Homework 1: Multipath in automotive radar imaging

Prof. Andrea Virgilio Monti Guarnieri
Dr. Marco Manzoni

A.Y. 2022/2023

Instructions

- The output of the homework is a set of MATLAB files and a report (Word, LaTeX) containing the answer to the proposed questions, the required figures and a brief explanation of them. The MATLAB files are demo so they must be error-free and they should not require any user input.
- The text defines requirements and parameters. Missing information -if any- are a free choice of the engineer.
- The deadline: **21/11/2022** (included).
- Compress all files in a ZIP package and give the name H1XXXXXXXX.zip where XXXXXXXX is the ID number of the student.
- The ZIP must be sent by email at marco.manzoni@polimi.it
- If you have any question or doubts, send a request at marco.manzoni@polimi.it
- The oral discussion of the homework will be scheduled later on.
- Partial solving of the following points is allowed.

1 Introduction

The objective of this homework is to understand the effect of multipath in automotive radar imaging. One of the main concerns of the automotive industry about the usage of radars on car is the effect of multipath. Multiple reflections of the electromagnetic wave can generate ghost targets in the focused image. These ghost targets may trigger dangerous maneuvers in advanced autonomous driving systems. In this homework you will learn how a radar system perceives the multipath and what are the risks associated with different antenna layout.

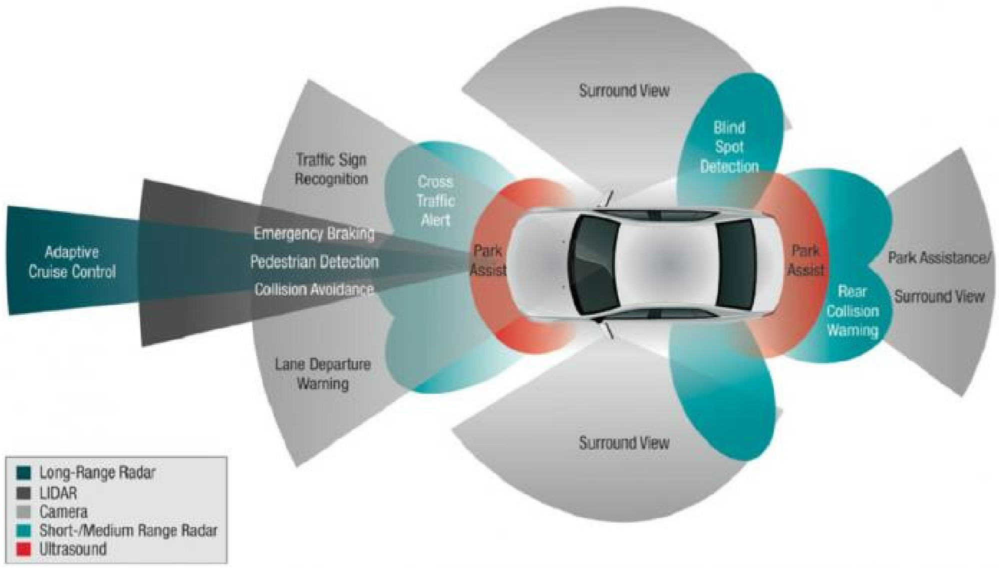

Figure 1: Radars mounted on a modern vehicle. Source: OpenPR.com

2 The front-looking automotive radar

The radar that you will design and simulate is a forward looking Uniform Linear Array (ULA) working at 77 GHz. For what concerns the direction of arrival (DoA), your client wants a maximum resolution (at the boresight of the array) of $\rho_\theta = 3.6$ degrees. The system must work at 77 GHz (W-Band) with a bandwidth $B = 2$ GHz and it will transmits linearly modulated chirp signals. First, you will design a simple monostatic array with these specifications and you will assess analytically the effect of multipath. You will confirm you hypothesis with a simulation of your designed system.

After that, you are required to design a bistatic array, where one antenna transmits an ULA receives the echo. Also in this case the effect of multipath must be analyzed and confirmed by simulations.

Lastly, a Multiple-Input Multiple-Output (MIMO) radar must be designed to reduce costs. In this case you will be required to design the number of TX/RX ULA with an appropriate number of antennas and an appropriate spacing between the elements. Also in this case the effect of multipath must be assessed via a simulation.

2.1 The monostatic array

Let's suppose to have a scenario like the one in Figure 2. The yellow car has a radar mounted in forward-looking geometry. The radar is formed by N_{TX} elements. Each one transmits and receives the electromagnetic wave. Suppose that the waveforms are orthogonal, thus they will not interfere with each other. The antennas are placed along the x direction. In the field of view there is a target, depicted in Figure 2 as a red car.

