Integrate-and-reset feedback and feedforward for a solenoid with unknown parameters

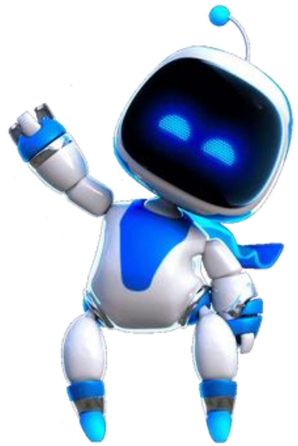
Riccardo Bertollo[1], Michael Schwegel[2], Andreas Kugi[2], Luca Zaccarian[3]

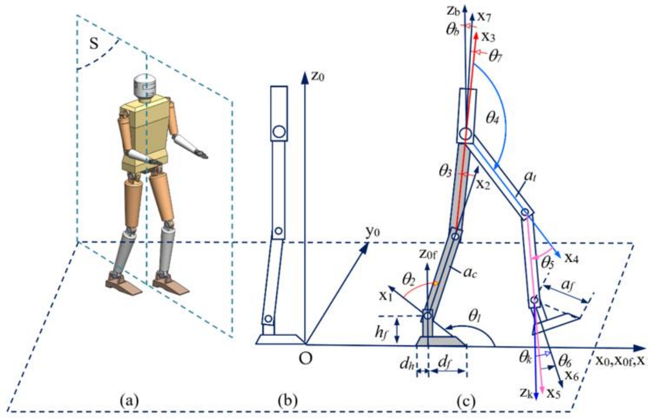
[1] Eindhoven University of Technology (NL)

[2] Vienna University of Technology (A)

[3] University of Trento (IT), LAAS-CNRS Tolouse (FR)

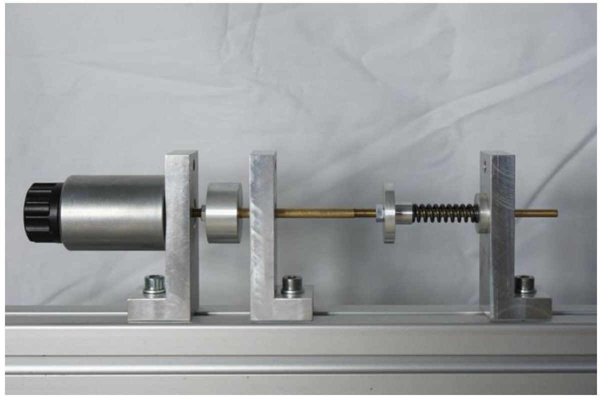
Why adaptive control?

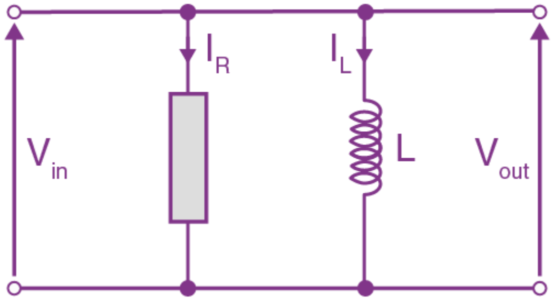




$$\dot{x} = f(x, u, \theta)$$
$$y = h(x)$$

Why adaptive control?

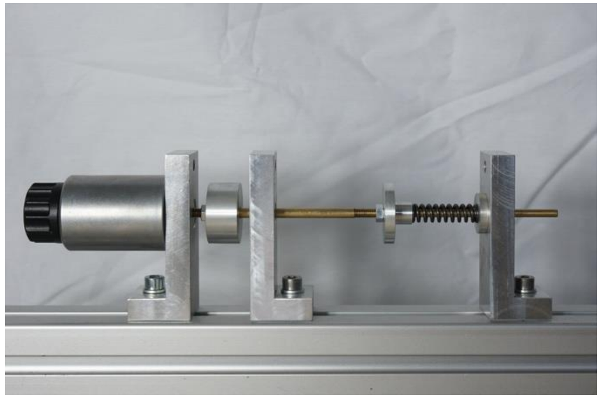


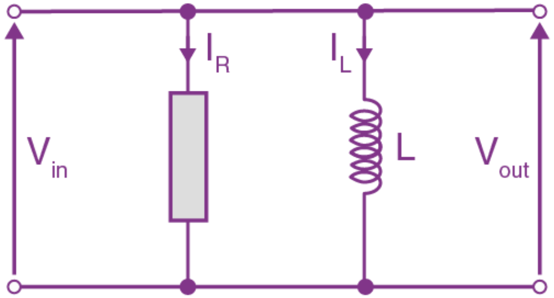


$$\dot{I} = -\frac{R}{L}I + \frac{1}{L}\Delta V + d$$

$$\Delta V = K(I)$$

Why adaptive control?



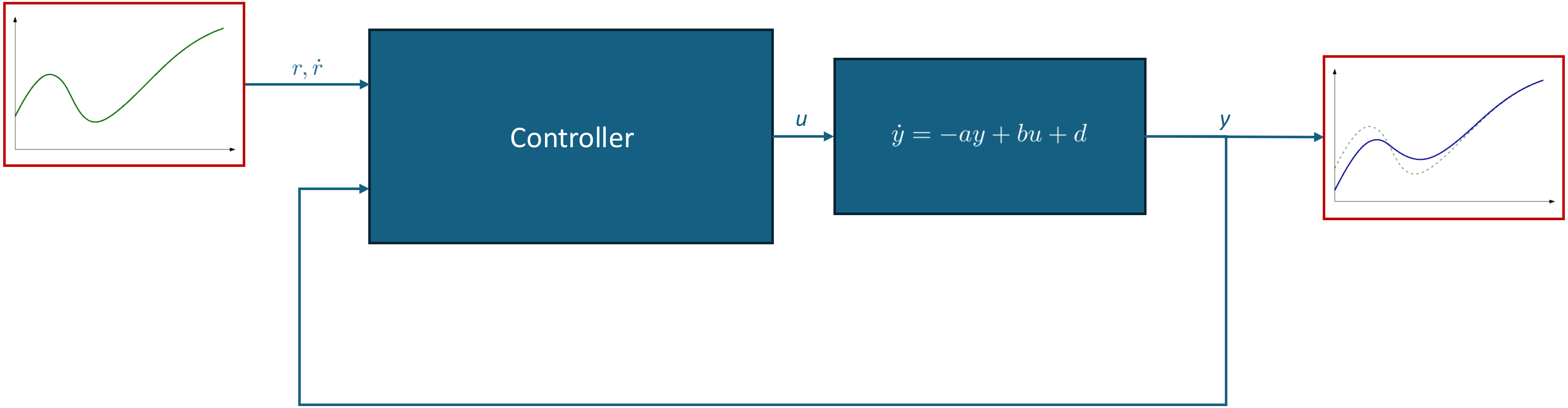


$$\dot{I} = -\frac{R}{L}I + \frac{1}{L}\Delta V + d$$

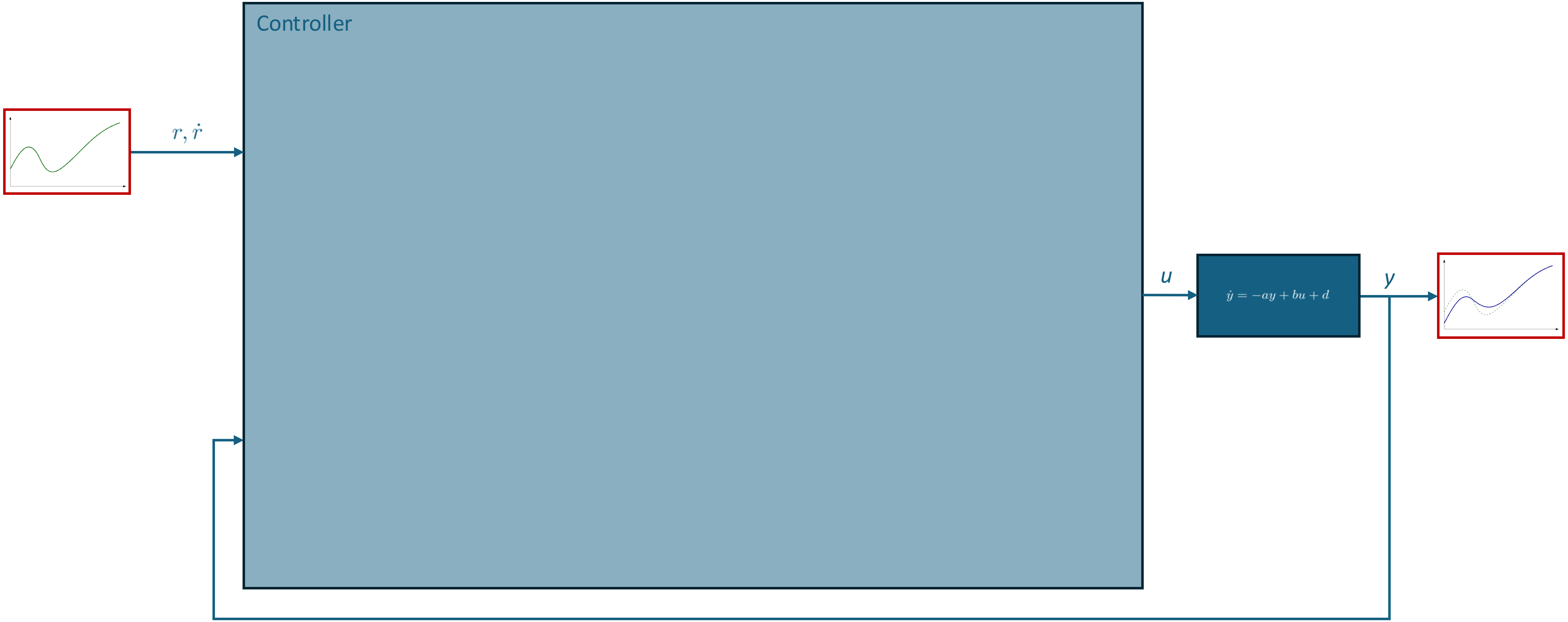
$$\Delta V = K(I)$$

(Slowly) time-varying!

