Musical Acoustics

HOMEWORK LAB 1

Group Members:
Riccardo Sebastiani Croce 10806858
Andrea De Fabritiis 10823661

November 3, 2022

1 Introduction

The goal of the homework is to characterize the vibration of a wine glass through an eigenfrequency analysis in COMSOL.

2 Wine glass modelling

We decide to model the following wine glass geometry.

Firstly, we assume a symmetry of the wine glass along the z-axis and so we design it starting from a 2D-axisymmetric model in rz plane. The geometry consists of a rectangle for the stem and then we mainly use sketch tools to realize the base and the upper part of the wine glass. Once it has been converted to solid in order to mesh the figure, it appears like Fig.1 and we assign a blank material with the characteristics of the glass (Young's Modulus $E = 73.1\,GPa$, Density $\rho = 2203\,kg/m^3$, Poisson's Ratio $\nu = 0.17$).

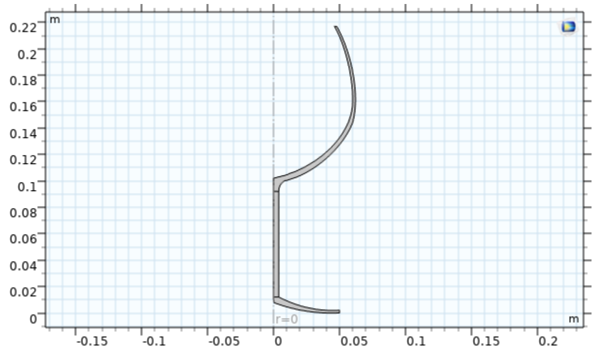


Figure 1: 2D-axisymmetric model of wine glass

We apply the pinned boundary condition for the base, imposing a *prescribed displacement* with $u_{0z} = 0$ so the bottom face is fixed only in the vertical direction.
Since we are more interested in the vibration of the upper part, we generate a coarse meshing for the entire system and then we refine it with 3 different *Free Triangular* point distributions that are finer as we go up to the top part.

Finally, we transform the 2D-axisymmetic model in a 3D one, by creating a work plane inside a 3D component and importing the previous 2D component, and obtaining the complete figure through a *Revolve* operation around the z-axis. Also in the 3D model we introduce the same boundary condition as before and the mesh follows the same principle.

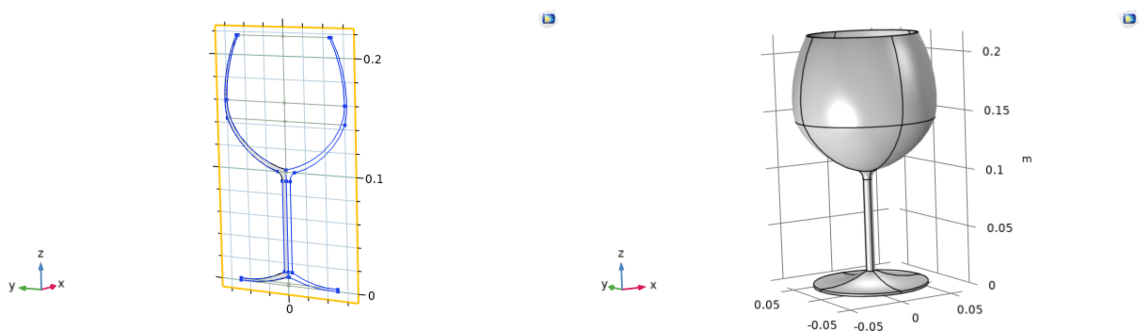


Figure 2: Plane geometry on yz plane (left) and final 3D model (right)

