Homework 2: SAR Interferometry (InSAR)

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Instructions

- The output of the homework is a set of MATLAB files and a report (Word, LaTeX) containing the answer to the proposed questions, the required figures and a brief explanation of them. The MATLAB files are demo so they must be error-free and they should not require any user input.
- The text defines requirements and parameters. Missing information -if any- are a free choice of the engineer.
- The deadline: **31/01/2023** (included).
- Compress all files in a ZIP package and give the name H1XXXXXXXX.zip where XXXXXXXX is the ID number of the student.
- The ZIP must be sent by email at marco.manzoni@polimi.it
- If you have any question or doubts, send a request at marco.manzoni@polimi.it
- The oral discussion of the homework will be scheduled later on.
- Partial solving of the following points is allowed.

1 Introduction

This homework aims to simulate a space-borne radar acquisition and perform SAR interferometry (InSAR) on two (or more) images. In particular, you will try to estimate the topography of the scene starting from an interferometric couple. Estimating the scene's elevation is not the only task in which InSAR excels. InSAR is useful also for deformation estimation, atmospheric mapping, change detection, and more. In Figure 1 you can see a so-called interferogram: this is how an earthquake is seen through the eyes of a radar satellite.

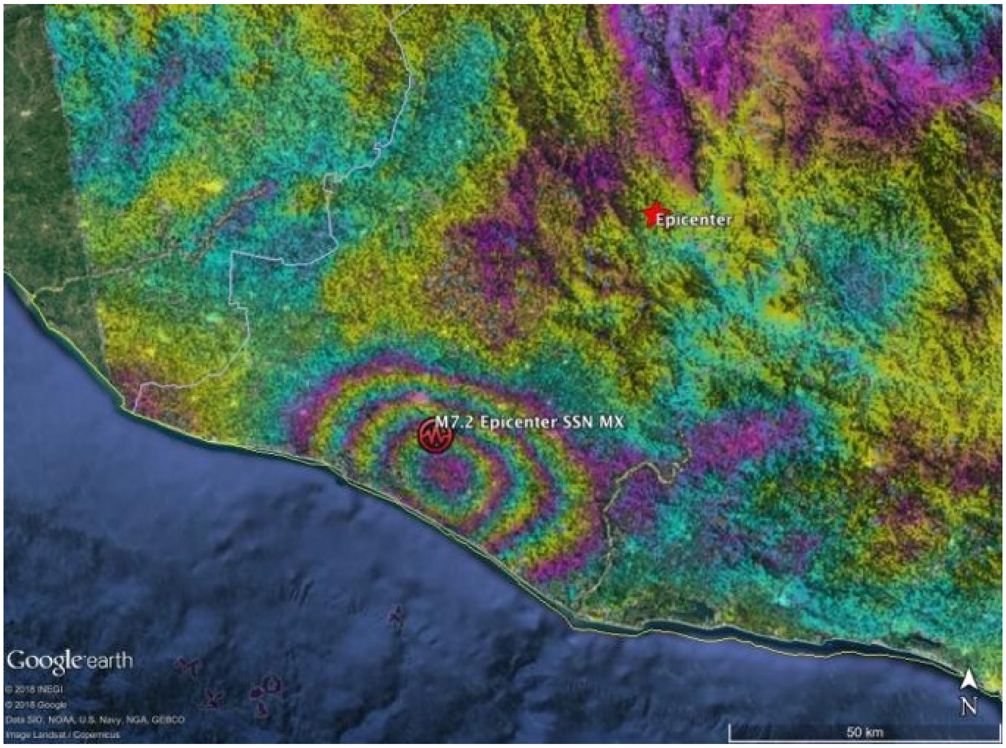

Figure 1: An interferogram showing the deformation induced by an earthquake. Source: NASA

2 Mono-dimensional SAR data simulation.

Let us now suppose to have a mono-dimensional SAR geometry like the one in Figure 2. In this figure, we see two satellites: they can acquire the scene simultaneously (single-pass interferometry) or on two different time instants (multi-pass interferometry). One of the two satellites is called a "master" satellite (M), and the other a "slave" satellite (S). Their nominal height over the ground is 693 km (as the European Satellite Sentinel-1), and they are separated by a horizontal baseline (B) of 200 meters. The operational frequency is 5.4 GHz with a bandwidth that guarantees 5 meters of slant range resolution (once again, precisely the same as Sentinel-1). The two satellites are imaging a scene that is not flat but shows some topography. The mean incidence angle (θ) over a flat reference surface is 35 degrees. The flat reference surface is depicted In Figure 2 as the y axis. The y axis is called the *ground range* in radar jargon, while the z axis is the height.

