Modeling and simulating opinion dynamics in social networks

Ricardo Wu

Part 1

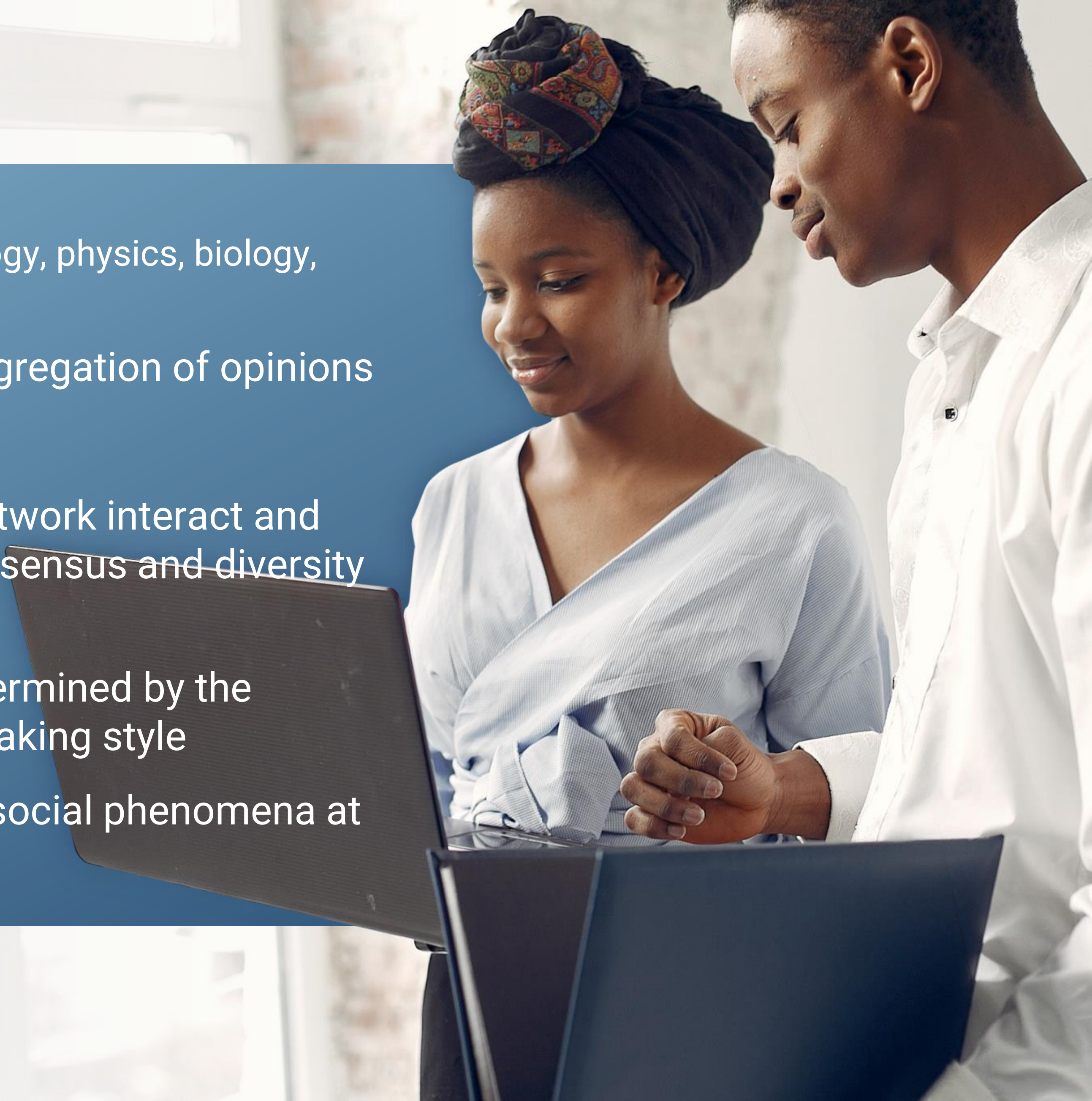
- **Background** — What is Opinion dynamic?
- **Aim** — Why and what I research?
- **Problem Statement** — What is opinion dynamic problem?
- **Simulation** — Simulation of other research papers
- **Innovative model designation** — What is my model?
- **Follow-up plan** — What is next?

Background

What is Opinion Dynamic?

Opinion dynamics

- An interdisciplinary research of cybernetics, sociology, physics, biology, economics, and computer science
- It focuses on the generation, diffusion and aggregation of opinions or behaviors in a social network
- Models capture how individuals in a social network interact and exchange opinions, including maintaining consensus and diversity of opinions
- It tells individuals' interactions are mainly determined by the influence network and individuals' decision-making style
- Simple local interactions can cause complex social phenomena at the macro level



Applications

Voting
GDM
Statistics
Business
Engineering
Sociocybernetics
Management
Sociology
Marketing
Finance
E-Commerce
Politics
Economics

Aim

Why and what I research?

Model types

Macro-level

Discrete opinion

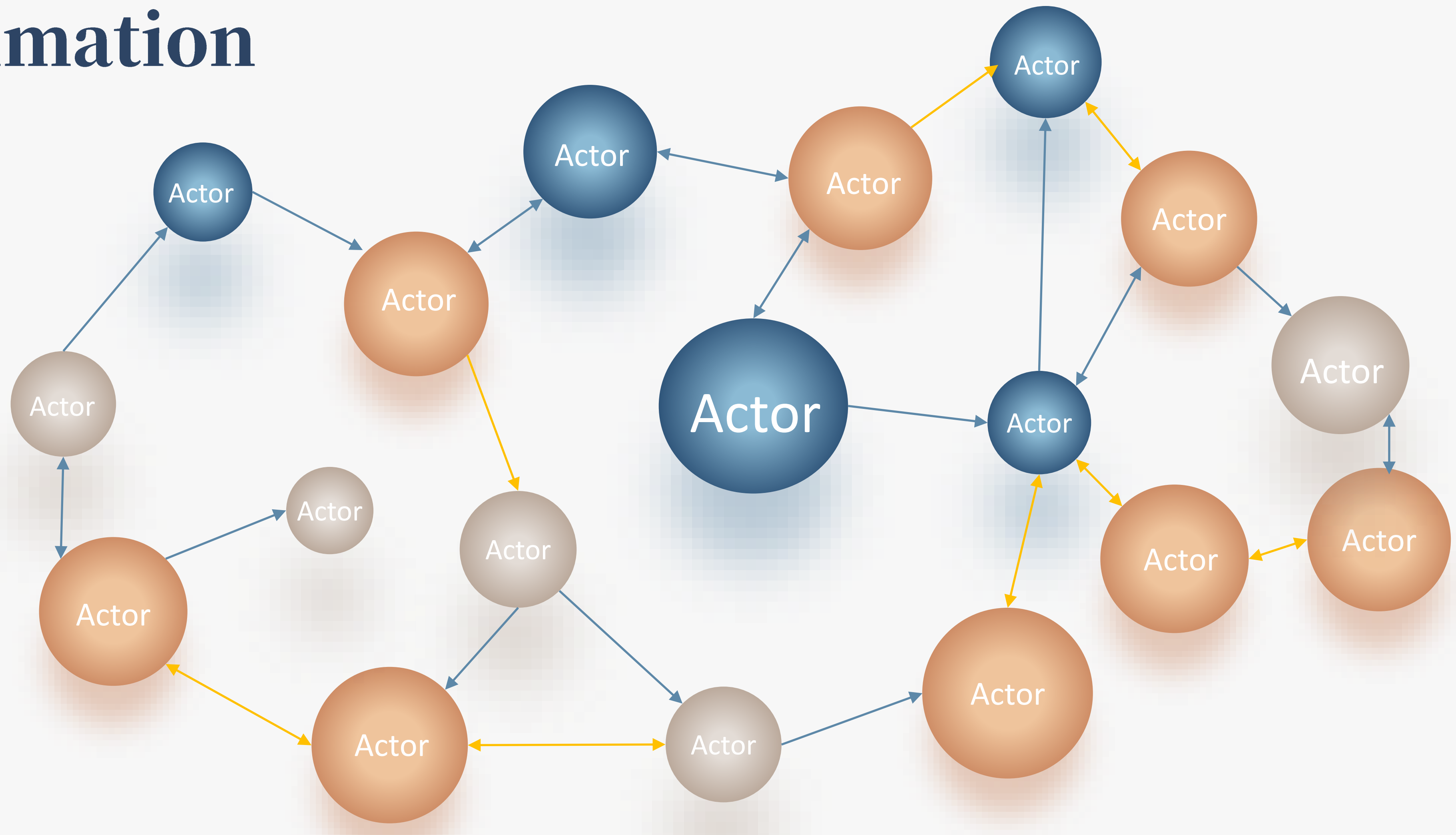
- Based on statistical physics
- Analyze large-scale social network
- E.g. Majority Rule model, Voter model

Micro-level

Continuous opinion

- Established on Mathematics
- Describe small-scale network and suitable to large-scale network
- E.g. Degroot model, Friedkin-Johnson model

Animation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Problem Statement

What is opinion dynamic problem?

