ESD - Elettronica dei Sistemi Digitali

Solutions on Boolean Algebra

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1 Boolean Algebra Exercises

1.1 Exercise 1

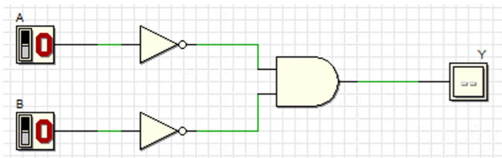
For each given truth table, write the Boolean equation in canonical **sum-of-products** form. Then simplify the expression to its minimal form using the theorems of Boolean algebra. Finally, implement the corresponding combinational circuit and verify its correctness by simulating it in DEEDS

1.1.1 (a)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

$\overline{A}\,\overline{B}$

The expression cannot be simplified any further using Boolean algebra theorems.



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

1.1.2 (b)

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$Y = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + AB\overline{C}$

Apply the Distributive Law
$= \overline{A}B(\overline{C} + C) + A\overline{B}C + AB\overline{C}$

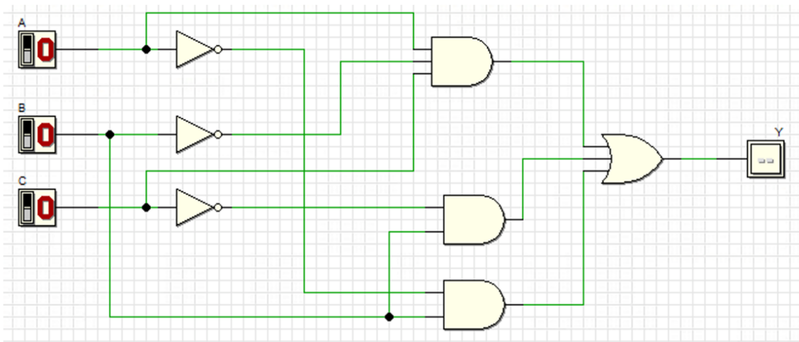
Apply the Complement Law:
$= \overline{A}B1 + A\overline{B}C + AB\overline{C}$

Apply the Identity Law:
$= \overline{A}B + A\overline{B}C + AB\overline{C}$

Apply the Distributive Law
$= A\overline{B}C + B(A\overline{C} + \overline{A})$

Apply the Absorption Law
$= A\overline{B}C + B(\overline{C} + \overline{A})$

Apply Distribution
$= A\overline{B}C + B\overline{C} + B\overline{A}$



C	B	A	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

1.1.3 (c)

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$Y = A\overline{B}C + AB\overline{C} + ABC$

Apply the Distributive Law:
$= AC(\overline{B} + B) + AB\overline{C}$

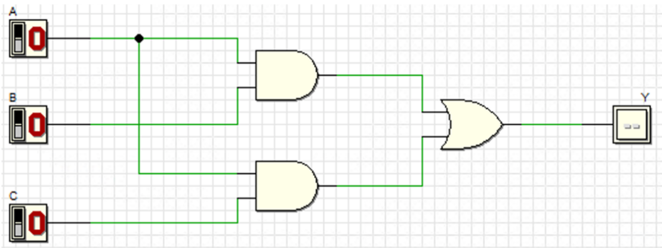
Apply the Complement Law:
$= AC1 + AB\overline{C}$

Apply the Identity Law:
$= AC + AB\overline{C}$

Apply the Distributive Law:
$= A(B\overline{C} + C)$

Apply the Absorption Law:
$= A(B + C)$

Apply Distribution
$= AB + AC$



A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

1.1.4 (d)

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

$Y = \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}B\overline{C}D + ABC\overline{D} + ABCD$

Apply the Distributive Law:
$= \overline{A}\,\overline{C}D(\overline{B} + B) + ABC\overline{D} + ABCD$

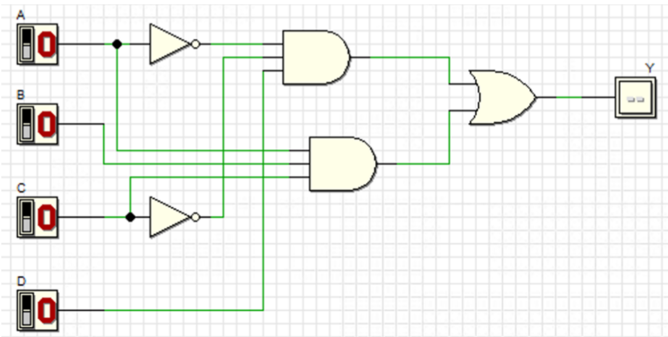
Apply the Complement Law:
$= \overline{A}\,\overline{C}D1 + ABC\overline{D} + ABCD$

Apply the Identity Law:
$= \overline{A}\,\overline{C}D + ABC\overline{D} + ABCD$

Apply the Distributive Law:
$= \overline{A}\,\overline{C}D + ABC(\overline{D} + D)$

Apply the Complement Law:
$= \overline{A}\,\overline{C}D + ABC1$

Apply the Identity Law:
$= \overline{A}\,\overline{C}D + ABC$



A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

1.1.5 (e)

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$Y = \overline{A}\,\overline{B}CD + \overline{A}BCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$

Apply the Distributive Law:
$= \overline{A}\,CD(\overline{B} + B) + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$

Apply the Complement Law:
$= \overline{A}\,CD1 + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$

Apply the Identity Law:
$= \overline{A}\,CD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$

Apply the Distributive Law:
$= \overline{A}\,CD + A\overline{B}\,\overline{C}(\overline{D} + D) + A\overline{B}C\overline{D}$

Apply the Complement Law:
$= \overline{A}\,CD + A\overline{B}\,\overline{C}1 + A\overline{B}C\overline{D}$

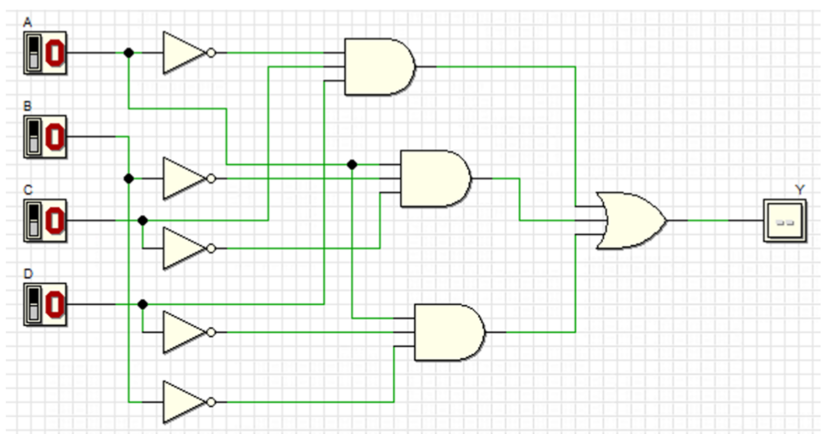
Apply the Identity Law:
$= \overline{A}\,CD + A\overline{B}\,\overline{C} + A\overline{B}C\overline{D}$

Apply the Distributive Law:
$= \overline{A}\,CD + A\overline{B}\,\overline{C} + A\overline{B}C\overline{D}$

Apply: Distribution Law:
$= \overline{A}\,CD + A\overline{B}(\overline{C} + C\overline{D})$

Apply the Absorption Law:
$= \overline{A}\,CD + A\overline{B}(\overline{C} + \overline{D})$

Apply: Distribution Law:
$= \overline{A}\,CD + A\overline{B}\,\overline{C} + A\overline{B}\,\overline{D}$

