Reset-control-based current tracking for a solenoid with unknown parameters

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Solenoid model

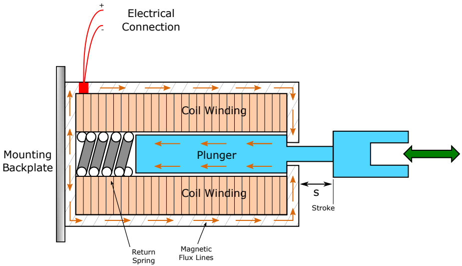
Solenoid-based actuators: first-order dynamics from input voltage to output current

$$\frac{\partial \Psi}{\partial i}\frac{\mathrm{d}i}{\mathrm{d}t} = u(t) - R_L i(t) - \frac{\partial \Psi}{\partial s}\dot{s}$$

$$\downarrow$$

$$\dot{y} = a_p y + b_p u + \bar{d},$$

where $a_p = -R_L/L < 0$, $b_p = 1/L > 0$ and $\bar{d}$ is a constant input bias.



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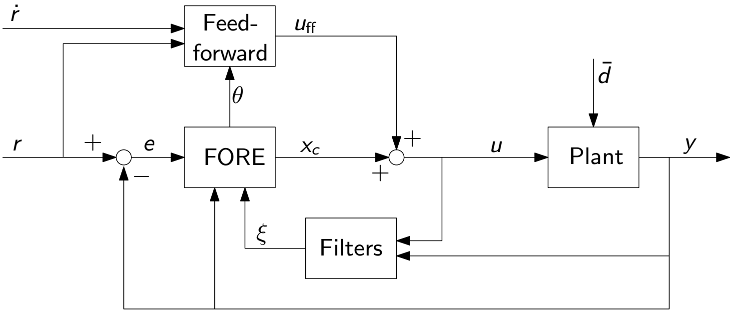
Known control strategies:

- PID, internal mode control, sliding mode control $\rightarrow$ robust but poor performance
- adaptive control (recursive least squares)

Proposed controller



Two main components of the hybrid scheme, similar to *Panni et al. (2014)*, *Cordioli et al. (2015)* and *Cocetti et al. (2019)*:

- first-order reset element (feedback) → ensures tracking with correct parameter estimation
- adaptive controller (feedforward) → stabilizes the parameter estimation error

Information gathered during flows, adaptation happens at jumps.

Feedback stabilizer: First-Order Reset Element

Main idea of the FORE feedback controller:

- Unstable continuous-time evolution $\rightarrow$ fast dynamics
- Suitable reset to zero $\rightarrow$ stability of the overall hybrid dynamics

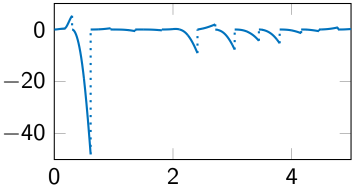


Figure: Example of the FORE state evolution.

FORE hybrid dynamics:

$$\begin{cases} \dot{x}_c = a_c x_c + b_c e, \\ \dot{\tau}_r = 1, \end{cases} \qquad (e, x_c, \tau_r) \in \mathcal{F}_r,$$

$$\begin{cases} x_c^+ = 0, \\ \tau_r^+ = 0, \end{cases} \qquad (e, x_c, \tau_r) \in \mathcal{J}_r,$$

where

$$\mathcal{F}_r := \{(e, x_c, \tau_r) : \varepsilon e^2 + 2 e x_c \geq 0 \text{ or } \tau_r \leq \rho_{\min}\},$$

$$\mathcal{J}_r := \overline{\mathbb{R}^2 \setminus \mathcal{F}_r}.$$

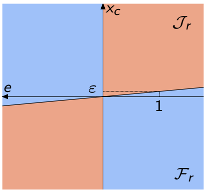
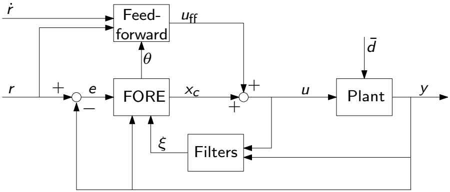


Figure: Typical shape of the flow/jump sets of a FORE.

