$$y' + xy = x$$

Factor integrante $h(x) = e^{\frac{x^2}{2}}$

$$e^{\frac{x^2}{2}} y' + x e^{\frac{x^2}{2}} y = e^{\frac{x^2}{2}} x$$

$$\Leftrightarrow \left(e^{x^2/2} y \right)' = e^{\frac{x^2}{2}} x$$

$$\Leftrightarrow e^{x^2/2} y = P\left[e^{\frac{x^2}{2}} x \right] + C$$

$$\Leftrightarrow y = e^{-x^2/2} \left(P\left[e^{\frac{x^2}{2}} x \right] + C \right)$$

$$= C e^{-\frac{x^2}{2}} + 1$$

$$y' + A(x) y = B(x)$$

Fator integrante: $h(x) = e^{P[A(x)]}$

$$e^{P[A(x)]} y' + A(x) e^{P[A(x)]} y = B(x) e^{P[A(x)]}$$

$\Leftrightarrow$ ...

$$\Leftrightarrow e^{P[A(x)]} y = P\left[B(x) e^{P[A(x)]} \right] + C$$

$$\Leftrightarrow y = e^{-P[A(x)]} \left(P\left[e^{P[A(x)]} \cdot B(x) \right] + C \right)$$

$$y' + xy = xe^{-x^2} y^{-3} \underset{y\neq 0}{\Leftrightarrow} y^3 y' + x y^4 = x e^{-x^2}$$

subst. $v = y^4 \Leftrightarrow v' = 4y^3y' \Leftrightarrow y = \pm\sqrt[4]{v}$

$$\Leftrightarrow 4y^3 y' + 4xy^4 = 4xe^{-x^2}$$

$$\Leftrightarrow v' + 4xv = 4xe^{-x^2}$$

Como estamos perante uma EDO linear de 1.ª ordem a solução geral é:

$$\Leftrightarrow v = e^{-P[4x]} \cdot \left(P[e^{P[4x]} \cdot 4xe^{-x^2}] + C \right)$$

$$v = e^{-2x^2} \left(P[e^{2x^2} \cdot 4xe^{-x^2}] + C \right)$$

$$v = e^{-2x^2} \left(P[e^{x^2} \cdot 4x] + C \right)$$

$$v = e^{-2x^2} \left(2e^{x^2} + C \right)$$

$$\Rightarrow y = \pm\sqrt[4]{v} = \pm e^{-\frac{x^2}{2}} \sqrt[4]{2e^{x^2} + C}$$