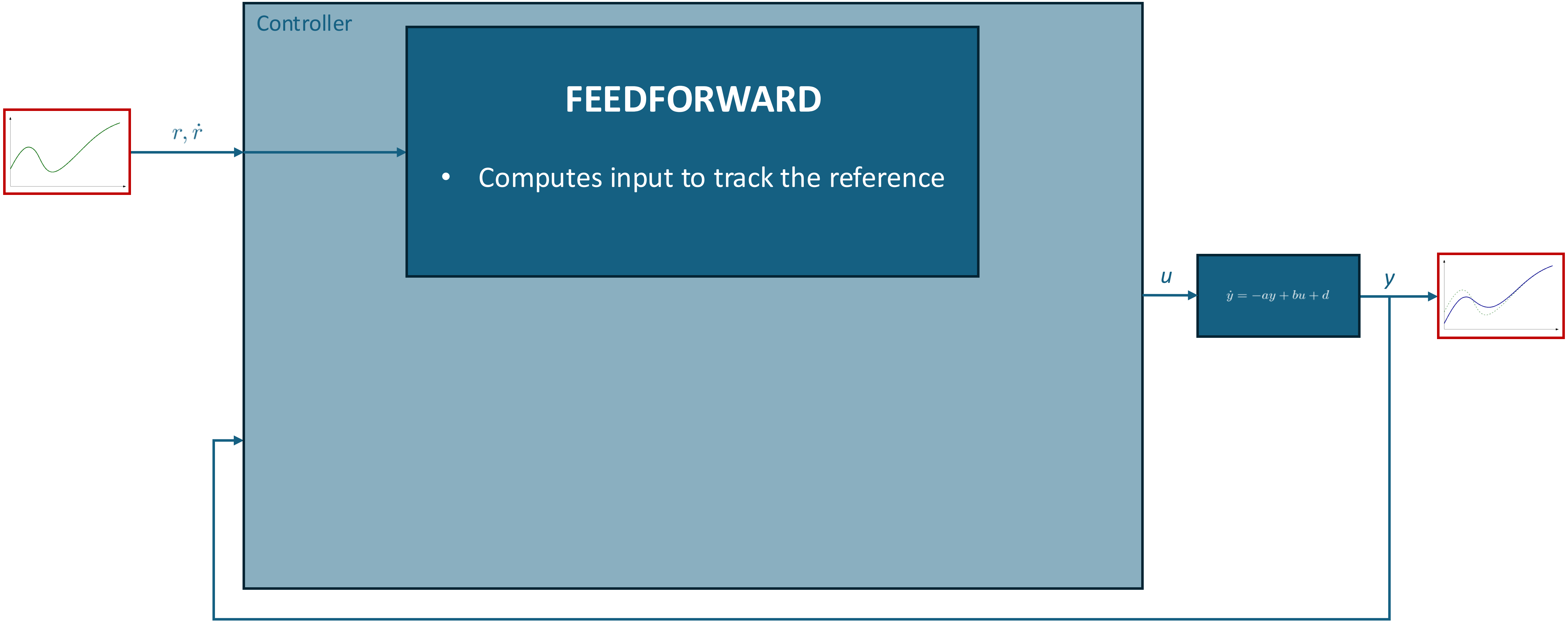
The reference tracking problem



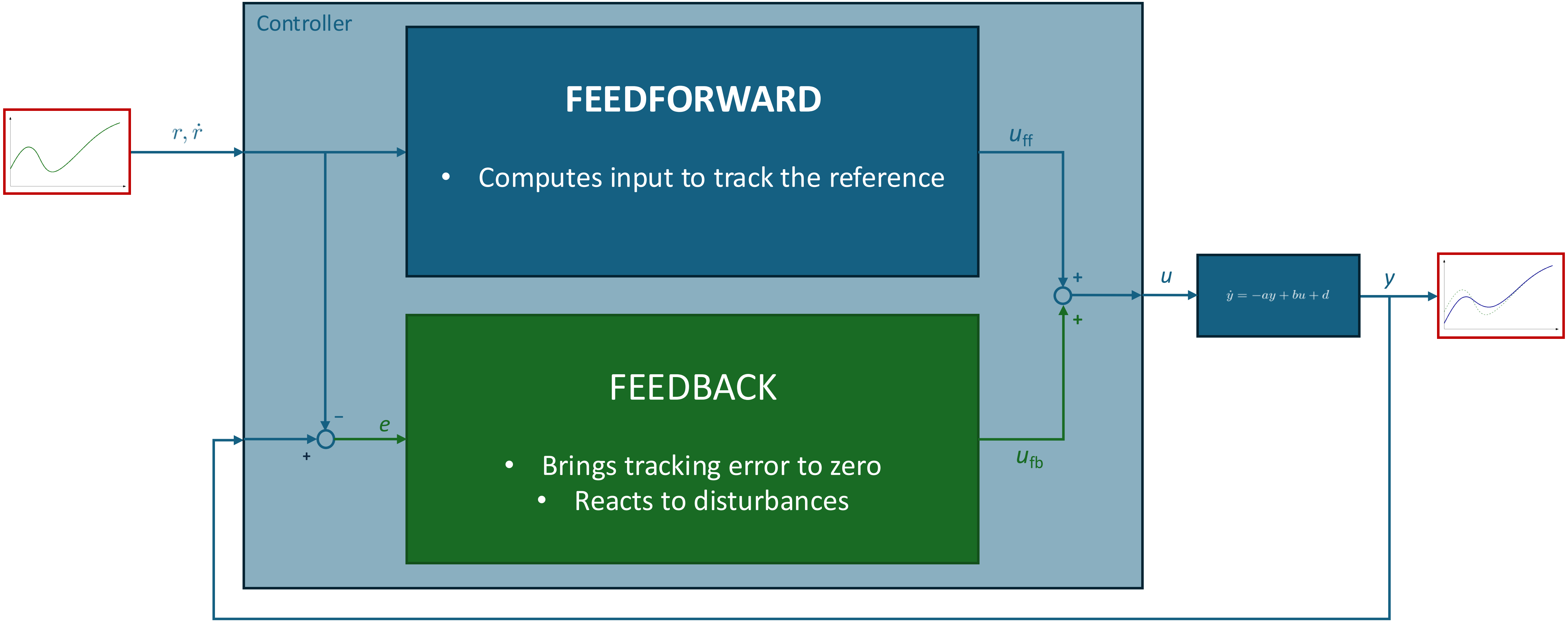
The reference tracking problem



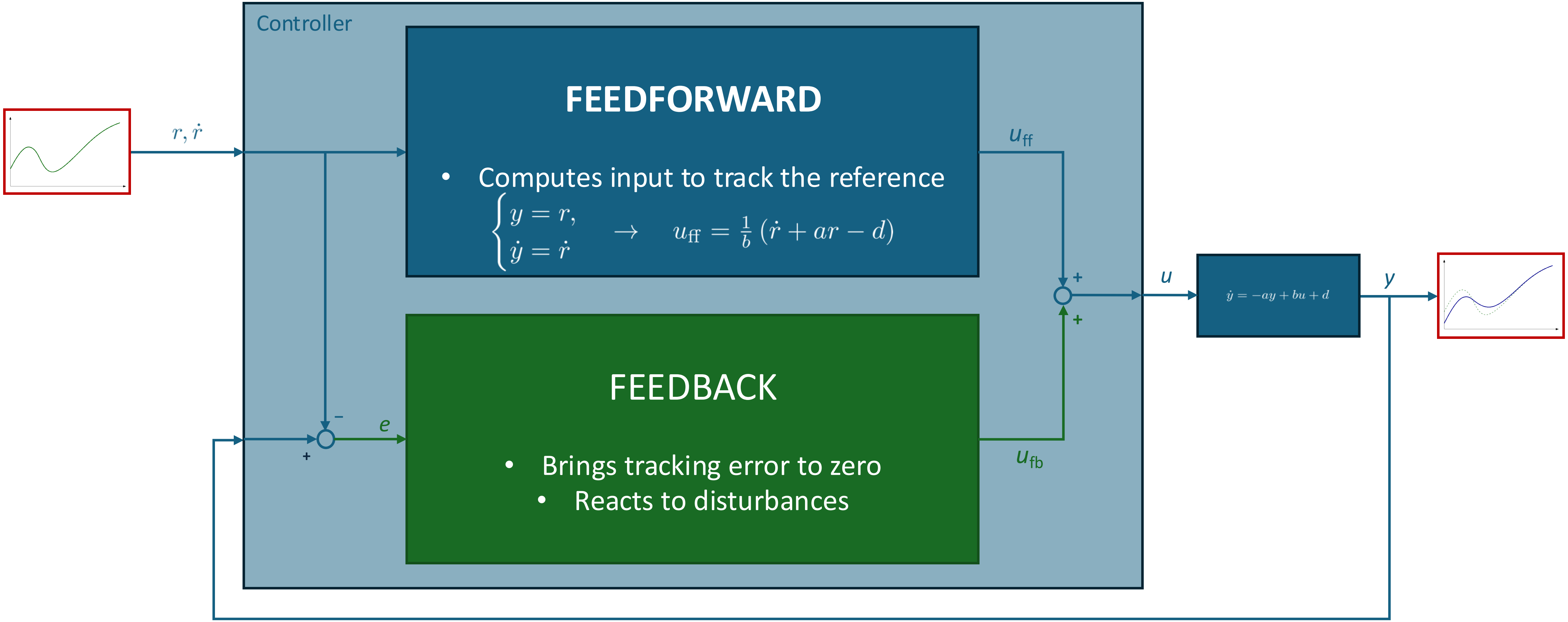
The reference tracking problem



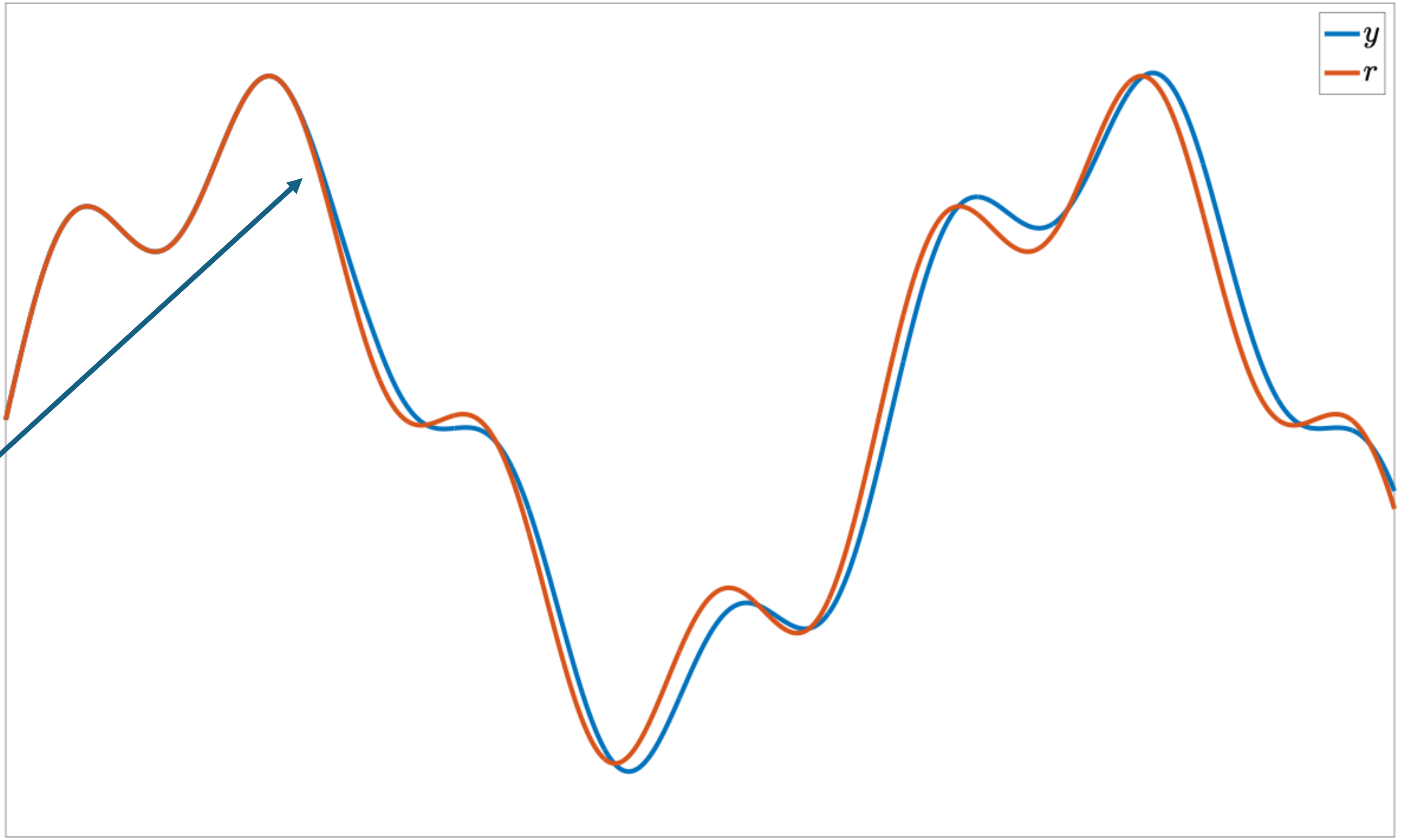
The reference tracking problem



The reference tracking problem

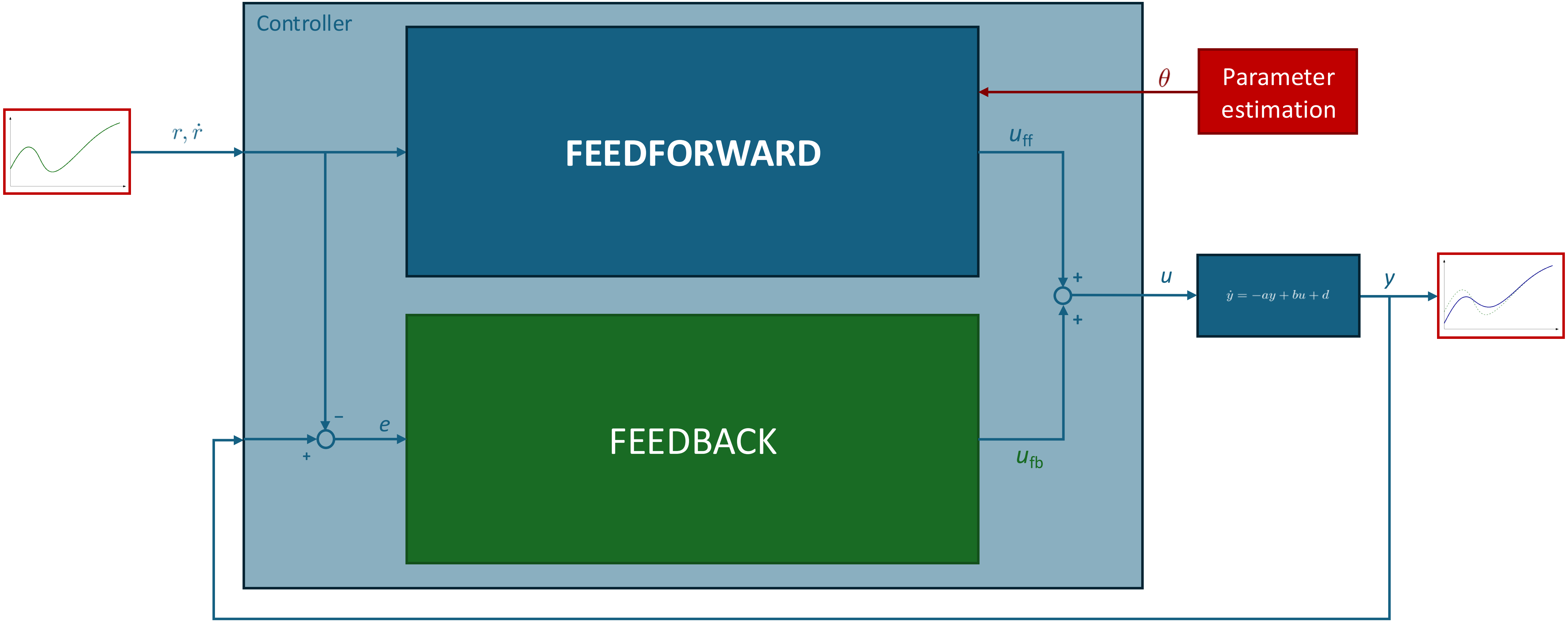


Wrongly estimated parameters

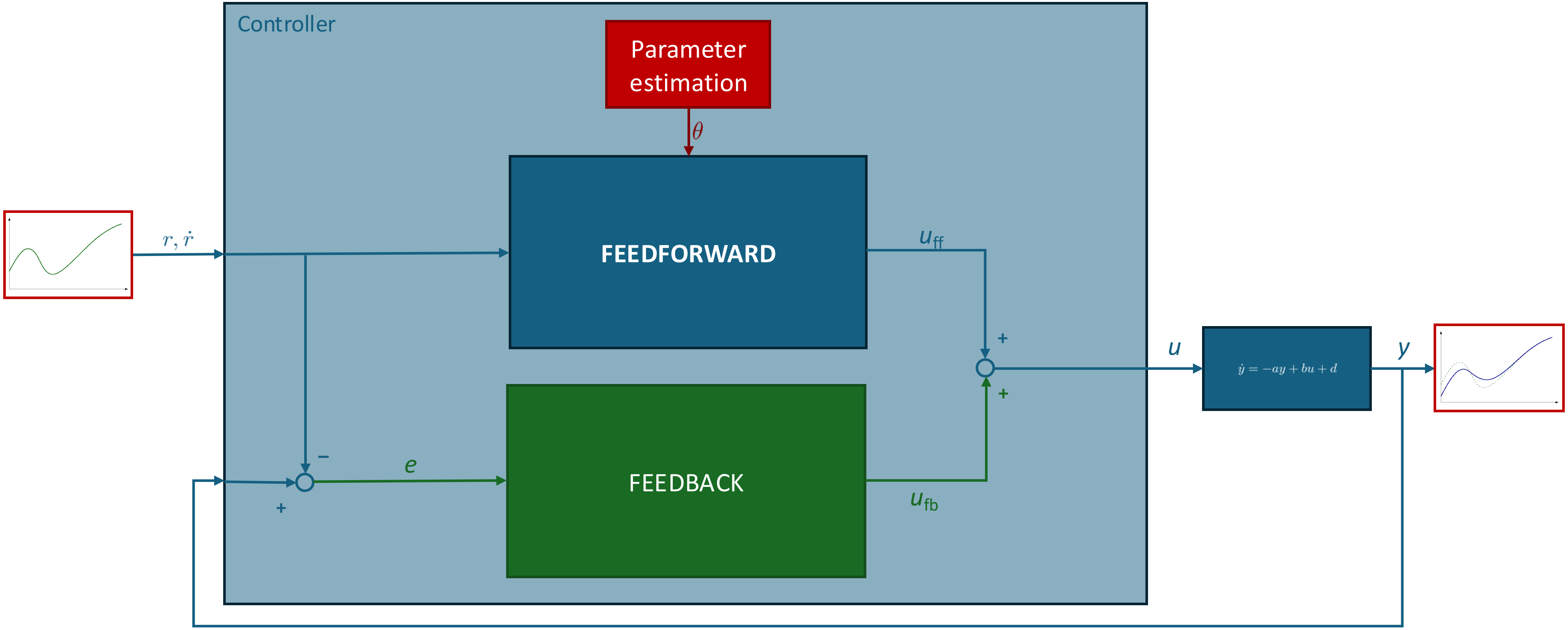


Parameters start changing

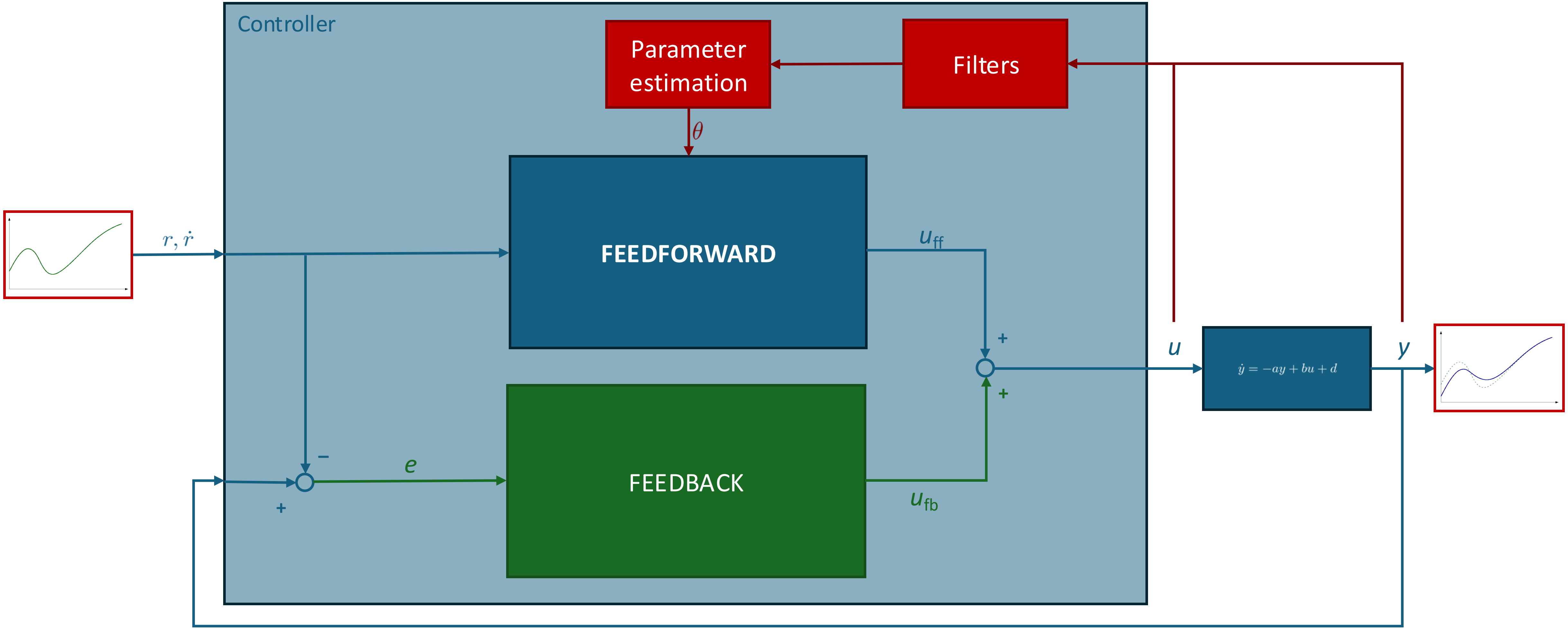
Online parameter estimation



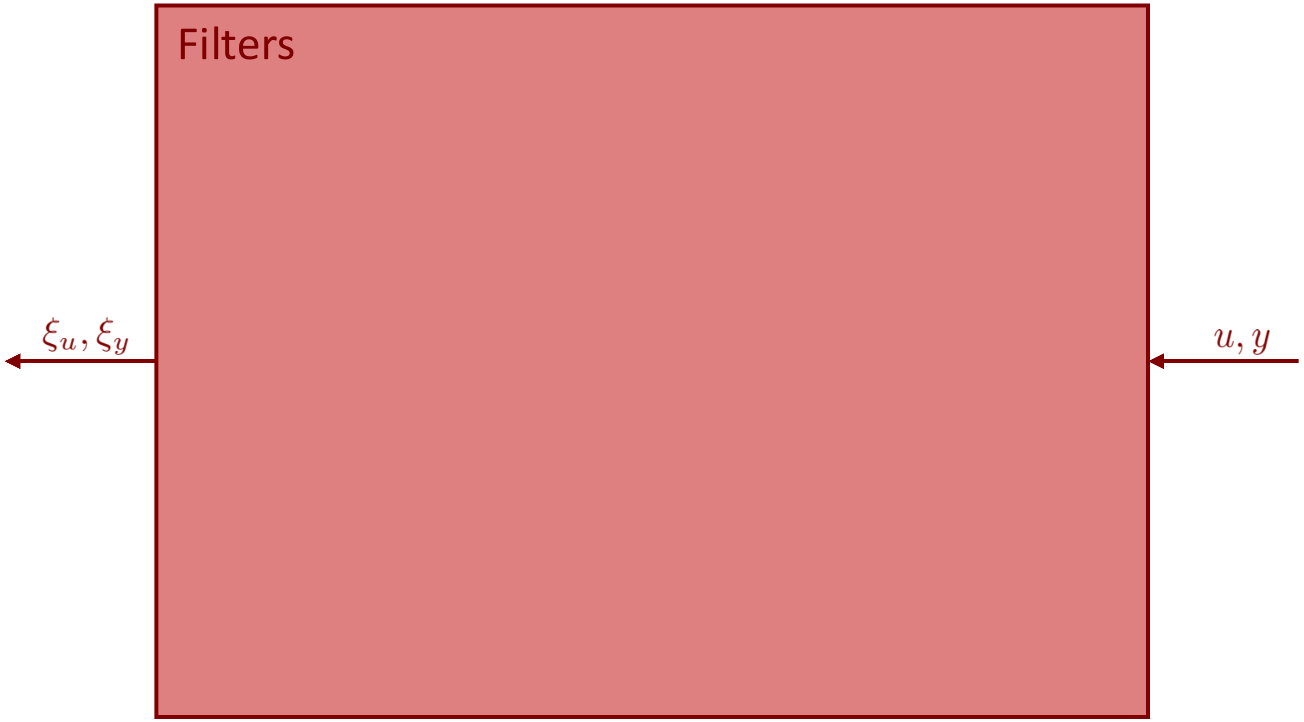
Online parameter estimation



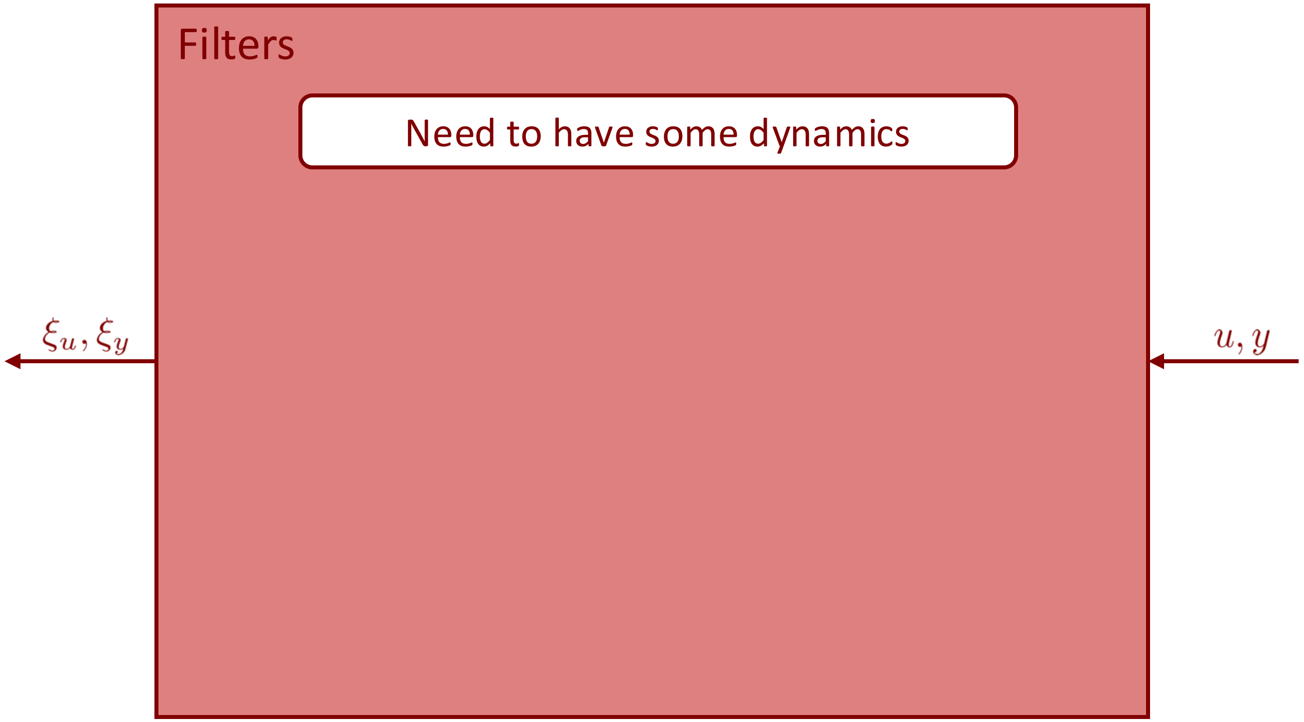
Online parameter estimation



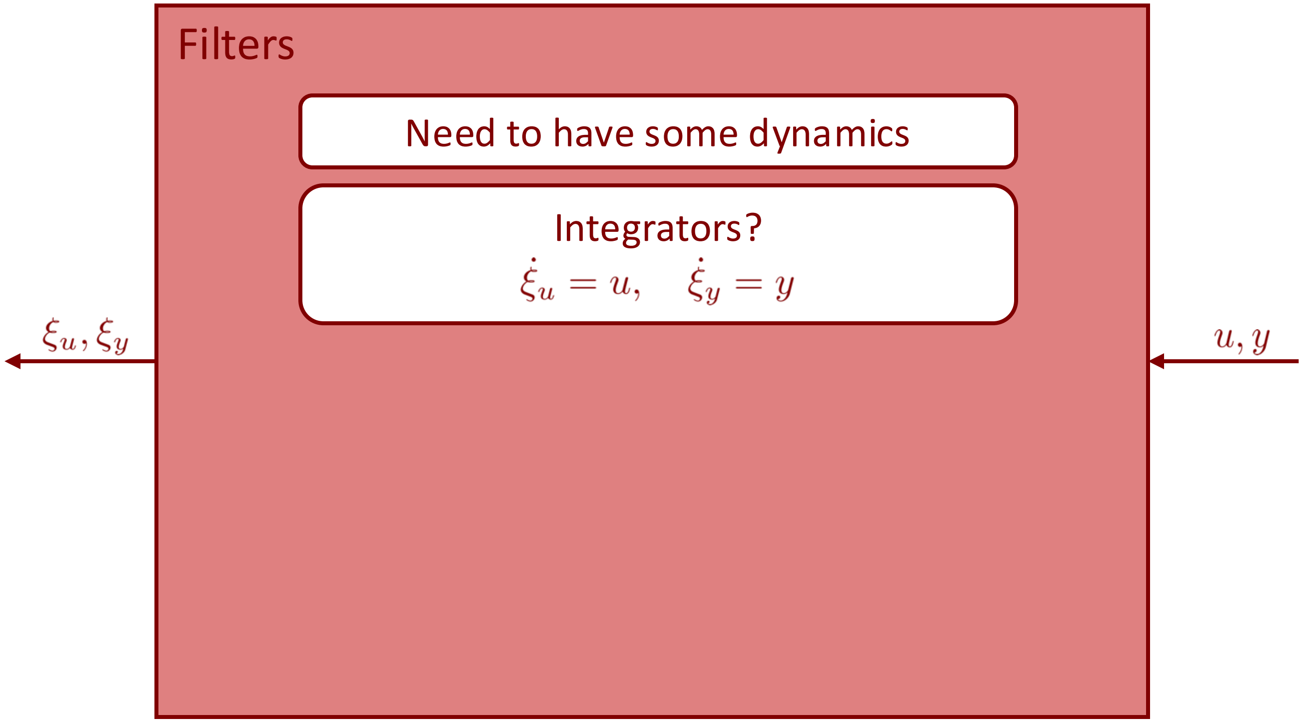
Filtering signals from the plant



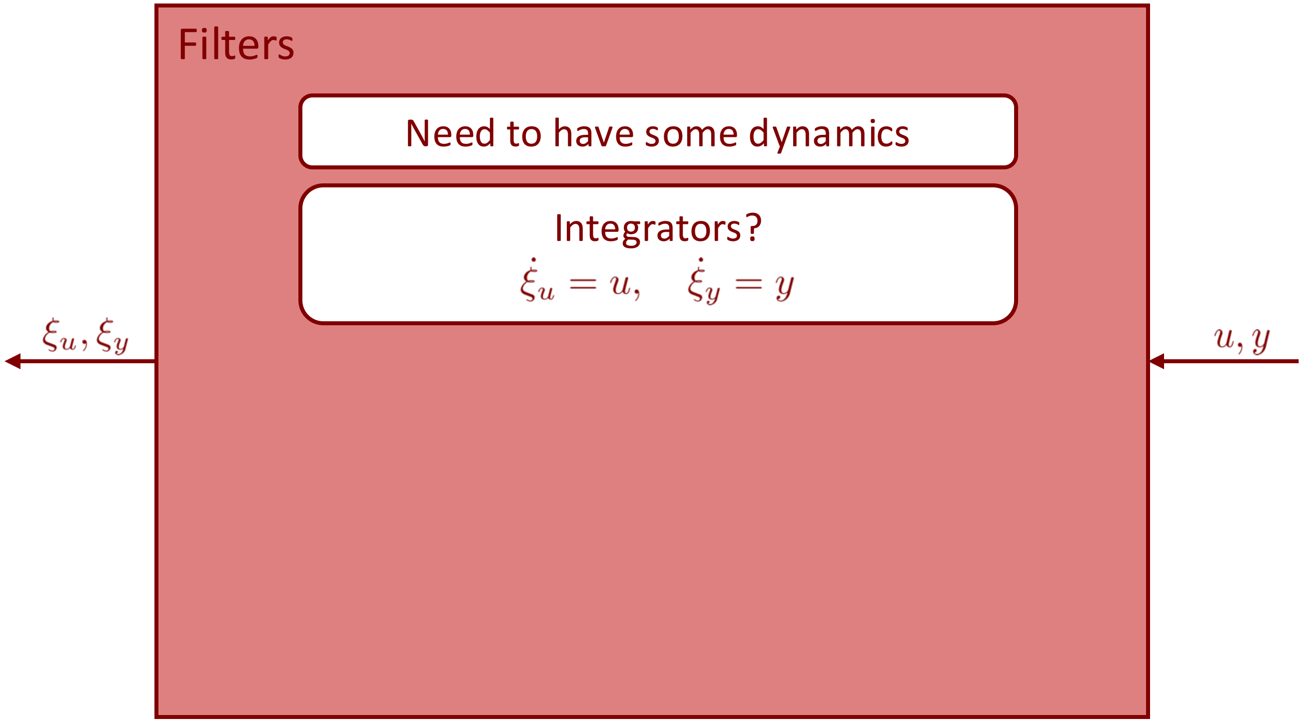
Filtering signals from the plant