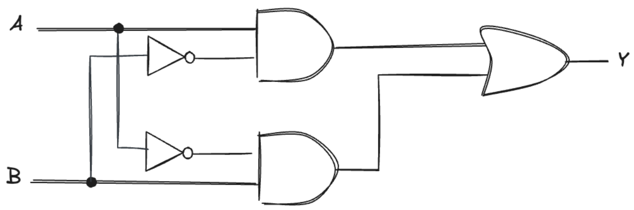


A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

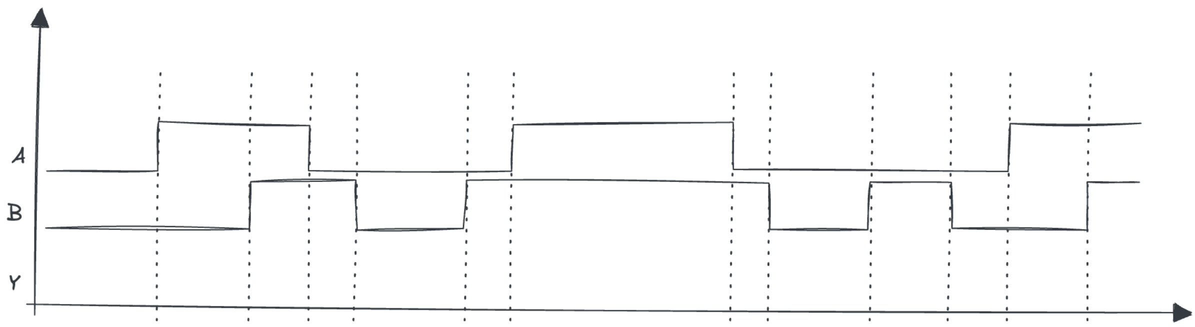
1.2 Exercise 2

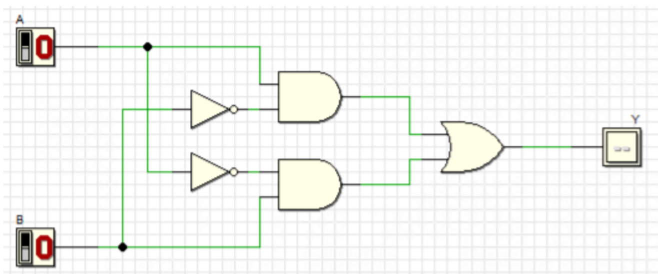
Complete the truth table and the timing diagram of the following digital circuits. Then derive the Boolean equation in canonical **sum-of-products** form and simplify it to its minimal expression using Boolean algebra theorems. Finally, implement the corresponding circuit in DEEDS and verify the correctness of your solution.

1.2.1 (a)



A	B	Y
0	0	
0	1	
1	0	
1	1	

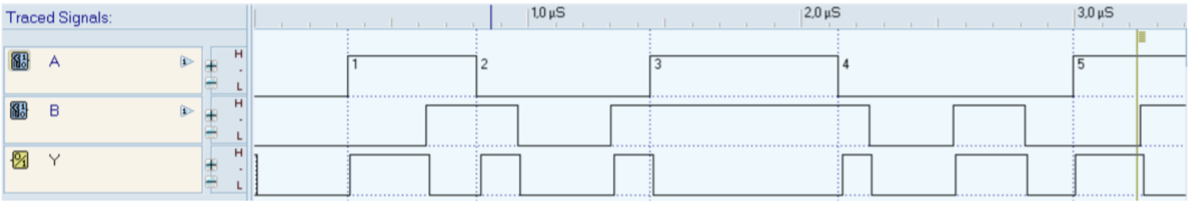




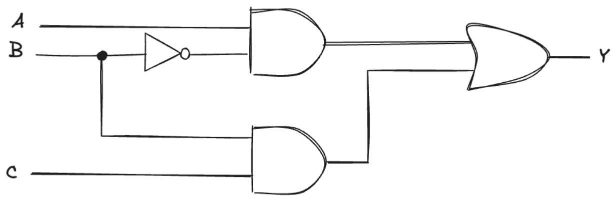
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

$Y = (A \cdot \overline{B}) + (\overline{A} \cdot B)$

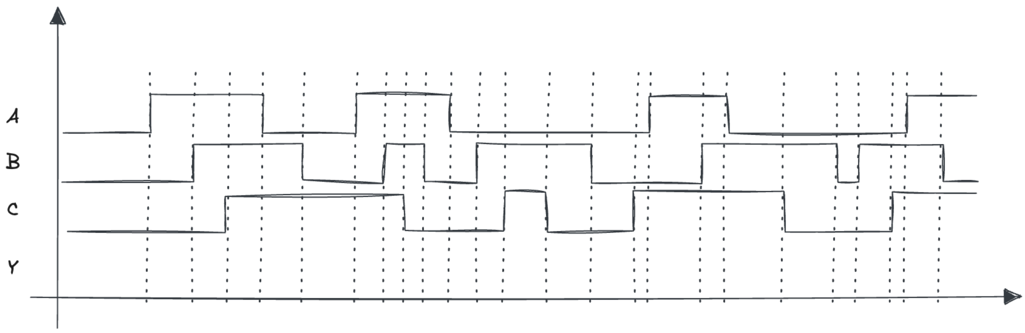
The expression cannot be simplified any further; however, by inspecting the truth table, we can see that it corresponds to an XOR

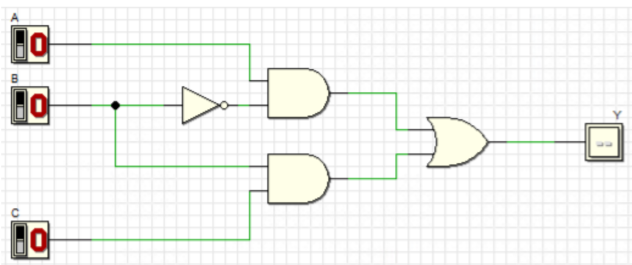


1.2.2 (b)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

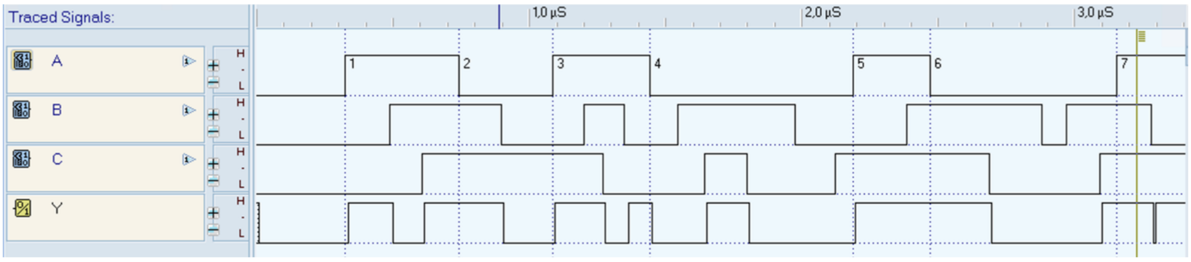
$Y = \overline{A}BC + A\overline{B}\,\overline{C} + A\overline{B}C + ABC$

$= \overline{A}BC + ABC + A\overline{B}\,\overline{C} + A\overline{B}C$

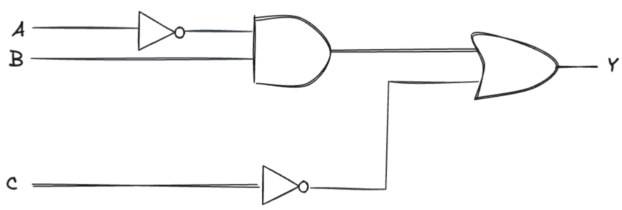
$= BC(\overline{A} + A) + A\overline{B}(\overline{C} + C)$

$= BC \cdot 1 + A\overline{B} \cdot 1$

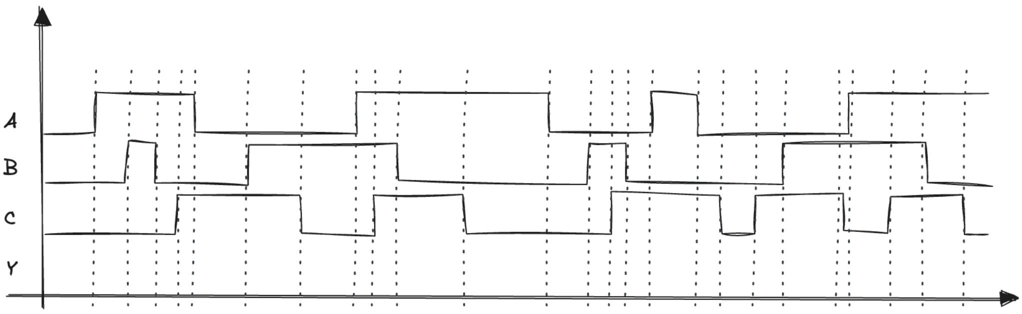
$= BC + A\overline{B}$

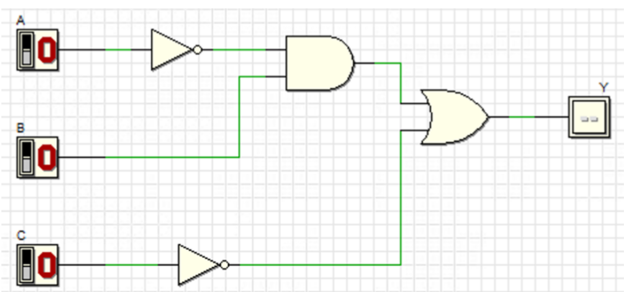


1.2.3 (c)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$

$$= \overline{A}\,\overline{C}(\overline{B} + B) + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$$