This chapter of the homework aims to simulate two mono-dimensional SAR images. To do so, the student is required to follow these steps:

1. First of all, you have to define a scenario. In particular, you must know the ground range, height, and complex reflectivity of each target in the scene. You can then define a pixel with three numbers and put them in a vector.

$$\mathbf{p}_i = [y_i, z_i, t_i]^T \tag{1}$$

   where the first element is the pixel ground range coordinate, the second is the pixel height, and the third is the complex reflectivity.

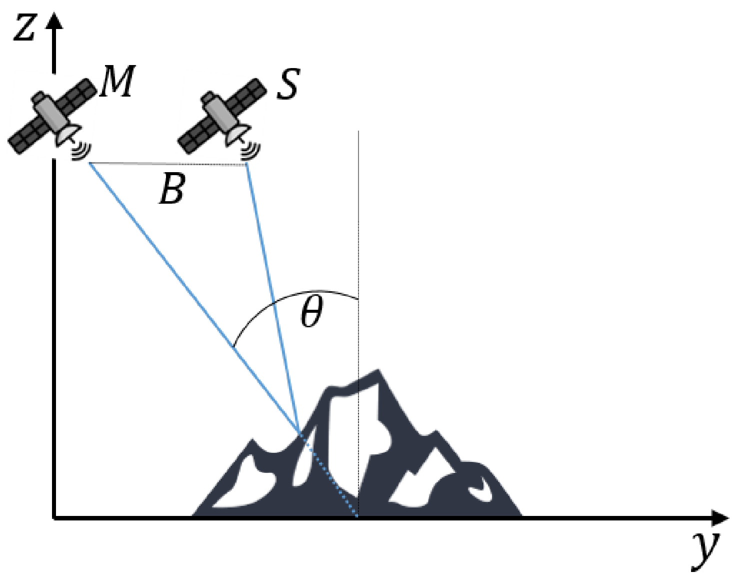

Figure 2: Geometry of the InSAR acquisitions.

> *Hint:* In a real scenario, the elementary scatterers will form a continuous surface on the ground. This is not possible on a PC. Thus, we will make the complex reflectivity function a finely sampled discrete function. It is suggested to generate points spanning 1 km of ground, with sampling in the ground range direction of $\rho_{rg}/4$ (a quarter of the range resolution) and with an elevation profile of your choice. For example, a Gaussian function (smooth topography).
> Remember that the scene's center should have an incidence angle of 35 deg. You know the height of the satellites, the incidence angle, the size of the scene...

2. Define a slant range axis (r). This axis is significantly different w.r.t. the ground range axis. The former is the natural radar line of sight axis of the distances. Notice that the slant range axis is defined once for all the acquisitions.

> *Hint:* When you define an axis, always ask yourself: at which number should this axis start? At which number should it end? What is the sampling spacing? In this specific context: at which distance do we have the closest target in the scene? At which distance do we have the farthest target in the scene? What is the range resolution?

3. Simulate the acquisition. You already know that a SAR image is formed by the coherent sum of all the single scatterers in the scene, properly weighted by the SAR Impulse Response Function (IRF) shifted in the slant range position of the scatterers and phase

shifted accordingly:

$$I^n(r) = \sum_{i=1}^{N_p} t_i \text{sinc}\left[\frac{r - R^n(\mathbf{p}_i)}{\rho_{rg}}\right] e^{-j\frac{4\pi}{\lambda}R^n(\mathbf{p}_i)} \tag{2}$$

where $I^n(r)$ is the n^{th} image at the range r, N_p is the number of scatterers in the scene, $R^n(\mathbf{p}_i)$ is the geometrical distance from the n^{th} sensor to the target in coordinates $\mathbf{p}_i$ and λ is the wavelength. The expression of $R^n(\mathbf{p}_i)$ is given by:

$$R^n(\mathbf{p}_i) = \sqrt{(Y^n - y_i)^2 + (Z^n - z_i)^2} \tag{3}$$

where Y^n and Z^n are the coordinates of the n^{th} satellite.

