# Sawtooth Wave Detection

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**Abstract** –.Sawtooth waves occur in Rapid Eye Movement (REM) sleep although their origin remains unknown. They have been used primarily to classify sleep stages. Classification of Sawtooth Waves is still done in a tedious manner, with pattern recognition experts working through hours of sleep recorded signals. The initial goal of this paper was to develop a method of automatically detecting Sawtooth waves but, because of several difficulties discussed in this paper and, besides the fact that many approaches were made, it was not possible to reach a valid conclusion about the feasibility of the problem.
***Index** - REM, Classification, Sawtooth, Decomposition*

## I. INTRODUCTION

As the human being needs to sleep to allow for certain processes in the brain to take place, sleep is a very important part in a subject's health. Sleep disorders are on the rise with an increasingly stressful world and, following this worldwide trend, the study of sleep has being progressively considered as a diagnostic tool.[1]

One of the most important parts in these studies is the Sleep Staging. In a full sleep cycle 5 different sleep stages occur and each one has its own characteristics. The purpose of this work was initially to develop an algorithm that could be potentially used to detect the REM cycles. Intense dreaming occurs during REM sleep as a result of heightened brain activity, but paralysis occurs simultaneously in the major voluntary muscle groups.

The percentage of REM sleep is highest during infancy and early childhood. During adolescence and young adulthood, the percentage of REM sleep declines. Infants can spend up to 50% of their sleep in the REM stage of sleep, whereas adults spend only about 20% in REM. [5]

REM sleep in EEG is characterized by several features. One of them is the presence of Sawtooth Waves which origin is still unknown.[2] These waveforms have been detected in children down to 5 weeks old and even in other species such as Papius Anubis and as such, is a rather universal feature. [3] Although it is a rather simple pattern, the high variability between different subjects and even the high inter and intra variability between different pattern recognition experts have been a major challenge in automatizing the waveform detection procedure.[4] In this study, several attempts regarding sawtooth classification have been made using EMD, STFT and Correlation using both K Nearest Neighbours and Naïve Bayes classification algorithms.

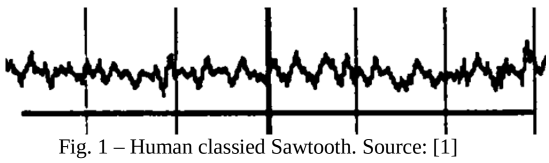

Fig. 1 – Human classied Sawtooth. Source: [1]

### *A. EMD*

An alternative data analysis tool was proposed by Norden E. Huang called the Hilbert-Huang Transform (HHT) [6]. The HHT technique for analyzing data consists of a decomposition algorithm called empirical mode decomposition (EMD). These decompositions of the signal can be used in order to find out the instantaneous frequencies present in the signal with the HHT.

The algorithm attempts to decompose nearly any signal into a finite set of functions. These functions are called intrinsic mode functions (IMFs). The algorithm utilizes an iterative sifting process which successively subtracts the local mean from a signal. The sifting process is as follows:

1. Determine the local extrema (maxima, minima) of the signal.
2. Connect the maxima with an interpolation function, creating an upper envelope about the signal.
3. Connect the minima with an interpolation function, creating a lower envelope about the signal.
4. Calculate the local mean as half the difference between the upper and lower envelopes.
5. Subtract the local mean from the signal.
6. Iterate on the residual.

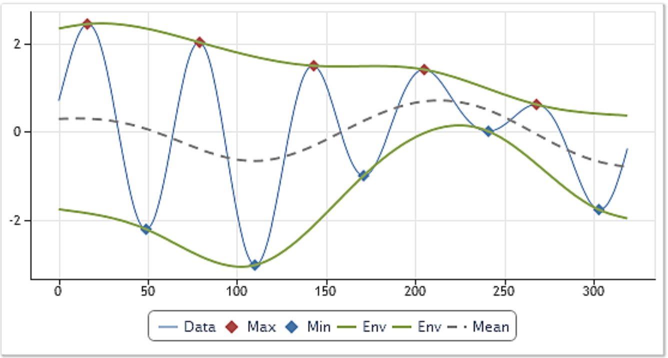

Fig. 2 – Ilustration depicting the EMD procedure. Source: [6]

The sifting process is repeated until the signal meets the definition of an IMF. Then, the IMF is subtracted from the original signal, and the sifting process is repeated on the remainder. This is repeated until the final residue is a monotonic function. [7] The last extracted IMF is the lowest frequency component of the signal, better known as the trend. The definition of an IMF is a signal which has a zero-mean, and whose number of extrema and zero-crossings differ by at most one [5].

