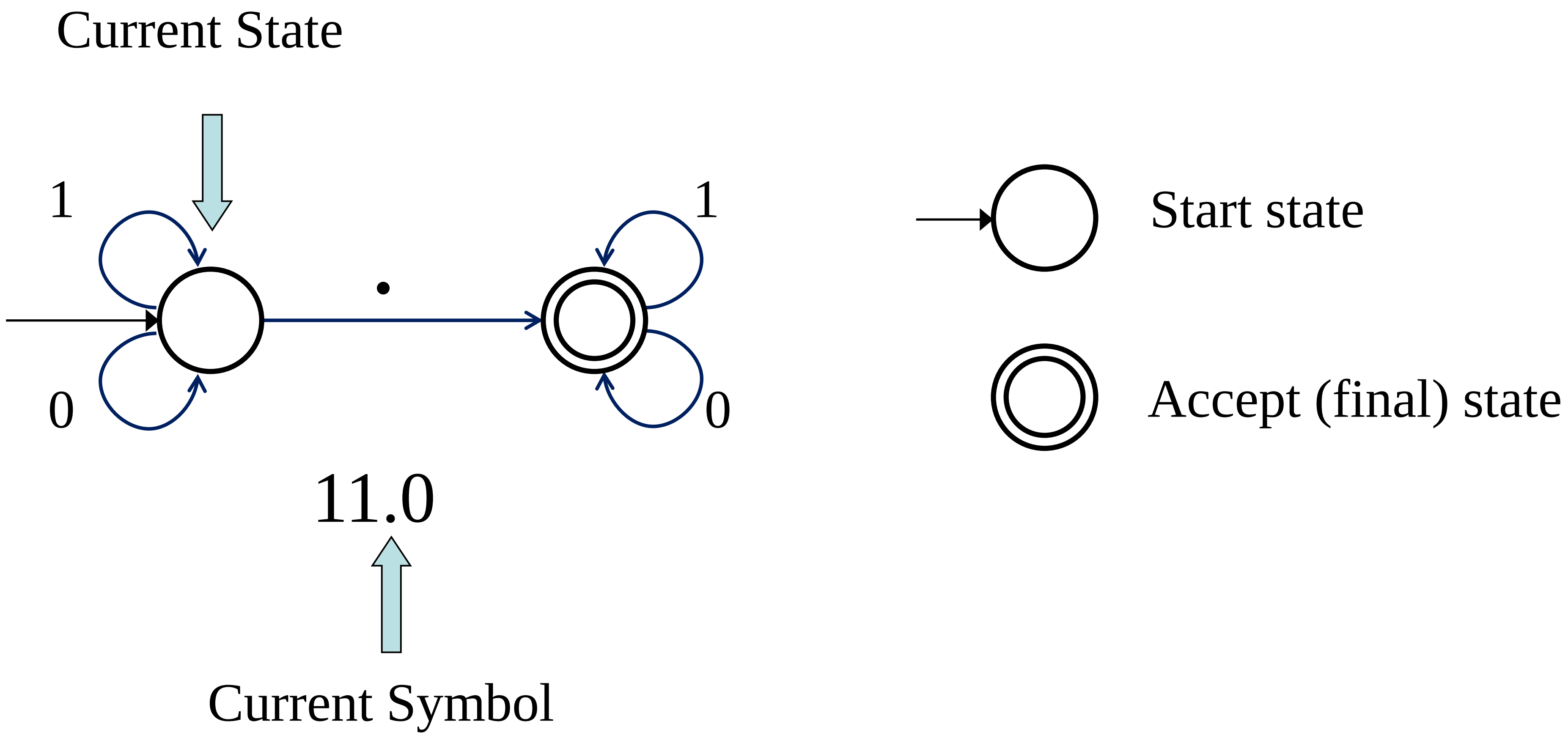
Example


Current State
1
1
•
0
0
11.0
Current Symbol
Start state
Accept (final) state

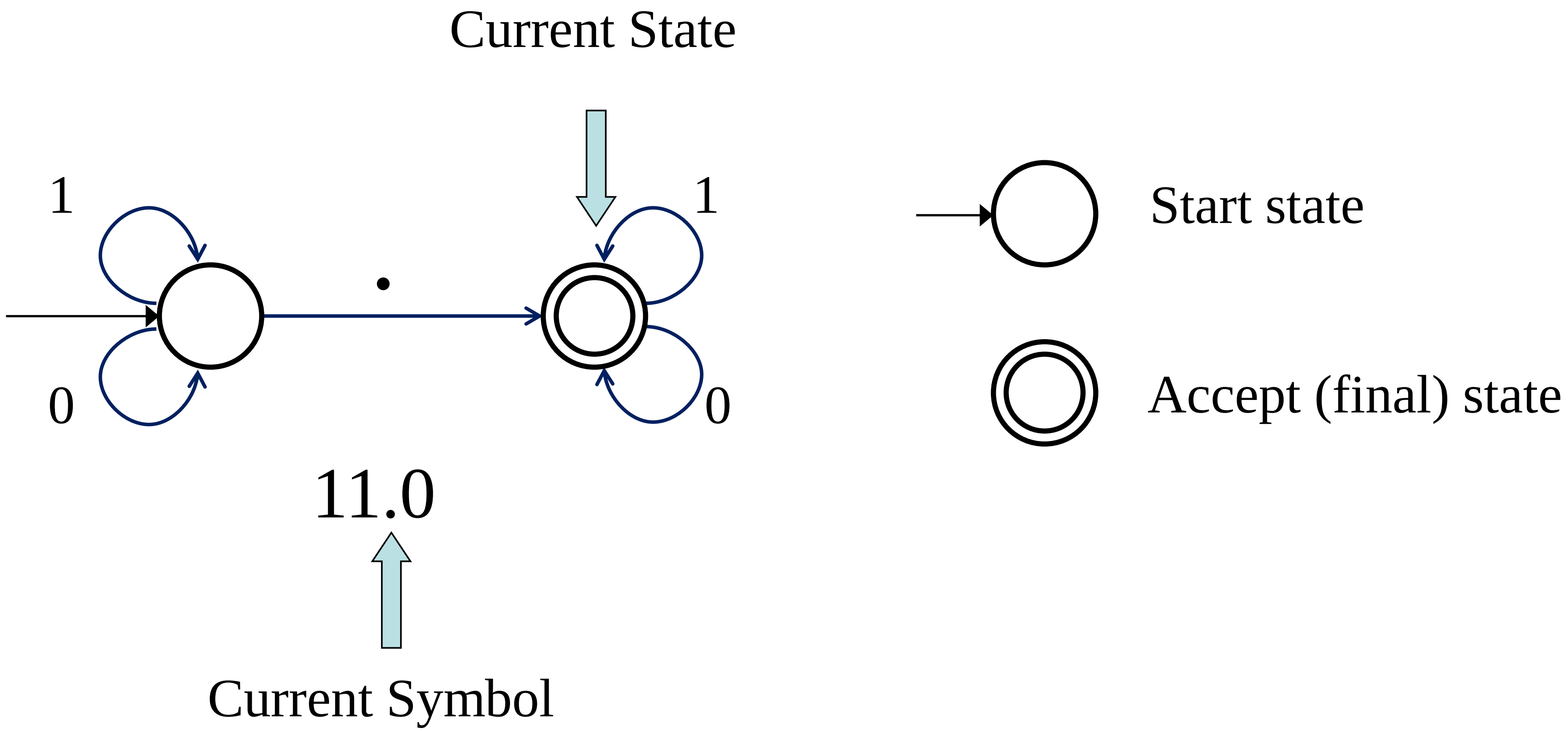
Example


Current State
1
1
.
0
0
11.0
Current Symbol
Start state
Accept (final) state

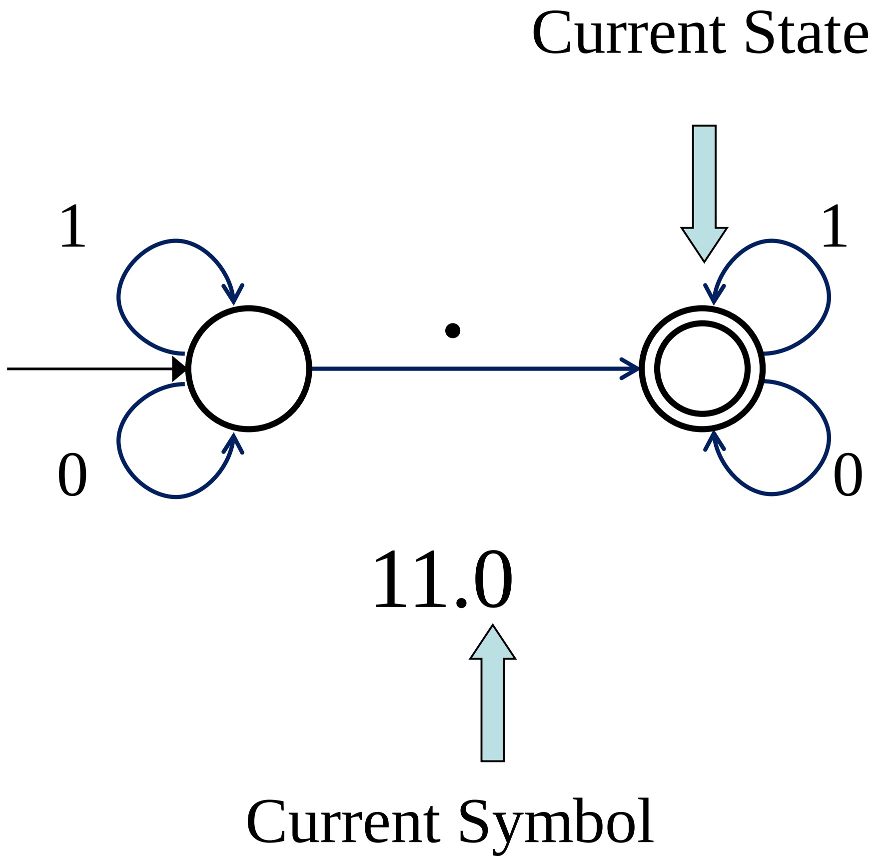
Example


Current State
1
1
.
0
0
11.0
Current Symbol
Start state
Accept (final) state

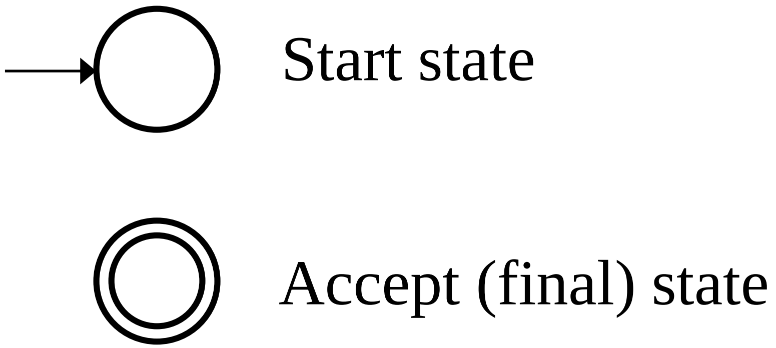
Example


Current State
1
1
.
0
0
11.0
Current Symbol
Start state
Accept (final) state

Example

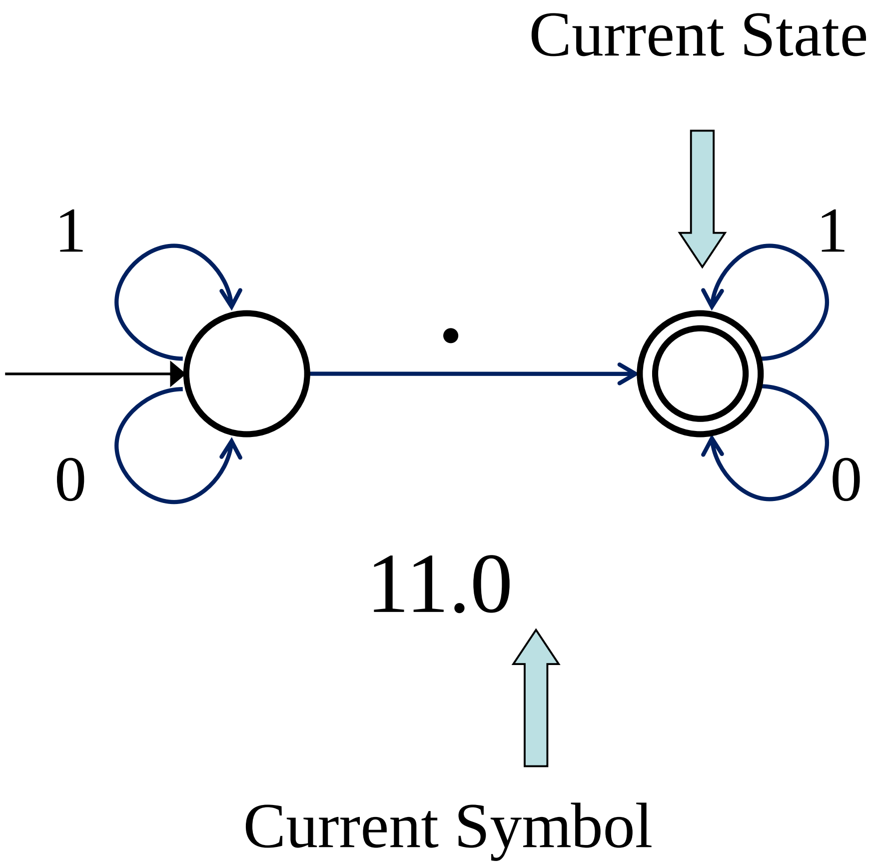


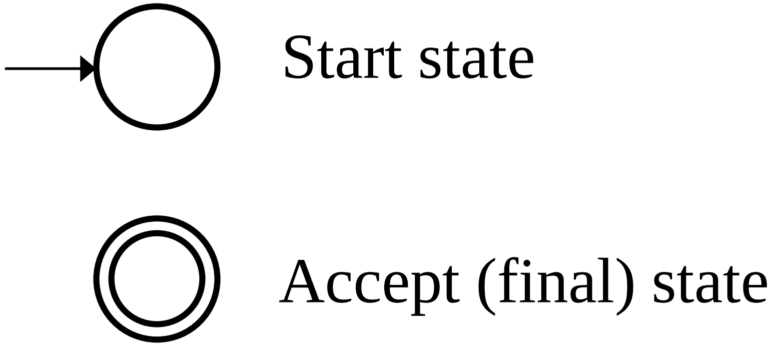
Current State
1
1
.
0
0
11.0
Current Symbol



Start state
Accept (final) state

Example





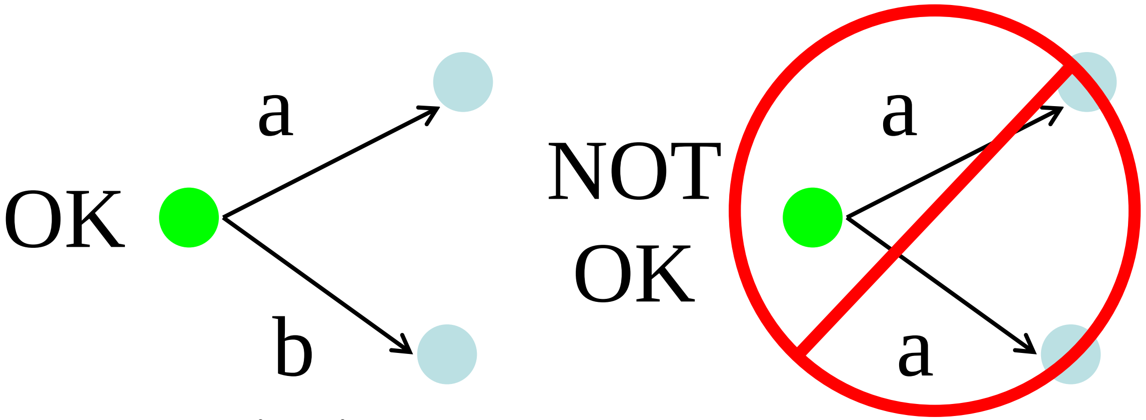
String accepted!

Finite Automata (FAs)

- NFA: Non Deterministic Finite Automata
  - A state may have more than one output transition labeled with the same symbol (the same occurence can lead to different states)
- DFA: Deterministic Finite Automata
  - The occurrence of a symbol cannot lead to different states
- NFAs may have ε transitions (sometimes these FAs are called ε-NFAs)
- In DFAs the input string is always fully processed - the execution of the DFA only finishes after matching the last input symbol

NFA vs DFA

- DFA
  - Without ε transitions
  - A maximum of one transition from each state for each symbol



- NFA – none of these restrictions

Finite Automata (FAs)

- Deterministic Finite Automata (DFAs)
  - Faster execution than NFAs, but
  - More complexity of the automaton (usually!)

