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Cellular Automata Networks

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Abstract

A small-world cellular automaton network has been formulated to simulate the long-range interactions of complex networks using unconventional computing methods in this paper. Conventional cellular automata use local updating rules. The new type of cellular automata networks uses local rules with a fraction of long-range shortcuts derived from the properties of small-world networks. Simulations show that the self-organized criticality emerges naturally in the system for a given probability of shortcuts and transition occurs as the probability increases to some critical value indicating the small-world behaviour of the complex automata networks. Pattern formation of cellular automata networks and the comparison with equation-based reaction-diffusion systems are also discussed.

Keywords: automata networks, cellular automata, nonlocal PDE, small-world networks, self-organized criticality.

1 Introduction

Theory and computation about complex networks such as the bacterial colonies, interacting ecological species, and the spreading of computer virus over the Internet are becoming very promising and they may have important applications in a wide range of areas. The proper modelling of these networks is a challenging task and the studies in this area are still at very early stage. However, various techniques and applications have been investigated, especially in the area of computational logic, the Internet network, and application of bio-inspired algorithms [1-7]. Since the pioneer work of Watts and Strogatz on small-world networks, a lot of interesting studies on the theory and application of small-world networks [7-9,12-18] have been initiated. More recently, the automata networks have been developed by Tomassini and his colleagues [14, 15] to study the automata network in noise environment. Their study shows that small-world automata networks are less affected by random noise. The properties of complex networks such as population interactions, Internet servers, forest fires, ecological species and financial transactions are mainly determined by the way of connections between the vertices or occupied sites.

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Network modelling and formulations are essentially discrete in the sense that they deal with discrete interactions among discrete nodes of networks because almost all the formulations are in terms of the degrees of clustering, connectivity, average nodal distance and other countable degrees of freedom. Therefore, they do not work well for interactions over continuous networks and media. In the later case, modelling and computations are usually carried out in terms partial differential equations (PDEs), however, almost all PDEs (except those with integral boundary conditions) are local equations because the derivatives and the dependent variable are all evaluated at concurrent locations. For example, the 3-D reaction-diffusion equation

$$u_t = D\nabla^2 u + r(u; x, y, z, t), \tag{1}$$

describes the variation of $u(x, y, z, t)$ such as temperature and concentration with spatial coordinates (x, y, z) and time t. While the diffusion coefficient D can be constant, but the reaction rate r may depends on u and location (x, y, z) as well as time t. This equation is local because u, u_t, r and $\nabla^2 u$ are all evaluated at the same point (x, y, z) at any given time t. Now if we introduce some long-distance shortcuts (e.g., a computer virus can spread from one computer to another computer over a long-distance, not necessary local computers), then the reaction rate can have a nonlocal influence in a similar manner. We can now modify the above equation as

$$u_t = D\nabla^2 u + q\Big(u(x, y, z, t), u(x_*, y_*, z_*, t); x, y, z, t\Big), \tag{2}$$

where q depends on the local point (x, y, z) and another point (x_*, y_*, z_*) far away. Obviously, q can be any function form. As a simple example in the 1-D case: $u_t = Du_{xx} + u(x, t) * [1 - u(x, t)] + \beta u(x - s, t)$, where s is simply a shift and $\beta \in [0, 1]$ is a constant. This equation is nonlocal since the reaction rate depends the values of u at both x and $x - s$ at t. This simple extension makes it difficult to find analytical solutions. Even numerical solutions are not easy to find because the standard numerical methods do not necessarily converge due to the extra nonlocal term. This paper will investigate this aspect in detail using unconventional solution methods such as small-world cellular automata networks.

The present work aims to develop a new type of small-world cellular automata by combining local updating rules with a probability of long-range shortcuts to simulate the interactions and behaviour of a complex system. By using a small fraction of sparse long-range shortcut interactions together with the local interactions, we can simulate the evolution of complex networks. Self-organized criticality will be tested based on the results from the cellular automata. The important implications in the modelling and applications will also be discussed.

