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Distributed hybrid observer with prescribed convergence rate for a linear plant using multi-hop decomposition

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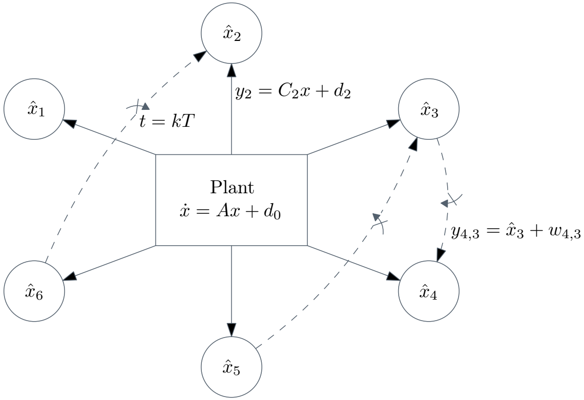
System description & problem formulation



Available information:

- continuous time $\to y_i(t)$
- sampled-data $\to y_{i,j}(kT)$

External disturbances:

- process noise d_0
- measurement noise d_i
- transmission noise $w_{i,j}$

Problem 1

Design a distributed observer whose solutions enjoy a **finite gain exponential ISS** property from the disturbances to the estimation errors, with **prescribed convergence rate $\alpha > 0$**.

Multi-hop decomposition

Iteratively define $\boldsymbol{C_{i,\varrho}}$**: output matrix** for agent i **at "hop"** $\boldsymbol{\varrho}$

$$C_{i,\varrho} := \begin{cases} C_i, & \text{if } \varrho = 0, \\ \begin{bmatrix} C_{i,\varrho-1} \\ \operatorname*{col}_{j \in \mathcal{N}_i} \left(C_{j,\varrho-1} \right) \end{bmatrix}, & \text{if } \varrho \geq 1, \end{cases}$$

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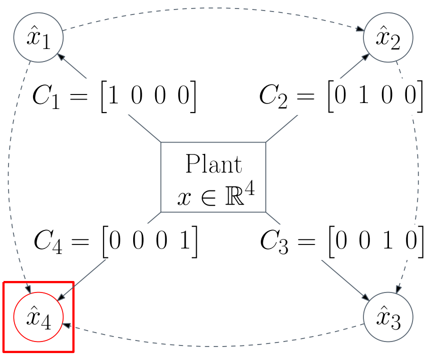

$C_{4,0} = C_4$

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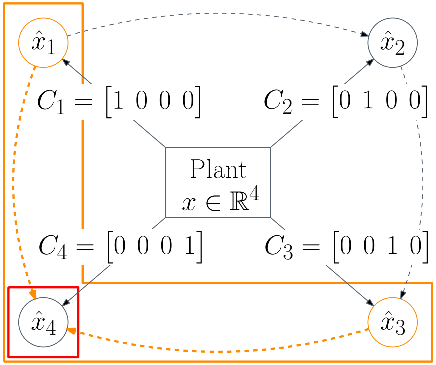

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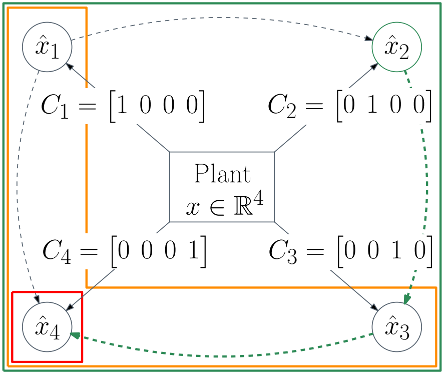

$$C_{4,0} = C_4 \qquad C_{4,1} = \begin{bmatrix} C_4 \\ \hline C_1 \\ C_3 \end{bmatrix} \qquad C_{4,2} = \begin{bmatrix} C_4 \\ C_1 \\ C_3 \\ \hline C_2 \end{bmatrix}$$

Multi-hop decomposition (cont'd)

Iteratively define $\boldsymbol{W_{i,\varrho}}$: **"innovation" matrix** for hop ϱ

Pair $(C_{i,\varrho}, A)$

$\overline{\mathcal{O}}_{i,\varrho}$ unobservable subspace

$\mathcal{O}_{i,\varrho}$ observable subspace

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$$\begin{cases} \mathrm{Im}\left(W_{i,0}\right) = \mathcal{O}_{i,0} \\ \boldsymbol{\mathrm{Im}\left(W_{i,\varrho}\right) = \mathcal{O}_{i,\varrho} \cap \overline{\mathcal{O}}_{i,\varrho-1}}, \quad \varrho = 1, \dots, \ell_i \\ \mathrm{Im}\left(W_{i,\ell_i+1}\right) = \overline{\mathcal{O}}_{i,\ell_i} \end{cases}$$

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How do we choose ℓ_i?

