Sound Analysis, Synthesis and Processing
Homework #4

Group components:

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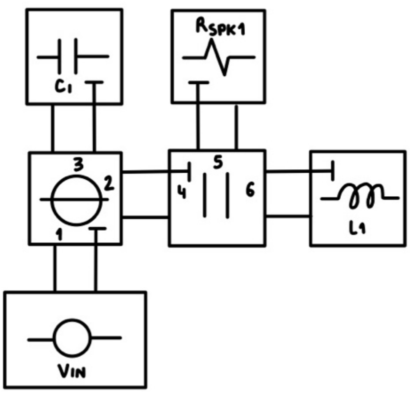
WDF scheme

HIGH



$Z_1 = Z_2 + Z_3$

$Z_2 = Z_4$

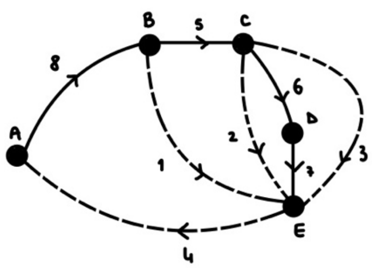
$Z_3 = \frac{T_s}{2C_1}$

$Z_4 = \frac{Z_5 \cdot Z_6}{Z_5 + Z_6}$

$Z_5 = R_{SPK1}$

$Z_6 = \frac{2L_1}{T_s}$

MID (symbolic circuit)



independent voltages: v_5, v_6, v_7, v_8

independent currents: i_1, i_2, i_3, i_4

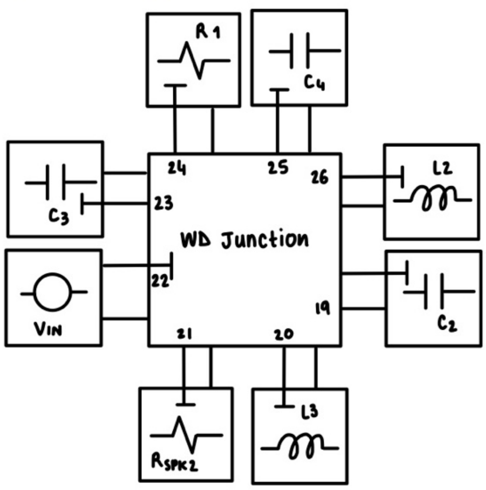
$q \geq p$ $\quad S = I - 2ZB^T(BZB^T)^{-1}B$

$$i = B^T i_1 \rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

I 8×8 identity matrix

Z diag$[Z_{19}, Z_{20}, \dots, Z_{27}]$

MID (WDF circuit)



$Z_{19} = \frac{T_s}{2C_2}$

$Z_{20} = \frac{2L_3}{T_s}$

$Z_{21} = R_{SPK2}$

$Z_{22} =$ solve(S(4,4) == 0, Z22)