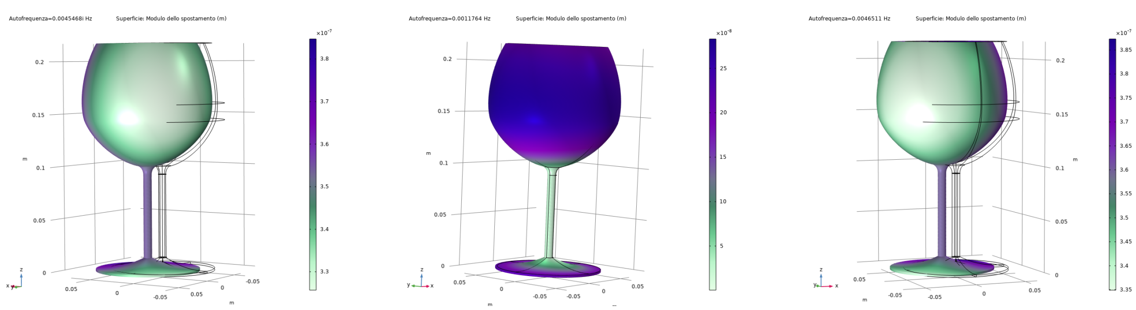
3 3D model evaluation

Once the model is built, the remaining part of the process consists in the study of the vibrational characteristics of the glass. In order to derive the eigenfrequencies of the model, we used the **Eigenfrequency study** in the *Solid mechanics* section. After evaluating the study using different center frequencies, we decided to center the study around a frequency of 1Hz in order to verify also its behaviour at low frequencies.

Autofrequenza (Hz)
0.0045468i
0.0011764
0.0046511
66.266
66.267
288.81
517.71
517.74
716.40
716.42
1772.4
1879.8
1879.8
3450.1
3450.2
5255.5
5256.0
5344.3
5344.3
7487.5

Figure 3: Eigenfrequency values according to the 3D model

The first eigenfrequencies that derive from this particular study are small valued and can also be complex valued, due to computational noise: these represent the **Rigid Modes**. When a body is not adequately supported, it can translate or rotate as a whole without deformation. A body without any constraint has six rigid body modes: 3 translations and 3 rotations.
In this particular case, the constraints are applied to the base of the glass, which is fixed for vertical displacement: the translational mode along the vertical axis z and the two rotational modes around the x and y axis are not allowed. Therefore, the first 3 eigenfrequencies found, which are approximately zero valued, represent a combination of the three roto-translational modes admitted.



(a) First rigid mode (b) Second rigid mode (c) Third rigid mode

Figure 4: First three rigid modes

Apart from these three initial modes, the other modes appear to have higher frequency values and describe classical expected motion from vibrating glass: deformation. All the other eigenfrequencies deducted from the study happen to describe some type of deformation of the glass, whether it concerns the cup, the stem or the base, although being fixed on the vertical axis, by a linear combination of the three.

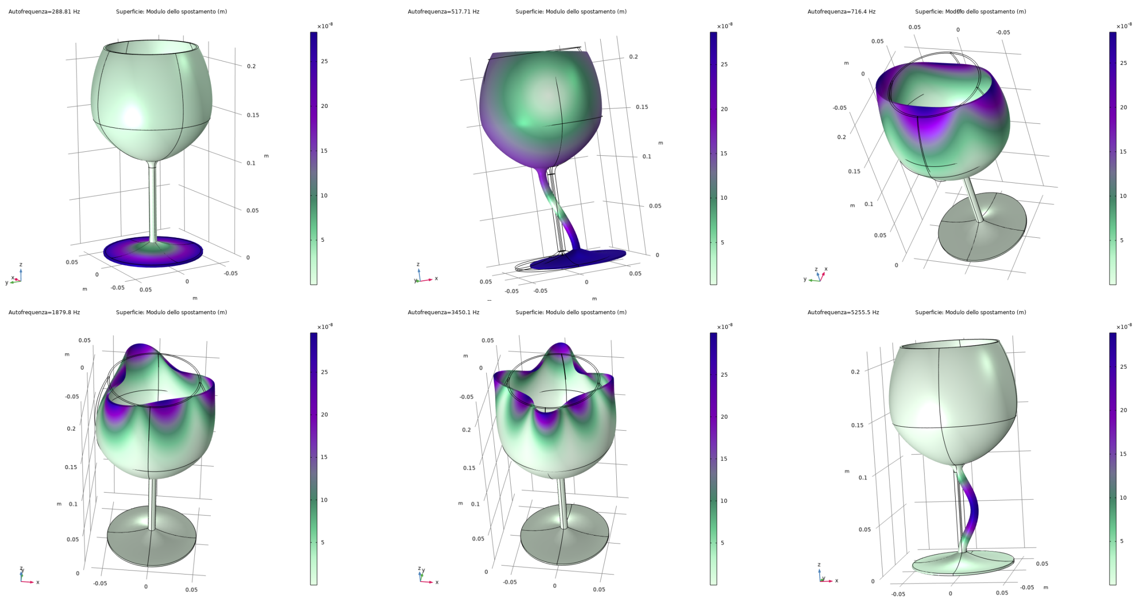


Figure 5: Animations of modes for the 3D model

4 2D model evaluation

4.1 Axisymmetric model

For the sake of evaluating the same 3D model in a 2D setting, both geometrical and mechanical properties must be taken into account. First of all, the model's geometrical properties such as, of course, symmetry: in order to reduce the dimensions of the setting, the figure must be symmetrical with respect to all axis. In second place, concerning mechanical properties, it appears to be necessary that the constraints are constant around the object's circumference.
If these requirements are fulfilled, it is possible to formulate the equations of motion using only a 2D cross section.
The 2D slice is sufficient to recover the full 3D stress state and strain state by integrating the governing equations over the full revolution.