Tasks:

1. Write the signal model of the demodulated and range compressed received signal at the n^{th} antenna. Assume to be in far field and neglect any amplitude considerations. For

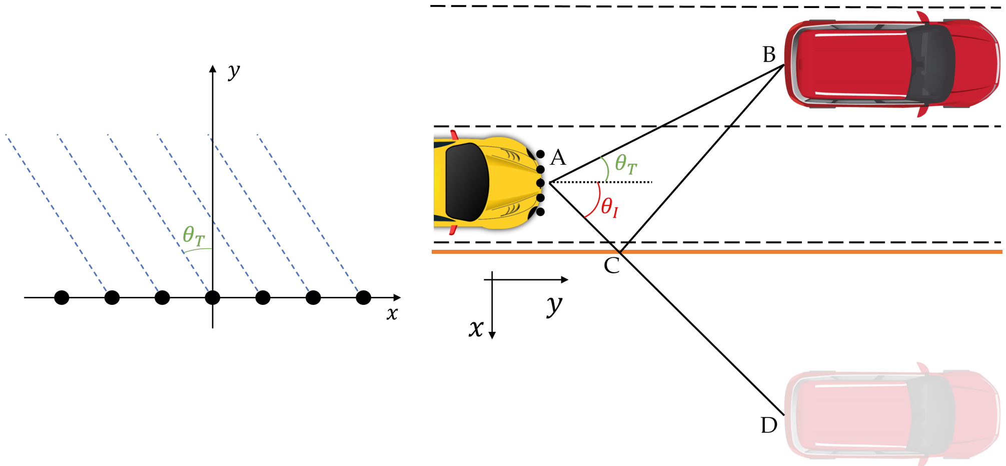

Figure 2: Right: scenario under test. The yellow car has a radar and in the field of view there is a target (the red car). The orange line represents a reflective surface. Left: a zoom over the radar.

any given range, read the signal along the channels: can you recognize this function? Can you propose a way to detected the DoA (in other words, to estimate θ_T)?

> *Hint:* Remember that the demodulated and range compressed received signal at the n^{th} antenna can be written as:
>
> $$s_{RX}^{nn}(r) = \text{sinc}\left(\frac{r - R^n(\mathbf{p}_T)}{\rho_r}\right) e^{-j\frac{4\pi}{\lambda}R^n(\mathbf{p}_T)} \tag{1}$$
>
> where $R^n(\mathbf{p}_T)$ is the distance from the n^{th} antenna to the target located in $\mathbf{p}_T = [x_t, y_t]^T$, ρ_r is the range resolution which can be derived from the bandwidth B. Expand in Taylor series the distance up to the first order, or equivalently use the plane wave approximation...

2. Given the specifications about the angular resolution and the carrier frequency, what is the required length of the array? What is the number of antenna required? What is their spacing?

> *Hint:* Look at the signal model written before and remember the Nyquist-Shannon sampling theorem.

3. Now you can simulate the system using MATLAB®. In the simulation process it is assumed that you know the position of each antenna element and the position of the target. The position of the target is a free choice of the engineer, but remember to respect the far field condition. First of all, plot the geometry of the scene (positions of the sensors and the target).

You can compute the distances from the target to the n^{th} element as:

$$R^n(\mathbf{p}_T) = \sqrt{(x_n - x_t)^2 + (y_n - y_t)^2} \tag{2}$$

Once you have generated the range compressed data, you can focus the signal using the method proposed by you in the previous task. Remember that the starting point of your focusing procedure should be a matrix with a number of rows equal to the length of your range axis r and a number of columns equal to the number of antennas in the array. This is called range-compress data matrix. Once you have estimated the range/angle position of the target from data, compare it with the ground truth. How can you prove that the resolution is the correct one?

4. Now it is time to add the multipath. In Figure 2 a reflective surface is depicted in orange. The EM wave travels the path $\overline{AB}$, then the path $\overline{BC}$ and finally $\overline{CA}$. Notice that the path length is equivalent to the path $\overline{AB}$ plus the path $\overline{DA}$. The target in light red is the so-called image target and it is specular w.r.t the real target. Write the signal model of the received signal. Can you predict what will be the position of the multipath signal?

> *Hint:* The signal model associated to the multipath can be written as:
>
> $$s_{RX}^{nn} = \text{sinc}\left(\frac{r - [R^n(\mathbf{p}_T) + R^n(\mathbf{p}_I)]}{\rho_r}\right) e^{-j\frac{2\pi}{\lambda}[R^n(\mathbf{p}_T) + R^n(\mathbf{p}_I)]} \tag{3}$$
>
> where this time $R^n(\mathbf{p}_I)$ is the distance from the n^{th} element to the image target. Expand both $R^n(\mathbf{p}_T)$ and $R^n(\mathbf{p}_I)$ in Taylor series...

5. Now simulate the multipath. In your simulation place properly an image target, compute all the distances and then focus again the signal to detected the DoA. Is it where you expected to be?

2.2 The bistatic array

The monostatic configuration is not the only one. One could think about a system where only one antenna transmits a waveform and an ULA receives the echo. In this way, since there is only one transmitter there is no need to have orthogonal waveforms. Figure 3 represents exactly this system. The central antenna in red transmits the signal and all the others (including the transmitting antenna) receive the echo. Repeat every task of the previous section. Can you spot the differences between the monostatic and the bistatic configuration? Is the multipath reconstructed in the same position for the two systems? Which one can be more dangerous?