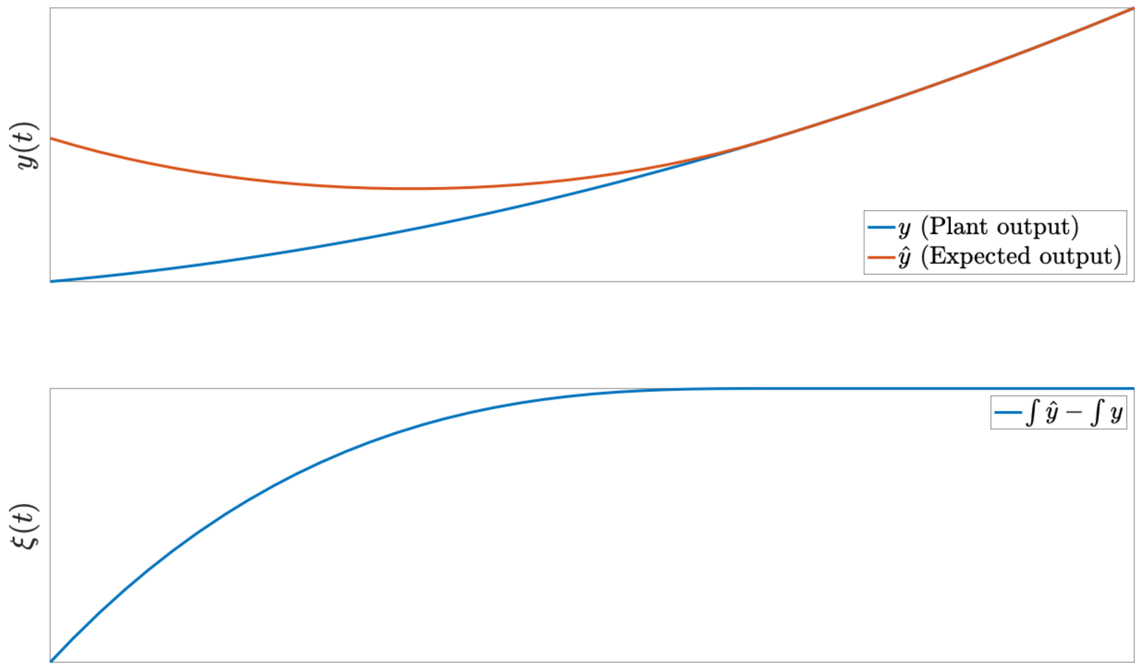


Filtering signals from the plant



Filtering signals from the plant





Filtering signals from the plant

ξ_u, ξ_y ← **Filters** ← u, y

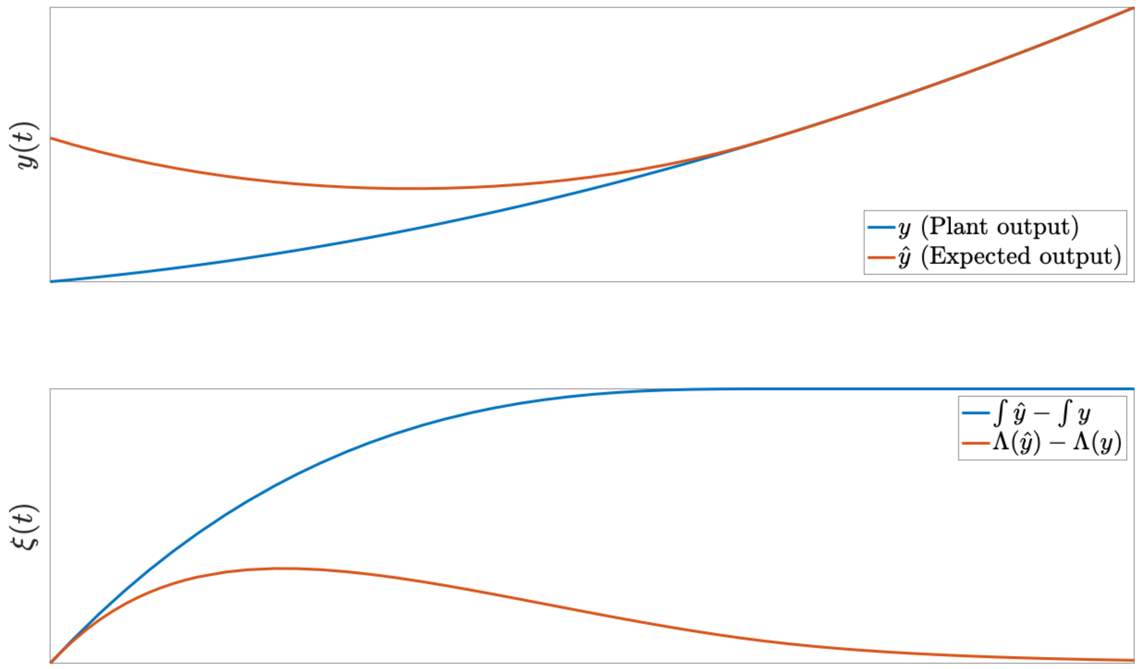
Need to have some dynamics

Integrators?

$$\dot{\xi}_u = u, \quad \dot{\xi}_y = y$$

Converging filters?

$$\Lambda : \quad \dot{\xi}_y = -\alpha \xi_y + y$$



Filtering signals from the plant

ξ_u, ξ_y ← **Filters** ← u, y

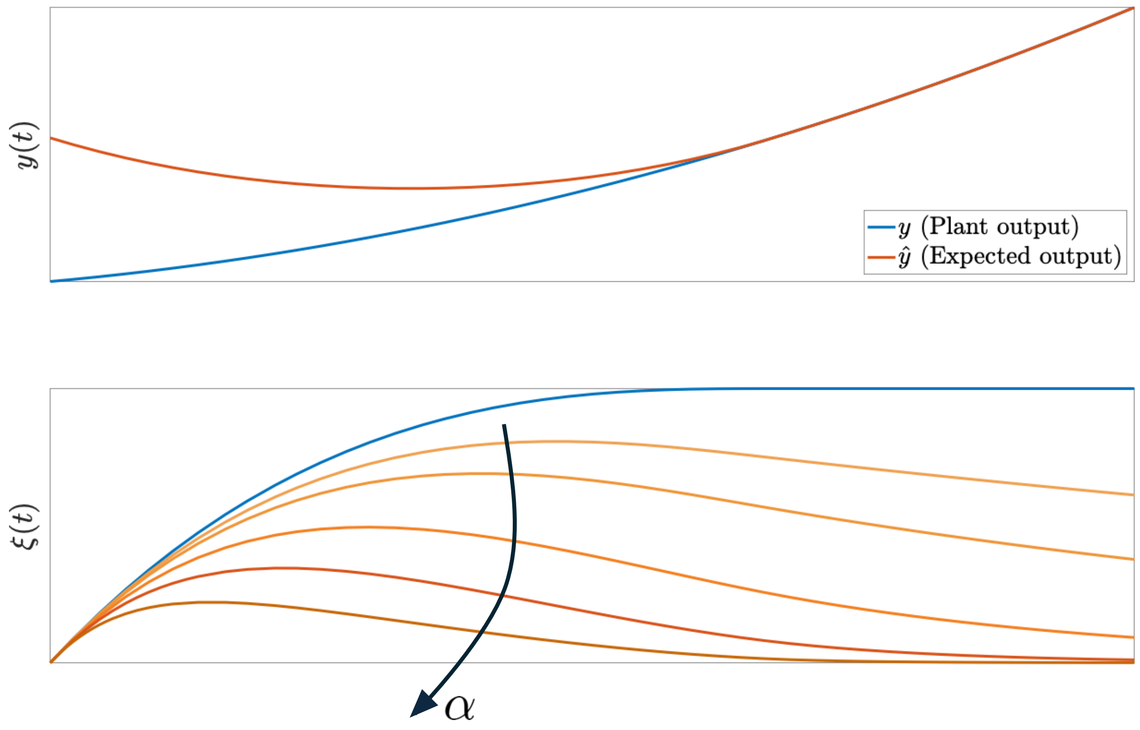
Filters

- Need to have some dynamics
- Integrators?

  $$\dot{\xi}_u = u, \quad \dot{\xi}_y = y$$

- Converging filters?

  $$\Lambda : \quad \dot{\xi}_y = -\alpha \xi_y + y$$



Filtering signals from the plant

Filters

$\xi_u, \xi_y \leftarrow$ Filters $\leftarrow u, y$

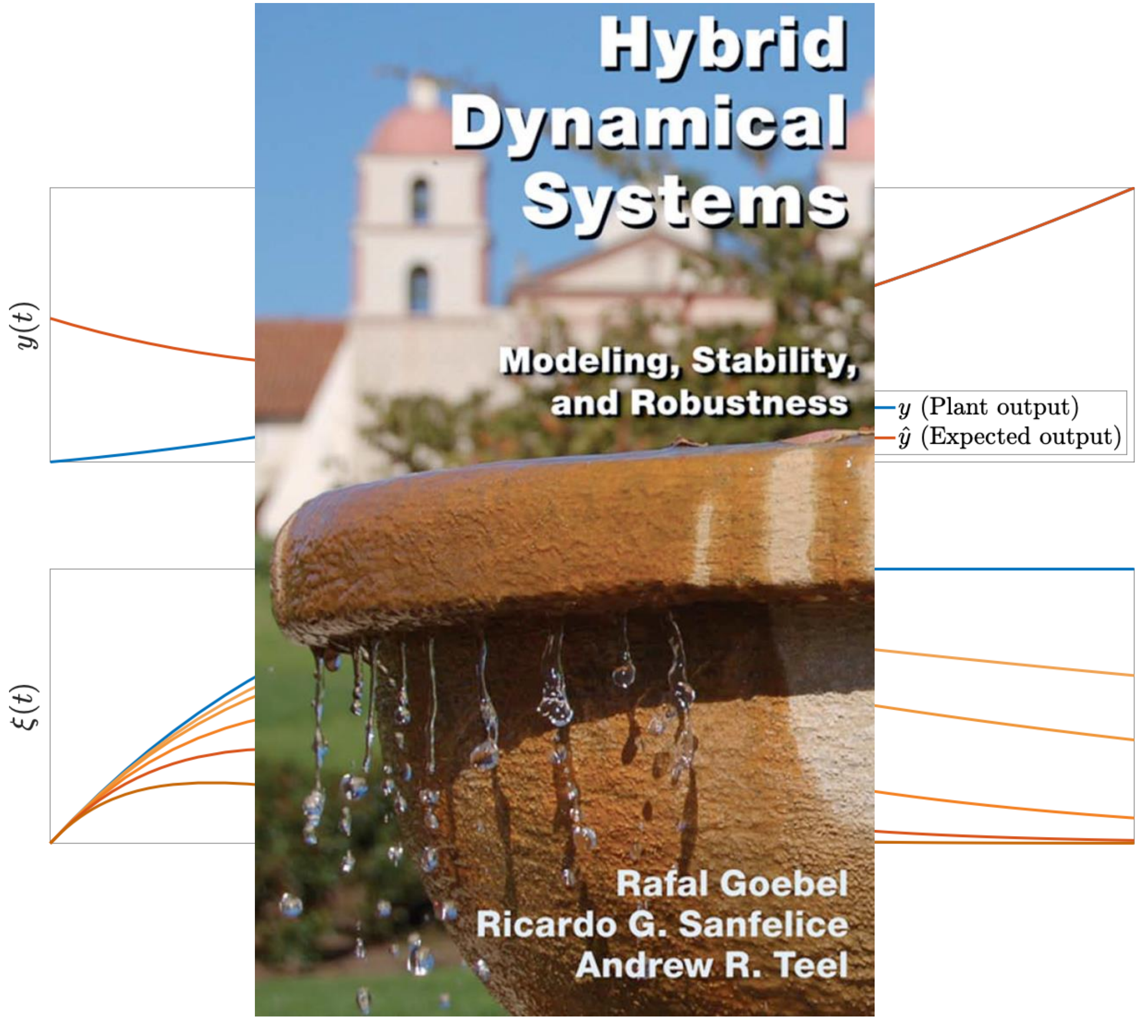
Need to have some dynamics

Integrators?