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How do we choose ℓ_i?

Assumption

The system is **collectively α-detectable**. Namely, for each i there exists ℓ_i such that the pair (C_{i,ℓ_i}, A) is α-detectable.

Collective α-detectability $\not\Longrightarrow$ connected graph

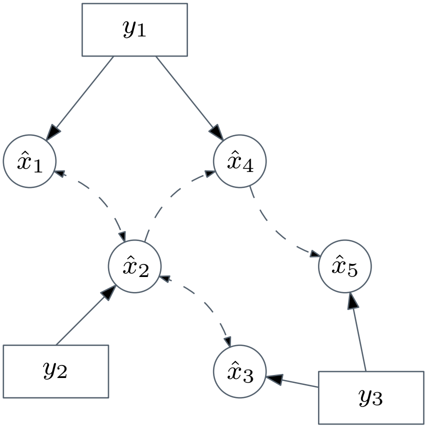

Figure [del Nozal et al., *Automatica*, 2019]: If pair $\left(\begin{bmatrix} C_1^\top & C_2^\top & C_3^\top \end{bmatrix}^\top, A\right)$ is α-detectable, collective α-detectability holds, but the graph is not (strongly) connected.

Distributed sampled-data observer

$$\dot{\hat{x}}_i = A\hat{x}_i + W_{i,0}L_i(y_i - C\hat{x}_i) \qquad t \notin \{kT, k \in \mathbb{N}\}$$

$$\hat{x}_i^+ = \hat{x}_i + \sum_{\varrho=1}^{\ell_i} \sum_{j \in \mathcal{N}_i} W_{i,\varrho} N_{i,j,\varrho} W_{j,\varrho-1}^\top (y_{i,j} - \hat{x}_i) \qquad t \in \{kT, k \in \mathbb{N}\}$$

Distributed sampled-data observer

Copy of the observed plant

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Consensus correction term (sampled data)

Cascade error dynamics

Define the **"transformed" error coordinate $\boldsymbol{\varepsilon_{i,\varrho}}$**

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Error dynamics

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\tau} \end{bmatrix} = \begin{bmatrix} A_\varepsilon \varepsilon + Rd \\ 1 \end{bmatrix}, \qquad (\varepsilon, \tau) \in \mathcal{C} := X,$$

$$\begin{bmatrix} \varepsilon^+ \\ \tau^+ \end{bmatrix} = \begin{bmatrix} J_\varepsilon \varepsilon + Sw \\ 0 \end{bmatrix}, \qquad (\varepsilon, \tau) \in \mathcal{D} := \mathbb{R}^{n_\varepsilon} \times \{T\},$$

$$X := \mathbb{R}^{n_\varepsilon} \times [0, T]$$

$$A_\varepsilon = \begin{bmatrix} A_{\overline{\ell}} & & \star \\ & \ddots & \\ 0 & & A_0 \end{bmatrix}$$

$$J_\varepsilon = \begin{bmatrix} \Delta_{\overline{\ell}} & & \star \\ & \ddots & \\ 0 & & \Delta_0 \end{bmatrix}$$

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$$\boxed{\Delta_\rho := \operatorname{diag}\left(I - \left(\sum_{j \in \mathcal{N}_i} N_{i,j,\varrho} W_{j,\varrho-1}^\top\right) W_{i,\varrho}\right)}$$

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Property 1

The **local observer gains $\boldsymbol{L_i}$** are chosen so that, for each i,

$$\overline{A}_{i,0} := \left(W_{i,0}^\top A - L_i C_i\right) W_{i,0}$$

has spectral abscissa $\overline{\alpha} \leq -\alpha$.