Generative vs Recognize

- Regular expressions are a mechanism to generate the strings of a language
- Finite automata (FAs) are a mechanism to recognize if a string belongs to the language
- Standard approach
  - Use regular expressions when defining the language (regular languages), usually the lexemes of a programming language
  - Translation of the regular expressions to FAs to implement the lexical analysis

From the Regular Expression to the FA

- Translating the regular expression to an FA
  - Construction using induction on the structure
  - Given an arbitrary regular expression **r**,
  - Assume we can convert it to an automaton with
    - one start state
    - one accept state
  - Using the method of **Thompson-McNaughton-Yamada** (aka **Thompson construction**)
- Implementation of the FA

Thompson-McNaughton-Yamada Method (aka **Thompson construction**)

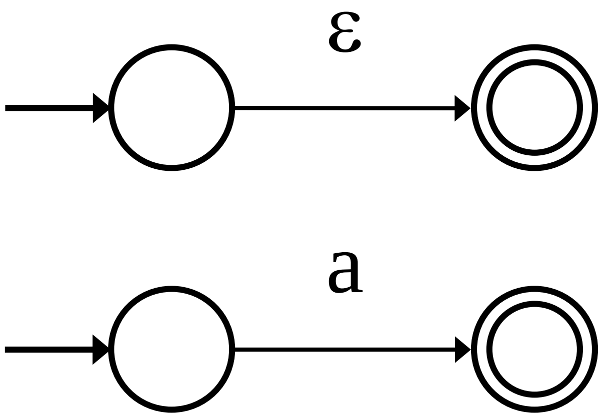
FROM REGULAR EXPRESSIONS TO FAS

Basic Constructs

- Empty expression and a symbol

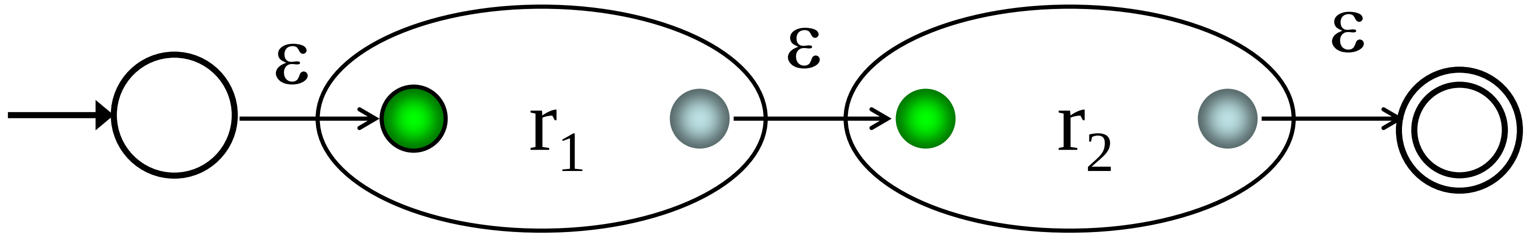
ε

$a \in \Sigma$



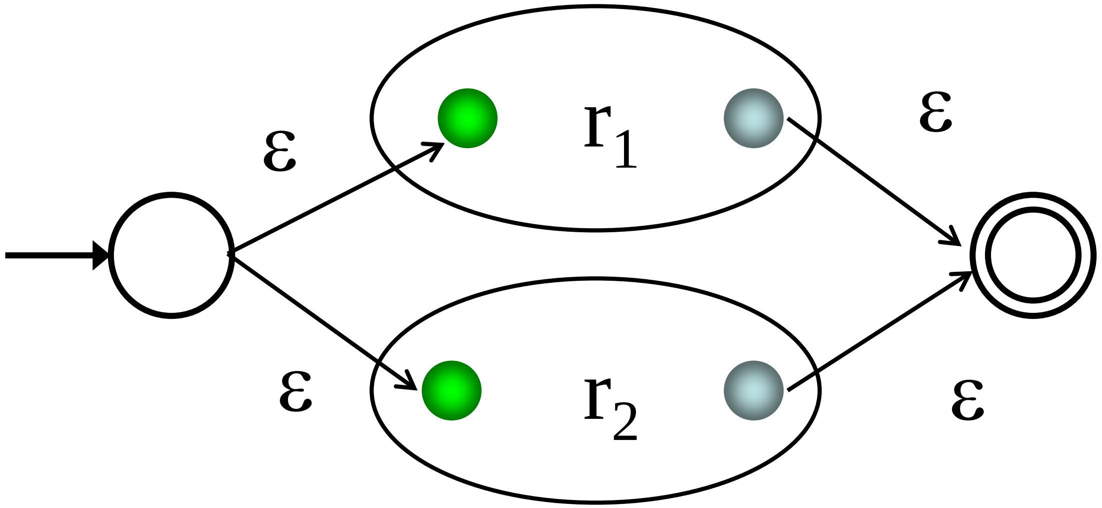
Concatenation

- r1.r2



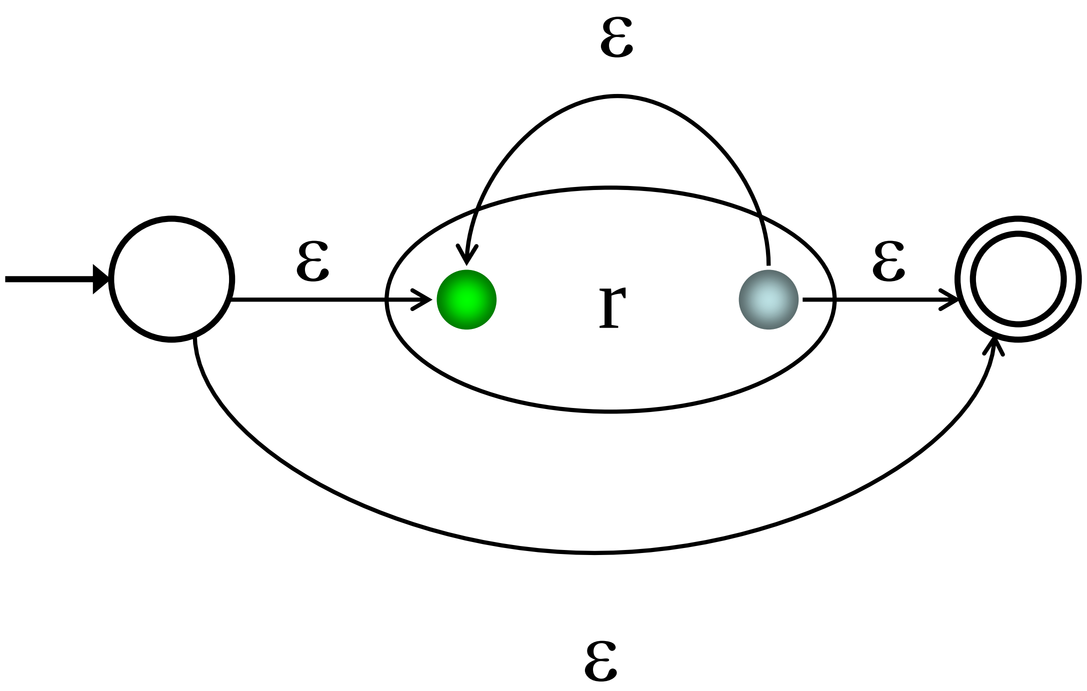
Union

- r1 | r2

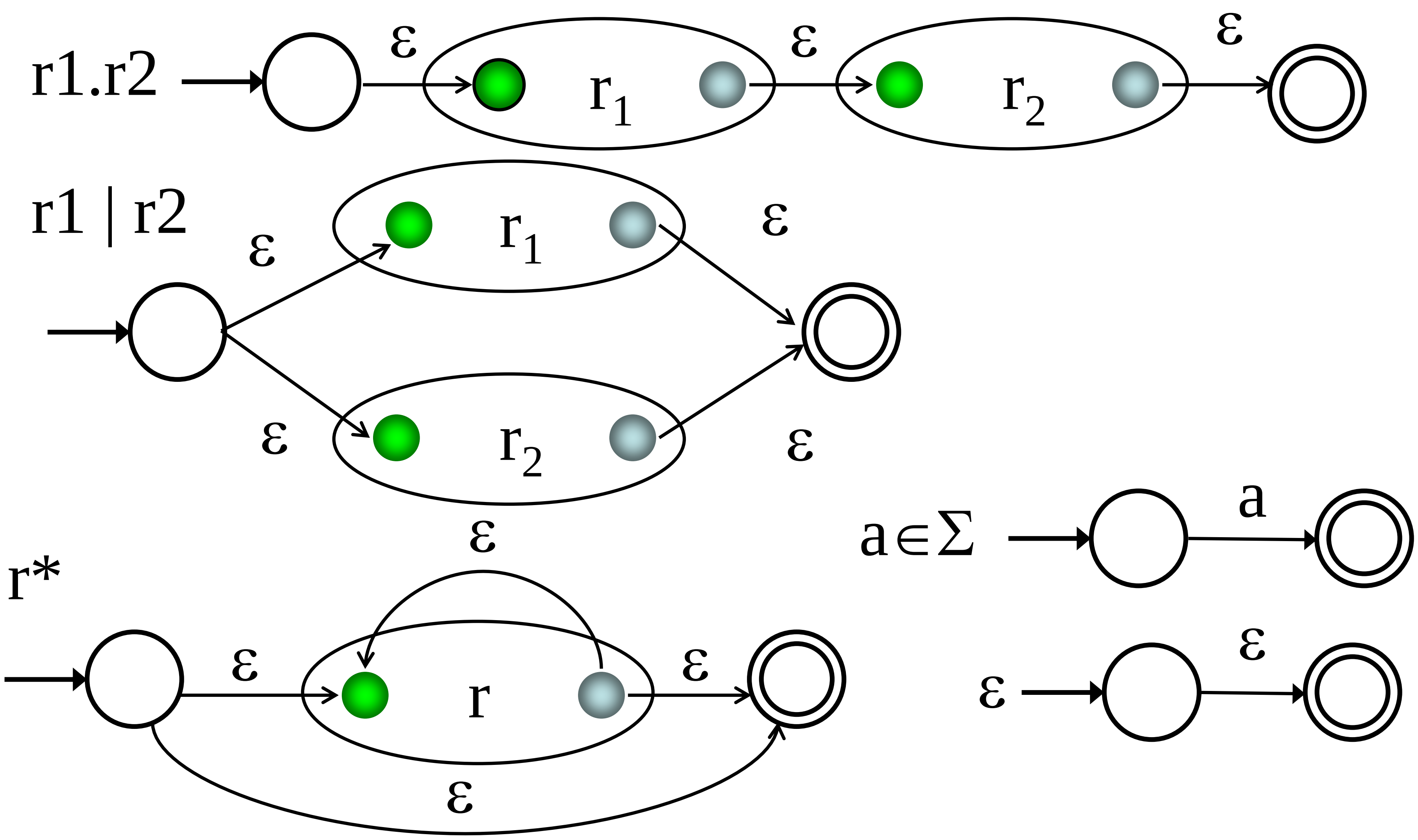


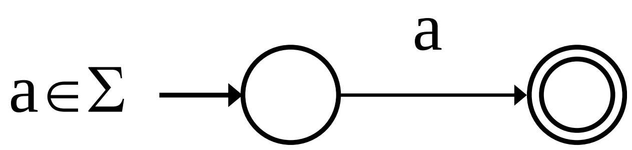
Kleene Star

- r*



Conversion Rules

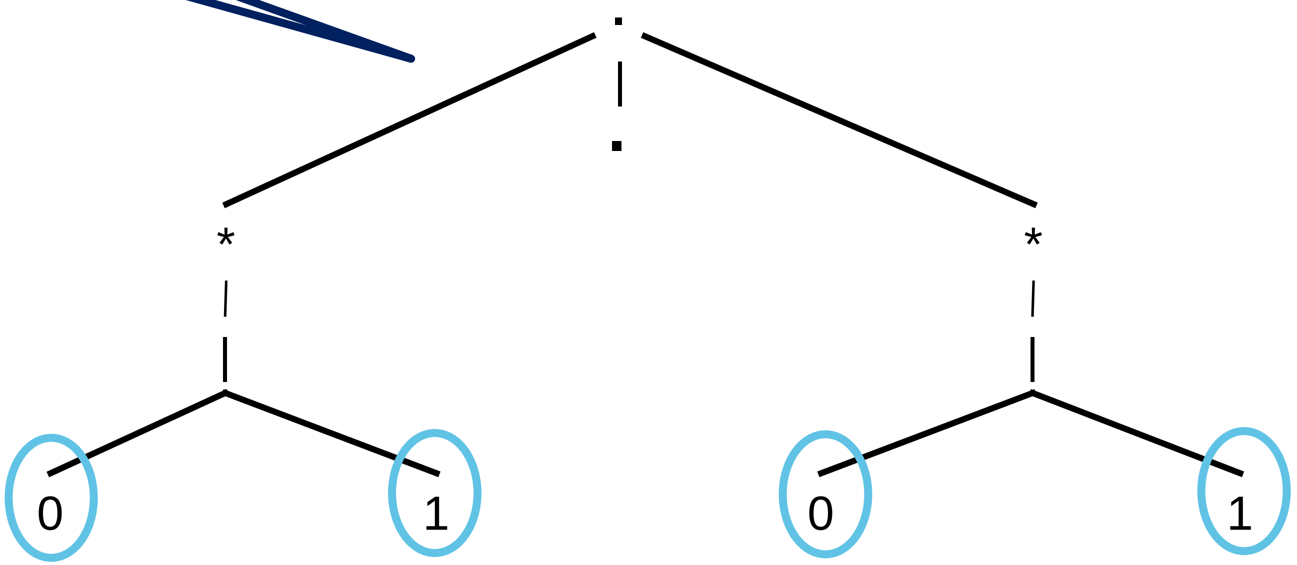


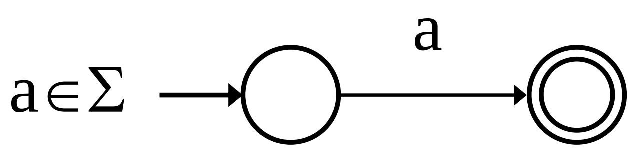


RE to DFA

- Example: (0 | 1)*.(0 | 1)*

Tree representing the input RE





RE to DFA

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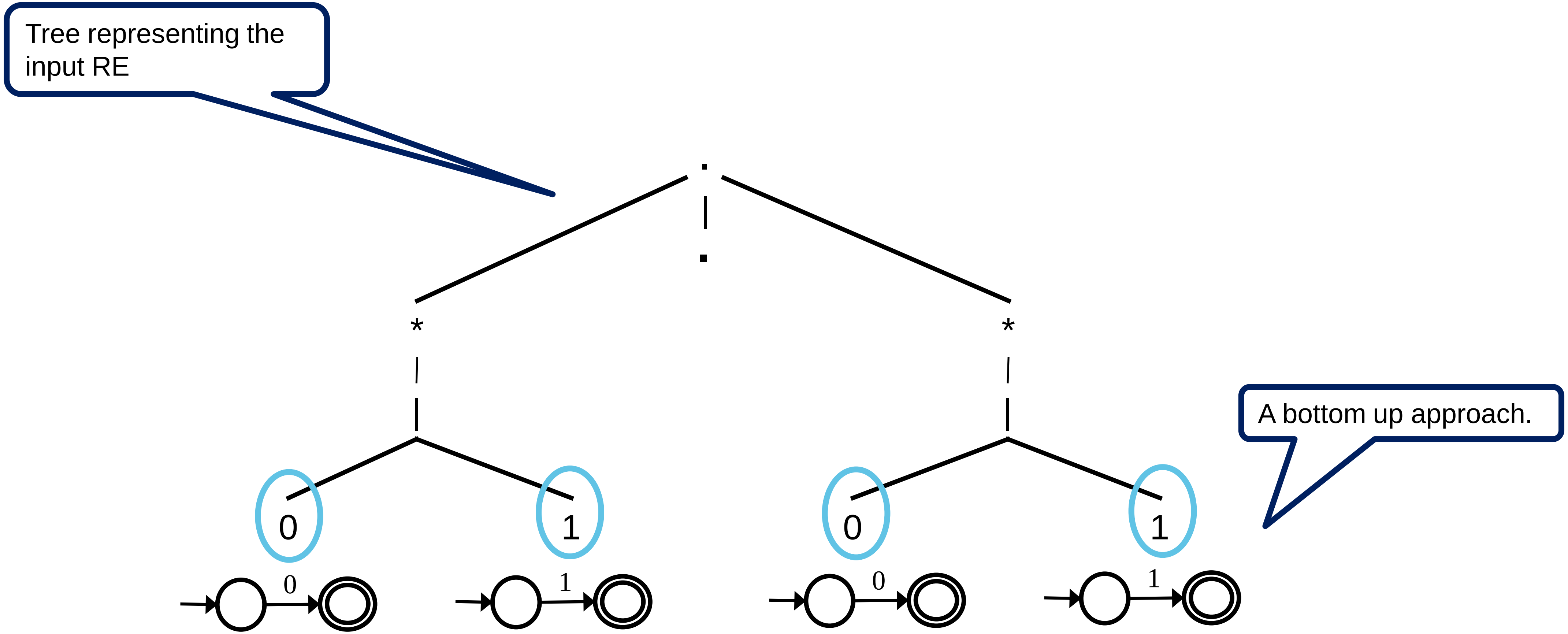
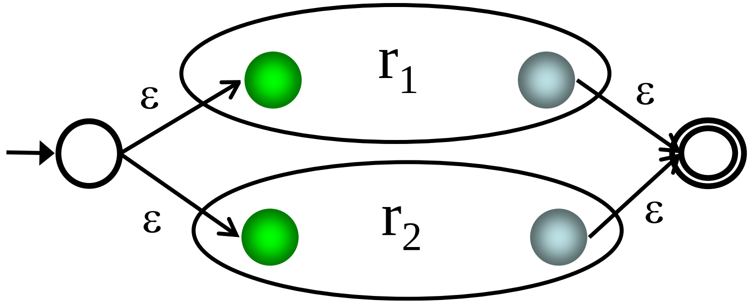


