MILP Constraints — Restaurant HVAC System

Task 1: Optimal in Hindsight Solution

Sets and Indices

Symbol	Description
T = {0, 1, ..., 9}	Set of time slots (hours)
R = {1, 2}	Set of rooms
t in T	Time index
r in R	Room index

Variables

Symbol	Type	Description
p[r,t] in [0, P_max]	Continuous	Heater power of room r at hour t (kW)
v[t] in {0,1}	Binary	Ventilation ON/OFF at hour t
temp[r,t] in R	Continuous	Temperature of room r at hour t (°C)
hum[t] >= 0	Continuous	Humidity of the whole place at hour t (%)
delta_low[r,t] in {0,1}	Binary	Low temperature overrule controller active
delta_high[r,t] in {0,1}	Binary	High temperature overrule controller active
delta_hum[t] in {0,1}	Binary	Humidity overrule controller active

Objective Function

Minimize the total electricity cost from heating and ventilation:

```
min SUM_{t in T} [ lambda_t * ( SUM_{r in R} p[r,t] + P_vent * v[t] ) ]
```

Constraints

1. Initial Temperature Condition

The temperature of each room is initialized at T0 at hour 0.

```
temp[r,0] = T0 for all r in R
```

2. Temperature Dynamics

The temperature of each room evolves as a linear function of thermal exchange, heat loss, heater power, ventilation cooling, and occupancy.

```
temp[r,t] = temp[r,t-1]
+ zeta_exch * ( temp[3-r, t-1] - temp[r, t-1] )
+ zeta_loss * ( T_out[t-1] - temp[r, t-1] )
+ zeta_conv * p[r, t-1]
- zeta_cool * v[t-1]
+ zeta_occ * occ[r, t-1]
```

```
for all r in R, t in T\{0}
```

3. Initial Humidity Condition

The humidity is initialized at H0 at hour 0.

```
hum[0] = H0
```

4. Humidity Dynamics

The humidity evolves as a function of total occupancy and ventilation.

```
hum[t] = hum[t-1]
+ eta_occ * ( occ[1,t-1] + occ[2,t-1] )
- eta_vent * v[t-1]
for all t in T\{0}
```

5. Low Temperature Overrule Controller — Activation

The controller activates when the room temperature drops below T_low.

```
M * delta_low[r,t] >= T_low - temp[r,t] for all r, t
```

6. Low Temperature Overrule Controller — Memory

The controller remains active if it was active and temperature is still below T_ok.

```
M * delta_low[r,t] >= delta_low[r,t-1] * T_ok - temp[r,t]
for all r, t in T\{0}
```

7. Low Temperature Overrule Controller — Deactivation

The controller deactivates when the temperature surpasses T_ok.

```
M * (1 - delta_low[r,t]) >= temp[r,t] - T_ok for all r, t
```

8. Low Temperature Overrule Controller — Force Heater to Maximum

When the low overrule controller is active, the heater is forced to maximum power.

```
p[r,t] >= P_max * delta_low[r,t] for all r, t
```

9. High Temperature Overrule Controller — Activation

The controller activates when the room temperature rises above T_high.

```
M * delta_high[r,t] >= temp[r,t] - T_high for all r, t
```

10. High Temperature Overrule Controller — Memory

The controller remains active if it was active and temperature is still above T_ok.

```
M * delta_high[r,t] >= delta_high[r,t-1] * T_high - temp[r,t]
for all r, t in T\{0}
```

11. High Temperature Overrule Controller — Deactivation

The controller deactivates when the temperature drops below T_ok.

```
M * (1 - delta_high[r,t]) >= T_ok - temp[r,t] for all r, t
```

12. High Temperature Overrule Controller — Force Heater to Zero

When the high overrule controller is active, the heater is forced to zero power.

```
p[r,t] <= P_max * (1 - delta_high[r,t]) for all r, t
```

13. Conflict Between Controllers

The low and high temperature overrule controllers cannot be active simultaneously.

```
delta_low[r,t] + delta_high[r,t] <= 1 for all r, t
```

14. Humidity Overrule Controller — Activation

The humidity overrule controller activates when humidity exceeds H_high.

```
M * delta_hum[t] >= hum[t] - H_high for all t
```

15. Humidity Overrule Controller — Force Ventilation ON

When the humidity overrule controller is active, ventilation is forced ON.

```
v[t] >= delta_hum[t] for all t
```

16. Ventilation Inertia

If ventilation is turned OFF at hour t, it must have been ON for the 3 preceding hours.

```
v[t-2] + v[t-1] + v[t] >= 3 * (v[t-1] - v[t]) for all t >= 2
```

17. Initial Condition — Low Temperature Controller

The low temperature overrule controller is inactive at hour 0.

```
delta_low[r,0] = 0 for all r in R
```

18. Initial Condition — High Temperature Controller

The high temperature overrule controller is inactive at hour 0.

```
delta_high[r,0] = 0 for all r in R
```