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Wireless scheduling with multiple data rates: From physical interference to disk graphs☆

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ABSTRACT

Scheduling of wireless transmissions is a core component of performance optimization of wireless ad-hoc networks. Current radio technologies offer multi-rate transmission capability, which allows to increase the network's throughput. Nevertheless, most approximability results of scheduling algorithms have focused on single-rate radios. In this paper, we propose two formulations for the problem of scheduling wireless requests with multiple data-rates, considering the physical interference model with uniform power assignment. The objective of both problems is to select a subset of communication requests to transmit simultaneously, such that the sum of their data rates is maximized and no collisions occur. In the first formulation, data-rates are given as part of the input. In the second formulation, the data-rate assignment is part of the solution. We show that, under certain constraints on the input, these problems can be approximated by a disk graph model. This means that, despite the global nature of the physical interference model, conflicts between simultaneous requests can be restricted to the local neighborhood of the transmitting nodes. We show how to build the corresponding disk graph instances and prove that a weighted maximum independent set in this graph-based model provides a constant-factor approximation in the physical interference model. Moreover, we implement a polynomial-time approximation scheme, as well as a parallel implementation of the algorithm, to obtain solutions that are within an arbitrarily small factor of being optimal in the disk graph model.

1. Introduction

Scheduling of wireless communication requests lies in the heart of performance optimization of wireless ad-hoc networks. Current wireless communication technologies, such as IEEE 802.11a/b/g/n and IEEE 802.16, allow data to be transmitted at multiple data rates. The higher the requested rate, the higher must be the signal-to-interference-plus-noise ratio at the receiver, which can be achieved either by increasing the transmitting power of the sender or by decreasing the interference of concurrent transmissions. Either way, the multi-rate functionality alters the spatial reuse constraints of the wireless channel, which in turn modifies the structure of the scheduling process of communication requests.

In this work we are interested in modelling and analyzing algorithms to solve the following problem. Given a set of wireless links that can transmit with multiple data rates, we want to select a subset of theses links, such that all of the selected receivers can successfully decode their messages and the overall data rate is provably close to the maximum possible, i.e., provide approximation guarantees for the obtained solutions. We propose two formulations for this problem. In the first formulation, referred to as the *Multi-Rate Scheduling Problem*, the data-rates are given as part of the input. In the second formulation, referred to as the *Variable-Rate Scheduling Problem*, the assignment of a data-rate to each link selected to transmit is part of the solution. Note that there is a trade-off between the total communication data-rate and the number of scheduled requests. Setting a communication request to a higher data-rate requires lower interference coefficient, which results in fewer number of concurrent transmissions. Setting a communication request to a lower data-rate results in fewer bits transmitted.

In this work we use two different interference models to analyze and solve these problems. On the one hand, we want to provide provably feasible solutions in a model as realistic as possible. On the other hand, we would like to be able to use a simple enough model to be able to derive concise theoretical bounds for our results. We start by defining the problems in the *physical interference model*. In this model, a transmission is considered successful iff the signal-to-interference-plus-noise ratio (SINR) at each

☆ This work is based on preliminary conference versions [1] and [2]

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receiver is above a certain threshold, which depends on the radio hardware and the data-rate at which the transmission must be scheduled. This model is among the most realistic ones used in theoretical studies, but is also quite complex, making the process of designing algorithms quite challenging and does not allow the application of many existing analytical tools, such as, for example, graph-theoretic techniques.

Recently, it has been shown that some wireless link scheduling problems in the physical model can be approximated, to within constant factors, by graph-based models, provided that some input parameters are restricted. For example, in [3], it was shown that single-rate transmission conflicts in the physical model can be approximated by a *unit disk graph*, as long as constant transmission power is used and link lengths differ at most by a factor of 2. Such model transformation allows one to simplify the global and cumulative nature of wireless interference, as captured by the physical model, by using local, distance-based relations/edges of a geometric graph.

In this work, we build upon these results and use the “physical-to-graph model approximation” technique to solve wireless scheduling problems. By constructing a *disk graph* representation of each problem instance, we show that if a maximum weight independent set (MWIS) is computed on the disk graph, it provides a constant approximation to each problem in the physical interference model. We solve the problems by implementing a polynomial-time approximation scheme (PTAS) (as well as a parallel version of it). As opposed to many previous results in wireless networks that use graph-based models, this approach allows the application of graph-theoretic algorithmic tools, while guaranteeing feasible solutions and approximation bounds in the physical interference model, which is closer to reality.

The contributions of this work can be summarized as follows. Firstly, we present the model transformation process, showing how a graph-based problem instance can be built for the *Multi-Rate* and the *Variable-Rate Scheduling Problems*, keeping the desired SINR properties of the original model. Next, we prove that a solution to the MWIS problem on the graph instance provides a constant approximation to the original problem. This is an important step, since it allows to apply many existing graph-based algorithmic tools to the physical interference model, preserving approximation guarantees. Finally, we describe a PTAS solution for both problems, and show that good-quality solutions can be obtained.

This paper is structured in the following way. In Section 2 we describe some related work. In Section 3 we present the details of the modelling process and problem definitions. In Section 4 we show how to build a problem instance of the *Multi-Rate Scheduling Problem* in the disk graph model and prove that this model reduction is correct and guarantees a constant-factor approximation. In Section 5 we describe the model transformation process for the *Variable-Rate Scheduling Problem*. In Section 6 we describe the PTAS implementation. Finally, in Section 7 we present extensive simulation results.

2. Related work

Studying wireless networks in graph-based models usually involves studying the problems of coloring and independent set. Coloring a general graph is not only an NP-complete problem, but is also hard to approximate to within factor of $n^{1-\epsilon}$, for any constant $\epsilon > 0$ [4]. Wireless networks, however, can usually be better modeled by more restricted classes of graphs, such as geometric graphs. In [5], Clark et al. proved that a series of closely related problems to scheduling of wireless links, such as coloring in graphs, independent set, domination, independent domination, and connected domination, are NP-complete in unit disk graphs (UDGs). The maximum independent set problem can be approximated to within factor 5 using an online greedy algorithm [6] (which is optimum for an online deterministic algorithm) and factor 3 by a greedy offline algorithm [7]. In the case of disk graphs, the online greedy approach yields a $(n-1)$-approximation, whereas the greedy offline algorithm that processes disks in non-decreasing size order achieves a 5-approximation. Finally, the problem of maximum independent set in (unit) disk graphs can be approximated to within a factor $(1-\epsilon)$, for any fixed $\epsilon > 0$ using a polynomial-time approximation scheme (PTAS) [8,9]. In particular, Erlebach et al. [9] provide a PTAS for the weighted independent set problem in disk graphs. A short survey on disk graphs can be found in [10].

Analytical results in more realistic models, such as the physical interference model, are more recent. In [11] it was shown that the problem of scheduling wireless transmissions with uniform power assignment is NP-complete in the physical interference model. In [12], a constant approximation algorithm was proposed for the (maximization) one-slot scheduling problem and a logarithmic approximation for the (minimization) multi-slot problem. These results were derived for the single-rate scenario. Several other works studied different aspects of the problem, such as linear power assignment [13] and power control [14,15].

In [3] it was shown that single-rate scheduling in the SINR model can be approximated to within a constant factor by coloring a unit disk graph, in the case when constant transmission power is used and the link set is “nearly-equilength”, i.e., link lengths vary by at most a factor of two. In this work we take a step further, and show that a constant approximation can be achieved by approximating multi-rate scheduling with a disk graph with variable disk radii.

Joint channel assignment and routing has been investigated in [16]. Efficient channel assignment schemes can greatly relieve the interference effect of simultaneous transmissions while routing schemes can alleviate potential congestion. In [17], Kodialam et al. characterized the capacity region in multi-radio multi-channel wireless mesh networks and derived the upper bounds on the capacity in terms of achievable throughput. We solve the link scheduling problem, which is a fundamental problem in any wireless network. It is a building block that can be integrated into larger problems. For instance, it appears as a sub-problem in other problems, such as routing.

In [18], several versions of the wireless capacity problem were analyzed. Among them, one is referred to as “scheduling with QoS generalization and uniform power”, and is similar to the *Multi-Rate Scheduling Problem* studied here. A constant approximation was obtained for the case of arbitrary link lengths and general metric space. In [19], the problems of power control and data-rate maximization are studied in one formulation. The proposed solution is proved to be $O(\log n)$-approximation and computes both the power levels and the data-rates assignment to the selected links. Our work, on the other hand, provides a constant approximation, although to a more restricted range of scenarios, and considers uniform power assignment, which one might argue represents most practical scenarios.

3. Model

In this work we study the problem of scheduling wireless communication requests (links) in the physical interference model using multiple and variable data rates.

A typical problem instance consists of a set $L = \{\ell_1, \dots, \ell_n\}$ of n wireless links (and, as explained later, a set of data rates), where each link ℓ_i represents a communication request from a sender s_i to a receiver r_i: $\ell_i = (s_i, r_i)$. The communication devices are viewed as nodes positioned in a Euclidean space. The distance between a sender s_j and a receiver r_i is denoted by $d_{ji} = d(s_j, r_i)$, and the length of a link ℓ_i is denoted by $d_{ii} = d(s_i, r_i)$.

