

Ní Ríordáin, M., Coben, D., & Miller-Reilly, B. (2015). What Do We Know about Mathematics Teaching and Learning of Multilingual Adults and Why Does it Matter? *Adults Learning Mathematics: An International Journal*, 10(1), 8-23

What Do We Know about Mathematics Teaching and Learning of Multilingual Adults and Why Does it Matter?

Máire Ní Ríordáin

National University of Ireland
<maire.niriordain@nuigalway.ie>

Diana Coben

University of Waikato
<dccoben@waikato.ac.nz>

Barbara Miller-Reilly

University of Auckland
<b.miller-reilly@auckland.ac.nz>

Abstract

The significant role of language in mathematics teaching and learning is not a new phenomenon. Given the growth of cultural and economic migration, the increasing international focus on education for economic development and the widespread use of English as a language for learning, we have become acutely aware of the importance of language in adults' mathematics learning. While investigation has been undertaken in relation to the role of language in the learning and teaching of mathematics at primary and second level, little research has been done on multilingual (including bilingual) adults' learning of mathematics and the ways in which teaching might support such learning. In this paper we investigate the role of language in the mathematics and numeracy education of bi/multilingual adults with a focus on the mathematics register and discourse; we address the relationship between language(s) and learning; we provide a review of available literature specific to adult learners; and discuss implications for adult mathematics education.

Keywords: language, adult mathematics education, numeracy, bilingual/multilingual learners

Introduction

This paper is the latest in a series of papers for which the impetus was an international comparative study of adult numeracy education, focusing initially on the UK and New Zealand. That endeavour has so far resulted in a series of papers presented at successive international conferences of Adults Learning Mathematics – A Research Forum (ALM), starting at ALM17 in Oslo, Norway. At ALM20 in Newport, South Wales, we investigated language policy and adult numeracy education in Wales and New Zealand, focusing on the Māori language in New Zealand and the Welsh language in Wales (Coben & Miller-Reilly, 2014). In our ALM20 paper we noted that while much has been written about the relationship between language and literacy, the relationship between language and numeracy - especially adult numeracy - has been less explored, in particular from a policy perspective, despite evidence of the importance of language for learning. Accordingly, in that paper we sought to

Volume 10(1) – August 2015

Copyright © 2015 by the authors. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution License](#), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

shed light on the policy context in which adult numeracy education is set in Wales and New Zealand with respect to those languages, viewed from a critical linguistic human rights perspective.

The need for further examination of the specific role of language in adult mathematics education is thus an important consideration emerging from our previous research. Language and communication are essential elements of teaching and learning mathematics, as is evident from research carried out in bi/multilingual settings (Gorgorió & Planas, 2001). Language facilitates the transmission of (mathematical) knowledge, values and beliefs, as well as cultural practices. Language is also the channel of communication within a mathematics classroom as language provides the tool for teacher-student and student-student interaction. Accordingly, we decided to examine the role of language in teaching and learning mathematics with and to adults. In this paper we are collaborating with Máire Ní Ríordáin, who has explored the role of language in mathematics learning with respect to Gaeilge, the Irish language (Ní Ríordáin, 2013; Ní Ríordáin & O'Donoghue, 2007, 2008; Ní Ríordáin & O'Donoghue, 2009, 2011). Our theoretical investigation focuses on the following main question: What is the role of language in the mathematics and numeracy education of bi/multilingual adults? This main question is broken down into the following sub-questions which are addressed in turn in the following sections:

- What can we learn from research on language and multilingualism?
- How should we understand the relationship between language and mathematics?
- What is meant by the ‘mathematics register’ and why is it important?
- What factors impede and promote mathematics learning in an additional language?
- What do we know about mathematics/numeracy education, language and adult learning?
- What are the implications for adult mathematics education in bi/multilingual settings?

Before answering these questions we wish to make clear that we recognise that there are differences between bilingualism and multilingualism, with their respective consequences for mathematics teaching and learning, and we use both terms in this paper as well as the composite term ‘bi/ multilingualism’ when we mean to indicate that these can be considered together. Similarly, terms such as ESL (English as a Second Language), ELF (English as a Lingua Franca), EAL (English as an Additional Language) and English as a Second or Other Language (ESOL) are used throughout this paper. We are aware that there are differences between these contexts for bilingual and multilingual learners and that more than one language may be involved (e.g., Breton as well as French in France, multiple languages in South Africa). However, this paper is presenting a theoretical perspective and is drawing on available literature published in English viewed through the lens of adult mathematics and numeracy education. We aim to be as comprehensive as possible within the scope of this paper because we want to contribute to opening up this important area for further research and development. We believe that to limit our investigation to either bilingualism or multilingualism, or to one context over another, might result in not presenting appropriate available research to help guide future research developments.

What can we learn from research on language and multilingualism?

There is growing recognition that language (and bilingualism/multilingualism) plays a key role in mathematics teaching and learning. Given the increase in international migration, the changing status of minority/indigenous groups and the dominance of English as a language for learning and teaching mathematics, many students face a transition to learning mathematics through the medium of another language (Barwell, Barton & Setati, 2007). Much diverse research has been undertaken on the effect of bilingualism/multilingualism on

mathematics education, but it is beyond the scope of this paper to address all aspects. However, research on language and multilingualism from a range of perspectives has been reviewed by Canagarajah and Wurr (2011). They highlight the burgeoning research on English as a Lingua Franca (ELF) in which ELF is a locally achieved practice adopting negotiation strategies to achieve intelligibility and constructing intersubjective norms that are sufficient to achieve their communicative objectives. They point to an emerging synthesis in the research literature which treats:

- Competence as an adaptive response of finding equilibrium between one's resources and the factors in the context (participants, objectives, situational details), rather than a cognitive mastery of rational control;
- Cognition as working in context, *in situ*, distributed across diverse participants and social actors;
- Proficiency as not applying mental rules to situations, but aligning one's resources to situational demands, and shaping the environment to match the language resources one brings.

(Canagarajah & Wurr, 2011, p. 11)

We take this synthesis as the starting point of our exploration of language and multilingualism in relation to adult mathematics education. It chimes with our professional and academic experience in that it situates the language user (in our case, the adult learner of mathematics) as actively seeking to adapt and align their language use with the demands of the context. We draw on Grosjean's (1999) concept of bilingualism as a *continuum of modes*, with bilingual individuals using their languages independently and together depending on the context and purpose. We think this may shed light on some of the factors that promote and impede the mathematics learning of bilingual and multilingual adults. In particular, we are concerned with how bi/multilingual adult learners may use their languages when engaged in mathematical discourse and the process of learning, and how these language(s) may provide a set of linguistic resources for social and cognitive purposes, as described by Zahner and Moschkovich (2011). If competence is "an adaptive response of finding equilibrium between one's resources and the factors in the context (participants, objectives, situational details)" as Canagarajah and Wurr (2011, p. 11) propose, then adult mathematics and numeracy educators need to accommodate this in their teaching and learners need to know this is an effective strategy, albeit with some pitfalls for the unwary, rather than 'wrong'. Accordingly, it is of importance to examine the role of language in teaching and learning mathematics.