Once a signal has been fully decomposed, it can be written as the finite sum of the IMFs and a final residue as shown in equation 1:

$$S(t) = R(t) + \sum_{j=1}^{n} IMF_j \tag{1}$$

### A.1 Hilbert Spectral Analysis

The purpose of HHT is to demonstrate an alternative method to present spectral analysis tools for providing the time-frequency-energy description of time series data. Also, the method attempts to describe nonstationary data locally. Rather than a Fourier or wavelet based transform, the Hilbert transform is used, in order to compute instantaneous frequencies and amplitudes and describe the signal more locally. Equation 2 displays the Hilbert transform $\tilde{y}$ that can be written for any function x(t) that is of LP class. PV denotes Cauchy's principle value integral. [7]

$$H[x(t)] = \tilde{y}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(t)}{t-\tau} d\tau \tag{2}$$

This transformation in the signal allows the representation of the signal z(t):

$$z(t) = x(t) + i\tilde{y}(t) = A(t)e^{i\theta(t)} \tag{3}$$

Where

$$A(t) = (x^2 + \tilde{y}^2)^{\frac{1}{2}}, \quad \theta(t) = tan^{-1}\left(\frac{\tilde{y}}{x}\right) \tag{4}$$

With this formalism it is easy to note that:

$$\omega = \frac{d\theta}{dt} \tag{5}$$

This will be the instantaneous frequency of the signal.

### *B.STFT*

In an effort to correct the major deficiency in the Fourier Transform that was the lack of time resolution in the frequencies, Dennis Gabor (1946) adapted the Fourier transform to analyze only a small section of the signal at a time -- a technique called signal windowing. Gabor's adaptation, called the Short-Time Fourier Transform (STFT), maps a signal into a two dimensional function of time and frequency as it can be seen in equation 6:

$$STFT\{x(N)\} = X(m, \omega) = \sum_{-\infty}^{+\infty} x[n]w[n-m]e^{-j\omega n} \tag{6}$$

The STFT represents a sort of compromise between the time- and frequency-based views of a signal. It provides some information about both when and at what frequencies a signal event occurs. However, you can only obtain this information with limited precision, and that precision is determined by the size of the window. While the STFT compromise between time and frequency information can be useful, the drawback is that once you choose a particular size for the time window, that window is the same for all frequencies. [8]

### *C.CORRELATION*

The cross-correlation function can be used to discover similarities between signals since it’s a measurment of the inner product of two signals over a sliding window. Theoretically, one can use a sliding sawtooth window and analyse where the peaks in the cross-correlated signals are.

The cross-correlation of two continuous functions $\emptyset_{xy}$ is defined by:

$$\emptyset_{xy} = \int_{-\infty}^{+\infty} x(\tau - t)y(\tau)d\tau \tag{7}$$

If we compare equation (7) with the convolution between two functions:

$$x(t) * y(t) = \int_{-\infty}^{+\infty} x(t-\tau)y(\tau)d\tau \tag{8}$$

We can see that the only difference is that for the cross-correlation, one of the two functions is not reversed. Thus,

$$\emptyset_{xy} = x(-t) * y(t) \quad (9)$$

Which leads to the final conclusion that

$$\emptyset_{xy} = FT^{-1}[X^*(f) * Y(f)] \quad (10)$$

### *D. CLASSIFICATION*

This project used two alternative methods for signal classification between signals with Sawtooths or not : K-Nearest Neighbours and Naive Baives. These two methods are described in this subsection.

**K-Nearest Neighbors**

The K-nearest neighbors algorithm uses a database in which data points are separated into different separate classes (training set), and uses the similarity (by means of distance) between new points and those from the classes to predict to which class they belong (classification).

The reason it is called the K-nearest neighbors is because the procedure it follows to classify a new sample, is the following: a k positive integer number is specified; the k data points closest to the sample to classify (with the lowest distance) are selected; the class of those data points from the database is accessed, and the class with the most points from the k selected is the class attributed to the new sample. [9]

**Naive Bayes**

The Naive Bayesian classifier is based on Bayes' theorem with independence assumptions between predictors. A Naive Bayesian model is easy to build, with no complicated iterative parameter estimation which makes it particularly useful for very large datasets. Despite its simplicity, the Naive Bayesian classifier often does surprisingly well and is widely used because it often outperforms more sophisticated classification methods.