Problem Statement

Opinion dynamic problem

Agents $A = \{A_1, A_2, \cdots, A_n\}$

Opinion of agent i at time t = $x_i(t)$, t ∈ N

Influence weight A_i to A_j: stochastic matrix w_{ij} and $\sum_{j=1}^{n} w_{ij} = 1$

Fusion process

$$x_i(k+1) = w_{i1}x_1(k) + w_{i2}x_2(k) + \cdots + w_{n1}x_n(k)\,, k = 0,1,2\cdots$$

Consensus

$$\lim_{n\to\infty} x_i(k) = c, \forall x_i(0) \in R^n\,, i = 1,2,\cdots, n.$$

Application example

The Mathematics of Marital Conflict by Gottman in 1999

$$W_{t+1} = a + r_1 W_t + I_{HW}(H_t)$$

$$H_{t+1} = b + r_2 W_t + I_{WH}(W_t)$$

r, a, b are analyzed from data
a, b: Individual emotions
$r_1 W_t, r_2 W_t$: emotions with the other
I: Influence functions and the nonlinear part of the equations.
I_{HW}: Influence of the husband on the wife at time t

Accuracy 90%

Novel Multidimensional Models of Opinion Dynamics in Social Networks

Sergey E. Parsegov, Anton V. Proskurnikov, *Member, IEEE*, Roberto Tempo, *Fellow, IEEE*, and Noah E. Friedkin

Simulation 1

Simulation of other research papers

Degroot Model

Most basic continuous opinion model in 1974

- Repeated communication – keep updating from previous opinion
- Information comes only once
- See how information disseminates
- Who has influence, convergence speed, network structure impact
- A consensus is reached if and only if the group is strongly connected and aperiodic

$$x_i(k+1) = \sum_{j=1}^{n} w_{ij}\, x_j(k)$$

Matrix form: $x\,(k+1) = Wx(k)$

$x = \{x_1, x_2, \cdots, x_n\}^T$ scalar opinions $x_i \in R$

Limitation: People's opinions are not merely composed of influence weight, which means only influenced by others; they should have their own tendency towards some topics.

Definition:

1. $\lim_{k\to\infty} x(k) = x^*, its\ convergent$
2. *If for any* $x(0), \alpha \in R, \lim_{k\to\infty} x(k) = \alpha 1_n$, *then its consistent*

Friedkin-Johnsen Model

Extension on Degroot model in 1999

Λ is diag(ξ), ξ = (ξ1 , ξ2 , . . . , ξn) ∈ R^n , ξi ∈ [0, 1] , 0 ≤ Λ ≤ I,

ξi: actor i 's susceptibilities to the other actors
1 − ξi : stubbornness of his initial opinion

If ξi = 1, i is non-stubborn/open-minded; if ξi = 0, i is totally stubborn

$$x(k+1) = \Lambda W x(k) + (I - \Lambda)x(0)$$

$x = \{x_1, x_2, \cdots, x_n\}^T$ scalar opinions $x_i \in R$

Limitation: Individuals in a real society doesn't only discuss on 1 topic, topics may be independent or dependent to each other

Definition:

1. *Its convergent if for any vector $u \in R^n$ the sequence x(k) has a limit*

$$x' = \lim_{k\to\infty} x(k) \rightarrow x' = \Lambda W x + (I - \Lambda)u$$

and FJ model is stable which means non- stubbornness and non-oblivion
In which ΛW is a Schur stable matrix: $\rho(\Lambda W) < 1$

1. *If for any $x(0), \alpha \in R$, $\lim_{k\to\infty} x(k) = \alpha 1_n$, then its consistent*

Multidimensional FJ Model

Extension on FJ model in 2017

F-J model: $x_{\mathrm{i}}(k+1) = \lambda_{ii} \sum_{j=1}^{n} w_{ij} x_j(k) + (1-\lambda_{ii}) u_i,\ u_i = x_i(0)$

$$vector\ opinion = x_1(k), x_2(k), \cdots, x_n(\mathrm{k}) \in R^m$$

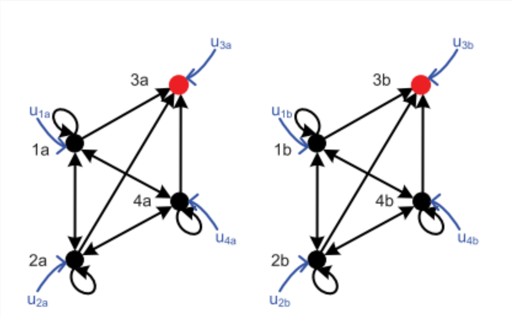
Each vector $x_{\mathrm{i}}(k) = (x_i^1(k), \ldots \ldots x_i^m(k))$ stands for the opinions of the i_{th} agent on m different issues

However, this doesn't take issues' interdependencies into consideration

$x_{\mathrm{j}}(k) \in R^m$, $x_i(k+1) = \lambda_{ii} C \sum_{j=1}^{n} w_{ij} x_j(k) + (1-\lambda_{ii}) u_i$
There the model adds a constant "coupling matrix" C $\in R^{mxm}$
Matrix C is defined a multi-issues dependence structure (MiDS)
It can be measured through estimation.

* W is a property of the social network(Influence); C
expresses the interrelations between different topics of interest

Multidimensional FJ Model



When C = I_2, Two independent topics

Example: $n = 4, W = \begin{matrix} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{matrix}$, Susceptibility matrix $\Lambda = I - diag\ W$

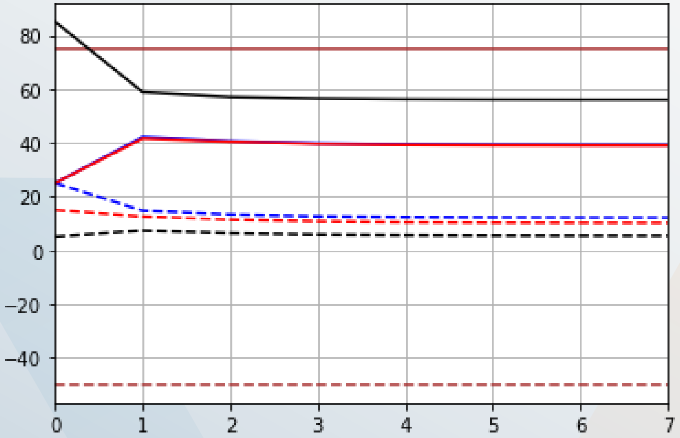
$$m = 2\ (2\ dimentional),\quad x_j(k) = \{x_j^1(k),\ x_j^2(k)\}^T,$$

Since the topic-specific opinions $x_j^1(k),\ x_j^2\ (k)$ evolve independently, their limits can be calculated independently

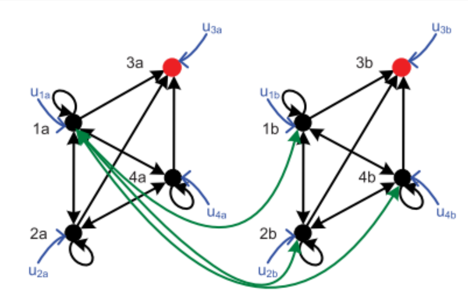
Applying $x' = \sum_{k=0}^{\infty}(\Lambda W)^k(1-\Lambda)u = (I-\Lambda W)^{-1}(I-\Lambda)u$ to $u^i = (x_1^i(0),\ x_2^i(0), x_3^i\ (0),\ x_4^i(0))^T$ i=1,2

$$x(0) = u = [\underbrace{25, 25,}_{u_1 = x_1(0)}\ \underbrace{25, 15,}_{u_2 = x_2(0)}\ \underbrace{75, -50,}_{u_3 = x_3(0)}\ \underbrace{85, 5}_{u_4 = x_4(0)}\ ]^\top$$

The vector of steady agents' opinion is $x' = [60, -19.3, 60, -21.5, 75, -50, 75, -23.2]^T$



Multidimensional FJ Model



When C = C_1, $C_1 = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$ Two positively couped topics

Example: $n = 4, W = \begin{matrix} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{matrix}$, Susceptibility matrix $\Lambda = I - diag\ W$