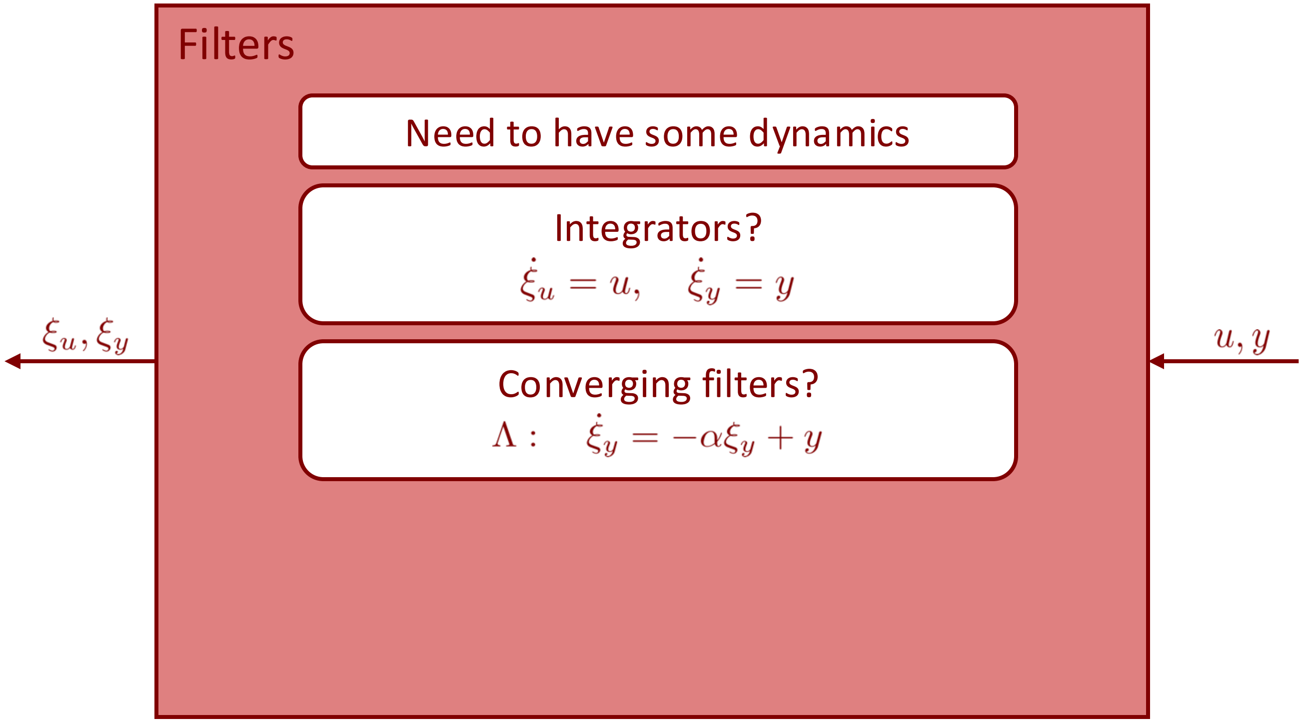
$$\dot{\xi}_u = u, \quad \dot{\xi}_y = y$$

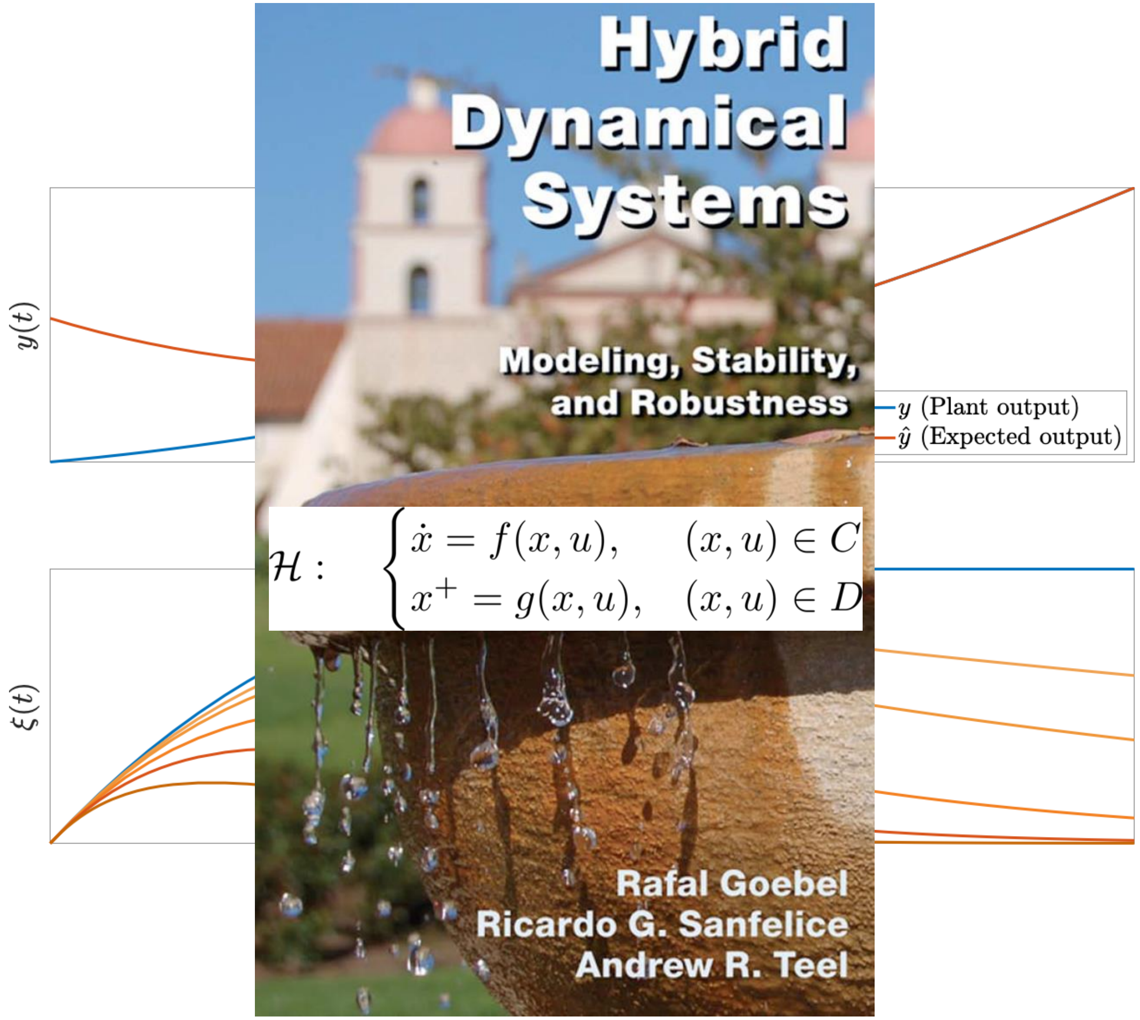
Converging filters?

$$\Lambda : \quad \dot{\xi}_y = -\alpha \xi_y + y$$



Filtering signals from the plant





Filtering signals from the plant

Filters

$\xi_u, \xi_y \leftarrow$ Filters $\leftarrow u, y$

Need to have some dynamics

Integrators?

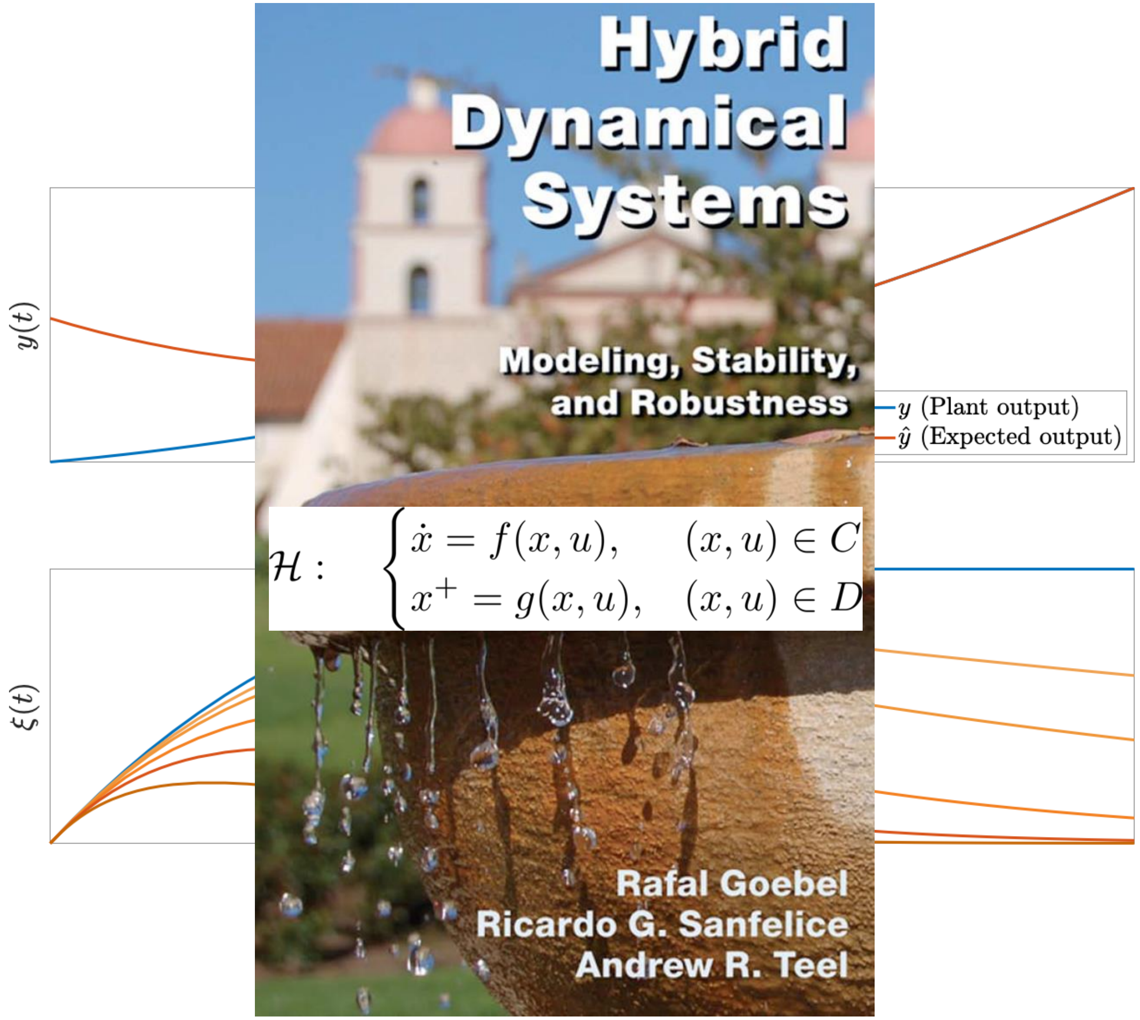
$$\dot{\xi}_u = u, \quad \dot{\xi}_y = y$$

Converging filters?

$$\Lambda : \quad \dot{\xi}_y = -\alpha \xi_y + y$$

Integrate and reset!

$$\mathcal{H} : \begin{cases} \dot{\xi}_y = y, & \xi_y \in C \\ \xi_y^+ = 0, & \xi_y \in D \end{cases}$$



Filtering signals from the plant

Filters

$\xi_u, \xi_y \leftarrow$ Filters $\leftarrow u, y$

Need to have some dynamics

Integrators?

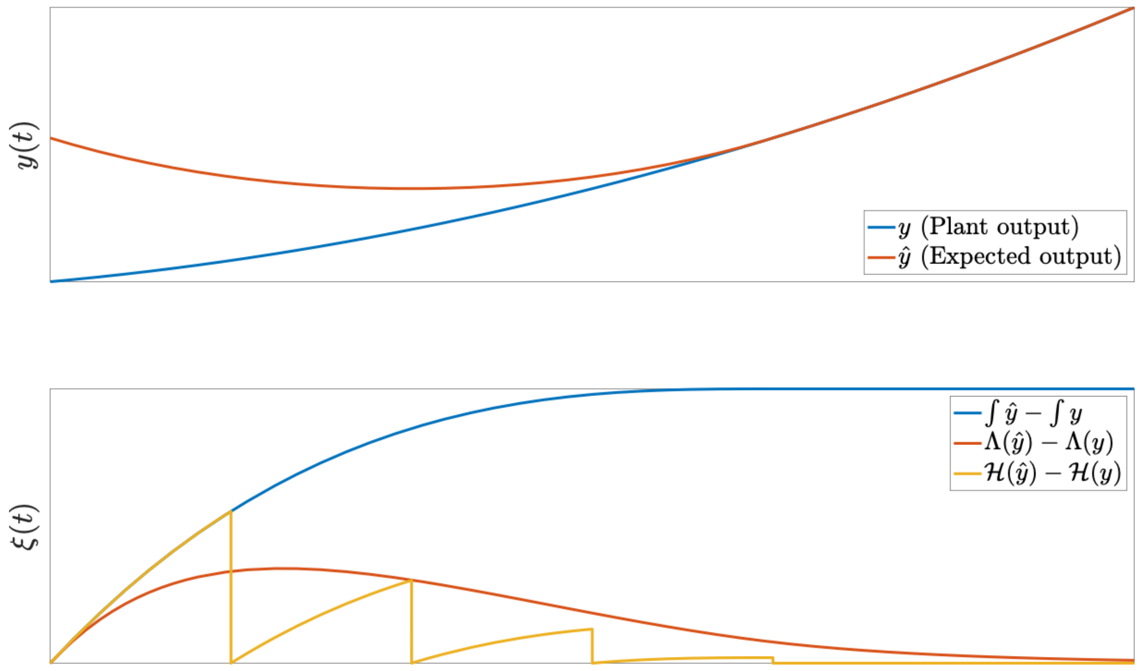
$$\dot{\xi}_u = u, \quad \dot{\xi}_y = y$$

Converging filters?

$$\Lambda : \quad \dot{\xi}_y = -\alpha \xi_y + y$$

Integrate and reset!

$$\mathcal{H} : \begin{cases} \dot{\xi}_y = y, & \xi_y \in C \\ \xi_y^+ = 0, & \xi_y \in D \end{cases}$$



Filtering signals from the plant

$\xi_u, \xi_y \longleftarrow$ **Filters** $\longleftarrow u, y$

Need to have some dynamics

Integrators?

$$\dot{\xi}_u = u, \quad \dot{\xi}_y = y$$

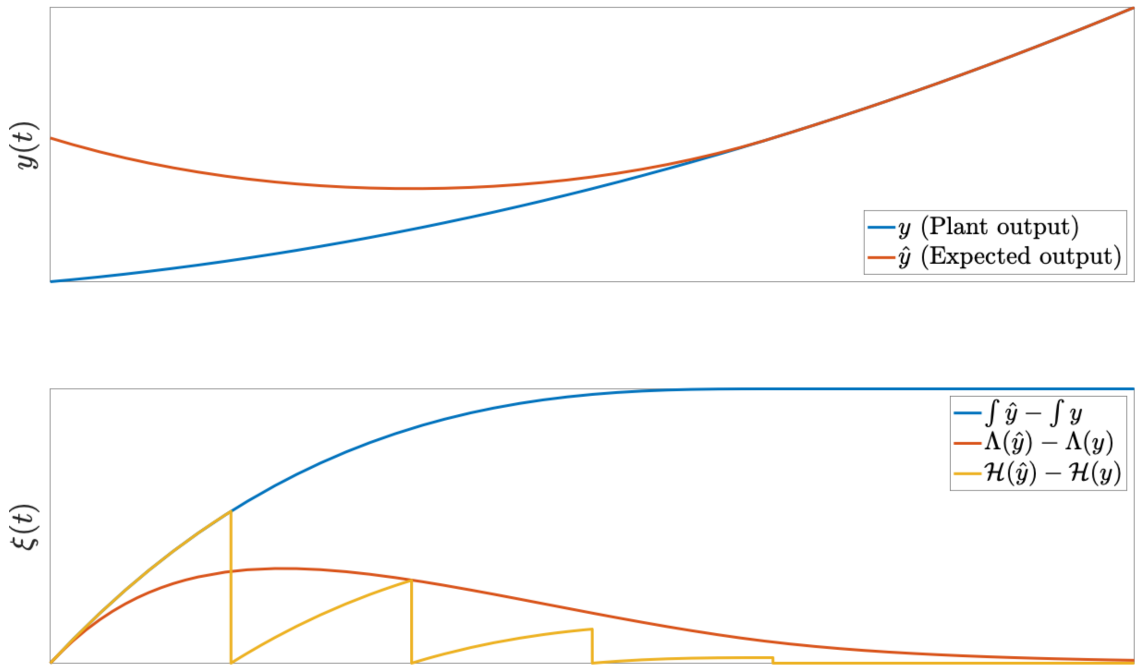
Converging filters?