$$= \overline{A}\,\overline{C}1 + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$$

$$= \overline{A}\,\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$$

$$= \overline{A}(BC + \overline{C}) + A\overline{B}\,\overline{C} + AB\overline{C}$$

$$= \overline{A}(B + \overline{C}) + A\overline{B}\,\overline{C} + AB\overline{C}$$

$$= \overline{A}(B + \overline{C}) + A\overline{C}(B + \overline{B})$$

$$= \overline{A}(B + \overline{C}) + A\overline{C}1$$

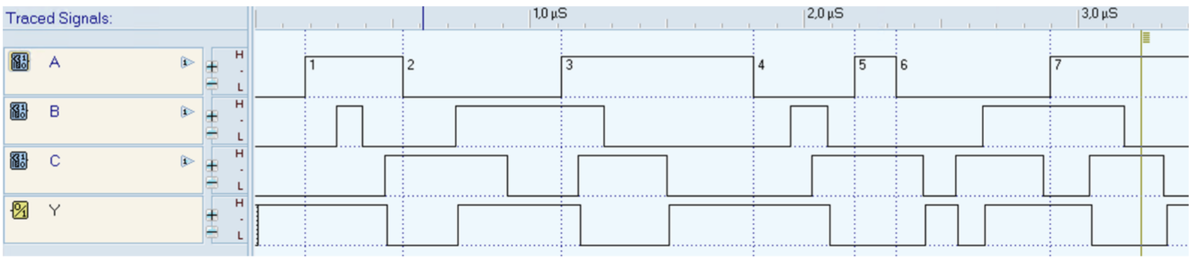
$$= \overline{A}(B + \overline{C}) + A\overline{C}$$

$$= \overline{A}B + \overline{A}\,\overline{C} + A\overline{C}$$

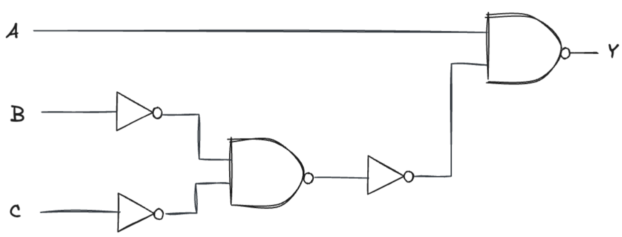
$$= \overline{A}B + \overline{C}(\overline{A} + A)$$

$$= \overline{A}B + \overline{C}1$$

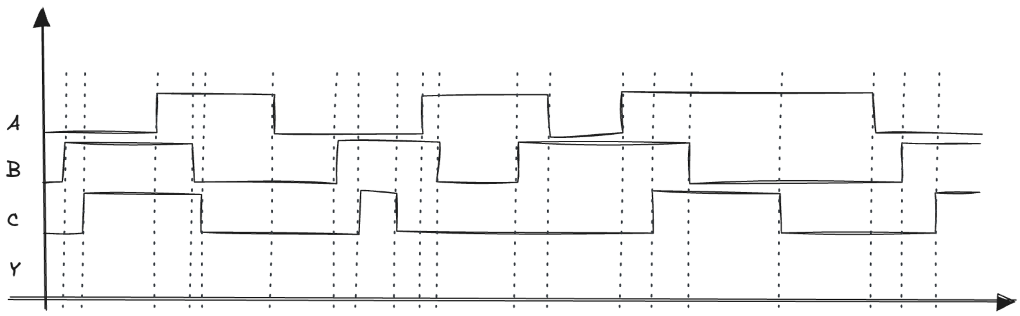
$$= \overline{A}B + \overline{C}$$

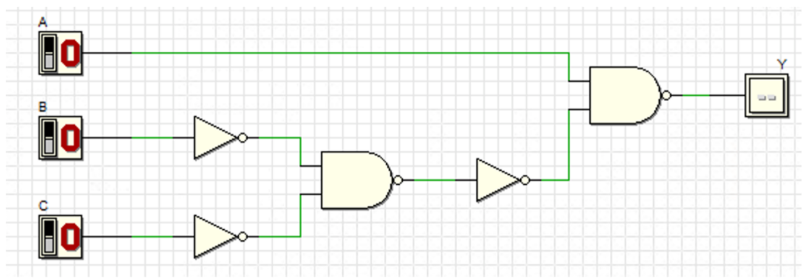


1.2.4 (d)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \overline{A}\,\overline{B}(C + \overline{C}) + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \overline{A}\,\overline{B} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \overline{A}(\overline{B} + B\overline{C}) + \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \overline{A}(\overline{B} + \overline{C}) + \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

$$= \overline{A}(\overline{B} + \overline{C}) + BC(A + \overline{A}) + A\overline{B}C + AB\overline{C}$$

$$= \overline{A}(\overline{B} + \overline{C}) + BC + A\overline{B}C + AB\overline{C}$$

$$= \overline{A}(\overline{B} + \overline{C}) + C(A\overline{B} + B) + AB\overline{C}$$

$$= \overline{A}(\overline{B} + \overline{C}) + C(A + B) + AB\overline{C}$$

$$= \overline{A}\,\overline{B} + \overline{A}\,\overline{C} + C(A + B) + AB\overline{C}$$

$$= \overline{A}\,\overline{B} + C(A + B) + \overline{C}(AB + \overline{A})$$

$$= \overline{A}\,\overline{B} + C(A + B) + \overline{C}(B + \overline{A})$$

$$= \overline{A}\,\overline{B} + CA + CB + \overline{C}B + \overline{C}\,\overline{A}$$

$$= \overline{A}\,\overline{B} + CA + B(C + \overline{C}) + \overline{C}\,\overline{A}$$

$$= \overline{A}\,\overline{B} + CA + B + \overline{C}\,\overline{A}$$

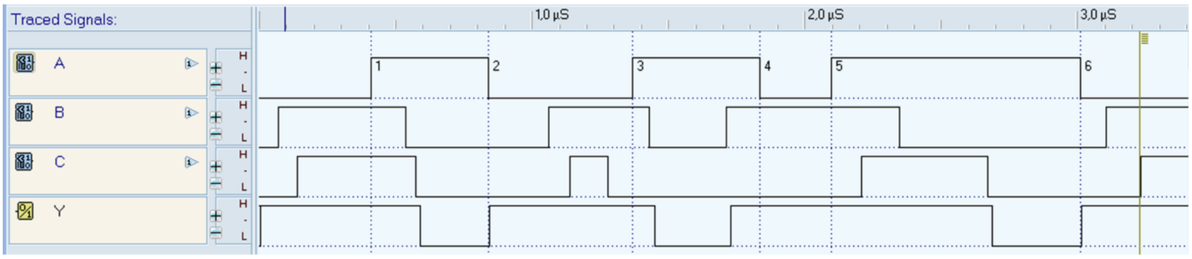
$$= \overline{A} + CA + B + \overline{C}\,\overline{A}$$

$$= \overline{A}(1 + \overline{C}) + CA + B$$

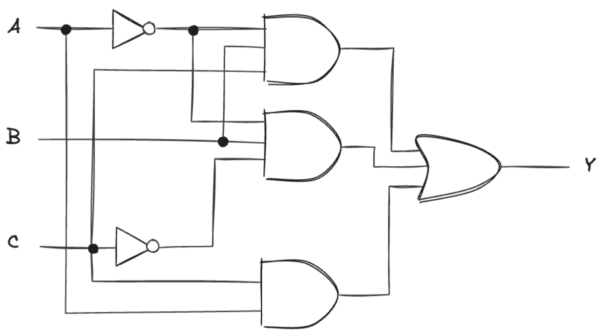
$= \overline{A} + CA + B$

$= \overline{A} + C + B$

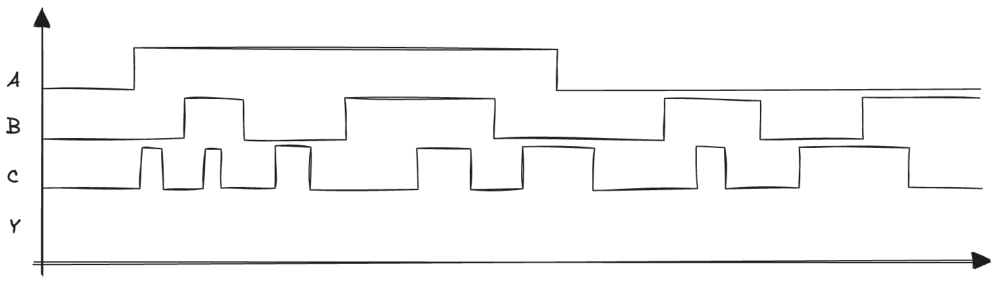
This is very tricky to simplify, however, if we start from the POS form (since we have just one 0 in the table and a lot of 1s), we can get the solution immediately.