For simplicity's sake and without loss of generality, we assume that transmissions are slotted into *synchronized time slots* of equal length and there are no primary conflicts in the transmission setup, i.e., each node is either a sender or a receiver and each receiver is associated with only one sender. Scenarios with this type of conflicts could be reformulated by introducing additional nodes at the same position, such that there is one sender-receiver pair for each link. In this way, if a node is scheduled to transmit at a certain time slot, the interference at that location will be infinite and, therefore, no co-located node can be scheduled as the receiver in the same time-slot. Moreover, we assume that all nodes transmit with the same power level P, i.e., we use *uniform power assignment* scheme.[1]

In the physical interference model, a receiver r_i successfully decodes a transmission from a sender s_i iff

$$SINR_{\mathcal{S}_t}(\ell_i) = \frac{\frac{P}{d_{ii}^{\alpha}}}{\sum_{s_j \in \mathcal{S}_t, s_j \neq s_i} \frac{P}{d_{ji}^{\alpha}} + N} \geq \beta, \tag{1}$$

where $\mathcal{S}_t$ is the set of nodes concurrently transmitting in time-slot t; $\alpha > 2$ is a constant path-loss exponent, with typical values in the range $2 < \alpha \leq 6$; $d_{ii}^{-\alpha}$ is the propagation attenuation (link gain); N is a constant ambient noise; and β is the minimum signal-to-interference-plus-noise-ratio (SINR) required for a successful message decoding. Typically, when performing theoretical analysis, it is assumed that $\beta > 1$ and has the same value for all links in L.

Now let $\mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$ be a set of data rates, where $|\mathcal{T}|$ is bounded by a constant, and let $\beta(t_k) > 1, t_k \in \mathcal{T}$ denote a data-rate-dependent minimum SINR threshold required for a successful transmission with data rate t_k (note that, the higher the data rate t_k, the higher is the hardware threshold $\beta(t_k)$). We say that a successful transmission with data rate $t(\ell_i)$ (and corresponding hardware threshold $\beta(t(\ell_i))$) occurs when the following condition is satisfied:

$$SINR_{\mathcal{S}_t}(\ell_i) \geq \beta_i, \tag{2}$$

where β_i is an abbreviation we use for the SINR threshold $\beta(t(\ell_i))$ of a link ℓ_i that transmits with data rate $t(\ell_i) \in \mathcal{T}$.

In this work, we present two different problem definitions that incorporate the use of multiple data rates. To the first problem we refer as the *Multi-Rate Scheduling Problem*, to the second problem we refer as the *Variable-Rate Scheduling Problem*. In both problems, the objective is to select a subset of links that transmit successfully at the same time, while maximizing the total sum of their data-rates.[2] In the first problem, each link $\ell_i \in L$ is assigned a fixed data rate $t(\ell_i)$ and then used as input to the maximization program. In the second problem, a set of links and a set of possible data rates are used as input, and the objective is to select a subset of links *and assign* each link the "best" data rate to transmit with.

Definition 3.1. The *Multi-Rate Scheduling Problem* can be formulated as follows. The problem's input is a set of n tuples $\{(\ell_1, t(\ell_1)), \ldots, (\ell_n, t(\ell_n))\}$, where $\ell_i \in L = \{\ell_1, \ldots, \ell_n\}$ and $t(\ell_i) \in \mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$, such that each link $\ell_i \in L$ is assigned a data rate $t(\ell_i)$ and a hardware threshold $\beta_i = \beta(t(\ell_i))$. We define a binary variable x_i, such that:

$$x_i \begin{cases} = 1 & \textit{if link } \ell_i \textit{ transmits} \\ = 0 & \textit{otherwise.} \end{cases}$$

The objective of the problem is to pick a subset of links $\mathcal{S} \subseteq L$, such that the total data rate is maximized over one time-slot and the *SINR* is at least β_i at every scheduled receiver r_i.

$$\text{maximize} \quad \sum_{i=1}^{n} x_i t(\ell_i)$$

subject to:

$$SINR_{\mathcal{S}}(\ell_i) \geq x_i \beta_i, \quad \forall\ \ell_i \in L, \mathcal{S} = \{\ell_k | k \neq i \text{ and } x_k = 1\} \tag{3}$$
$$x_i \in \{0, 1\}.$$

Definition 3.2. The *Variable-Rate Scheduling Problem* can be defined as follows. The input is comprised by a set of links $L = \{\ell_1, \ldots, \ell_n\}$ and a set of data rates $\mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$, where $|\mathcal{T}|$ is bounded by a constant. A variable x_{ij} is defined as:

$$x_{ij} \begin{cases} = 1 & \textit{if link } \ell_i \in L \textit{ transmits with data rate } t_j \in \mathcal{T} \\ = 0 & \textit{otherwise.} \end{cases}$$

The objective of the problem is to select a subset of links $\mathcal{S} \subseteq L$ *and assign* each link $\ell_i \in \mathcal{S}$ a data rate $t(\ell_i) = t_j \in \mathcal{T}$, such that the total number of transmitted bits is maximized over one time-slot and the *SINR* is at least $\beta_i = \beta(t(\ell_i))$ at every scheduled receiver $r_i | \ell_i \in \mathcal{S}$. More formally:

$$\text{maximize} \quad \sum_{i=1}^{n} \sum_{j=1}^{|\mathcal{T}|} x_{ij} t_j$$

subject to:

$$\sum_{j=1}^{|\mathcal{T}|} x_{ij} \leq 1, \quad \forall\ \ell_i \in L,$$

$$SINR_{\mathcal{S}}(\ell_i) \geq \sum_{j=1}^{|\mathcal{T}|} x_{ij} \beta_i, \quad \forall\ \ell_i \in L, \mathcal{S} = \{\ell_k | k \neq i \text{ and } \sum_{j=1}^{|\mathcal{T}|} x_{kj} = 1\}$$
$$x_{ij} \in \{0, 1\}. \tag{4}$$

We make use of a series of definitions introduced in [3,12,20].

Definition 3.3. The *affectance* of link ℓ_v caused by a link ℓ_w is

$$a_w(\ell_v) = \frac{\frac{P}{d_{wv}^{\alpha}}}{\frac{P}{d_{vv}^{\alpha}}} = \left(\frac{d_{vv}}{d_{wv}}\right)^{\alpha}. \tag{5}$$

For convenience, let $a_v(\ell_v) = 0$. The combined affectance of a set S on link ℓ_v is denoted by $a_S(\ell_v) = \sum_{\ell_w \in S} a_w(\ell_v)$.

Note that, as proved in [20], the affectance function satisfies the following properties for a set S of links:

1. *(Range)* S is SINR-feasible if and only if, for all $\ell_v \in S$, $a_S(\ell_v) \leq 1/\beta_v$ (otherwise, there would be a link $\ell_v \in S$ such that $SINR_{\mathcal{S}}(\ell_v) < \beta_v$).
2. *(Additivity)* $a_S(\ell_v) = a_{S_1}(\ell_v) + a_{S_2}(\ell_v)$, whenever (S_1, S_2) is a partition of S.

4. Disk graph model approximation

In this section we show that when an instance of the *Multi-Rate Scheduling Problem* is composed by links that vary in length by at most a constant factor and are assigned data rates that also vary by at most a constant factor, it can be approximated by a disk graph, i.e., many-to-many interference relationships of the SINR model can be simplified by pairwise relationships, modulo small constant factors.

[1] In reality there exist many wireless networks where nodes can choose different transmission powers, however, either just from a small set of possible power levels, or where the power range is bounded. Apart from constants, the analytical results of this paper hold for both extensions.

[2] Note that maximizing the sum of data rates is equivalent to maximizing the total data transmitted, since we assume all transmissions occur for exactly one time slot.

The proof works by constructing two disk graphs where each sender has an associated disk. The graph used to derive an upper bound utilizes (small) disks that represent inevitable conflicts in the SINR model. The graph used to derive a lower bound utilizes (large) disks used to construct feasible schedules. We begin with a series of definitions: In Definitions 4.1 and 4.2, we characterize a set of links according to their affectance and relative distances, respectively. In Definition 4.3, we define a conflict graph of a set of links based on their relative distances. In Definition 4.4, we define a disk graph derived from a set of links, which will be used to prove the lower bound on the achievable data rate. In Definition 4.5, we define a unit disk graph derived from a set of links, which will be used to show the upper bound on the total achievable data rate.

Definition 4.1. Given a set of links $L = \{\ell_1, \dots, \ell_n\}$ and an (ordered) vector $p = \{p(l_1), \dots, p(l_n)\}$, we define L to be a ***p*-signal set** if the affectance of any link $\ell_i \in L$ is at most $1/p(\ell_i)$, i.e., $a_L(\ell_i) \le 1/p(\ell_i)$. Note that if $p = \{\beta_1, \dots, \beta_n\}$ then a p-signal set L is SINR-feasible, i.e., $SINR(\ell_i) \ge \beta_i$, $\forall \ell_i \in L$.

Definition 4.2. Given a set of links $L = \{\ell_1, \dots, \ell_n\}$ and a set of values $r = \{r(l_1), \dots, r(l_n)\}$, we define a set of links L to be ***r*-independent** if any two links ℓ_v, $\ell_w \in L$ satisfy the constraint $d_{vw} \cdot d_{wv} \ge r(\ell_v) r(\ell_w) d_{vv} d_{ww}$.

Definition 4.3. Given a set of links $L = \{\ell_1, \dots, \ell_n\}$ and a set of values $r = \{r(\ell_1), \dots, r(\ell_n)\}$, we define an ***r*-conflict graph $G_r(L)$** on the link set L to be a graph with a vertex for each link in L and an edge between two vertices (ℓ_v, ℓ_w) iff ℓ_v and ℓ_w are not r-independent.

Definition 4.4. Given a set of links $L = \{\ell_1, \dots, \ell_n\}$ and a set of values $z = \{z(\ell_1), \dots, z(\ell_n)\}, z(\ell_i) \ge 1, \forall \ell_i \in L$, we define a **disk graph *$DG_z(L)$*** on the link set L to be a graph with a vertex for each sender in L and an edge between two vertices (s_v, s_w) iff $d(s_v, s_w) < z(\ell_v) d_{vv} + z(\ell_w) d_{ww}$.