4. Plot the absolute value of the two SAR images (the master and the slave).

> *Hint:* Use a subplot to show that the two images are not aligned in the slant range domain. This is expected since the slant range axis represents the distances from the radar to the targets. If the radars are in different positions (i.e. they have a baseline), the targets will be in different positions in the slant range axis. In Figure 3 an example is depicted.

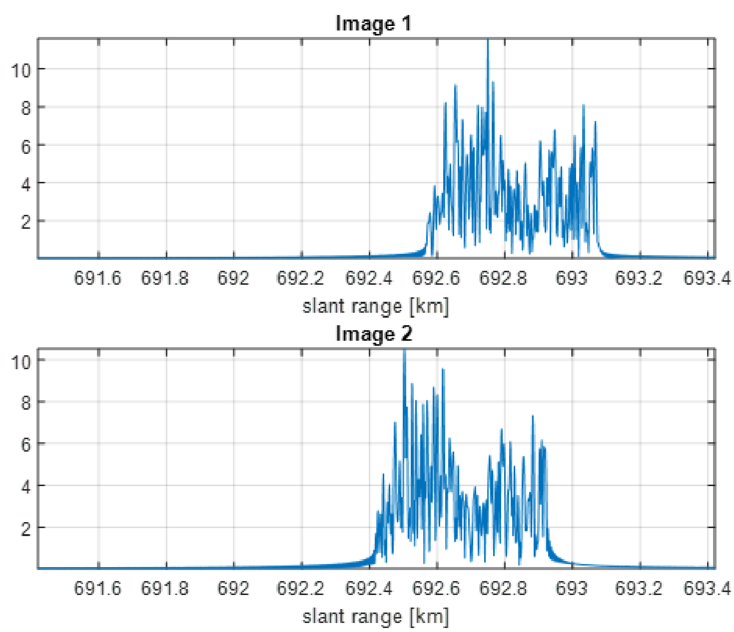

Figure 3: The two mono-dimensional SAR images are not aligned (they are not coregistered).

You have now completed the data simulation part! Now it is time to coregister the images.

3 Image coregistration

As you can see from Figure 3, the two images are not aligned. To clarify this concept, let us suppose to have a target at a specific ground range/height position. The radar image, however, is not in ground range/height coordinates, but in slant range coordinates, or, in other words, in the *distance-from-the-sensor* coordinates. If, as in Figure 2, the two satellites are displaced, the distances will be different. Therefore, the target will appear in different slant range positions in the two SAR images.

3.1 Image coregistration in slant range

> **Warning:** This is the trickiest part of the homework. If you can't figure it out, skip to Section 3.2.

To fix this problem, the two images must be coregistered. There are several methods to coregister images. Here, we will rely on a reference (known) elevation model. Follow the steps below to implement the coregistration.

1. Define the coordinates of a reference surface. This could be, for example, a digital elevation model of the scene or even a simple flat surface with a height equal to zero. Each pixel of the reference surface is localized in the ground range/height coordinate system by two values, namely y_i^{REF} and z_i^{REF} where this i indicates the i^{th} pixel of the reference surface. It is suggested to use as y_i^{REF} the same value used in the mono-dimensional SAR data simulation. The values of z_i^{REF} can be set to zero for all the values of i.

2. Now you can calculate the distances from the master satellite (choose one of the two) to the reference surface, namely:

$$R^M(\mathbf{p}_i^{REF}) = \sqrt{(Y^M - y_i^{REF})^2 + (Z^M - z_i^{REF})^2} \tag{4}$$

   where Y^M and Z^M are the ground range/height coordinates of the master satellite.

3. Write the location of each pixel of the reference surface as a function of the slant range of the master image. Remember that for each pixel of the reference surface, you know the height, the ground range position, and the distance from the master. What we want to do is to project the height and the ground range position in radar coordinates (slant range). This is done by simple interpolation. In pseudocode:

```
z(r) = interp1(R_ref, z_ref, r)
y(r) = interp1(R_ref, y_ref, r)
```

   where the first element in the interpolation function indicates the distances calculated in Equation 4, the second element is the vector containing all the reference heights

(z_i^{REF}) or the reference ground ranges (y_i^{REF}), while the last one in the slant range axis defined in point 2 of section 2. You just did a projection of the reference surface into the radar coordinates.