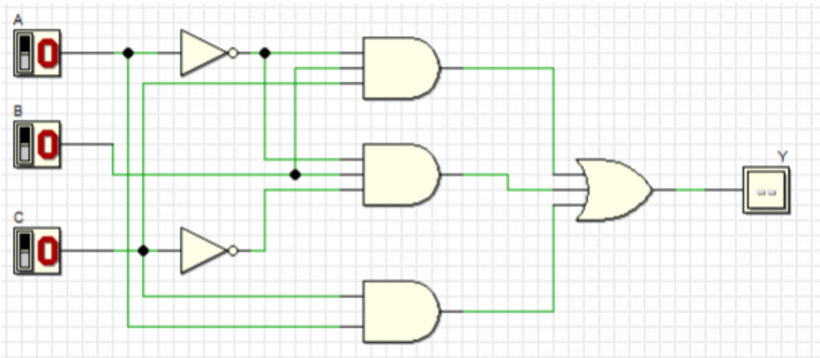


1.2.5 (e)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Check with the simulator what happens around 2.5 µs and 3.2 µs, and try to provide an explanation for the phenomenon.

$Y = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$

$= \overline{A}B(\overline{C} + C) + A\overline{B}C + ABC$

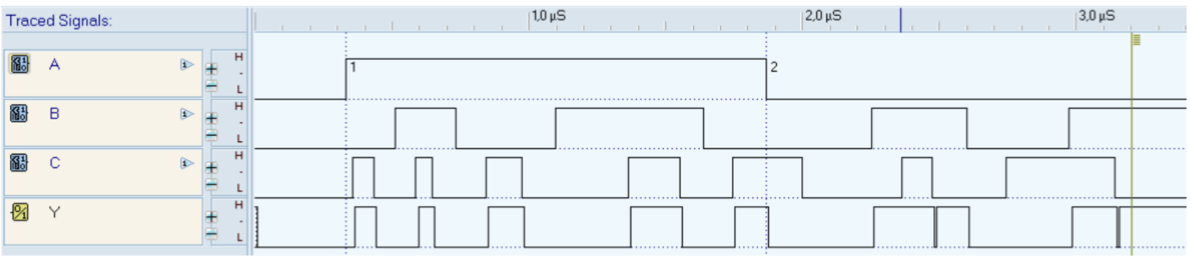
$= \overline{A}B(1) + A\overline{B}C + ABC$

$= \overline{A}B + A\overline{B}C + ABC$

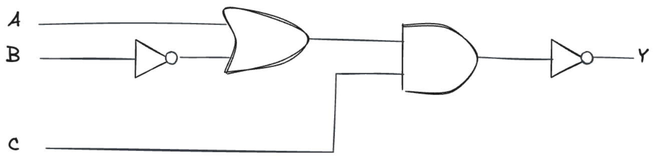
$= \overline{A}B + AC(\overline{B} + B)$

$= \overline{A}B + AC(1)$

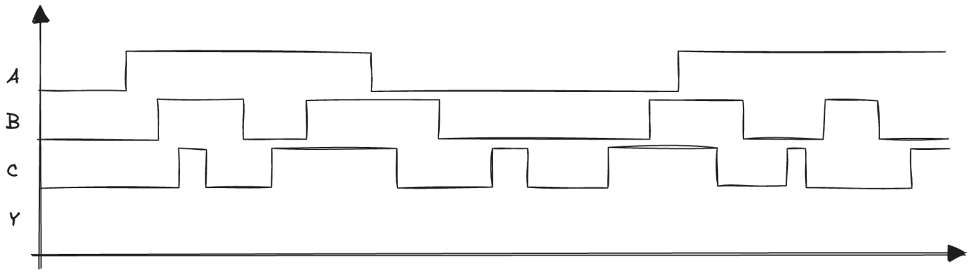
$= \overline{A}B + AC$

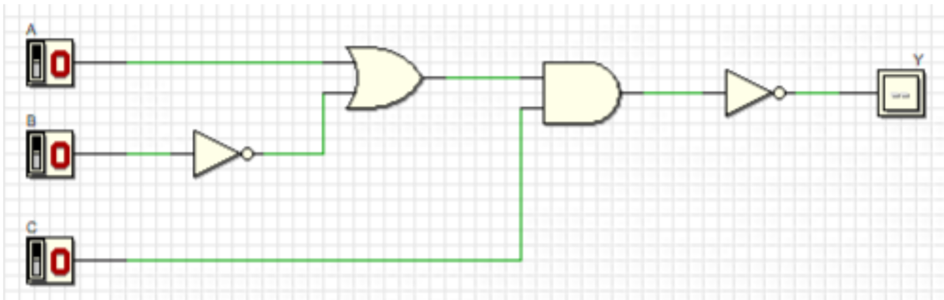


1.2.6 (f)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$

$= \overline{A}\,\overline{C}(\overline{B} + B) + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$

$= \overline{A}\,\overline{C}1 + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$

$= \overline{A}\,\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$

$= \overline{A}(\overline{C} + BC) + A\overline{B}\,\overline{C} + AB\overline{C}$

$= \overline{A}(\overline{C} + B) + A\overline{B}\,\overline{C} + AB\overline{C}$

$= \overline{A}(\overline{C} + B) + A\overline{C}(\overline{B} + B)$

$= \overline{A}(\overline{C} + B) + A\overline{C}1$

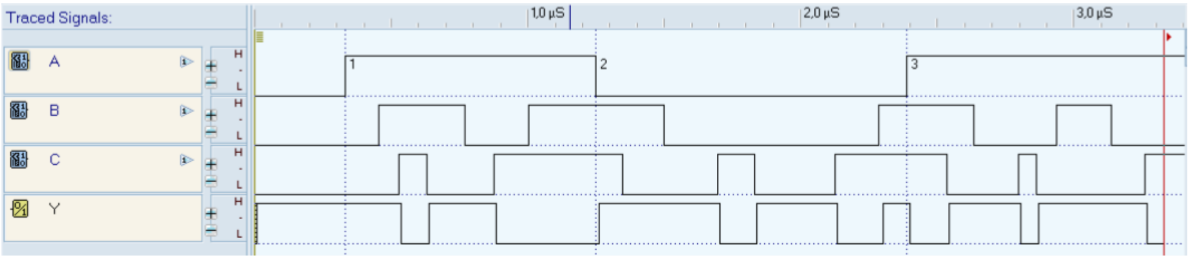
$= \overline{A}(\overline{C} + B) + A\overline{C}$

$$= \overline{A}\,\overline{C} + \overline{A}B + A\overline{C}$$

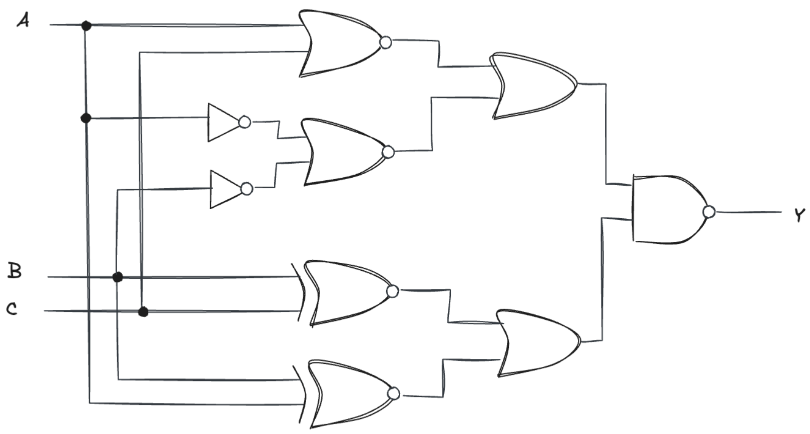
$$= \overline{C}(\overline{A} + A) + \overline{A}B$$