Definition 4.5. Given a set of links L with minimum link length $d_{\min}$ and a constant u, we define a **unit disk graph *$UDG_u(L)$*** on the link set L, to be a graph with a vertex for each sender in L and an edge between two vertices (s_v, s_w) iff $d(s_v, s_w) < u \cdot d_{\min}$.

In the following lemma, we establish a relation between affectance and relative distance of links.

Lemma 4.1. *If S is a p-signal set of links, where $p(\ell_i) = \beta_i, \forall \ell_i \in S$, then S is r-independent, where $r(\ell_i) = \beta_i^{1/\alpha}, \forall \ell_i \in S$.*

Proof. Since S is a p-signal set, any two links ℓ_v, $\ell_w \in S$ satisfy $\frac{P}{d_{vv}^{\alpha}} / \frac{P}{d_{wv}^{\alpha}} \ge \beta_v$ and $\frac{P}{d_{ww}^{\alpha}} / \frac{P}{d_{vw}^{\alpha}} \ge \beta_w$. By multiplying the inequalities, we get $d_{vw} \cdot d_{wv} \ge \beta_v^{1/\alpha} \beta_w^{1/\alpha} d_{vv} d_{ww}$, which means that they are r-independent. □

In Lemma 4.2, we provide the lower bound by proving that, if a function $z(\ell_i)$ is appropriately defined, and the corresponding disk graph ***$DG_z(L)$*** is built, then an independent set of disks corresponds to a conflict-free schedule of links in the SINR model.

Lemma 4.2. *Consider a set of links S, where each link ℓ_i is assigned a data rate $t(\ell_i) = t_k \in \mathcal{T}$ with corresponding hardware threshold $\beta_i = \beta(t_k)$, and uniform power assignment is used. If every two senders s_v, $s_w \in S$ are separated by a distance $d(s_v, s_w) \ge z(\ell_v) d_{vv} + z(\ell_w) d_{ww}$ (i.e., S is an independent set in the disk graph $DG_z(S)$), then S forms a p-signal set, where $p(\ell_i) = \beta_i, \forall \ell_i \in S$ (i.e., a set that can be scheduled concurrently without collisions in the SINR model). The values $z(\ell_i)$ are defined as follows:*

$$z(\ell_i) = \frac{g_i w}{d_{ii}}, \tag{6}$$

$$g_i = \left[\beta_i \cdot \left(\frac{d_{ii}}{w} \right)^{\alpha} \cdot \frac{\alpha - 1}{\alpha - 2} \cdot \alpha \cdot 4C \right]^{\frac{1}{\alpha - 2}}, \tag{7}$$

$$w = d_{\min} \cdot z_{\min}, \tag{8}$$

$$d_{\min} = d(r_{\min}, s_{\min}), \tag{9}$$

$$z_{\min} = z(l_{\min}) = \left(\frac{\beta(t(\ell_{\min}))\alpha \cdot 4C}{\alpha - 2} \right)^{\frac{1}{\alpha}}, \tag{10}$$

$$\ell_{\min} = \mathrm{argmin}_{\ell_i \in S} (\beta_i^{1/\alpha} d_{ii}), \tag{11}$$

$$r_{\min}, s_{\min} = \text{receiver and sender of } \ell_{\min} \tag{12}$$

$$C = \pi \sqrt{3}/6. \tag{13}$$

Proof. The proof is provided in the Appendix. □

In Lemma 4.3 we provide the upper bound by proving that if an appropriate UDG is built for a set of links, this UDG is a subgraph of the conflict graph based on relative distances of links.

Lemma 4.3. *Given a set of links L with minimum link length $d_{\min}$, a constant $u \ge 1$, and a set of values $r(\ell_i) = u, \forall \ell_i \in L$. If we consider an r-conflict graph $G_r(L)$ and a unit disk graph $UDG_{u-1}(L)$, we have that $UDG_{u-1}(L) \subseteq G_r(L)$.*

Proof. To prove the claim, let v and w be two adjacent nodes in $UDG_{u-1}(L)$. By Definition 4.5 of a unit disk graph, $d(s_v, s_w) < (u-1) d_{\min}$. By triangular inequality, $d_{vw} \le d(s_v, s_w) + d_{ww} < (u-1) d_{\min} + d_{ww} \le (u-1) d_{ww} + d_{ww} = u \cdot d_{ww}$. So is $d_{wv} < u \cdot d_{vv}$. Multiplying the two inequalities, we have that $d_{vw} d_{wv} < u^2 d_{vv} d_{ww} = r(\ell_v) r(\ell_w) d_{vv} d_{ww}$. Therefore, ℓ_v and ℓ_w are neighbors in the r-conflict graph $G_r(L)$. □

Theorem 4.4 summarizes the lower and upper bounds used in our model transformation.

Theorem 4.4. *Consider a set of links S with minimum link length $d_{\min}$, where each link ℓ_i is assigned a data rate $t(\ell_i) = t_k \in \mathcal{T}$ with corresponding hardware threshold $\beta_i = \beta(t_k)$ and uniform power level. Moreover, consider a constant $u = \beta_{\min}^{1/\alpha}$, where $\beta_{\min} = \min_{\ell_i \in L} \beta_i$, a set of values p, defined as $p(\ell_i) = \beta_i, \forall \ell_i \in L$ and a set of values z, defined in (6). Claim 1: Any independent set in the disk graph $DG_z(L)$ is a p-signal set, i.e., a set that can be scheduled concurrently without collisions in the SINR model. Claim 2: Any p-signal subset of L is an independent set in the unit disk graph $UDG_{u-1}(L)$.*

Proof. Claim 1: By Lemma 4.2, an independent set in $DG_z(L)$ is a p-signal set and, therefore, can be scheduled successfully in the SINR model. Claim 2: By Lemma 4.1, a p-signal subset of links is also r-independent, where $r(\ell_i) = p(\ell_i)^{1/\alpha} = \beta_i^{1/\alpha}, \forall \ell_i \in S$. Note that $u = \min_{\ell_i \in S} r(\ell_i)$ and, if a set is r-independent, it is also u-independent, since $d_{vw} \cdot d_{wv} \ge \beta_v^{1/\alpha} \beta_w^{1/\alpha} d_{vv} d_{ww} \ge \beta_{\min}^{2/\alpha} d_{vv} d_{ww}$. Therefore, any p-signal subset of L is an independent set in the conflict graph $G_u(L)$. By Lemma 4.3, it is then an independent set in $UDG_{u-1}(L)$ (since this is equivalent to the claim $UDG_{u-1}(L) \subseteq G_u(L)$). □

Finally, in Theorem 4.5 we prove the approximation ratio of the model transformation.

Theorem 4.5. *Consider two constants $\Delta_l \ge 1$, $\Delta_\beta \ge 1$, and a set of links L with link lengths in the range $[d_{\min}, \dots, \Delta_l \cdot d_{\min})$, where each link ℓ_i is assigned uniform power and a data rate $t(\ell_i) = t_k \in \mathcal{T}$ with corresponding hardware threshold $\beta_i = \beta(t_k)$ in the range $[\beta_{\min}, \dots, \Delta_\beta \cdot \beta_{\min})$. A solution to the maximum weight independent* *set problem MWIS in the disk graph $DG_z(L)$, z being a function defined in (6), is a* ***constant approximation*** *to the Multi-Rate Scheduling Problem in the SINR model. The approximation ratio ρ is defined as*

$$\rho = C(1+2a/b)^2, \tag{14}$$

$$b = \beta_{\min}^{1/\alpha} - 1, \tag{15}$$

$$a = 2\Delta_l \cdot w \cdot \max_{\ell_i \in L} g_i, \tag{16}$$

$$\max_{\ell_i \in L} g_i = \max_{\ell_i \in L} (\beta_i d_{ii}^{\alpha})^{\frac{1}{\alpha-2}} \left(\frac{4C\alpha(\alpha-1)}{(\alpha-2)w^{\alpha}} \right)^{\frac{1}{\alpha-2}},$$

$$\leq (\Delta_\beta \beta_{\min} (\Delta_l d_{\min})^{\alpha})^{\frac{1}{\alpha-2}} \left(\frac{4C\alpha(\alpha-1)}{(\alpha-2)w^{\alpha}} \right)^{\frac{1}{\alpha-2}}, \tag{17}$$

where C, w and $z_{\min}$ are defined in (13), (8), and (10), respectively.

Proof. *Upper Bound*: By Theorem 4.4, we know that for $p(\ell_i) = \beta_i, \forall \ell_i \in L$ and $u = \beta_{\min}^{1/\alpha}$, any *p*-signal subset of *L* is an independent set in the *unit disk graph* $UDG_{u-1}(L)$. In other words, if two links are too close to each other, i.e., are neighbors in $UDG_{u-1}(L)$, they are also neighbors in the conflict graph $G_u(L)$ and, therefore, not *u*-independent. By Lemma 4.1, they do not form a u^{α}-signal set, and, since $u^{\alpha} = \beta_{\min}$, they cannot be scheduled concurrently in the SINR model. This means that "small" disks of radius $d_{\min}(u-1)$ capture inevitable conflicts in the SINR model.

Lower Bound: By Theorem 4.4, we know that for *z* defined in (6), any independent set in the disk graph $DG_z(L)$ is a *p*-signal set, i.e., can be scheduled concurrently without collisions in the SINR model. This means that a solution to the MWIS problem in the disk graph model renders a feasible solution to the Multi-Rate Scheduling Problem in the SINR model.