How should we understand the relationship between language and mathematics?

Given the significant role of language for the teaching and learning of mathematics, the language through which we initially learn mathematics will provide the mathematical foundations to be built upon and developed within that language (Gorgorió & Planas, 2001). A significant body of research exists supporting the view that different linguistic characteristics may impact on cognitive processing. Influential psychologists and educationalists, including Vygotsky and Bruner, have investigated the nature and relationship between language and thought. The primary concern to emerge from this research is whether language follows thought, thus making language a means for expressing our thoughts, or whether language determines and is a prerequisite for our thoughts (Brodie, 1989). The relationship between language and thought is extremely complex and conflicting views exist in the literature. However, the general consensus in cognitive science is to presume that thinking is occurring in some language (Sierpinska, 1994). Vygotsky was one of the earliest theorists to begin researching the area of learning and its association with language. He concluded that language is inextricably linked with thought – "the concept does not attain to individual and independent life until it had found a distinct linguistic embodiment"

(Vygotsky, 1962, p. 4). Although a thought comes to life in external speech, in inner speech energy is focused on words to facilitate the generation of a thought. If this is the case, it raises an important question – does the nature of the language used affect the nature of the thought processes themselves? The transition from thought to language is complex as thought has its own structure. It is not an automatic process and thought only comes into being through meaning and fulfils itself in words. Thought is mediated both externally by signs and internally by word meanings (Vygotsky, 1962). Bruner (1975) emphasises that it is the use of language as an instrument of thinking that is of importance, as well as its effect on cognitive processing. Therefore, thought is intimately linked with language and ultimately conforms to it.

Mathematical language is considered as a distinct ‘register’ within a natural language. Therefore, the mathematics register in Irish will be different from the mathematics register in English, with each language possessing distinct ways and structures for expressing mathematical meaning and concepts. Of concern here is what the consequences of differences in languages are for adult mathematics learners. For example, do the (mathematical) thinking processes of those learning mathematics through the medium of Irish differ from those learning mathematics through the medium of English? How are languages utilised in and impact on developments in mathematical discourses in adult education? The concept of the structure of a language impacting on thought processes is referred to as the linguistic-relativity hypothesis (Sapir, 1949; Whorf, 1956). The basic premise of this hypothesis is that the vocabulary and phraseology of a particular language influences the thinking and perception of speakers of this language, and that conceptions not encoded in their language will not be available to them. Hence, they are proposing that each language will have a different cognitive system and that this cognitive system will influence the speaker’s perception of concepts (Whorf, 1956). Therefore, in theory, an Irish speaker/learner should have a different cognitive system to that of an English speaker/learner, influence our actions and accordingly may influence mathematical understanding. For example, Miura et al. (1994, p. 410) contend that ‘numerical language characteristics (*East-Asian languages*) may have a significant effect on cognitive representation of number’. However, other researchers have questioned argued for the difficulty in applying the linguistic-relativity hypothesis and the difficulty in testing such claims in relation to mathematical thinking (Towse & Saxton, 1997). We acknowledge that this may be too strong of a way of viewing the influence of language on the mathematical thinking and less severe forms of this hypothesis have been proposed. We support the premise that language may not shape and determine our entire mathematical thinking, but that it may influence it to a certain degree and facilitates our thinking and perception (Sternberg & Sternberg, 2012). It follows from interpretation of this theory that the language through which we speak/learn facilitates our thinking and perception. When working with bilingual/multilingual learners, we need to be acutely aware of their languages and how these languages may impact on their mathematical thinking and learning as language is necessary to facilitate mental representation and manipulation of written mathematical text (Sierpinska, 1994).

‘Understanding can be thought of as an actual or a potential mental experience’ (Sierpinska, 1994, p. 1). Sierpinska defines these mental experiences as ‘acts of understanding’ as distinct from ‘an understanding’, which is the potential to experience an act of understanding. These acts of understanding occur at a particular time and are short in duration. In education, understanding is often correlated with cognitive activity over a longer period of time. In this ‘process of understanding’, ‘acts of understanding’ represent the important steps while the attained ‘understandings’ represent the supports for further development (Sierpinska, 1994). For many, understanding is often associated with meaning and/or understanding why (e.g., Piaget, 1978). Understanding can be described in relation to meaning, while meaning can be described in terms of understanding, thus heightening the confusion surrounding the topic. In order to be consistent in explaining the association between meaning and understanding, we consider that ‘the object of understanding is the same as the object of meaning: it is the sign broadly understood’ (Sierpinska, 1994, p. 23). Therefore, the concept/thought forms the basis

of our understanding, while what we seek to understand are the signs that embody these concepts/thoughts. Because language and thought are interrelated (Bruner, 1975; Vygotsky, 1962) and thought is engaged in our understanding, then language is involved in developing our mathematical understanding. Understanding unveils a meaning: learners move from what the text states to grasping what the text is articulating (Sierpinska, 1994).

In 1979 Cummins refined his Threshold Hypothesis and this led to the development of his Developmental Interdependence Hypothesis, which had a more in-depth focus on the relationship between a student's two (or more) languages. The Interdependence Hypothesis proposed that the level of proficiency and use already achieved by a student in their first language would have an influence on the development of the student's proficiency and use of their second language. Cummins (1980) also addresses the importance of recognising that both languages interact and are stored together internally (Common Underlying Proficiency). Therefore, the impact of the first (and second, third, etc.) language of learning for mathematics is significant and needs examination when investigating bilingual/multilingual students. In particular, investigation is needed into how a particular language and its syntactical structure may impact on mathematical activity and reasoning (Morgan, Tang & Sfard, 2011). Galligan's (2001) extensive literature review in relation to differences between English and Chinese is the most significant review to be undertaken in relation to mathematics. She found that considerable differences exist in orthography, syntax, semantics and phonetics between the Chinese and English languages and that these differences may impact on the processing of mathematical text. However, few other studies specifically exist in relation to a comparison of languages and impact on mathematics learning (Cai, 1998). In particular, there is a dearth of quality research in the relationship of adult numeracy teaching and learning and language, and it is to this issue that we turn next.

