CS 237B: Principles of Robot Autonomy II Problem Set 02

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Problem 1: Form and Force Closure.

(i) Force closure can be seen as a "generalization" of form closure. Form closure considers only normal forces applied in the body contacts, but force closure considers friction-applied forces. Therefore force closure contains the span of form closure wrench Space. We can see this by the wrench basis for a point contact frictionless and with friction examples:

$$\vec{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, f_3 \ge 0.$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \sqrt{f_1^2 + f_2^2} \leq \mu f_3, f_3 \geq 0.$$

In the case of $\mu=0$, each contact can provide forces only along the normal direction, and force closure reduces to form closure.

- (ii) To span the \mathbb{R}^n wrench linearly, we will need n contacts, but to get form closure, we need to span the space positively. Therefore, we will need n+1 contacts to apply a wrench to achieve the positive span (with the correct position and direction, verifying using a planar graph, e.g.) together with another set of wrenches. For 2D, $w = (f_x, f_y, w_z) \in \mathbb{R}^3$, we need 4 contacts. For 3D, $w = (f_x, f_y, f_z, w_x, w_y, w_z) \in \mathbb{R}^6$, we need 7 contacts.
- (iii) Form closure analysis:

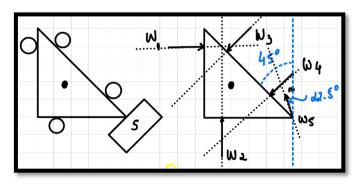


Figure 1 - Problem 1 Diagram.

For our analysis, we use a planar graph, evaluating the set of signals rotation. The (+) for counterclockwise and (-) for clockwise movement when one of the five forces is cut off.

• F1 out:

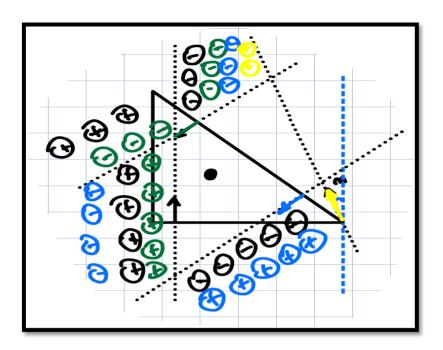


Figure 2 - F1 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (2,3,4,5) finger subset.

• F2 out:

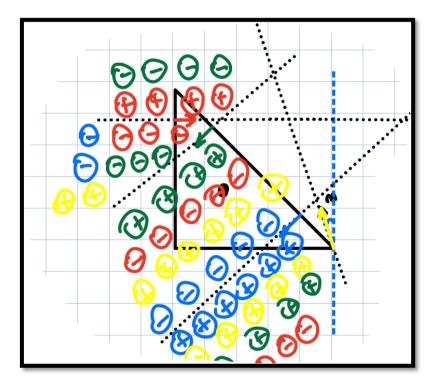


Figure 3 - F2 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (1,3,4,5) finger subset.

• F3 out:

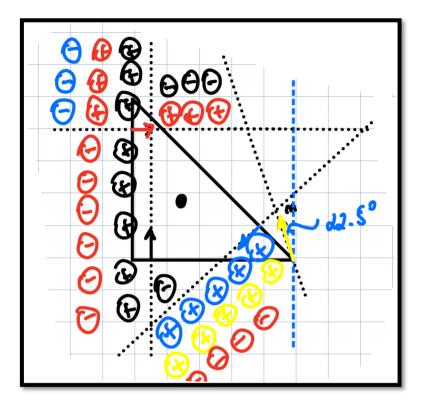


Figure 4 - F3 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (1,2,4,5) finger subset.

• F4 out:

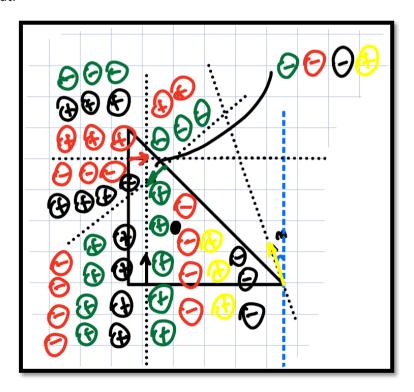


Figure 5 - F4 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (1,2,3,5) finger subset.

• F5 out:

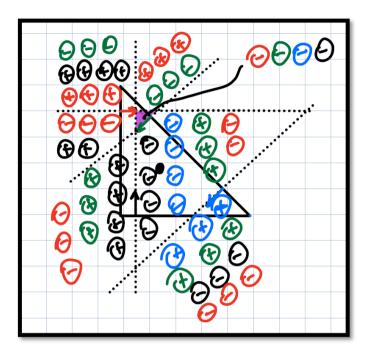


Figure 6 - F5 out.

