

# Temporal\_Difference

January 2, 2019

## 1 Mini Project: Temporal-Difference Methods

In this notebook, you will write your own implementations of many Temporal-Difference (TD) methods.

While we have provided some starter code, you are welcome to erase these hints and write your code from scratch.

### 1.0.1 Part 0: Explore CliffWalkingEnv

Use the code cell below to create an instance of the [CliffWalking](#) environment.

```
In [35]: import gym
         env = gym.make('CliffWalking-v0')
```

The agent moves through a  $4 \times 12$  gridworld, with states numbered as follows:

```
[[ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11],
 [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23],
 [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35],
 [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47]]
```

At the start of any episode, state 36 is the initial state. State 47 is the only terminal state, and the cliff corresponds to states 37 through 46.

The agent has 4 potential actions:

```
UP = 0
RIGHT = 1
DOWN = 2
LEFT = 3
```

Thus,  $\mathcal{S}^+ = \{0, 1, \dots, 47\}$ , and  $\mathcal{A} = \{0, 1, 2, 3\}$ . Verify this by running the code cell below.

```
In [36]: print(env.action_space)
         print(env.observation_space)
```

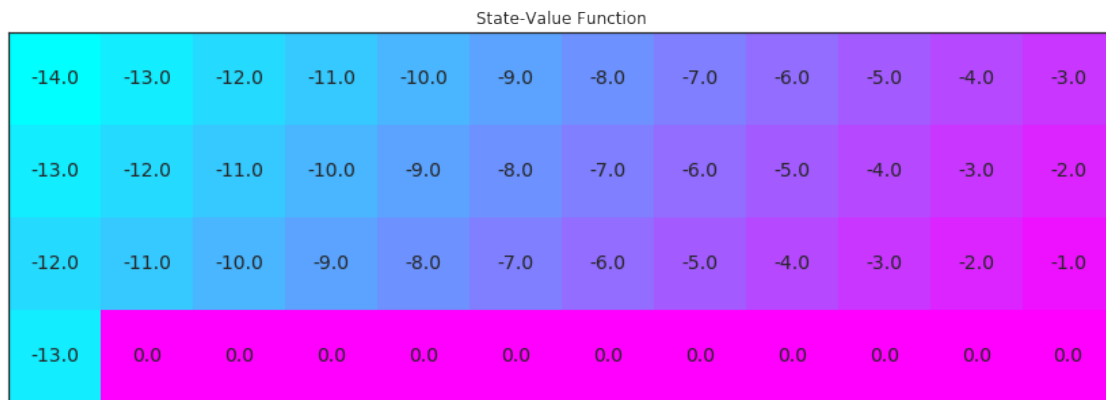
```
Discrete(4)
Discrete(48)
```

In this mini-project, we will build towards finding the optimal policy for the CliffWalking environment. The optimal state-value function is visualized below. Please take the time now to make sure that you understand *why* this is the optimal state-value function.

```
In [37]: import numpy as np
         from plot_utils import plot_values

         # define the optimal state-value function
         V_opt = np.zeros((4,12))
         V_opt[0:13][0] = -np.arange(3, 15)[::-1]
         V_opt[0:13][1] = -np.arange(3, 15)[::-1] + 1
         V_opt[0:13][2] = -np.arange(3, 15)[::-1] + 2
         V_opt[3][0] = -13

         plot_values(V_opt)
```



## 1.0.2 Part 1: TD Prediction: State Values

In this section, you will write your own implementation of TD prediction (for estimating the state-value function).

We will begin by investigating a policy where the agent moves: - RIGHT in states 0 through 10, inclusive,  
 - DOWN in states 11, 23, and 35, and - UP in states 12 through 22, inclusive, states 24 through 34, inclusive, and state 36.