Approximation Ratio: Firstly we observe that if we define a maximum disk radius as $d_{\min} \cdot R$, $R = 2\Delta_l \max_{\ell_i \in L} w \cdot g_i$, where *w* and g_i were defined in (8),(7), then the disk graph $DG_z(L)$, *z* defined in (6), is contained in the unit disk graph $UDG_R(L)$, i.e., $DG_z(L) \subseteq UDG_R(L)$. Let $a = R$ and $b = u - 1 = \beta_{\min}^{a/\alpha} - 1$. Note that $a \geq b$. Consider two unit disk graphs $UDG_a(L)$ and $UDG_b(L)$ with different radii but on the same set of links *L*. Now consider a link $l_v \in L$ with closed neighborhood $N_a = N_{UDG_a(L)}[l_v]$ in $UDG_a(L)$ and the induced subgraph $UDG_b(L)[N_a]$ on N_a in $UDG_b(L)$. The cardinality of a maximum independent set

$$|MIS(UDG_b(L)[N_a])| \leq C(1+2a/b)^2.$$

This is due to the fact that the nodes in an independent set in $UDG_b(L)$ form disjoint balls of radius $bd_{\min}/2$ centered at senders of the links. Senders in N_a are all contained in the ball $B(s_v, ad_{\min})$. This means that the (small) balls in the independent set $MIS(UDG_b(L)[N_a])$ are completely contained in the larger ball $B(s_v, (a+b/2)d_{\min})$. Therefore, we can apply the packing argument (19) to show that only a limited number of smaller disjoint balls can fit inside the larger ball, namely $|MIS(UDG_b(L)[N_a])| \leq \mathcal{P}(B(s_v, (a+b/2)d_{\min}), bd_{\min}/2) \leq C(1+2a/b)^2$.

This means that the Multi-Rate Scheduling Problem reduces, within constant factors, to MWIS in disk graphs. So any algorithm to solve MWIS on $DG_z(L)$ is a constant approximation for the Multi-Rate Scheduling Problem. The performance ratio of such algorithm is bounded by $C(1+2a/b)^2$. □

5. Variable-rate disk graph

In this section we extend the model transformation technique, proposed in Section 4, to transform an instance of the *Variable-Rate Scheduling Problem* in the physical model into a problem instance in a disk graph model. In contrast to what was done in Section 4 for the *Multi-Rate Scheduling Problem*, we need to include different data-rate assignment options in each problem instance, so that the solution selects an assignment that maximizes the total data-rate of transmitting links. In order to do this, instead of adding one disk for each link, we add $|\mathcal{T}|$ disks for each link, i.e., one disk for each combination of link $\ell_i \in L$ and data-rate $t_k \in \mathcal{T}$.

In order to construct a disk-graph problem instance, we use the following definition (which is an extension of Definition 4.4):

Definition 5.1. Given a set of links $L = \{\ell_1, \ldots, \ell_n\}$, a set of data-rates $\mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$, and a function $z'(\ell_i, \beta_k)$. Consider a set of disks $\mathcal{D}'_{z'} = \{D_{\ell_1,t_1}, \ldots, D_{\ell_1,t_{|\mathcal{T}|}}, D_{\ell_2,t_1}, \ldots, D_{\ell_2,t_{|\mathcal{T}|}}, \ldots, D_{\ell_n,t_{|\mathcal{T}|}}\}$ of size $n \times |\mathcal{T}|$, such that every disk D_{ℓ_i,t_k} has radius $z'(\ell_i, \beta_k)d_{ii}$, $\ell_i \in L$, $\beta_k \in \{\beta(t_1), \ldots, \beta(t_{|\mathcal{T}|})\}$ and is centered at the sender of each link $\ell_i \in L$. We define a **variable-rate disk graph$DG'_{z'}(L)$** to be a graph with a vertex for each disk center in $\mathcal{D}'_{z'}$ and an edge between two vertices $(v(\ell_i, t_k), v(\ell_j, t_m))$ iff $d(v(\ell_i, t_k), v(\ell_j, t_m)) < z'(\ell_i, t_k)d_{ii} + z'(\ell_j, t_m)d_{jj}$, i.e., if the two disks intersect. The weight of each vertex $v(\ell_i, t_k) \in DG'_{z'}(L)$ is assigned the value t_k.

In this way, each link in the problem's input is assigned $|\mathcal{T}|$ disks, and the radius of each disk is proportional to length d_{ii} of the link, multiplied by a factor (function *z*), which depends on the SINR threshold $\beta(t(\ell_k))$ and the link length (and the global path-loss parameter α). The higher the data-rate, the larger will be the radius of the corresponding disk. Higher values of α, on the other hand, require lower radii for the disks. Function z' is defined as follows (note that $r_{\min}$ and $s_{\min}$ are the receiver and the sender of link $\ell_{\min}$, respectively):

$$z'(\ell_i, \beta_k) = \frac{g_{i,k} w}{d_{ii}},$$

$$g_{i,k} = \left[\beta_k \cdot \left(\frac{d_{ii}}{w} \right)^{\alpha} \cdot \frac{\alpha-1}{\alpha-2} \cdot \alpha \cdot 4C \right]^{\frac{1}{\alpha-2}}, \tag{18}$$

where *w*, $d_{\min}$, $z_{\min}$, $\ell_{\min}$, and *C* are defined in (8), (9), (10), (11), and (13), respectively.

Note that a problem instance of size *n* in the SINR model is transformed into a problem instance of size $n \times |\mathcal{T}|$ in the disk model. This increase in problem size adds extra computation cost, but we will show that a solution to the MWIS problem in this new model results in a correct solution in the SINR model. Moreover, we show that the approximation ratio of the MWIS solution also holds for the *Variable-Rate Scheduling Problem* in the SINR model, up to a constant factor.

Firstly, we provide the lower bound in Lemma 5.1.

Lemma 5.1. *Consider a set of wireless links $L = \{\ell_1, \ldots, \ell_n\}$ that transmit under uniform power assignment and a set of data-rates $\mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$. Now consider a variable-rate disk graph $DG'_{z'}(L)$ on L and $\mathcal{T}$, as defined in 5.1. Any Independent Set (IS) on $DG'_{z'}(L)$ represents a feasible solution to the Variable-Rate Scheduling Problem, i.e., $IS(DG'_{z'}(L))$ will only contain disks corresponding to links that can be scheduled concurrently without collisions in the SINR model.*

Proof. We know from Section 4 that for a given assignment of data-rates to links, an independent set in the corresponding disk graph with fixed data-rates represents a feasible solution, i.e., no collisions in the SINR model. It is left to show that more than one disk corresponding to different data-rates but the same link $\ell_i \in L$ are never contained in the same independent set on $DG'_{z'}(L)$. Lets assume that it is true, i.e., $IS(DG'_{z'}(L))$ contains two disks D_{ℓ_i,t_k} and D_{ℓ_i,t_m} corresponding to the same link ℓ_i. This means their centers overlap and therefore the two disks intersect, and there is an edge between the corresponding vertices in $DG'_{z'}(L)$. This contradicts the assumption that $IS(DG'_{z'}(L))$ is an independent set, which completes the proof. □

Finally, we prove the approximation factor in Theorem 5.2.

Theorem 5.2. *Consider a set of wireless links $L = \{\ell_1, \ldots, \ell_n\}$ that transmit under uniform power assignment and a set of data-rates $\mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$. Now consider a variable-rate disk graph $DG'_{z'}(L)$ on L and $\mathcal{T}$. A solution to the maximum weight independent set problem MWIS in the disk graph $DG'_{z'}(L)$, z' being a function defined in (18), is a* ***constant approximation*** *to the* Variable-Rate Scheduling Problem *in the SINR model.*

Proof. By Lemma 5.1 we know that any independent set in $DG'_{z'}(L)$ is a feasible solution to the *Data-Rate Maximization Problem*. Also, from Section 4, we know that for a given assignment of data-rate to links, a solution to *MWIS* in the disk graph with fixed data-rates $DG_z(L) \subseteq DG'_{z'}(L)$ is a constant approximation to the link scheduling problem with fixed data rates. It remains to show that by having a separate disk for every possible data-rate at every link leads to an approximation for the *Variable-Rate Scheduling Problem*. Let's assume that a solution $MWIS_1$ to the MWIS problem on $DG'_{z'}(L)$ is more than a constant smaller than the optimum. This means that there is a subset of links $S_{opt} \in L$ with some data-rate assignment $t : \mathcal{T} \to L$ that is more than a constant "heavier" than $MWIS_1$. If we consider the links and the corresponding data-rates of S_{opt} and use it as input to build a disk graph $DG_z(S_{opt})$, as defined in 4.4, we know that a solution to MWIS on $DG_z(S_{opt})$, say $MWIS_2$, is a constant approximation to the *Multi-Rate Scheduling Problem*. We also know that $MWIS_2 \subseteq DG_z(S_{opt}) \subseteq DG'_{z'}(L)$ and that $MWIS_2$ is heavier than $MWIS_1$. This contradicts the assumption that $MWIS_1$ is a maximum-weight independent set of $DG'_{z'}(L)$ and is not a constant approximation to the *Variable-Rate Scheduling Problem*, which completes the proof. □

This means that the *Variable-Rate Scheduling Problem* reduces, within constant factors, to MWIS in disk graphs. So any algorithm to solve MWIS on $DG'_{z'}(L)$ is a constant approximation for the *Variable-Rate Scheduling Problem*.