We doesn't have anymore a mix of plus and minus signals in the regions (purple's region), therefore, the object is NOT in form closure yet for (1,2,3,4) finger subset.

(vi) Figure 2, analyzing the range of μ that the grasp yields force closure.

$$f_1=(0,1)$$
, with friction coef. μ

Therefore,

 $f_{1,1}=(\mu,1)$ and $f_{1,2}=(-\mu,1)$

And finally,

 $f_2=(0,-1)$ and $f_3=(-1,0)$

For point contact coordinates,

 $p_1=(0,0), p_2=(c,1)$ and $p_3=(0.5,h)$

So, we can calculate the wrenches,

$$W_{1,1}=\big(f_{1,1},\tau_{1,1}\big), W_{1,2}=\big(f_{1,2},\tau_{1,2}\big), W_2=(f_2,\tau_2) \ and \ W_3=(f_3,\tau_3)$$
 The wrench matrix $\mathcal F$,

$$\mathcal{F} = [W_{1,1} \quad W_{1,2} \quad W_2 \quad W_3],$$

The cross matrices (vector in 2D),

$$P_{[x]}^1 = [0 \quad 0], P_{[x]}^2 = [-1 \quad c], P_{[x]}^3 = [0 - h \quad 0.5],$$

Follow to wrench evaluation,

$$W_{1,1} = \begin{bmatrix} \mu \\ 1 \\ 0 \end{bmatrix}, W_{1,2} = \begin{bmatrix} -\mu \\ 1 \\ 0 \end{bmatrix}, W_2 = \begin{bmatrix} 0 \\ -1 \\ -c \end{bmatrix}, W_3 = \begin{bmatrix} -1 \\ 0 \\ h \end{bmatrix}.$$

And \mathcal{F} evaluation,

$$\mathcal{F} = \begin{bmatrix} \mu & -\mu & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & -c & h \end{bmatrix},$$

To force closure conditions,

(1) rank (
$$\mathcal{F}$$
) = 3 (μ , c and $h \neq 0$)

(2)
$$\mathcal{F}k = 0, k_i > 0$$

$$\begin{bmatrix} \mu & -\mu & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & -c & h \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = 0,$$

Now we have the set of equations above,

$$\mu k_1 - \mu k_2 - k_4 = 0 \Rightarrow k_4 = \mu(k_1 - k_2)(I)$$
$$k_1 + k_2 - k_3 = 0 \Rightarrow k_3 = k_1 + k_2(II)$$
$$-ck_3 + hk_4 = 0(III)$$

Substituting (I), (II) in (III),

$$-c(k_1 + k_2) + hk_4 = 0$$

$$-ck_1 - ck_2 + h\mu k_1 - h\mu k_2 = 0$$

$$-k_1(c - h\mu) - k_2(c + h\mu) = 0$$

And finally,

$$\frac{k_1}{k_2} = \frac{-(c + h\mu)}{(c - h\mu)} > 0,$$

Now we have two ranges,

$$(c + h\mu) < 0$$
 and $(c - h\mu) > 0$

Or

$$(c + h\mu) > 0$$
 and $(c - h\mu) < 0$.

Where c = 0.25 and h = 0.5,

$$\mu < -0.5$$
 and $\mu < 0.5$ (first condition)

$$\mu > -0.5$$
 and $\mu > 0.5$ (second condition)

Then the μ range will be the intersection of the intervals above,

$$-0.5 < \mu < 0.5 \Rightarrow |\mu| < 0.5$$

The range of
$$\mu(c,h) \Rightarrow \frac{-c}{h} < \mu < \frac{c}{h}$$
.

Problem 2: Grasp Force Optimization.

(i) Rewriting equations (4) and (5) as $\phi f + w^{ext} = 0$:

$$\phi f = -w^{ext}$$

Where
$$w^{ext} = (f^{ext}, \tau^{ext})$$
,

From eqs (4) and (5),

$$f^{ext} = -\sum_{i=1}^{M} T^{(i)} f^{(i)}$$
 and $\tau^{ext} = -\sum_{i=1}^{M} -P_{[x]}^{(i)} T^{(i)} f^{(i)}$.

