

# *Particle Detectors*

*Lecture 11*

*12/04/17*

a.a. 2016-2017

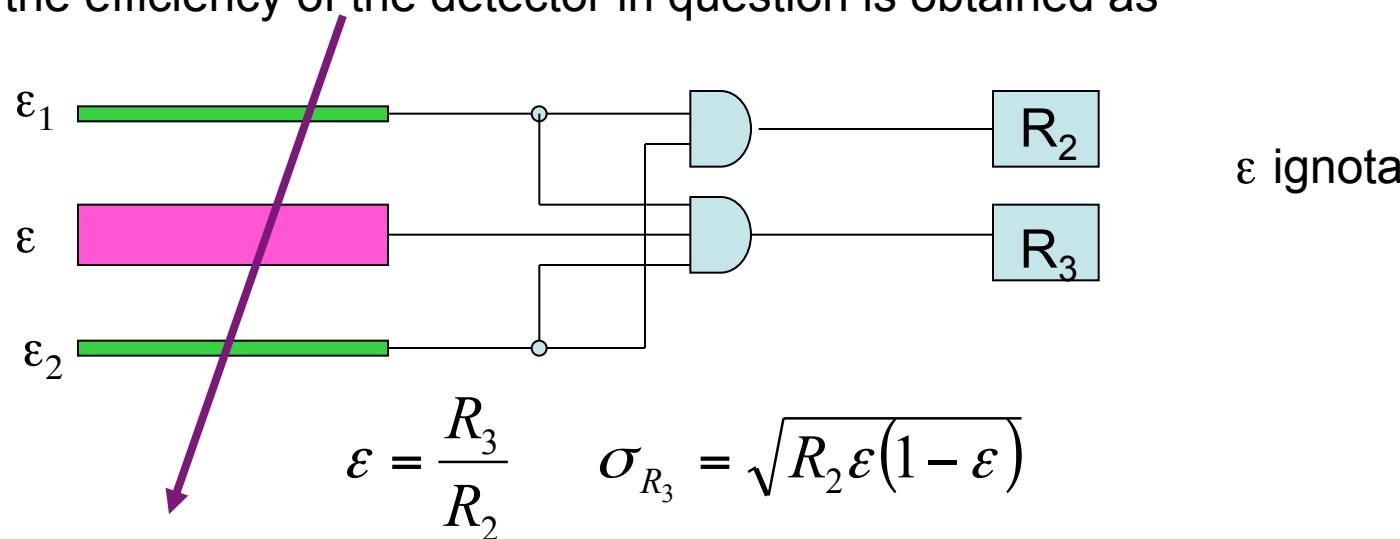
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# Caratteristiche degli apparati

Exercise: How to determine the intrinsic efficiency of a detector

The detector of unknown efficiency  $\epsilon$  is placed between two trigger counters with efficiencies  $\epsilon_1$  and  $\epsilon_2$ ; one must make sure that particles which fulfil the trigger requirement, which in this case is a twofold coincidence, also pass through the sensitive volume of the detector under investigation.

The twofold coincidence rate is  $R_2 = \epsilon_1 \cdot \epsilon_2 \cdot N$ , where  $N$  is the number of particles passing through the detector array. Together with the threefold coincidence rate  $R_3 = \epsilon_1 \cdot \epsilon_2 \cdot \epsilon \cdot N$ , the efficiency of the detector in question is obtained as



Dove  $R_2$  corrisponde al numero di prove,  $\epsilon=p$  (numero di successi),  $1-\epsilon=q$  (numero di insuccessi)

# **Caratteristiche degli apparati**

## **Accettanza geometrica**

In realtà esiste anche un'altra efficienza, l'efficienza geometrica spesso chiamata accettanza.

In altre parole l'apparato non solo deve essere intrinsecamente efficiente, ma deve anche coprire geometricamente la zona che mi interessa, cioe'  $\Delta\Omega/4\pi \rightarrow 1$

Questo definisce il cosi' detto "potere di raccolta" o gathering power del rivelatore. Dipende solo dalla geometria del rivelatore o degli elementi del rivelatore in relazione al flusso di particelle incidenti.

Agli acceleratori e' sinonimo di "ermeticita'", negli esperimenti di astroparticelle o passivi e' sinonimo di "exposure factor"

# **Caratteristiche degli apparati**

The **coincidence counting rate** of any particle telescope depends upon the effective dimensions and relative positions, i.e. **the geometry**, of the telescope sensors as well as the **intensity of radiation in the surrounding space and the sensor efficiencies**.

The experimentalist's task is to compute the intensity of radiation given the coincidence counting rate and the parameters (e.g. sensor dimensions) of his telescope. This is the task not only of the space scientist with instruments in an unknown radiation environment but also of the nuclear physicist with his collimated beams.

For an ideal telescope the factor of proportionality relating the counting rate  $C$  to the intensity  $I$  is defined as the ***gathering power*  $\Gamma$**  of the telescope.

When the intensity is isotropic, i.e.,  $I = I_0$ , the factor of proportionality is called the ***geometrical factor G***. That is  $C = GI_0$ .

Also called geometrical acceptance.

# Caratteristiche degli apparati

The  $C(x,t)$  in a time period  $T$  in a detector is given by

J.D. Sullivan,  
GEOMETRICAL FACTOR AND DIRECTIONAL RESPONSE OF  
SINGLE AND MULTI-ELEMENT PARTICLE TELESCOPES,  
NUCLEAR INSTRUMENTS AND METHODS 95 (1971) pp. 5-11

where

- $C$  = coincidence counting rate ( $\text{sec}^{-1}$ ),  
 $\alpha$  = label for kind of particle,  
 $J_\alpha$  = spectral intensity of the  $\alpha$ th kind of particle  
( $\text{sec}^{-1} \text{cm}^{-2} \text{sr}^{-1} E^{-1}$ ),  
 $\varepsilon_\alpha$  = detection efficiency for the  $\alpha$ th kind of particle,  
 $t$  = time,  
 $t_0$  = time at start of observation,  
 $T$  = total observation time,  
 $d\sigma$  = element of surface area of the last telescope sensor to be penetrated,  
 $S$  = total area of the last telescope sensor,  
 $d\omega = d\phi d \cos \theta =$  element of solid angle ( $\theta$  polar angle,  $\phi$  azimuth),  
 $\Omega$  = domain of  $\omega$ , this is limited by the other telescope sensors,  
 $x$  = spatial coordinate of the telescope,  
 $r$  = unit vector in direction  $\omega$ , and  
 $\hat{r} d\sigma$  = effective element of area looking into  $\omega$ .

$$C(x, t_0) = (1/T) \int_{t_0}^{t_0 + T} dt \int_S d\sigma \cdot \hat{r} \int_\Omega d\omega \int_0^\infty dE \times \\ \times \sum_\alpha \varepsilon_\alpha(E, \sigma, \omega, t) J_\alpha(E, \omega, x, t),$$

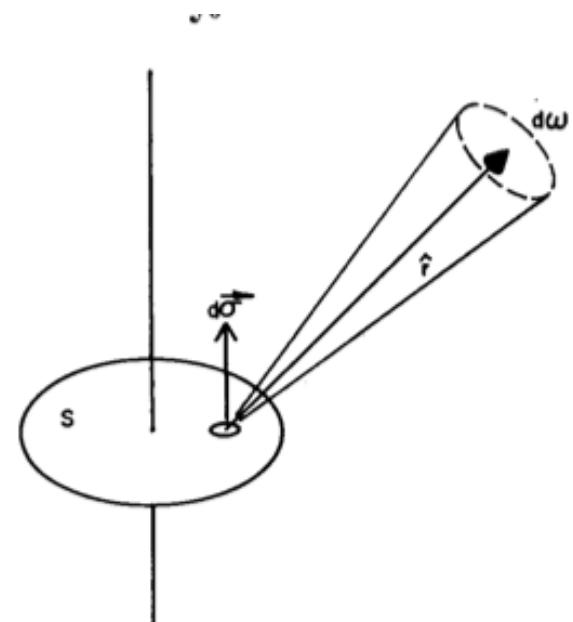


Fig. 1. A telescope with a single plane detector viewing one hemisphere.

Semplifichiamo assumendo che:

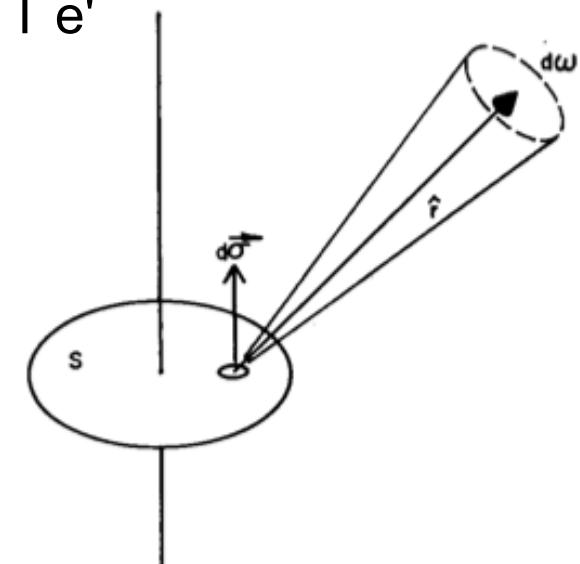
$$C(\mathbf{x}, t_0) = (1/T) \int_{t_0}^{t_0+T} dt \int_S d\sigma \cdot \hat{r} \int_{\Omega} d\omega \int_0^{\infty} dE \times \\ \times \sum_{\alpha} \varepsilon_{\alpha}(E, \sigma, \omega, t) J_{\alpha}(E, \omega, \mathbf{x}, t),$$

il flusso incidente sia indipendente da  $\mathbf{x}$ ,  
fattorizzabile come  $J(E, \omega, t) = J_o(E, t)F(\omega)$ ,  
ci sia una sola specie di particelle  $\alpha = 1 \rightarrow$

il # di particelle con  $E$  fra  $E$  e  $E+dE$  contate nel tempo  $T$  e'

$$\frac{dN}{dE} = \int_t^{t+T} dt \int_{\Omega} d\omega \int_S d\vec{\sigma} \cdot \hat{r} F(\omega) \varepsilon(E, t) J_o(E, t)$$

e' il fattore o accettanza geometrica del rivelatore  $G(E, t)$  in unita' di area x sr



La funzione di risposta direzionale del rivelatore e'  
definita da

$$A(\omega, E) = \int_S d\vec{\sigma} \cdot \hat{r}$$

il term. nell'integrale e' l'area "vista" da una particella che arriva da  $\theta, \phi$

Nel caso di distribuzione isotropa,  $F(\omega)d\omega = d\omega/4\pi$

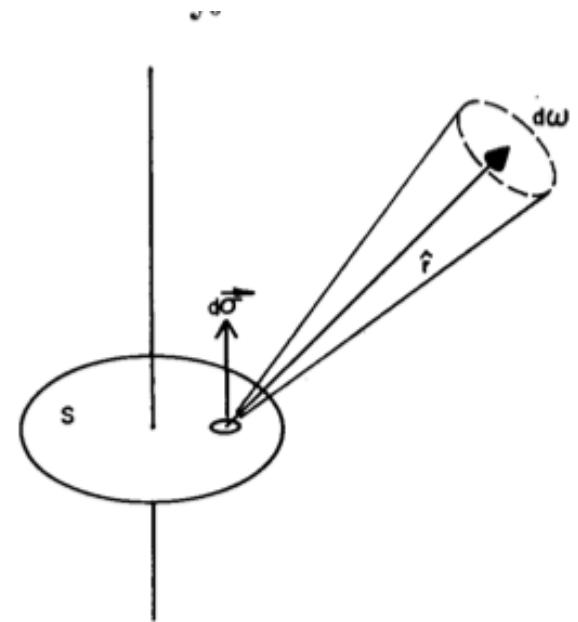
# Caratteristiche degli apparati

$$\frac{dN}{dE} = \int_t^{t+T} dt \int_{\Omega} d\omega \int_S d\vec{\sigma} \cdot \hat{r} F(\omega) \epsilon(E, t) J_o(E, t)$$

$$A(\omega, E) = \int_S d\vec{\sigma} \cdot \hat{r}$$

→

$$\frac{dN}{dE} = \int_t^{t+T} dt \int_{\Omega} d\omega A(\omega, E) F(\omega) \epsilon(E, t) J_o(E, t)$$