$$\Lambda : \quad \dot{\xi}_y = -\alpha \xi_y + y$$

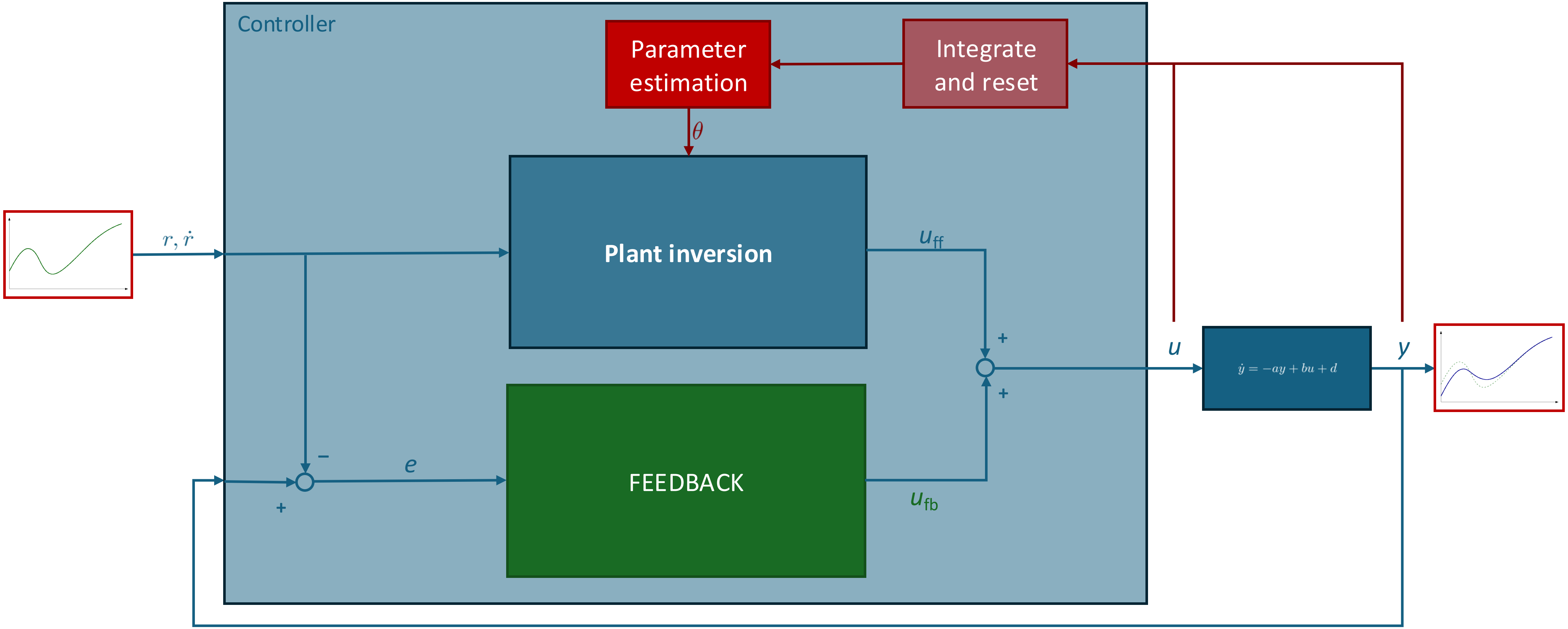
Integrate and reset!

$$\mathcal{H} : \begin{cases} \dot{\xi}_y = y, & \xi_y \in C \\ \xi_y^+ = 0, & \xi_y \in D \end{cases}$$

It can be triggered periodically, but it does not have to!



Good ideas come from feedback!
First Order Reset Elements



Good ideas come from feedback!
First Order Reset Elements

FEEDBACK

L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

Good ideas come from feedback!
First Order Reset Elements

FEEDBACK

Inspired by the Clegg integrator [1958]

$$\begin{cases} \dot{x}_c = e, & \text{"normally"} \\ x_c^+ = 0, & \text{"at the right moment"} \end{cases}$$

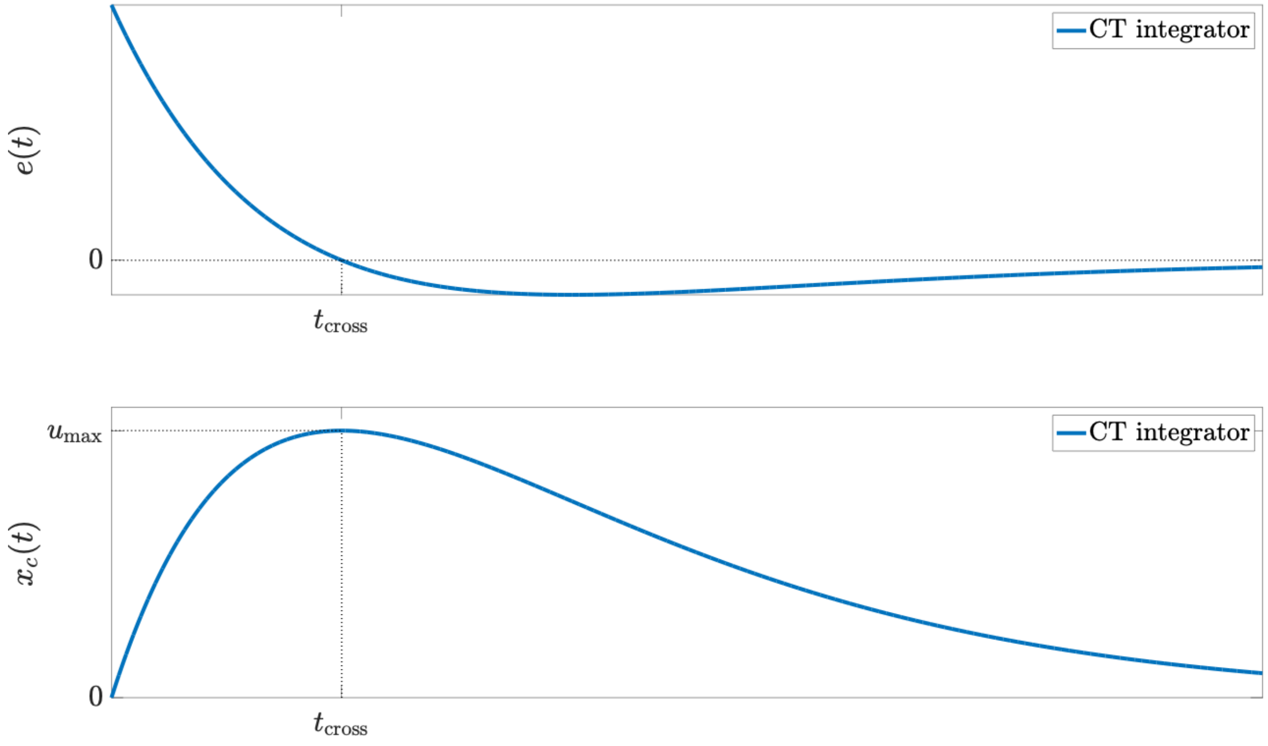
L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

Good ideas come from feedback!
First Order Reset Elements

FEEDBACK

Inspired by the Clegg integrator [1958]

$$\begin{cases} \dot{x}_c = e, & \text{"normally"} \\ x_c^+ = 0, & \text{"at the right moment"} \end{cases}$$



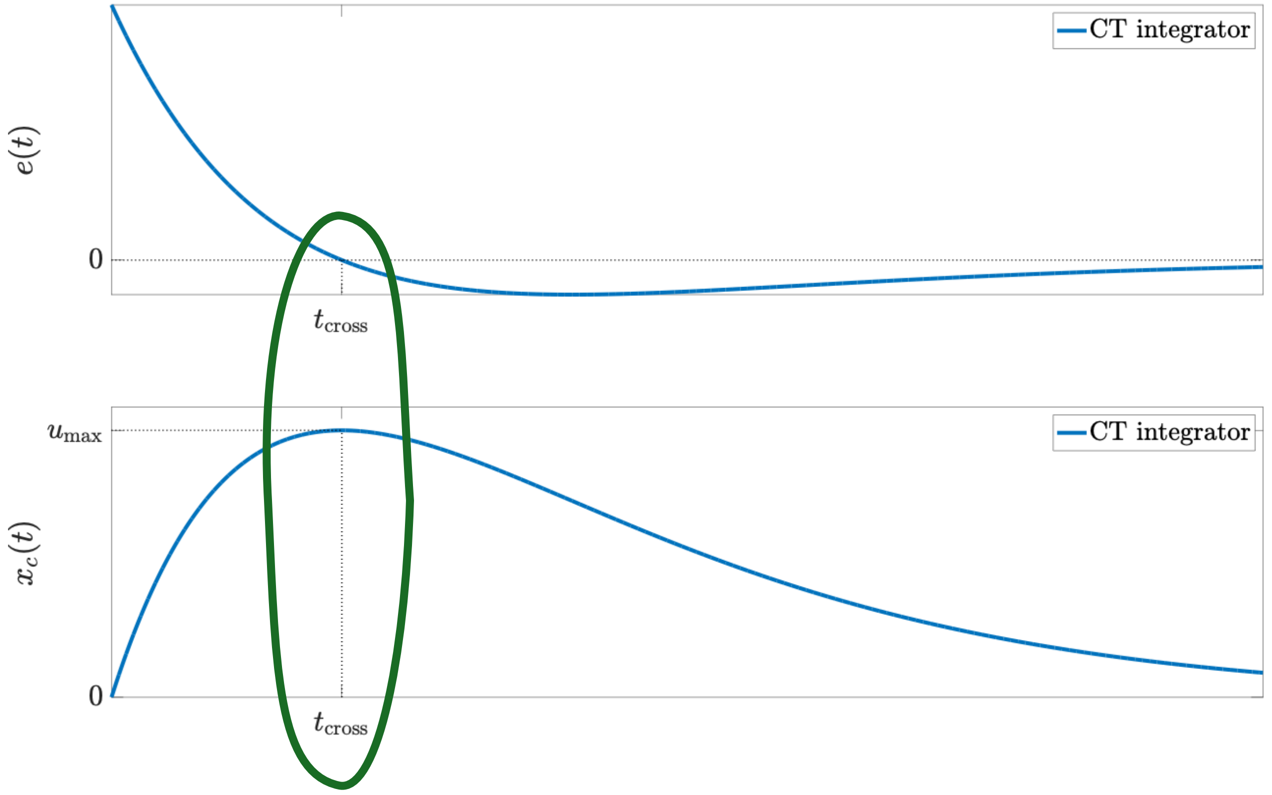
L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

Good ideas come from feedback!
First Order Reset Elements

FEEDBACK

Inspired by the Clegg integrator [1958]

$$\begin{cases} \dot{x}_c = e, & \text{“normally”} \\ x_c^+ = 0, & \text{“at the right moment”} \end{cases}$$



L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

Good ideas come from feedback!
First Order Reset Elements

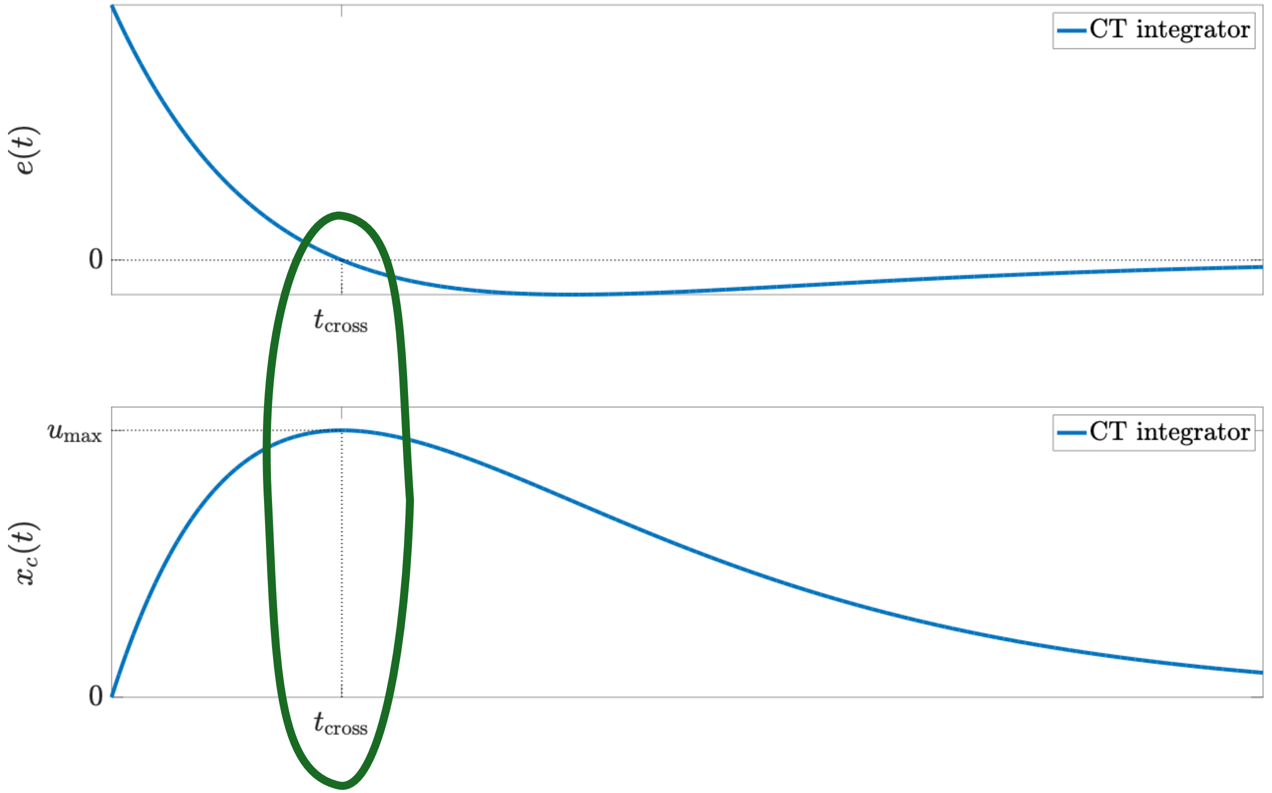
FEEDBACK

Inspired by the Clegg integrator [1958]

$$\begin{cases} \dot{x}_c = e, & \text{“normally”} \\ x_c^+ = 0, & \text{“at the right moment”} \end{cases}$$

Clegg integrator dynamics:

$$\begin{cases} \dot{x}_c = e, & x_c e \geq 0 \\ x_c^+ = 0, & x_c e \leq 0 \end{cases}$$



L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

Good ideas come from feedback!
First Order Reset Elements

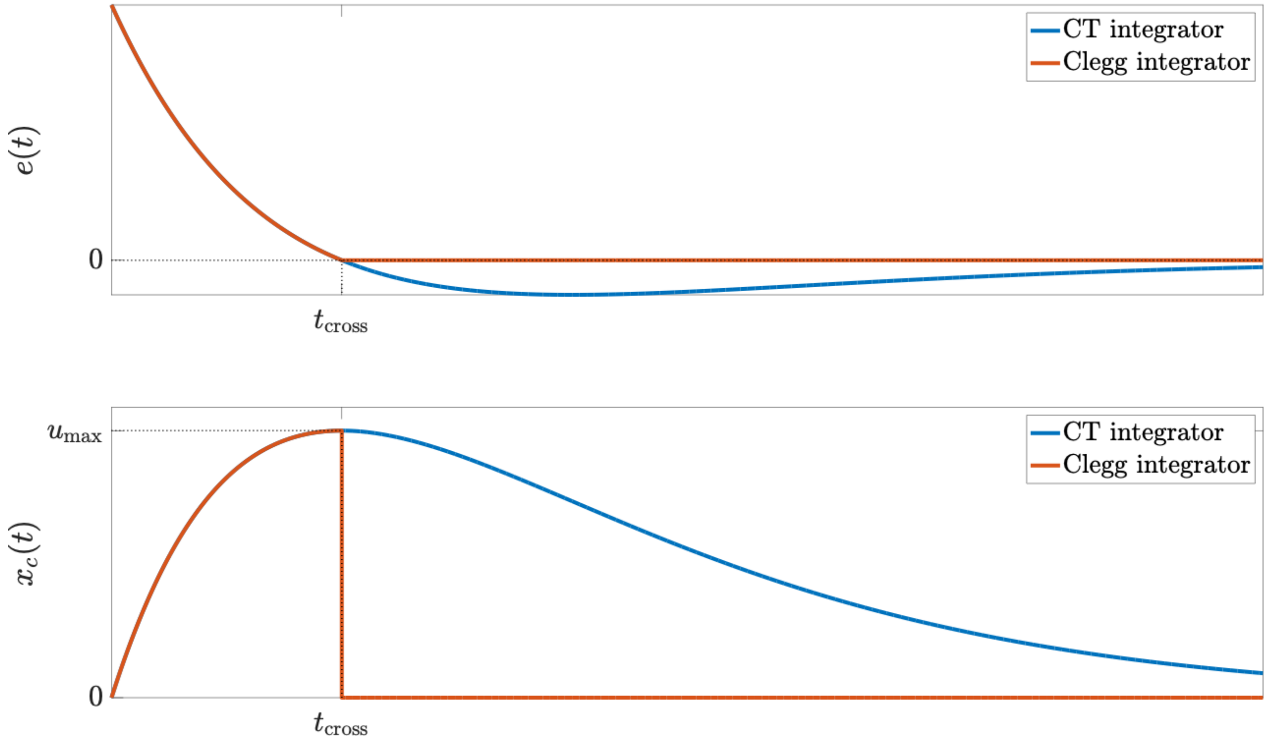
FEEDBACK

Inspired by the Clegg integrator [1958]

$$\begin{cases} \dot{x}_c = e, & \text{“normally”} \\ x_c^+ = 0, & \text{“at the right moment”} \end{cases}$$

Clegg integrator dynamics:

$$\begin{cases} \dot{x}_c = e, & x_c e \geq 0 \\ x_c^+ = 0, & x_c e \leq 0 \end{cases}$$