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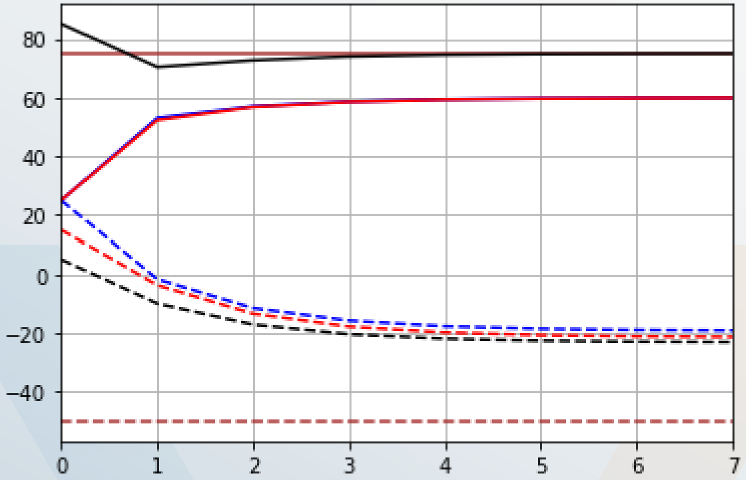
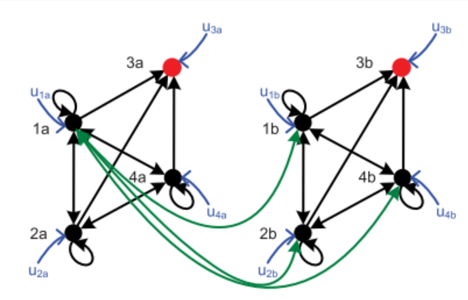
$$x(0) = u = [\underbrace{25, 25,}_{u_1 = x_1(0)}\ \underbrace{25, 15,}_{u_2 = x_2(0)}\ \underbrace{75, -50,}_{u_3 = x_3(0)}\ \underbrace{85, 5}_{u_4 = x_4(0)}\ ]^T$$

The vector of steady agents' opinion is x' = [39.2, 12, 39, 10.1, 75, −50, 56, 5.3]^T



Multidimensional FJ Model



When C = C_2, $C_2 = \begin{bmatrix} 0.8 & -0.2 \\ -0.3 & 0.7 \end{bmatrix}$ Two negatively couped topics

Example: $n = 4, W = \begin{matrix} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{matrix}$, Susceptibility matrix $\Lambda = I - diag\ W$

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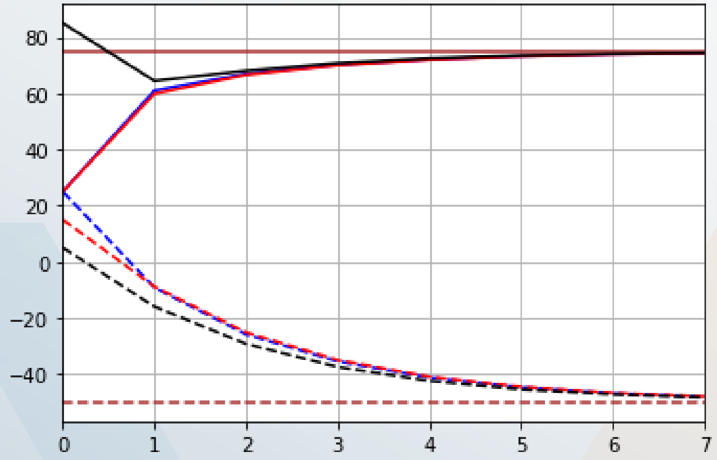
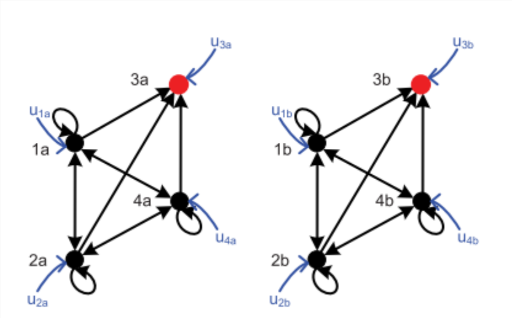
The vector of steady agents' opinion is $x' = [52.3, -30.9, 52.1, -33.3, 75, -50, 68.4, -33.2]^T$



Degroot-like Dynamic Model



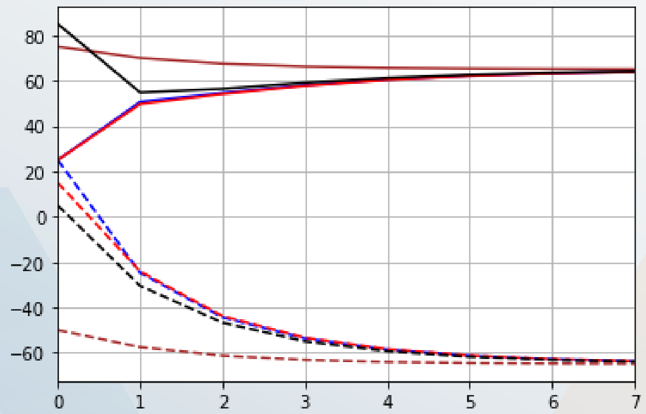
Susceptibility matrix $\Lambda = I_n$,When C = I_2, Two independent topics

$$\text{Example: n} = 4, \text{W} = \begin{matrix} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{matrix},$$

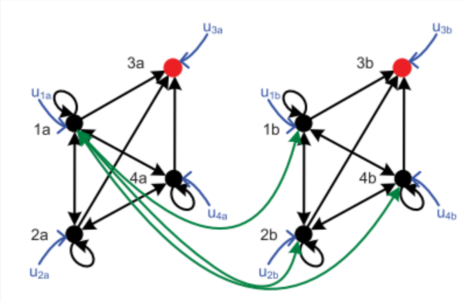
$$m = 2\ (2\ dimentional),\ \ x_j(k) = \{x_j^1(k),\ x_j^2(k)\}^T,$$

$$x(0) = u = [\ \underbrace{25, 25,}_{u_1 = x_1(0)}\ \underbrace{25, 15,}_{u_2 = x_2(0)}\ \underbrace{75, -50,}_{u_3 = x_3(0)}\ \underbrace{85, 5}_{u_4 = x_4(0)}\ ]^\top$$

$$\lim_{k \to \infty} x(k) = [75,\ -50,\ 75,\ -50,\ 75,\ -50,\ 75,\ -50]^T$$



Degroot-like Dynamic Model



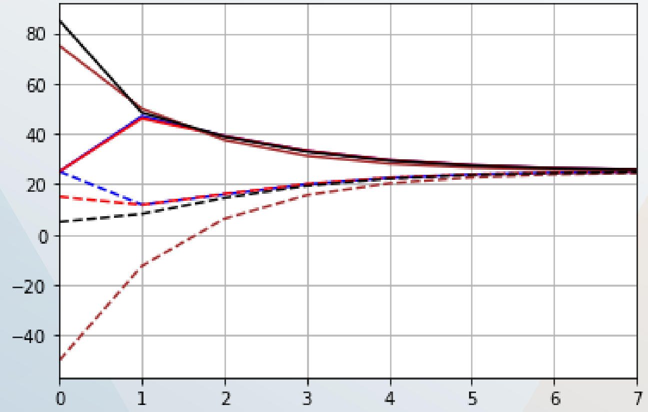
Susceptibility matrix $\Lambda = I_n$,When C = C_1, Two positively couped topics

$$\text{Example: n} = 4, \text{W} = \begin{matrix} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{matrix}$$

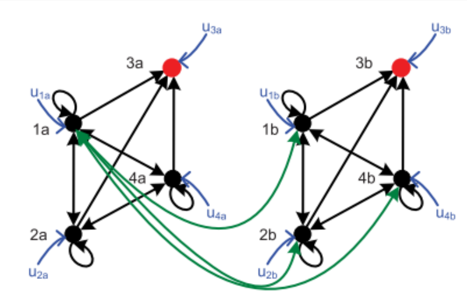
$$m = 2\ (2\ dimentional),\quad \mathrm{x_j}(k) = \{x_j^1(k),\ x_j^2(k)\}^T,$$

$$x(0) = u = [\ \underbrace{25, 25,}_{u_1 = x_1(0)}\ \underbrace{25, 15,}_{u_2 = x_2(0)}\ \underbrace{75, -50,}_{u_3 = x_3(0)}\ \underbrace{85, 5}_{u_4 = x_4(0)}\ ]^\top$$

$$\lim_{k \to \infty} x(k) = [25,\ 25,\ 25,\ 25,\ 25,\ 25,\ 25,\ 25]^T$$



Degroot-like Dynamic Model



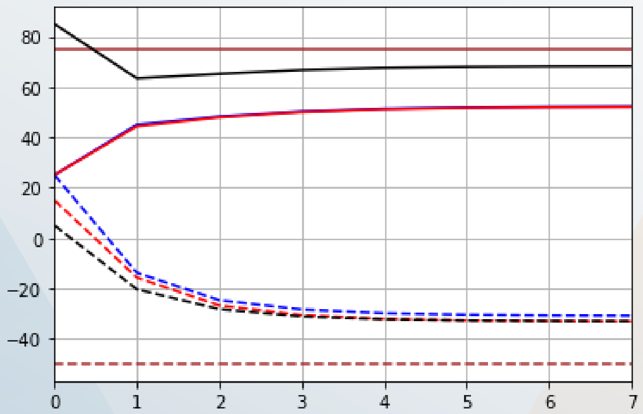
Susceptibility matrix $\Lambda = I_n$,When C = C_2, Two negatively couped topics

$$\text{Example: } \mathrm{n} = 4, \mathrm{W} = \begin{matrix} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{matrix}$$