Nel caso di distribuzione isotropa,  $F(\omega)d\omega = d\omega/4\pi$

$$\frac{dN}{dE} = \int_t^{t+T} dt \left[ \int_{\Omega} d\omega \frac{A(\omega, E)}{4\pi} \right] \epsilon(E, t) J_o(E, t)$$

$\underbrace{\qquad\qquad\qquad}_{=G(E)}$

→ nel caso di flusso isotropo l'accettanza dipende solo dalla geometria del rivelatore e dall'energia della particella

# Caratteristiche degli apparati

Nel caso semplice di un telescopio a singolo piano di un rivelatore qualsiasi su cui incide un flusso isotropo di particelle e' semplice calcolare i fattori geometrici del rivelatore

$$A(\omega, E) = \int_S d\vec{\sigma} \cdot \hat{r} = \int_S \cos\theta d\sigma = S \cos\theta$$

$$G(E) = \int_{\Omega} A(E, \omega) d\omega = 2\pi S \int_0^1 \cos\theta d\cos\theta = \pi S$$

E' l'accettanza geometrica di un rivelatore a singolo piano: non e' l'area  $S$  ma  $\pi S$  perche' "raccoglie" particelle con dir di incidenza fra 0 e  $\pi/2$ , cioe' con accettanza differenziale  $S \cos\theta$

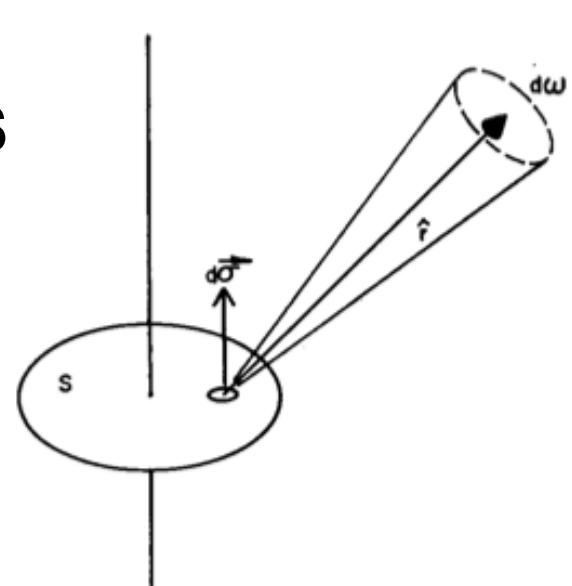


Fig. 1. A telescope with a single plane detector viewing one hemisphere.

# Caratteristiche degli apparati

Legato al concetto di risposta del rivelatore c'e' la

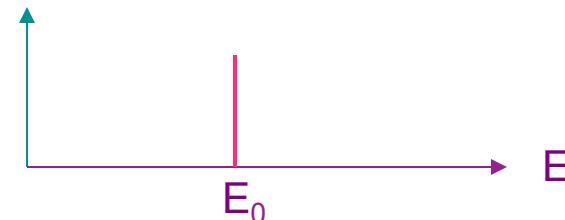
## D. Risoluzione Energetica

capacità del rivelatore di discriminare energie vicine

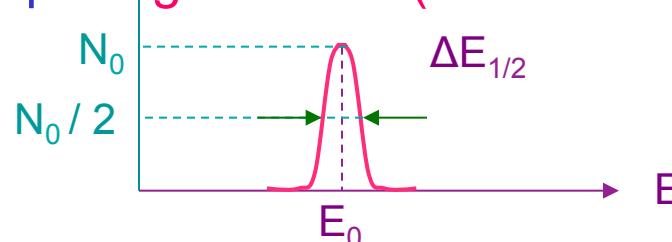
Importante quando la misura dell'energia e' richiesta

Misura: fascio monocromatico ( $E_0$ ) di particelle che incide sul rivelatore → osservazione dello spettro d' ampiezza risultante:

- Caso ideale: δ di Dirac



- Caso reale: fluttuazioni nel n. di ionizzazioni e nell'energia dep → gaussiana (nel caso di assorbimento completo)



$$R = \frac{\Delta E_{1/2}}{E_0} \quad (\text{FWHM})$$

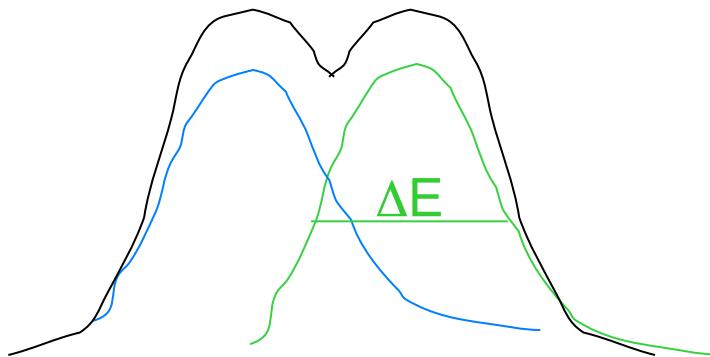
Lo sparpagliamento delle altezze di impulso intorno a E0 costituisce la funzione di risposta del rivelatore

# **Caratteristiche degli apparati**

## Risoluzione in energia.

Per apparati costruiti per misurare l'energia della particella è fondamentale la risoluzione in energia.

La risoluzione in energia può essere misurata usando un fascio monoenergetico ed osservando lo spettro risultante.



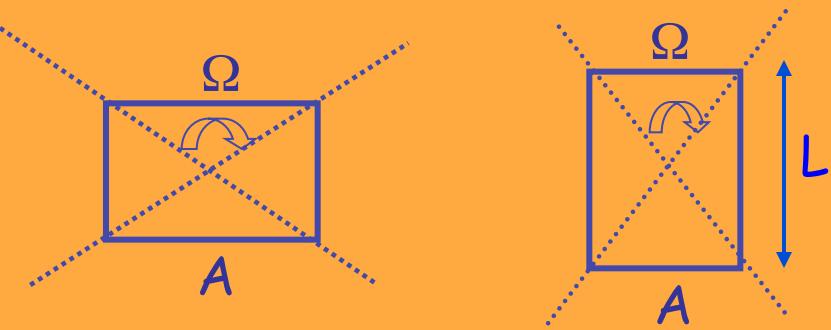
Per energie entro  $\Delta E(\text{fwhm})$   
non possiamo determinare  $E$

# **Caratteristiche degli apparati**

- La risposta in energia dice qual'è la probabilità che una particella con energia “vera”  $E$  sia misurata con energia  $E_{\text{mis}}$ ,  $R(E_{\text{mis}}, E)$
- Entra nella determinazione del rate  
 $dN(E_{\text{mis}})/dtdE_{\text{mis}} = \int dE G(E, t) \varepsilon(E, t) R(E_{\text{mis}}, E) J_o(E, t)$   
occorre invertire l'equazione per ottenere  $J_o(E, t)$   
 $G, \varepsilon, R$  sono in genere ottenute dai dati e da simulazioni MC

# Acceptance

- Exposure factor:  $\text{Area} \cdot \Omega \cdot T$
- Counts:  $\text{Flux} \cdot \text{Area} \cdot \Omega \cdot T$

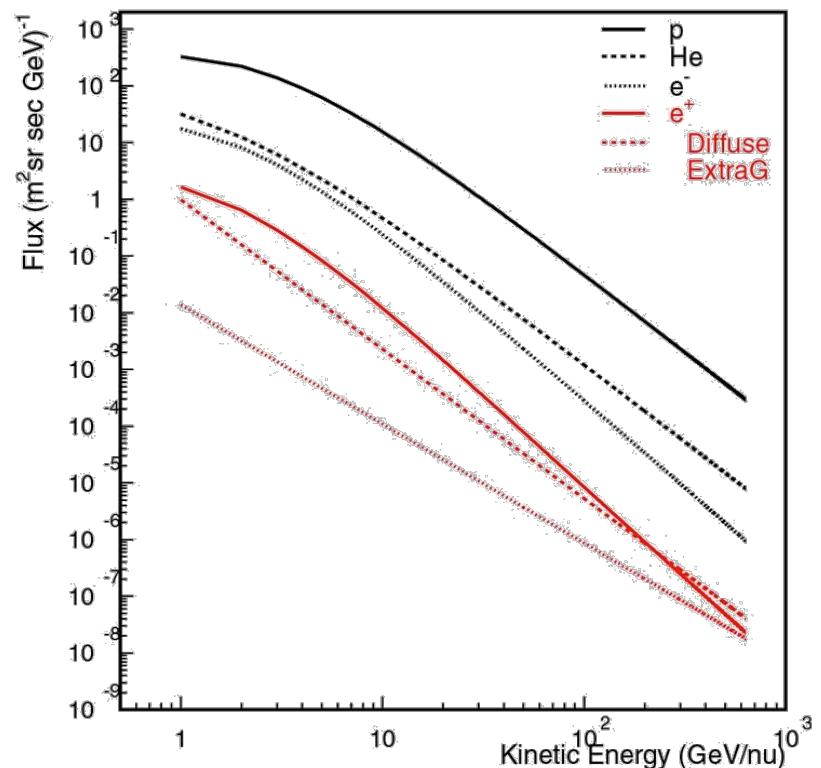


Time needed for 10% stat error (sec)

$$\text{Area} \cdot \Omega = 1 \text{ m}^2\text{sr}$$

	10 GeV	100 GeV	200 GeV
p	4	200	700
e-	500	$10^5$	$5 \cdot 10^5$
p/e+ → factor	$10^4 - 10^5$		

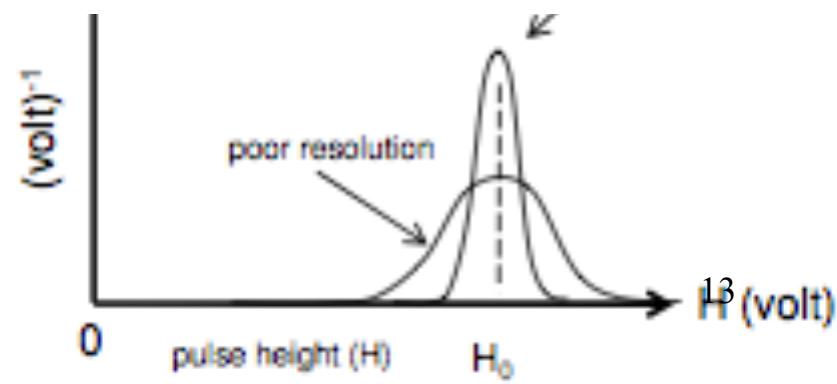
Selection Efficiency assumed 1



# Caratteristiche degli apparati

- There are number of Sources of fluctuations:
  - a) Drift of detector operating characteristics ( HV, gain ..)
  - b) random noise in detector & electronics
  - c) statistical noise intrinsic to nature of signal  
(discrete number of charge carriers, fluctuations in energy deposition in detector)
- C) represents an irreducible minimum amount of fluctuation that will always be present in the detector signal no matter how perfect the remainder of the system is made. In a wide category of detector applications, the statistical noise represents the dominant source of fluctuation in the signal and thus sets an important limit on detector performance.

