

$$\begin{aligned}\Delta S_N &= S_N - \bar{S}_N \\ \Delta \bar{S}_N &= \bar{S}_N - S^* \\ \Delta S_k &= S_k - \bar{S}_k\end{aligned}$$

$$\Delta \bar{S}_k = \bar{S}_k - S^*$$

$$C(\Delta S_k, \Delta u_k)$$

$$\begin{aligned}C_k &= \frac{1}{2} (\Delta \bar{S}_N + \Delta S_N)^T Q_N (\Delta \bar{S}_N + \Delta S_N) \\ &\quad + \frac{1}{2} \sum_{k=0}^{N-1} (\Delta \bar{S}_k + \Delta S_k)^T Q (\Delta \bar{S}_k + \Delta S_k) \\ &\quad + \cancel{\frac{1}{2} \sum_{k=0}^{N-1} (\Delta \bar{u}_k + \Delta u_k)^T R (\Delta \bar{u}_k + \Delta u_k)}\end{aligned}$$

$$q_k = C_{S,k}^T$$

$$C_{S,k} = \frac{\partial C}{\partial S} (\bar{S}_k, \bar{u}_k)$$

$$+ \cancel{(\Delta \bar{u}_k + \Delta u_k)^T R (\Delta \bar{u}_k + \Delta u_k)} + (u_k + \Delta u_k)^T R (u_k + \Delta u_k)$$

$$= Q (\Delta \bar{S}_k + \Delta S_k)$$

$$\begin{aligned}C_{S,k}^T &= (Q (\Delta \bar{S}_k + \Delta S_k))^T \\ &= (\Delta \bar{S}_k + \Delta S_k)^T Q\end{aligned}$$

$$q_k = (\Delta \bar{S}_k + \Delta S_k)^T Q \rightarrow (1)$$

$$q_N = \frac{\partial C}{\partial S} (\bar{S}_N, \bar{u}_N) \rightarrow (2)$$

$$= Q_f (\Delta \bar{S}_N + \Delta S_N)$$

$$r_k = C_{u,k}^T$$

$$C_{u,k} = \frac{\partial C}{\partial u} (\bar{S}_k, \bar{u}_k) = R \cancel{u_k} \Delta \cancel{u_k}$$

$$r_k = \cancel{\Delta \bar{u}_k^T R \Delta \bar{u}_k} \rightarrow (3)$$

$$= R (u_k + \Delta u_k)$$