Moreover, we see mathematics as a discourse and a type of communication (Sfard, 2012). Discourse is more than just language. Our usage is close to that of Gee's Discourse, which he distinguishes with an upper-case D, while discourse (with a lower-case d) refers to language-in-use (Gee, 1990). As defined by Gee (1996, p. 131):

A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts,' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role.

By employing this definition, Discourses are more than verbal and written language and the use of technical language; Discourses also involve communities, points of view, beliefs and values, and pieces of work. Moschkovich (2012, p. 95) utilises the phrase 'mathematics Discourse practices' to draw attention to the fact that Discourses are embedded in sociocultural practices. The following description is provided:

On the one hand, mathematical Discourse practices are social, cultural and discursive because they arise from communities and mark membership in different Discourse communities. On the other hand, they are also cognitive, because they involve thinking, signs, tools and meanings. Mathematical Discourses are embedded in sociocultural practices. Words, utterances or texts have different meanings, functions and goals depending on the practices in which they are embedded. Mathematical Discourses occur in the context of practices and practices are tied to communities. Mathematical Discourse practices are constituted by actions, meanings for utterances, foci of attention and goals: these actions, meanings, foci and goals are embedded in practices.'

(Moschkovich, 2012, p. 95)

Therefore, mathematical Discourse practices involve multi-semiotic systems (e.g. speech, writing, gestures, images, etc.) and thus are of importance when analysing mathematical teaching and learning in relation to bi/multilingual adults. Accordingly, we stress the importance of other factors such as exposure to mathematics, teaching strategies employed

and culture as influencing attainment in mathematics, not just language (Towse & Saxton, 1997).

What is meant by the ‘mathematics register’ and why is it important?

We consider mathematical language as a distinct ‘register’ within a natural language, e.g., Gaeilge or English or French, which is described as “a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings” (Halliday, 1975, p. 65). One aspect of the mathematics register consists of the special vocabulary used in mathematics (Gibbs & Orton, 1994); it is the language specific to a particular situation type (Lemke, 1989). However, the mathematics register is more than just vocabulary and technical terms. It also contains words, phrases and methods of arguing within a given situation, conveyed through the use of natural language (Pimm, 1987). The grammar and vocabulary of the specialist language are not a matter of style but rather methods for expressing very diverse things (Ellerton & Wallace, 2004). Therefore, each language will have its own distinct mathematics register, encompassing ways in which mathematical meaning is expressed in that language. As is evident, the complex ‘register’ of mathematics is similar to a language *per se* and requires learning skills similar to those used in learning a language. This adds another dimension to mathematics learning and reinforces the view that the content of mathematics is not taught without language. The process of learning mathematics inevitably involves the mastery of the mathematics register (Setati, 2005). This allows students to communicate their mathematical findings in a suitable manner; “without this fluency, students are restricted in the ways that they can develop or redefine their mathematical understandings” (Meaney, 2005, p. 129). Developing a learner’s mathematical register provides them with analytical, descriptive and problem solving skills within a language and a structure through which they can explain a wide range of experiences. Once the register is mastered, learners will have the ability to listen, question and discuss, together with an ability to read, record and participate in mathematics. However, when working with adult bilingual/multilingual learners we need to be cognisant of each language having its own distinct mathematics register and ways of communicating mathematics. Similarly, registers exist in many disciplines (e.g., science, technology) but likewise ordinary/everyday English language can be classified as a register. The mathematics and ordinary language registers can interfere, often in subtle ways, in a learning environment. Thus learners need to recognise each of these registers so as to identify which is being used at any given time (Sierpinska, 1994) and this is a challenge many bi/multilingual learners encounter. We need to be aware of how different languages and registers, and use of multiple languages and registers, may help support the development of mathematical knowledge (Ní Ríordáin, 2013).

What factors impede and promote mathematics learning in an additional language?

It follows from the foregoing discussion of the mathematics register that mathematics is far from ‘language free’ and its particular vocabulary, syntax and discourse can cause problems for students learning it in a second language (Barton & Neville-Barton, 2003). Similarly, the concepts presented here are not limited to second language learners – these language features can impact on monolingual learners of mathematics. However, while many students who learn mathematics in their mother-tongue (e.g. Gaeilge) have difficulty in acquiring the mathematics register, this is heightened for those who must learn it in a second language (e.g., English). Learners have to cope with the new mathematics register, as well as the new language in which the mathematics is being taught (Setati & Adler, 2000). Some of the language features that may impede mathematical learning are discussed in the following sections. We propose that knowledge of such language factors can be utilised by teachers of bilingual/multilingual adult mathematics learners in order to promote and support mathematics learning.

A key issue that causes significant problems for second language learners (as well as monolingual learners) is the number of ‘borrowed’ words from everyday English (Pimm, 1987). These words tend to be ambiguous due to having one meaning in the mathematics register, while having another meaning in everyday use (Yushau & Bokhari, 2005). Examples of such words include average, degree, even, odd, operation, etc. The non-mathematical meanings of these terms can influence mathematical understanding, as well as being a source of confusion. Also, Rudner (1978) found that language features such as conditions (if, when); comparatives (greater than, the most); negatives (not, without); inferential (should, could); low information pronouns (it, something) can be sources of difficulty and hinder students’ interpretation and understanding of mathematical word problems. Similarly, the use of specialist terms can lead to misinterpretation of mathematical tasks. Students tend to only encounter these terms within the mathematics classroom (for example, ‘quadrilateral’, ‘parallelogram’ and ‘hypotenuse’) and they are unlikely to be reinforced outside of it (Pimm, 1987). If second language learners do not acquire their correct meaning then this can lead to difficulties within the mathematics context. Second language learners have a tendency to translate new mathematical terms/vocabulary into their mother-tongue. There may be no equivalent translation and/or the translation may be done incorrectly, thus resulting in further confusion and misinterpretation (Graham, 1988). However, what is also worth considering is that specialist terms in some languages actually facilitate access to meaning and accordingly could be incorporated into teaching and supporting mathematics development for bilingual/multilingual adult learners. Such examples include the development of terms in Māori (Barton, Fairhall & Trinick, 1998), Chinese (Galligan, 2001) and terminology in the Irish language where, for example, velocity is ‘treolus’ which when directly translated is ‘direction speed’ (Ní Ríordáin, 2013).