$$= \overline{C}1 + \overline{A}B$$

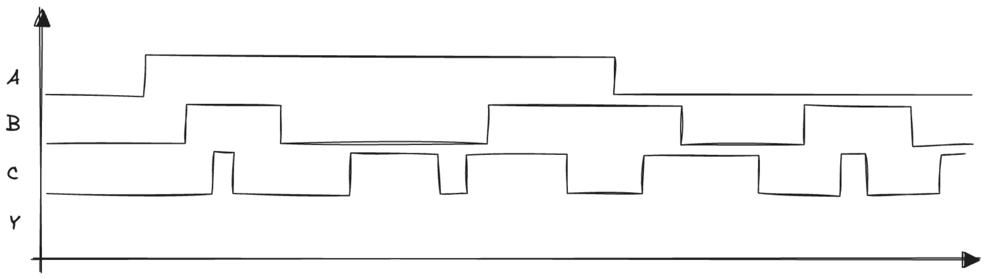
$$= \overline{C} + \overline{A}B$$

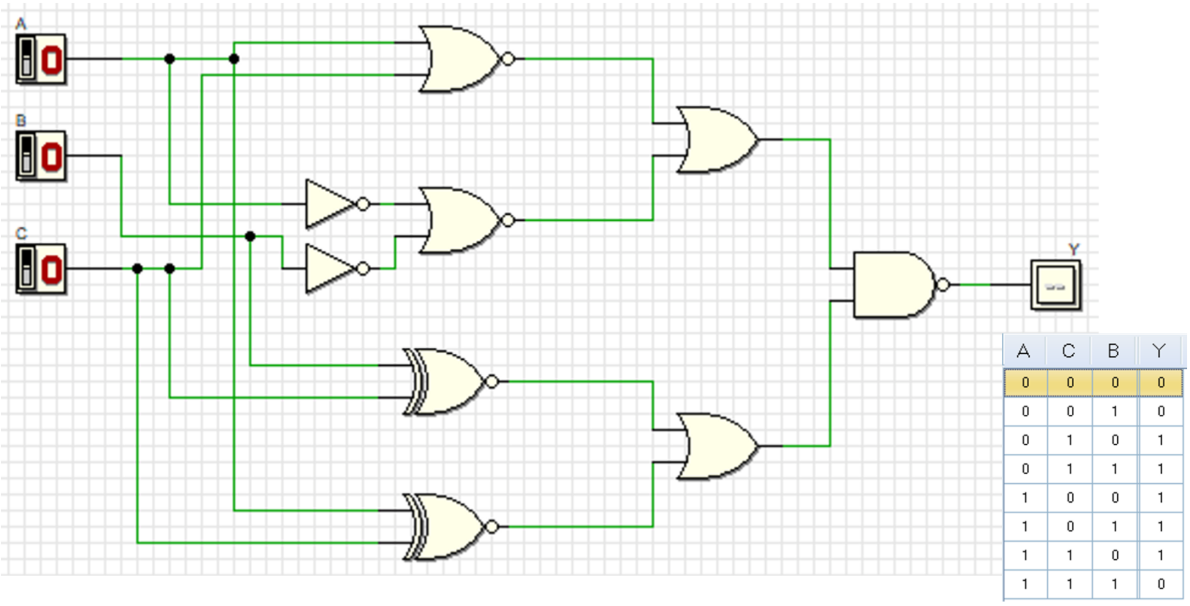


1.2.7 (g)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	C	B	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$Y = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C}$$

$$= \overline{A}B(\overline{C} + C) + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C}$$

$$= \overline{A}B1 + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C}$$

$$= \overline{A}B + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C}$$

$$= \overline{A}B + A\overline{B}(\overline{C} + C) + AB\overline{C}$$

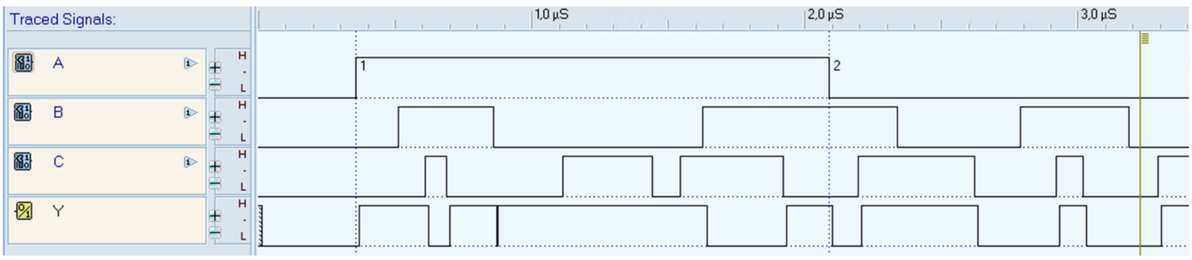
$$= \overline{A}B + A\overline{B}1 + AB\overline{C}$$

$$= \overline{A}B + A\overline{B} + AB\overline{C}$$

$$= A\overline{B} + B(A\overline{C} + \overline{A})$$

$$= A\overline{B} + B(\overline{C} + \overline{A})$$

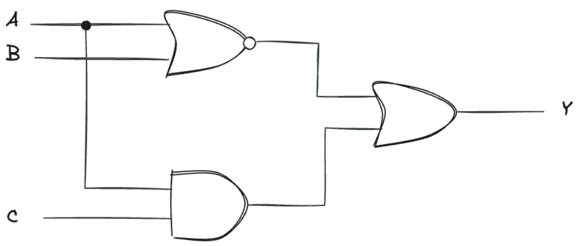
$$= A\overline{B} + B\overline{C} + B\overline{A}$$



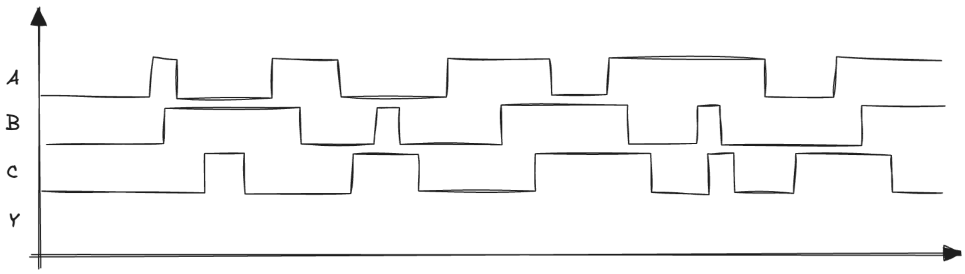
1.3 Exercise 3

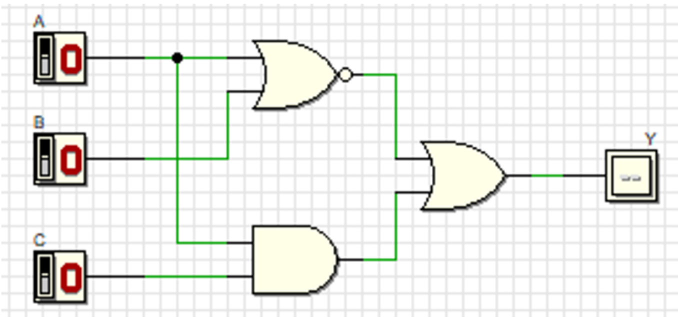
Complete the truth table and the timing diagram of the following digital circuits. Then derive the Boolean equation in canonical **product-of-sums** form and simplify it to its minimal expression using Boolean algebra theorems. Finally, implement the corresponding circuit in DEEDS and verify the correctness of your solution.