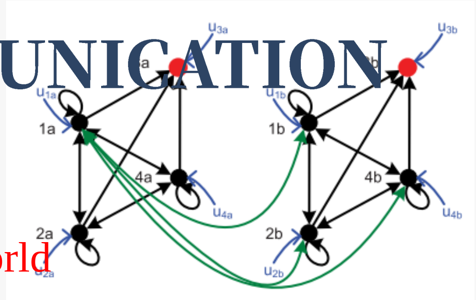
$$m = 2 \; (2\ dimentional), \quad \mathrm{x_j}(k) = \{x_j^1(k),\ x_j^2(k)\}^T,$$

$$x(0) = u = [\ \underbrace{25, 25,}_{u_1 = x_1(0)}\ \underbrace{25, 15,}_{u_2 = x_2(0)}\ \underbrace{75, -50,}_{u_3 = x_3(0)}\ \underbrace{85, 5}_{u_4 = x_4(0)}\ ]^\top$$

$$\lim_{k\to\infty} x(k) = [65,\ -65,\ 65,\ -65,\ 65,\ -65,\ 65,\ -65]^T$$



OPINION DYNAMICS UNDER GOSSIP-BASED COMMUNICATION



Resolution on complex sequences of interpersonal influences

The multi-dimensional model still cannot represent the real world
Why?

Time inconsistency

modified neighbors' opinion

Your initial opinion with stubbornness

Assuming that only two agents interact during each step:

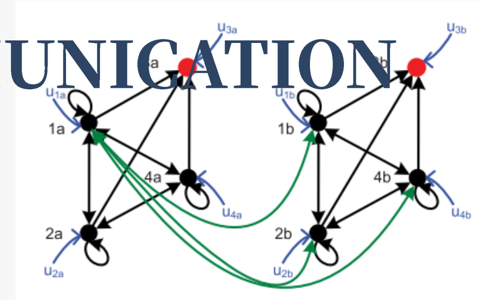
$$x_i(k+1) = \left(1 - \gamma_{ij}^1 - \gamma_{ij}^2\right)x_i(k) + \gamma_{ij}^1 C x_j(k) + \gamma_{ij}^2 u_i$$

New opinion of the agent is a weighted average of his/her previous opinion $x_i(k)$, the prejudice and the neighbor's previous opinion $u_i = x_i(0)$.

γ_{ij}^1 and γ_{ij}^2 are the elements of matrix Γ^1 and Γ^2, respectively, where $\Gamma^1 = \Lambda W$ and $\Gamma^2 = (I - \Lambda)W$.

Each vector $x_\mathrm{i}(k) = (x_i^1(k), \dots \dots x_i^m(k))$ stands for the opinions of the i_{th} agent on m different issues

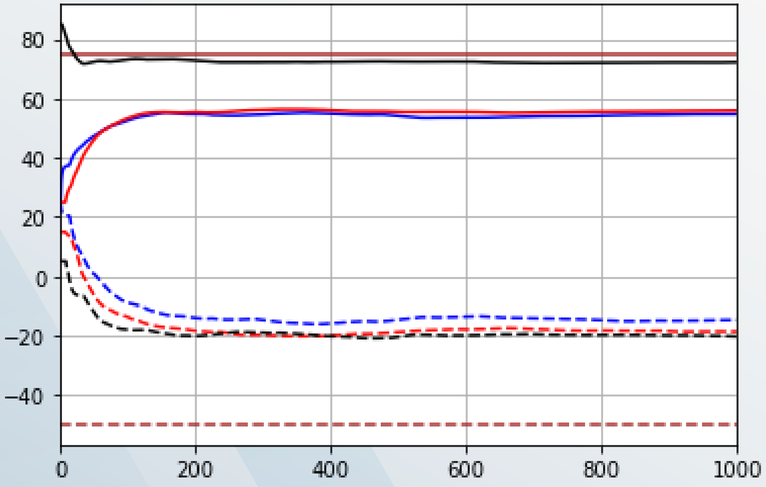
OPINION DYNAMICS UNDER GOSSIP-BASED COMMUNICATION



$$Example\text{:}\ \mathrm{n} = 4, \mathrm{W} = \begin{matrix} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{matrix},\ \text{Susceptibility matrix}\ \Lambda = I\ -\ diag\ W$$

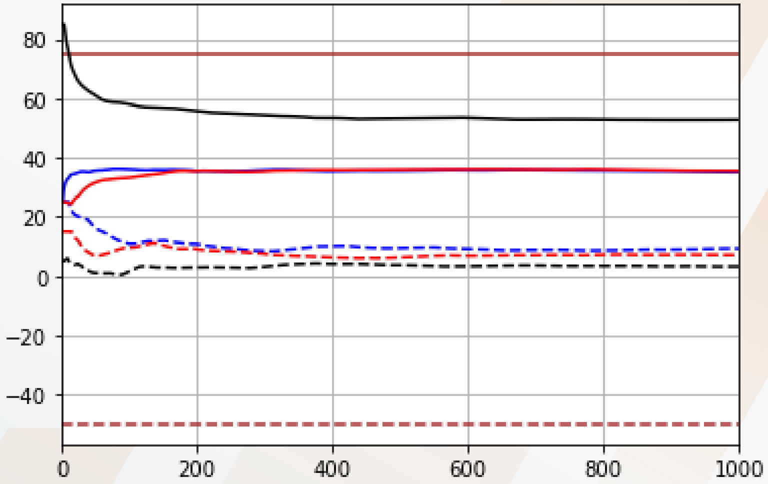
$$m = 2\ (2\ dimentional),\ \ x_{\mathrm{j}}(k) = \{x_j^1(k),\ x_j^2(k)\}^T$$

Averages



(a)

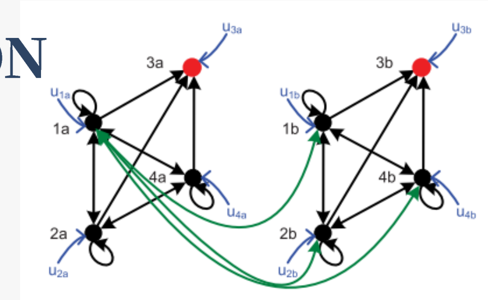
Averages



(b)

Convergence of the Cesaro-Polyak averages. (a) the MiDS matrix $C = I_2$ (b) the MiDS matrix $C = C_1$

OPINION DYNAMICS UNDER GOSSIP-BASED COMMUNICATION



$When\ C = C_1,\ topics\ are\ positively\ coupled.$

$Example\!:\mathrm{n} = 4, \mathrm{W} = \begin{array}{cccc} 0.220 & 0.120 & 0.360 & 0.300 \\ 0.147 & 0.125 & 0.344 & 0.294 \\ 0 & 0 & 1 & 0 \\ 0.090 & 0.178 & 0.446 & 0.286 \end{array},\ \text{Susceptibility matrix}\ \Lambda = I - diag\ W$