The policy is specified and printed below. Note that states where the agent does not choose an action have been marked with -1.

```
In [38]: policy = np.hstack([1*np.ones(11), 2, 0, np.zeros(10), 2, 0, np.zeros(10), 2, 0, -1*np.
         print("\nPolicy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):")
         print(policy.reshape(4,12))
```

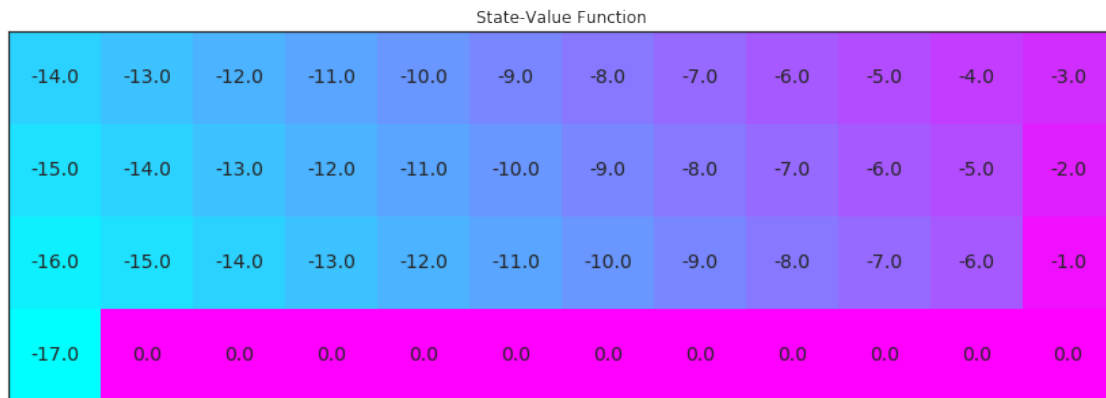
Policy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):

```
[[ 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  2.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  2.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  2.]
 [ 0. -1. -1. -1. -1. -1. -1. -1. -1. -1. -1. -1.]]
```

Run the next cell to visualize the state-value function that corresponds to this policy. Make sure that you take the time to understand why this is the corresponding value function!

```
In [39]: V_true = np.zeros((4,12))
         for i in range(3):
             V_true[0:12][i] = -np.arange(3, 15)[::-1] - i
         V_true[1][11] = -2
         V_true[2][11] = -1
         V_true[3][0] = -17

         plot_values(V_true)
```



The above figure is what you will try to approximate through the TD prediction algorithm.

Your algorithm for TD prediction has five arguments: - env: This is an instance of an OpenAI Gym environment. - num\_episodes: This is the number of episodes that are generated through agent-environment interaction. - policy: This is a 1D numpy array with policy.shape equal to the number of states (env.nS). policy[s] returns the action that the agent chooses when in state s. - alpha: This is the step-size parameter for the update step. - gamma: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: 1).

The algorithm returns as output: - V: This is a dictionary where V[s] is the estimated value of state s.

Please complete the function in the code cell below.

```
In [40]: from collections import defaultdict, deque
         import sys

         def td_prediction(env, num_episodes, policy, alpha, gamma=1.0):
             # initialize empty dictionaries of floats
```

```

V = defaultdict(float)
# loop over episodes
for i_episode in range(1, num_episodes+1):
    # monitor progress
    if i_episode % 100 == 0:
        print("\rEpisode {}/{}".format(i_episode, num_episodes), end="")
        sys.stdout.flush()

    ## TODO: complete the function
    state = env.reset()
    while True:
        # choose action A
        action = policy[state]
        # take action A, observe R, S'
        next_state, reward, done, info = env.step(action)
        # perform updates
        V[state] = V[state] + (alpha * (reward + (gamma * V[next_state]) - V[state])
        # S <- S'
        state = next_state
        # end episode if reached terminal state
        if done:
            break

    return V