Feedback stabilizer: tracking error GES and ISS

Tracking error flow dynamics:

$$
\begin{aligned}
\dot{e} &= \dot{r} - \dot{y} \\
&= \dot{r} - a_p y - b_p(x_c + u_{\text{ff}}) - \bar{d} \\
&= a_p e - b_p x_c - (b_p u_{\text{ff}} - \dot{r} + a_p r + \bar{d}) \\
&= a_p e - b_p x_c - d,
\end{aligned}
$$



where $d := b_p u_{\text{ff}} - \dot{r} + a_p r + \bar{d}$ should be canceled by the feedforward term.

Assumption 1 (Physical parameters)

The plant is such that $a_p < 0$ and $b_p > 0$.

Proposition 1

Under Assumption 1, for any choice of $(a_c, b_c) \in \mathbb{R}_{>0} \times \mathbb{R}_{>0}$, the point $(x_c, e) = (0, 0)$ is GES for the reset feedback, and it is finite gain ISS from d to (x_c, e).

Feedforward adaptation: parameter estimation error

Ideal feedforward control is such that $b_p u_{\mathrm{ff}}^* - \dot{r} + a_p r + \bar{d} = 0$, thus

$$u_{\mathrm{ff}}^* = \frac{1}{b_p}\left(\dot{r} - a_p r - \bar{d}\right)$$

$$= \begin{bmatrix} 1 & r & \dot{r} \end{bmatrix} \frac{1}{b_p} \begin{bmatrix} -\bar{d} \\ -a_p \\ 1 \end{bmatrix}$$

$$= \chi^\top \theta^*$$

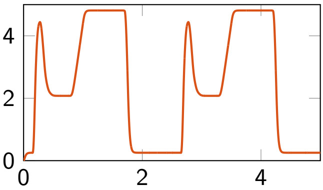


Figure: Shape of the reference input used in simulations.

We introduce the parameter estimate θ, so that $u_{\mathrm{ff}} := \chi^\top \theta \to u_{\mathrm{ff}}^*$ when $\theta \to \theta^*$.

Assumption 2

The reference input $t \mapsto r(t)$ is differentiable; both $r(t)$ and $\dot{r}(t)$ are available signals and uniformly bounded.

Feedforward adaptation: (hybrid) recursive least squares

Adaptation performed at jumps → additional states $\xi = \begin{bmatrix}\xi_y & \xi_u & \xi_s\end{bmatrix}^\top$ to gather information during flows

$$\begin{cases} \dot{\theta} = 0 \\ \dot{\xi} = \begin{bmatrix} y & u & 0 \end{bmatrix}^\top \\ \dot{\tau}_a = 1 \\ \dot{R} = 0 \end{cases} \qquad \begin{cases} \theta^+ = g_\theta\left(\theta, R, \xi, \tau_a, y\right) \\ \xi^+ = \begin{bmatrix} 0 & 0 & y \end{bmatrix}^\top \\ \tau_a^+ = 0 \\ R^+ = g_R\left(R, \xi, \tau_a, y\right) \end{cases}$$

$$\tau_a \in [0, \rho_{\max}] \qquad \tau_a \in [\rho_{\min}, \rho_{\max}]$$

Recursive Least Squares (RLS) with Directional Forgetting (DF) → information on the evolution collected in a vector $\varphi := \begin{bmatrix}\tau_a & \xi_y & y - \xi_s\end{bmatrix}$, which defines the updates

$$g_\theta\left(\theta, R, \xi, \tau_a, y\right) := \theta - \left(R^+\right)^{-1} \frac{\varphi}{\varphi^\top \varphi} \left(\varphi^\top \theta - \xi_u\right)$$