6. Approximation algorithms

In this section we describe a polynomial-time approximation scheme (PTAS) that computes a maximum weight independet set (MWIS) of a disk graph. As shown in Sections 4 and 5, if we carefully construct the input graph instances, the MWIS solution represents a constant-factor approximation for the *Multi-Rate* and the *Variable-Rate Scheduling Problems*. We present two implementations of this algorithm: the first implementation, to which we refer as Disk-MRS, is used to solve the *Multi-Rate Scheduling Problem*; the second implementation, to which we refer as Data rate PPTAS, is a parallel implementation of Disk-MRS and is used to solve the *Variable-Rate Scheduling Problem*, given that it results in significantly larger disk graph instances.

6.1. Disk-MRS

In this section we describe the Disk-MRS algorithm and show how it can be used to solve the *Multi-Rate Scheduling Problem*.

The input to the problem is a set of n tuples $\{(\ell_1, t(\ell_1)), \ldots, (\ell_n, t(\ell_n))\}$, where $\ell_i \in L = \{\ell_1, \ldots, \ell_n\}$ and $t(\ell_i) \in \mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$, such that each link $\ell_i \in L$ is assigned a data rate $t(\ell_i)$ and a hardware threshold $\beta_i = \beta(t(\ell_i))$. Using results from Section 4, this input is transformed into a disk graph ***$DG_z(L)$***, comprised by a set of disks $\mathcal{D} = \{D_1, \ldots, D_n\}$, according to Definition 4.4, where each disk is assigned a weight $w(D_i) = t(\ell_i)$ and a radius $R(D_i) = d_{ii} \cdot z(\ell_i)$, where z is a function defined in 6.

Next, ***$DG_z(L)$*** is used as input to the algorithm proposed by Erlebach et al. in [9] for computing MWIS in disk graphs. We refer to this algorithm as MWIS-PTAS and outline its mechanism in Appendix B. The goal of MWIS-PTAS is to compute, for a given set of disks with variable weights and radii, a subset of disjoint (non-overlapping) disks with maximum total weight. Given an integer $K > 1$, the algorithm achieves an approximation ratio of $(1 + \frac{1}{K-1})^2$, i.e., as K gets larger, the approximation ratio gets arbitrarily close to 1. The output solution $\mathcal{S} \subseteq L$ is formed by links l_i, where l_i is part of the solution if disk D_i is part of the MWIS (see Algorithm 1).

Algorithm 1 Disk-MRS

Require: A set $L = \{\ell_1, \ldots, \ell_n\}$ of links with data ratest$(\ell_i) = t_k$, $t_k \epsilon \mathcal{T} = \{t_1, \ldots, t_{|\mathcal{T}|}\}$, and a parameter K.

Ensure: A subset $\mathcal{S} \subseteq L$ in which every link ℓ_i can betransmitted successfully with data rate $t(\ell_i)$ and the total weight$w(\mathcal{S})$ is maximized.

1: Generate $DG_z(L) = \{D_1, \ldots, D_n\}$, according to the Definition 4.4 of adisk graph and the function z (6);

2: Assign weights $w(D_i) = t(\ell_i), \forall D_i \in DG_z(L)$;

3: $\mathcal{S} =$ MWIS-PTAS(DG_z, K);

4: **return** $\mathcal{S}$.

6.2. Data-rate parallel PTAS

Data-Rate Parallel PTAS is a parallel implementation of Disk-MRS, which we apply to solve the *Variable-Rate Scheduling Problem*, given that it results in significantly larger disk graph instances.

The first step is to apply the results of Section 5 to transform the original problem input into a set of disks $\mathcal{D'}_{z'}$, using the Definition 5.1.

Parallel algorithms usually consist of a map phase and a reduction phase. In the map phase, which consists of mapping parallel instances to threads, each thread has a unique identifier. We represent every possible subset of n elements by a binary number. Each thread calculates the next potential solution, transforms it to a subset of links and checks if the potential solution is an independent set. If it is, the thread computes its weight and stores its local maximum solution. Obviously, sets with more than $16k^4$ elements are not computed since we know they can not be a solution, a property from the PTAS algorithm.

Each thread computes what the next potential solution is independently from the other threads, since the only information needed is the total number of threads, which is pre-configured, and the thread's identifier. The computation of the next potential solution is completely distributed and guarantees load balance among threads. The evaluation of each potential solution depends only on the disk graph structure. The disk graph structure is used only for reading and is distributed among all threads. Thus, the map phase is completely distributed.

The second phase is the reduction phase, which consists of synchronizing the solutions of the threads. This phase happens when all threads finished their local computation. In this phase, we collect the maximum value of each local thread and compute the maximum solution.

6.3. ApproxDiversity

In order to evaluate the performance of our disk-based approach, we compared the solution returned by Disk-MRS (see Algorithm 1) with the algorithm proposed in [12], to which we refer as ApproxDiversity. To the best of our knowledge, ApproxDiversity is the only algorithm to solve the weighted one-slot scheduling problem in the SINR model. It is an approximation algorithm for one-slot scheduling initially designed for single rate with SINR model without power control. We slightly modified it to handle the multi-rate case, as shown in Algorithm 2.

Algorithm 2 ApproxDiversity

Require: A set $L = \{\ell_1, \dots, \ell_n\}$ of links with data rate $t(l_i) = t_k$, $t_k \epsilon \mathcal{T} = \{t_1, \dots, t_{|\mathcal{T}|}\}$.

Ensure: Asubset L_j^k in which every link can be transmitted successfullyand the total weight $w(L_j^k)$ is maximized

1: $w(\ell_i) = t(\ell_i), \forall \ell_i \in L$;
2: Let $R = R_0, \dots, R_{\log(l_{\max})}$ such that R_k is the set of linksl_i of length $2^k \le d_{ii} < 2^{k+1}$;
3: $\mu = 4\left(\frac{8\beta_{max}(\alpha-1)}{\alpha-2}\right)^{\frac{1}{\alpha}}$;
4: **for all** $R_k \neq \emptyset$ **do**
5: Partition the plane into squares of width $\mu \cdot 2^k$;
6: 4-color the cells such that no two adjacent cells have the same color.
7: **for** $j = 1$ **to** 4 **do**
8: For each square A of color j, pick the *heaviest* link$l_i \in R_k$ with receiver r_i in A, assign it to L_j^k ($L_j^k = L_j^k \cup l_i$);
9: **end for**
10: **end for**
11: **return** $\text{argmax}_{L_j^k} \sum_{l_i \in L_j^k} w(l_i)$;

Table 1
IEEE 802.11b required SINR per data-rate.

Rates (Mbps)	1	2	5.5	11
SINR (dB)	4	6	8	10

Table 2
IEEE 802.11n 5Ghz 40MHZ required SINR per data-rate.

Rates (Mbps)	30	60	90	120	180	240	270	300
SINR (dB)	14	17	19	22	26	30	31	32

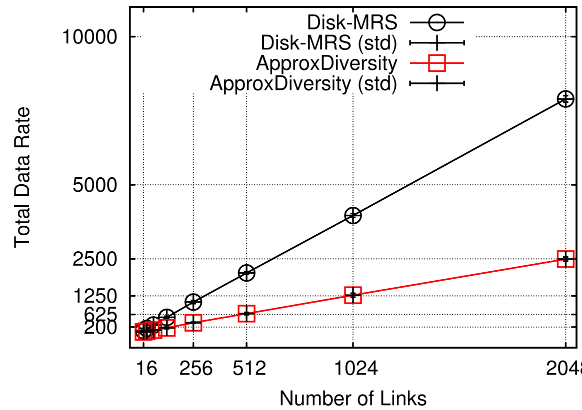

Fig. 1. Multi-Rate Results with $\alpha = 3, K = 4, 4 \le \beta_i \le 10$ (IEEE 802.11b).

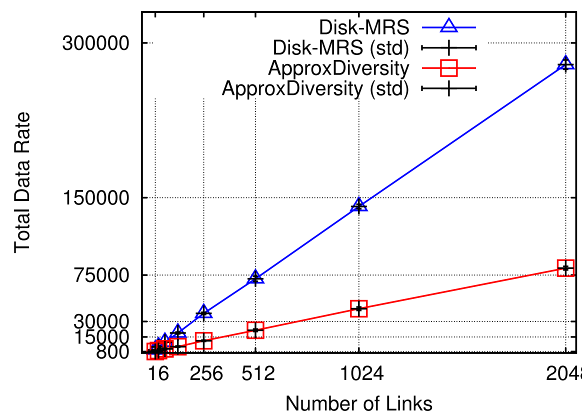

Fig. 2. Multi-Rate Results with $\alpha = 3, K = 4, 10 \le \beta_i \le 32$ (IEEE 802.11n).

ApproxDiversity works on the original link set L (no intermediate disk graph instance is built). The links are divided into link length classes $\{R_0, \dots, R_{\log(l_{\max})}\}$. Each link length class is partitioned into square cells of side μ, computed as a function of link length and parameters α and β of the SINR model. We used the maximum value $\beta_{\max}$ to compute the cell size, in order to guarantee no interference among cells no matter what data rate (β_i) is used (see line 2 of Algorithm 2). Thereafter, each cell is colored with 4 colors, such that no two adjacent cells have the same color. Then, for each cell, the heaviest link is chosen. In the end, the set of links with maximum total weight among all possible partitions (cell sizes) and colorings is returned.

7. Simulation results

In this section, we present simulations results, including a comparative study for the algorithms described in Section 6.

We generated random topologies by placing n receiver nodes uniformly distributed over a plane field of size 10^4 x 10^4 units. The respective n senders were uniformly distributed inside a disk of radius $l_{\max}$ around each of the receivers. In all experiments, the number of simulation runs was chosen to obtain sufficiently small confidence intervals. The standard deviation is plotted for all data points.