Ne parleremo piu' avanti



# **Caratteristiche degli apparati**

- Estimation for fluctuation, can be given by assuming a Poisson's distribution of events  $N$ .
- Standard deviation is  $\sqrt{N}$
- $N$  is usually large, Then the response function is a Gaussian shape

$$G(H) = \frac{A}{\sigma\sqrt{2\pi}} e^{-\frac{(H-H_0)^2}{2\sigma^2}}$$

- $FWHM = 2.35\sigma$

# Caratteristiche degli apparati

The response of many detectors is ideally linear →

- Average pulse  $H_0 = KN$  where  $K = \text{constant}$

standard deviation  $\sigma = K\sqrt{N}$  and

$$FWHM = 2.35K\sqrt{N} \quad \text{assuming poisson stat}$$

$$\text{Energy resolution } R_{\text{Poisson Limit}} = \frac{FWHM}{H_0}$$

$$R = \frac{2.35K\sqrt{N}}{KN} = \frac{2.35}{\sqrt{N}}$$

$N = \text{nbr. of charges}$

$K = e/C$

Remember that in a read-out RC circuit with  $RC \gg t_c$

$$H_0 = V = Q/C = Ne/C (= e\Delta E/wC)$$

where  $e$  is the electron charge,  $C$  is the readout capacitance,  $w$  is the effective ionization potential and  $\Delta E$  is the deposited energy

Note that this limiting value of  $R$  depends only on the number of charge carriers  $N$ , and the resolution improves ( $R$  will decrease) as  $N$  is increased.

From Eq. 3-5 we see that in order to achieve an energy resolution better than 1 percent, one must have  $N$  greater than 55,000. An ideal detector would have as many charge carriers generated per event as possible, so that this limiting resolution be as small a percentage as possible. The great popularity of semiconductor, diode detectors stems from the fact that a very large number of charge carriers are generated in these devices per unit energy lost by the incident radiation.

# Fano Factor

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- surprisingly, when you look at the energy resolution of radiation detectors, like gas ionization detectors, you find that the resolution is **better than  $\text{sqrt}(N)$ ?**
- this was studied by Fano and the explanation goes like this...
- the **energy deposited by the particle** equals **# of ionizations** times the **average energy required to ionize** plus **# of excitations** times the **average energy per excitation**  $E = N_{\text{ion}} \bar{I} + N_{\text{exc}} \bar{E}_{\text{exc}} = N_{\text{ion}} w$ 
  - simplified as  $E = N_{\text{ion}} w$ ,  $w$  is the average energy loss per electron-ion pair produced
    - remember:  $w$  is around 30 eV per e-i pair in a gas (e.g. we calculated for 300 keV that's about 10,000 electron-ion pairs)  $\longrightarrow \frac{1}{\sqrt{N}}$  or 1%
- the energy deposited is a fixed quantity though; it's the total energy in the gamma ray photopeak, for example
- thus, the uncertainty  $\Delta N_{\text{ion}}$  is correlated with  $\Delta N_{\text{exc}}$

$$E = N_{ion} \bar{I} + N_{exc} \bar{E}_{exc} = N_{ion} w$$

## Fano Factor cont'd

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- Poisson statistics applies for independent variables but these are constrained in that their weighted sum must equal the total energy deposited
  - e.g. so you might happen to have a few more ionization events; that means you must have a few less excitation events, and vice versa
- make up some more numbers:
  - 2 excitations for every ionization
  - 300 keV deposited is 10,000 ionizations and 20,000 excitations
  - consider it as 1 independent variable and 1 correlated one

$$|\Delta N_{ion}| = \frac{\sqrt{N_{exc} \bar{E}_{exc}}}{\bar{I}} = \frac{5\sqrt{20000}}{20} \doteq 35.4$$

- smaller than  $\sqrt{10,000} = 100!$

Fano factor definition:

$$F \equiv \frac{\sigma_{actual}^2}{\sigma_{Poisson}^2}; \quad F \leq 1$$

$$\sigma_{actual} = \sqrt{FN}$$

# **Caratteristiche degli apparati**

- The departure of the observed statistical fluctuations in the number of charge carriers from pure Poisson's statistics is defined by Fano factor,  $F$

$$F = \frac{\text{observed variance in } N}{\text{Poisson predicted variance}}$$

- Because variance is  $\sigma^2$ , the equivalent equation

$$R_{limit} = \frac{2.35K\sqrt{N}\sqrt{F}}{KN} = 2.35\sqrt{\frac{F}{N}}$$

- $F < 1$  for semiconductor detectors and proportional counters,  $F \approx 1$  for scintillators

# **Caratteristiche degli apparati**

Distribuzione Poissoniana del numero di coppie  $e^- - \text{ione}$  prodotte dalla particella incidente → la risoluzione migliora crescendo l'energia depositata.

Se  $w$  è l'energia media richiesta per ogni ionizzazione (= per tutte) → crescendo l'energia depositata  $E$ , cresce il numero di ionizzazioni  $J=E/w$  → minori fluttuazioni.

- i. **Assorbitori sottili:**  $\sigma^2=J$  (Poisson).  $J$  è il numero medio di eventi prodotti. →  $R(\text{fwhm})=2.355(J^{1/2}/J)=2.355(w/E)^{1/2}$
- ii. **Assorbitori spessi:** migliore risoluzione se assorbo tutta l'energia.  $R(\text{fwhm})=2.355(Fw/E)^{1/2}$ . Migliore risoluzione se  $F < 1$   
 $F=0.06$  nei semiconduttori;  $F = 0.17$  nei gas nobili ed  $F = 1$  negli scintillatori.

# **Caratteristiche degli apparati**

1. Dobbiamo distinguere fra le fluttuazioni occasionali alla Landau (molto grandi) della perdita di energia che avviene in assorbitori sottili e le fluttuazioni del numero delle coppie ione elettrone prodotte per una perdita di energia fissa.  
*(quest'ultimo caso è vero per tutte le particelle che depositano tutta l'energia nell'apparato).*
2. Il fattore di Fano è una funzione di tutti i processi fondamentali che possono portare ad un trasferimento di energia al materiale. Questo include anche tutti i processi che non generano una ionizzazione come ad esempio eccitazione di fononi etc. → **è quindi una costante intrinseca del mezzo.**  
Teoricamente è molto difficile da calcolare accuratamente in quanto richiede la conoscenza di tutte le reazioni che possono avvenire nel mezzo considerato.

# **Caratteristiche degli apparati**

Ulteriori sorgenti di fluttuazione:

- rumore associato all' elettronica
- deriva delle caratteristiche del sistema

Ipotesi : fluttuazioni indipendenti

$$\longrightarrow (\Delta E_{1/2}^{\text{tot.}})^2 = (\Delta E_{1/2}^{\text{stat.}})^2 + (\Delta E_{1/2}^{\text{elet.}})^2 + (\Delta E_{1/2}^{\text{der.}})^2 + \dots$$

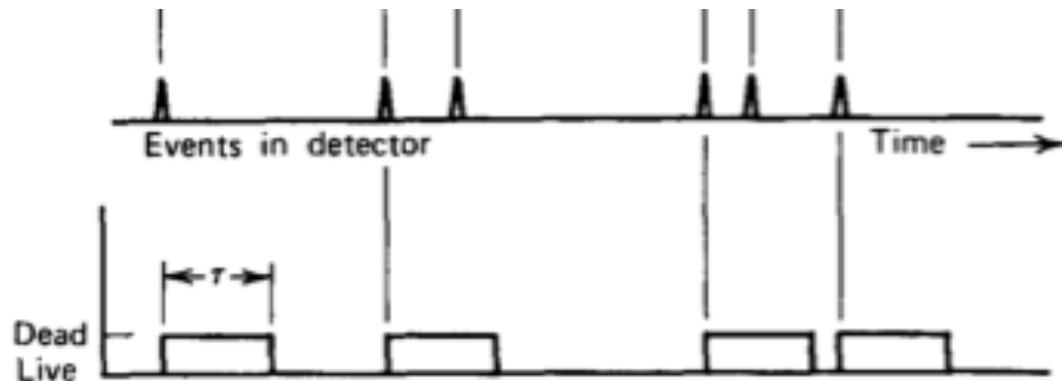
Esempio: fotoni da 1 MeV

$$\left. \begin{array}{l} \text{NaI : } R \sim 8 \% \\ \text{GeLi : } R \sim 0.1 \% \end{array} \right\}$$

# **Caratteristiche degli apparati**

In nearly all detector systems, there will be a minimum amount of time that must separate two events in order that they can be recorded as two separate pulses. In some cases the limiting time may be set by processes in the detector itself, and in other cases the limit may arise in the associated electronics.

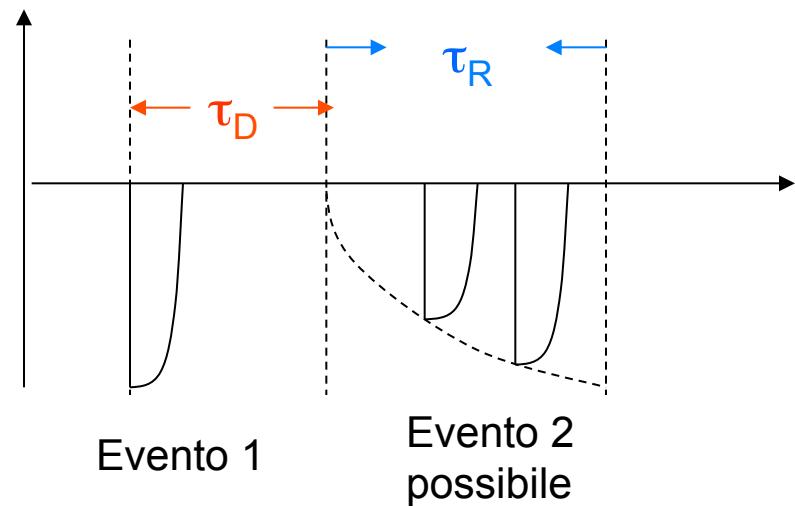
**This minimum time separation is usually called the *dead time* of the counting system.**



During the dead time, the passage of another particle is not recorded

# **Caratteristiche degli apparati**

The dead time, in which no further particles can be detected, is followed by a phase where particles can again be measured; however, the detector may not respond to the particle with full sensitivity. After a further time, the recovery time  $\tau_R$ , the detector can again supply a signal of normal amplitude.

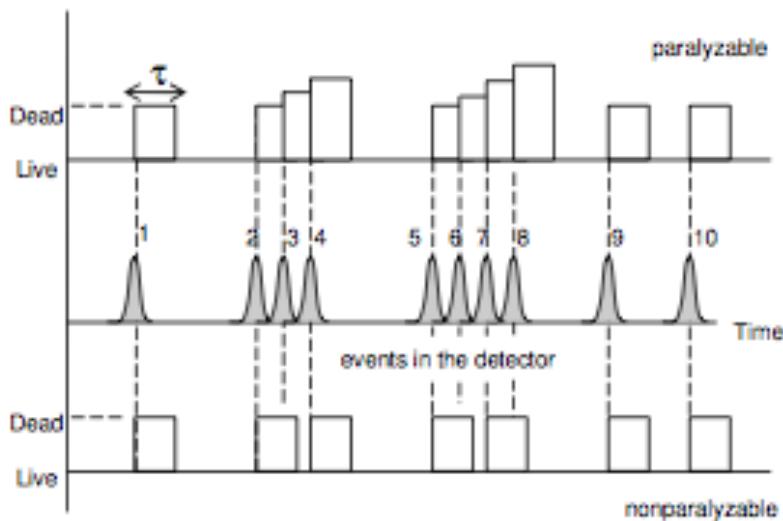


# **Caratteristiche degli apparati**

- In a random sample, two events may occur very close in time, and some true events may be lost due to the dead time
- There are two methods to determine the true number of events
  1. paralyzable detector method
  2. nonparalyzable detector method
- Dead time  $\tau$  is set after each true event that occurred during the “live period”
  - Paralyzable detector method:  
Any event occurred during dead period not recorded as counts, but it extends the dead period  $t$  following the lost event.
  - Non paralyzable detector method:  
it just ignore the other event occurred during dead period  $t$   
Following example shows the difference between paralyzable and nonparalyzable events

The middle line represents 10 events along the time axis as they come.

Assume events 3,4 and 6,7,8 come very close in time (i.e. within the dead time of previous event)



Five events in  
paralyzable method

Seven events in  
nonparalyzable method

Events 1,9 and 10 are recorded by both detectors.