$\Phi(\varphi)$, projection matrix induced by the available direction φ

$$g_R\left(R, \xi, \tau_a, y\right) := R - \eta \frac{R\varphi\varphi^\top R}{\varphi^\top R \varphi} + \frac{\varphi\varphi^\top}{\varphi^\top \varphi}$$

R is used as the weight for the update

ΔR, forgetting of old data, weighted by η

Overall system: complete error dynamics

Using the following state

$$x := \begin{bmatrix} x_{\text{fb}} & \tilde{\theta} & \tilde{\varphi} & \tilde{\xi}_a & R & \tau_r & \tau_a \end{bmatrix}^\top$$

$$x_{\text{fb}} := [-e \;\; x_c]^\top$$

Scalar error s.t. $\dot{\tilde{\xi}}_a = 0$ and $\tilde{\xi}_a^+ = 0$

$$\tilde{\theta} := \theta - \theta^* \qquad \tilde{\varphi} := \varphi - \chi_{\text{i}} := \varphi - \int_{t_j}^{t} \chi(s,j)ds$$

we obtain the following error dynamics for the complete system

$$\begin{cases} \dot{x}_{\text{fb}} = A_F x_{\text{fb}} + B b_p \chi^\top \tilde{\theta} \\ \dot{\tilde{\theta}} = 0 \\ \dot{\tilde{\varphi}} = C x_{\text{fb}} + F b_p \chi^\top \tilde{\theta} \\ \dot{\tilde{\xi}}_a = 0, \quad \dot{R} = 0, \quad \dot{\tau}_r = 1, \quad \dot{\tau}_a = 1 \end{cases} \qquad x \in \mathcal{C} := \left\{ x \in X : x_{\text{fb}}^\top M x_{\text{fb}} \geq 0 \text{ or } \tau_r \leq \rho_{\min} \right\}$$

$$\begin{cases} x_{\text{fb}}^+ = x_{\text{fb}} \\ \tilde{\theta}^+ = \left(I - \left(R^+\right)^{-1} \Phi\left(\tilde{\varphi} + \chi_{\text{i}}\right) \right) \tilde{\theta} - \left(R^+\right)^{-1} \frac{(\tilde{\varphi}+\chi_{\text{i}})}{(\tilde{\varphi}+\chi_{\text{i}})^\top (\tilde{\varphi}+\chi_{\text{i}})} \tilde{\xi}_a \\ \tilde{\varphi}^+ = 0, \quad \tilde{\xi}_a^+ = 0 \\ R^+ = R - \eta \Delta R\left(\tilde{\varphi} + \chi_{\text{i}}\right) + \Phi\left(\tilde{\varphi} + \chi_{\text{i}}\right) \\ \tau_r^+ = \tau_r, \quad \tau_a^+ = 0 \end{cases} \qquad x \in \mathcal{D}_a := \{ x \in X : \tau_a \geq \rho_{\min} \}$$

$$\begin{cases} x_{\text{fb}}^+ = A_J x_{\text{fb}} \\ \tilde{\theta}^+ = \tilde{\theta}^+ \\ \tilde{\varphi}^+ = \tilde{\varphi}, \quad \tilde{\xi}_a^+ = \tilde{\xi}_a \\ R^+ = R \\ \tau_r^+ = 0, \quad \tau_a^+ = \tau_a \end{cases} \qquad x \in \mathcal{D}_r := \left\{ x \in X : x_{\text{fb}}^\top M x_{\text{fb}} \leq 0 \text{ and } \tau_r \geq \rho_{\min} \right\}$$

Overall system: main stability result

We need the following conjecture, suggested by repeated simulations

Conjecture 1

Given $\alpha_m \in (0,1)$ and the compact set $X_R := \left\{ R \in \mathbb{R}^3 : \alpha_m I \leq R \leq \frac{I}{\eta} \right\}$, we have

$$R(0,0) \in X_R \implies R(t,j) \in X_R, \quad \forall (t,j) \in \operatorname{dom} R.$$

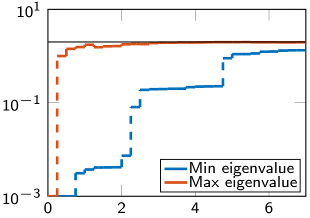


Figure: Evolution of minimum and maximum eigenvalue of R in one of the simulations.