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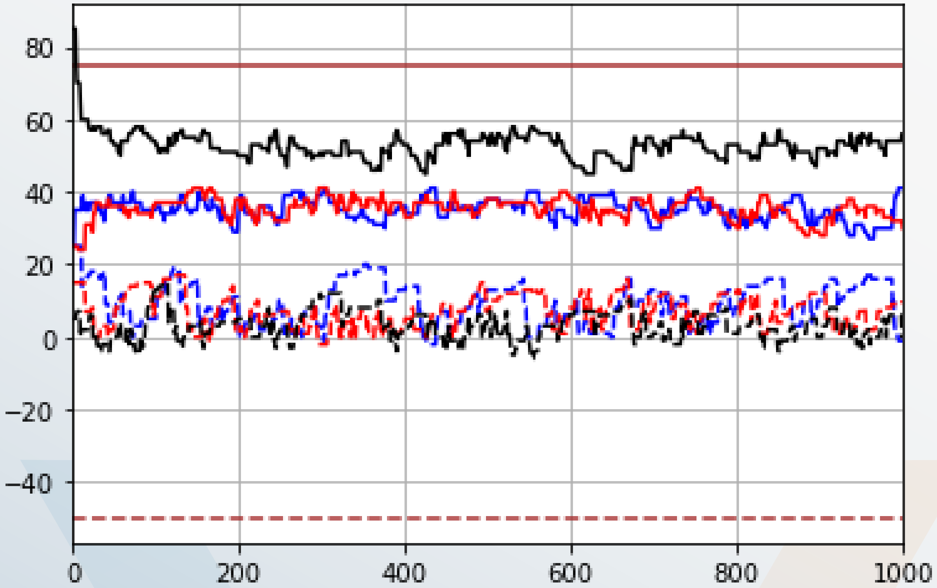
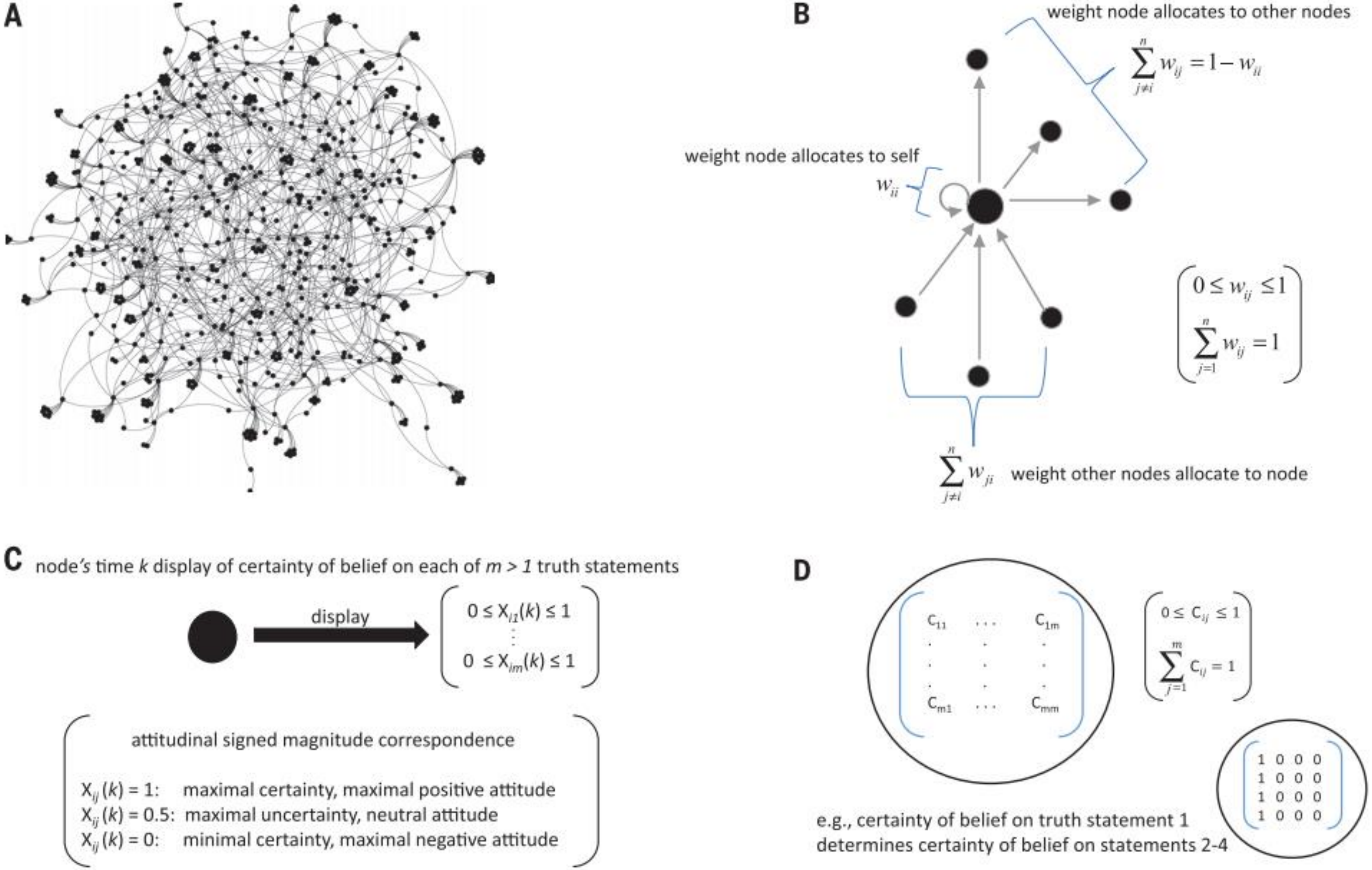


Network science on belief system dynamics under logic constraints

Noah E. Friedkin,[1*] Anton V. Proskurnikov,[2,3] Roberto Tempo,[4] Sergey E. Parsegov[5]

Simulation 2

Simulation of other research papers



E tensor matrix equation of belief system dynamics

$$\mathbf{X}(k+1) = \mathbf{AWX}(k)\mathbf{C}^T + (\mathbf{I} - \mathbf{A})\mathbf{X}(0),\ k = 0, 1, \ldots$$

$$\left(\mathbf{A}_{n \times n},\ a_{ii} = 1 - w_{ii}\ \forall i,\ a_{ij} = 0\ \forall i \neq j\right)$$

Network science on belief system dynamics under logic constraints

Model

$$X(k+1) = \lambda_{ii} C \sum_{j=1}^{n} w_{ij} x_j(k) + (1-\lambda_{ii}) u_i$$

X(0) = n x m matrix of n individuals and m truth statements with truth value on which individuals have heterogeneous certainties of belief in the[0,1]

Statement

Attitudinal signed magnitude correspondence

$$\begin{aligned} x_{ij}(k) &= 1: && maximal\ certainty, maximal\ positive\ attitude \\ x_{ij}(k) &= 0.5: && maximal\ uncertainty, neutral\ attitude \\ x_{ij}(k) &= 0: && maximal\ uncertainty, maximal\ negative\ attitude \end{aligned}$$

2001 Powell’s UN speech logic structure

Statement 1. Saddam Hussein has a stockpile of weapons of mass destruction.

Statement 2. Saddam Hussein’s weapons of mass destruction are real and present dangers to the region and to the world.

Statement 3. A preemptive invasion of Iraq would be a just war.

Network science on belief system dynamics under logic constraints

Condition

Consider a population:

(i) is attentive to Powell’s UN speech logic structure,

(ii) maximally open to interpersonal influence,

(iii) accepts its logic structure,

(iv) connected in a regular influence network structure that allows direct or indirect flows of influence from every individual i to every individual j of the population.

Experiment

Data generation

The data used in the simulation are generated randomly, and the technical details are showed as follows.

Based on the F-J model assumption $a_{ii} = 1 - w_{ii}$ for all i,

$$W = AR + I - A,$$

where $R = [r_{ij}]$ is the matrix of relative weights among the interpersonal allocations of weights to others. That is, $r_{ii} = 0$, $0 \le r_{ij} \le 1$, and $\sum_{j=1}^{n} r_{ij} = 1$ for all i and all $i \neq j$. This decomposition allows adjustments in A, holding R constant, and adjustments of R, holding A constant. We hold R constant and adjust A.

Experiment

Data generates

Fig. 1-4 are based on the same realization of the initial certainty of beliefs of n individuals on $m = 3$ statements $X(0)$. The matrix of initial beliefs $X(0)$ contains heterogeneous certainties of belief on each statement with mean values $0.90, 0.50$ and 0.10, for statements 1-3, respectively.

To generate it, we draw three sets of $n = 1000$ real values x_i from the normal (Gaussian) distribution $N(\mu_i, 1)$, $\mu_i = \ln(\pi_i/(1-\pi_i))$, where $\pi_1 = 0.90, \pi_2 = 0.50$, and $\pi_3 = 0.10$ respectively for the three statements. For each statement $i = 1,2,3$ with the mean μ_i and variance 1, the ith column of $X(0)$ is then a distribution of the corresponding certainties of beliefs $\frac{\exp(x_i)}{1+\exp(x_i)}$.

Simulations 1-4 are also based on the same realization of R. It is a sparse valued Gilbert (n, p) random graph, $p = 0.011$, normalized to obtain a row stochastic matrix, that allows direct or indirect flows of influence occur from every individual i to every individual j of the population. In other words, the employed R is one realization of a random low density aperiodic irreducible row stochastic matrix.