2 Small-World Networks

Small-world networks are a special class of networks with a high degree of local clustering as well as a small average distance, and this small-world phenomenon can be achieved by adding randomly only a small fraction of the long-range connections, and some common networks such as power grids, financial networks and neural networks behave like small-world networks [10, 11]. The application of small-world networks into the modelling of infection occurring locally and at a distance was first carried out by Boots and Sasaki [7] with some interesting results. The dynamic features such as spreading and response of an influence over a network have also been investigated in recent studies [11] by using shortest

paths in system with sparse long-range connections in the frame work of small-world models. The influence propagates from the infected site to all uninfected sites connected to it via a link at each time step, whenever a long-range connection or shortcut is met; the influence is newly activated at the other end of the shortcut so as to simulate long-range sparkling effect. These phenomena have successfully been studied by Newman and Watts model [12] and Moukarzel [10]. Their models are linear in the sense that the governing equation is linear and the response is immediate as there is no time delay in their models [20]. More recently, one of the most interesting studies has been carried out by De Arcaneglis and Herrmann [8] using the classic height model on a lattice, which implied the self-organized criticality in the small-world system concerned.

On the other hand, cellular automata have been used to simulate many processes such as lattice gas, fluid flow, reaction-diffusion and complex systems [18, 19, 20, 21, 22] in terms of interaction rules rather than the conventional partial differential equations. Compared to the equation-based models, simulations in term of cellular automata are more stable due to their finite states and local interacting rules [18]. In fact, in most cases, the PDE models are equivalent to rule-based cellular automata if the local rules can be derived from the corresponding PDE models [19, 9], and thus both PDE models and CA rules can simulate the same process [22]. However, we will show that cellular automata networks are a better approach for solving nonlocal equation-based models.

The rest of the present paper will focus on: 1) to formulate a cellular automaton network on a 2-D lattice grid with sparse long-range shortcuts; 2) to simulate the transition and complexity concerning small-world nonlocal interactions; 3) to test the self-organized criticality of the constructed network systems; 4) to find the characteristics of any possible transition.

3 Cellular Automata Networks

Earlier studies on cellular automata use local rules updating the state of each cell and the influence is local. That is to say, the state at the next time step is determined by the states of the present cell concerned and those of its immediate surrounding neighbour cells. Even the simple rules can produce complex patterns [19]. The rule and its locality determine the characteristics of the cellular automata. In fact, we do not have to restrict that the rules must be local, and in general the influence can be either local or nonlocal. Thus, we can assume the rules of cellular automata can be either local or nonlocal or even global. The state of a cell can be determined by m cells consisting of m_i immediate neighbour cells and $m_o = m - m_i$ other cells at longer distance. In the case of local rules only, $m = m_i$ and $m_o = 0$. If $m_o \neq 0$, then the rules are nonlocal. If m is the same order of the total cells $N \times N$ of the cellular automaton, then rules are global. Nonlocal interactions rule for lattice-gas system was first developed by Appert and Zaleski in the discussion of a new momentum-conserving lattice-gas model allowing the particles exchange momentum between distant sites [2]. Some properties of local and nonlocal site exchange deterministic cellular automata were investigated by researchers [15, 5]. As the nonlocal rules are different from the local rules, it is naturally expected that the nonlocal rules may lead to different behavior from conventional local rule-based cellular automata. Furthermore, self-organized criticality has been found in many systems in nature pioneered by Bak and his colleagues [3, 4]. One can expect that there may be cases when self-organized criticality, cellular automata, and small-world phenomena can occur at the same time. More specifically, if a

finite-state cellular automaton with a small-fraction of long-range shortcuts is formulated, a natural question is: Do the self-organized criticality exist in the small-world cellular automaton? Is there any transition in the system?

3.1   Local Cellular Automata

A cellular automaton is a finite-state machine defined on a regular lattice in d-dimensional case, and the state of a cell is determined by the current state of the cell and the states of its neighbour cells [18, 19]. For simplicity, we use 2-D in our discussions. A state $\phi_{i,j}$ of a cell (i,j) at time step $n+1$ can be written in terms of the previous states

$$\phi_{i,j}^{n+1} = \sum_{k,l=-r}^{r} c_{k,l}\phi_{i+k,j+l}^{n}, \quad i,j = 1,2,...,N \tag{3}$$

where summation is over the $4r(r+1)$ Moore neighbourhood cells. In the d-dimensional case, there are $(2r+1)^d - 1$ Moore neighbourhood cells. N is the size of the 2-D automaton, and $c_{k,l}$ are the coefficients. For the simplest and well-known 2-D Conway's game of life $c_{k,l} = 1$ for 8 neighbour cells ($r = 1$). Now let us introduce some nonlocal influence from some sparse long-range cells (see Fig. 1) by combining small-world long-range shortcuts and conventional cellular automata to form a new type of cellular automata networks.