RE to DFA



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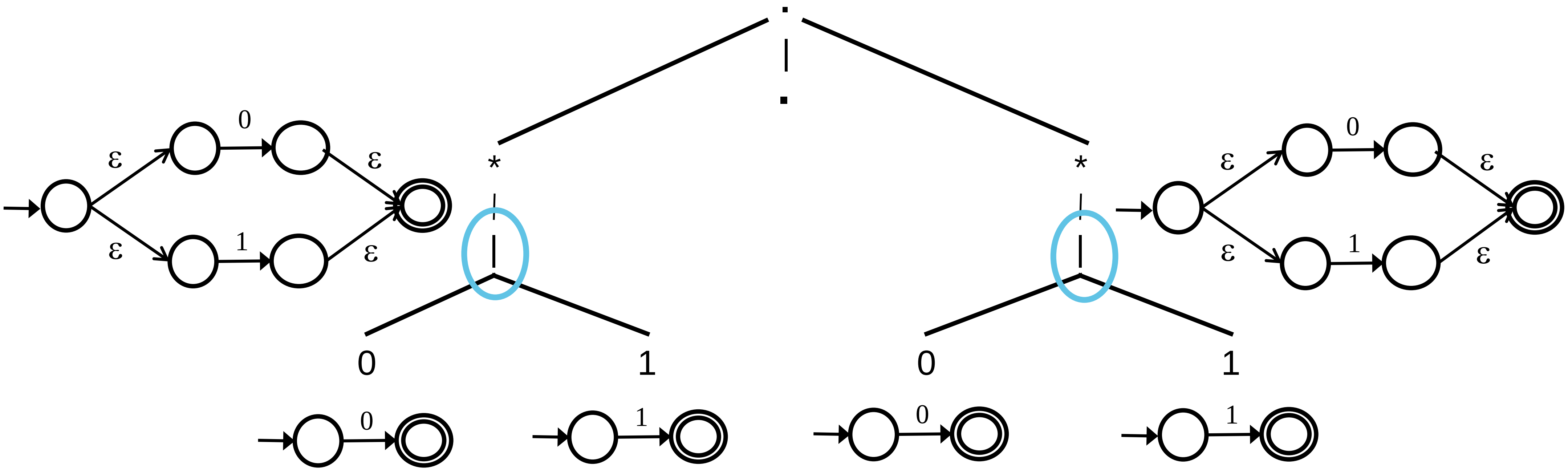
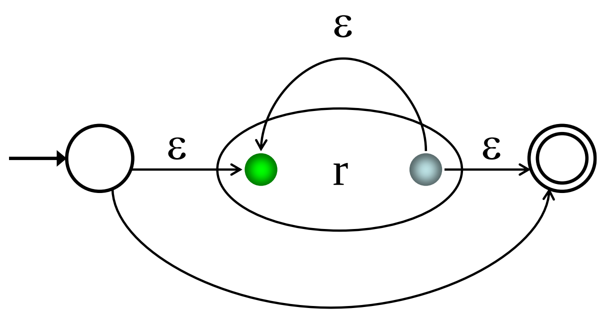
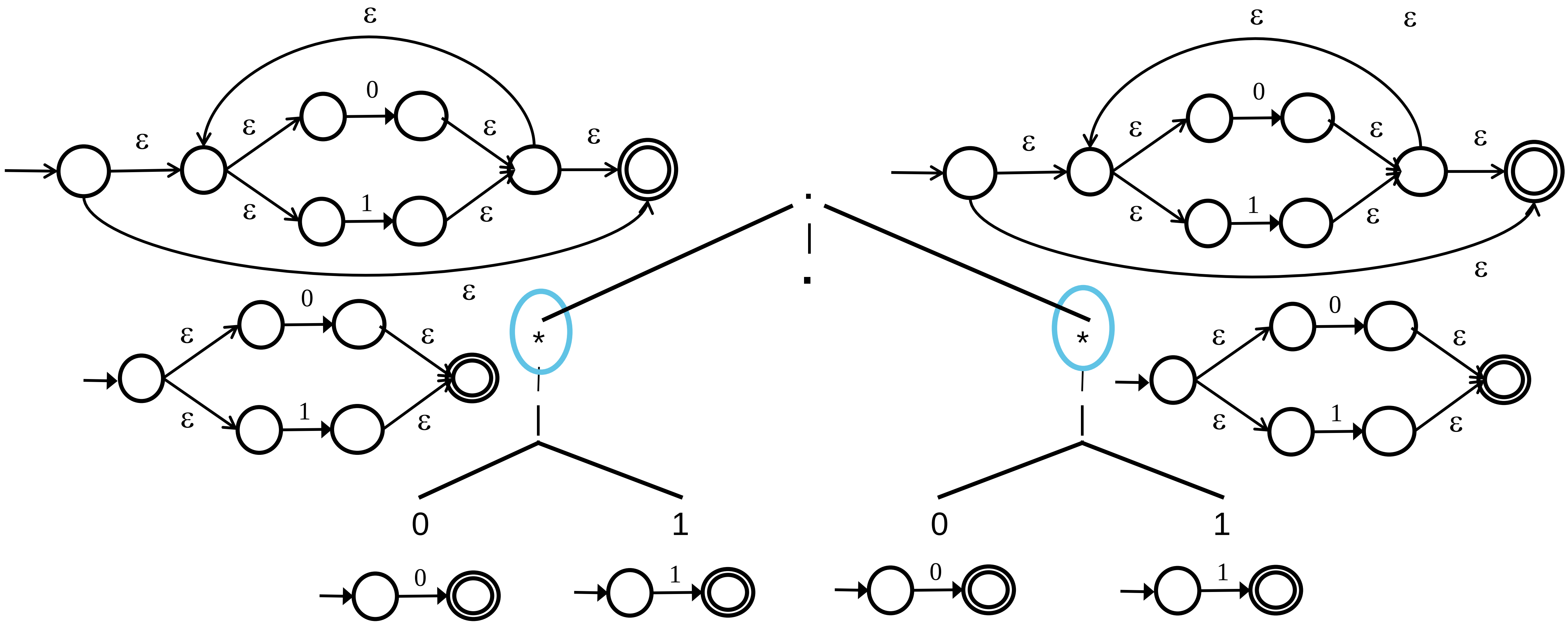