After event 2 is registered, event 3 and 4 restart the dead period for paralyzable detector which misses both event and 4.

In non paralyzable method, after event 2 is registered, it recovers to register event 4. ( event 3 is lost since it is within dead time of event 2 and event 4 is outside the dead time of 2)

- After event 5 is recorded, paralyzable detector extends the dead period from events 6,7, and 8. As a result, all events 6,7,8 are lost.
- In non-paralyzable detector, after event 5 is recorded, it recovers to record the event 7. Only event 6 and 8 are lost as they are within dead time of event 5 and 7 respectively.

- Let us obtain an expression for true interaction rate. Dead time is a fixed value for each event in this method.

assume  $R$  = rate of true interactions

$R_{\text{meas}}$  = rate of measured events

$\tau$  = dead time for one event

If  $m$  is the measured nbr of events during the time  $T$ , the measured rate is  $R_{\text{meas}}$  and the dead time cumulated is  $t_d = m\tau$ .

If  $n$  is the true nbr of incident particles and  $R$  is the true rate, we have lost  $Rt_d = Rm\tau$  events in the  $T$  interval, then

$$n = m + m\tau R \rightarrow \text{divide by } T$$

$$\rightarrow R = R_{\text{meas}} + RR_{\text{meas}}\tau \rightarrow R = R_{\text{meas}} / (1 - R_{\text{meas}}\tau)$$

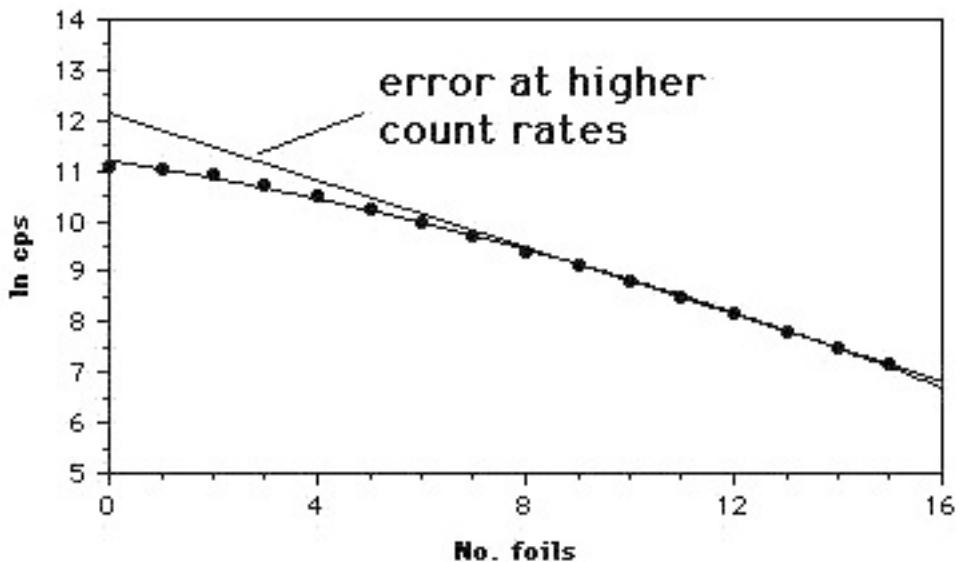
O anche

$$\rightarrow R_{\text{meas}} = R / (1 + R\tau)$$

# Non paralizzabile

Esempio: un flusso di fotoni incide su una pila di fogli di spessore dato

L'attenuazione aspettata e' un esponenziale in funzione del numero di fogli attraverasti, con una lunghezza di attenuazione  $\mu = N_A / A\sigma$



$$I/I_0 = \exp(-\mu*x)$$

$$\ln I = \ln I_0 - \mu*x$$

/

straight line

$$R_{\text{meas}} = R / (1 + R\tau)$$

Le correzioni diventano importanti ad alti rates R, quando  $R\tau$  non e' piu'  $\ll 1$   
Il numero dei conteggi misurato devia da quello aspettato.

# **Caratteristiche degli apparati**

## **Esempio per la misura del tempo morto.**

Il metodo classico per misurare un tempo morto è il cosiddetto metodo delle due sorgenti.

Supponiamo che  $n_1$  ed  $n_2$  sono i conteggi **veri** delle 2 sorgenti e  $R_1$ ,  $R_2$  ed  $R_{12}$  sono i conteggi **misurati** per le 2 sorgenti separate ed assieme rispettivamente. Assumiamo inoltre che non ci sia fondo. Allora:

$$n_1 = \frac{R_1}{1 - R_1\tau} \quad n_2 = \frac{R_2}{1 - R_2\tau} \quad n_1 + n_2 = \frac{R_{12}}{1 - R_{12}\tau}$$

$\Rightarrow$

$$\frac{R_{12}}{1 - R_{12}\tau} = \frac{R_1}{1 - R_1\tau} + \frac{R_2}{1 - R_2\tau}$$

$\Rightarrow$

$$\tau = \frac{R_1 R_2 - [R_1 R_2 (R_{12} - R_1)(R_{12} - R_2)]^{1/2}}{R_1 R_2 R_{12}}$$

# **Caratteristiche degli apparati**

- ❖ **Tempo di sensibilità  $\tau_s$** ; intervallo di tempo in cui i segnali possono essere raccolti indipendentemente dalla loro correlazione con il **trigger**. È importante per tutti gli apparati pulsati.  
e.g. Se in un esperimento l'apparato è comandato da una interazione del fascio nella targhetta, si apre normalmente una finestra di tempo  $\tau_s$  durante la quale raccolgo l'evento. Se per caso durante quell' intervallo di tempo passa un raggio cosmico anche questo evento di fondo viene raccolto.
- ❖ **Tempo di lettura** : tempo richiesto per leggere l'evento.
- ❖ **Tempo di ripetizione** : strettamente connesso al tempo di lettura determina il minimo intervallo di tempo necessario perché 2 eventi successivi possano essere distinti.
- ❖ **Tempo di memoria** : massimo ritardo di tempo fra il trigger e la raccolta dell'evento in modo da avere ancora un' efficienza del 50%
- ❖ **Tempo di risoluzione** : caratterizza il minimo intervallo di tempo per distinguere 2 eventi; molto simile al tempo di ripetizione. Mentre il tempo di ripetizione si riferisce a tutto l'apparato sperimentale il **tempo di risoluzione** si riferisce ad una parte dell'apparato ( **camera calorimetro ...** )

# Caratteristiche degli apparati

The criterion by which to judge the quality of a detector is its resolution for the quantity to be measured (energy, time, spatial coordinates, etc.).

If a quantity with true value  $z_0$  is given (e.g. the monoenergetic  $\gamma$  radiation of energy  $E_0$ ), the measured results  $z_{\text{meas}}$  of a detector form a distribution function  $D(z)$  with  $z = z_{\text{meas}} - z_0$ ; the expectation value for this quantity is

$$\langle z \rangle = \frac{\int z D(z) dz}{\int D(z) dz}$$

This normalised function is usually referred to as the probability density function (PDF), an other name for the response function. The variance of the measured quantity is

$$\sigma_z^2 = \frac{\int (z - \langle z \rangle)^2 D(z) dz}{\int D(z) dz}$$

# Caratteristiche degli apparati

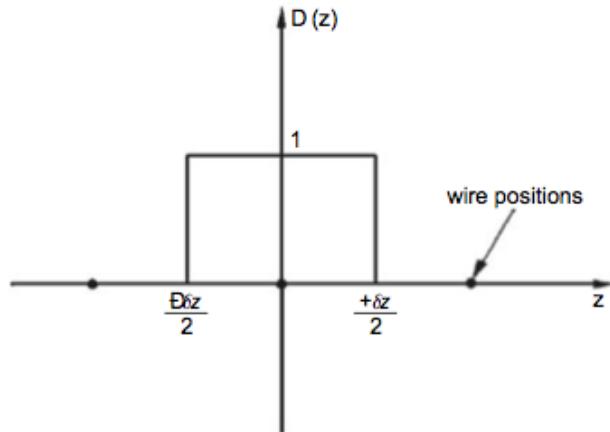
As an example, the expectation value and the variance for a rectangular distribution will be calculated. In a multiwire proportional chamber with wire spacing  $\delta z$ , the coordinates  $\perp$  to the wires of charged particles passing through the chamber are to be determined. Only a hit on a particular wire with number  $n_w$  is recorded (assuming only one hit per event) and its discrete coordinate,  $z_{\text{meas}} = z_{\text{in}} + n_w \delta z$  is measured.

The distribution function  $D(z)$  is constant = 1 from  $-\delta z/2$  up to  $+\delta z/2$  around the wire which has fired, and outside this interval the distribution function is zero

$$\Rightarrow \langle z \rangle = 0; \quad \sigma_z = \delta z / (12)^{1/2}$$

Infatti:

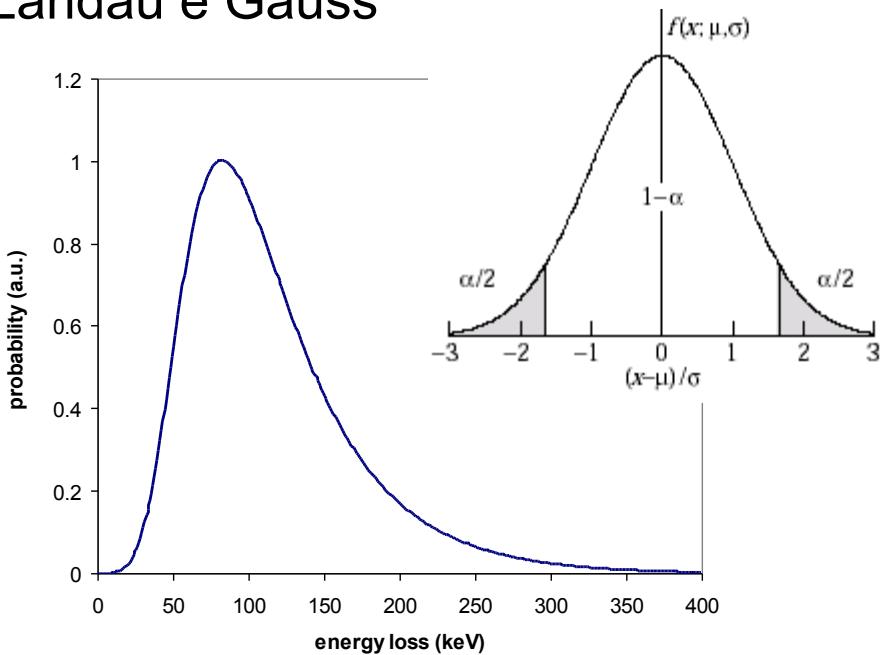
$$\begin{aligned} \langle z \rangle &= \int_{-\delta z/2}^{\delta z/2} z dz = \frac{z^2}{2} \Big|_{-\delta z/2}^{\delta z/2} = 0 \\ \sigma_z^2 &= \frac{\int_{-\delta z/2}^{\delta z/2} (z - 0)^2 dz}{\delta z} = \frac{\frac{z^3}{3} \Big|_{-\delta z/2}^{\delta z/2}}{\delta z} = \frac{\delta z^2}{12} \Rightarrow \sigma_z = \frac{\delta z}{\sqrt{12}} \end{aligned}$$



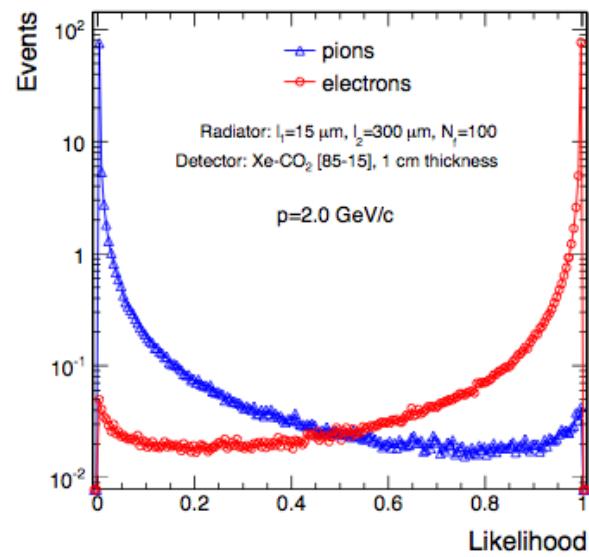
# Caratteristiche degli apparati

In generale la PDF di un rivelatore rappresenta la distribuzione di probabilita' dei possibili risultati di una misura di una data grandezza fisica.