Experiment

Belief heterogeneity on three truth statements

Assume individuals' levels of openness to interpersonal influence are all maximal

Parameters

$A = I$, and the diagonal values of $W = 0$, $w_{ii} = 0$,

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathrm{I}$$

The three statements of belief are independent

Parameters

$A = I$,

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The first statements of belief related to the second and third statements

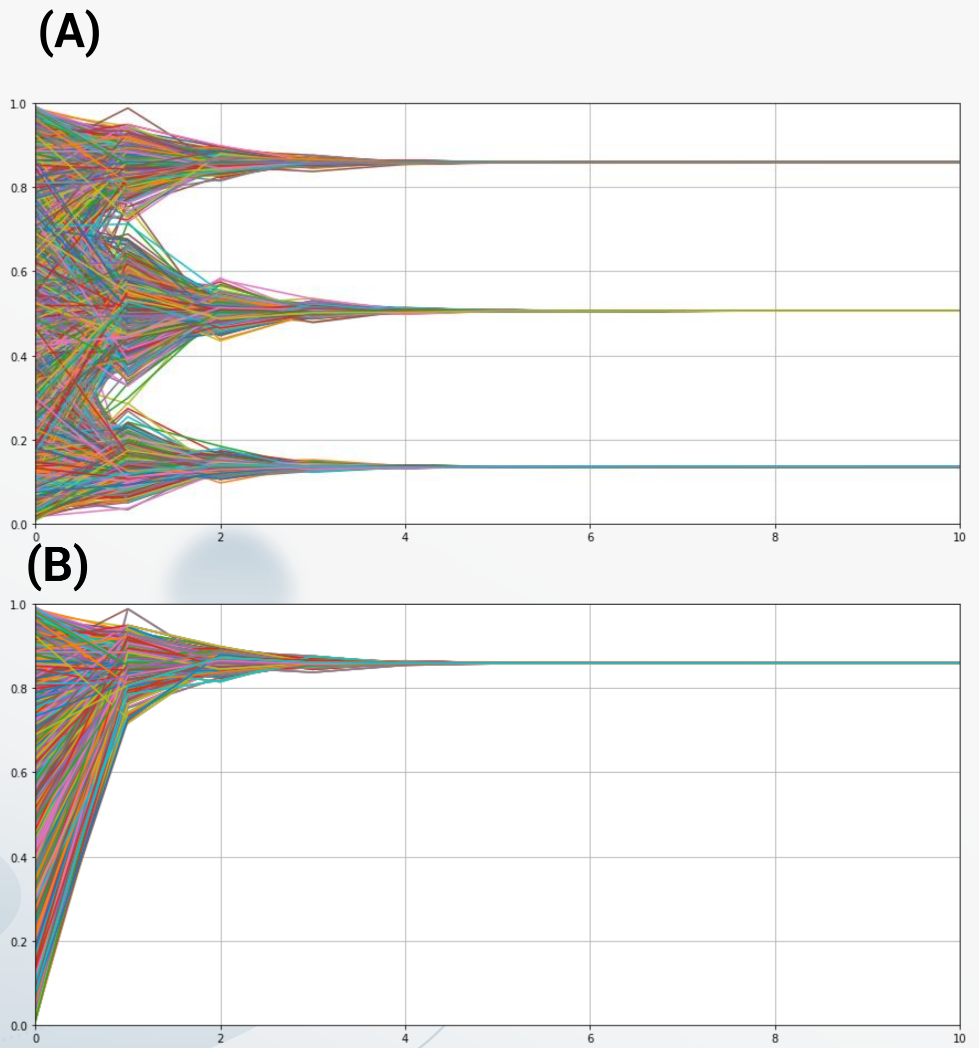


Fig. 1

Experiment

Belief heterogeneity on three truth statements

Introduces a level of closure to interpersonal influence that modestly anchors individuals on their initial beliefs

Parameters

$A = 0.85I$,

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

The three statements of belief are independent

Parameters

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The first statements of belief related to the second and third statements

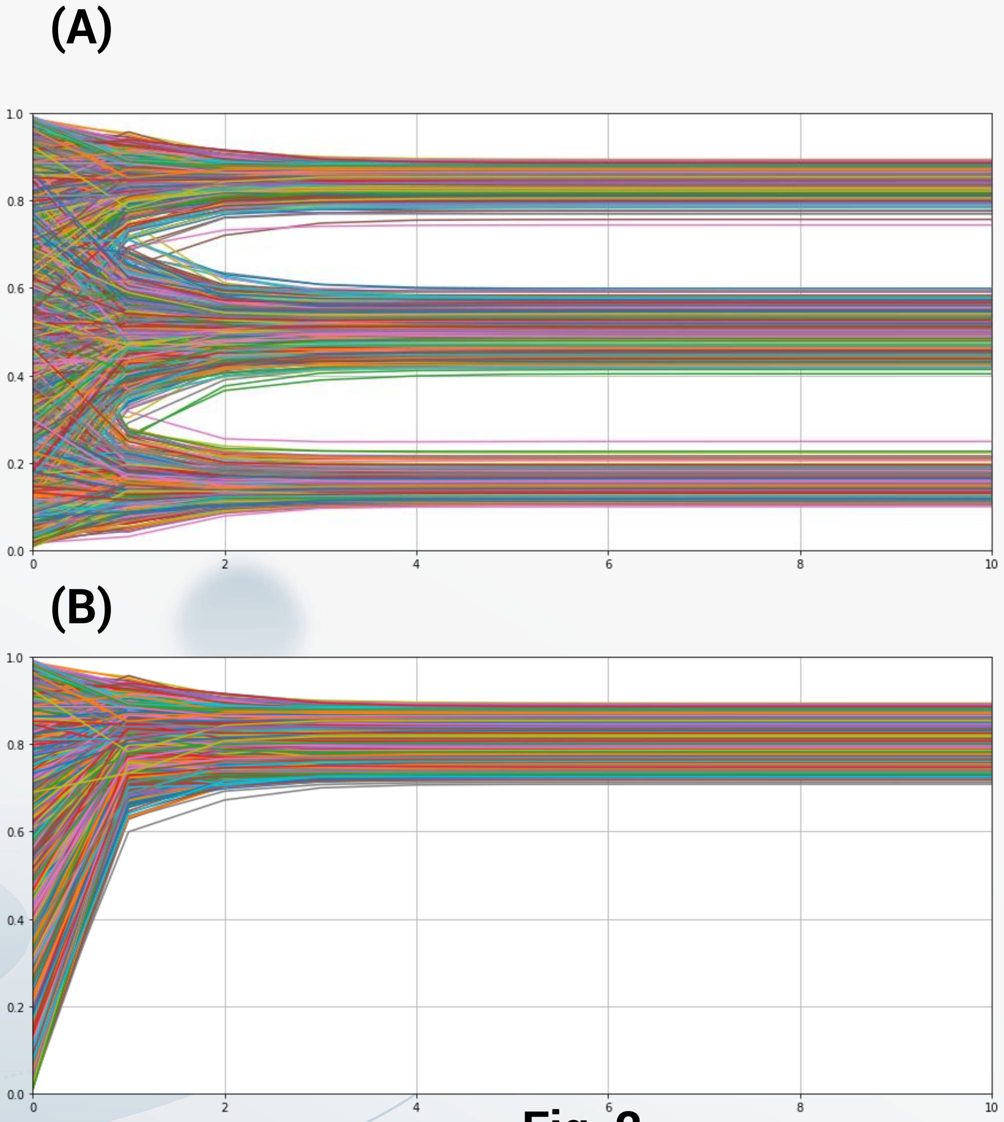


Fig. 2

Experiment

Belief heterogeneity on three truth statements

Introduces a level of closure to interpersonal influence that modestly anchors individuals on their initial beliefs

Parameters

Begins with the setup for Figure 2

$$A = I, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathrm{I}$$

(a) Alter the a_{ii} of an arbitrary subset of 100 individuals to $a_{ii} = 0$;

(b) Alters each of these 100 individual's three row values in $X(0)$ to a uniform value of 0.10.