4. You have your reference surface projected in the radar coordinates of the master. It is time to calculate distances again, this time not (as in the previous case) from the master satellite to each pixel of the reference surface in ground-range/height coordinates, but from each satellite to each pixel in radar coordinates.

$$r_{REF}^n = \sqrt{(Y^n - y(r))^2 + (Z^n - z(r))^2} \tag{5}$$

At this point, for each image and each pixel of the SAR image (in slant range coordinates), you know the distance from the reference surface. Since the two images are acquired from different positions in space, the distances will not be the same. There will be a difference that we want to compensate for.

5. It is time to perform the coregistration itself. Let's calculate the difference in distances with respect to a fixed master image:

$$\Delta r^n = r_{REF}^n - r_{REF}^M; \tag{6}$$

in this way, we find the shift for every pixel, then we want to apply this shift, and we can do this via interpolation again. In pseudo-code:

```
I_n_c_r = interp1(r, I_n_r, r+delta_r_n);
```

where the element at the left of the equal sign is the coregistered n^{th} image, the first input to the interpolation function is the range axis, the second element is the n^{th} non-coregistered image and the last is the query points of the interpolation function, in other words, the corrected slant range axis.

6. Plot the absolute value of the two coregistered SAR images (the master and the slave). They should now be aligned.

3.2 Images coregistration by geocoding

Warning: This section must be completed only if you couldn't complete the previous section. This section, indeed, is a loophole that allows for easier implementation of the coregistration procedure. If you already did the coregistration in the previous section, skip to Section 4.

There is an easier and more intuitive way to coregister the two images. You can use this method as a loophole if you can't figure out the way to coregister the two images in the previous section.

1. Define the coordinates of a reference surface. This could be, for example, a digital elevation model of the scene or even a simple flat surface with a height equal to zero. Each pixel of the reference surface is localized in the ground range/height coordinate system by two values, namely y_i^{REF} and z_i^{REF} where this i indicates the i^{th} pixel of the reference surface. It is suggested to use as y_i^{REF} the same value used in the mono-dimensional SAR data simulation. The values of z_i^{REF} can be set to zero for all the values of i.

2. Now you can calculate the distances from each satellite to the reference surface, namely:

$$R^n(\mathbf{p}_i^{REF}) = \sqrt{(Y^n - y_i^{REF})^2 + (Z^n - z_i^{REF})^2} \tag{7}$$

   where Y^n and Z^n are the ground range/height coordinates of the n^{th} satellite.

3. Project each image in ground range coordinates. This operation is also called *geocoding* and it is as simple as a single mono-dimensional interpolation:

```
I_n_c_gr = interp1(r, I_n_r, R_n);
```

   where the element at the left of the equal sign is the coregistered n^{th} image in ground range coordinates, the first input to the interpolation function is the range axis, the second element is the n^{th} non-coregistered image and the last is the query points of the interpolation function, in other words, the distances calculated in Equation 7.

4. Plot the absolute value of the two coregistered SAR images (the master and the slave). They should now be aligned. Notice that now they are in ground range coordinates and not in slant range as in the previous case.

4 The interferometric phase

Up to now, you formed two SAR images and coregistered them. It is time to form an interferogram and estimate the topography. This part of the homework will be less guided. Nevertheless, some suggestions will be given as a guideline:

1. Form the interferogram by complex conjugate multiplications between the images. Plot the interferometric phase as a function of range. Can you indicate the interferometric fringes due to the presence of the reference surface?

2. Compensate the interferogram for the reference surface. This is called interferogram flattening. *Hint:* remember that you already computed in the coregistration section the distances from each sensor to the reference surface...

3. Plot again the compensated interferogram. Are the interferometric fringes disappeared?

4. Filter the interferometric phase. You can use the "movmean" function to average, but read the function's documentation carefully and choose wisely an averaging window.