Esempi: distribuzione dei depositi di energia in assorbitori sottili e spessi:  
Landau e Gauss



Likelihood per particelle leggere e pesanti in un TRD



Va determinata "sperimentalmente" e/o con tecniche di MonteCarlo.  
*Questo e' un altro degli incubi del fisico sperimentale.*

# ***Caratteristiche degli apparati***

In general, efficiency and resolution of a detector are strongly correlated. Therefore one has to find an optimum for these two quantities also under consideration of possible backgrounds.

If, for example, in an experiment with an energy-loss, Cherenkov, or transition-radiation detector a pion–kaon separation is aimed at, this can in principle be achieved with a low misidentification probability.

However, for a small misidentification probability one has to cut into the distribution to get rid of the unwanted particle species. This inevitably results in a low efficiency: one cannot have both high efficiency and high two-particle resolution at the same time

# Cfr.: Radiazione di transizione-considerazioni per i rivelatori

The electron (or any other particle) identification performance of a TRD is quantified in terms of the pion efficiency at a given electron efficiency and is the fraction of pions wrongly identified as electrons.

The e- efficiency is the fraction of e- passing through the detector that are identified as electrons.

The pion efficiency depends strongly on the electron efficiency, which is a parameter that can be adjusted at the stage of the data analysis.

It is chosen such that the best compromise between electron efficiency and purity is reached. Usually 90% electron efficiency is the default value used to quote a TRD rejection power

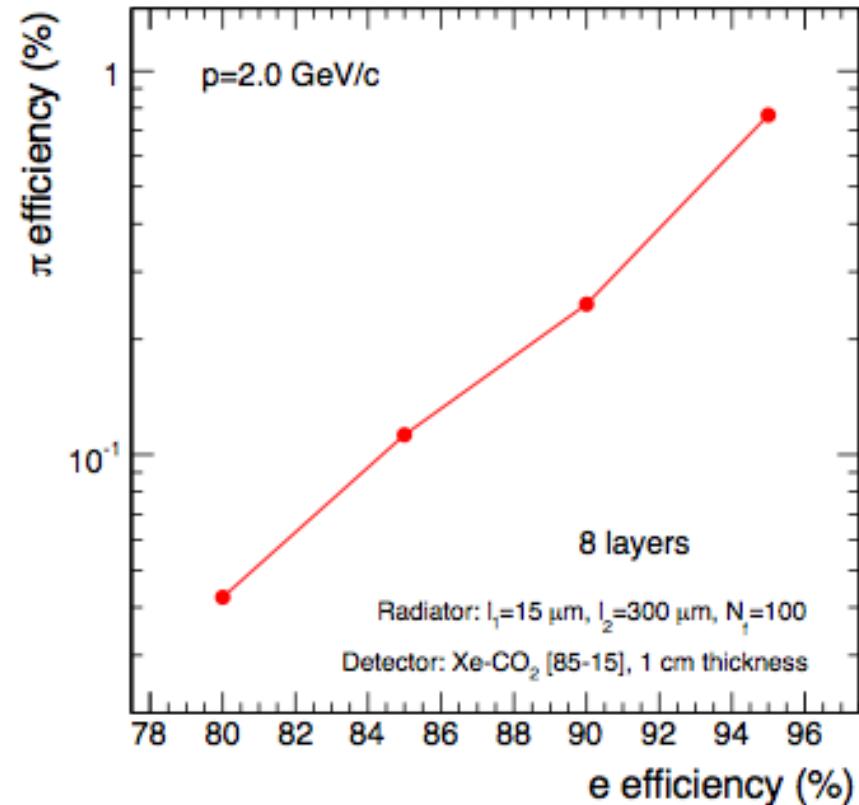


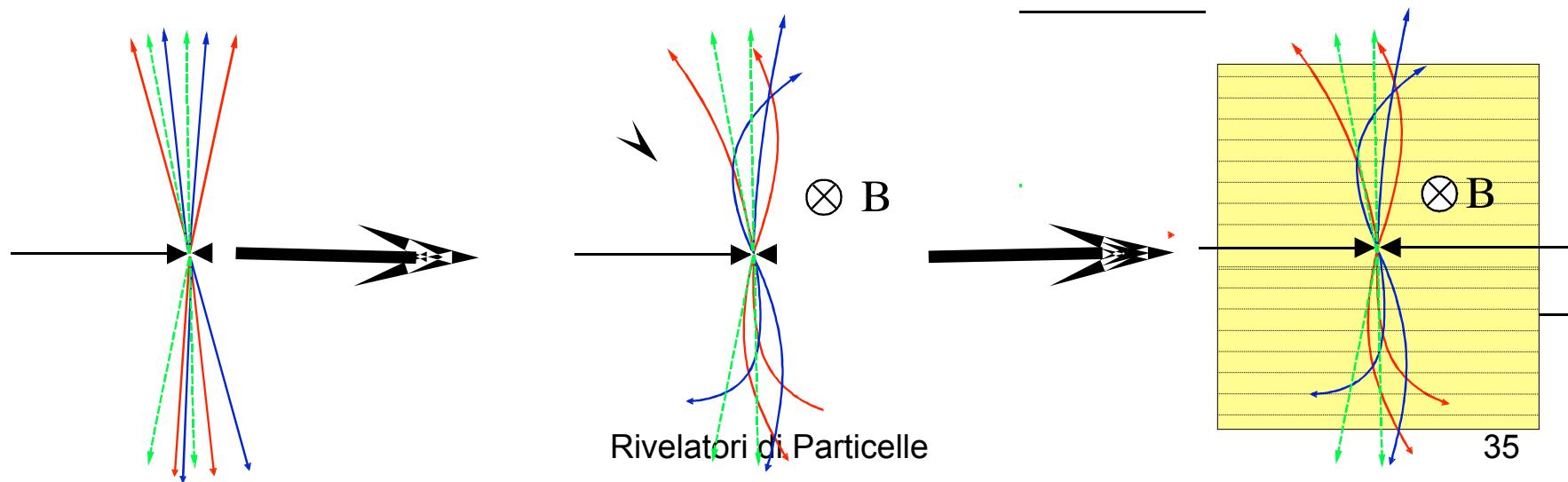
Figure 7: Pion efficiency as a function of the electron number of layers (right panel).

# *Misure d'impulso*

Per ottenere l'impulso  $\mathbf{p}$  di una particella si misura generalmente:

-la deflessione della particella tramite la misura della traiettoria della particella in un campo magnetico  $B$

E' necessario un apparato che permette di misurare le coordinate del punto di passaggio della particella in un numero sufficiente di punti, cioe' un apparato che mi permette la misura di tracce (insieme di camere **MWPC** o **a deriva** o **silici**) posto in un campo magnetico (possibilmente uniforme), genericamente definiti **spettrometri magnetici**.



# Charged Particle Tracking and Momentum Resolution

We measure the momentum of a charged particle by determining its trajectory in a known magnetic field.

We measure the trajectory of the charged particle by measuring its coordinates ( $x, y, z$  or  $r, \theta, \phi$ ) at several points in space.

Simplest case: determine radius of circle with 3 points

coordinates in space are usually measured using one or more of the following devices:

Wire Chamber	low spatial resolution ( $\approx 1\text{-}2\text{ mm}$ )
Drift Chamber (or TPC)	moderate spatial resolution ( $50\text{-}250\mu\text{m}$ )
Silicon detector	high spatial resolution ( $\approx 5\text{-}20\text{ }\mu\text{m}$ )

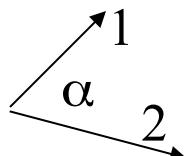
*Better momentum resolution  $\Rightarrow$  better mass resolution  $\Rightarrow$  better physics*

Many particles of interest are observed via their decay products:  $Z^0 \rightarrow e^-e^+$ ,  $D^+ \rightarrow K^-\pi^+\pi^+$ ,  $K^0 \rightarrow \pi^+\pi^+$

By measuring the momentum of the decay products we measure the mass of the parent.

$$m \rightarrow m_1 + m_2 \Rightarrow m^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = m_1^2 + m_2^2 + 2[(m_1^2 + p_1^2)^{1/2} (m_2^2 + p_2^2)^{1/2} - p_1 p_2 \cos\alpha]$$

For fixed  $\alpha$ :  $\sigma_{m^2}/m^2 \propto \sigma_p/p$



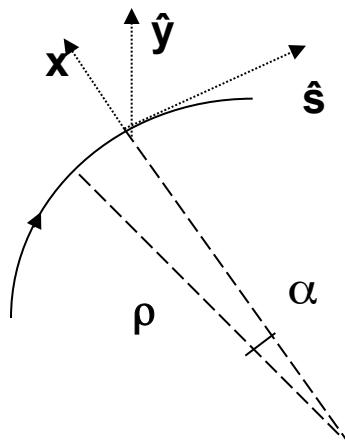
# Misure d'impulso

L'equazione del moto, nel caso di  $\mathbf{p} \perp \mathbf{F}$  e'  
Con  $|\mathbf{p}|$  costante.

$$\frac{\vec{p}}{dt} = q(\vec{v} \times \vec{B})$$

La forma di questa equazione cioè  $d\mathbf{v}/dt$  ortogonale a  $\mathbf{v}$  ed a  $\mathbf{B}$  implica  
moto circolare nel piano  $\perp \mathbf{B}$ .

Per ricavare il raggio di curvatura conviene utilizzare un sistema di  
coordinate curvilineo:



con  $x$ ,  $y$  ed  $s$  sistema destro,  $y \perp$  al piano  $x,s$ .  
 $\rho$  raggio di curvatura  
 $s$  coordinata curvilinea  
 $B$  diretto lungo l'asse  $y$  ( $B_y$ )

L'equazione di Lorentz diventa:

$$\frac{d\vec{p}}{dt} = q(\vec{v} \times \vec{B}) \Rightarrow \frac{dp}{dt} \hat{s} + p \frac{d\hat{s}}{dt} = -qvB_y \hat{x}$$

# Misure d'impulso

L'equazione di Lorentz :