3.2   Small-World Automata Networks

For simplicity, we define a small-world cellular automaton network as a local cellular automaton with an additional fraction or probability p of sparse long-range nonlocal shortcuts (see Fig. 1). For $m_i = 4r_m + 1$ immediate Neumann neighbours and $m_o = 2r_o$ nonlocal cells, the updating rule for a cell becomes

$$\phi_{i,j}^{n+1} = \sum_{k,l=-r_m}^{r_m} c_{k,l}\phi_{i+k,j+l}^{n} + \delta(p) \sum_{s,q=-r_o}^{r_o} c_{s,q}\phi_{i+d_i+s,j+d_j+q}^{n}, \tag{4}$$

where $\delta(p)$ is a control parameter that can turn the long-range cells on ($\delta = 1$) or off ($\delta = 0$) depending on the probability p.

The probability p is the fractions of long-range shortcuts in the total of every possible combinations. For $N \times N$ cells, there are $N^2(N^2 - 1)$ possible connections. The simplest form of δ can be written as $\delta(p) = pH(p - p_0)$ where H is a Heaviside function. p_0 is a critical probability, and p_0 can be taken as to be zero in most simulations in this paper. The updating rules are additive and thus form a subclass of special rules. We can extend the above updating rules to a generalized form, but we are only interested in additive rules here because they may have interesting properties and can easily be transformed to differential equations. In addition, the neighourhood can be either extended Moore neighourhood or Neumann neighbourhood. For Moore neighbourhood, $m_i = 4r_m(r_m + 1)$ and $m_o = 4r_o$. Our numerical experiments seem to indicate that Moore neighbourhood is more sensitive for avalanche and Neumann neighbourhood is more stable for pattern formation. A simple case for a small-world cellular automaton in 2-D case is $r_m = 1$ and $r_0 = 5$ (or any $r_0 > r_m$) so that it has 5 immediate Neumann neighbour cells and 2 shortcuts.

The distance between the nonlocal cells to cell (i,j) can be defined as
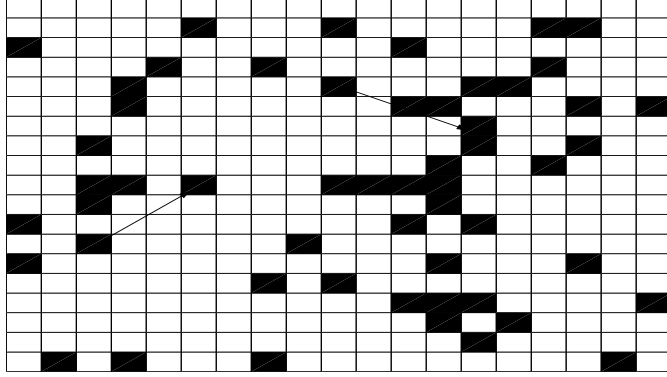
$$S = \sqrt{s_i^2 + s_j^2}. \tag{5}$$

Figure 1: A cellular automaton network with a probability p of long-range shortcuts

The nonlocality requires

$$\min(|s_i|, |s_j|) > r_m. \tag{6}$$

The nonlocal influence can also be introduced in other ways. Alternatively, we can use the conventional local rule-based cellular automaton and adding the long-distance shortcuts between some cells in a random manner. The probability p of the long-range shortcuts in all the possible connections is usually very small. Under certain conditions, these two formulations are equivalent. More generally, a finite-state cellular automaton with a transition rule $G = [g_{ij}]$ $(i, j = 1, 2, ..., N)$ from one state $\Phi^n = [\phi_{ij}^n]$ $(i, j = 1, 2, ..., N)$ at time level n to a new state $\Phi^{n+1} = [\phi_{ij}^{n+1}]$ $(i, j = 1, 2, ..., N)$ at time level $n + 1$ can be written as

$$G : \Phi^n \mapsto \Phi^{n+1}, \ g_{ij} : \phi_{ij}^n \mapsto \phi_{ij}^{n+1}, \tag{7}$$

where g_{ij} takes the same form as equation (4) for small-world cellular automata.