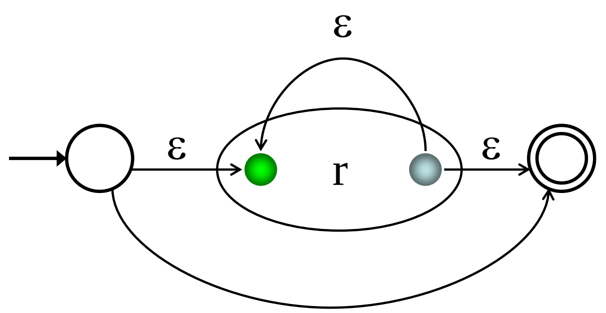
RE to DFA



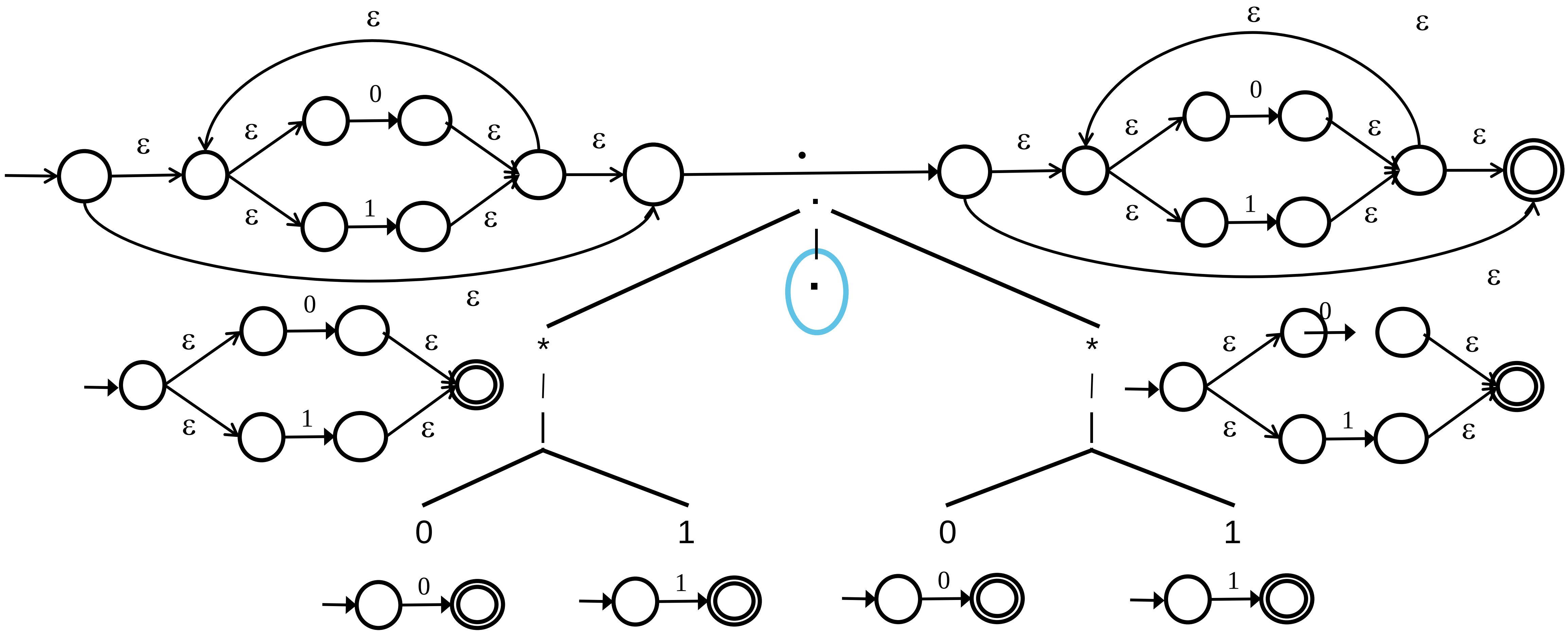
- Example: (0 | 1)*.(0 | 1)*



RE to DFA



- Example: (0 | 1)*.(0 | 1)*



Conversion

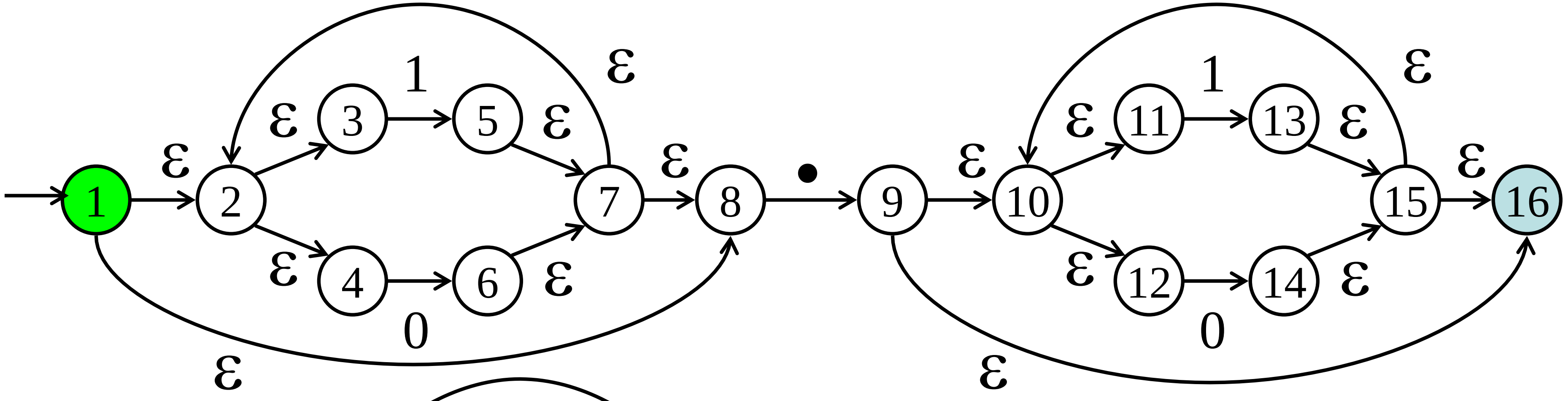
- The conversion RE→FA with the previous rules produces an NFA
- The resultant NFA can be automatically transformed into a DFA
  - The DFA can be exponentially larger than the NFA
    - An NFA with N stathes can result in a DFA with 2^N-1 states (2^N states if we count the dead state)
  - Simplification of the DFA involves its minimization
    - *See the method used in Theory of Computation course*

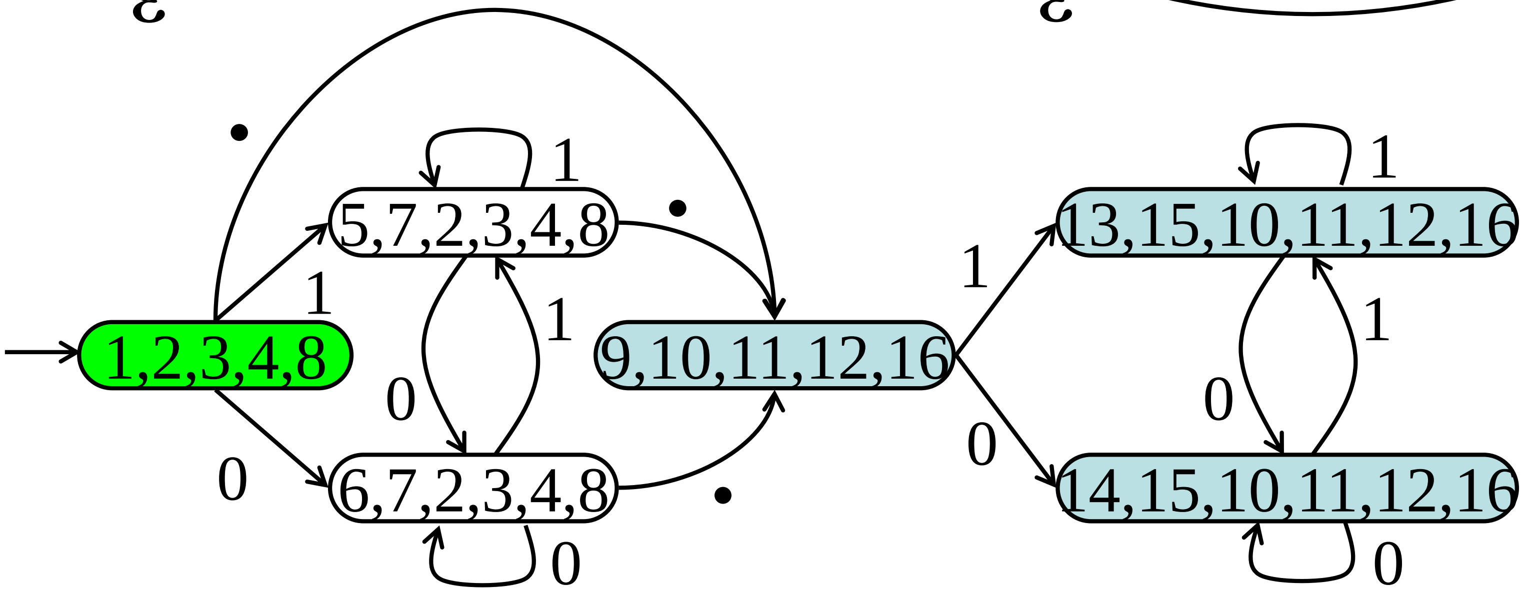
Conversion NFA to DFA

- The DFA has a state for each subset of states of the NFA
  - The start state of the DFA corresponds to the states reached following the ε transitions from the NFA start state
  - A state *qi* of the DFA is an accept state if an accept state of the NFA is included in the group of states associated to *qi*
- To determine the transition with symbol "a" of a state D of the DFA
  - Consider S an empty state
  - Find a set N of states D in the NFA
    - For all the states of the NFA in N
      - Determine the set of states N' in which the NFA can be after matching "a"
      - Update S with the union of S with N'
  - If S is not empty
    - there is a transition "a" from D to the DFA state which has the set of states S of the NFA
  - Else
    - there is none transition "a" from D

NFA to DFA

- Example: (0 | 1)*.(0 | 1)*





IMPLEMENTING THE LEXICAL ANALYZER

Lexical Structure of the Programming Languages

- Each language has various categories of classes.
- In a programming language:
  - Keywords (if, while)
  - Arithmetic operations (+, -, *, /)ord
  - Integer numbers (1, 2, 45, 67)
  - Foating point numbers (1.0, .2, 3.337)
  - Identifiers (abc, i, j, ab345)
- Typically we have a lexical category for each keyword
- Each lexical category is defined by regular expressions

Examples of Lexical Categories

- If-keyword = if
- While-keyword = while
- Operator = + | - | * | /
- Integer = [0-9] [0-9]*
- Float = [0-9]*. [0-9]*
- Identifier = [a-z]([a-z] | [0-9])* or [a-z][a-z0-9]*
- In the syntactic analysis we will use these categories

From the Regular Expressions to the Lexical Analyzer

- Translation of the regular expression to an NFA
- Translation of the NFA to a DFA
- State minimization of the DFA
- Implementing in software the DFA