1.3.1 (a)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



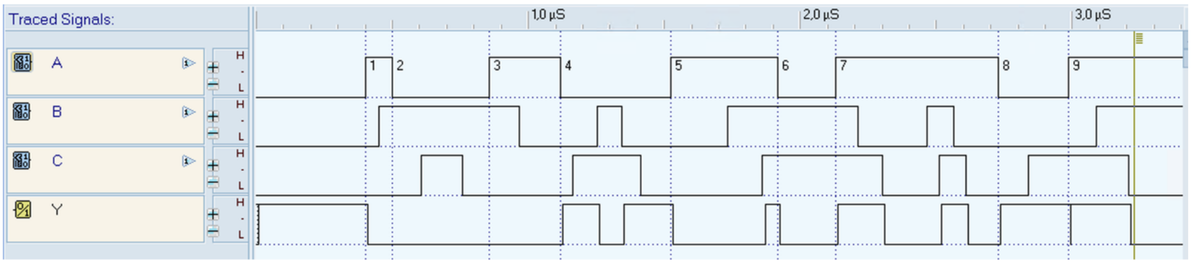


A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

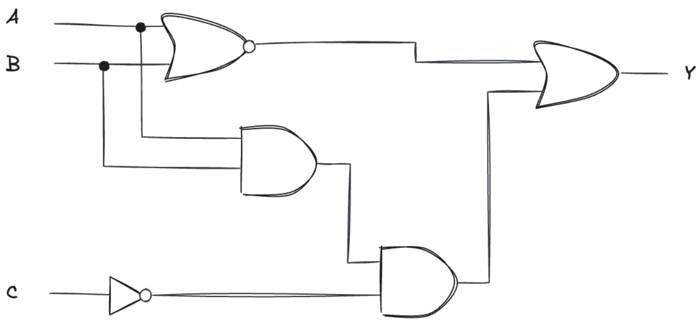
$Y = (A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$

$= (A + \overline{B})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$

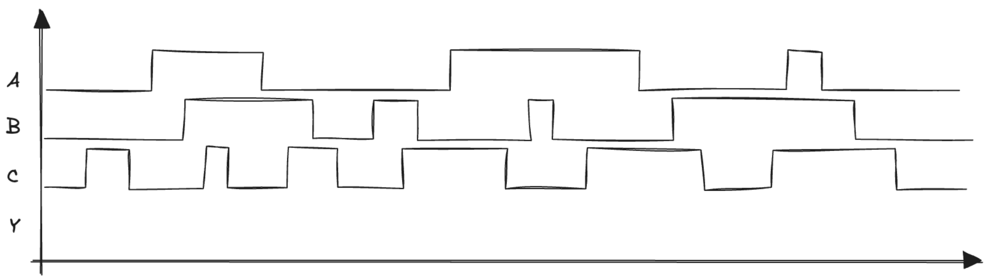
$= (A + \overline{B})(\overline{A} + C)$

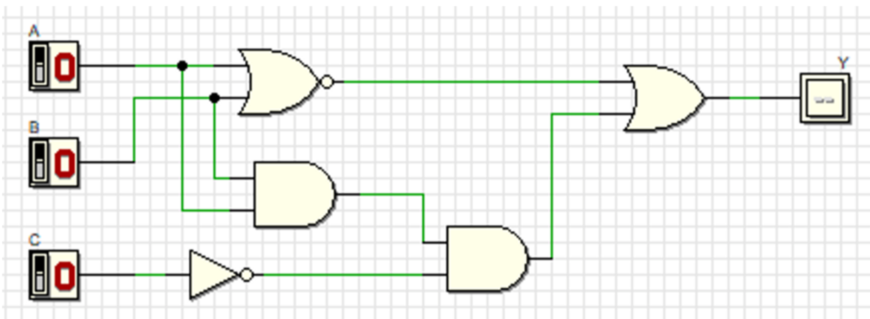


1.3.2 (b)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$$Y = (A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$$

$$= (A + \overline{B})(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$$

$$= (A + \overline{B})(\overline{A} + B)(\overline{A} + \overline{B} + \overline{C})$$

Convert so SOP:

$= (A\overline{A} + AB + \overline{B}\,\overline{A} + \overline{B}B)(\overline{A} + \overline{B} + \overline{C})$

$= (0 + AB + \overline{A}\,\overline{B} + 0)(\overline{A} + \overline{B} + \overline{C})$

$= (AB + \overline{A}\,\overline{B})(\overline{A} + \overline{B} + \overline{C})$

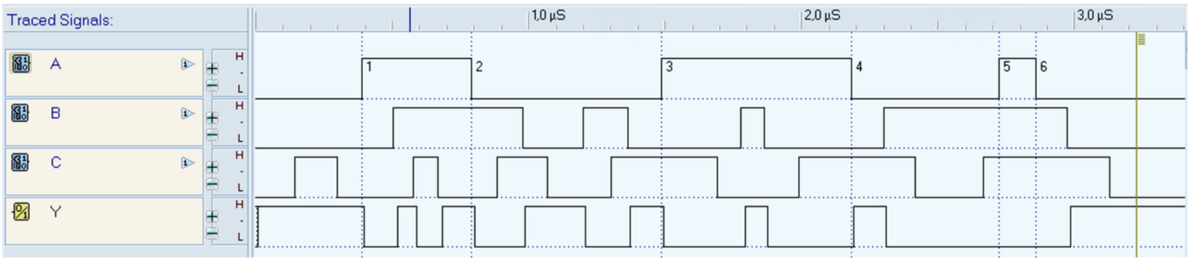
$= AB\overline{A} + AB\overline{B} + AB\overline{C} + \overline{A}\,\overline{B}\,\overline{A} + \overline{A}\,\overline{B}\,\overline{B} + \overline{A}\,\overline{B}\,\overline{C}$

$= 0 + 0 + AB\overline{C} + 0 + \overline{A}\,\overline{B}\,\overline{B} + \overline{A}\,\overline{B}\,\overline{C}$

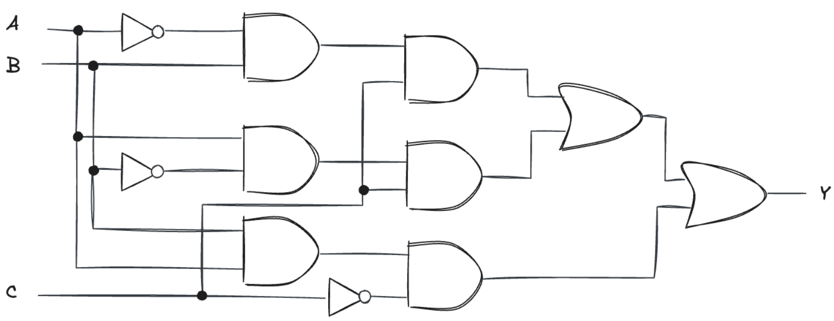
$= AB\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,\overline{B}\,\overline{C}$

$= AB\overline{C} + \overline{A}\,\overline{B}(1 + \overline{C})$

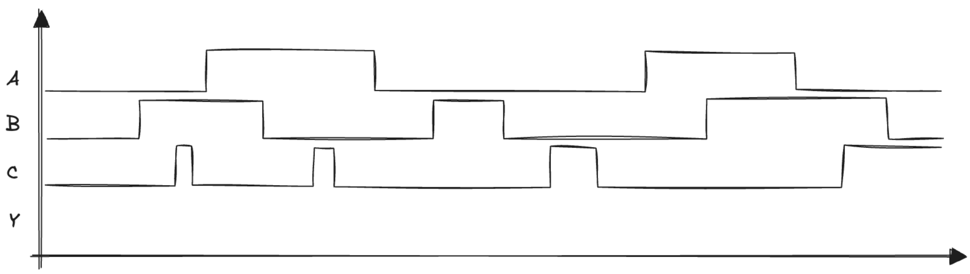
$= AB\overline{C} + \overline{A}\,\overline{B}$

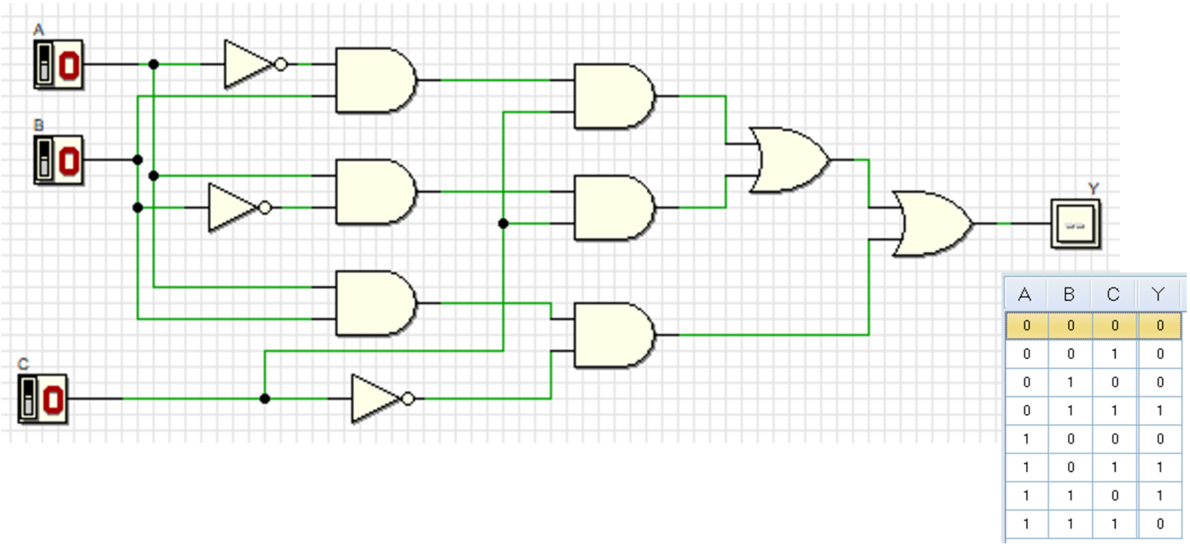


1.3.3 (c)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$Y = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + \overline{B} + \overline{C})$