```

Run the code cell below to test your implementation and visualize the estimated state-value function. If the code cell returns **PASSED**, then you have implemented the function correctly! Feel free to change the `num_episodes` and `alpha` parameters that are supplied to the function. However, if you'd like to ensure the accuracy of the unit test, please do not change the value of `gamma` from the default.

```
In [41]: import check_test
```

```

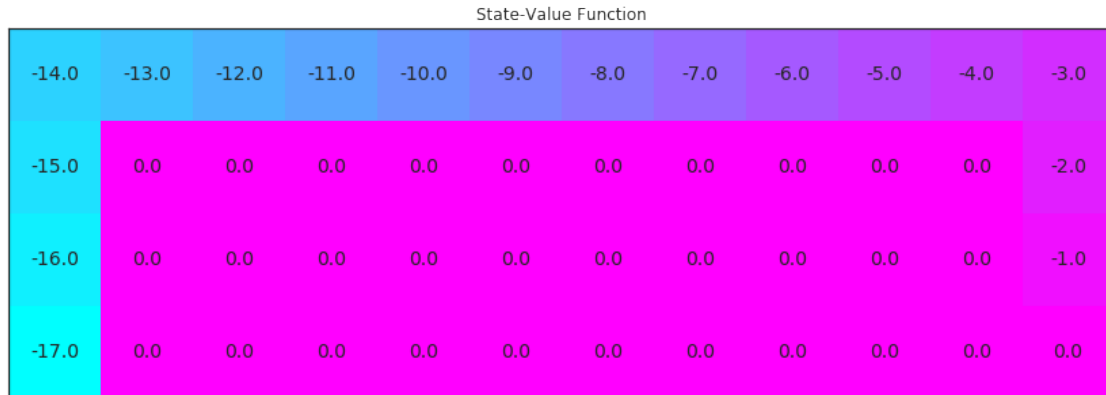
# evaluate the policy and reshape the state-value function
V_pred = td_prediction(env, 5000, policy, .01)

# please do not change the code below this line
V_pred_plot = np.reshape([V_pred[key] if key in V_pred else 0 for key in np.arange(48)])
check_test.run_check('td_prediction_check', V_pred_plot)
plot_values(V_pred_plot)

```

Episode 5000/5000

**PASSED**



How close is your estimated state-value function to the true state-value function corresponding to the policy?

You might notice that some of the state values are not estimated by the agent. This is because under this policy, the agent will not visit all of the states. In the TD prediction algorithm, the agent can only estimate the values corresponding to states that are visited.

### 1.0.3 Part 2: TD Control: Sarsa

In this section, you will write your own implementation of the Sarsa control algorithm.

Your algorithm has four arguments: - env: This is an instance of an OpenAI Gym environment.  
 - num\_episodes: This is the number of episodes that are generated through agent-environment interaction. - alpha: This is the step-size parameter for the update step. - gamma: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: 1).

The algorithm returns as output: - Q: This is a dictionary (of one-dimensional arrays) where  $Q[s][a]$  is the estimated action value corresponding to state  $s$  and action  $a$ .

Please complete the function in the code cell below.