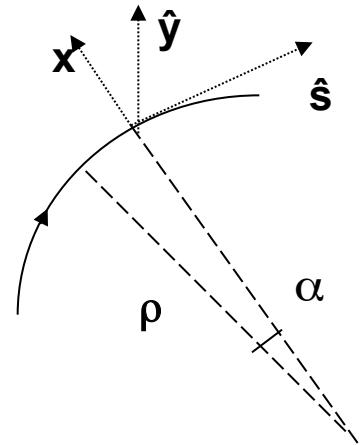
$$\frac{d\vec{p}}{dt} = \frac{dp}{dt} \hat{s} + p \frac{d\hat{s}}{dt} = -qvB_y \hat{x} \quad v = \text{velocità}$$

può essere semplificata osservando che  $|\mathbf{p}| = \text{costante}$  e

$$\frac{d\hat{s}}{dt} = -\frac{d\alpha}{dt} \hat{x} = -\frac{d\alpha}{ds} \frac{ds}{dt} \hat{x} = \frac{-v}{\rho} \hat{x}$$

$$-\frac{p}{\rho} \hat{x} = -qvB \hat{x}$$

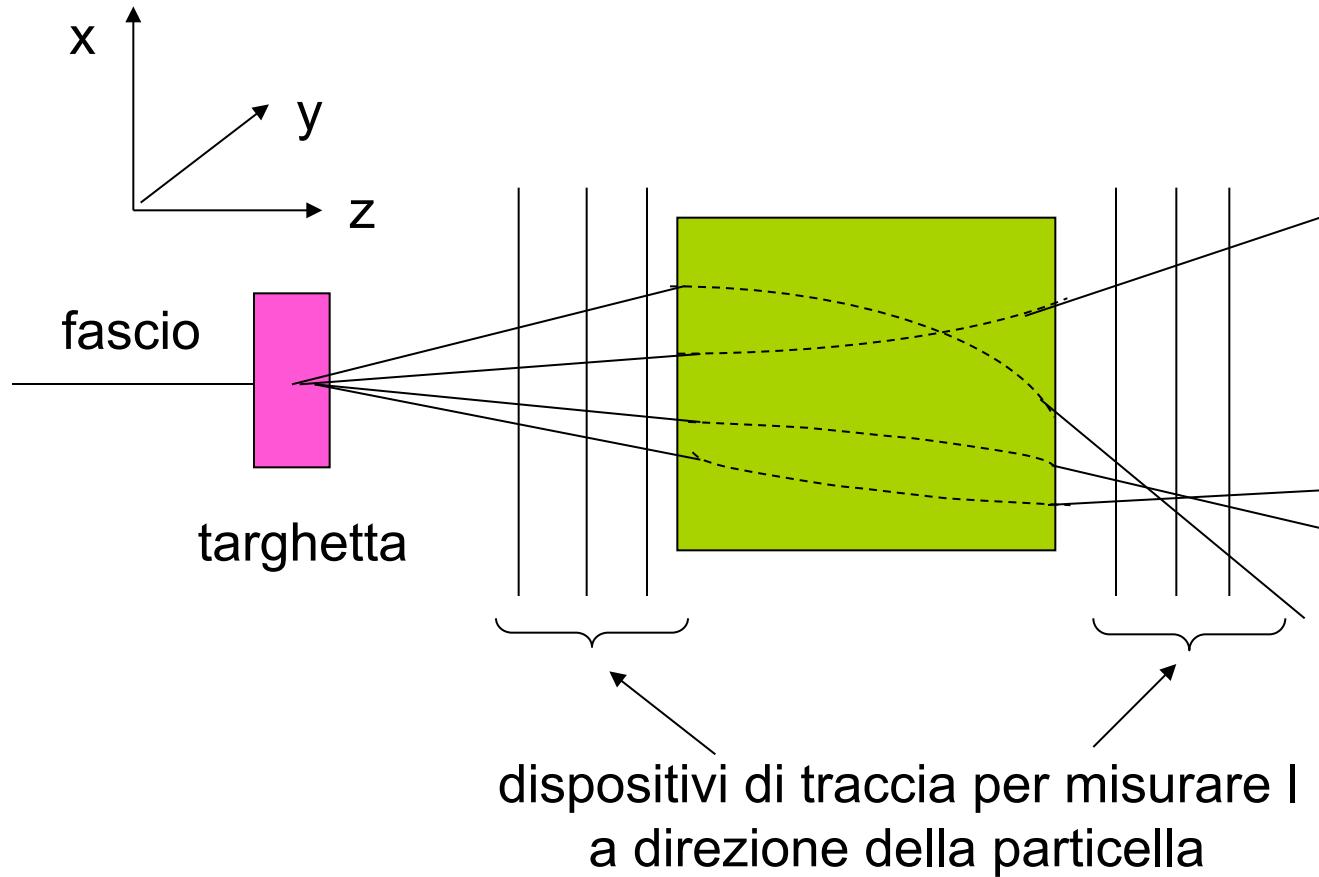
$$\rho = (p/qB_y)$$



- Fissato  $B$ , il raggio di curvatura della particella dipende da  $P/Q$ .  
 **$R = P/Q$  e' detta rigidita' (magnetica) della particella.**
- Particelle con la stessa rigidita' hanno la stessa traiettoria nel campo  $B$ .
- Per determinare il momento  $P$  dalla misura del raggio di curvatura occorre conoscere il valore assoluto della carica
- Il verso della curvatura dipende dal segno della carica

# *Misure d'impulso*

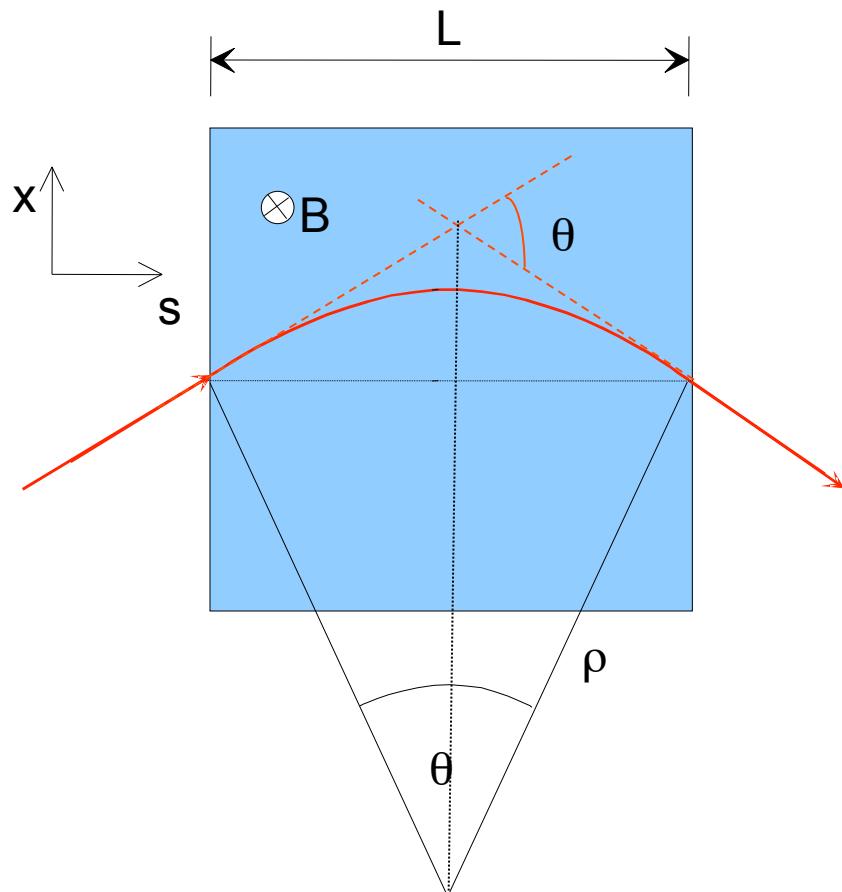
Misura della deflessione: la direzione della particella e' misurata prima e dopo la regione in cui c'e' campo magnetico



Rappresentazione schematica di uno spettrometro magnetico

# Misure d'impulso

La deflessione nel piano  $xs$  si vede dalla figura:



$$\sin(\theta/2) = L/2\rho$$

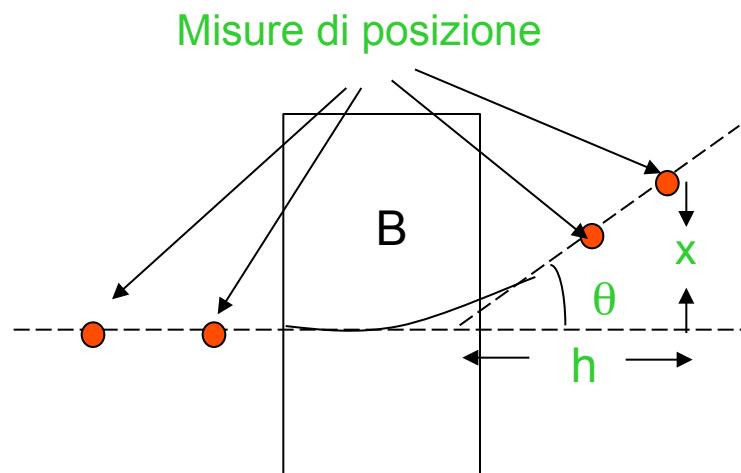
$$\rho = p/qB, \text{ perciò } \sin(\theta/2) = LqB/2p$$

Per piccole deflessioni  
 $\sin(\theta/2) \approx \theta/2$ , quindi  
 $\theta \approx LqB/p$ , cioè  $p = qBL/\theta$   
o  
 $R = BL/\theta$       Con  $R = p/q$

# Misure d'impulso

## Precisione dell'apparato tracciante.

Le traiettorie prima di entrare nel magnete e dopo essere uscite sono rettilinee  
→ misura di  $\theta$ .



$$\text{da } p = qB_y L \frac{1}{\theta} \Rightarrow$$

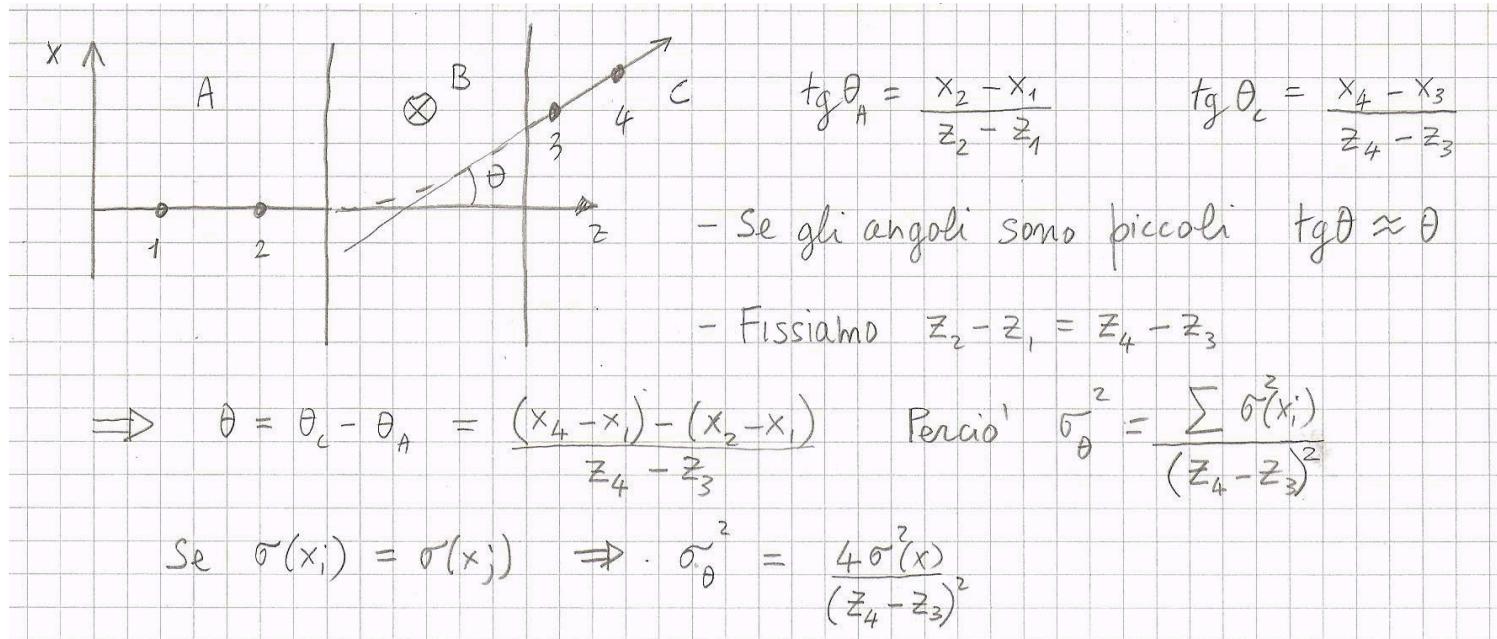
$$\left| \frac{dp}{d\theta} \right| = qB_y L \frac{1}{\theta^2} = \frac{p}{\theta}$$

$$\Rightarrow \frac{dp}{p} = \frac{d\theta}{\theta} \Rightarrow \frac{\sigma(p)}{p} = \frac{\sigma(\theta)}{\theta}$$

Ovviamente dato che  $p = qR \rightarrow dp/p = dR/R$

Per determinare  $\theta$  devo avere almeno 4 punti ( 2 prima e 2 dopo il magnete), perché mi servono 2 direzioni.