L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

Good ideas come from feedback!
First Order Reset Elements

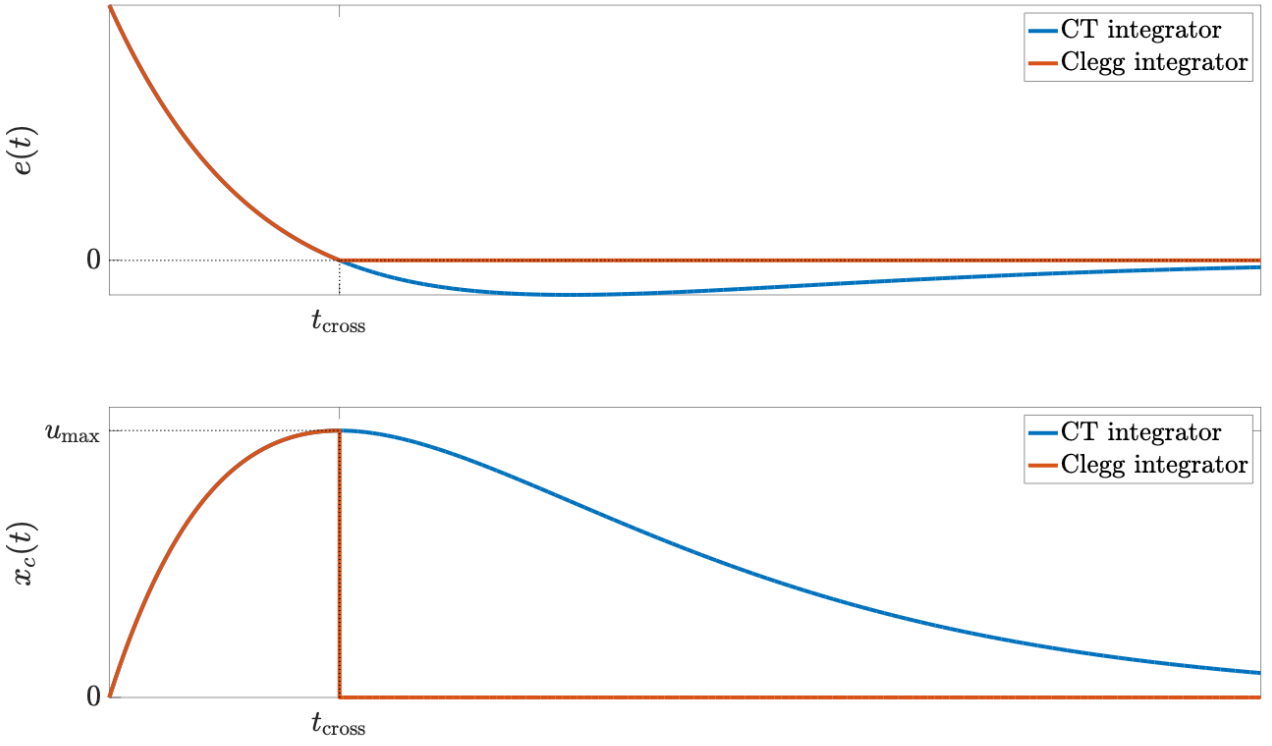
FEEDBACK

Inspired by the Clegg integrator [1958]

$$\begin{cases} \dot{x}_c = e, & \text{“normally”} \\ x_c^+ = 0, & \text{“at the right moment”} \end{cases}$$

FORE dynamics:

$$\begin{cases} \dot{x}_c = a_c x_c + b_c e, & x_c e \geq 0 \\ x_c^+ = 0, & x_c e \leq 0 \end{cases}$$



L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

Good ideas come from feedback!
First Order Reset Elements

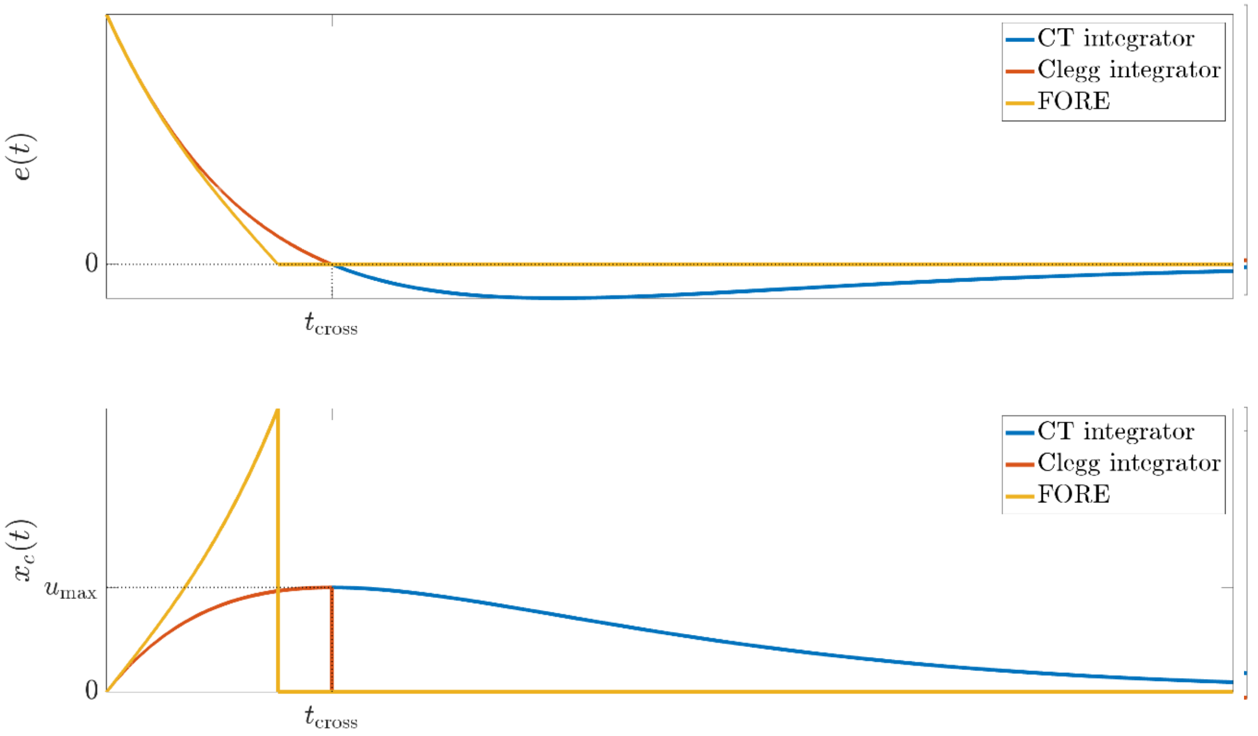
FEEDBACK

Inspired by the Clegg integrator [1958]

$$\begin{cases} \dot{x}_c = e, & \text{“normally”} \\ x_c^+ = 0, & \text{“at the right moment”} \end{cases}$$

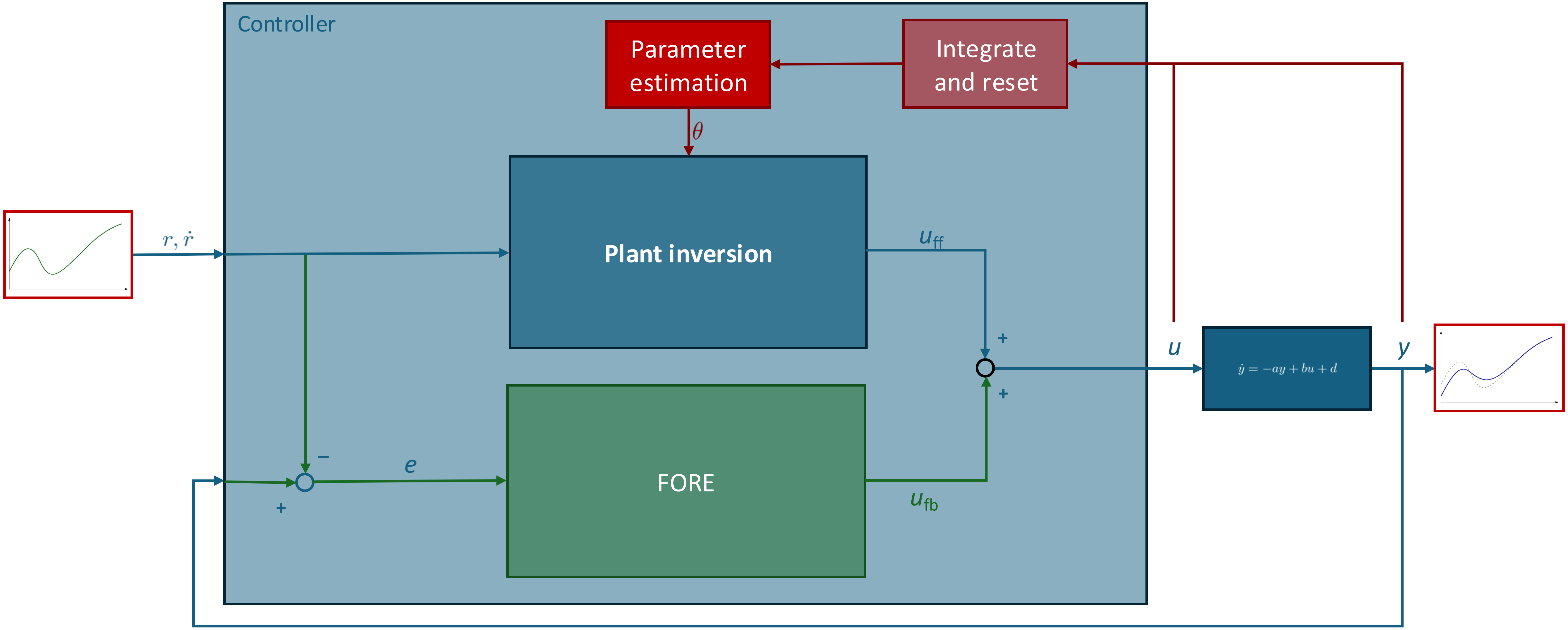
FORE dynamics:

$$\begin{cases} \dot{x}_c = a_c x_c + b_c e, & x_c e \geq 0 \\ x_c^+ = 0, & x_c e \leq 0 \end{cases}$$

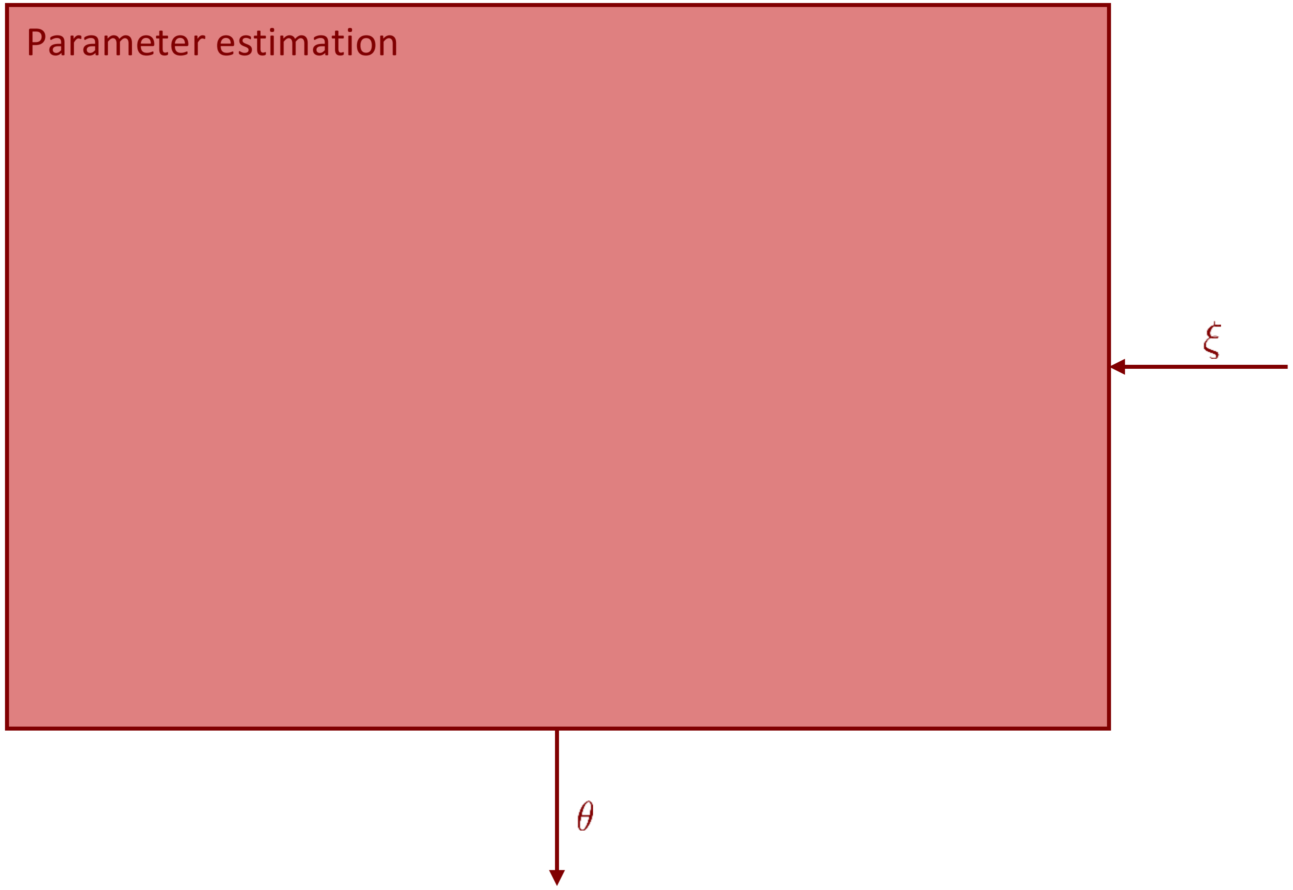


L. Zaccarian, D. Nesic and A. R. Teel, "First order reset elements and the Clegg integrator revisited," *Proceedings of the 2005, American Control Conference, 2005.*

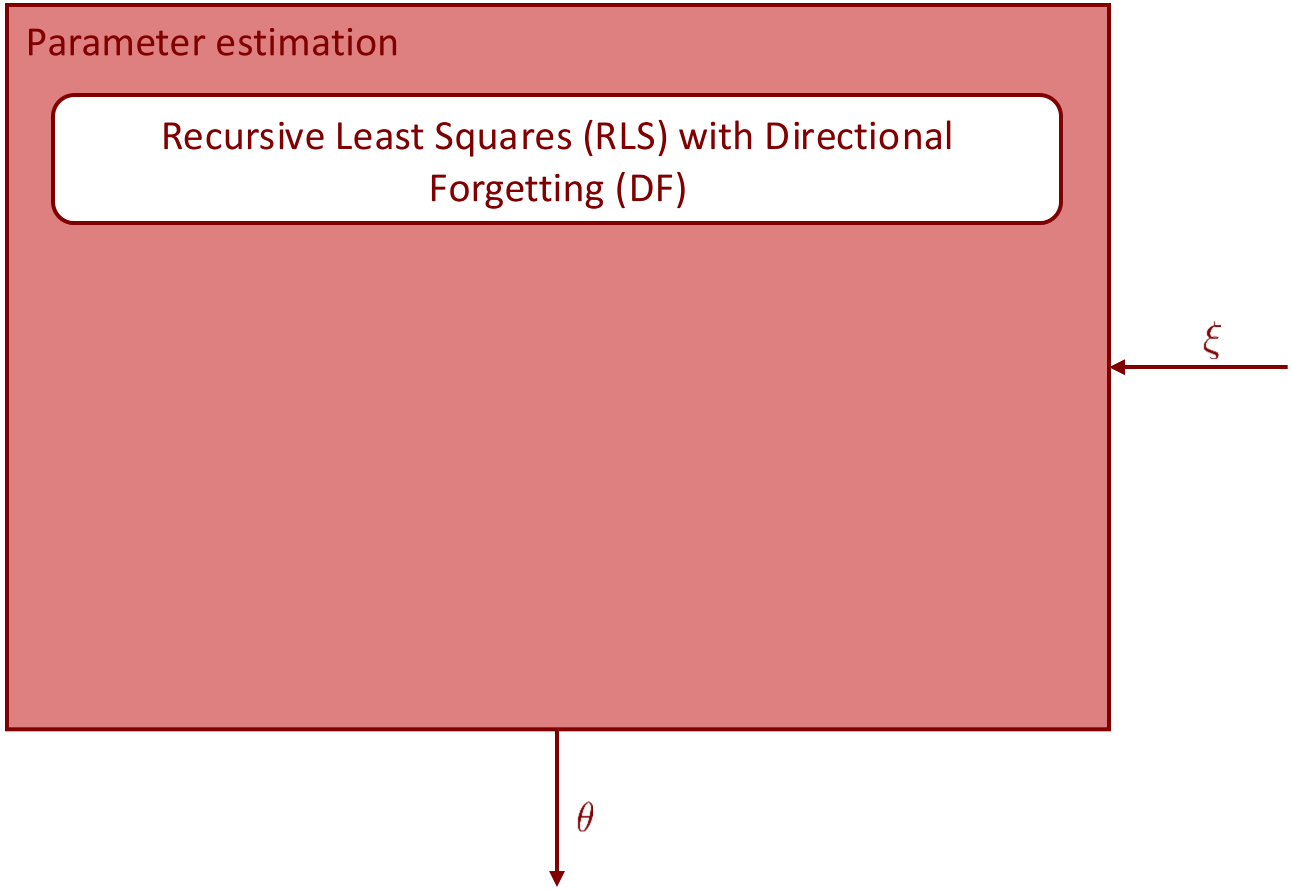
Last ingredient: the adaptation algorithm



Last ingredient: the adaptation algorithm



Last ingredient: the adaptation algorithm



L. Cao, H. Schwartz. “A directional forgetting algorithm based on the decomposition of the information matrix.” *Automatica, vol. 36, n. 11, pp. 1725-1731. 2000.*

Last ingredient: the adaptation algorithm

Parameter estimation

Recursive Least Squares (RLS) with Directional Forgetting (DF)

RLS: $\xi \to \varphi, \quad \begin{cases} R^+ = g_R(R, \varphi) \\ \theta^+ = g_\theta(\theta, R, \varphi) \end{cases}$

ξ

θ

L. Cao, H. Schwartz. “A directional forgetting algorithm based on the decomposition of the information matrix.” *Automatica, vol. 36, n. 11, pp. 1725-1731. 2000.*

Last ingredient: the adaptation algorithm

Parameter estimation

Recursive Least Squares (RLS) with Directional Forgetting (DF)

RLS: $\xi \to \varphi, \quad \begin{cases} R^+ = g_R(R, \varphi) \\ \theta^+ = g_\theta(\theta, R, \varphi) \end{cases}$

DF: $R^+ = g_R(R, \varphi) - \eta \Delta R(\varphi), \quad \eta \in (0, 1)$

ξ

θ

L. Cao, H. Schwartz. “A directional forgetting algorithm based on the decomposition of the information matrix.” *Automatica, vol. 36, n. 11, pp. 1725-1731. 2000.*

Last ingredient:
the adaptation algorithm

Parameter estimation

Recursive Least Squares (RLS) with Directional Forgetting (DF)

