Wave Dispersion Equation – Linear Gravity Waves (Airy Theory)

This repository provides a comprehensive suite of solutions and approximations for analyzing wave dispersion, which is essential for wave prediction, oceanographic calculations, and coastal engineering design. It includes a reference "exact" solution using the Newton-Raphson method, classical and contemporary explicit approximations, and high-order Padé approximants for high precision.

Background

The **linear wave dispersion equation** relates wave frequency (or period) to wavenumber and water depth for gravity waves:

$\omega^2 = g \cdot k \cdot \tanh(k \cdot h)$

where:

- **ω (omega)** is the angular frequency:
  $\omega = 2\pi / T$
  (where T is the wave period)

- **g** is the gravitational acceleration.

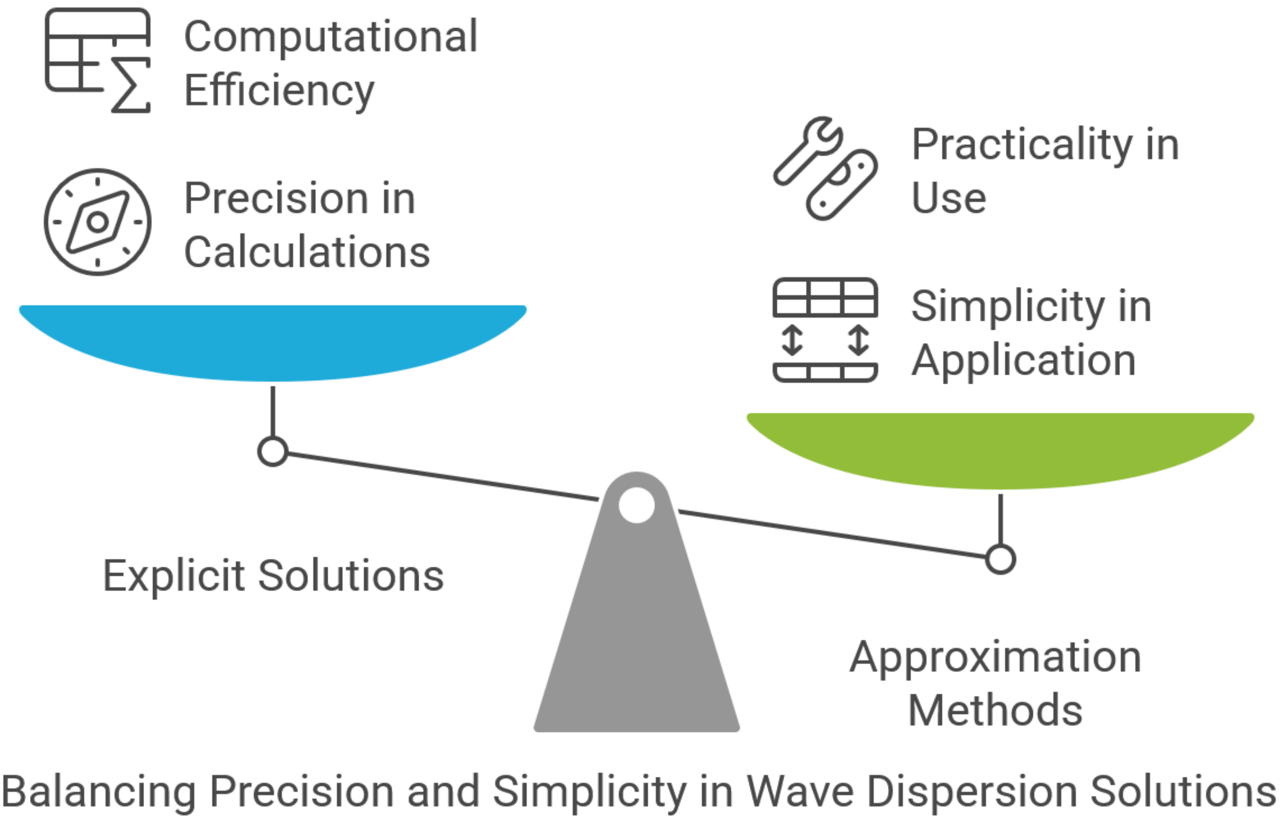
- **k** is the wavenumber:
  $k = 2\pi / L$
  (where L is the wavelength)

- **h** is the water depth.

The transcendental nature of this equation prevents closed-form solutions for *k*. Therefore, it is nondimensionalized to:

$k_0h = kh \cdot \tanh(kh)$

This equation is solved iteratively for the dimensionless wavenumber *kh* ($k_0 = \omega^2/g$). An accurate *kh* evaluation is vital for computing wave phase speed, group velocity, and understanding various nearshore processes. Explicit approximations bypass the need for iteration but must be chosen carefully based on accuracy requirements.



Module Contents

- **Reference "Exact" Solution:** `kh_numeric()` implements the Newton-Raphson iteration method for a highly precise solution of wave dispersion, acting as a benchmark for other techniques.

- **Classical Approximations:** Established methods from researchers like Hunt, Eckart, Nielsen, and Gilbert.
- **Contemporary Approximations:** Recent techniques from researchers such as Guo, Beji, Vatankhah & Aghashariatmadari, Simarro & Orfila, Yu, Fenton & McKee, Guan & Ju, and Iwagaki.
- **High-Order Padé Approximations:** Carvalho's 2025 high-order Padé approximants deliver exceptional precision, addressing increasing complexity in wave calculations as a robust alternative to simpler methods.

References

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These references span classical methods (e.g., Eckart, 1951; Hunt, 1979), modern explicit approximations (e.g., Guo, 2002; Beji, 2013; Vatankhah & Aghashariatmadari, 2013; Simarro & Orfila, 2013; Yu, 2014), pivotal contributions from Fenton and colleagues (Fenton & McKee, 1990; Fenton, 1972), and innovative computational approaches (Carvalho, 2006 & 2025) to improve the dispersion relation accuracy.