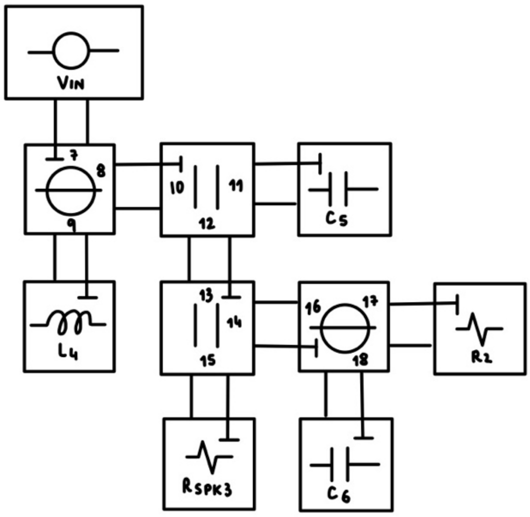
$Z_{23} = \frac{T_s}{2C_3}$

$Z_{24} = R_1$

$Z_{25} = \frac{T_s}{2C_4}$

$Z_{26} = \frac{2L_2}{T_s}$

LOW



$$Z_7 = Z_8 + Z_9$$

$$Z_8 = Z_{10}$$

$$Z_9 = \frac{2L_4}{T_S}$$

$$Z_{10} = \frac{Z_{11} \cdot Z_{12}}{Z_{11} + Z_{12}}$$

$$Z_{11} = \frac{T_S}{2C_S}$$

$$Z_{12} = Z_{13}$$

$$Z_{13} = \frac{Z_{14} \cdot Z_{15}}{Z_{14} + Z_{15}}$$

$$Z_{14} = Z_{16}$$

$$Z_{15} = R_{SPK3}$$

$$Z_{16} = Z_{17} + Z_{18}$$

$$Z_{17} = R_2$$

$$Z_{18} = \frac{T_S}{2C_6}$$

SCATTERING MATRIXES

$$\begin{bmatrix} a_1[k] \\ a_2[k] \\ a_3[k] \end{bmatrix} = S_{S1} \begin{bmatrix} b_1[k] \\ b_2[k] \\ b_3[k] \end{bmatrix} \qquad S_{S1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2 \cdot \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_4[k] \\ a_5[k] \\ a_6[k] \end{bmatrix} = S_{P1} \begin{bmatrix} b_4[k] \\ b_5[k] \\ b_6[k] \end{bmatrix} \qquad S_{P1} = 2 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_4 & 0 & 0 \\ 0 & Z_5 & 0 \\ 0 & 0 & Z_6 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_4 & 0 & 0 \\ 0 & Z_5 & 0 \\ 0 & 0 & Z_6 \end{bmatrix}^{-1} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_7[k] \\ a_8[k] \\ a_9[k] \end{bmatrix} = S_{S2} \begin{bmatrix} b_7[k] \\ b_8[k] \\ b_9[k] \end{bmatrix} \qquad S_{S2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2 \cdot \begin{bmatrix} Z_7 & 0 & 0 \\ 0 & Z_8 & 0 \\ 0 & 0 & Z_9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_7 & 0 & 0 \\ 0 & Z_8 & 0 \\ 0 & 0 & Z_9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{10}[k] \\ a_{11}[k] \\ a_{12}[k] \end{bmatrix} = S_{P2} \begin{bmatrix} b_{10}[k] \\ b_{11}[k] \\ b_{12}[k] \end{bmatrix} \qquad S_{P2} = 2 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_{10} & 0 & 0 \\ 0 & Z_{11} & 0 \\ 0 & 0 & Z_{12} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_{10} & 0 & 0 \\ 0 & Z_{11} & 0 \\ 0 & 0 & Z_{12} \end{bmatrix}^{-1} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

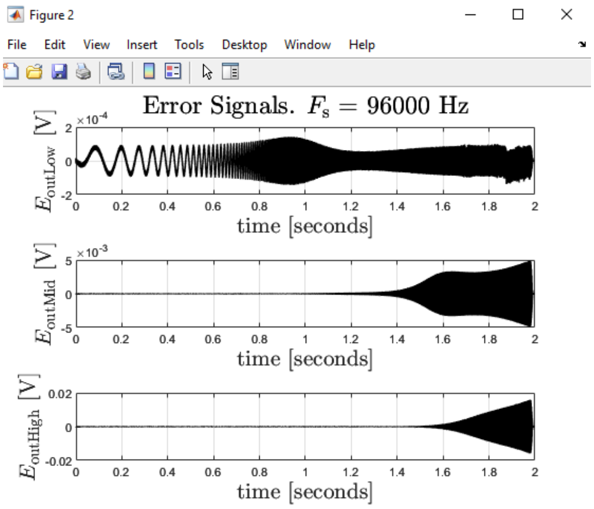
$$\begin{bmatrix} a_{13}[k] \\ a_{14}[k] \\ a_{15}[k] \end{bmatrix} = S_{P3} \begin{bmatrix} b_{13}[k] \\ b_{14}[k] \\ b_{15}[k] \end{bmatrix} \qquad S_{P3} = 2 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_{13} & 0 & 0 \\ 0 & Z_{14} & 0 \\ 0 & 0 & Z_{15} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_{13} & 0 & 0 \\ 0 & Z_{14} & 0 \\ 0 & 0 & Z_{15} \end{bmatrix}^{-1} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

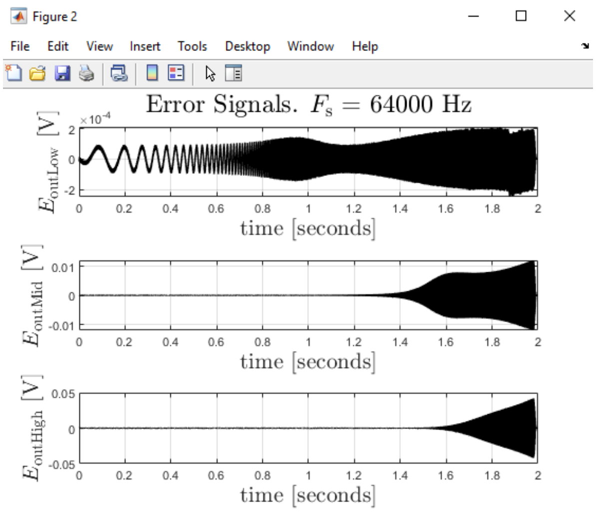
$$\begin{bmatrix} a_{16}[k] \\ a_{17}[k] \\ a_{18}[k] \end{bmatrix} = S_{S3} \begin{bmatrix} b_{16}[k] \\ b_{17}[k] \\ b_{18}[k] \end{bmatrix} \qquad S_{S3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2 \cdot \begin{bmatrix} Z_{16} & 0 & 0 \\ 0 & Z_{17} & 0 \\ 0 & 0 & Z_{18} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} Z_{16} & 0 & 0 \\ 0 & Z_{17} & 0 \\ 0 & 0 & Z_{18} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