$= (A + B)(A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + \overline{B} + \overline{C})$

$= (A + B)(C + (A + \overline{B})(\overline{A} + B))(\overline{A} + \overline{B} + \overline{C})$

$= (A + B)(C + AB + \overline{A}\,\overline{B})(\overline{A} + \overline{B} + \overline{C})$

$= ((A + B)C + (A + B)AB + (A + B)\overline{A}\,\overline{B})(\overline{A} + \overline{B} + \overline{C})$

$= (AC + BC + AAB + BAB + A\overline{A}\,\overline{B} + B\overline{A}\,\overline{B})(\overline{A} + \overline{B} + \overline{C})$

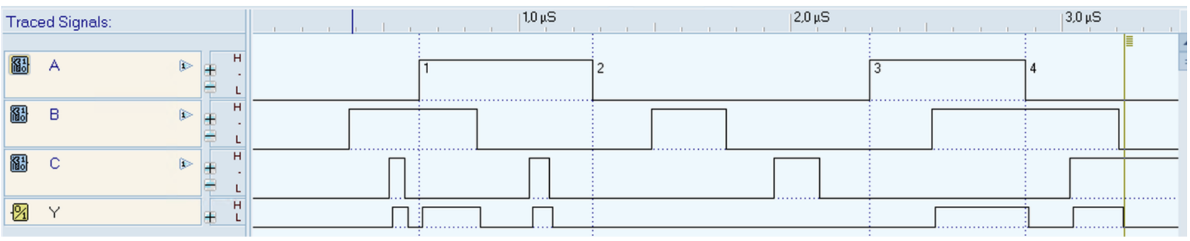
$= (AC + BC + AB + AB + 0 + 0)(\overline{A} + \overline{B} + \overline{C})$

$= (AC + BC + AB)(\overline{A} + \overline{B} + \overline{C})$

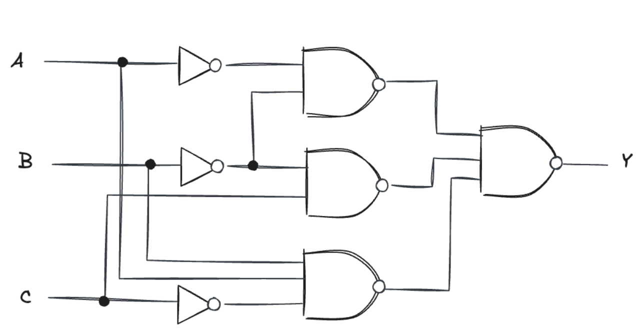
$= AC\overline{A} + AC\overline{B} + AC\overline{C} + BC\overline{A} + BC\overline{B} + BC\overline{C} + AB\overline{A} + AB\overline{B} + AB\overline{C}$

$= 0 + AC\overline{B} + 0 + BC\overline{A} + 0 + 0 + 0 + 0 + AB\overline{C}$

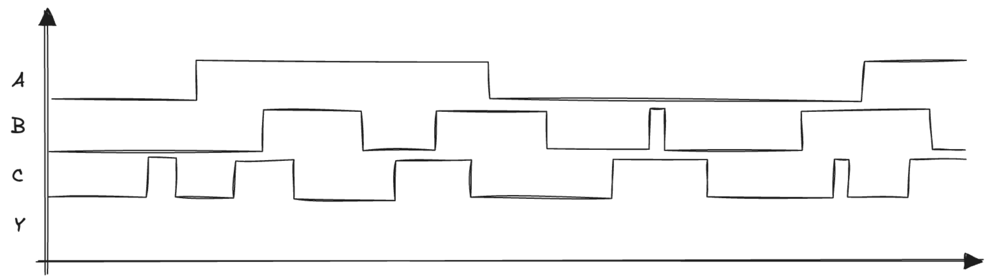
$= AC\overline{B} + BC\overline{A} + AB\overline{C}$

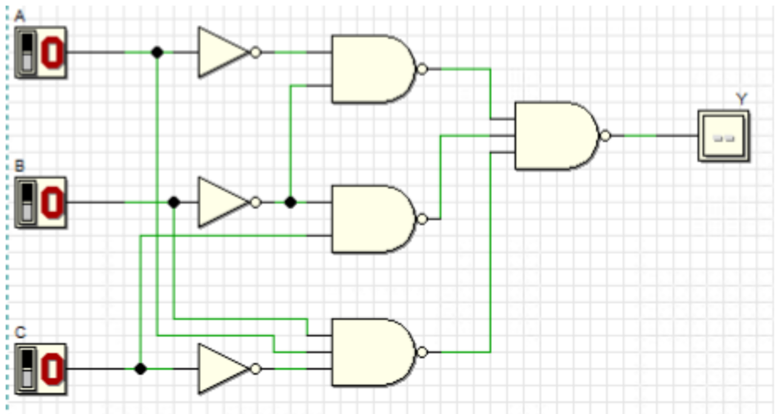


1.3.4 (d)



A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	





A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$Y = (A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+C)(\overline{A}+\overline{B}+\overline{C})$

$= (A+\overline{B})(\overline{A}+B+C)(\overline{A}+\overline{B}+\overline{C})$

$= (A+\overline{B})(\overline{A}+(B+C)(\overline{B}+\overline{C}))$

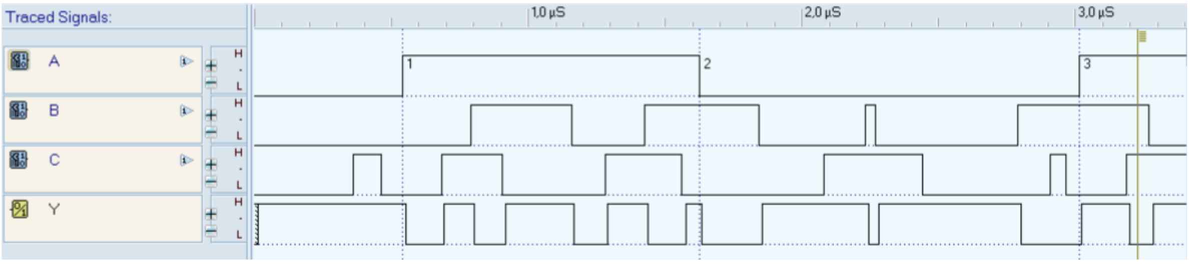
$= (A+\overline{B})(\overline{A}+B\overline{B}+B\overline{C}+C\overline{B}+C\overline{C})$

$= (A+\overline{B})(\overline{A}+B\overline{C}+C\overline{B})$

$= A\overline{A}+AB\overline{C}+AC\overline{B}+\overline{B}\,\overline{A}+\overline{B}B\overline{C}+\overline{B}C\overline{B}$

$= 0+AB\overline{C}+AC\overline{B}+\overline{A}\,\overline{B}+0+0$

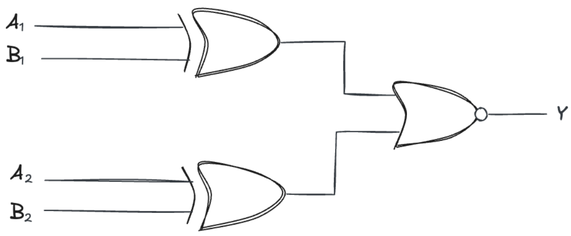
$= AB\overline{C} + AC\overline{B} + \overline{A}\,\overline{B}$