Then w^{ext} .

$$w^{ext} = - \begin{bmatrix} T_{dxd}^{(1)} & T_{dxd}^{(2)} & \cdots & T_{dxd}^{(M)} \\ P_{[x]}^{(1)} T^{(1)} & P_{[x]}^{(2)} T^{(2)} & \cdots & P_{[x]}^{(M)} T^{(M)} \end{bmatrix} \begin{pmatrix} f^{(1)} \\ f^{(2)} \\ \vdots \\ f^{(M)} \end{pmatrix} = \phi f.$$

So we can find the matrix ϕ and f,

$$\phi = \begin{bmatrix} T_{dxd}^{(1)} & T_{dxd}^{(2)} & \cdots & T_{dxd}^{(M)} \\ P_{[x]}^{(1)}T^{(1)} & P_{[x]}^{(2)}T^{(2)} & \cdots & P_{[x]}^{(M)}T^{(M)} \end{bmatrix}_{p \times n} and f = \begin{pmatrix} f^{(1)} \\ f^{(2)} \\ \vdots \\ f^{(M)} \end{pmatrix}_{n \times 1}.$$

Where.

$$p = wrench \ size - 3 (2D) \ or 6 (3D)$$

$$n = M * D_i D - dimension of^{(i)}$$

M = # points of normal forces, each of them couls generate 1,2 or 4 wrenches.

(ii)Recast objective function in linear form:

Adding a scalar s and constraint $||f^i|| \le s, i = 1, ..., M$.

For norm-2,

$$||f^{(i)}||_2 = \sqrt{f_x^{(i)^2} + f_y^{(i)^2} + f_z^{(i)^2}} \le s$$

The quadratic cone constraint is:

$$\sqrt{{f_x^{(i)}}^2 + {f_y^{(i)}}^2} \le \mu_i f_z^{(i)}$$

$$f_x^{(i)^2} + f_v^{(i)^2} \le \mu_i^2 f_z^{(i)^2}$$
,

Add $f_z^{(i)^2}$ for each side of equation,

$$\begin{split} f_x^{(i)^2} + f_y^{(i)^2} + f_z^{(i)^2} &\leq \mu_i^2 f_z^{(i)^2} + f_z^{(i)^2}, \\ & \left\| f^{(i)} \right\|_2^2 \leq s^2, \\ & \left\| f^{(i)} \right\|_2^2 \leq \left(\mu_i^2 + 1 \right) f_z^{(i)^2}, \\ & \left\| f^{(i)} \right\|_2 \leq \left[0 \quad 0 \quad \sqrt{\left(\mu_i^2 + 1 \right)} \right] f^{(i)}, \\ & S = \left[0 \quad 0 \quad \sqrt{\left(\mu_i^2 + 1 \right)} \right], \end{split}$$
 Then $h^T = \begin{pmatrix} 0 \quad 0 \quad \sqrt{\left(\mu_1^2 + 1 \right)} \quad \cdots \quad 0 \quad 0 \quad \sqrt{\left(\mu_n^2 + 1 \right)} \right)_{1xn}.$ $n = M * D, D - dimension of^{(i)}$

(iii) Define the variable x and the SOCP parameters:

$$x = \text{cpx variable (forces)} \in \mathbb{R}^n$$
.

Defining A_i matrix,

$$\begin{split} \|A_{i}f + b_{i}\|_{2} & \leq {c_{i}}^{T}f + d_{i}, \\ \text{Comparing against} \sqrt{{f_{x}^{(i)}}^{2} + {f_{y}^{(i)}}^{2}} & \leq \left[0 \quad 0 \quad \sqrt{\left(\mu_{i}^{2} + 1\right)}\right] f^{(i)}, \end{split}$$

$$b_i = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}_{n_i \times 1}, d_i = 0,$$

$$||A_i f||_2 \le c_i^T f,$$

$$A_i = \begin{pmatrix} \cdots & 1 & 0 & 0 & \cdots \\ \cdots & 0 & \cdots & 0 & \cdots \\ \cdots & 0 & 0 & 1 & \cdots \end{pmatrix}_{n_i \times n},$$

$$c_i^T = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & \sqrt{(\mu_{i,1}^2 + 1)} & \cdots & 0 & 0 & \sqrt{(\mu_{i,k_i}^2 + 1)} & 0 & 0 \end{pmatrix}_{1xn},$$

$$g_{p x 1} = -w^{ext},$$

$$F = \phi$$
.

Where,

$$n = M * D, D - dimension of^{(i)},$$

$$n_i = D$$
,

 $p = wrench \ size - 3 \ (2D) \ or \ 6 \ (3D).$

Problem 3: Learning Intuitive Physics.