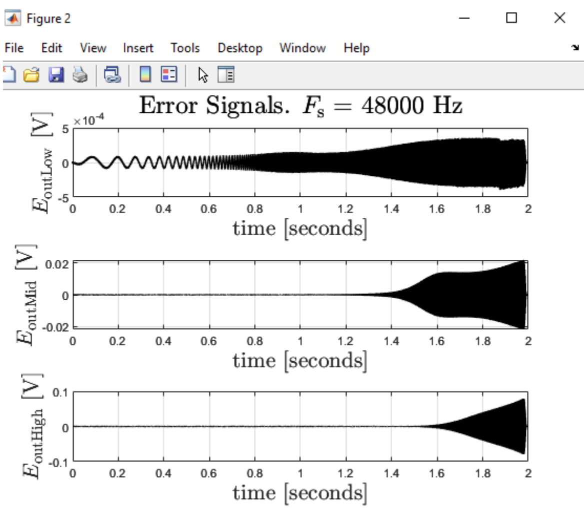
WDF scheme comments

The mid-section was implemented through a single adapter. We did this by creating a symbolic scheme for the circuit where we used a tree-cotree decomposition to find the independent voltages and currents. From this we applied the scattering matrix equations to adapt the ideal voltage generator impedance.

Subplots of the error signals







Questions

1. The Highpass filter has a bigger error with respect to the Lowpass and the Bandpass. This means that the Highpass section is the least accurate one and this is caused by the frequency warping effect. With the Trapezoidal Rule the frequency is mapped as follows:

$$j\omega \leftarrow j\frac{2}{T_s}\tan\left(\tilde{\omega}\frac{T_s}{2}\right)$$

For $\tilde{\omega}$ Ts / 2 that goes to zero, the tangent is approximated as $\tilde{\omega}$ Ts / 2 which results in j ω is mapped as j $\tilde{\omega}$. The higher the frequency the more it is warped resulting in a loss of accuracy.

2. The higher the sampling frequency Fs = 1/Ts , the more the difference between ω and $\tilde{\omega}$ becomes negligible in the whole frequency range of interest. This is because the lower the Ts is, the lower the argument of the tangent is, resulting in a wider band where the tangent approximation is accurate.

3. If we add a single diode it is not possible to compute the WDF without iterative methods because we would have an ideal generator and a non-linear element that cannot be adapted, resulting in delay free loops. If we instead replace the ideal voltage source with a resistive voltage source the circuit becomes computable even without iterative methods because the resistive voltage source can be adapted by setting Z[k] = R_g which results in a B[k] = V_g [k] giving us the possibility to set the diode as root of the WDF.

4. Start by considering the constitutive equation of an inductor in the continuous-time domain:
V(s) = s*L*I(s).

Express the approximation based on the Backward Euler Method of such constitutive equation in the discrete-time domain:

$$V(K) = \frac{1-z^{-1}}{Ts} L\ i(K) = \frac{L}{Ts} i(K) - \frac{L}{Ts} i(K-1)$$

Derive the corresponding (non-adapted) scattering relation in the WD domain:

$$V(K) = R_e(K) \cdot i(K) + V_e(K)$$

$$b(K) = \frac{R_e(K) - z(K)}{R_e(K) + z(K)} \cdot a(K) + \frac{2z(K)}{R_e(K) + z(K)} \cdot V_e(K)$$

$$R_e(K) = \frac{L}{Ts}$$

$$V_e(K) = -\frac{L}{Ts} i(K-1)$$

Derive the adapted scattering relation along with the corresponding adaptation condition on the free parameter Z[k]:

$$z(K) = R_e(K) \longrightarrow b(K) = V_e(K) = -\frac{L}{Ts} i(K-1) = -\frac{L}{Ts} \cdot \frac{a(K-1) - b(K-1)}{2 \cdot \frac{L}{Ts}} = \frac{-a(K-1) + b(K-1)}{2}$$

WD Inductor Model based on Backward Euler Method:

CONSTITUTIVE EQ.	WAVE MAPPING IN CASE OF ADAPTATION	ADAPTATION CONDITION
$v(t) = L\frac{di(t)}{dt}$	$b(K) = \frac{-a(K-1) + b(K-1)}{2}$	$Z(K) = \frac{L}{T_S}$