The state of each cell can be taken to be discrete or continuous. From simplicity, we use n_v-valued discrete system and for most of the simulations in the rest of the paper, we use $n_v = 2$ (thus, each cell can only be 0 or 1) for self-criticality testing, and $n_v = 1024$ for pattern formation. Other numbers of states can be used to meet the need of higher accuracies.

4 Simulations and Results

By using the small-world cellular automaton formulated in the previous section, a large number of computer simulations have been carried out in order to find the statistic characteristics of the complex patterns and behaviour arising from cellular automata networks with different probabilities p of long-range shortcuts. Numerical simulations are carried out on an $N \times N$ lattice in 2-D setting, and usually, $N \geq 40$, or up to 5000. Different simulations with different lattice size are compared to ensure the simulated results are independent of the lattice size and time steps. In the rest of the paper, we present some results concerning the features of transition and self-organized criticality of small-world cellular automata.

4.1 Self-organized criticality

For a lattice size of $N = 2000 \times 2000$ with a fixed p, a single cell is randomly selected and perturbed by flipping its state in order to simulate an event of avalanche in 2-D automata
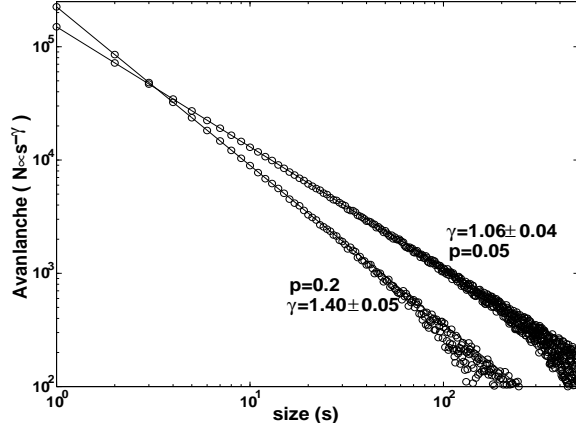
Figure 2: Avalanche size distribution for a small-world cellular automaton.

networks with the standard Moore neighbourhood and Game-of-life updating rules, but a probability p is used to add long-range shortcuts to the cellular automaton. A shortcut forces the two connecting cells having the same state. Figure 2 shows the avalanche size distribution for two different values of $p = 0.05$ and $p = 0.2$, respectively. The avalanche size is defined as the number of cells affected by any single flipping perturbation.

In the double logarithmic plot, the data follows two straight lines. It is clearly seen that there exists a power law in the distribution, and the gradient of the straight line is the exponent of the power-law distribution. A least-square fitting of $N \propto s^{-\gamma}$, leads to the exponents of $\gamma = 1.06 \pm 0.04$ for $p = 0.05$ and $\gamma = 1.40 \pm 0.05$ for $p = 0.2$. Although a power-law distribution does not necessarily mean the self-organized criticality. Self-organized criticality has been observed in other systems [1-4,26]. The pattern formed in the system is quasi-stable and a little perturbation to the equilibrium state usually causes avalanche-like readjustment in the system imply the self-organized criticality in the evolution of complex patterns of the cellular automaton. This is the first time of its kind by using computer simulations to demonstrate the feature of self-organized criticality on a *cellular automaton network*. We can also see in Figure 2 that different probabilities p will lead to different values of exponents. The higher the probability, the steeper the slope.

4.2 Transition of small-world systems

For a fixed grid of 2000×2000 cells, we can vary the probability p to see what can happen. For a single event of flipping state, the fraction of population affected is plotted versus p in a semi-logarithmic plot as shown in Figure 3 where the fraction of population is defined as the number (N_a) of cells affected among the whole population N^2, that is N_a/N^2. The sharp increase of the fraction versus the probability p indicates a transition in the properties of cellular automata networks. For a very small probability $p < 0.004$, the influence of the event mainly behaves in a similar way as the conventional local cellular automata. As the probability increases, a transition occurs at about $p = 0.01$. For $p > 0.05$, any event will affect the whole population. This feature of transition is consistent with the typical small-world networks [12, 21].

