CS 237B: Principles of Robot Autonomy II  
Problem Set 02

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Problem 1: Form and Force Closure.

(i) Force closure can be seen as a “generalization” of form closure. Form closure considers only normal  
forces applied in the body contacts, but force closure considers friction-applied forces. Therefore force  
closure contains the span of form closure wrench Space. We can see this by the wrench basis for a  
point contact frictionless and with friction examples:

In the case of μ = 0, each contact can provide forces only along the normal direction, and force closure  
reduces to form closure.

(ii) To span the wrench linearly, we will need n contacts, but to get form closure, we need to span the  
space positively. Therefore, we will need n+1 contacts to apply a wrench to achieve the positive span  
(with the correct position and direction, verifying using a planar graph, e.g.) together with another set  
of wrenches. For 2D, , we need 4 contacts. For 3D, ,  
we need 7 contacts.

(iii) Form closure analysis:

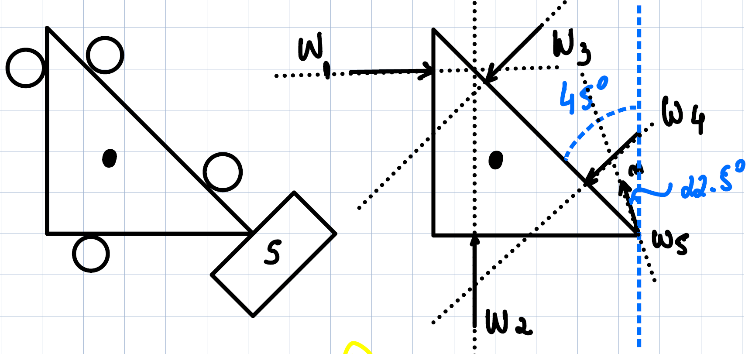


Figure - Problem 1 Diagram.

For our analysis, we use a planar graph, evaluating the set of signals rotation. The (+) for counterclockwise and (-) for clockwise movement when one of the five forces is cut off.

* F1 out:

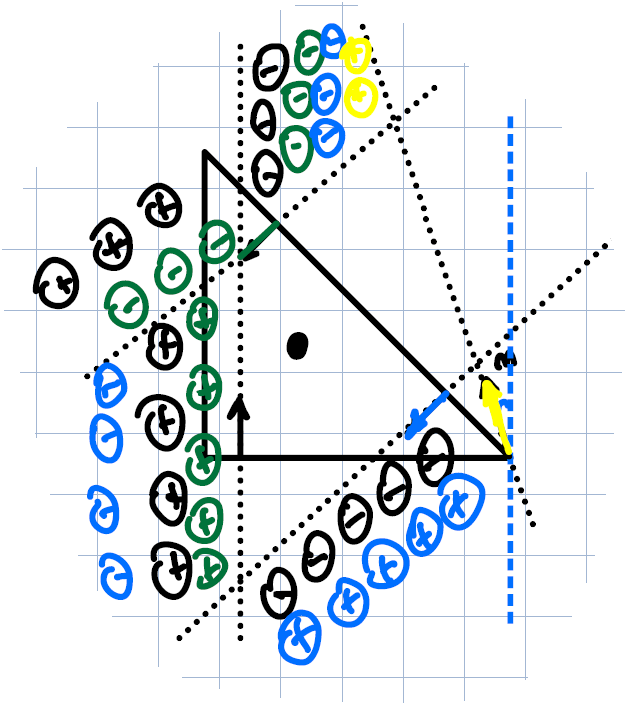


Figure - F1 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (2,3,4,5) finger subset.