In view of this, we can state the following theorem

Theorem 1

The attractor

$$\mathcal{A} := \left\{ x \in X : \left(x_{\text{fb}}, \tilde{\theta}, \tilde{\varphi}, \tilde{\xi}_a \right) = 0 \right\}$$

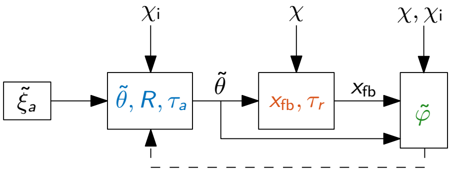
is UGS for the error dynamics. Moreover, if $\tilde{\theta} \to 0$ then $x_{\text{fb}} \to 0$.

where $X := \mathbb{R}^9 \times X_R \times \mathbb{R} \times [0, \rho_{\max}]$.

Overall system: sketch of the main proof

The proof relies on the pseudo-cascade structure of the overall error dynamics:

- $\tilde{\xi}_a(t,j) = 0$ for all $t \geq t_1, j \geq 1$, it poses no problem
- the adaptation subsystem is UGS, independently of $\chi_\mathrm{i}, \tilde{\varphi}$ $\big($proved with the Lyapunov function $V\big(\tilde{\theta}, R\big) := \tilde{\theta}^\top R\tilde{\theta}$ $\big)$



- the feedback subsystem is ISS from input $\tilde{\theta}$ to x_fb
- the information gathering subsystem integrates bounded signals over a bounded time

Combining these facts $\rightarrow$ exponential ISS bounds uniform in χ, χ_i for the second and third subsystem $\rightarrow$ both statements are proven.

Simulation results

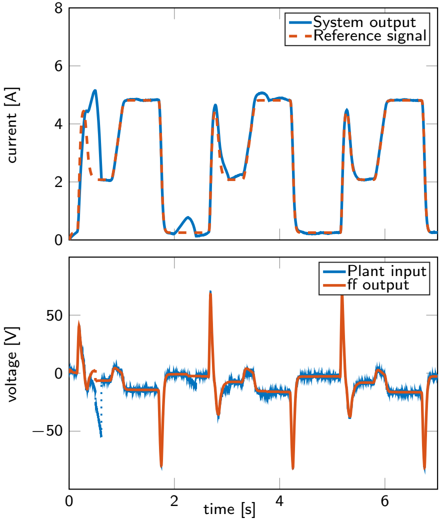


Figure: Top: plant output (blue), current reference (orange). Bottom: plant input (blue), feedforward input (orange).

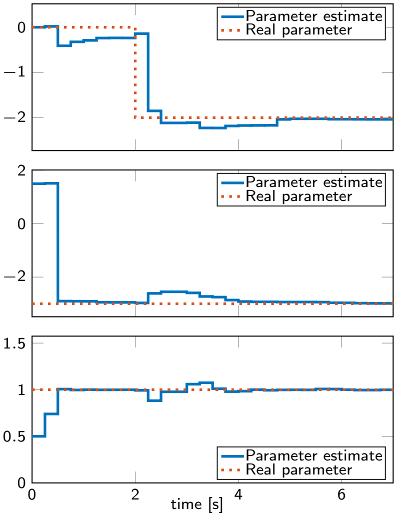


Figure: Parameter estimates θ (blue) and real values θ^* (dotted orange).

Conclusions and future work

- We proposed an hybrid adaptive control scheme for a solenoid with unknown parameters, to track a reference current
- The controller ensures stability of the parameter estimation error, and convergence of the tracking error when the parameter estimation error converges
- Simulations have confirmed the theoretical results, suggesting that a persistently exciting signal could ensure convergence of the error coordinates to the origin
- Future developments will aim at proving convergence under PE conditions, propose an optimal choice of the adaptation instants and provide an experimental validation of the results