Comparing with the local rule-based cellular automata, the transition in small-world cellular automata is an interesting feature. Without such shortcuts, there was no transition observed in the simulations. However, self-organized criticality was still observed in finite-
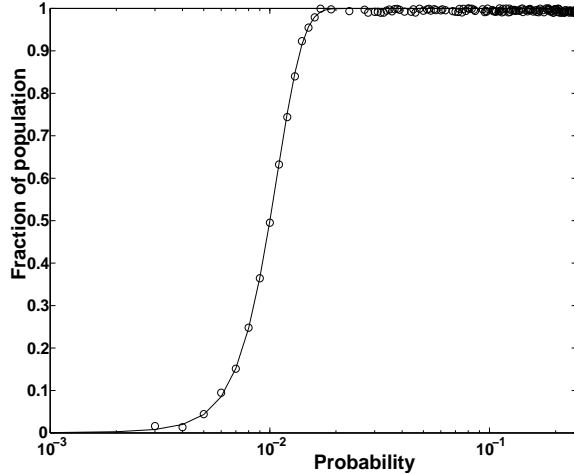
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Figure 3: Transition of influence with different probabilities p of long-range shortcuts.

state cellular automata [1, 21] without transition. In the present case, both self-organized criticality and transition emerge naturally. Thus, the transition in cellular automata networks suggests that this transition may be the result of nonlocal interactions by long-range shortcuts.

This feature of transition may have important implications when applied to the modelling of real-world phenomena such as the Internet and social networks. For a system with few or no long-range interactions, there is no noticeable change in its behavior in transition. However, as the long-range shortcuts or interacting components increase a little bit more, say to $p = 0.01$, then a transition may occur and thus any event can affect a large fraction of the whole population. For example, to increase the speed of finding information on the Internet, a small fraction of long-range shortcuts in terms of website portals and search engine (e.g., Google) and high-capacity/bandwidth connections could significantly increase the performance of the system concerned. In addition, the self-organized criticality can also imply some interesting properties of the Internet and other small-world networks, and these could serve as some topics for further research.

4.3   Nonlocal Partial Differential Equations

The evolution of a system can usually be described by two major ways: rule-based systems and equation-based systems. The rule-based systems are typically discrete and use local rules such as cellular automata or finite difference system. As discussed by many researchers [18, 22], the finite difference systems are equivalent to cellular automata if the updating rules for the cellular automata are derived directly from their equation-based counterpart. On the other hand, the equation-based systems are typically continuous and they are often written as partial differential equations. Sometimes, the same system can described using these two different ways. However, there is no universal relationship between a rule-based system and an equation-based system [22]. Given differential equation, it is possible to construct a rule-based cellular automaton by discretizing the differential equations, but it is far more complicated to formulate a system of partial differential equations for a given cellular automaton. For example, the following 2-D partial differential equation for nonlinear
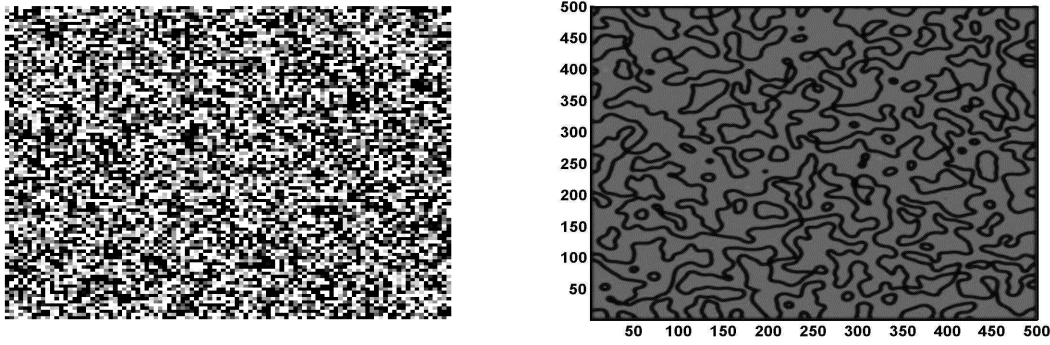
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Figure 4: a) Initial random configuration on a 500×500 grid at $t = 0$; b) 2-D Pattern formation and distribution displayed at $t = 100$.

