Assignment 2

Due: 12. November 2021, 10:15 CET

General information: This assignment should be completed in groups of **three to four people**. Submit your assignment via the corresponding LernraumPlus activity. You can only upload one file, therefore make sure to place all your files required to run your solution (.py, .ipynb, .pdf etc) into **one archive** and just upload that archive.

Exercise information: This assignment may be the only one without any programming tasks. You may either write the solutions digitally (LaTex, Markdown, etc) or scan/photo your handwritten solutions (make sure the quality is readable). Regardless, you should convert your solution to a pdf document before submitting.

For all tasks, make sure not to only write down the final answer but also key steps in order to justify your answer.

Exercise 1: (5 Points)

Consider the following joint distribution over three binary variables:

world	A	B	C	$p(w_i)$
w_1	true	true	true	0.050
w_2	true	true	false	0.125
w_3	true	false	true	0.225
w_4	true	false	false	0.175
w_5	false	true	true	0.025
w_6	false	true	false	0.050
w_7	false	false	true	0.100
w_8	false	false	false	0.250

Task 1: What is $p(A = true)$? $p(B = true)$? $p(C = false)$?

Task 2: What are the probabilities for the different worlds when conditioned on the event $C = true$, i.e. $p(w_i|C = true)$?

Task 3: What is $p(A = false|C = true)$? $p(B = true|C = true)$?

Exercise 2: (5 Points)

Consider the following joint distribution over three binary variables:

world	A	B	C	$p(w_i)$
w_1	true	true	true	0.24
w_2	true	true	false	0.04
w_3	true	false	true	0.16
w_4	true	false	false	0.06
w_5	false	true	true	0.06
w_6	false	true	false	0.16
w_7	false	false	true	0.04
w_8	false	false	false	0.24

Task 1: Test for each of the three variable pairs (A,B; A,C and B,C) if they are conditionally independent given the third variable.

Exercise 3: (5 Points)

Bayes' theorem is an important prerequisite for a lot of methods dealing with uncertain data. Given the likelihood $P(B|A)$, a prior belief $P(A)$, and the marginal likelihood $P(B)$ the posterior belief $P(A|B)$ can be calculated:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Using Bayes' theorem one has not to rely on mere truth/false statements anymore, as is the case in most traditional AI approaches. Instead, probability values that represent ones degrees of belief, are assigned to statements. Bayes' theorem can then be used to calculate how this degree of belief changes under incoming new information that itself has a specific uncertainty.

Consider the following classical example:

`It is raining:`	$p(A = true)$	$= 0.6$
`It is not raining:`	$p(A = false)$	$= 0.4$
`The lawn is wet, if it rains:`	$p(B = true \mid A = true)$	$= 0.1$
`The lawn is not wet, if it rains:`	$p(B = false \mid A = true)$	$= 0.9$
`The lawn is wet, if it does not rain:`	$p(B = true \mid A = false)$	$= 0.85$
`The lawn is not wet, it it does not rain:`	$p(B = false \mid A = false)$	$= 0.15$

Task 1: Calculate by hand: $p(B = false)$

Task 2: Calculate by hand: $p(A = true|B = true)$

Exercise 4: (5 Points)

A standard XOR logic gate is given by the table below. If we observe that the output of the XOR gate is 0, we know that either A and B were both 0, or A and B were both 1, we do not however know which state A was in, without knowing anything about B.

A	B	A xor B
0	0	0
0	1	1
1	0	1
1	1	0

Now, consider a *soft* version of the XOR gate given by the next table, so that the gate stochastically outputs $C = 1$ depending on its inputs. Additionally, consider $A \perp\!\!\!\perp B$ and $p(A = 1) = 0.6$ and $p(B = 1) = 0.8$.

A	B	$p(C = 1 \mid A, B)$
0	0	0.15
0	1	0.95
1	0	0.8
1	1	0.25

Task 1: What is the probability that A was turned on, given that the soft XOR outputs 0, i.e. what is p(A=1 | C=0)?