# Misure d'impulso



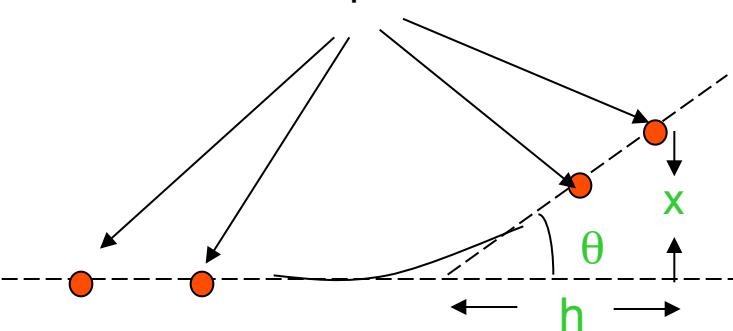
Ricordando che

$$\frac{\sigma(p)}{p} = \frac{\sigma(\theta)}{\theta} \Rightarrow$$

$$\frac{\sigma(p)}{p} = \frac{2\sigma(x)/h}{qB_y L} \cdot p \quad h = z_4 - z_3$$

$\sigma(p) \propto \sigma(x)p^2/BLh$

- dipende dal quadrato di  $p$
- dipende dalla risoluzione spaziale della coordinata nel piano di curvatura
- dipende dal braccio di leva del magnete



$$\frac{\sigma(p)}{p} = \frac{\sigma(\theta)}{\theta} \quad \Rightarrow$$

## Misure d'impulso

$$\frac{\sigma(p)}{p} = \frac{2\sigma(x)/h}{qB_y L} \cdot p$$

$p = qR$  Per esempio con  $L = 1m$ ,  $h = 1m$ ,  $B = 1T$  e  $\sigma x = 0.2 \text{ mm}$ , si ha

$$\frac{\sigma(p)}{p} = 1.3 \times 10^{-3} \cdot p \quad [GeV/c]$$

A seconda della qualità dell'apparato si possono ottenere risoluzioni :

$$\frac{\sigma(p)}{p} = (10^{-3} \div 10^{-4}) \cdot p \quad [GeV/c]$$

In esperimenti di astroparticelle e' tradizione definire la Maximum Detectable Rigidity, cioe' la rigidita' o momento tale che

$$\frac{\sigma(p_{\max})}{p_{\max}} = 1$$

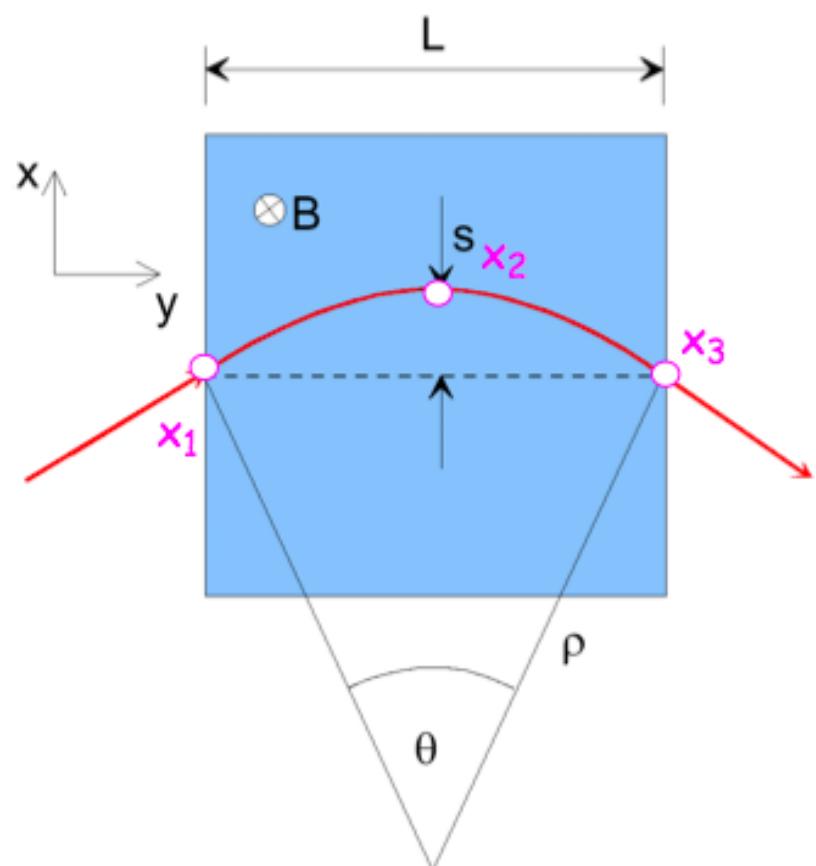
Uno spettrometro magnetico con risoluzione data dalla (1) tipicamente può misurare impulsi fino a :

$$p_{\max} = 1 \div 10 [Tev/c]$$

Un altro metodo utilizzato per determinare l'impulso è la misura della sagitta ( $s$ ), in cui le coordinate lungo la traiettoria sono misurate NELL campo magnetico.

Momentum is determined by measurement of **track curvature**  $\kappa = 1/\rho$  in  $B$  field:

Measure **sagitta  $s$**  of the track. For the momentum component transverse to  $B$  field:



$$p_T = qB\rho \quad q = ez$$

Units:  $p_T[\text{GeV}] = 0.3B[\text{T}]\rho[\text{m}] z$

$$\frac{L/2}{\rho} = \sin \frac{\theta}{2} \approx \frac{\theta}{2} \text{ (for small } \theta) \Rightarrow \theta \approx \frac{L}{\rho} = \frac{0.3B \cdot L}{p_T} z$$

$$s = \rho \left(1 - \cos \frac{\theta}{2}\right) \approx \rho \left(1 - \left(1 - \frac{1}{2} \frac{\theta^2}{4}\right)\right) = \rho \frac{\theta^2}{8} \approx \frac{0.3L^2B}{8p_T} z$$

For the simple case of **three measurements**:

$$s = x_2 - (x_1 + x_3)/2 \Rightarrow ds = dx_2 - dx_1/2 - dx_3/2$$

with  $\sigma_x = dx_i$  uncorrelated error of single measurement:

$$\sigma_s^2 = \sigma_x^2 + \frac{\sigma_x^2}{4} \cdot 2 = \frac{3}{2} \sigma_x^2$$

$$\frac{\sigma_{p_\perp}}{p_\perp} = \frac{\sigma_s}{s} = \frac{\sqrt{3/2}\sigma_x}{Z(0.3BL^2)/(8p_\perp)} = \frac{8p_\perp\sqrt{3/2}\sigma_x}{0.3BL^2 Z} = 32.6 \frac{p_\perp\sigma_x}{BL^2 Z} \text{ (m, GeV/c, T)}$$

For 3 points the relative momentum resolution is given by:  $\frac{\sigma(p_T)}{p_T} = \frac{\sigma_s}{s} = \sqrt{\frac{3}{2}}\sigma_x \cdot \frac{8p_T}{0.3BL^2}$

- degrades linearly with transverse momentum
- improves linearly with increasing B field
- improves quadratically with radial extension of detector

In the case of  $N$  equidistant measurements according to Gluckstern [NIM 24 (1963) 381]:

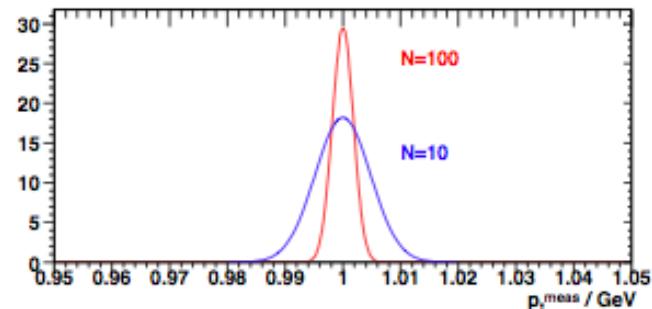
$$\boxed{\frac{\sigma(p_T)}{p_T} = \frac{\sigma(\kappa)}{\kappa} = \frac{\sigma_x \cdot p_T}{0.3BL^2} \sqrt{\frac{720}{(N+4)}}} \quad (\text{for } N \geq 10, \text{ curvature } \kappa = 1/\rho)$$

- Increases only as  $N^{1/2}$  with the nbr. of coord measurement

Example: For  $p_T = 1\text{GeV}$ ,  $L = 1\text{m}$ ,  $B = 1\text{T}$ ,  $\sigma_x = 200\mu\text{m}$  and  $N = 10$  one obtains:

$$\frac{\sigma(p_T)}{p_T} \approx 0.5\% \quad \text{for a sagitta } s \approx 3.8\text{cm}$$

Important track detector parameter:  $\frac{\sigma(p_T)}{p_T^2}$  (%/GeV)



# Cfr.: Effect of Multiple Scattering on Resolution

The position measurement contribution to the momentum resolution is only part of the story. We also have contributions from multiple scattering (MS) and angular resolution. Previously we saw that the momentum resolution contribution to MS was given by:

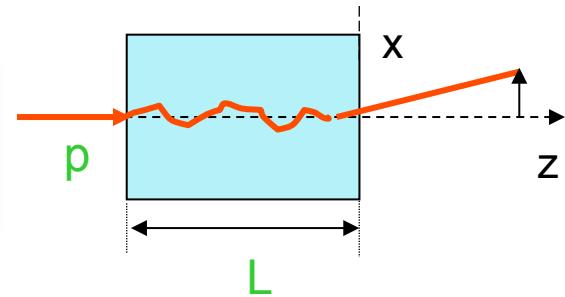
- Approximate relation (PDG):

$$\theta_0 = \theta_{plane}^{RMS} \approx \frac{13.6 \text{ MeV}}{p\beta c} z \sqrt{\frac{L}{X_0}}$$

i.e.  $\theta_0 \propto \frac{1}{\beta p} \sqrt{\frac{L}{X_0}}$

Charge of incident particle

Radiation length of absorbing material



- Apparent sagitta due to multiple scattering (from PDG):

$$s_{app} = r\theta^2/8 \text{ e } r = L/\theta \text{ per cio' } s_{app} = L\theta/8$$

Il calcolo esatto da'

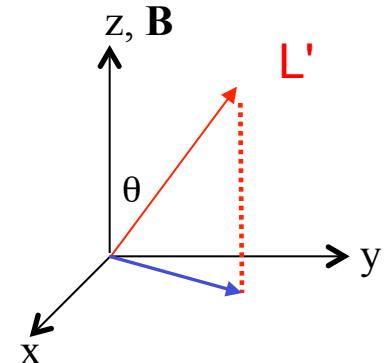
$$s_{plane} = \frac{L\theta_0}{4\sqrt{3}}$$

# More on Momentum Resolution

$$\frac{\sigma_{p_T}}{p_T} = \frac{s_{plane}^{rms}}{s_B} = \frac{\frac{L'}{4\sqrt{3}} \frac{13.6 \times 10^{-3}}{p\beta} z \sqrt{L'/X_0}}{0.3BL^2z/(8p_\perp)} \quad \text{with } L' = L/\sin\theta, p_\perp = p\sin\theta$$

If a particle enters the B field with an angle  $\theta$

$$\frac{\sigma_{p_T}}{p_T} = \frac{52.3 \times 10^{-3}}{\beta B \sqrt{LX_0 \sin\theta}}$$



**MS depends on the total path length ( $L'$ ) and momentum ( $p$ ).**

**Bending in the magnetic field depends on  $p_\perp$  ( $= p\sin\theta$ ) and projected path length ( $L$ ).**

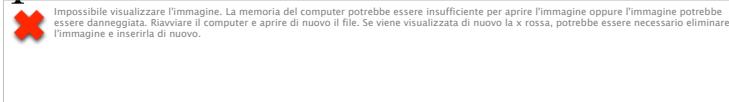
combined resolution is the two added in quadrature:

$$\left(\frac{\sigma_{p_\perp}}{p_\perp}\right)^2 = \left(\sqrt{\frac{720}{n+4}} \frac{\sigma_y p_\perp}{(0.3BL^2)}\right)^2 + \left(\frac{52.3 \times 10^{-3}}{\beta B \sqrt{LX_0 \sin\theta}}\right)^2 \quad (\text{m, GeV/c, T})$$

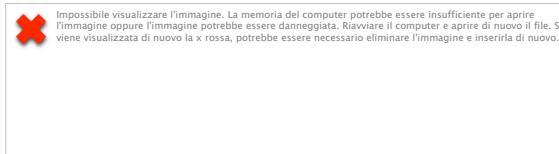
Technically speaking, the above is only the transverse momentum ( $p_\perp$ ) resolution.  
We want an expression for the total momentum resolution!