A DFA Interpreter

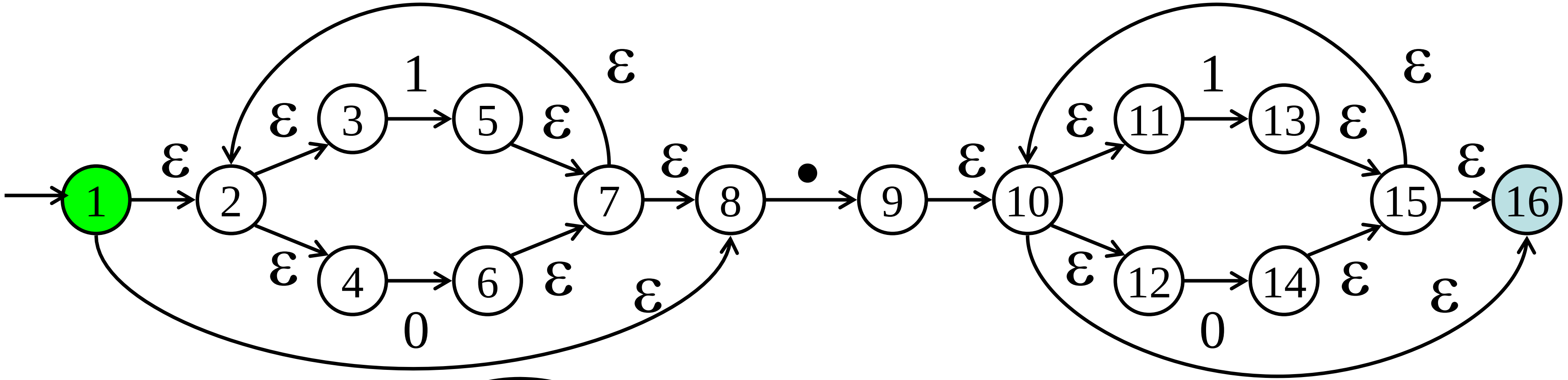
- Pseudo-code of the interpreter

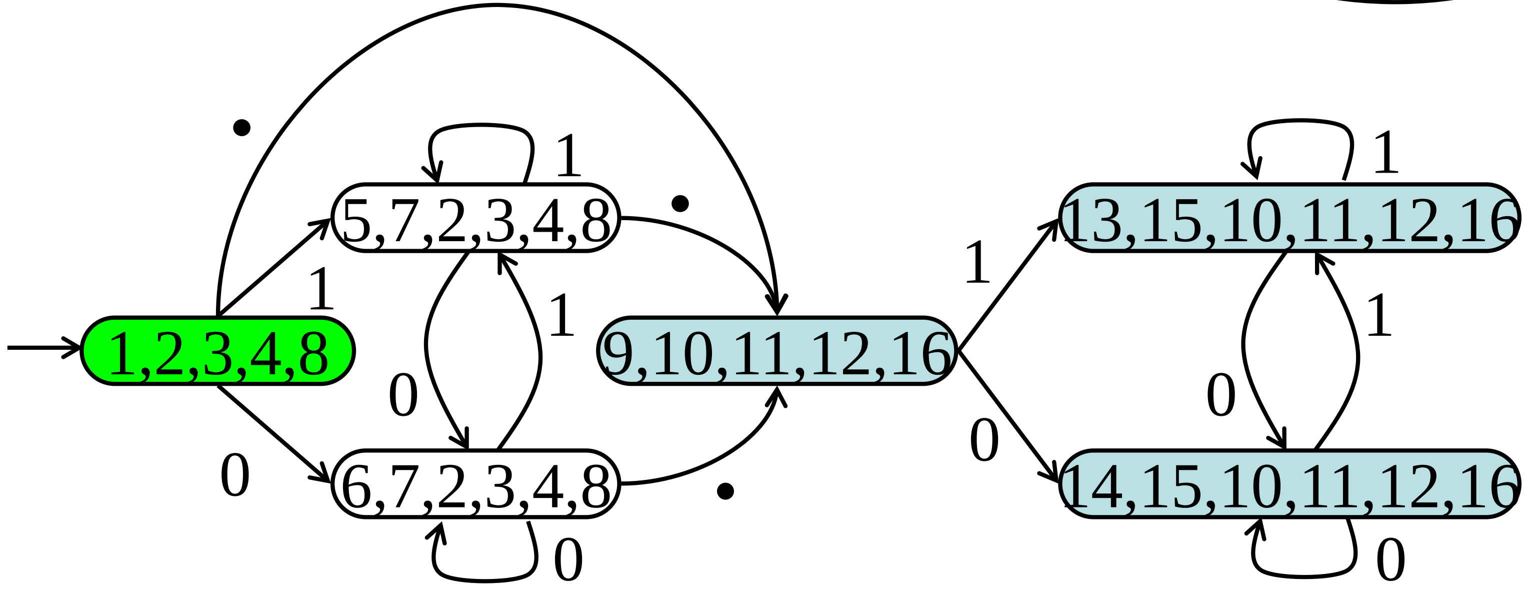
```
Input: DFA

State = DFA initial state;
inputChar = getchar();
While(inputChar) {
    State = trans(State, inputChar);
    inputChar = getchar();
}
If(State is an accept state)
  do  action related to accept state (recognize String)
Else
   do other action
```

What about NFAs?

NFA to DFA

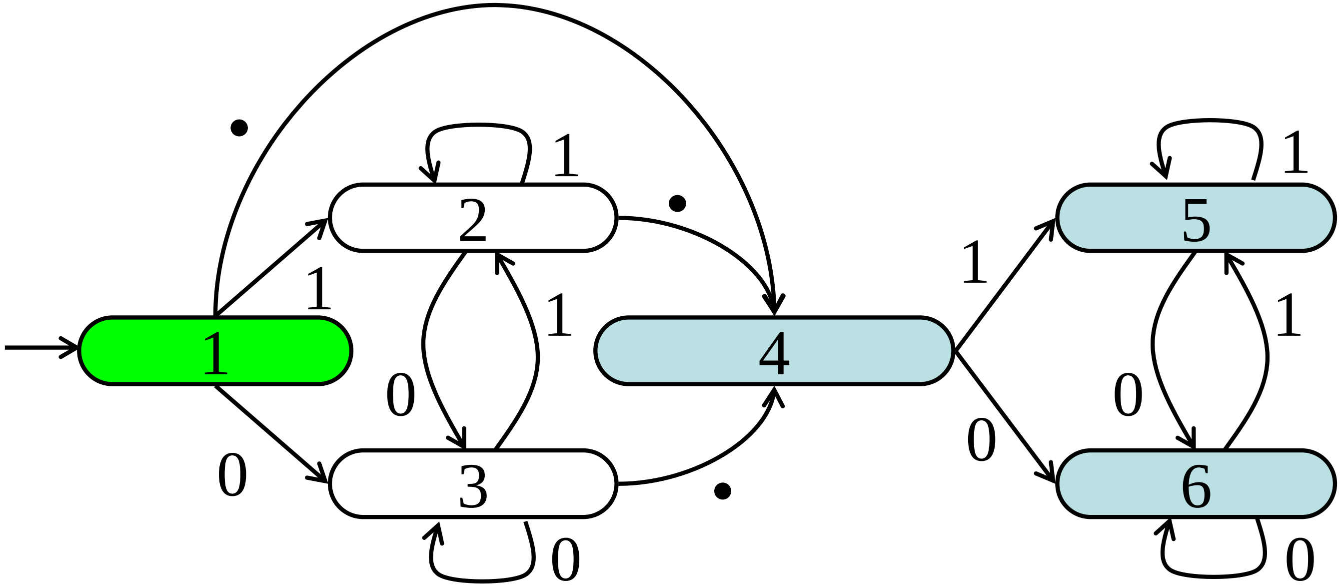
- Example: (0 | 1)*.(0 | 1)*





DFA

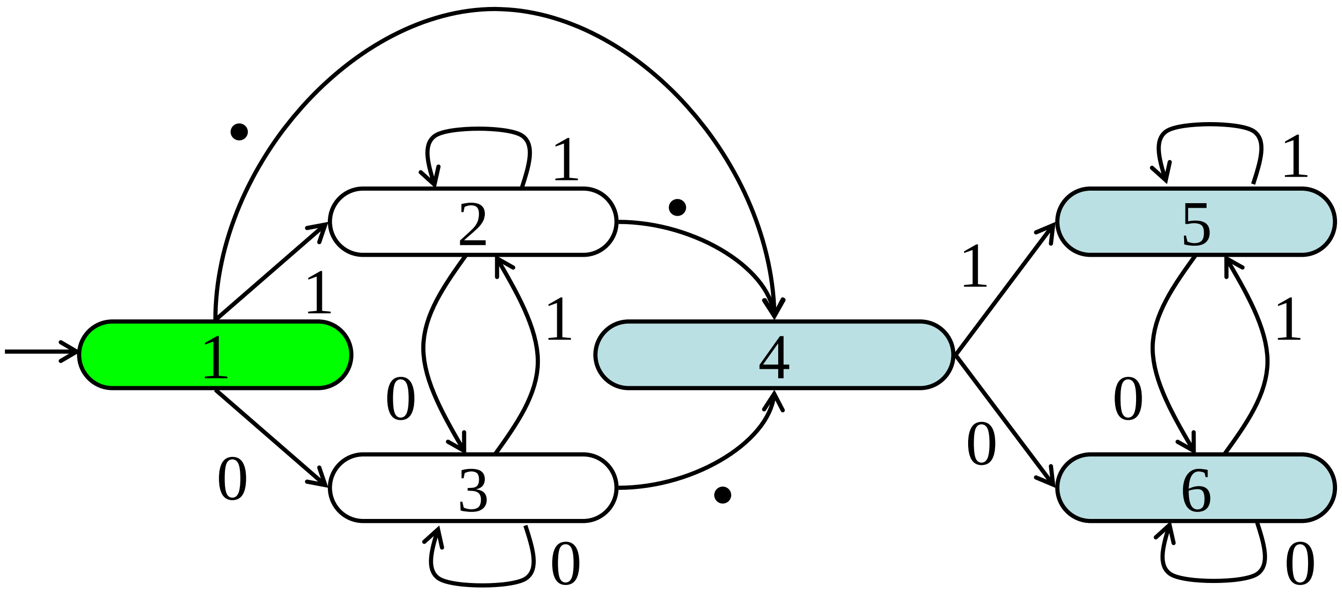
- State transition table (added state 0 – dead state for the transitions not present)



Current state	Next state		
	“0”	“1”	“.”
0	0	0	0
→ 1	3	2	4
2	3	2	4
3	3	2	4
*4	6	5	0
*5	6	5	0
*6	6	5	0