To model the multiple data rate links, we obtained their necessary SINR values from the IEEE 802.11 Standard [21], which defines the minimum SINR needed to decode a transmission at a given rate. Table 1 [22] shows the required SINR values for IEEE 802.11b and the respective data-rates. Table 2 [23] presents the necessary SINR values for the IEEE 802.11n with 40 MHz channel width on 5 GHz.

In our input sets, each link's data rate was uniformly chosen from Tables 1 or 2. The weight of a link was modeled as the data rate of the link, since it indicates how much data can be transmitted.

To begin with, we compare the performance of Disk-MRS (see Algorithm 1) to the performance of ApproxDiversity, proposed in [12] (see Algorithm 2).

Initially, we analyze the total data rate of the schedule as a function of the number of links. The number of links varied from 16 to 2048, $l_{\max} = 6\sqrt{2}$, Disk-MRS parameter $K = 4$ and $\alpha = 3$ unless otherwise noticed. Fig. 1 shows the total data rate as a function of the number of nodes scheduled by Disk-MRS and ApproxDiversity using IEEE 802.11b data rates (Table 1). Fig. 2 depicts the same experiment with IEEE 802.11n data rates (Table 2). Fig. 3 illustrates the results in terms of Disk-MRS gain, showing that Disk-MRS achieves an average factor 3 gain over ApproxDiversity. For all densities, the Disk-MRS algorithm achieved better results.

In Fig. 4, we study the influence of the path-loss exponent α on the performance of the algorithms. For all α, Disk-MRS algorithm computes schedules with higher total data rates.

Next, we evaluate the algorithms for instances where the data rate is constant (fixed $\beta_i = 4, \forall \ell_i \in L$). Fig. 5 illustrates the performance of both algorithms, once again Disk-MRS has a performance gain factor of about three.

Next, we analyze the performance of Disk-MRS algorithm by varying the approximation parameter K. The number of links is set to 512, β_i varies from 4 to 10, and $\alpha = 3$. Fig. 6 shows that, as K increases, a better approximation is achieved, as expected.

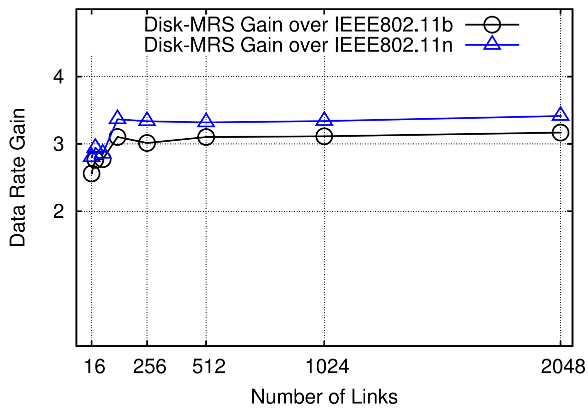

Fig. 3. Multi-Rate Gain with $\alpha = 3, K = 4, 4 \le \beta_i \le 10$.

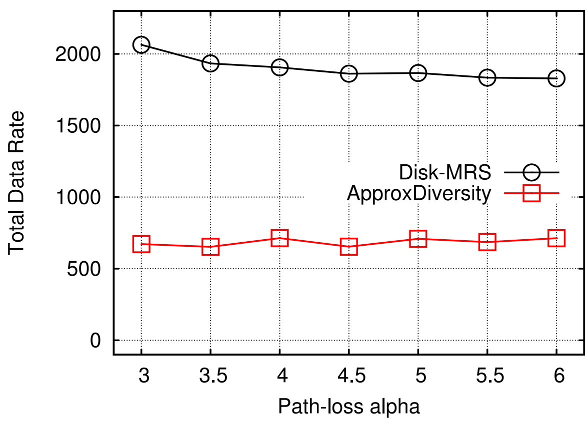

Fig. 4. Path-loss α Results with 512 links, $K = 4,\ 4 \le \beta_i \le 10$.

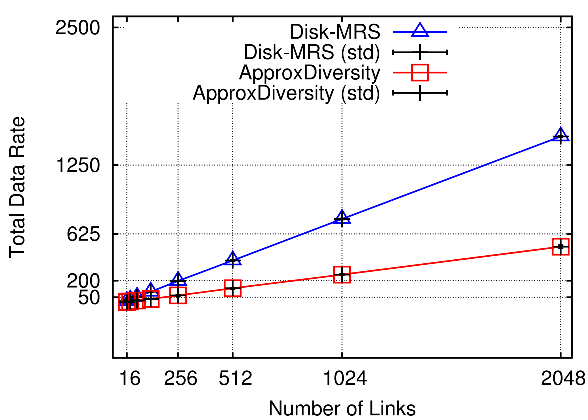

Fig. 5. Single Rate with $\alpha = 3, K = 4, \beta_i = 10, \forall \ell_i \in L$.

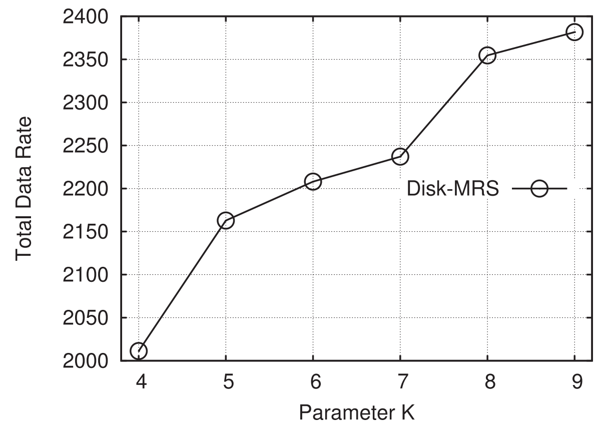

Fig. 6. PTAS parameter K evaluation with 512 links, $\alpha = 3,\ 4 \le \beta_i \le 10$.

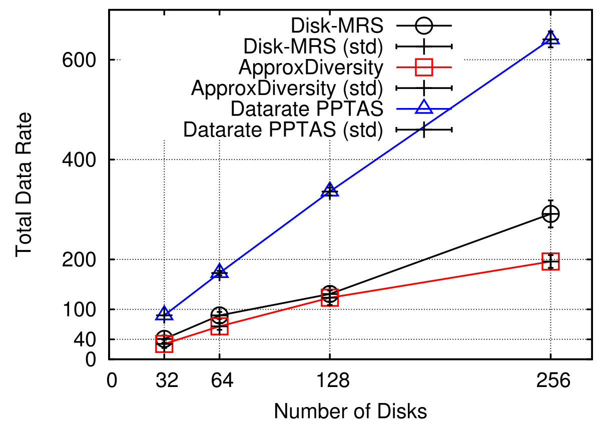

Fig. 7. Data-Rate results with $\alpha = 3, K = 4, 4 \le \beta_i \le 10$ (IEEE 802.11b).

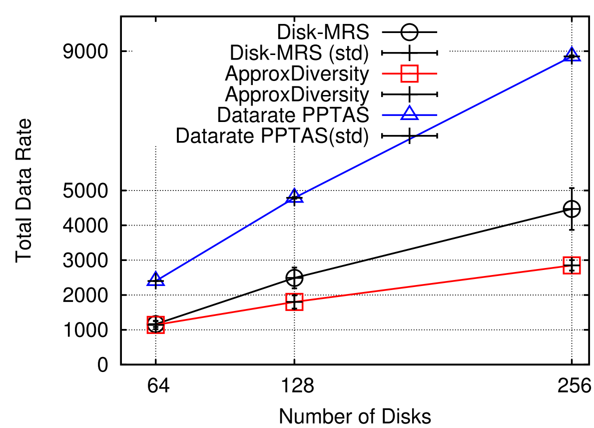

Fig. 8. Data-Rate results with $\alpha = 3, K = 4, 10 \le \beta_i \le 32$ (IEEE 802.11n).

In order to evaluate the performance of Data-Rate Parallel PTAS, we compared it both to ApproxDiversity and to Disk-MRS. We start by analyzing the total data-rate of the schedule as a function of the number of disks.[3] The number of links was set from 8 to 64, $l_{\max} = 6\sqrt{2}$, the Data-Rate Parallel PTAS parameter $K = 4$, and $\alpha = 3$, unless otherwise noticed. Fig. 7 depicts the total data-rate as a function of the number of disks scheduled by Data-Rate Parallel PTAS, Disk-MRS and ApproxDiversity using data-rates (set $\mathcal{T}$) from Table 1 (IEEE 802.11b). Fig. 8 shows the same experiment with data-rates from Table 2 (IEEE 802.11n). Since IEEE 802.11n has greater variety of data-rates, the Data-Rate Parallel PTAS gain over Disk-MRS and ApproxDiversity is greater than the gain obtained when comparing with the IEEE 802.11b. But, IEEE 802.11n has more data-rate choices, which is (significantly) more expensive to compute than 802.11b data-rates. Fig. 9 illustrates the results in terms of gain.

Next, we analyze the path-loss exponent α influence over the performance of the algorithms. Fig. 10 depicts their performance for α varying from 3 to 6. In all cases, Data-Rate Parallel PTAS algorithm obtained higher total data rate.

Figs. 11 and 12 illustrate the distribution of different values of data-rates for instances using 802.11b and 802.11n, respectively.

[3] Note that here we use the number of disks as the x-axis instead of the number of links, since it determines the actual size of the problem's input.