Parameters

Begins with the setup for Figure 2, and alters the logic constrain structure,

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0.80 & 0 & 0.20 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 1 & 0 & 0 \\ 0.20 & 0 & 0.80 \\ 0 & 0 & 1 \end{bmatrix}$$

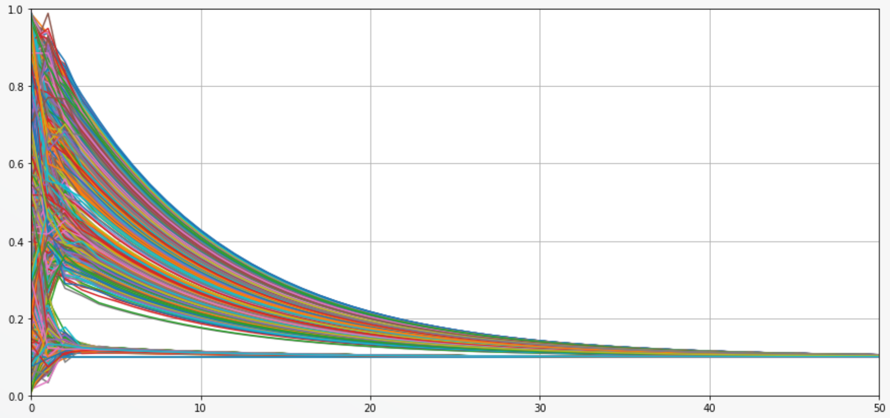


Fig. 3

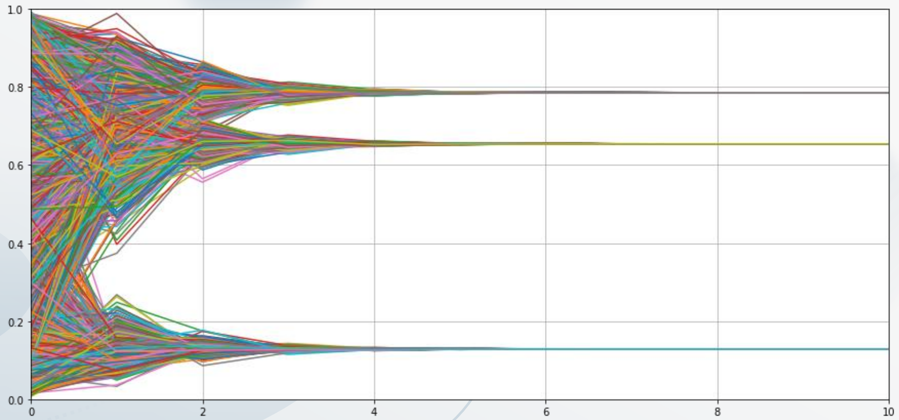


Fig. 4

Experiment

Belief heterogeneity on three truth statements

$$X(k+1) = AWY(k), \qquad Y(k) = \begin{pmatrix} x^1(k)C_1^T \\ x^n(k)C_2^T \end{pmatrix}$$

Parameters

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \mathrm{I} \qquad C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Parameters

$$X(0) = \begin{pmatrix} 0.96 & 0.56 & 0.16 \\ 0.56 & 0.54 & 0.14 \\ 0.92 & 0.52 & 0.12 \\ 0.88 & 0.48 & 0.08 \\ 0.86 & 0.46 & 0.06 \\ 0.84 & 0.44 & 0.04 \end{pmatrix}$$

Opinions

Parameters

$$W = \begin{bmatrix} 0 & \underline{0.80} & 0.20 & 0 & \underline{0} & 0 \\ 0.50 & 0 & 0.50 & 0 & 0 & 0 \\ 0.20 & 0.80 & 0 & 0 & 0 & 0 \\ 0 & 0.80 & 0 & 0 & 0.10 & 0.10 \\ 0 & 0.80 & 0 & 0.10 & 0 & 0.10 \\ 0 & 0.80 & 0 & 0.10 & 0.10 & 0 \end{bmatrix}$$

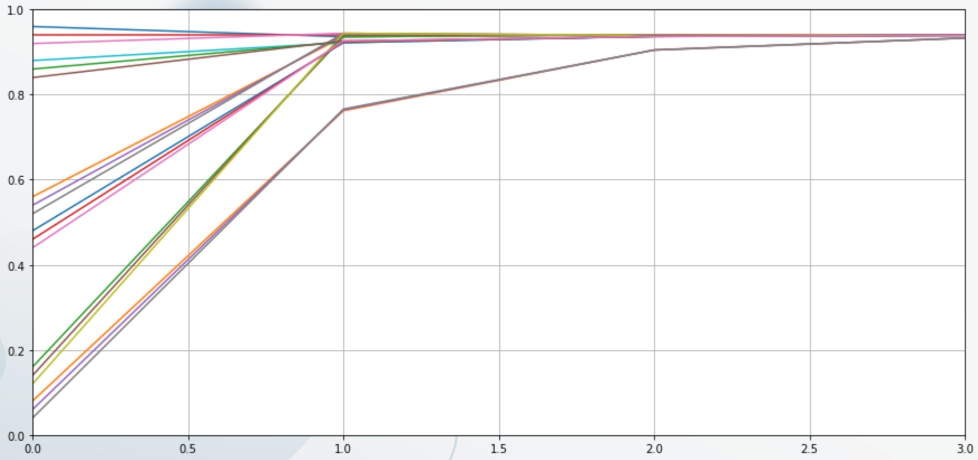


Fig. 5

Innovative model designation

What is my model?

Work progress

01

Simulations

02

Innovation

03

Testing

04

Data crawling

Opinion dynamics of heterogeneous multidimensional models under batch gossip-based communication

Model

$$x_i(k+1) = \left(N - \sum_{j}^{(i,j)\in E'} \gamma_{ij}^1 - \sum_{j}^{(i,j)\in E'} \gamma_{ij}^2\right) x_i(k) + \sum_{j}^{(i,j)\in E'} \gamma_{ij}^1 y_j(k) + \sum_{j}^{(i,j)\in E'} \gamma_{ij}^2 u_i$$

Interaction graph $G[W] = (V, E)$

Batch E' $|E'| = N \le |E|$

Initial opinion $u_i = x_i(0)$

Matrix form

$$X'^{(k+1)} = X'(k) - \left(\Gamma^{1'} + \Gamma^{2'}\right) X'(k) + \Gamma^{1'} Y'(k) + \Gamma^{2'} X'(0)$$

$$where,\ Y'^{(k)} = \begin{pmatrix} y^1(k) \\ \cdots \\ y^N(k) \end{pmatrix} = \begin{pmatrix} x^1(k) C_1^T \\ \cdots \\ x^N(k) C_N^T \end{pmatrix}$$

$$\Gamma^1 = \Lambda W \qquad \Gamma^2 = (I - \Lambda) W$$

$y_j(k)$ is the displayed opinions of others to which agent i may be responding are those agents' internal integrations of their own positions on the m issue dependency constrains

Give two matrices Γ^1, Γ^2 such that $\gamma_{ij}^1, \gamma_{ij}^2 \ge 0$ and $\gamma_{ij}^1 + \gamma_{ij}^2 \le 1$, for each arc $(i, j) \in E'$, the i_{th} agents update its opinion $x_i(k)$ about m issues at step $k + 1$ in accordance with the model.

where $y_i(k)$ is the displayed opinions of others to which agent i may be responding are those agents' $(j = 1, \cdots, n)$ internal integrations of their own positions on the m issue dependency constrains, which are denoted by $y_{j1}(k), \cdots, y_{jm}(k)$ and defined as

$$y_{jm}(k) = \sum_{u=1}^{m} c_{mu} x_{mu}(k)$$

$$y_i = x_i(K)\, C^{\mathrm{T}}$$

Then the influence system operates with these displayed positions with the model. Hence during each interaction, the agent's opinion is averaged with its own *prejudice* and modified neighbors' opinion $y_j(k)$. The other opinions remain unchanged as step k.