Bayes theorem provides a way of calculating the posterior probability, P(c|x), from P(c), P(x), and P(x|c). The Naive Bayes classifier assumes that the effect of the value of a predictor (x) on a given class (c) is independent of the values of other predictors. This assumption is called class conditional independence. [10]

$$P(c|x) = \frac{P(x|c)P(c)}{P(x)} \quad (11)$$

And:

$$P(c|x) = P(x_1|c) \times P(x_1|c) \ldots \times P(x_n|c) \times P(c)$$

Where:
P(c|x) is the posterior probability of class (target) given predictor (attribute).
P(c) is the prior probability of class.
P(x|c) is the likelihood which is the probability of predictor given class.
P(x) is the prior probability of predictor.

## II. METHODOLGY

### *A.PREPROCESSING*

The signals that this Project had at its disposal to work on were in edf format. This format is used to store medical signals since it allows reading in several channels at different sampling rates. The file has two major features: the Header, which contains information about the number and name of the channels, the date and time of the record and other characteristics of the signal such as the sampling rate and the actual data itself which comes measured in milivolts.

Since the EEG signal is very susceptible to noise, two filters were applied to the signal in order to remove some of that noise. Both the filters were a 10th order Butterworth filter with double passage. The filters applied were a 25 Hz lowpass (to remove high frequency noise and a 0.5 Hz highpass (to remove DC drift).

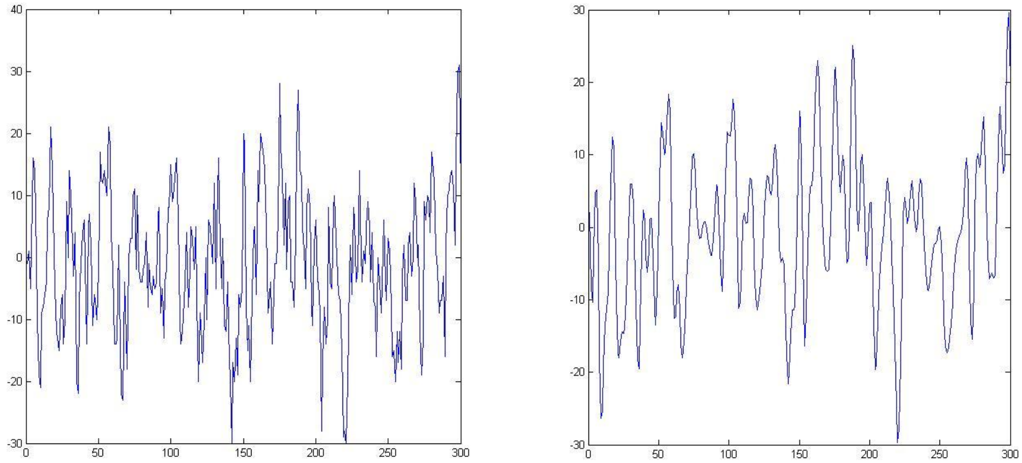


Fig. 3 – Ilustration depicting the difference between filtered(right) and unfiltered(left) signals. As one could expect, the filtered signal is smoother.

After the signal was filtered, the approach was to only isolate the REM zones in the sleep signals (with data given on the stages of the signals used) since it is known that the sawtooth waveform is only found within REM sleep.

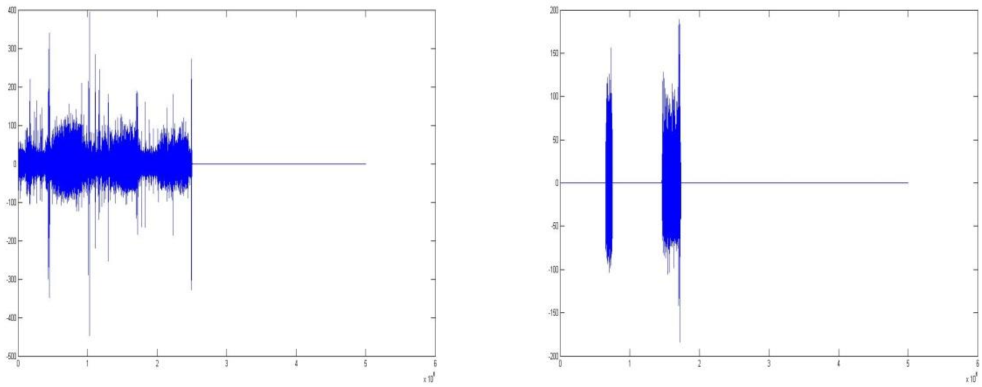

Fig. 4– Ilustration depicting the difference between REM staged (right) and raw (left) signals.