5. Plot the filtered interferometric phase. Does it wrap? Remember that a phase is always known as modulus 2π. Can you propose a way to unwrap the interferometric phase? If you can't find a way, use the unwrap function in MATLAB. In Figure 4, you can observe a wrapped and unwrapped function. Notice how the wrapped signal is in between $-\pi$ and π.

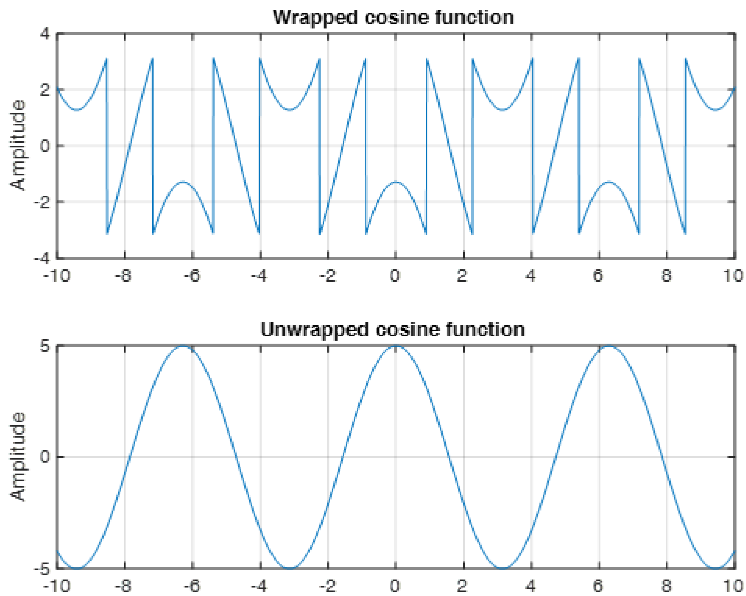

Figure 4: A wrapped signal (notice the amplitude between $-\pi$ and π and the corresponding unwrapped signal.

6. Now you can convert the interferometric phase into heights. How? First of all, you have to compute the conversion factor, namely:

$$\phi = \frac{4\pi}{\lambda} \frac{B_{\perp}}{Rsin(\theta)} z = k_z z \tag{8}$$

where ϕ is the filtered and unwrapped interferometric phase, $B_{\perp}$ is the normal baseline, R is the range from the master satellite to the reference surface, and θ the incidence angle on the reference surface. Notice that the value of k_z changes for each pixel. Moreover, you don't need to compute $B_{\perp}$... what is $B_{\perp}/R$?

7. Can you compare the true topography with the estimated one? Remember also that the estimated topography is always intended as the topography over the reference surface you chose in the flattening process. *Hint:* If your image is in radar coordinates, take the true topography and project it in radar coordinates. If you choose the second coregistration procedure, you don't need to make the projection. Now plot the true topography and overlap the estimated one.

5 (optional) decorrelation, subsidence and multi-baseline interferometry

If you are reading, it means that you are curious about InSAR (and have already completed the points above). The points below are a little bit more complicated, and they are optional.

5.1 Decorrelation

From the previous points, it seems that InSAR can always estimate the topography of the scene. However, it is so only if we have *coherence* between images, in other words, if they are correlated. In Equation 2 the complex correlation coefficient depends only upon the pixel index (i) and not on the acquisition (n). In this way, we simulated a perfectly stable scene in which, between the two acquisitions, the only thing that changes is the satellite's position. But what happens if the scene changes, or, in other words, if the complex reflectivity also depends upon time? What happens if t_M (the master acquisition) and t_S (the slave acquisition) are uncorrelated? What happens if there is a partial correlation between them? Are you still able to estimate topography? Repeat the same processing above, but this time introduce two more scenarios: a totally decorrelation in time and a partial decorrelation in time (choose a value of the correlation coefficient between 0 and 1).

5.2 Subsidence

Imagine there is no topography but the terrain sink between the two acquisitions. Can you repeat all the processing to estimate how much the terrain sank?

5.3 Multi-baseline interferometry

As the most complicated case, suppose an unknown topography and subsidence. For simplicity's sake, suppose that the subsidence can be modeled as a constant velocity sinking of the terrain. You can now use more than 2 SAR images (simulate how many images you want). Can you propose and implement a method to estimate topography and subsidence rate at the same time? *Hint:* as always, start to write the signal model of the interferometric phase. *Second hint:* look at your book...