The **consensus gains $\boldsymbol{N_{i,j,\varrho}}$** are chosen so that, for each i and each $\varrho \in \{1, \dots, \ell_i\}$,

$$\overline{A}_{i,\varrho} := \mathrm{e}^{(W_{i,\varrho}^\top A W_{i,\varrho})T} \left(I - \Big(\sum_{j \in \mathcal{N}_i} N_{i,j,\varrho} W_{j,\varrho-1}^\top\Big) W_{i,\varrho}\right)$$

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Observer tuning

Property 1

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In practice, good to have

$$\left.\begin{array}{l} \blacktriangleright\ \overline{\alpha} \ll -\alpha \\ \blacktriangleright\ \overline{\beta} \approx \mathrm{e}^{-\alpha T} \end{array}\right\} \implies \text{local estimation errors converge faster than consensus errors}$$

Main results

Theorem 1 (Input-to-State Stability)

If the system is collectively α-detectable and the gains satisfy Property 1, then the proposed observer solves Problem 1.

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If the system is collectively α-detectable, there always exist gains satisfying Property 1.

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Theorem 2 (Feasibility)

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Remark

From Theorem 1+2, collective α-detectability is **sufficient** to obtain ISS.

Sketch of proof of Theorem 1

Consider the attractor

$$\mathcal{A} := \{(\varepsilon, \tau) \in X : \varepsilon = 0\}$$

and the Lyapunov function

$$V(\varepsilon, \tau) := \sqrt{\varepsilon^\top \mathrm{e}^{A_\varepsilon^\top (T-\tau)} P \mathrm{e}^{A_\varepsilon (T-\tau)} \varepsilon}$$

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Lemma

There exist $c_1, c_2, M, c_C, c_D \in \mathbb{R}_{>0}$ and $\eta \leq \mathrm{e}^{-\alpha T}$ such that

- $c_1|\varepsilon| \leq V(\varepsilon, \tau) \leq c_2|\varepsilon|, \quad$ for all $(\varepsilon, \tau) \in X$,
- $|\nabla_\varepsilon V(\varepsilon, \tau)| \leq M, \quad$ for all $(\varepsilon, \tau) \in X \setminus \mathcal{A}$,
- $\dot{V} \leq c_C|d|, \quad$ for all $(\varepsilon, \tau) \in \mathcal{C} \setminus \mathcal{A}$,
- $V^+ \leq \eta V + c_D|w|, \quad$ for all $(\varepsilon, \tau) \in \mathcal{D} \setminus \mathcal{A}$,

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- $V^+ \leq \eta V + c_D|w|$, for all $(\varepsilon, \tau) \in \mathcal{D} \setminus \mathcal{A}$,

Using this, the integration of V along any solution gives

$$\boxed{|\mathrm{col}\,(e_i(t,k))\,| \leq \kappa \mathrm{e}^{-\alpha t} |\mathrm{col}\,(e_i(0,0))\,| + \gamma_C \|d\|_\infty + \gamma_D \|w\|_\infty}$$

Sketch of proof of Theorem 2

Local gains

Place eigenvalues of

$$\overline{A}_{i,0} = W_{i,0}^{\top} A W_{i,0} - \boldsymbol{L_i} C_i W_{i,0}$$

Sketch of proof of Theorem 2

Local gains

Place eigenvalues of

$\overline{A}_{i,0} = W_{i,0}^\top A W_{i,0} - \boldsymbol{L_i} C_i W_{i,0}$

$\Downarrow$

Trivially feasible

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Trivially feasible

Consensus gains

Place eigenvalues of

$$\begin{aligned}\overline{A}_{i,\varrho} &= \mathrm{e}^{(W_{i,\varrho}^{\top} A W_{i,\varrho})T}\Big(I - \sum_{j\in\mathcal{N}_i} \boldsymbol{N_{i,j,\varrho}} W_{j,\varrho-1}^{\top} W_{i,\varrho}\Big)\\ &= E_{i,\rho} - \left(E_{i,\rho}\mathrm{row}\left(\boldsymbol{N_{i,j,\rho}}\right)\right)\mathrm{col}\left(W_{j,\varrho-1}^{\top} W_{i,\varrho}\right)\\ &= E_{i,\rho} - \overline{\boldsymbol{N}}_{\boldsymbol{i,\rho}} \overline{C}_{i,\rho}\end{aligned}$$

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⇓

Feasible if $\begin{bmatrix} E_{i,\rho} - \lambda I \\ \overline{C}_{i,\rho} \end{bmatrix}$ is full rank