### *B.EMD*

A first approach for solving the Classification of Sawtooth wave was the Empirical Mode Decomposition (EMD) with 5 levels and the Huang Hillbert Transform. Using the approach described in the previous section, it was possible to construct two classification models (KNN and Naïve Bayes in this case) with the following steps:

The signal was separated in two parts, (one with human classified sawtooths-data possesed and the other without).

Both these parts were fed into the EMD algorithm with a 3 second window with 50% overlap between consecutive EMD's.

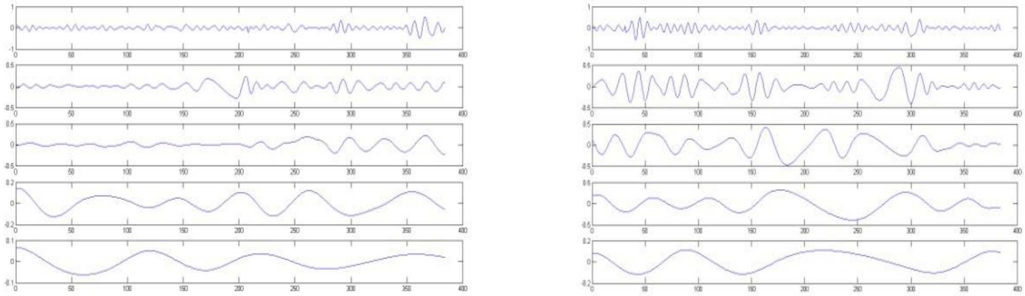

Fig. 4– Ilustration showing the EMD decompositions for two different 3 second windows, one (on the right) with sawtooth waveforms and the other (on the left) without

The resulting EMD's (only the 2nd an 3rd level because they're the one with more information – as seen in the literature [1-3] since the 1st is normally just noise and the last two are the residue and very low frequencies which are not a characteristic of sawtooth waveforms) were transformed using the Huang Hillbert Transform which were fed into the classifiers properly classified for training purposes.

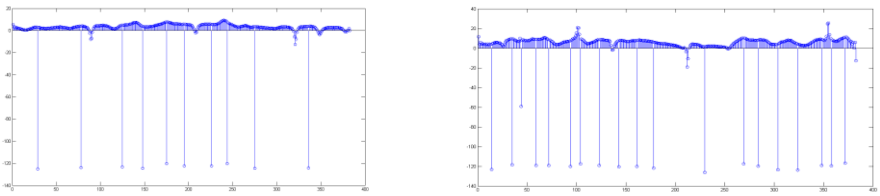

Fig.5– Ilustration showing the Hilbert Huang transform for the sum of the 2nd and 3rd level of the EMD Decomposition (with sawtooth-right, without sawtooth – left)

The data used for training were the 100.edf, 101.edf and 17.edf files and the test data was the 50.edf data.

The results of the classification are depicted in the next figure:

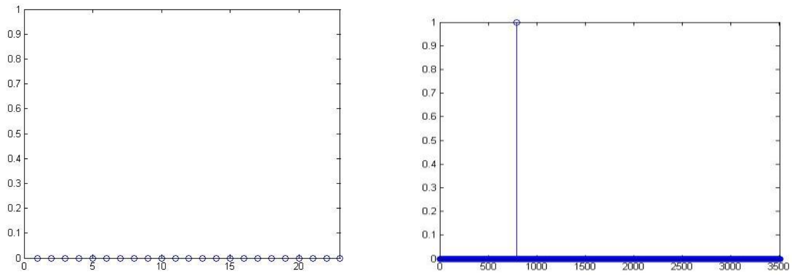

Fig.6– Results for the classification -signal with sawtooths(left) and signal without sawtooths (right) KNN classifier

As one can see, and considering 1's are sawtooth detection an 0 is classification of non-sawtooth wave, the true negative ratio is quite high but unforntunatelly so is the false negative ratio.