* F2 out:

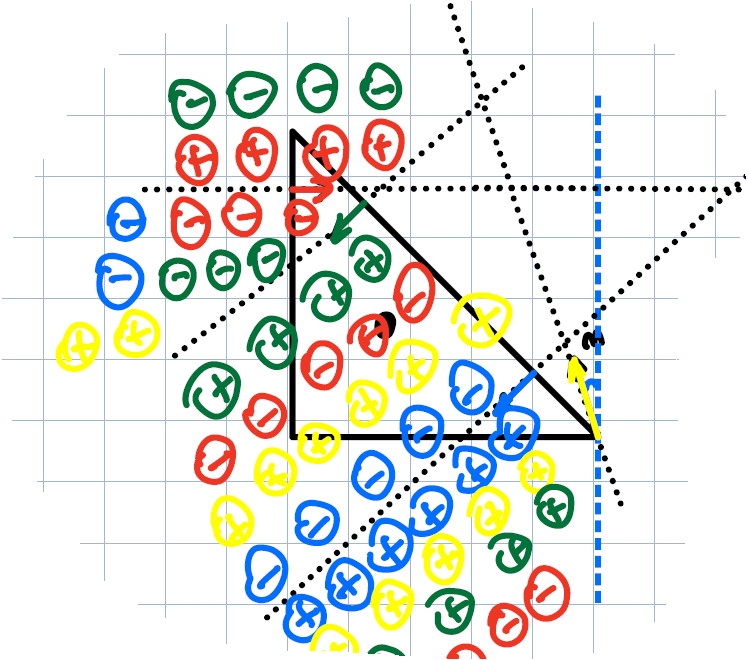


Figure 3 - F2 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (1,3,4,5) finger subset.

* F3 out:

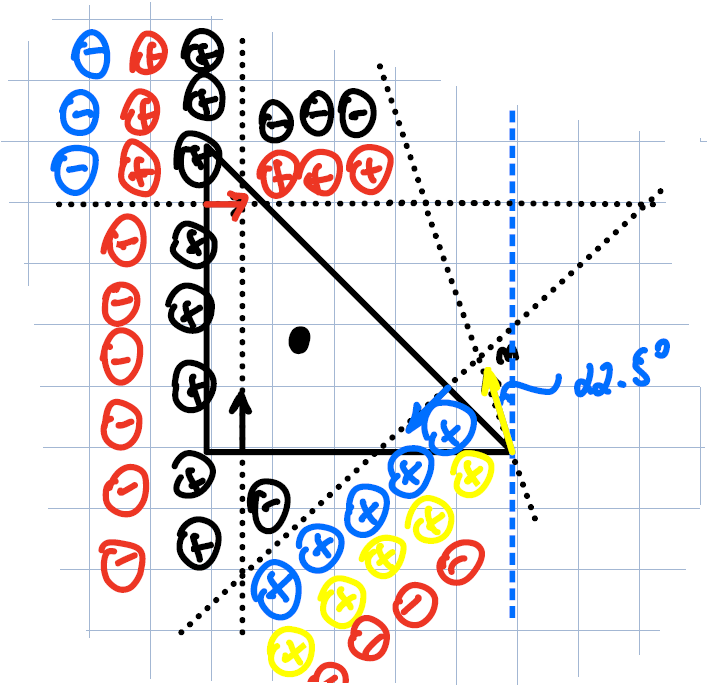


Figure - F3 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (1,2,4,5) finger subset.

* F4 out:

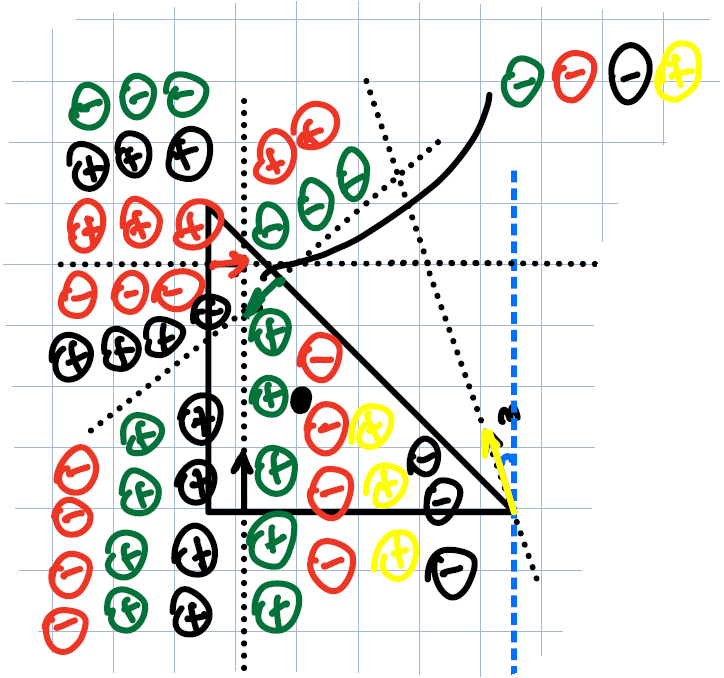


Figure 5 - F4 out.

We still have a mix of plus and minus signals in the regions, therefore, the object is in form closure yet for (1,2,3,5) finger subset.

* F5 out:

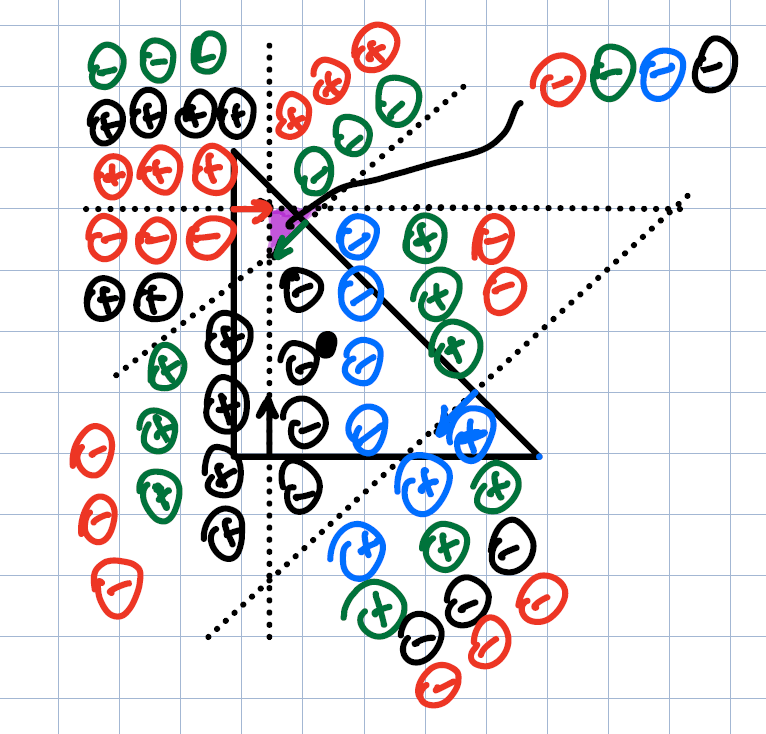


Figure 6 - F5 out.

We doesn’t have anymore a mix of plus and minus signals in the regions (purple’s region), therefore, the object is NOT in form closure yet for (1,2,3,4) finger subset.

(vi) Figure 2, analyzing the range of μ that the grasp yields force closure.

Therefore,

And finally,

For point contact coordinates,

So, we can calculate the wrenches,

The wrench matrix ,

,

The cross matrices (vector in 2D),

,

Follow to wrench evaluation,

And evaluation,

,

To force closure conditions,

(1) rank () = 3 ()

(2)

Now we have the set of equations above,

Substituting (I), (II) in (III),

And finally,

Now we have two ranges,

Or

,

Where c = 0.25 and h = 0.5,

Then the range will be the intersection of the intervals above,

The range of

Problem 2: Grasp Force Optimization.

(i) Rewriting equations (4) and (5) as :

Where

From eqs (4) and (5),

Then ,

So we can find the matrix and ,

(ii)Recast objective function in linear form:

Adding a scalar and constraint

For norm-2,

The quadratic cone constraint is:

Add for each side of equation,

,

S = ,

Then .

(iii) Define the variable x and the SOCP parameters:

= cpx variable (forces)

Defining matrix,

Comparing against ,

,

,

,

.

Problem 3: Learning Intuitive Physics.

(iv)

(v)

(vii)