Context is also a key issue: ‘Words can change their meaning depending on their context within the mathematics lesson’ (Gibbs & Orton, 1994, p. 98). In terms of language analysis, this is known as semantics – establishing the meaning in language, or the relationship and representation between signs and symbols. Due to the multiple meanings that various words can have, the context is vital in determining the correct interpretation. This is very much connected to the use of specialist terms in mathematics and multiple meanings of words (as discussed in the previous paragraph). This is by no means limited to the English language. For example, in Chinese context is essential as the language has relatively little grammar (Galligan, 2001). Findings from a review of literature found that children experience more difficulties with the semantic structure of word problems than with other contributing factors such as the vocabulary and symbolism of mathematics and standard arithmetic (Ellerton & Clarkson, 1996). Working within a bi/multilingual mathematics learning context would require attention being given to context and semantics and impact on mathematics learning. Finally, symbolism is one of the most distinctive features of mathematics, for example $>$, $<$, $/$, Σ . It is crucial for the construction and development of mathematics. Unfortunately “symbolism can accordingly cause considerable difficulties to those whose mother language has different structures” (Austin & Howson, 1979, p. 176). One of the requirements for mathematical learning is that students can interpret the mathematical text and convert it to an appropriate symbolic representation, and perform mathematical operations with these symbols (Brodie, 1989). Thus if students cannot understand the text (due to the language medium) they will be unable to convert it to the appropriate mathematical construction needed to solve the problem. Symbols provide structure, allow manipulation, and provide for reflection on the task completed.

What do we know about mathematics/numeracy education, language and adult learning?

While research has been done on language and mathematics learning, this has tended to focus on children's rather than adults' learning (e.g. Barwell, Leung, Morgan, & Street, 2002; Chval, Pinnow, & Thomas, 2015; Cuevas, 1984). A literature review by Benseman, Sutton and Lander (2005a) in New Zealand designed to provide a "critical evaluation" of the available research evidence on effective practices in literacy, numeracy and language (LNL) teaching and educational programme provision found "a dearth of specific research relating to this area in New Zealand and the situation is only marginally better overseas" and they were not able to identify any research that met the criteria for their review with regard to "factors associated with progress in numeracy" (Benseman et al., 2005a, p. 5). Earlier, Coben stated in the first major literature review of research in, and relevant to, adult numeracy:

The research domain of adult numeracy is fast-developing but still under-researched and under-theorised. It may be understood in relation to mathematics education, as well as to adult literacy and language, and to lifelong learning generally.

(Coben et al., 2003, p. 117)

Similarly, a review of the literature in adult numeracy conducted in the U.S.A. found, in common with the findings of earlier reviews (Coben et al., 2003; Tout & Schmitt, 2002), that "very few research studies have used Adult Basic Education students to study the effects of adult numeracy instruction, and the research that does exist is neither theory-driven nor guided by any systematic approach" (Condelli et al., 2006, p. 23).

In the years following these reviews, the National Research and Development Centre for Adult Literacy and Numeracy (NRDC) in England undertook a series of 'effective practice' studies, including a study of effective practice in adult numeracy (Coben et al., 2007). The summary of findings for that study highlighted the diversity of adult numeracy learners and education, including the fact that many learners were bi/multilingual:

The heterogeneous nature of adult numeracy teaching, the range of learners and the number of variables amongst teachers and learners, make it difficult to identify effective practices and factors that can be generalised with confidence across the whole sector. [...] There were particular difficulties for adults with lower ability levels, and with reading or language difficulties (two in every five learners in the sample spoke English as an additional language).

(Coben et al., 2007, p. 10)

Also, over the period from 1998 to 2007, the Longitudinal Study of Adult Learning (LSAL) investigated the development of 'literacy' (subsuming 'numeracy' into 'literacy') in adult life of a target population of about 1000 high school dropouts. At the start of the study, participants were aged 18-44, proficient but not necessarily native English speakers, and residents of Portland, Oregon, USA. Perhaps because the participants were all proficient English speakers, language issues do not feature in the findings, that: literacy development varies and continues to develop in adult life after leaving school: age has an effect on literacy growth; literacy measures are correlated; life history events have effects on literacy development; participation in programmes and self-study have patterns of effect on literacy development; and there are strong effects of programme participation on adults' subsequent perceptions of improved literacy (Reder, 2012). Also, long-term effects on proficiency bear out the predictions of practice engagement theory (Reder, 1994; Sheehan-Holt & Smith, 2000) in that engagement in literacy practices leads to growth in literacy proficiency and literacy development in adulthood affects employment and earnings (Reder, 2012). The NRDC 'Effective Practice in Adult Numeracy' study and LSAL have to an extent filled the gap identified by Coben et al. in 2003 and Benseman et al. in 2005a, but still much remains to be done before we can say that understanding of instructional impacts on adults learning numeracy/mathematics and language is sufficiently strong to support clear guidance on teaching and learning.

Research is also limited on the impact of workplace numeracy programmes, and on workplace ESOL (English for Speakers of Other Languages) and ESL (English as a Second Language) programmes. For example, Barker writes with regard to workplace literacy: “Empirical studies on the impact of workplace literacy programmes are not common, indeed the whole area of evaluation of training is underdeveloped” (Barker, 2001, p. 28). Gray (2006, p. 45) states that “information on outcomes from workplace ESL instruction is also lacking. The research studies that do exist are generally case studies or qualitative research.” A review by the American Institutes for Research (AIR) which focused on ESL students in Adult Basic Education also found very few studies with Adult Basic Education and ESL students which demonstrated a statistically significant impact of instruction (Condelli, Wrigley & Yoon, 2008). As was the case also for Benseman, Sutton and Lander’s literature review (Benseman et al., 2005a), the scope of the criteria used to select studies was extended. Condelli, Wrigley and Koon found that over the last 30 years there had been extensive international research into second language, as distinct from (first language) literacy, teaching and learning, but this was not the case in the numeracy field. A search for research about numeracy for ESL students by an AIR team in 2006 revealed none: “There exists no research base at all on how numeracy is taught in ESL classes, let alone studies that examine instructional approaches and their impact on these learners” (Condelli et al., 2006, p. 34).

In the first major review of adult numeracy only a few studies were found that addressed research on numeracy and ESL students and it was concluded that “Very little is known about learning and teaching adult numeracy with adults who speak languages other than English; research is needed in this area” (Coben et al., 2003, p. 118). This conclusion is echoed by AIR researchers, who suggested five areas for further research in order to move the field of Adult Basic Education forward. The fifth area suggested by the AIR researchers for further research is relevant to this paper, namely, to “examine (instruction for) ESL learners and students with learning disabilities” since these areas are “completely neglected” (Condelli et al., 2006, p. 61). The review goes on to state:

We found no research on how to provide instruction to these learners, on how they learn, or on how to address the challenges these learners face in learning mathematics. Research needs to pay particular attention to instruction for adult ESL learners, who make up over 40 percent of students in adult literacy programmes.