*(Feel free to define additional functions to help you to organize your code.)*

```
In [42]: def update_Q(Qsa, Qsa_next, reward, alpha, gamma):
          return Qsa + (alpha * (reward + (gamma * Qsa_next) - Qsa))

          def epsilon_greedy_probs(env, Q_s, i_episode, eps=None):
              epsilon = 1.0 / i_episode
              if eps is not None:
                  epsilon = eps
              policy_s = np.ones(env.nA) * epsilon / env.nA
              policy_s[np.argmax(Q_s)] = 1 - epsilon + (epsilon / env.nA)
              return policy_s

In [43]: import matplotlib.pyplot as plt
          %matplotlib inline

          def sarsa(env, num_episodes, alpha, gamma=1.0):
              # initialize action-value function (empty dictionary of arrays)
```

```

Q = defaultdict(lambda: np.zeros(env.nA))
# initialize performance monitor
plot_every = 100
tmp_scores = deque(maxlen=plot_every)
scores = deque(maxlen=num_episodes)
# loop over episodes
for i_episode in range(1, num_episodes+1):
    # monitor progress
    if i_episode % 100 == 0:
        print("\rEpisode {}/{}".format(i_episode, num_episodes), end="")
        sys.stdout.flush()

    ## TODO: complete the function
    # initialize score
    score = 0
    # begin an episode, observe S
    state = env.reset()
    # get epsilon-greedy action probabilities
    policy_s = epsilon_greedy_probs(env, Q[state], i_episode)
    # pick action A
    action = np.random.choice(np.arange(env.nA), p=policy_s)
    # limit number of time steps per episode
    for t_step in np.arange(300):
        # take action A, observe R, S'
        next_state, reward, done, info = env.step(action)
        # add reward to score
        score += reward
        if not done:
            # get epsilon-greedy action probabilities
            policy_s = epsilon_greedy_probs(env, Q[next_state], i_episode)
            # pick next action A'
            next_action = np.random.choice(np.arange(env.nA), p=policy_s)
            # update TD estimate of Q
            Q[state][action] = update_Q(Q[state][action], Q[next_state][next_action],
                                       reward, alpha, gamma)

            # S <- S'
            state = next_state
            # A <- A'
            action = next_action
        if done:
            # update TD estimate of Q
            Q[state][action] = update_Q(Q[state][action], 0, reward, alpha, gamma)
            # append score
            tmp_scores.append(score)
            break
    if (i_episode % plot_every == 0):
        scores.append(np.mean(tmp_scores))
# plot performance

```

```

plt.plot(np.linspace(0,num_episodes,len(scores),endpoint=False),np.asarray(scores))
plt.xlabel('Episode Number')
plt.ylabel('Average Reward (Over Next %d Episodes)' % plot_every)
plt.show()
# print best 100-episode performance
print(('Best Average Reward over %d Episodes: ' % plot_every), np.max(scores))

return Q

```

Use the next code cell to visualize the *estimated* optimal policy and the corresponding state-value function.

If the code cell returns **PASSED**, then you have implemented the function correctly! Feel free to change the `num_episodes` and `alpha` parameters that are supplied to the function. However, if you'd like to ensure the accuracy of the unit test, please do not change the value of `gamma` from the default.

```

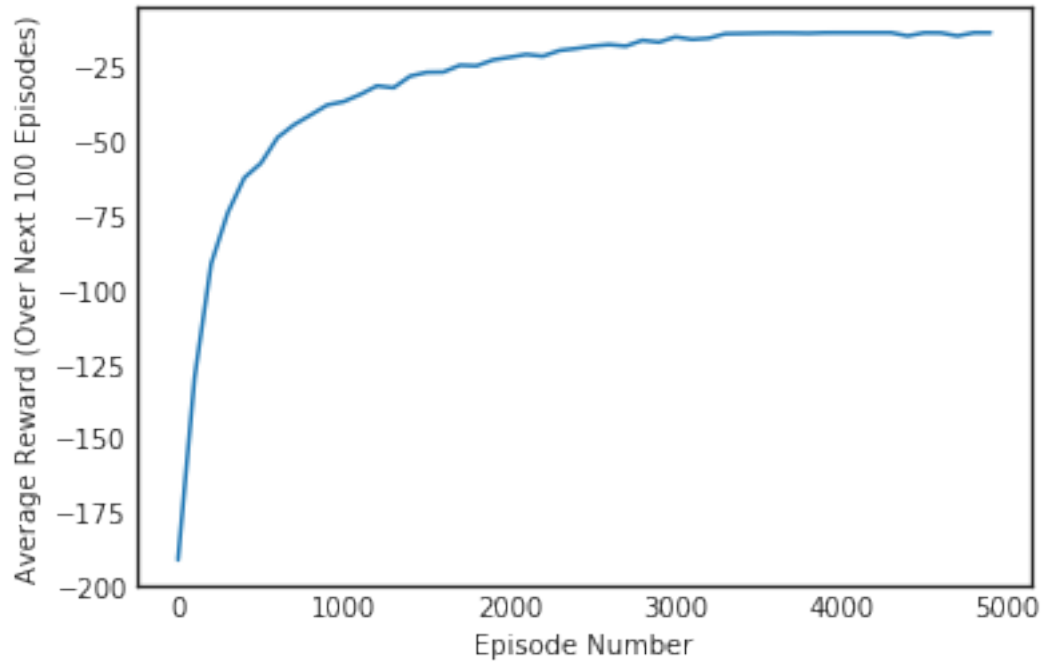
In [44]: # obtain the estimated optimal policy and corresponding action-value function
Q_sarsa = sarsa(env, 5000, .01)

# print the estimated optimal policy
policy_sarsa = np.array([np.argmax(Q_sarsa[key]) if key in Q_sarsa else -1 for key in n
check_test.run_check('td_control_check', policy_sarsa)
print("\nEstimated Optimal Policy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):")
print(policy_sarsa)

# plot the estimated optimal state-value function
V_sarsa = ([np.max(Q_sarsa[key]) if key in Q_sarsa else 0 for key in np.arange(48)])
plot_values(V_sarsa)