RLS: $\xi \to \varphi, \quad \begin{cases} R^{+} = g_R(R, \varphi) \\ \theta^{+} = g_\theta(\theta, R, \varphi) \end{cases}$

DF: $R^{+} = g_R(R, \varphi) - \eta \Delta R(\varphi), \quad \eta \in (0,1)$

ξ

θ

Theorem 1

Consider the set

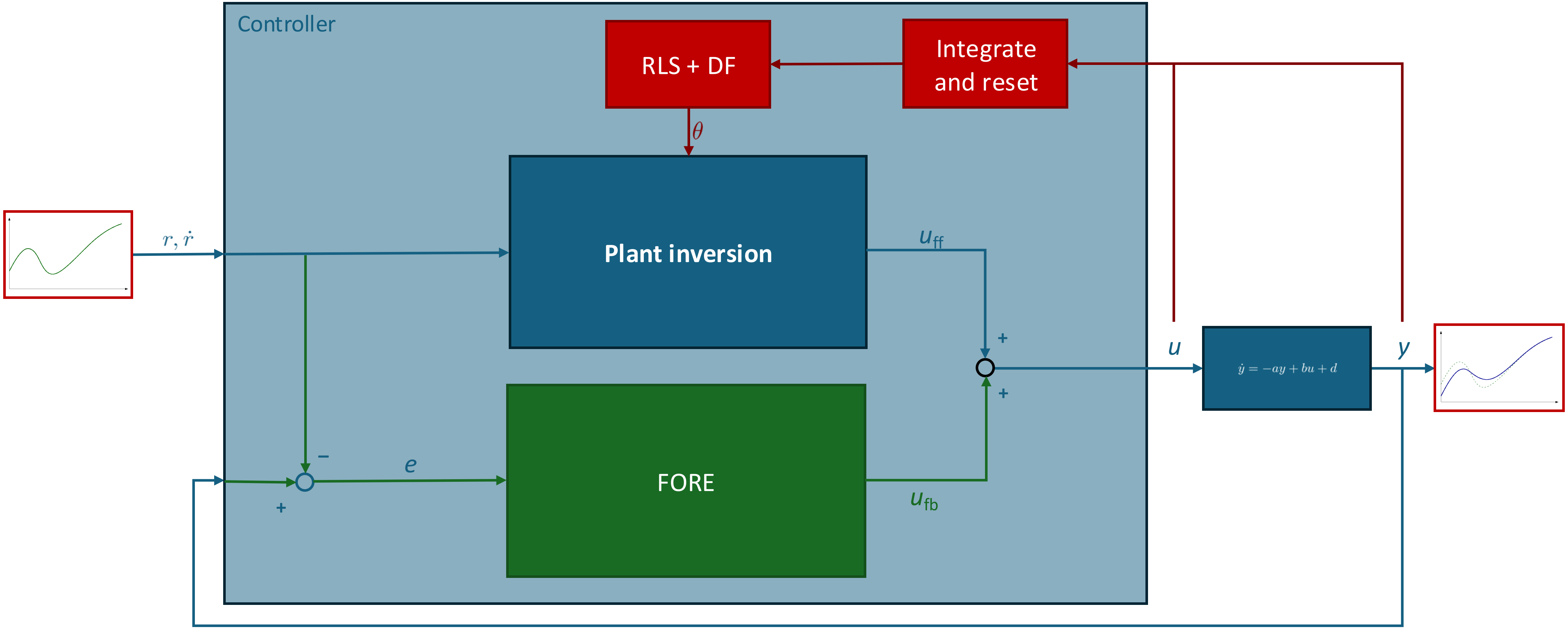
$$X_R := \left\{ R \in \mathbb{R}^{3\times 3} : \alpha_m I \leq R \leq \eta^{-1} I, \ \alpha_m \in (0,1) \right\}$$

We have that

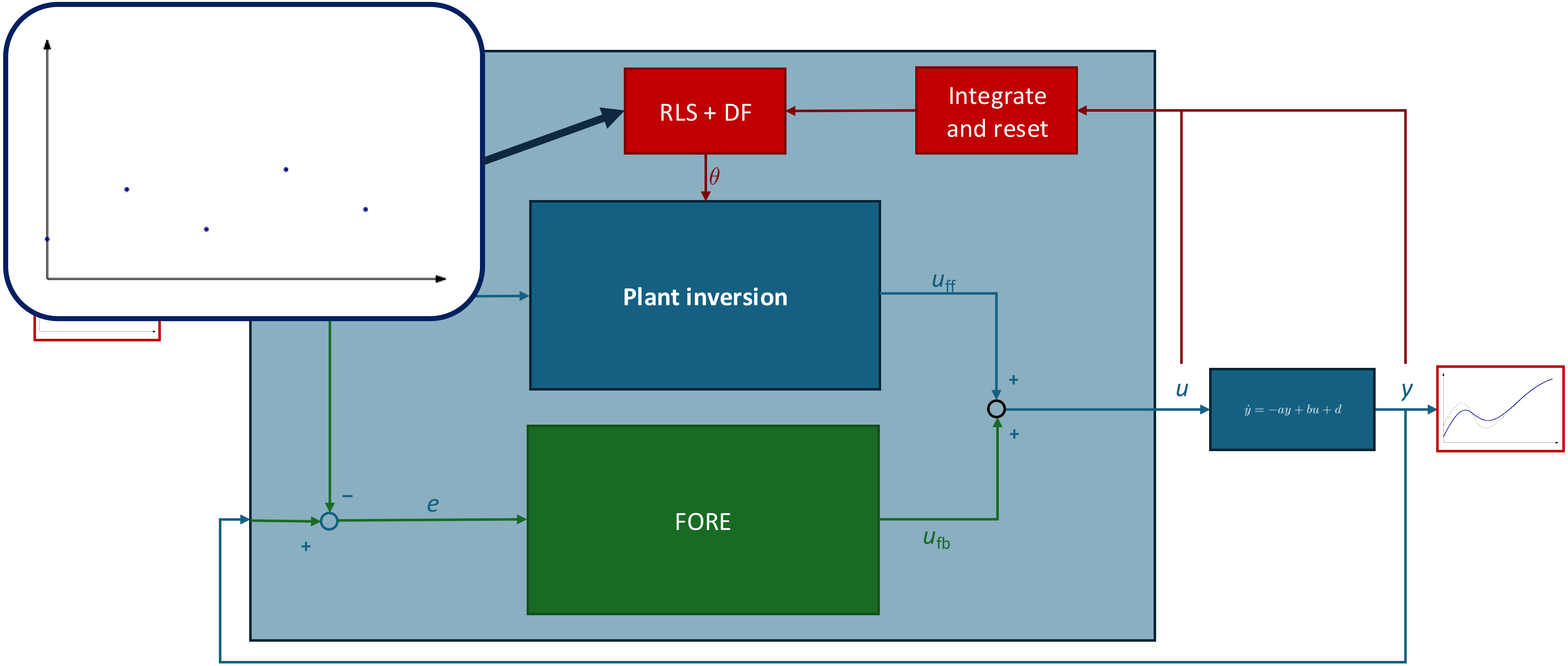
$$R(0,0) \in X_R \implies R(t,j) \in X_R,$$
$$\text{for all } (t,j) \in \mathrm{dom}(R)$$

L. Cao, H. Schwartz. “A directional forgetting algorithm based on the decomposition of the information matrix.” *Automatica, vol. 36, n. 11, pp. 1725-1731. 2000.*