(Condelli et al., 2006, p. 62)

Benseman, Sutton and Lander (Benseman et al., 2005a) found that various factors appear likely to enhance learner gain in literacy, numeracy and language. These include appropriately skilled teachers able to identify learners’ strengths and weaknesses in speaking, reading, writing and numeracy and undertaking deliberate, explicit and sustained acts of teaching, clearly focused on learners’ diagnosed needs, using a curriculum linked to learners’ lived experience. Programmes of over 100 hours of tuition are recommended, allowing for high levels of participation, using a range of clearly structured teaching methods. They recommend ongoing assessment, taking account of the variation in learners’ skills and a focus on efforts to retain learners, including pro-active management of the positive and negative forces that help and hinder persistence as well as family-focused programmes with a clear focus on literacy and numeracy development. They also contend that ESOL programmes may need to be longer and should be structured to maximise oral communication, discussion and group work based on real-world situations, texts and tasks. Computers and multi-media technology can provide useful support, with bi-lingual teaching to explain concepts and learning tasks.

The authors note that an “emphasis on individualized teaching and learning may not support the needs of adult ESOL learners” for talking and interaction and such an individualised approach may be more common in adult numeracy classrooms than in other areas of education (Benseman et al., 2005a, p. 78). In addition, they state that: “the use of everyday, culturally-specific situations to contextualise maths problems may act as a barrier to attainment for ESOL learners in numeracy classes, when they don’t have either sufficient

language knowledge or contextual experience" (Benseman et al., 2005a, p. 78). Such cautionary notes are necessary, since, as Ciancone (1996) acknowledges, some educators working with adult learners may not be experienced with the teaching and learning of mathematics in a contextualised manner and linked with literacy. Marr's (2000) work demonstrates "that aspects of language acquisition will develop when supplemented with conceptual tasks and activities that focus on the written and oral use of mathematical understandings" (Coben et al., 2003, p. 113). Numeracy taught in this way might benefit all adult students as they learn the mathematics register, and might particularly help students learning English as an additional language in the numeracy classroom. A kit designed to develop the adult numeracy skills of literacy and mathematics trained teachers in Australia using a participatory workshop approach includes a chapter on 'Language and Maths' which includes theoretical and background information and provides guidelines for integrating literacy and numeracy with adult learners (but not ESOL learners) in the classroom (Marr & Helme, 1991).

In particular, studies focused on the teaching of mathematics/numeracy to multilingual adult learners are limited. The aim of an observational study of 15 literacy, numeracy and language teachers (Benseman, Sutton & Lander, 2005b, p. 92) was to gain an overview of how teachers teach literacy, numeracy and language in New Zealand. In the event, little numeracy teaching was observed in this study. These teachers came from tertiary institutions, community organisations, workplaces and private training establishments. The authors of the review comment on the diversity of the sector and the challenges of teaching adults literacy, numeracy and language. They conclude that teachers' commitment, empathy and support for their learners stem from the teachers' strong belief in the value of their programmes and their intrinsic interest in their work. The range of teaching methods observed was limited (both generic and literacy, numeracy and language-related). The authors note that generic teaching and classroom management skills play a significant role in literacy, numeracy and language teaching and the teachers appeared to use the same teaching strategies for ESOL as for others for whom English was a first language.

Ciancone gives guidelines for teaching numeracy in an ESL literacy programme, drawing on a range of sources (Ciancone & Jay, 1991; Kallenbach, 1994; Leonelli & Schwendeman, 1994; Lucas, Dondertman & Ciancone, 1991). He recommends: encouraging learners to look for patterns rather than just finding the right answer; pointing out to them that there may be many ways to solve the same problem; encouraging peer-group collaboration: he argues that the best way to clarify one's own understanding of a concept is to explain it to someone else. He also recommends encouraging learners to write journals about their mathematics learning and their feelings about learning mathematics, because using the language of mathematics reinforces both the mathematical concepts taught and the learner's proficiency in English. He notes that although numeracy is an everyday coping skill, mathematical concepts can be quite abstract and advocates using more concrete and visual explanations to facilitate understanding of the abstract concept. He advocates that each numeracy lesson should provide a balance between skill building and functional needs. For example, a lesson might begin with a problem (e.g. a mistake on a pay check) that provides a context for learning new skills (such as subtracting decimals), or it might start with a skill (e.g. adding decimals) followed by practical applications (such as adding sales tax to a fast food bill). He argues that mathematics should be included in literacy instruction from the beginning and that even learners who have almost no proficiency in English need to learn numbers for such basic activities as shopping and riding the bus (Ciancone, 1996, p. 4).

Finally, consideration must be given to EAL adults in post-compulsory mathematics education (e.g. undergraduate). Barton, Chan, King, Neville-Barton and Sneddon (2005), investigating the issue surrounding the learning of mathematics at university by students who have English as an Additional Language (EAL students), showed that the problems experienced by these students are not experienced by students whose first language is English (L1 students). EAL students struggle with their learning of mathematics in English at

undergraduate level much more than has been appreciated. The effect is masked at Year 1 undergraduate level because of the better mathematical prior knowledge of EAL students and the relatively low language requirements at this level. However, the effect in the third year is much greater. This study of third-year undergraduate students confirms for the first time that specific features of mathematical discourse cause difficulty for EAL students. Discourse density and logical structure are particularly confirmed as critical in this study, although comprehending mathematical discourse as a whole is also found to be much more complex than anticipated. In addition, these EAL students are unaware of their difficulties. These authors suggest that Departments of Mathematics need to acknowledge and address this issue in realistic ways.

What are the implications for adult mathematics education in bi/multilingual settings?

As we have seen, the literature specifically dealing with *adult* mathematics education in bi/multilingual settings is sparse. The considerably greater body of research on children's mathematics learning in relation to language, and on language learning more generally, may give some indications of ways forward. For example, Winslow concludes that

The view of mathematics as characterized by a specific linguistic register, including an analysis of the nature and functions of this *mathematical register*, enabled me to provide several arguments for mathematics teaching in an ambitious sense which emphasises communicative abilities developed thereby, and to formulate the arguments in a way which indicate the position of mathematics among other forms of communication in which human knowledge (and school subjects) appear. In particular, I have stressed the importance of general fluency in the mathematical register for global, noise-free human interaction, for participation in human society and for popular engagement in the dialogue between humanity and nature. I have pointed out the contours of possible need for a complete revision of mathematics teaching, both in the choice of emphasis and in its temporal placement within the educational system.