```

Episode 5000/5000



Best Average Reward over 100 Episodes: -13.0

## PASSED

Estimated Optimal Policy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):

```
[ [ 2  3  2  2  1  2  2  2  1  2  1  2]
  [ 1  2  3  1  1  0  2  0  1  0  1  2]
  [ 1  1  1  1  1  1  1  1  1  1  1  2]
  [ 0 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1]]
```

State-Value Function

-10.838	-10.363	-9.728	-9.017	-8.269	-7.495	-6.705	-5.91	-5.116	-4.325	-3.556	-2.85
-11.276	-10.61	-9.852	-9.048	-8.218	-7.373	-6.512	-5.639	-4.756	-3.857	-2.94	-1.998
-12.003	-11.003	-10.003	-9.002	-8.002	-7.001	-6.0	-5.0	-4.0	-3.0	-2.0	-1.0
-13.003	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



### 1.0.4 Part 3: TD Control: Q-learning

In this section, you will write your own implementation of the Q-learning control algorithm.

Your algorithm has four arguments: - env: This is an instance of an OpenAI Gym environment.  
- num\_episodes: This is the number of episodes that are generated through agent-environment interaction. - alpha: This is the step-size parameter for the update step. - gamma: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: 1).

The algorithm returns as output: - Q: This is a dictionary (of one-dimensional arrays) where  $Q[s][a]$  is the estimated action value corresponding to state  $s$  and action  $a$ .

Please complete the function in the code cell below.

(Feel free to define additional functions to help you to organize your code.)

```
In [45]: def q_learning(env, num_episodes, alpha, gamma=1.0):
    # initialize empty dictionary of arrays
    Q = defaultdict(lambda: np.zeros(env.nA))
    # initialize performance monitor
    plot_every = 100
    tmp_scores = deque(maxlen=plot_every)
    scores = deque(maxlen=num_episodes)
    # loop over episodes
    for i_episode in range(1, num_episodes+1):
        # monitor progress
        if i_episode % 100 == 0:
            print("\rEpisode {}/{}".format(i_episode, num_episodes), end="")
            sys.stdout.flush()

        ## TODO: complete the function
        # initialize score
        score = 0
        # begin an episode, observe S
        state = env.reset()
        while True:
            # get epsilon-greedy action probabilities
            policy_s = epsilon_greedy_probs(env, Q[state], i_episode)
            # pick next action A
            action = np.random.choice(np.arange(env.nA), p=policy_s)
            # take action A, observe R, S'
            next_state, reward, done, info = env.step(action)
            # add reward to score
            score += reward
            # update Q
            Q[state][action] = update_Q(Q[state][action], np.max(Q[next_state]), reward)
            # S <- S'
            state = next_state
            # until S is terminal
            if done:
                # append score
                tmp_scores.append(score)
```

```

        break
    if (i_episode % plot_every == 0):
        scores.append(np.mean(tmp_scores))
    # plot performance
    plt.plot(np.linspace(0, num_episodes, len(scores), endpoint=False), np.asarray(scores))
    plt.xlabel('Episode Number')
    plt.ylabel('Average Reward (Over Next %d Episodes)' % plot_every)
    plt.show()
    # print best 100-episode performance
    print(('Best Average Reward over %d Episodes: ' % plot_every), np.max(scores))
    return Q