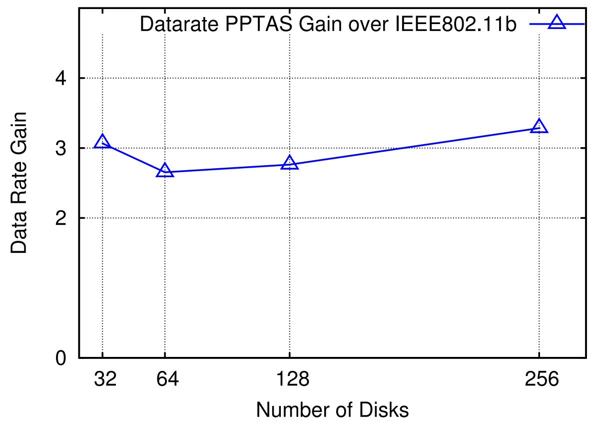

Fig. 9. Data-Rate results with $\alpha = 3, K = 4, 4 \leq \beta_i \leq 10$ (IEEE 802.11b).

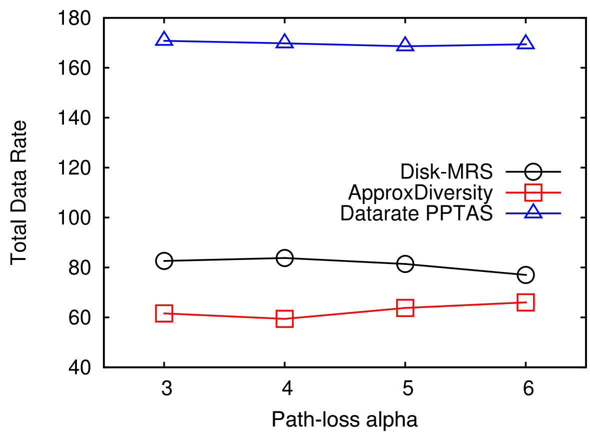

Fig. 10. Path-loss α results with 16 links, $K = 4$, $4 \leq \beta_i \leq 10$.

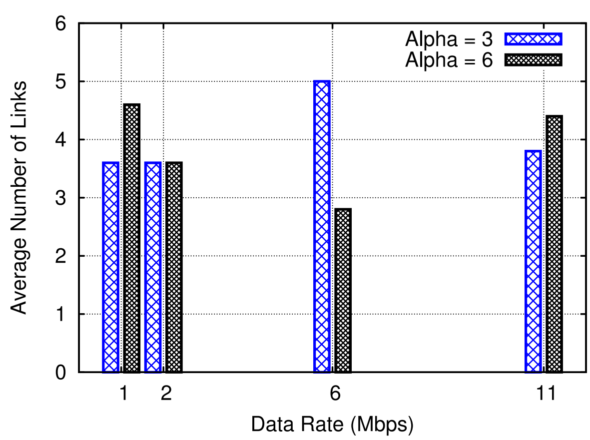

Fig. 11. Histogram of rates in the solution with link=16, $\alpha = 3$ and $\alpha = 6, K = 3, 4 \leq \beta_i \leq 10$ (IEEE 802.11b).

First, we can see that Data-Rate Parallel PTAS selects a variety of data-rates. We compare solutions with $\alpha = 3$ and $\alpha = 6$. As α increases, the energy dissipation coefficient also increases, decreasing interference from other links and bringing the opportunity to choose links with higher data-rates. This also allows to choose links with small data-rate that are not affected by links with high data-rate. In other words, the option of picking one big disk and several small ones over two medium disks becomes possible when the interference decays quicker (α values are higher).

We evaluate the scalability of the Parallel PTAS by running on a computer with 16 processors. Fig. 13 shows the execution time

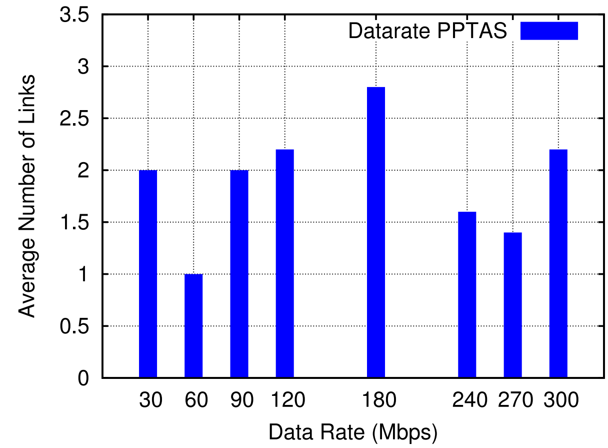

Fig. 12. Histogram of rates in the solution with link=16, $\alpha = 3, K = 3, 10 \leq \beta_i \leq 32 (IEEE802.11n)$.

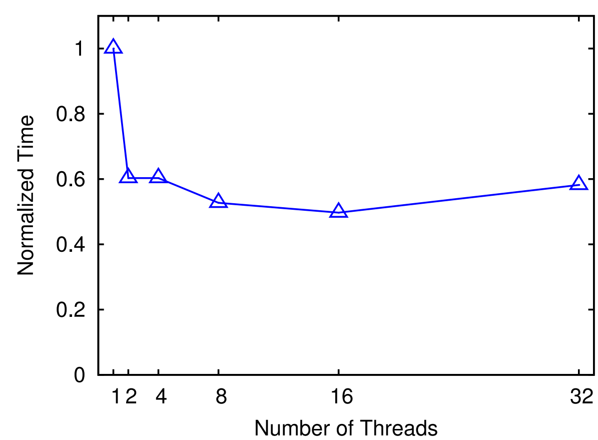

Fig. 13. Normalized time for Parallel PTAS with various number of threads with link $= 32$, $\alpha = 3, K = 3, 4 \leq \beta_i \leq 10$ (IEEE 802.11b).

per number of threads. The y-axis is the completion time normalized by executing time of 1 thread. As we increase the number of threads, the execution time decreases. There is no benefit running with 32 threads since the computer has 16 processors and there is the overhead of thread context switch.

Note that Figs. 1–4 and 7 fig0008 fig0009–10, which analyze the performance of the proposed algortihm with variable SINR parameters β_i and α, demonstrate the robustness (low sensitivity) of this solution to changes in the SINR model parameters.

To sum up, the simulations indicated that the proposed approach is both practical, since we were able to implement and solve various problem instances, and presents advantages in various scenarios, including different densities of links, number of data-rates and path-loss exponent.

8. Conclusion

In this paper, we studied the problem of scheduling wireless requests in the physical interference model with multiple and variable data rates. We proposed a method of solving two versions of the problem by providing intermediate representations as disk graphs. As opposed to the majority of previous results on graph-based models, our approach allows the application of graph-theoretic algorithmic tools, while guaranteeing feasible solutions in the physical interference model, which is closer to reality than graph models.

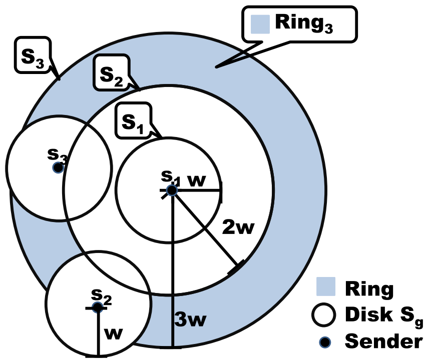

Fig. 14. Balls of radius w are disjoint.

We showed that, in a Euclidean space, where the path loss exponent is strictly larger than two, the problem can be modeled as a disk graph if the data rates and sender-receiver distances differ by a contact factor between communication requests. We showed how to build the corresponding disk graph instances. Moreover, we implemented a polynomial-time approximation scheme algorithm.

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Appendix A. Proof of Lemma 4.2

Proof. Let S' denote the set of senders in S. Note that, by assumption, senders in S' are separated by distance at least $2w$ (since $z(\ell_v)d_{vv} + z(\ell_w)d_{ww} \geq 2z_{\min}d_{\min} = 2w$)

We will use the result from [24], where it is proved that the maximum number $\mathcal{P}$ of balls of radius r that can be "packed" into a ball $B(x, t \cdot r)$ of radius $t \cdot r$, $t > 0$ centered at any point x (in a 2-dimensional doubling metric space) is bounded by

$$\mathcal{P}(B(x, tr), r) \leq Ct^2, \quad \text{where} \quad C = \pi\sqrt{3}/6. \tag{19}$$

We will first prove the result for the "minimum-radius" link $\ell_{\min}$ (11) and then extend it to an arbitrary link $\ell_i \in S$. Let $g \geq 1$ be a number and $s_{\min} \in S'$ be the sender of $\ell_{\min}$. Let $S_g = \{s_y \in S' | d(s_{\min}, s_y) < gw\}$ be the set of senders within distance less than gw from $s_{\min}$, and let $Ring_g = S_g \setminus S_{g-1}$. By construction, $S_2 = \emptyset$ (see Fig. 14). The senders in $Ring_g$ are of distance at least $(g-1)w$ from $s_{\min}$, so the affectance of each sender s_y in $Ring_g$ on $\ell_{\min}$ is at most

$$\begin{aligned} a_{s_y \in Ring_g}(l_{\min}) &= \frac{1/d(s_y, r_{\min})^\alpha}{1/d_{\min}^\alpha} \\ &\leq \frac{1/(w(g-1))^\alpha}{1/d_{\min}^\alpha} \\ &= \frac{1}{(z_{\min}(g-1))^\alpha}, \end{aligned} \tag{20}$$