pattern formation for $u(x, y, t)$

$$\frac{\partial u}{\partial t} = D(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + \gamma f(u, x, y, t), \tag{8}$$

can always be written as an equivalent cellular automaton if the local rules of cellular automaton are obtained from the PDE. Conversely, a local cellular automaton can lead to a local system of partial differential equation (PDE), if the construction is possible [22]. A local PDE can generally be written as

$$\mathcal{F}(u, u_x, u_y, ..., x, y, t) = 0. \tag{9}$$

A nonlocal PDE can be written as

$$\mathcal{F}(u, u_x, u_y, ..., x, y, t, x + S(x, y, p), y + S(x, y, p)) = 0, \tag{10}$$

where $S(x, y, p)$ is the the averaged distance of long-range shortcuts and p is the probability of nonlocal long-range shortcuts. In order to show what a nonlocal equation means, we modify the above equation for pattern formation as

$$u_t = D\nabla^2 u(x, y, S, p, t) + \gamma u(x, y, S, p, t)[1 - u(x, y, S, p, t)]. \tag{11}$$

This nonlocal equation is far more complicated than equation (8). For the proposed cellular automaton networks, a system of nonlocal partial differential equations will be derived, though the explicit form of a generic form is very difficult to obtain and this requires further research.

4.4 Pattern Formation

Even for simple nonlinear partial differential equations, complex pattern formation can arise naturally from initially random states. For example, the following nonlinear partial differential equation for pattern formation $u(x, y, t)$

$$\frac{\partial u}{\partial t} = D(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + \gamma u(1 - u) + \beta u(x - S), \tag{12}$$
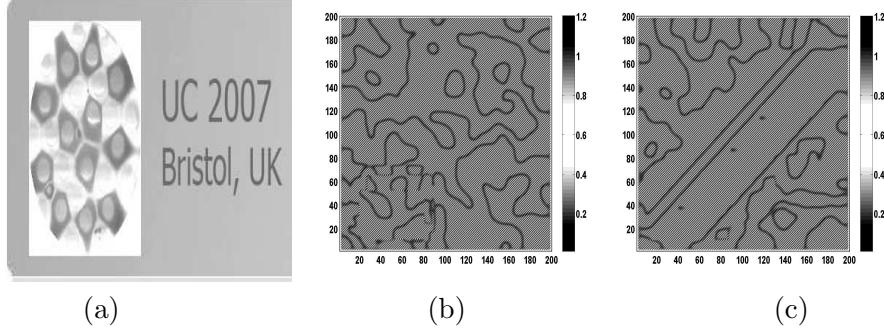
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(a)   (b)   (c)

Figure 5: a) The UC2007 logo has been mapped onto a 200×200 grid as the initial condition at $t = 0$; b) 2-D pattern distribution with a short-cut probability $p = 0.0025$ and $S = 5$ (or $\beta = 0.05$) displayed at $t = 200$; c) 2-D pattern with a short-cut probability $p = 0.01$ and $S = 50$ (or $\beta = 0.2$) at $t = 200$.

can be discretized using central finite difference scheme in space with $\Delta x = \Delta y = \Delta t = 1$. Then, it is equivalent to

$$u_{i,j}^{n+1} = \sum_{k,l=-r}^{r} a_{k,l} u_{i+k,j+l}^{n} + \gamma u_{i,j}^{n}(1 - u_{i,j}^{n}) + \beta u_{i-S,j}^{n}, \tag{13}$$

where $r = 1$, $a_{0,0} = 1 - 4D$, $a_{-1,0} = a_{+1,0} = a_{0,-1} = a_{0,+1} = D$. It is a cellular automaton with the standard Neumann neighbourhood for this PDE.

The formed patterns and their distribution resulting from the system on a 500×500 grid are shown in Fig. 4 where $D = 0.2$, $S = 5$, $\gamma = 0.5$ and $\beta = 0.01$. We can see that stable patterns can be formed from initial random states.

The formed patterns using the Neumann neighbourhood ($r = 1$) are very stable and are almost independent of the initial conditions. In fact, the initial state does not matter and the only requirement for the initial state is some degree of randomness. If we run the same program using a photograph or the UC2007 conference logo, similar patterns can also form naturally as shown in Figure 5. In our simulations, we have used $D = 0.2$ and $\gamma = 0.5$. Other parameters S and β can vary. For the case shown in Fig. 5b, $S = 5$ and $\beta = 0.05$, while $S = 50$ and $\beta = 0.2$ are used in Fig. 5c. This means that the initial state does not affect the characteristics of pattern formation and this is consistent with the stability analysis [22].