2.3 The MIMO radar

A MIMO radar is a system in which we have N_{TX} transmitting and N_{RX} receiving antennas. Each transmitting antenna send a signal and all the receiving ones record the echo. The

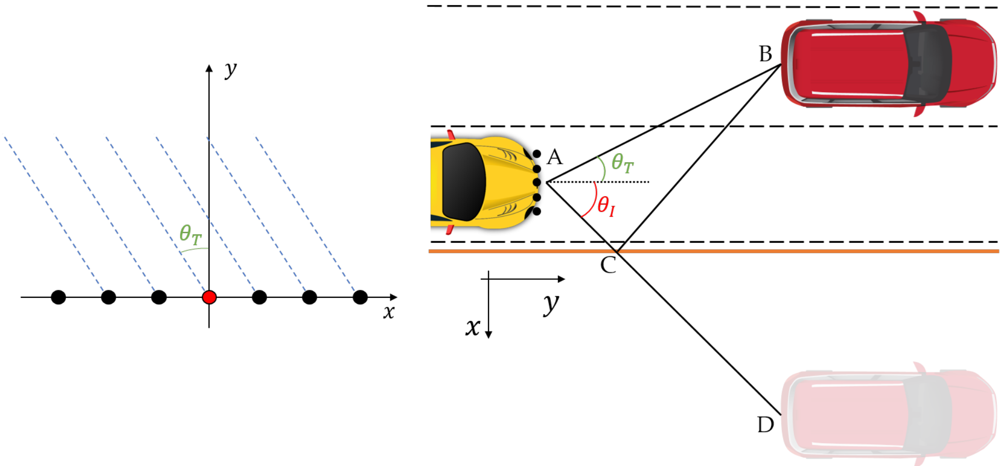

Figure 3: Bistatic geometry

transmission may happen simultaneously using orthogonal waveforms or one after the other (Time Division Multiplexing).

It can be demonstrated that a system composed by a transmitting antenna in a generic position $\mathbf{x}_{TX}$ and a receiving one in a position $\mathbf{x}_{RX}$ is equivalent to a monostatic system where a virtual antenna transmits and received the signal from the position:

$$\mathbf{x}_V = \frac{\mathbf{x}_{TX} + \mathbf{x}_{RX}}{2} \tag{4}$$

This position is also called *virtual phase center*. Ax example is shown in Figure 4 where the red dots are TX elements, the blue ones are RX elements and the virtual phase centers are depicted in purple.

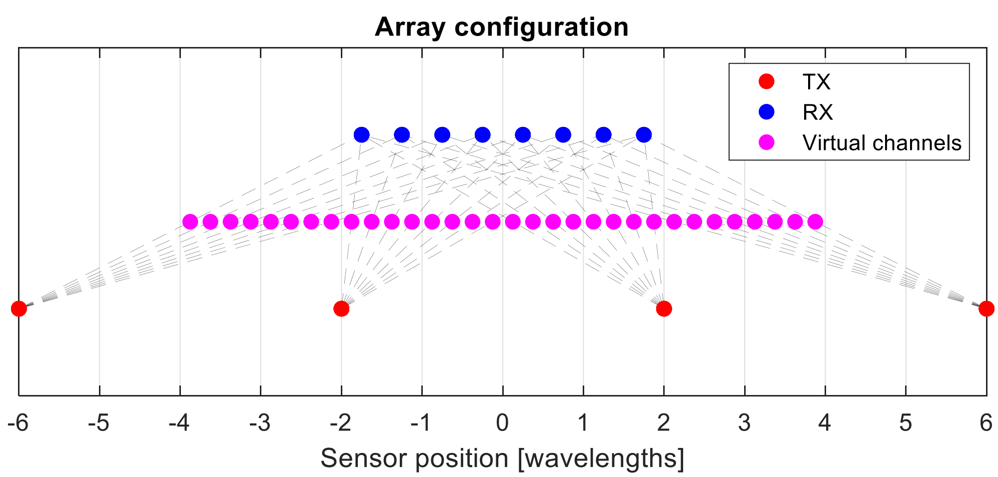

Figure 4: An example of MIMO radar layout.

Tasks:

1. (optional) Demonstrate that the virtual phase center is located in the mid-point between the transmitting and receiving elements (i.e. demonstrate equation 4).

2. By leveraging the concept of virtual phase center, create a virtual array as the one in Section 2.1 with the least number of TX and RX elements. Remember that the number of virtual channels is $N_V = N_{TX} \times N_{RX}$.

3. Simulate again the acquisition using the MIMO radar and focus the image (Attention: do not use the monostatic equivalent). Simulate the multipath environment with both the real target and the image one. Is it different w.r.t the monostatic case?