### *B.STFT*

A second approach for solving the Classification of Sawtooth waveforms was using the Short Time Fourier Transform (STFT) of the signals in a 3 second blackman window with 50 % overlap just like in the previous example so that comparisons about both methods could be made. The training and testing set were exactly the same.

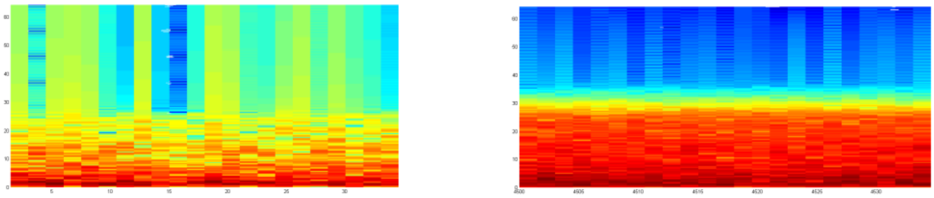

Fig 7– Spectrogram for a signal with(left) and without (right) Sawtooth waveforms

The results of the classification are depicted in the next figure:

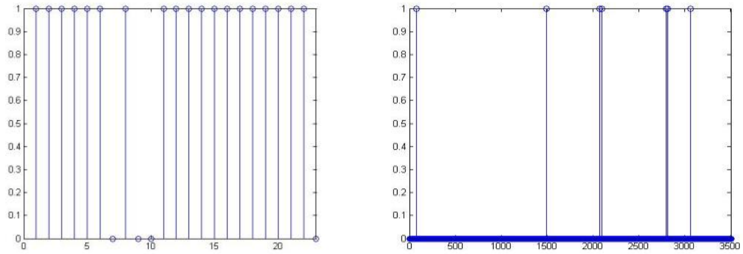

Fig 8– Results for the classification -signal with sawtooths(left) and signal without sawtooths (right) KNN classifier

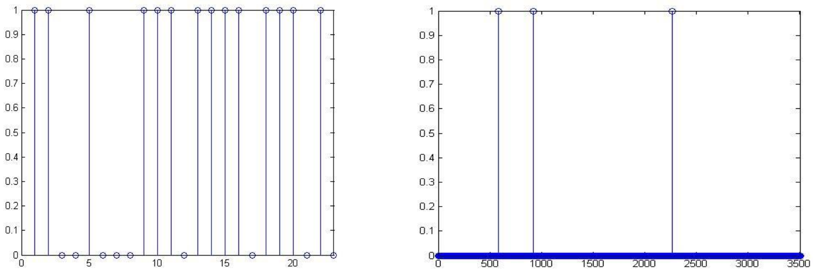

Fig 9– Results for the classification -signal with sawtooths(left) and signal without sawtooths (right) Naïve Bayes classifier

As one can see, the performance using the STFT as feature for classification has a marginal increase if compared with the previous method. The performance of the Naïve Bayes Classifier is similar but slightly worse.

### *C.CORRELATION*

A third approach for solving the Classification of Sawtooth waveforms was using the Correlation of the signals in a 3 second window with 50 % overlap with sawtooths varying from 2 to 5hz and with an angle from 79 to 90º just like in the previous example so that comparisons about both methods could be made. The training and testing set were exactly the same. The data used as feature was the mean, median, variance and standart deviation. Even when combining these features with other ones like the one described above the classification results were not satisfactory.

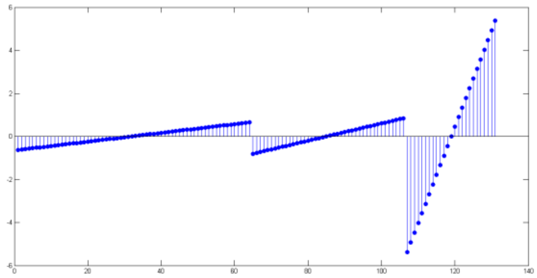

Fig 10– Examples of the sawtooths used to evaluate the correlation

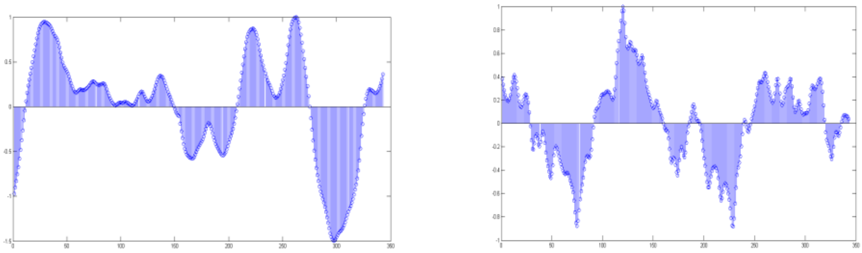

Fig 11– Result of the correlation of a sawtooth of frequency 3hz and 83º angle with a signal with (left) and without(right) sawtooth

It can be seen though, that the correlation with signals with sawtooth present is a lot “smoother” that with a signal without them suggesting future possible approaches.