(Winslow, 1998, p. 23)

Winslow's 'linguistic approach to the justification problem in mathematics education' chimes with the findings of Schleppegrell's (2007) review of research, which suggests that focusing on the features of the language through which mathematics is constructed can be a strategy for engaging students and supporting their learning. Schleppegrell stresses that "focus on meaning, not form, is key to all discussion about mathematics concepts" (Schleppegrell, 2007, p. 151). However, she points out that exposure to the mathematics register through teacher's talk or textbook, or interaction with peers, will not in itself lead to learners developing the mathematics register. She cites research by Adams (2003) who suggests that teachers can help students to move from everyday language into the mathematics register by helping students recognize and use technical language rather than informal language when they are defining and explaining concepts; by working to develop connections between the everyday meanings of words and their mathematical meanings, especially for ambiguous terms, homonyms, and similar-sounding words; and by explicitly evaluating students' ability to use technical language appropriately. There is a tension between the formal register of academic mathematics and the 'everyday' or 'functional' focus of many adult numeracy programmes. Evidence-based ways forward have been proposed by various researchers. For example, contributors to an edited collection exploring the mathematics education of Latinos/as present research that grounds mathematics instruction with and for these learners in the resources to be found in culture and language. Language (e.g., bilingualism) is thus not framed as an obstacle to learning, but as a resource for mathematical reasoning and learning, in and out of school (Téllez, Moschkovich & Civil, 2011). Prediger, Clarkson and Bose (2012) also propose a way forward for teaching in multilingual contexts. They do this through purposefully relating multilingual registers. They explore the overlap between the three

different strong ideas related to different language registers and discourses: code switching, transitions between informal and academic (mathematical) forms of language within a given language, and transitions between different mathematical representations. They argue that integrating these ideas has the potential to enhance language-sensitive teaching strategies in multilingual classrooms that aim for conceptual understanding: an insight that might be fruitful for adult educators. Schleppegrell argues for more active engagement with the mathematics register and concludes that:

We have seen that features of the mathematics register can be identified and analysed by students to see how meaning is made in mathematics. Teachers can support the development of the multi-semiotic mathematics register through oral language that moves from the everyday to the technical mode. Students can be encouraged to produce extended discourse in mathematics classrooms, engage in discussion about the language through which word problems are constructed, and practice the writing of mathematics concepts in authentic ways. Teachers can become aware of the linguistic issues in learning and teaching mathematics and can develop tools for talking about language in ways that enable them to engage productively with students in constructing mathematics knowledge. Further research by applied linguists and mathematics educators can explore the linguistic challenges of mathematics learning in its multi-semiotic complexity to provide more support for teachers who want to engage struggling learners.

(Schleppegrell, 2007, pp. 156-157)

We conclude that there is a clear need to develop specific recommendations in relation to multilingual adult learners of mathematics in order to address their specific needs and to facilitate participation in mathematical discourse.

Summary – why does it matter?

This paper explored aspects of practice concerned with mathematics teaching and learning in relation to multilingual adults. But why does this matter? The importance of language for the teaching, learning, understanding and communication of mathematics cannot be ignored. Educational objectives require students to understand mathematical concepts and to possess an ability to express their understanding of these concepts in written format (Gerber, Engelbrecht, Harding & Rogan, 2005). However, the function of language does not lie solely in the representation of mathematical knowledge. Language is required for and engaged in bringing this knowledge into existence (Halliday & Martin, 1993). Furthermore, mathematics learners are required to possess competency both in everyday language and mathematics-specific language, but competency in the natural language does not necessarily contribute to competency in the mathematics specific language (Lemke, 1989). Clearly the intricate relationship between mathematics learning and a student's language is highly complex. This is further complicated when the language of instruction/learning changes, as is the situation faced by many multilingual adult mathematics/numeracy learners. Moreover, we need to consider mathematics as a discourse (or Discourse, in Gee's terms) and one that is not singular or homogenous (Moschkovich, 2012). Accordingly, mathematical learners use multiple resources from their experiences (both in and outside of the learning context) and we need to be cognisant of multiple registers co-existing in the learning environment. Therefore, addressing the needs of multilingual adult learners is of paramount importance. Bi/multilingual learners should not be viewed in a deficit mode. Rather, their language(s) should be viewed as a resource for learning mathematics. However, as demonstrated in this research paper, this area is under-researched and under-theorised. Research practices/findings generated from participants from a dominant group (e.g. monolingual speakers) assumes these to be the norm for all adult learners. We endorse a call for more research in relation to multilingual adult learning, and we suggest that ALM, as an international and (to an extent) multilingual organization, is ideally placed to generate such studies and in so doing to increase understanding of the important and neglected area of the relationships between language and mathematics learning and teaching with and for adults. The need for more

research and debate, as well as language-informed and language-friendly policy, is evident. Only then can we tackle these issues through pedagogic and support measures.