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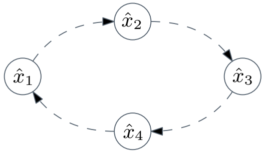
Feasible if $\begin{bmatrix} E_{i,\rho} - \lambda I \\ \overline{C}_{i,\rho} \end{bmatrix}$ is full rank

From multi-hop decomposition, $\overline{C}_{i,\rho}$ always full column rank

Simulation results



$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix},$$

$$C_i = e_i$$

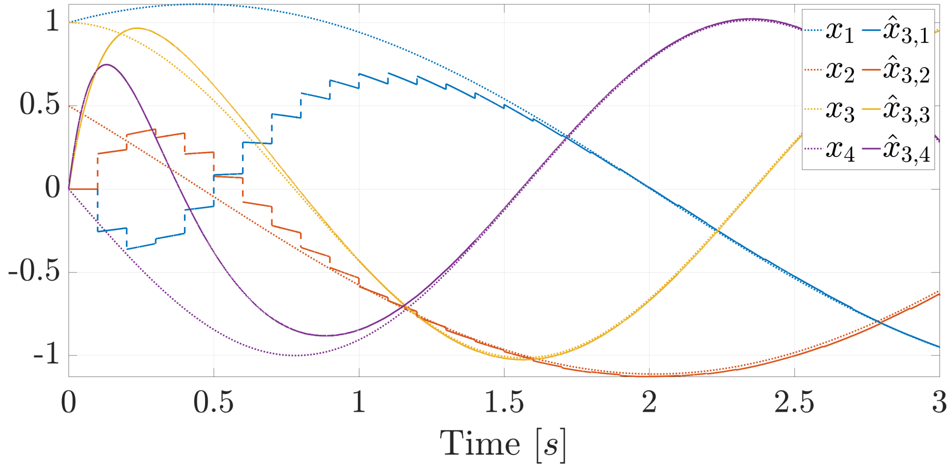
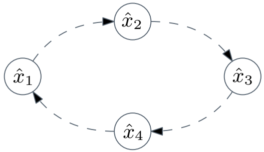


Figure: Evolution of the estimates of agent 3.

Simulation results



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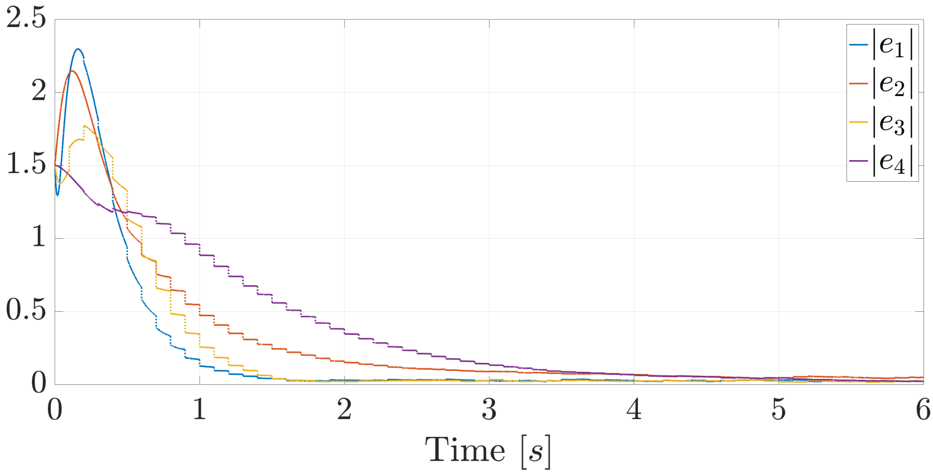

Figure: Evolution of the tracking error norm for each agent.

Conclusions

- We considered a continuous-time plant and a sensor network
  - local measurements are available continuously
  - exchange of information is sampled-data
  - process noise, measurement noise and transmission noise were considered
- We proposed a hybrid observer architecture based on the multi-hop decomposition
- We proved that collective α-detectability is necessary and sufficient to obtain prescribed α-exponential finite gain ISS from the disturbances to the estimation error
- Simulations illustrated the effectiveness of the proposed observer
- Future works might include the analysis of
  - asynchronous communication
  - time-varying sampling periods
  - energy minimization at a fog layer level