Implementing the DFA

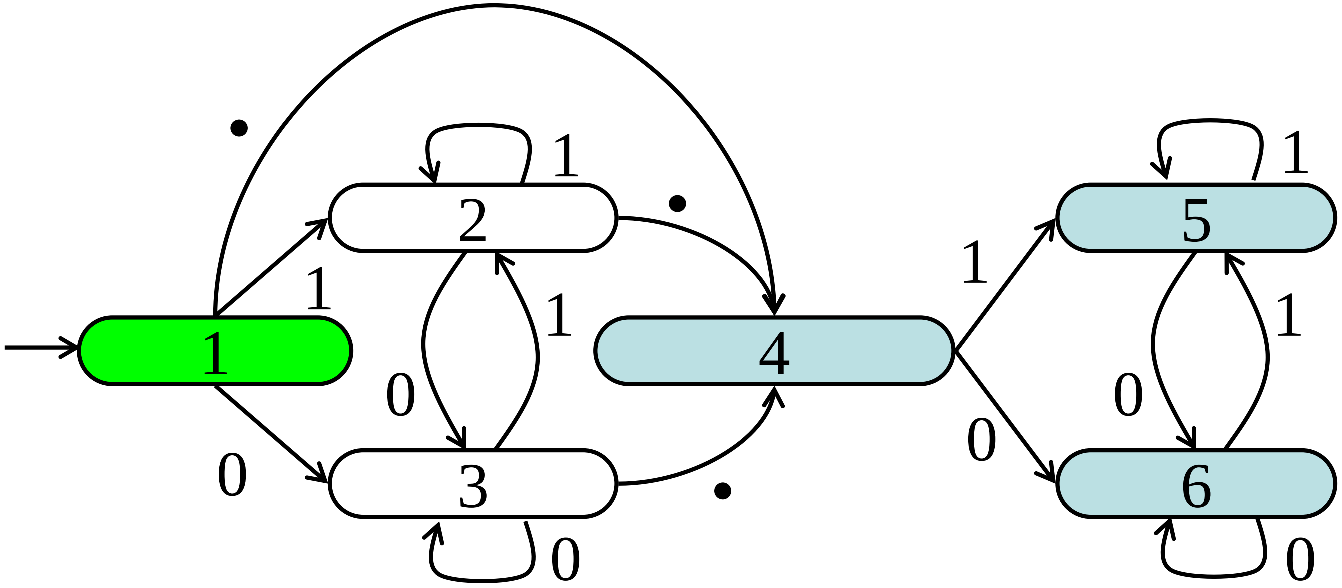
- Implementing the state transition table with a 2D array



Current state	Next state		
	"0"	"1"	"."
0	0	0	0
→ 1	3	2	4
2	3	2	4
3	3	2	4
*4	6	5	0
*5	6	5	0
*6	6	5	0

Implementing the DFA

- The array will output an index based on the state number and symbol
- Supposing the possibility of 256 symbols:
  - int Edge[NumStates][256] (NumStates = 7)



Current state	Next state		
	"0"	"1"	"."
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→ 1	3	2	4
2	3	2	4
3	3	2	4
*4	6	5	0
*5	6	5	0
*6	6	5	0

Implementing the DFA

```
int Edge[NumStates][256] =
{                  /*...  0  1  2  ...  9 ...   "." ... */
/* estado 0 */   {..., 0, 0, 0, ..., 0, ..., 0, ...},
/* estado 1 */   {..., 3, 2, 0, ..., 0, ..., 4, ...},
/* estado 2 */   {..., 3, 2, 0, ..., 0, ..., 4, ...},
/* estado 3 */   {..., 3, 2, 0, ..., 0, ..., 4, ...},
/* estado 4 */   {..., 6, 5, 0, ..., 0, ..., 0, ...},
/* estado 5 */   {..., 6, 5, 0, ..., 0, ..., 0, ...},
/* estado 6 */   {..., 6, 5, 0, ..., 0, ..., 0, ...}
}
```

Example:

Edge[3][(int) '.'] $\leftarrow$ 4

Current state	Next state		
	"0"	"1"	"."
0	0	0	0
$\rightarrow$ 1	3	2	4
2	3	2	4
3	3	2	4
*4	6	5	0
*5	6	5	0
*6	6	5	0

Implementing the DFA

- A 1D array translates the state number into the action number to do:

```
// state                     0  1  2  3  4  5  6
int final[NumStates] = {0, 0, 0, 0, 1, 1, 1}
```

Example:

final[4] ← 1 (thus, action 1 must be done)

Current state	Next state		
	"0"	"1"	"."
0	0	0	0
→ 1	3	2	4
2	3	2	4
3	3	2	4
*4	6	5	0
*5	6	5	0
*6	6	5	0

Implementing the DFA

- Pseudo-code of the interpreter

```
State = 1; // DFA initial state
inputChar = input.read();
While(inputChar) {
    State = edge[State][inputChar];
    inputChar = input.read();
}
If(final[State] == 1)  // 1 identifies an action
   action (recognize String)
Else
   other action
```

Optimizations of the DFA Implementation

- Table of transitions implemented by a 2D array can be further optimized
- Indirect mapping:

```
int Edge[NumStates][4] = {
/* estado 0 */    {0, 0, 0, 0},
/* estado 1 */    {3, 2, 4, 0},
/* estado 2 */    {3, 2, 4, 0},
/* estado 3 */    {3, 2, 4, 0},
/* estado 4 */    {6, 5, 0, 0},
/* estado 5 */    {6, 5, 0, 0},
/* estado 6 */    {6, 5, 0, 0}
}
                    /*… "0"  "1" "2" "3"… 9 …  "." … */
Int map[256] = { …,  0,   1,   3,   3, … 3, …   2, …}
Exemplo:
    Edge[3][(int) '.'] is translated to:
    Edge[3][map[(int) '.']]
```

Another possibility is to use if or switch constructs for the map function:

```
Switch(ch1) {
    case '0': return 0; break;
    case '1': return 1; break;
    case '.':  return 2; break;
    otherwise: return 3;
}
```

Optimizations of the DFA Implementation

- For the presented example and using tables:
  - Optimization allows to go from an array with 256 × 7 (1,792) elements to an array with 256 and other with 7 × 4 (28) elements
  - From 1,792 to 284 elements
- There are other optimization techniques...

With JavaCC

- The Lexical Analyzer is not implemented using tables
- Instead, code using switch and if-else constructs is generated
- Use of tables might be better when developing a lexical analyzer manually (easy to understand, change, and maintain)

ex1.jj

```
…
TOKEN: {
        < REALBIN: ("0"|"1")*"."("0"|"1")*>
}
…
```

ex1TokenManager.java:

```
…
      switch(jjstateSet[--i])
      {
        case 3:
          if ((0x3000000000000L & l) != 0L)
            { jjCheckNAddTwoStates(0, 1); }
          else if (curChar == 46)
          {
            if (kind > 1)
              kind = 1;
            { jjCheckNAdd(2); }
          }
          break;
        case 0:
          if ((0x3000000000000L & l) != 0L)
            { jjCheckNAddTwoStates(0, 1); }
          break;
        case 1:
          if (curChar != 46)
            break;
          kind = 1;
          { jjCheckNAdd(2); }
          break;
        case 2:
          if ((0x3000000000000L & l) == 0L)
            break;
          if (kind > 1)
            kind = 1;
          { jjCheckNAdd(2); }
          break;
        default : break;
      }
…
```

JavaCC

- Example of Java Integer Literal definition in JavaCC (Java1.5.jj)

```
< INTEGER_LITERAL:
        <DECIMAL_LITERAL> (["l","L"])?
        | <HEX_LITERAL> (["l","L"])?
        | <OCTAL_LITERAL> (["l","L"])?  >

|  < #DECIMAL_LITERAL: ["1"-"9"] (["0"-"9"])* >
|  < #HEX_LITERAL: "0" ["x","X"] (["0"-"9","a"-"f","A"-"F"])+ >
|  < #OCTAL_LITERAL: "0" (["0"-"7"])* >
```

JavaCC

- Example of Java identifier definition in JavaCC (Java1.5.jj)

```
< IDENTIFIER: <LETTER> (<PART_LETTER>)* >
|
  < #LETTER:
      [
         "$",
         "A"-"Z",
         "_",
         "a"-"z",
         "\u00a2"-"\u00a5",
        ...
         "\uffe5"-"\uffe6"
      ]
  >
|
  < #PART_LETTER:
      [
         "\u0000"-"\u0008",
         "\u000e"-"\u001b",
         "$",
         "0"-"9",
         "A"-"Z",
         "_",
         "a"-"z",
         "\u007f"-"\u009f",
         "\u00a2"-"\u00a5",
         ...
         "\ufff9"-"\ufffb"
      ]
  >
```

Summary

- Lexemes/strings of the language are specified using regular expressions (REs)
  - They are grouped in categories
  - Note that each token must be identified and its value may need to be stored
- Lexical analysis can be efficiently implemented using Finite Automata

Further Reading

- About regular expressions:
  - Jeffrey E.F. Friedl, ***Mastering Regular Expressions, Third Edition***, O'Reilly Media, Inc., Aug. 2006.
  - PERL Compatible Regular Expression (PCRE): **http://www.pcre.org/**
- Programming languages may have built-in support to regular expressions or may provide libraries/APIs:
  - Regular Expressions in Java: java.util.regex