References

- Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher*, 56(8), 786–795.
- Austin, J. L., & Howson, A. G. (1979). Language and mathematical education. *Educational Studies in Mathematics*, 10(2), 161-197.
- Barker, K. (2001). Return on Training Investment: An environmental scan and literature review. Vancouver, BC: FuturEd.
- Barton, B., Chan, R., King, P., Neville-Barton, P., & Sneddon, J. (2005). EAL undergraduates learning mathematics. *International Journal of Mathematical Education in Science and Technology. Special Issue: The Fifth Southern Hemisphere Conference on Undergraduate Mathematics and Statistics Teaching and Learning, Fraser Island, Australia, 22–26 November, 2005 Edited by Michael Bulmer*, 36(7), 721-729. doi: 10.1080/00207390500270950.
- Barton, B., Fairhall, U., & Trinick, T. (1998). Tikanga Reo Tatai: Issues in the development of a Māori mathematics register. *For The Learning of Mathematics*, 18(1), 3-9.
- Barton, B., & Neville-Barton, P. (2003). Language issues in undergraduate mathematics: A report of two studies. *New Zealand Journal of Mathematics*, 32(Supplementary Issue), 19–28.
- Barwell, R., Barton, B. & Setati, M. (2007). Multilingual issues in mathematics education: Introduction. *Educational Studies in Mathematics*, 64(2), 113-119.
- Barwell, R., Leung, C., Morgan, C., & Street, B. (2002). The language dimension of maths teaching. *Mathematics Teaching*, 180, 12-15.
- Benseman, J., Sutton, A., & Lander, J. (2005a). “Working in the Light of Evidence, as well as Aspiration”. A Literature review of the best available evidence about effective adult literacy, numeracy and language teaching. Phase 5 Report. Wellington, NZ: Ministry of Education, New Zealand.
- Benseman, J., Sutton, A., & Lander, J. (2005b). *Pedagogy in Practice: An observational study of literacy, numeracy and language teachers*. Auckland, NZ: Auckland UniServices Ltd.
- Brodie, K. (1989). Learning mathematics in a second language. *Educational Review*, 41(1), 39-15. doi: 10.1080/0013191890410105.
- Bruner, J. S. (1975). *Toward a Theory of Instruction*. Cambridge: Belknap/Harvard.
- Cai, J. (1998). An investigation of US and Chinese students' mathematical problem posing and problem solving. *Mathematics Education Research Journal*, 10(1), 37-50.
- Canagarajah, A. S. & Wurr, A. J. (2011). Multilingual communication and language acquisition: New research directions. *The Reading Matrix*, 11(1).
- Chval, K. B., Pinnow, R., & Thomas, A. (2015). Learning how to focus on language while teaching mathematics to English language learners: A case study of Courtney. *Mathematics Education Research Journal*, 27(1), 103-127. doi: 10.1007/s13394-013-0101-8.
- Ciancone, T. (1996). *Numeracy in the Adult ESL Classroom*. ERIC Digest. Washington DC: National Clearinghouse for ESL Literacy Education.
- Ciancone, T., & Jay, C. (1991). *Planning Numeracy Lessons for an ESL Literacy Classroom. A beginning*. Toronto, Ontario, Canada: Toronto Board of Education, Adult Basic Education Unit.
- Coben, D., Brown, M., Rhodes, V., Swain, J., Ananiadou, K., Brown, P., ... Storey, V. (2007). *Effective Practice: Numeracy*. London: NRDC.
- Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). *Adult Numeracy: Review of research and related literature* (pp. 173). London: National Research and Development Centre for Adult Literacy and Numeracy (NRDC).
- Coben, D., & Miller-Reilly, B. (2014). Numbers Talk – Words Count: Language policy and adult numeracy education in Wales and New Zealand. In A. Hector-Mason, J. Gibney, & G. Griffiths (Eds.), *Critical Moments in Mathematics Education. Proceedings of the 20th International Conference of Adults Learning Mathematics - A Research Forum (ALM), 1-4 July 2013* (pp. 78-93). Newport: University of South Wales in association with ALM.
- Condelli, L., Safford-Ramus, K., Sherman, R., Coben, D., Gal, I., & Hector-Mason, A. (2006). *A Review of the Literature in Adult Numeracy: Research and conceptual issues*. Washington, DC: American Institutes for Research.
- Condelli, L., Wrigley, H. S., & Yoon, K. S. (2008). The What Works Study: Instruction, literacy and language learning for adult ESL literacy students. In S. Reder & J. Bynner (Eds.), *Tracking*

- Adult Literacy and Numeracy Skills: Findings from longitudinal research* (pp. 132-159). London & New York: Routledge.
- Cuevas, G. J. (1984). Mathematics learning in English as a second language. *Journal for Research in Mathematics Education*, 15, 134–144.
- Cummins, J. (1979). Linguistic interdependence and the educational development of bilingual children. *Review of Educational Research*, 49(2), 222-251.
- Cummins, J. (1980). The cross-lingual dimensions of language proficiency: Implications for bilingual education and the optimal age issue. *TESOL Quarterly*, 14, 175-187. doi: 10.2307/3586312.
- Ellerton, N. F., & Clarkson, P. C. (1996). Language factors in mathematics teaching and learning. In A. J. Bishop (Ed.), *International Handbook of Mathematics Education* (Vol. 4, pp. 987-1033). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Galligan, L. (2001). Possible effects of English-Chinese language differences on the processing of mathematical text: A review. *Mathematics Education Research Journal*, 13(2), 112–132.
- Gerber, A., Engelbrecht, J., Harding, A., & Rogan, J. (2005). The influence of second language teaching on undergraduate mathematics performance. *Mathematics Education Research Journal*, 17(3), 3–21.
- Gee, J. P. (1990). *Social Linguistics and Literacies: Ideology in discourses*. London: Falmer Press.
- Gee, J. P. (1996). *Social Linguistics and Literacies: Ideology in discourses* (2nd ed.). London: Taylor & Francis.
- Gibbs, W., & Orton, J. (1994). Language and mathematics. In J. Orton & G. Wain (Eds.), *Issues in Teaching Mathematics* (pp. 95-116). London: Cassell.
- Gorgorió, N., & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics*, 47(1), 7-33.
- Graham, B. (1988). Mathematical education and aboriginal children. *Educational Studies in Mathematics*, 19(2), 119-135.
- Gray, A. (2006). *Upskilling Through Foundation Skills: A literature review. A report prepared for the Department of Labour*. Wellington: Department of Labour.
- Grosjean, F. (1999). Individual bilingualism. In B. Spolsky (Ed.), *Concise Encyclopedia of Educational Linguistics* (pp. 284-290). London: Elsevier.
- Halliday, M. A. K. (1975). *Learning How to Mean: Explorations in the development of language*. London: Edward Arnold.
- Halliday, M. A. K., & Martin, J. R. (1993). *Writing Science: Literacy and discursive power*. London: Falmer Press.
- Kallenbach, S. (1994). Massachusetts ABE Math Standards. *NELRC News*.
- Lemke, J. L. (1989). *Using Language in the Classroom* (2 ed.). New York: Oxford University Press.
- Leonelli, E., & Schwendeman, R. (1994). The Massachusetts Adult Basic Education Math Standards. Volume 1 *The ABE Math Standards Project*. Malden, MA: The Massachusetts Department of Education.
- Lucas, K., Dondertman, B., & Ciancone, T. (1991). *A Sequencing Guide for Numeracy: Whole numbers*. Toronto, Ontario, Canada: Toronto Board of Education, Adult Basic Education Unit.
- Marr, B. (2000). How can they belong if they cannot speak the language? Enhancing students' language use in the adult mathematics classroom. *Literacy and Numeracy Studies*, 10(1/2), 55-70.
- Marr, B., & Helme, S. (1991). *Breaking the Maths Barrier – A kit for building staff development skills in adult numeracy*. Melbourne, Australia: Language Australia.
- Meaney, T. (2005). Mathematics as text. In A. Chronaki & I. M. Christiansen (Eds.), *Challenging Perspectives in Mathematics Classroom Communication* (pp. 109–141). Westport, CT: Information Age.
- Miura, I. T., Okamoto, Y., Kim, C. C., Chang, C-M., Steere, M., & Fayol, M. (1994). Comparisons of children's cognitive representation of number: China, France, Japan, Korea, Sweden, and the United States. *International Journal of Behavioral Development*, 17, 401-411.
- Morgan, C., Tang, S., & Sfard, A. (2011). Grammatical structure and mathematical activity: comparing examination questions. In C. Smith (Ed.), *Proceedings of the British Society for Research into Learning Mathematics* (pp. 113-118) Retrieved from <http://www.bsrlm.org.uk/IPs/ip31-3/BSRLM-IP-31-3-Full.pdf>
- Moschkovich, J. N. (2012). How equity concerns lead to attention to mathematical discourse. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics – Theories, practices and policy* (pp. 89-105). New York: Springer.
- Ní Ríordáin, M. (2013). *A comparison of Irish and English language features and the potential impact of differences*. Paper presented at the Eighth Congress of European Research in Mathematics