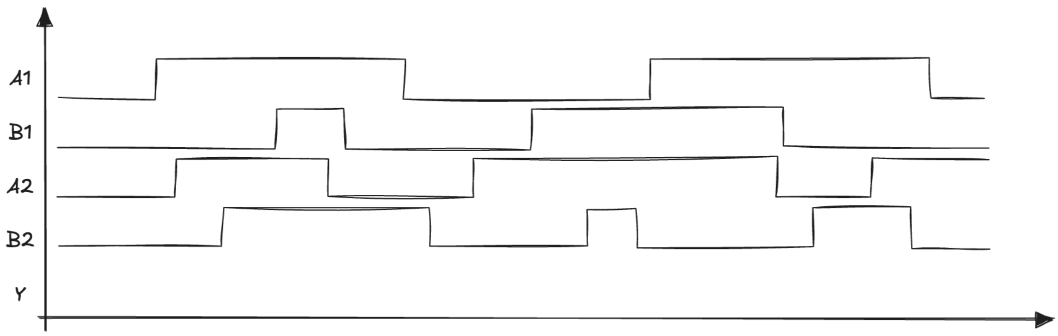
1.4 Exercise 4

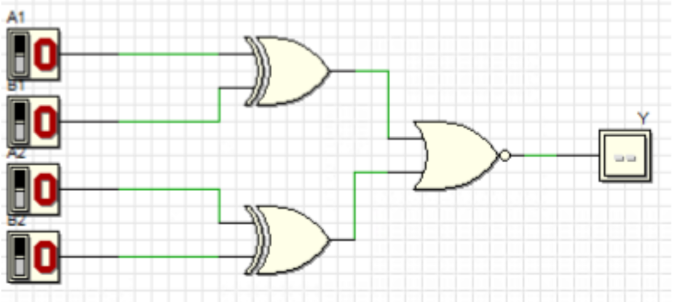
Consider the following circuits. The first one is a **2-bit comparator**, which determines whether the value of A is equal to the value of B. The second one is a **1-bit adder**, which adds A, B, and the input carry, generating both the sum and the carry outputs. Complete the truth table and the timing diagram. Explain the operation of the two circuits in light of the truth table, and verify that they behave as expected. Then implement the corresponding circuits in DEEDS and verify the correctness of your solutions through simulation.

1.4.1 (a) 2-bit comparator



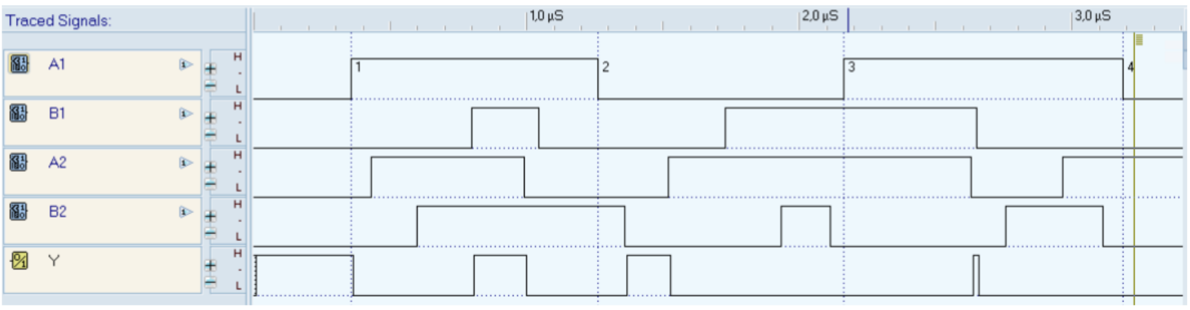
A1	B1	A2	B2	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	



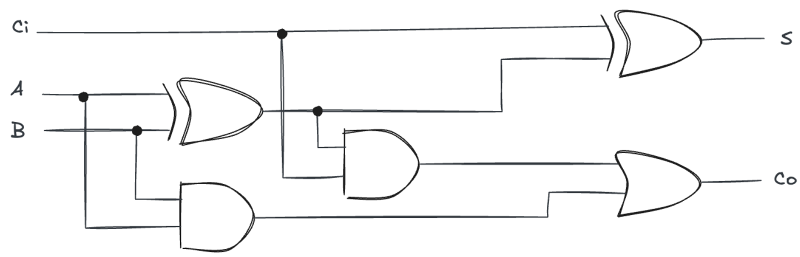


A1	B1	A2	B2	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

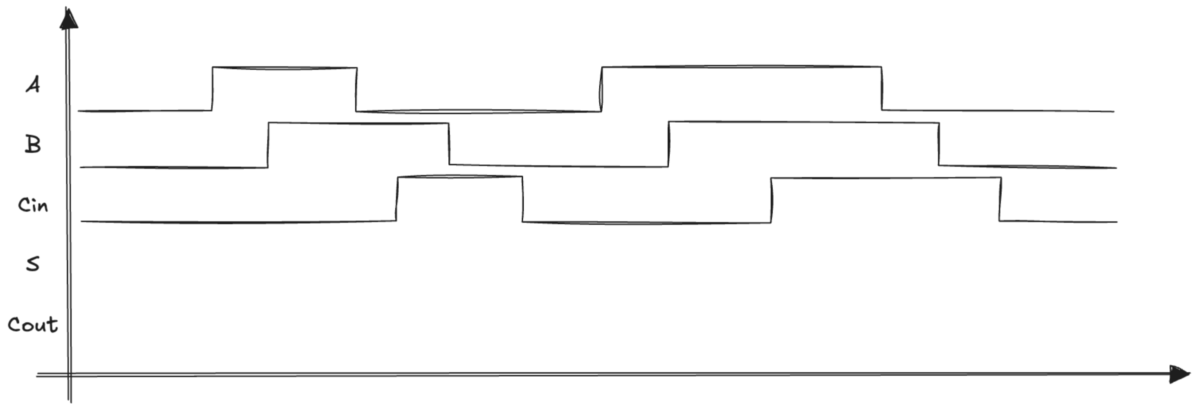
Each row lists all possible combinations of the two 2-bit inputs, A = A2A1 and B = B2B1. The output Y is 1 only when A and B are equal (A1=B1 and A2=B2). By comparing the output Y in the table, we confirm that it equals 1 only when both corresponding bits of A and B match — meaning the comparator works correctly and behaves as expected.

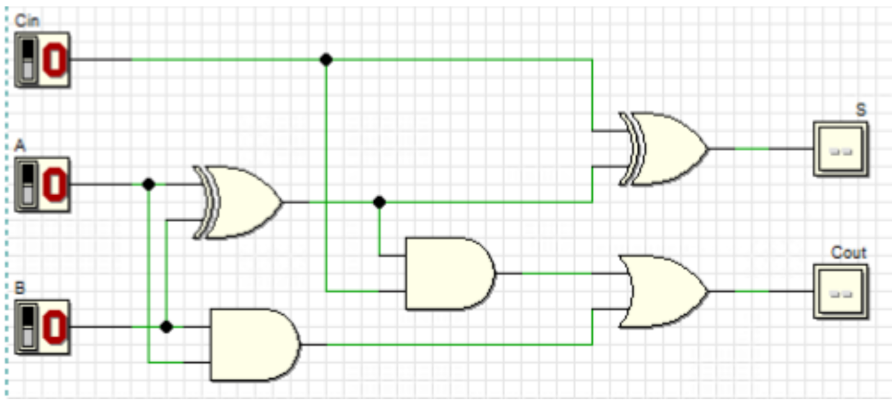


1.4.2 (b) 1-bit adder



A	B	Ci	S	Co
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

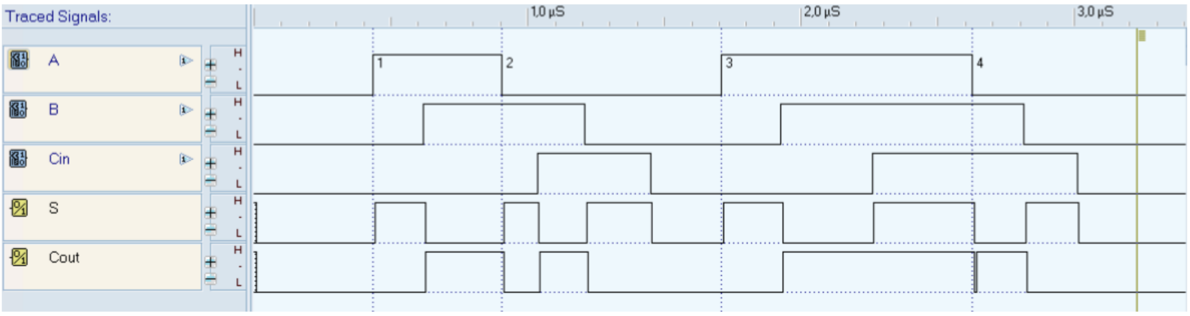




Cin	A	B	S
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

A	B	Cin	Cout
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

The sum output is 1 when an odd number of inputs among A, B, and Cin are 1. The carry output is 1 when at least two of the inputs are 1. The truth tables confirm that the circuit correctly performs binary addition of three inputs, producing the expected sum and carry outputs.


Traced Signals:
A
B
Cin
S
Cout
1,0 µS
2,0 µS
3,0 µS
1
2
3
4