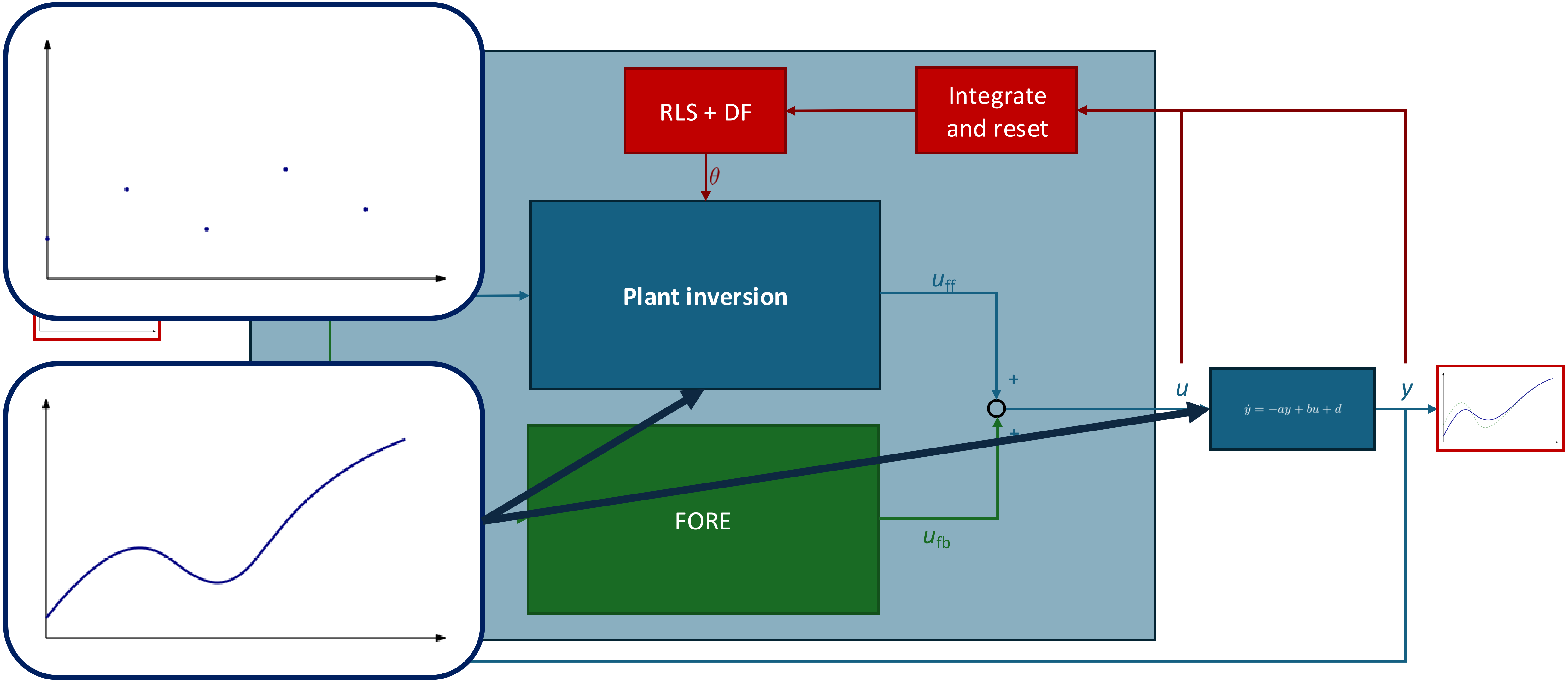
Guarantees for the closed loop



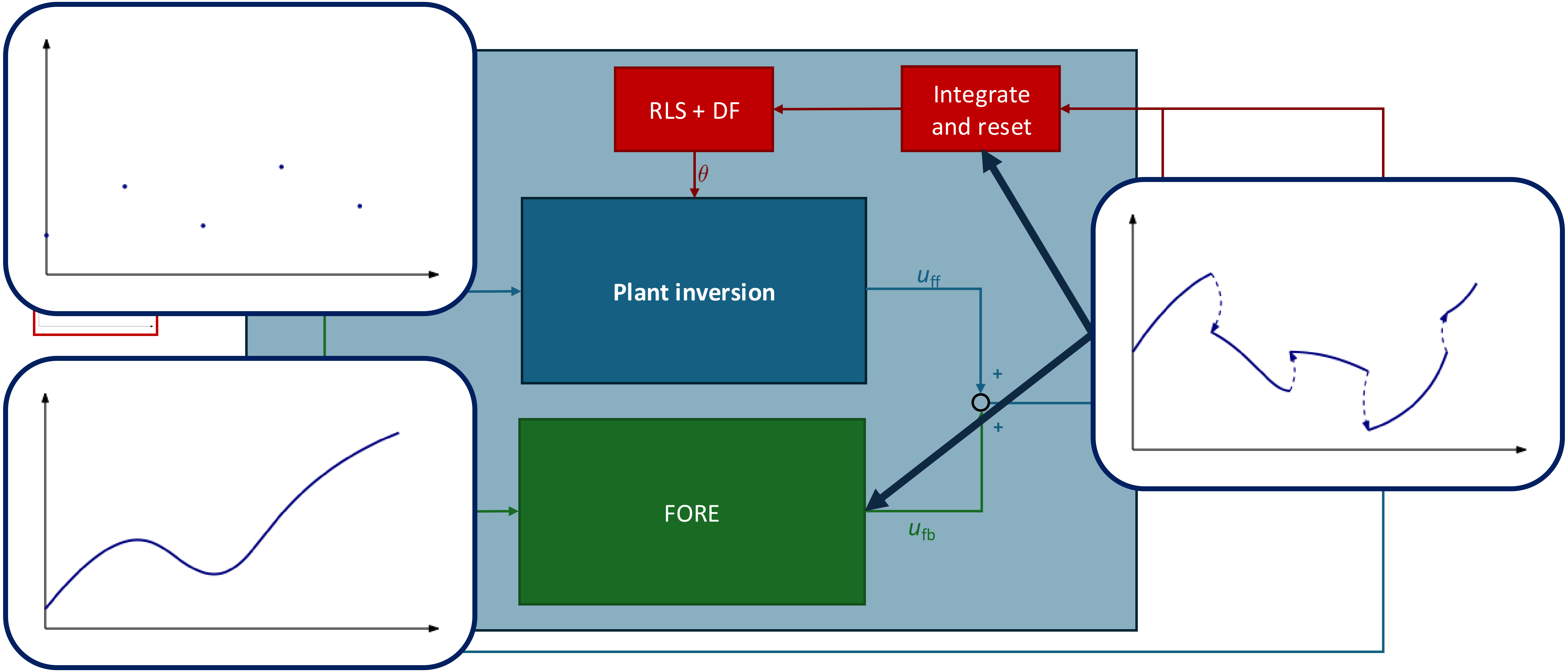
Guarantees for the closed loop



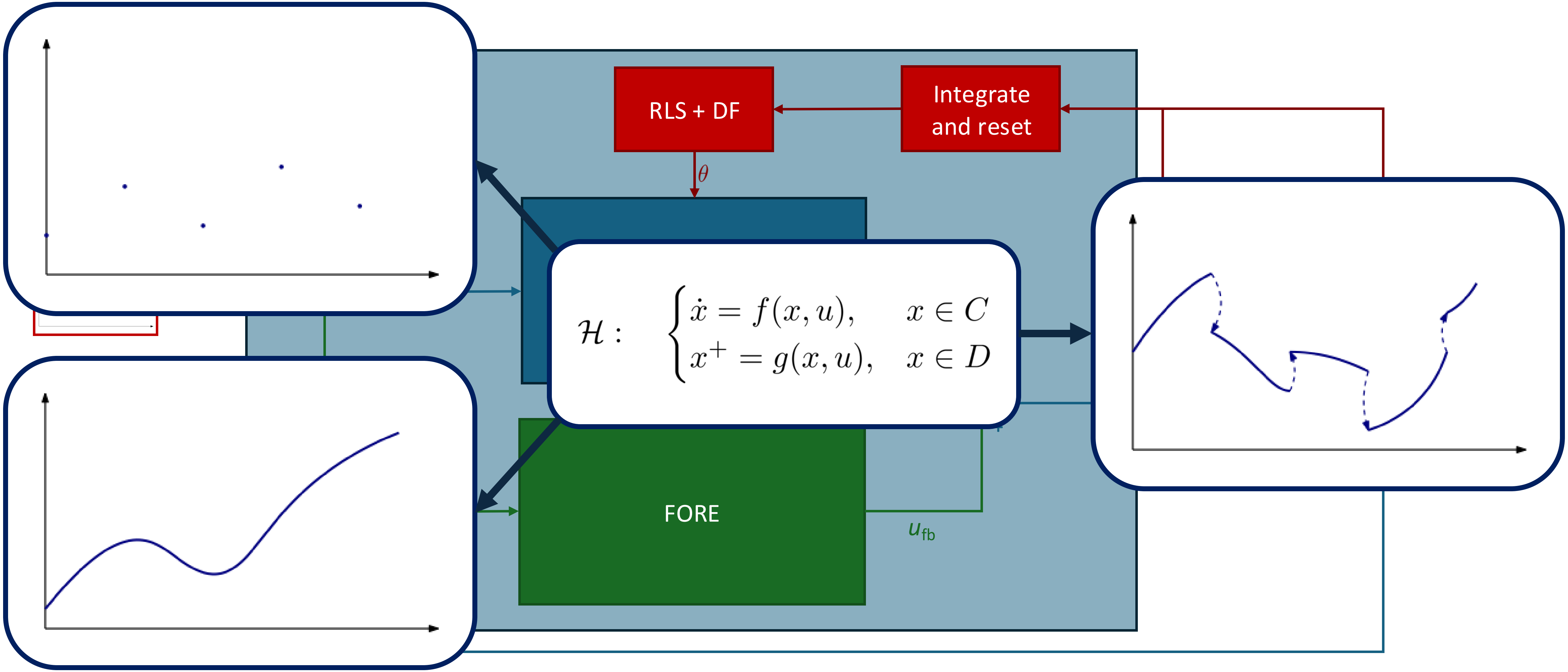
Guarantees for the closed loop



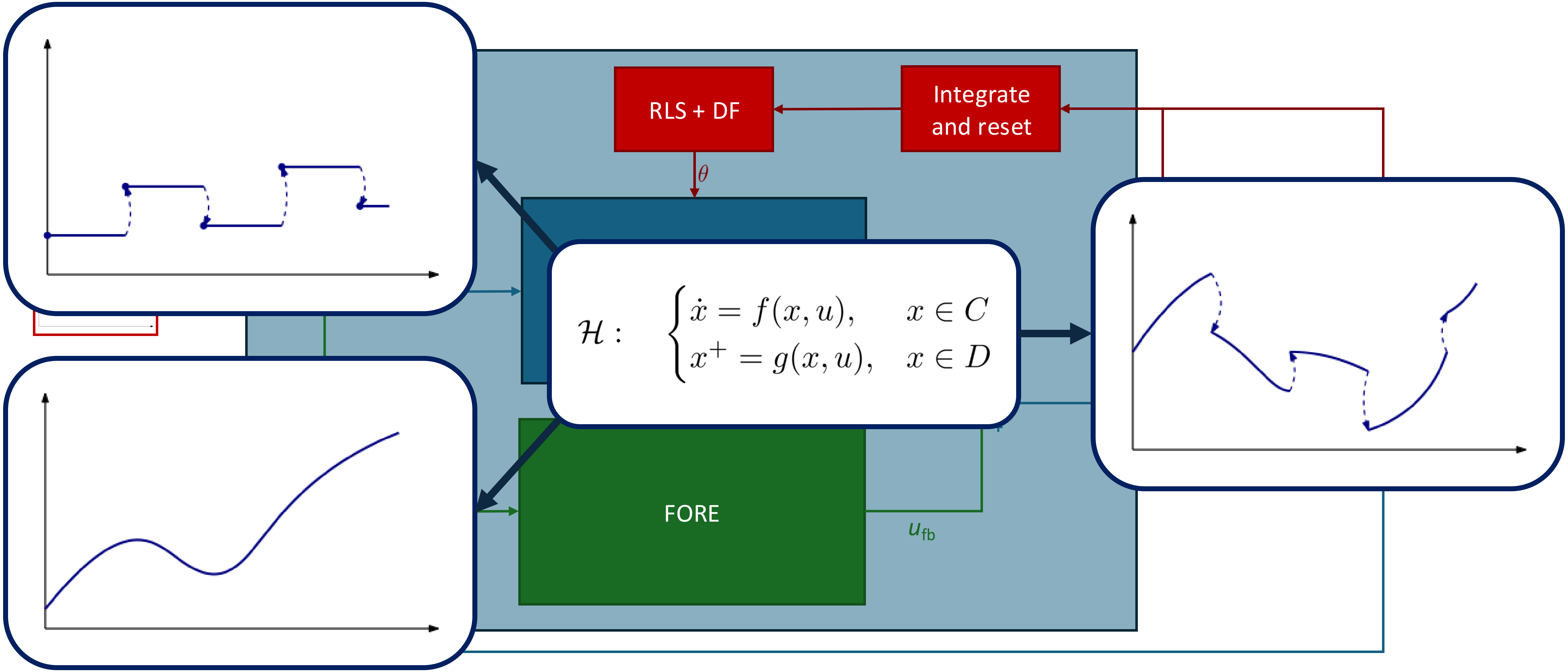
Guarantees for the closed loop



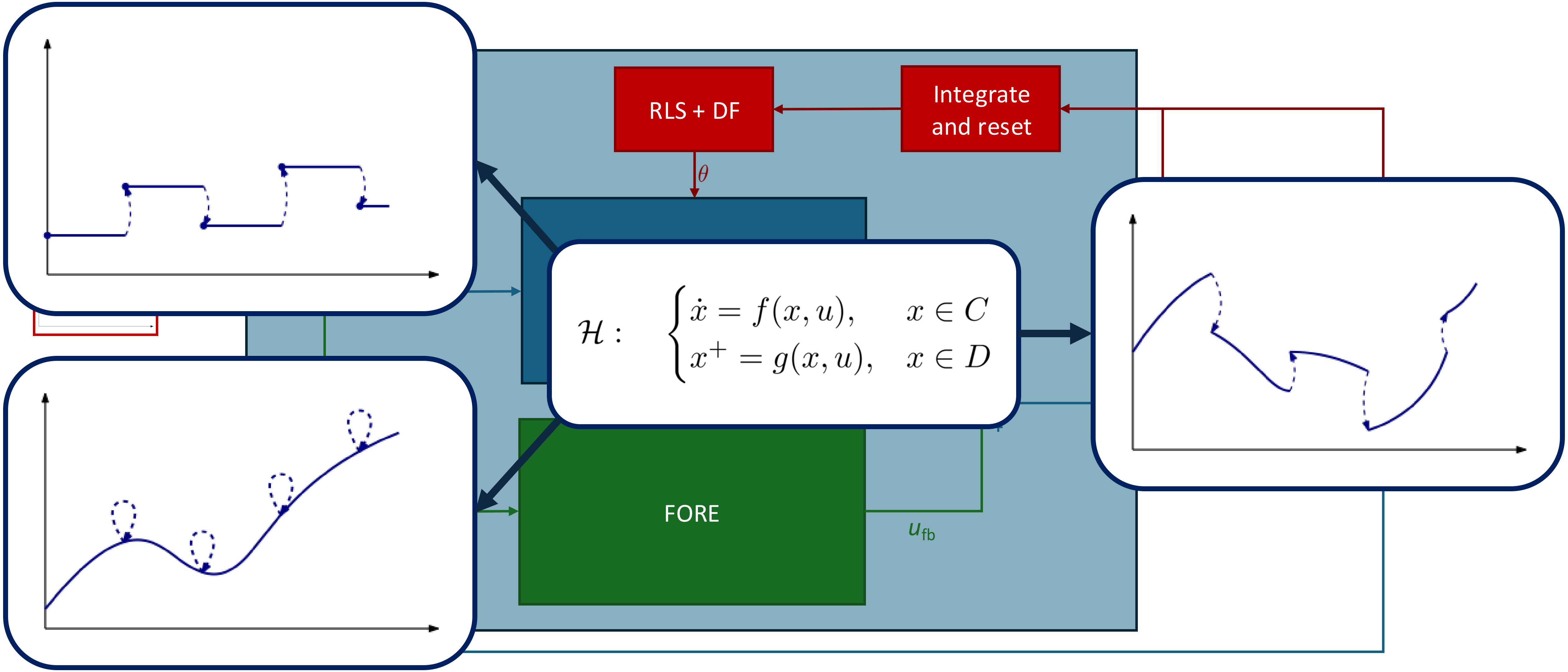
Guarantees for the closed loop



Guarantees for the closed loop



Guarantees for the closed loop



Guarantees for the closed loop

Theorem 2

If adaptations are persistently triggered, the reference tracking **error** e and the parameter estimation error $\tilde{\theta}$ are **bounded**.

Moreover, if $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$, then also $e \to \mathbf{0}$

Guarantees for the closed loop

Theorem 2

If adaptations are persistently triggered, the reference tracking **error** e and the parameter estimation error $\tilde{\theta}$ are **bounded**.

Moreover, if $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$, then also $\boldsymbol{e} \to \mathbf{0}$

Theorem 3

In the **input** is **persistently exciting**, then $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$

Guarantees for the closed loop

Persistency of excitation

Theorem 3

In the **input** is **persistently exciting**, then $\tilde{\theta} \to 0$

Guarantees for the closed loop

Persistency of excitation

$$\dot{y} = ay + bu, \quad y(0) = 1$$

Theorem 3

In the **input** is **persistently exciting**, then $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$

Guarantees for the closed loop

Persistency of excitation

$$\dot{y} = ay + bu, \quad y(0) = 1$$

$$(a, b) = (-2, 1), \qquad (\hat{a}, \hat{b}) = (-4, 2)$$

Theorem 3

In the **input** is **persistently exciting**,
then $\tilde{\boldsymbol{\theta}} \to \mathbf{0}$

Guarantees for the closed loop

Persistency of excitation

$$\dot{y} = ay + bu, \quad y(0) = 1$$

$$(a, b) = (-2, 1), \qquad (\hat{a}, \hat{b}) = (-4, 2)$$

$$r(t) = 1 \;\rightarrow\; \dot{r}(t) = 0$$

Theorem 3

In the **input** is **persistently exciting**, then $\tilde{\boldsymbol{\theta}} \rightarrow \mathbf{0}$

Guarantees for the closed loop

Persistency of excitation

$$\dot{y} = ay + bu, \quad y(0) = 1$$

$$(a, b) = (-2, 1), \qquad (\hat{a}, \hat{b}) = (-4, 2)$$

$$r(t) = 1 \rightarrow \dot{r}(t) = 0$$

$$u_{\text{ff}}(t) = \dot{r}(t) - \frac{\hat{a}}{\hat{b}} r(t) = 2$$

Theorem 3

In the **input** is **persistently exciting**, then $\tilde{\boldsymbol{\theta}} \rightarrow \mathbf{0}$

Guarantees for the closed loop

Persistency of excitation

$$\dot{y} = ay + bu, \quad y(0) = 1$$

$$(a, b) = (-2, 1), \qquad (\hat{a}, \hat{b}) = (-4, 2)$$

$$r(t) = 1 \;\rightarrow\; \dot{r}(t) = 0$$

$$u_{\mathrm{ff}}(t) = \dot{r}(t) - \frac{\hat{a}}{\hat{b}} r(t) = 2$$

$$\dot{y}(0) = ay(0) + bu_{\mathrm{ff}} = 0 \;\rightarrow\; y(t) = 1$$

Theorem 3

In the **input** is **persistently exciting**, then $\tilde{\boldsymbol{\theta}} \rightarrow \mathbf{0}$

Guarantees for the closed loop

Persistency of excitation

$$\dot{y} = ay + bu, \quad y(0) = 1$$

$$(a, b) = (-2, 1), \qquad (\hat{a}, \hat{b}) = (-4, 2)$$

$$\boxed{r(t) = 1} \rightarrow \dot{r}(t) = 0$$

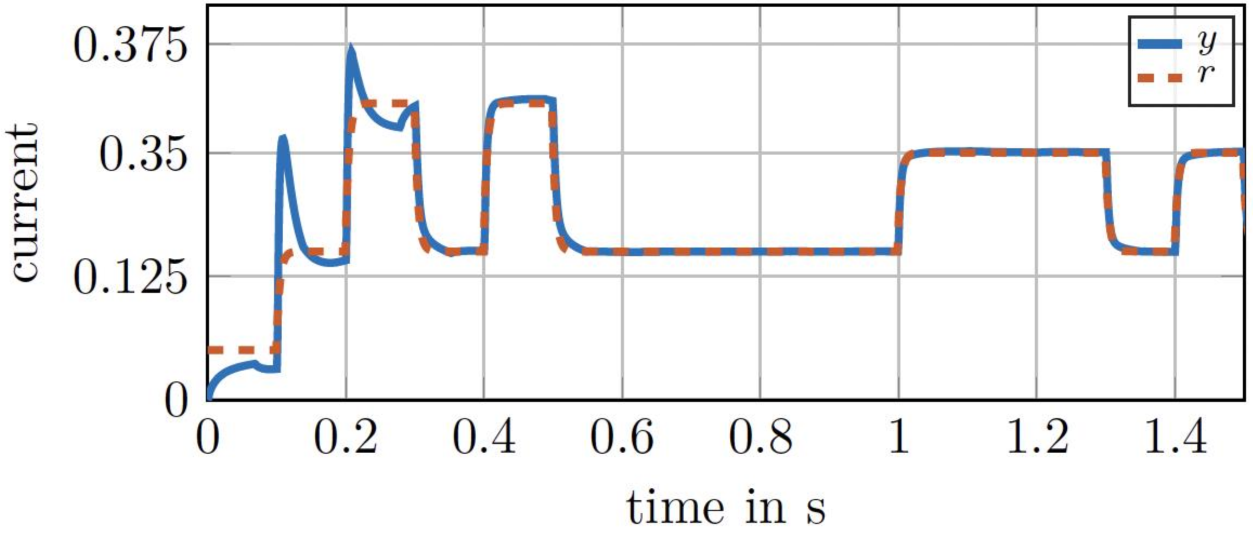
$$u_{\text{ff}}(t) = \dot{r}(t) - \frac{\hat{a}}{\hat{b}} r(t) = 2$$

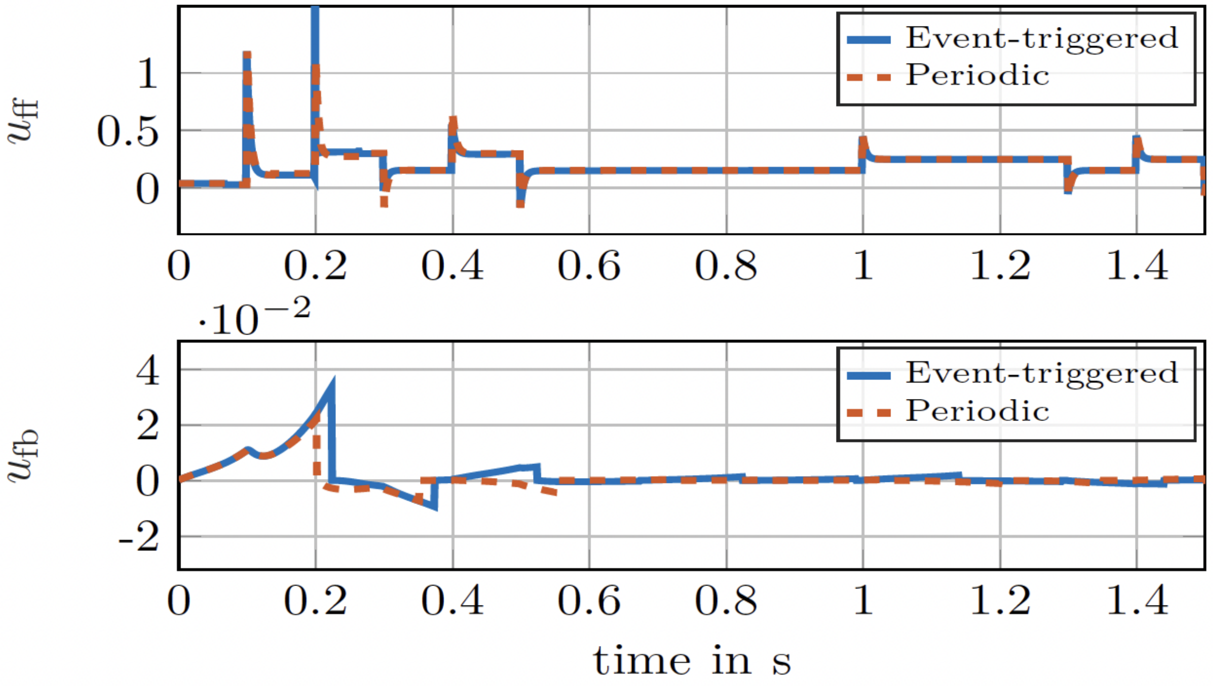
$$\dot{y}(0) = ay(0) + bu_{\text{ff}} = 0 \rightarrow \boxed{y(t) = 1}$$

Theorem 3

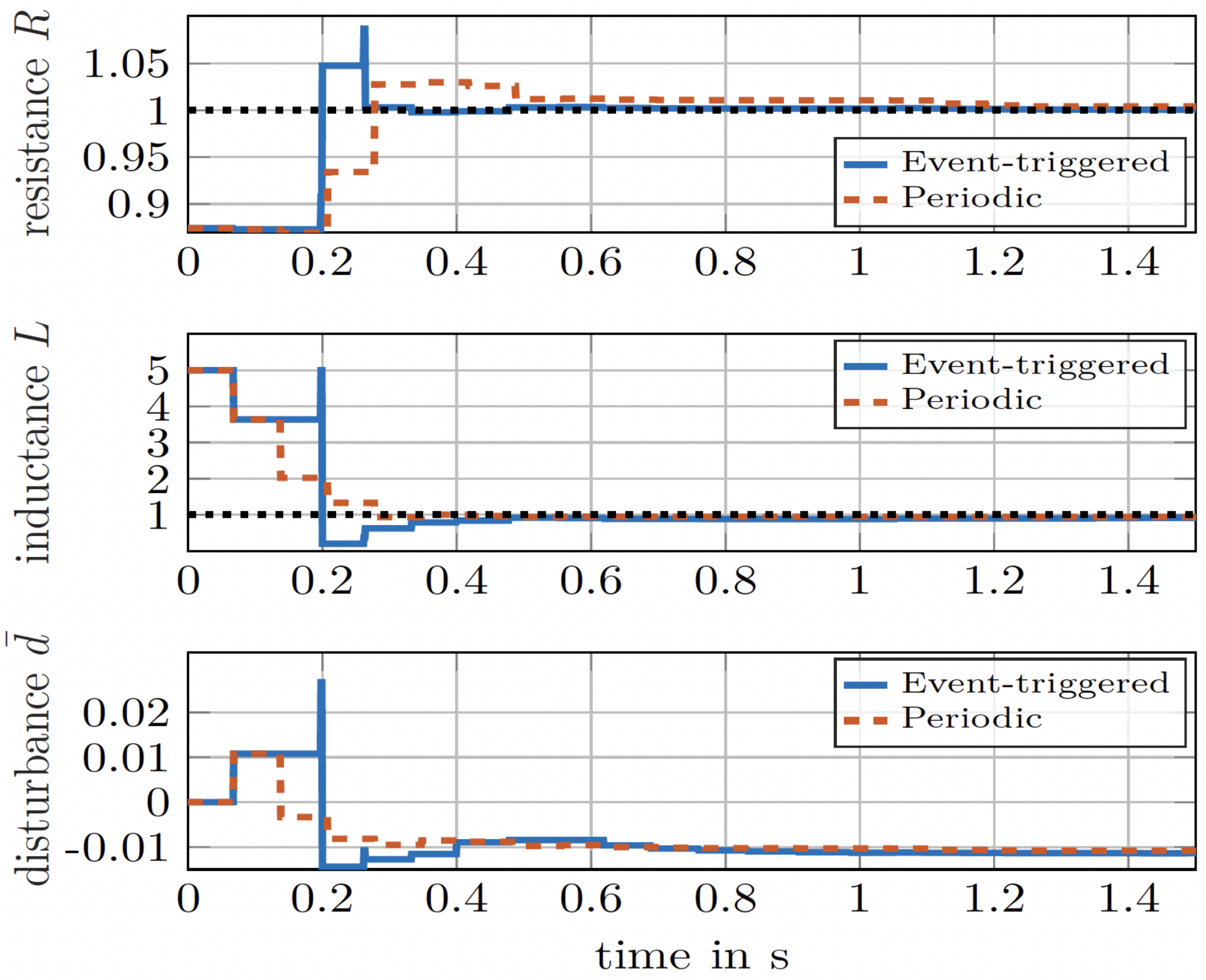
In the **input** is **persistently exciting**, then $\tilde{\boldsymbol{\theta}} \rightarrow \mathbf{0}$

Experimental results (input-output)





Experimental results (parameter estimates)



Conclusions

- We applied the integrate-and-reset paradigm both in the feedforward and tl feedback element of a model-reference adaptive control
- We proved an explicit bound for the information matrix in the recursive least squares algorithm with directional forgetting
- We proved stability of the control loop in case of persistent adaptations
- We proved convergence of the error to zero in case of a persistently exciting input
- Experiments confirmed the effectiveness of the proposed algorithm