```

Use the next code cell to visualize the *estimated* optimal policy and the corresponding state-value function.

If the code cell returns **PASSED**, then you have implemented the function correctly! Feel free to change the `num_episodes` and `alpha` parameters that are supplied to the function. However, if you'd like to ensure the accuracy of the unit test, please do not change the value of `gamma` from the default.

```

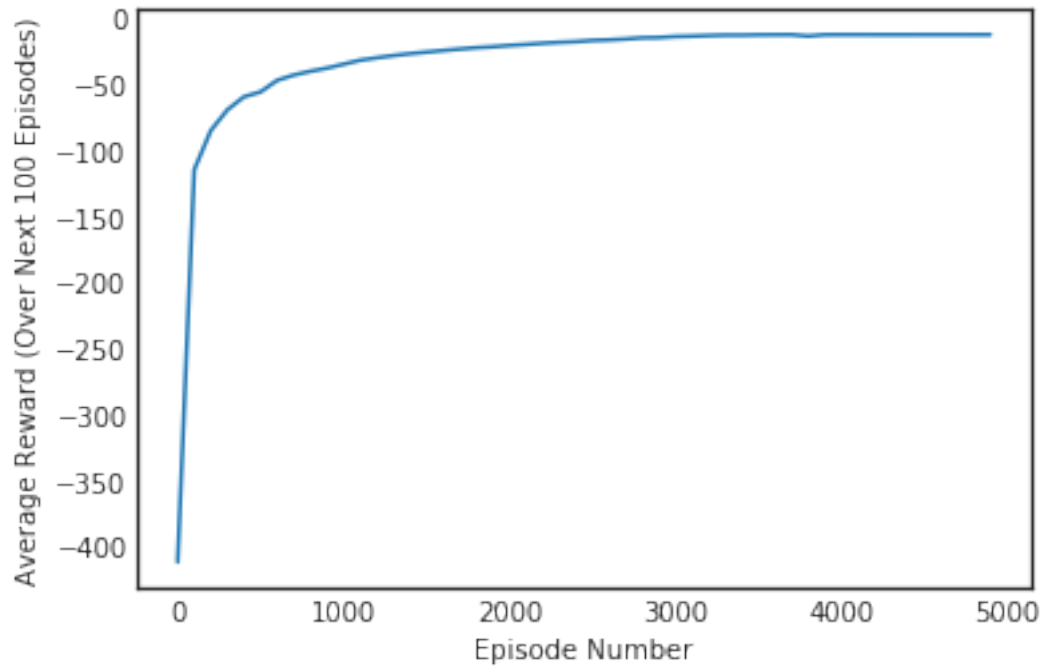
In [46]: # obtain the estimated optimal policy and corresponding action-value function
        Q_sarsamax = q_learning(env, 5000, .01)

        # print the estimated optimal policy
        policy_sarsamax = np.array([np.argmax(Q_sarsamax[key]) if key in Q_sarsamax else -1 for
        check_test.run_check('td_control_check', policy_sarsamax)
        print("\nEstimated Optimal Policy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):")
        print(policy_sarsamax)

        # plot the estimated optimal state-value function
        plot_values([np.max(Q_sarsamax[key]) if key in Q_sarsamax else 0 for key in np.arange(4