$\forall l_y \in Ring_g$. Then, the overall affectance on $\ell_{\min}$ can be bounded by

$$\begin{aligned} a_{S'}(\ell_{\min}) &= \sum_{g \geq 3} a_{Ring_g}(\ell_{\min}) \\ &\leq \sum_{g \geq 3} |S_g \setminus S_{g-1}| \cdot \left(\frac{1}{z_{\min}(g-1)}\right)^\alpha \\ &= \left(\frac{1}{z_{\min}}\right)^\alpha \cdot \sum_{g \geq 3} |S_g| \left(\frac{1}{(g-1)^\alpha} - \frac{1}{g^\alpha}\right) \end{aligned} \tag{21}$$

$$\leq \left(\frac{1}{z_{\min}}\right)^\alpha \cdot \sum_{g \geq 3} |S_g| \frac{\alpha}{(g-1)^{\alpha+1}}. \tag{22}$$

Equality (21) follows from the definition of $Ring_g$, which is comprised by the senders in ball S_g minus the senders in ball S_{g-1}, and inequality (22) follows from

$$\frac{1}{(g-1)^\alpha} - \frac{1}{g^\alpha} = \frac{g^\alpha - (g-1)^\alpha}{g^\alpha (g-1)^\alpha} \leq \frac{\alpha g^{\alpha-1}}{g^\alpha (g-1)^\alpha} < \frac{\alpha}{(g-1)^{\alpha+1}}$$

Using the packing bound (19) and the fact that all balls of radius w are contained in the ball $B(s_{\min}, (g+1)w)$, for $g \geq 3$ we have that $|S_g| \leq \mathcal{P}(B(s_{\min}, (g+1)w), w) \leq C(g+1)^2$, and therefore

$$\frac{|S_g|}{(g-1)^{\alpha+1}} \leq \frac{C(g+1)^2}{(g-1)^{\alpha+1}} \leq \frac{4C}{(g-1)^{\alpha-1}}.$$

The overall affectance on $\ell_{\min}$ can therefore be bounded by

$$\begin{aligned} a_{S'}(\ell_{\min}) &\leq \left(\frac{1}{z_{\min}}\right)^\alpha \cdot \alpha \cdot 4C \cdot \sum_{g \geq 2} \frac{1}{g^{\alpha-1}} \\ &\leq \left(\frac{1}{z_{\min}}\right)^\alpha \cdot \alpha \cdot 4C \cdot \frac{1}{\alpha-2} \\ &= \frac{1}{\beta(t(\ell_{\min}))}. \end{aligned} \tag{23}$$

Inequality (23) follows from the fact that $\sum_{x=2}^{\infty} 1/x^k \leq \int_1^\infty 1/x^k dx = 1/(k-1)$, and the last equality follows by plugging in the value of $z_{\min}$, defined in (10).

This proves our claim for the "minimum-radius" link, i.e., that $\ell_{\min}$ is part of a p-signal set, $p(\ell_i) = \beta_i, \forall \ell_i \in S$. Now let's consider an arbitrary link $\ell_i \in S$. Note that, by assumption, an area of radius $d_{ii} \cdot z(\ell_i) = g_i \cdot w = g_i \cdot d_{\min}z_{\min}$ (see definition (6) of $z(\ell_i)$) around any sender s_i does not contain any other senders in S'. Therefore, the interference on ℓ_i comes from rings $Ring_g$, $g > g_i$. Using the same reasoning as for $\ell_{\min}$, we can bound the affectance on ℓ_i by senders $y \in Ring_g$ as

$$a_y(\ell_i) = \frac{1/d_{yi}^\alpha}{1/d_{ii}^\alpha} \leq \frac{1/(w(g-1))^\alpha}{1/d_{ii}^\alpha} = \left(\frac{d_{ii}}{w(g-1)}\right)^\alpha$$

$$\begin{aligned} a_{S'}(\ell_i) &\leq \left(\frac{d_{ii}}{w}\right)^\alpha \cdot \alpha \cdot 4C \cdot \sum_{g \geq g_i} \frac{1}{(g)^{\alpha-1}} \\ &\leq \left(\frac{d_{ii}}{w}\right)^\alpha \cdot \alpha \cdot 4C \cdot \left(\frac{\alpha-1}{\alpha-2} \cdot \frac{1}{g_i^{(\alpha-2)}}\right) \leq \frac{1}{\beta_i}. \end{aligned}$$

The last inequality follows by plugging in the values of w, defined in (8) and g_i, defined in (7). Since all links $\ell_i \in S$ are affected by at most $1/\beta_i$, set S forms a p-signal set, $p(\ell_i) = \beta_i, \forall \ell_i \in S$, i.e., a set that can be scheduled concurrently without collisions in the SINR model, which completes the proof. □

Appendix B. MWIS-PTAS algorithm

The MWIS-PTAS algorithm works as follows. First it partitions the plane into quadrants. Then it applies the so-called shifting strategy to compute an approximated solution. The finite area of the quadrants provides an upper bound for the computation cost of the MWIS. Since the PTAS technique is quite expensive in terms of processing power, we propose a parallel implementation. Now, we describe each of these steps in more details.

Initially the various disks are partitioned into levels according to their diameters as following. Let $k > 1$ be a fixed positive integer. First, we scale all disks in such a way that the largest disk will have diameter equal to 1. Then, let $d_{\min}$ be the smallest diameter among all scaled disks. The disks $\mathcal{D}'_z$ are partitioned into $l+1$

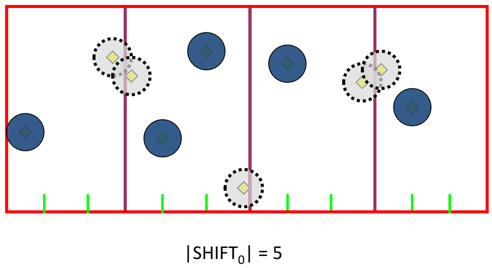

Fig. 15. Shift strategy from line 0.

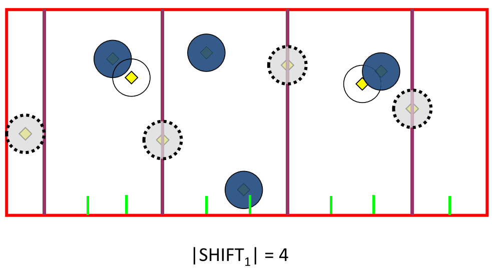

Fig. 16. Shift strategy from line 1.

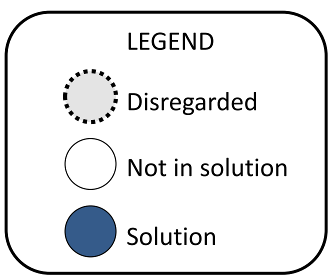

Fig. 17. Meanings of colors.

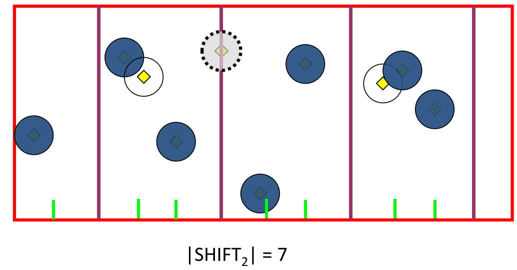

Fig. 18. Shift strategy from line 2.

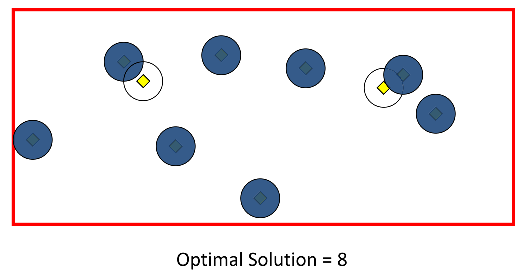

Fig. 19. Optimal solution.

levels, where l is the floor of $\log_{k+1}(1/d_{\min})$. Level j, $0 \le j \le l$, consists of disks D_i satisfying $(k+1)^{-j} \le R(D_i) \le (k+1)^{-(j+1)}$, where $R(D_i)$ is radius of disk D_i.

Next, we subdivide the plane into a grid, comprised of vertical and horizontal lines that are $(k+1)^{-j}$ apart from each other. Then, we apply the shifting strategy to compute the approximation solution into each square region of the grid.

The shifting strategy works as follows. We illustrate it with an instance of the MWIS problem (Fig. 15). The first step of the shifting strategy divides the plane into stripes (actually, it divides into a grid but for simplicity we explain the strategy in one dimension). Then, the shifting strategy combines the stripes into sub-areas of k continuous stripes (call it k-stripes). Let $k = 3$ for our example. The size of the stripes was previously explained in Section 8. The Shifting Strategy consists of shifting the combined k continuous stripes, trying all possible k-stripes partitions. In this way, the strategy tries many divisions of the space. Fig. 15 illustrates the case when we combine three stripes and the offset is 0. Fig. 16 shows the division when the offset is 1. Finally, Fig. 18 depicts the regions when the offset is 2.

When dividing the area into stripes, disks that touch the line are disregarded for computation since these disks could conflict with other disks in the other area. Fig. 17 shows the disk legend. The disregarded disk are indicated by light gray and dotted lines.

The second step consists of computing the optimal solution for each k-stripes. Computing the optimal solution might be computationally expensive, but the key idea here is that the quadrant-to-disk-area ratio is bounded, so the number of possible independent sets per quadrant is also bounded (we give more details later on). The dark blue (dark grey) disks indicate the disks that are part of the solution. The white disks are the disks that are not part of the MWIS solution.

The third step is to compute the final solution for each offset, which is the union of the k-stripes solutions. Figs. 15, 16 and 18 have the following solutions, respectively: 5, 4 and 7 (assuming all disks have weight 1).

The final solution for the shifting strategy is the one with the highest total weight, in our example, it is Fig. 18. Observe that the optimal solution is 8 (Fig. 19) but the shifting strategy solution is 7, since it is an approximation algorithm.

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