- Education (CERME 8) - Working Group 9. Retrieved from http://cerme8.metu.edu.tr/wgpapers/WG9/WG9_Ni_Riordain.pdf
- Ní Riordán, M., & O'Donoghue, J. (2007). Language issues for adults learning mathematics through a second language. In T. Maguire, N. Colleran, O. Gill, J. O'Donoghue, & K. Safford-Ramus (Eds.), *The Changing Face of Adults Mathematics Education: Learning from the Past, Planning for the Future. Proceedings of the 14th International Conference of Adult Learning Mathematics (ALM)* (pp. 241-250). Limerick: Adults Learning Mathematics (ALM) – A Research Forum, with CAMET (Ireland) & Institute of Technology Tallaght Dublin.
- Ní Riordán, M., & O'Donoghue, J. (2008). *An investigation into bilingual adult mathematics learners within an Irish context*. Paper presented at the ICME11 Conference, TSG8, Monterrey, Mexico.
- Ní Riordán, M., & O'Donoghue, J. (2009). The relationship between performance on mathematical word problems and language proficiency for students learning through the medium of Irish. *Educational Studies in Mathematics* 71(1), 43–64. doi: 10.1007/s10649-008-9158-9
- Ní Riordán, M., & O'Donoghue, J. (2011). Tackling the transition—the English mathematics register and students learning through the medium of Irish. *Mathematics Education Research Journal*, 23(1), 43–65. doi: 10.1007/s13394-011-0003-6
- Piaget, J. (1978). *Success and Understanding*. London: Routledge & Kegan Paul.
- Pimm, D. (1987). *Speaking Mathematically: Communication in mathematics classrooms*. New York: Routledge and Kegan Paul.
- Prediger, S., Clarkson, P., & Bose, A. (2012). *A way forward for teaching in multilingual contexts: Purposefully relating multilingual registers*. Paper presented at the 12th International Congress on Mathematical Education. TSG 30: Mathematics education in a multilingual and multicultural environment 8-15 July, 2012, Seoul, Korea. Retrieved from http://nime.hbcse.tifr.res.in/indian-participants-at-icme-2012/ArindamSusannephilip_TSG3018Teachinginmultilingualcontext.pdf
- Reder, S. (1994). Practice-engagement theory: A sociocultural approach to literacy across languages and cultures. In B. Ferdman, R.-M. Weber, & A. G. Ramirez (Eds.), *Literacy Across Languages and Cultures* (pp. 33-73). New York: State University of New York Press.
- Reder, S. (2012). Research Brief. The Longitudinal Study of Adult Learning: Challenging assumptions *Perspectives on Literacy and Essential Skills*. Montreal, Quebec: Centre for Literacy.
- Rudner, L. M. (1978). Using standard tests with the hearing impaired: The problem of item bias. *Volta Review*, 80, 31–40.
- Sapir, E. (1949). *Culture, Language and Personality*. Berkeley, CA: University of California Press.
- Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23(2), 139-159. doi: 10.1080/10573560601158461.
- Setati, M. (2005). Mathematics education and language: Policy, research and practice in multilingual South Africa. In R. Vithal, J. Adler, & C. Keitel (Eds.), *Researching Mathematics Education in South Africa: Perspectives, Practices and Possibilities* (pp. 73-109). South Africa: HSRC Press.
- Setati, M., & Adler, J. (2000). Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics*, 43(3), 243-269.
- Sfard, A. (2012). Introduction: Developing mathematical discourse – Some insights from communicational research *International Journal of Educational Research*, 51-52(3), 1-9.
- Sheehan-Holt, J., & Smith, C. (2000). Does basic skills education affect adults' literacy proficiencies and reading practices? *Reading Research Quarterly*, 35(2), 226-243.
- Sierpinska, A. (1994). *Understanding in Mathematics*. London: Falmer Press.
- Sternberg, R. J., & Sternberg, K. (2012). *Cognitive Psychology* (6 ed.). Belmont, CA: Wadsworth.
- Téllez, K., Moschkovich, J. N. & Civil, M. (Eds.) (2011). *Latinos/as and Mathematics Education: Research on learning and teaching in classrooms and communities*. Information Age Publishing, Inc.: Charlotte, NC.
- Tout, D., & Schmitt, M. J. (2002). The inclusion of numeracy in adult basic education. In J. Comings, B. Garner, & C. Smith (Eds.), *The Annual Review of Adult Learning and Literacy* (Vol. 3, pp. 152-202). San Francisco: Jossey-Bass.
- Towse, J. N. & Saxton, M. (1997). Linguistic influences on children's number concepts: Methodological and theoretical considerations. *Journal of Experimental Child Psychology*, 66, 362-375.
- Vygotsky, L. S. (1962). *Thought and Language* (E. Hanfmann & G. Vakar, Trans.). Cambridge, MA: MIT Press.
- Whorf, B. L. (1956). *Language, Thought, and Reality; Selected writings*. Cambridge, MA: Technology Press of Massachusetts Institute of Technology.

- Winslow, C. (1998). A linguistic approach to the justification problem in mathematics education. *For the Learning of Mathematics*, 18(1), 17-23.
- Yushau, B., & Bokhari, M. A. (2005). Language and mathematics: A mediational approach to bilingual Arabs. *International Journal of Mathematics Teaching and Learning*, 1-18.
- Zahner, W., & Moschkovich, J. N. (2011). Bilingual students using two languages during peer mathematics discussions. What does it mean? In K. Tellez & J. Moschkovich (Eds.), *Latinos and Mathematics Education: Research on learning and teaching in classrooms and communities* (pp. 37-62). Charlotte, NC: Information Age Publishing.