```

Episode 5000/5000



Best Average Reward over 100 Episodes: -13.0

## PASSED

Estimated Optimal Policy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):

```
[[ 0  2  1  3  1  1  1  0  1  1  1  1]
 [ 1  2  0  1  2  1  1  1  2  2  1  2]
 [ 1  1  1  1  1  1  1  1  1  1  1  2]
 [ 0 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1  0]]
```

State-Value Function											
-10.837	-10.357	-9.723	-9.014	-8.265	-7.492	-6.704	-5.91	-5.113	-4.325	-3.555	-2.85
-11.273	-10.606	-9.846	-9.043	-8.214	-7.368	-6.509	-5.638	-4.754	-3.856	-2.94	-1.998
-12.0	-11.0	-10.0	-9.0	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0	-2.0	-1.0
-13.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

### 1.0.5 Part 4: TD Control: Expected Sarsa

In this section, you will write your own implementation of the Expected Sarsa control algorithm.

Your algorithm has four arguments: - env: This is an instance of an OpenAI Gym environment.  
- num\_episodes: This is the number of episodes that are generated through agent-environment interaction. - alpha: This is the step-size parameter for the update step. - gamma: This is the discount rate. It must be a value between 0 and 1, inclusive (default value: 1).

The algorithm returns as output: - Q: This is a dictionary (of one-dimensional arrays) where  $Q[s][a]$  is the estimated action value corresponding to state  $s$  and action  $a$ .

Please complete the function in the code cell below.

(Feel free to define additional functions to help you to organize your code.)

```
In [49]: def expected_sarsa(env, num_episodes, alpha, gamma=1.0):
    # initialize empty dictionary of arrays
    Q = defaultdict(lambda: np.zeros(env.nA))
    # initialize performance monitor
    plot_every = 100
    tmp_scores = deque(maxlen=plot_every)
    scores = deque(maxlen=num_episodes)
    # loop over episodes
    for i_episode in range(1, num_episodes+1):
        # monitor progress
        if i_episode % 100 == 0:
            print("\rEpisode {}/{}".format(i_episode, num_episodes), end="")
            sys.stdout.flush()

        ## TODO: complete the function
        # initialize score
        score = 0
        # begin an episode
        state = env.reset()
        # get epsilon-greedy action probabilities
        policy_s = epsilon_greedy_probs(env, Q[state], i_episode, 0.005)
        while True:
            # pick next action
            action = np.random.choice(np.arange(env.nA), p=policy_s)
            # take action A, observe R, S'
            next_state, reward, done, info = env.step(action)
            # add reward to score
            score += reward
            # get epsilon-greedy action probabilities (for S')
            policy_s = epsilon_greedy_probs(env, Q[next_state], i_episode, 0.005)
            # update Q
            Q[state][action] = update_Q(Q[state][action], np.dot(Q[next_state], policy_s))
            # S <- S'
            state = next_state
            # until S is terminal
            if done:
```

```

        # append score
        tmp_scores.append(score)
        break
    if(i_episode % plot_every == 0):
        scores.append(np.mean(tmp_scores))
# plot performance
plt.plot(np.linspace(0,num_episodes,len(scores),endpoint=False),np.asarray(scores))
plt.xlabel('Episode Number')
plt.ylabel('Average Reward (Over Next %d Episodes)' % plot_every)
plt.show()
# print best 100-episode performance
print(('Best Average Reward over %d Episodes: ' % plot_every), np.max(scores))
return Q

```

Use the next code cell to visualize the *estimated* optimal policy and the corresponding state-value function.