*D.COMBINATIONS*

In order to try to improve the efficacy of the classification, combination of different features were experimented. Those were EMD+STFT+Correlation and EMD+STFT.

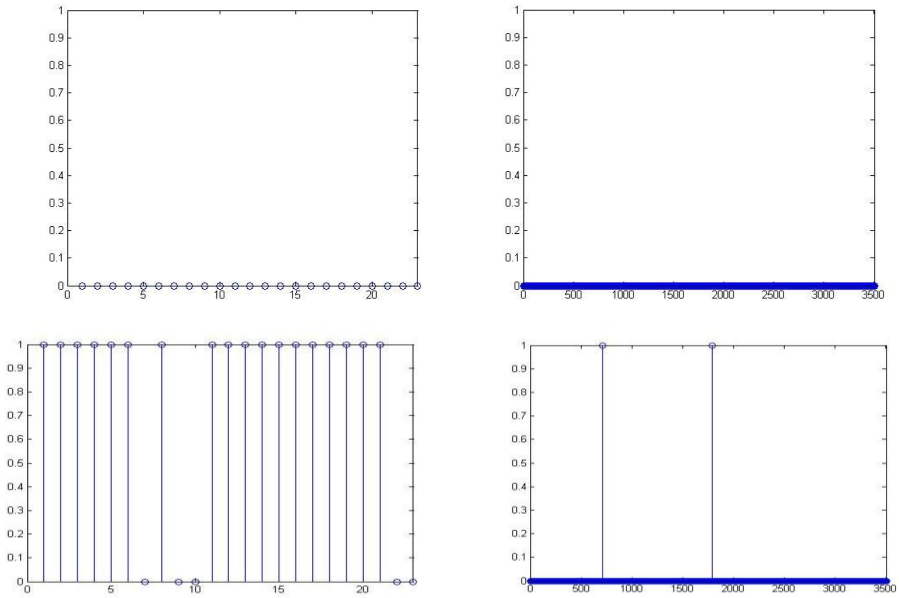

Fig 12– Results for the classification -signal with sawtooths(left) and signal without sawtooths (right) KNN classifier – EED+STFT+Correlation – up and EED + STFT –down

As one can see, the rate of true positives is the best one for the EMD+STFT based classifier with a low rate of false negatives making this combination the best one yet. These results could also prove that the mean, median, square deviation and variance after cross correlation are not good features for sawtooth classification.

## III. DISCUSSION

The results are not at all satisfactory. In the best case scenario, the false negative ratio was about 20% which is still quite high. In general this can be explained by several factors (noisy signals, high intervariability between different individuals, signals acquired in different channels and at different sampling rates, poor classification of sawtooth events and/or REM stages) but also by the fact that this is a highly unexplored field making every strategy used towards the goal, in fact a novelty.

Additionally: For the EMD approach, one can see that the Hillbert Transform introduces some random discontinuities (errors) at random intervals which can be one of the causes of it's low performance; the STFT approach was the one that had a better performance when used *solo* but it is obvious that there is some blurring in the time domain caused by the great frequency resolution. This can be a source of errors and the use of a wavelet transform could help overcome this problem; the correlation approach didn't offer good results even when used in a combination of features but our work has shown that the smoothness of the correlation function could be used as a discriminative feature in future works; the combination of techniques wielded better results in the EMD+STFT approach as it had the better true positive ratio and a very high true negative ratio. Thus, in this case the high variability between subjects could be (partially) overcome by the combination of multiple features.

## IV. CONCLUSION

This Project aimed at constructing a model that could be used to detect Sawtooth waveforms in a EEG signal. For the reasons explained in the Discussion section, this was not possible, but as also noticed in the Discussion section, there is room for improvement in this field and future investigations should be aware of the difficulties this work encountered. A model that could efficiently find these waveforms would be of paramount usefulness for a sleep staging model.

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