Details about Model

Based on $F - J\ model$

$$W = AR - (1 - A)$$

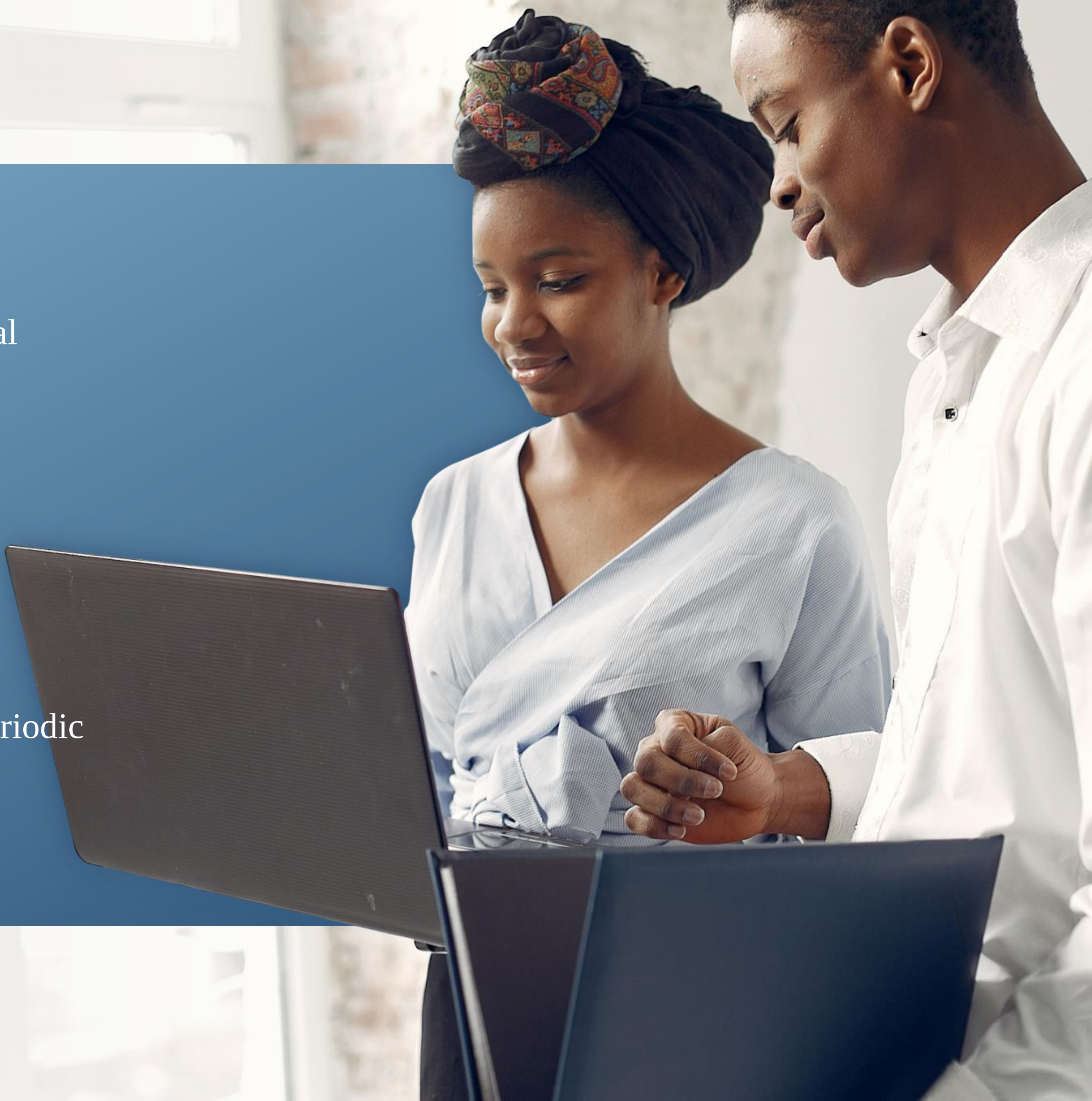
$R = r_{ij}$ is the matrix of relative weights among the interpersonal allocations of weights

$$r_{ii} = 0, 0 \le r_{ij} \le 1, \sum_{j=1}^{n} r_{ij} = 1\forall_{i}, all\ i \ne j$$

We hold R constant and adjust A

Sparse valued Gilbert (n, p) random graph , p = 0.011

The employed R is one realization of a random low density aperiodic irreducible row stochastic matrix. And $\Gamma = \mathrm{I} - \mathrm{diag}W$.



Data crawler

Install GoPUP from PIP (pre-installed in python 3)
Very easy to use

What is GoPUP? A web crawler to crawl public data from indexes or websites in its data warehouse using very simple codes

```
import gopup as gp
index_df = gp.google_index(keyword=“?", start_date=‘?', end_date=‘?’)
print(index_df)
```

Data Sources

知乎

有问题 上知乎

For simplicity, I chose a Chinese academic forum ZHIHU in GoPUP lists, the port in GoPUP is zhihu_hot_search_list, to adopt questions and answers of “Where to find analysis and research reports by industry”

The Zhihu users are actors and user v reply to a user u in a thread corresponds to an interaction (u, v) between two actors. We sample both sides from users and question with answers to create our data set. In order to study the evolution process of the popularity of the answers, we choose the top 10 answers as the 10-dimension feature of the actor opinion according to the number of likes. We sample the users posting a minimum of forty questions or answers per month in the forum , which give us 478 users. And the initial opinion of each actor is a 3-dimension vector $D_i = [d_1 , d_2 , d_3]^T$, and the $d_i \in \{0, 1\}$, i = 1, 2, 3
$d_j = 1$ means the actor i like the answer j. $D_{10} = [0, 0, 1]^T$ represents the initial opinion of actor 10.

Result

$$C_1 = I,\ \ C_2 = \begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}$$

Simulation 1

A = I, w_{ii} = 0, which eliminates all attachments to initial opinions

The number of interactions in each batch is N = 10

Simulation 2

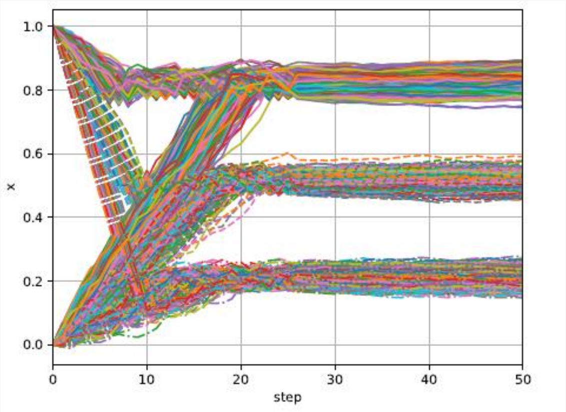
A = 0.85I, which modestly anchors all individuals on their initial certainties of belief , w_{ii} = 0,

The number of interactions in each batch is N = 10

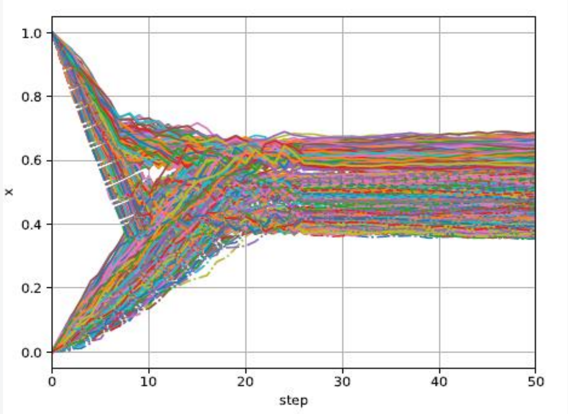
Simulation 3

A = 0.85I, which modestly anchors all individuals on their initial certainties of belief , w_{ii} = 0,

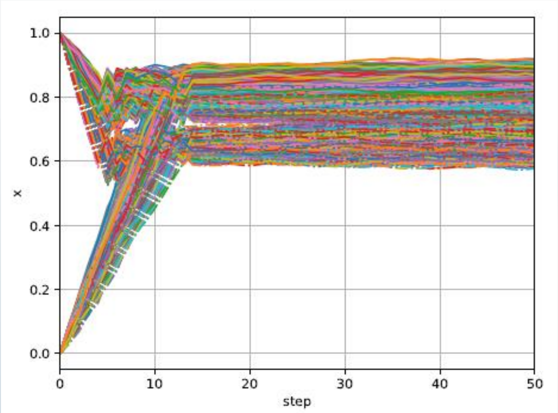
The number of interactions in each batch is N = 100



1. Independent case with batch 10



2. dependent case with batch 10



3. dependent case with batch 100

Conclusion and Limitation

1. Current research on opinion dynamics generally assumes that individuals make convex decisions, so that individuals in a social network will necessarily converge in their opinions under certain circumstances, but many groups make non-convex decisions. Therefore, these factors can be introduced into the construction of an opinion dynamics model to provide a more detailed picture of the opinion of groups in a social network.

2. Current research on opinion dynamics has generally focused on one of the three dimensions of individual attributes, interaction styles and decision-making, whereas in fact the evolution of group opinion is influenced by all three dimensions together. For example, If any two dimensions are combined, the model becomes very complex and it is difficult to derive relevant conclusions, and these issues need to be investigated.

Further development

It is difficult to quantize dependencies and stubbornness, even you ask most of the people, the answers may differ, such a phenomenon can be explained by book ’Noise’ of Daniel Kahneman. We are continuously influenced by noise all over us. In order to drive the development of opinion dynamic, quantization of these two parameters and multi-dimension consideration are of paramount importance.

The answer is almost revealed – advance in multi-disciplinary technology i.e., Neuroscience and Artificial intelligence. Recently, these technology are occupying the newspaper, like Neuralink established by Elon Musk makes monkeys able to play games through brainwave or Strentrode brain-computer interface helps patients with severe Paralysis to text, email, shop or even bank online. With these technology, opinion can one day be fully quantized, Metaverse is probably the main way.

End