If the code cell returns **PASSED**, then you have implemented the function correctly! Feel free to change the num\_episodes and alpha parameters that are supplied to the function. However, if you'd like to ensure the accuracy of the unit test, please do not change the value of gamma from the default.

```

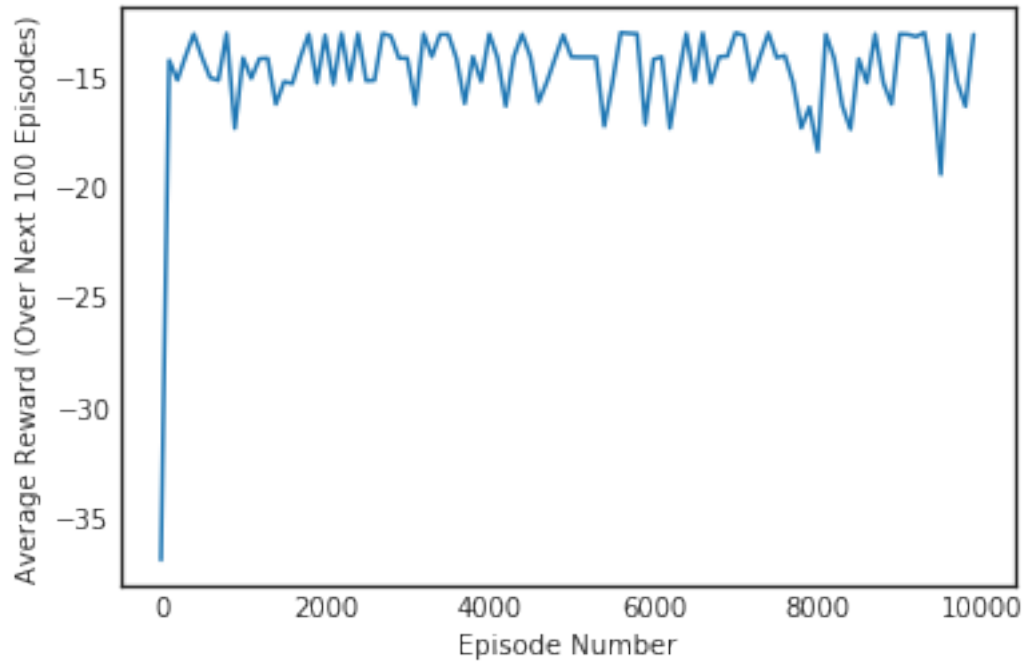
In [50]: # obtain the estimated optimal policy and corresponding action-value function
Q_expsarsa = expected_sarsa(env, 10000, 1)

# print the estimated optimal policy
policy_expsarsa = np.array([np.argmax(Q_expsarsa[key]) if key in Q_expsarsa else -1 for
check_test.run_check('td_control_check', policy_expsarsa)
print("\nEstimated Optimal Policy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):")
print(policy_expsarsa)

# plot the estimated optimal state-value function
plot_values([np.max(Q_expsarsa[key]) if key in Q_expsarsa else 0 for key in np.arange(4

```

Episode 10000/10000



Best Average Reward over 100 Episodes: -13.0

## PASSED

Estimated Optimal Policy (UP = 0, RIGHT = 1, DOWN = 2, LEFT = 3, N/A = -1):

```
[[ 2  1  1  1  1  0  1  1  2  2  1  2]
 [ 1  1  1  1  1  1  1  1  1  1  1  2]
 [ 1  1  1  1  1  1  1  1  1  1  1  2]
 [ 0 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1  0]]
```

State-Value Function											
-12.161	-12.033	-11.035	-10.285	-10.006	-9.006	-8.006	-7.005	-6.006	-5.014	-4.015	-3.011
-13.051	-12.046	-11.042	-10.04	-9.036	-8.032	-7.028	-6.025	-5.021	-4.017	-3.011	-2.006
-13.394	-12.263	-11.13	-9.995	-8.858	-7.72	-6.581	-5.44	-4.297	-3.152	-2.006	-1.0
-14.399	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

```
In [ ]:
```