5   Discussions

Small-world cellular automata networks have been formulated to simulate the interactions and behaviour of multi-agent systems and small-world complex networks with long-range shortcuts. Simulations show that power-law distribution emerges for a fixed probability of long-range shortcuts, which implies self-organized criticality in the avalanche and evolving complex patterns. For a given size of cellular grid, the increase of the probability of long-range shortcuts leads to a transition, and in this case, a single even can affect a large fraction of the whole population. In this sense, the characteristics of small-world cellular automota are very different from the conventional locally interacting cellular automata.

The nonlocal rule-based network systems in terms of cellular automata can have other complicated features such as its classifications compared with the conventional automata

and its relationship its partial differential equations. In addition, cellular automota networks could provide a new avenue for efficient unconventional computing for simulating complex systems with many open questions such as the relationship between cellular automata networks and nonlocal PDEs, and the potential implication on the parallelism of these algorithms. These are open problems to be investigated in the future research.

References

[1] Adami, C, Self-organized criticality in living systems, *Phys. Lett. A*, **203**, 23 (1995).

[2] Appert C. and Zaleski S., Lattice gas with a liquid-gas transition, *Phys. Rev. Lett.*, **64**, 1-4 (1990).

[3] Bak P, Tang C and Wiesenfeld K, Self-organized criticality: an explanation of 1/f noise, *Phys. Rev. Lett.*, **59**, 381-384 (1987).

[4] Bak P, *How nature works:the science of self-organized criticality*, Springer-Verlag, (1996).

[5] Boccara N. and M. Roger, Some properties of local and nonlocal site exchange deterministic cellular automata, *Int. J. Modern Phys.*, **C5**,581-588 (1994).

[6] Bollobas B., *Random graphs*, Academic Press, New York, (1985).

[7] Boots M., Sasaki A., Small worlds and the evolution of virulence: infection occurs locally and at a distance, *Proc Roy Soc Lond*, B **266**, 1933-1938 (1999).

[8] De Arcaneglis, L. and Herrmann, H. J., Self-organized criticality on small world networks, *Physica A*, **308**, 545-549 (2002).

[9] Guinot V., Modelling using stochastic, finite state cellular automata: rule inference from continuum model, *Appl. Math. Model.*, **26**, 701-714(2002).

[10] Moukarzel C. F., Spreading and shortest paths in systems with sparse long-range connections, *Phys. Rev. E*, **60**, 6263-6266, (1999).

[11] Newman M. E. J., Moore C., Watts D. J., Mean-field solution of the small-world network model, *Phys. Rev. Lett.*, **84**, 3201-3204 (2000).

[12] Newman M. E. J., Watts D. J., Scaling and percolation in the small-world network model, *Phys. Rev. E*, **60**,7332-7342 (1999).

[13] Pandit S. A., Amritkar R. E., Characterization and control of small-world networks, *Phys. Rev. E*, **60**, 1119-1122 (1999).

[14] Tomassini M., *Generalized automata networks*, 7th Int. Conference on Cellular Automata for Research and Industry, ACRI 2006, France, *Lecture Notes in Computer Sciences*, **4173**, 14-28 (2006).

[15] Tomassini M., Giacobini M., Darabos C., Evolution and dynamics of small-world cellular automata, *Complex Systems*, **15**, 261-284 (2005).

[16] Watts D. J., *Small worlds: The dynamics of networks between order and randomness*, Princeton Univ. Press, (1999).

[17] Watts D. J., Strogatz S. H., Collective dynamics of small-world networks, *Nature* (London), **393**, 440-442 (1998).

[18] Weimar J. R., Cellular automata for reaction-diffusion systems, *Parallel computing*, **23**, 1699-1715 (1997).

[19] Wolfram, S., *Cellular automata and complexity*, Reading, Mass: Addison-Wesley (1994).

[20] Yang X. S., Chaos in small-world networks, *Phys. Rev. E.*, **63**, 046206 (2001).

[21] Yang X. S. and Young Y., Cellular automata, PDEs, and pattern formation, in: Handbooks of Bioinspired Algorithms and Applications (eds. Olarius S. and Zomaya A. Y.),Chapman & Hall /CRC Press, 271-282 (2005).

[22] Yang X. S., Computational modelling of nonlinear calcium waves, *Applied Maths. Modelling*, **30**, 200-208 (2006).