# Even More on Momentum Resolution

We can get an expression for the total momentum ( $p$ ) resolution using:

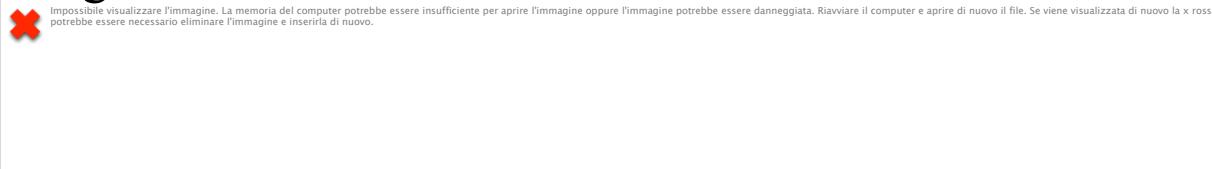


& treating  $\theta$  and  $p_{\perp}$  as independent variables. Using propagation of errors we find:



Often detectors measure the  $r\phi$  coordinate independently of the  $z$  coordinate. In these cases  $p_{\perp}$  and  $\theta$  are independent.

Putting it all together we have for the total momentum resolution:



GeV/c, T, m, radians

Position resolution

Multiple scattering

Angular resolution

While the above expression is only approximate it illustrates many important features:

- $p_{\perp}$  resolution improves as  $B^{-1}$  and depends on  $p$  as  $L^{-2}$  or  $L^{-1/2}$ .
- For low momentum ( $\beta \rightarrow 0$ ), MS will dominate the momentum resolution.
- Improving the spatial resolution ( $\sigma_y$ ) only improves momentum resolution if the first term dominate.
- Angular resolution is not usually the most important term since  $\theta_{\min} \approx 30-45^\circ$  and  $\sigma_\theta \approx 10^{-3}$  rad.

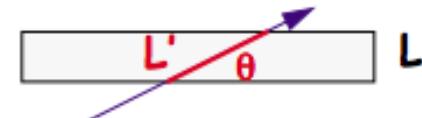
For more detailed information must do a Monte Carlo simulation (GEANT+detector).

Include: hit efficiencies, discrete scattering off of wires, non-gaussian tails, etc, etc....

The contribution to the momentum error from MS is given by:

$$\left. \frac{\sigma(p_T)}{p_T} \right|_{MS} = \frac{\sigma^{MS}(s)}{s} = 4 \frac{\frac{L'}{\sqrt{3}} \frac{13.6 \times 10^{-3}}{p\beta} z \sqrt{\frac{L'}{X_0}}}{\frac{0.3BL^2z/(8p_T)}{\beta B \sqrt{LX_0 \sin \theta}}} = \frac{0.053}{\beta B \sqrt{LX_0 \sin \theta}} \quad \text{with} \quad \begin{aligned} L' &= L/\sin \theta && \text{total path} \\ p_T &= p \sin \theta && \end{aligned}$$

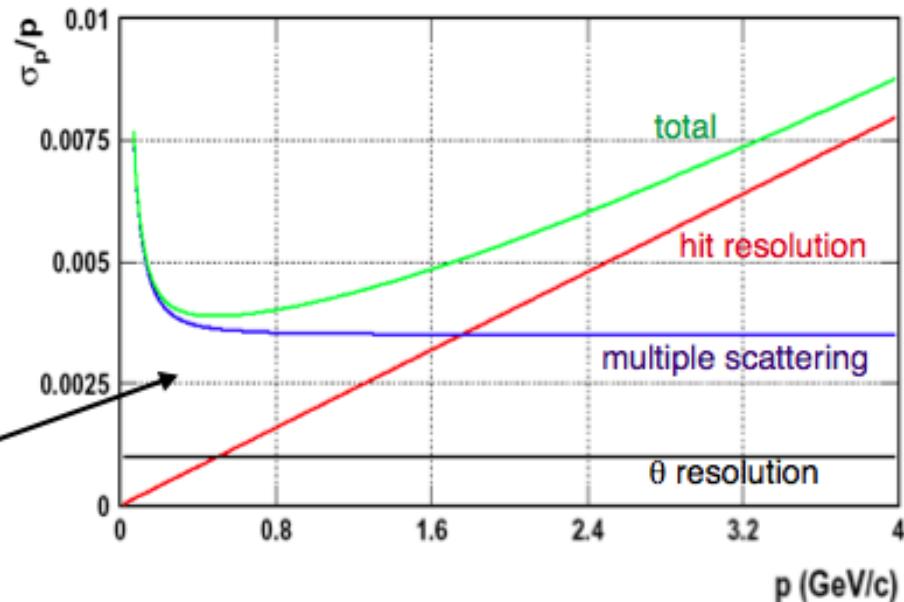
for  $\beta \rightarrow 1$  this part is momentum independent!



The combined total momentum error is:

$$\left( \frac{\sigma_p}{p} \right)^2 = \left( \sqrt{\frac{720}{N+4}} \frac{\sigma_x p \sin \theta}{0.3BL^2} \right)^2 + \left( \frac{0.053}{\beta B \sqrt{LX_0 \sin \theta}} \right)^2 + (\cot \theta \sigma_\theta)^2$$

Example for momentum dependence  
of individual contributions



# Still More on Momentum Resolution

Let's examine the momentum resolution equation for a **typical particle spectrometer**:

$$B=1.5\text{ T}$$

$$\sigma_y = 1.5 \times 10^{-4} \text{ m}$$

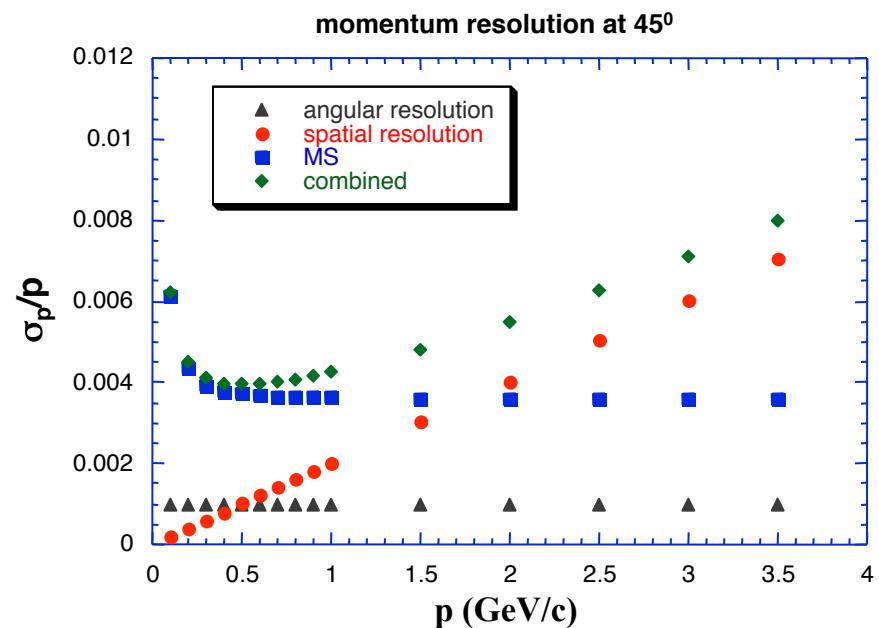
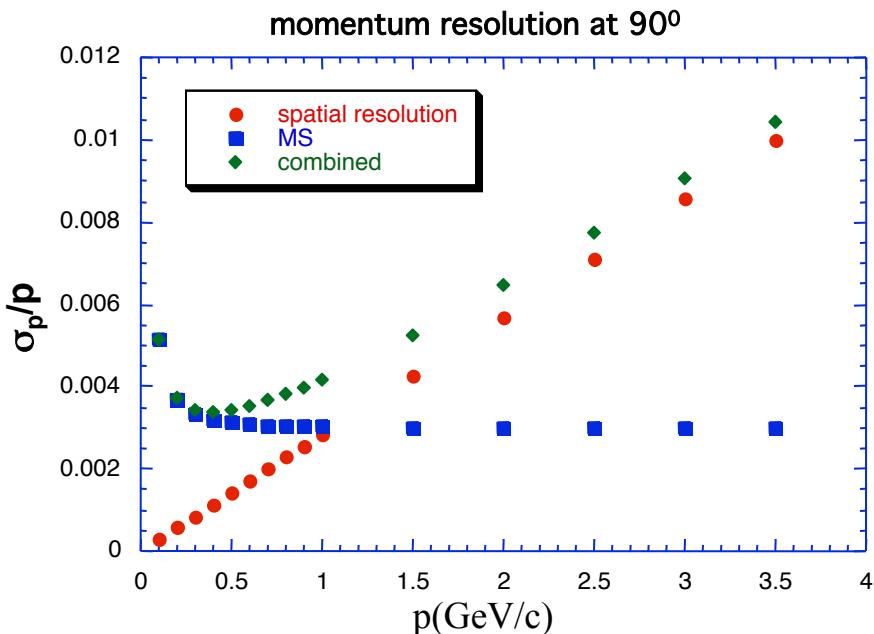
$$N=50$$

$$\sigma_\theta = 10^{-3} \text{ radians}$$

$$L=0.8 \text{ m}$$

$$L_r = 166.7 \text{ m (gas+wires)}$$

$$\left(\frac{\sigma_p}{p}\right)^2 = \left(2.85 \times 10^{-3} p \sin \theta\right)^2 + \left(\frac{3.0 \times 10^{-3}}{\beta \sqrt{\sin \theta}}\right)^2 + \left(3.0 \times 10^{-3} \cot \theta\right)^2$$

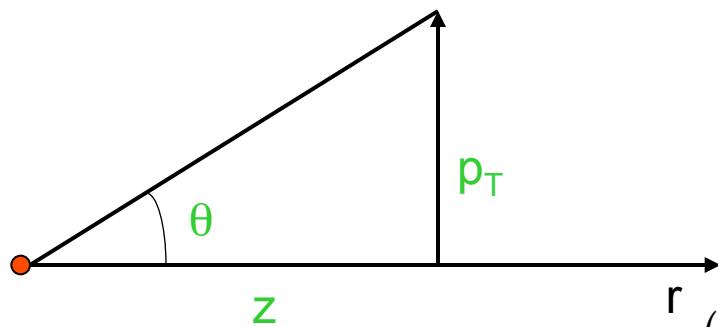


# Misure d'impulso

L'impulso totale della particella è ottenuto da  $p_T$  e dall'angolo polare  $\theta$ :

$$p = \frac{p_T}{\sin \vartheta}$$

Come nel caso del piano  $r\phi$  (trasverso) anche la misura dell'angolo polare ha un errore, sia dovuto alla risoluzione del tracciatore, sia allo scattering multiplo.



Nel caso di una misura di 2 sole  $z$

$$\sigma(\vartheta) = \frac{\sigma(z)}{z} \sqrt{2}$$

Se la traccia è misurata in  $N$  punti equidistanti si avrà:

$$\sigma(\vartheta)^{tr} = \frac{r}{z} \cdot \sqrt{\frac{12(N-1)}{N(N+1)}}$$