Epoch 13/50

(iv) Training process and validation loss: Epoch 1/50 Epoch 2/50 Epoch 3/50 Epoch 4/50 Epoch 5/50 20/20 [=============] - ETA: 0s - loss: 2.5681 Epoch 5: saving model to trained_models\cp-005.ckpt Epoch 6/50 Epoch 7/50 Epoch 8/50 Epoch 9/50 Epoch 10/50 20/20 [============ - - ETA: 0s - loss: 1.8394 Epoch 10: saving model to trained_models\cp-010.ckpt Epoch 11/50 Epoch 12/50

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Epoch 14/50
Epoch 15/50
Epoch 15: saving model to trained_models\cp-015.ckpt
Epoch 16/50
Epoch 17/50
Epoch 18/50
20/20 [=============================] - 27s 1s/step - loss: 0.8499 - val loss: 1.5926
Epoch 19/50
Epoch 20/50
Epoch 20: saving model to trained_models\cp-020.ckpt
Epoch 21/50
Epoch 22/50
Epoch 23/50
Epoch 24/50
Epoch 25/50
20/20 [============= - - ETA: 0s - loss: 0.7100
Epoch 25: saving model to trained_models\cp-025.ckpt
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Epoch 26/50
Epoch 27/50
Epoch 28/50
Epoch 29/50
Epoch 30/50
20/20 [========================] - ETA: 0s - loss: 0.6149
Epoch 30: saving model to trained_models\cp-030.ckpt
Epoch 31/50
Epoch 32/50
Epoch 33/50
Epoch 34/50
Epoch 35/50
Epoch 35: saving model to trained_models\cp-035.ckpt
Epoch 36/50
Epoch 37/50
Epoch 38/50
Epoch 39/50
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Epoch 40/50
Epoch 40: saving model to trained_models\cp-040.ckpt
Epoch 41/50
Epoch 42/50
Epoch 43/50
Epoch 44/50
Epoch 45/50
Epoch 45: saving model to trained_models\cp-045.ckpt
Epoch 46/50
Epoch 47/50
Epoch 48/50
Epoch 49/50
Epoch 50/50
20/20 [============= - - ETA: 0s - loss: 0.2353
Epoch 50: saving model to trained_models\cp-050.ckpt
(v) The predictions regarding to \mu_{pred} match with the dot product \sum_i p_i \mu_i. The layer "mu" is a
linear activation layer a(x) = x, therefore if the input is negative, output will be negative. To
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force a neural network to output positive μ_i we can choose another activation layer, e.g., relu layer.

(vii) Training process and validation loss:

Epoch 1/50

Epoch 2/50

Epoch 3/50

Epoch 4/50

Epoch 5/50

20/20 [========] - ETA: 0s - loss: 2.2200

Epoch 5: saving model to trained_models\cp-005.ckpt

Epoch 6/50

Epoch 7/50

Epoch 8/50

Epoch 9/50

Epoch 10/50

20/20 [=======] - ETA: 0s - loss: 1.5673

Epoch 10: saving model to trained_models\cp-010.ckpt

Epoch 11/50

Epoch 12/50

```
Epoch 13/50
Epoch 14/50
Epoch 15/50
20/20 [=========================] - ETA: 0s - loss: 1.2996
Epoch 15: saving model to trained_models\cp-015.ckpt
Epoch 16/50
Epoch 17/50
Epoch 18/50
Epoch 19/50
Epoch 20/50
20/20 [============ - - ETA: 0s - loss: 0.9990
Epoch 20: saving model to trained_models\cp-020.ckpt
Epoch 21/50
Epoch 22/50
Epoch 23/50
Epoch 24/50
Epoch 25/50
20/20 [============== - ETA: 0s - loss: 0.8532
Epoch 25: saving model to trained_models\cp-025.ckpt
```

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Epoch 26/50
Epoch 27/50
Epoch 28/50
Epoch 29/50
Epoch 30/50
20/20 [=================] - ETA: 0s - loss: 0.8045
Epoch 30: saving model to trained_models\cp-030.ckpt
Epoch 31/50
Epoch 32/50
Epoch 33/50
Epoch 34/50
Epoch 35/50
Epoch 35: saving model to trained_models\cp-035.ckpt
Epoch 36/50
Epoch 37/50
Epoch 38/50
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Epoch 39/50
Epoch 40/50
Epoch 40: saving model to trained_models\cp-040.ckpt
Epoch 41/50
Epoch 42/50
Epoch 43/50
Epoch 44/50
Epoch 45/50
20/20 [============================] - ETA: 0s - loss: 0.5089
Epoch 45: saving model to trained_models\cp-045.ckpt
Epoch 46/50
Epoch 47/50
Epoch 48/50
Epoch 49/50
Epoch 50/50
Epoch 50: saving model to trained_models\cp-050.ckpt
20/20 [============================] - 26s 1s/step - loss: 0.4774 - val loss: 1.8546
```

Comparing against the physical network the baseline NN performs worst. The baseline used predictions from data to evaluate the acceleration, the physical network evaluate the acceleration analytically, prediction the friction coefficient using the network only, this argument could be used to explain the difference in performing and results.