Autofrequenza (Hz)
1785.2
11920
14691
15760
16990
17680
18859
21186
24161
25471
27698
31856
35674
37038
40852
41637
45551
51140
56390
57456

Figure 6: Eigenfrequency values according to the 2D model

The evaluation of the model's eigenfrequencies takes into account only the radial and axial displacements, while the circumferential displacement is assumed to be zero.

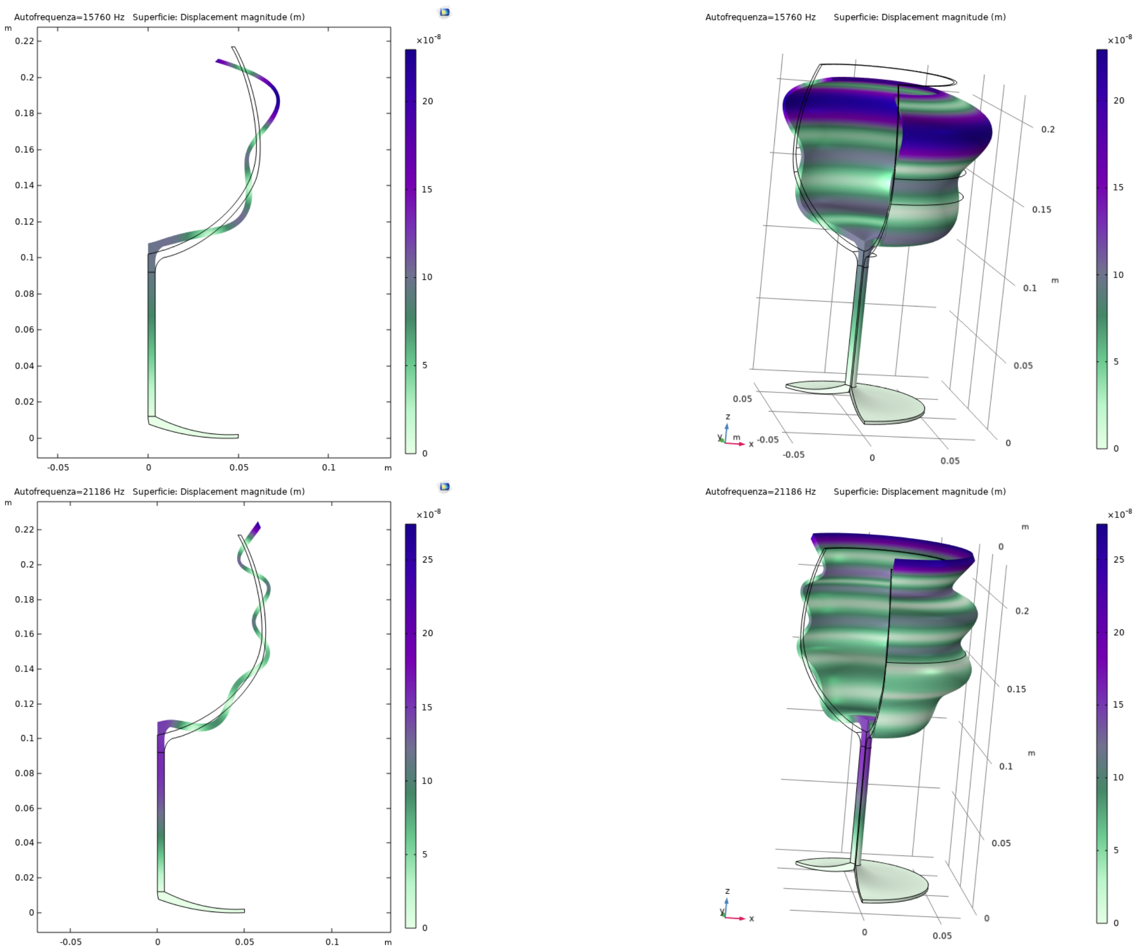


Figure 7: Animations of modes for the 2D model in both two and three dimensional space

In order to compare the 3D analysis to the 2D axisymmetric one it is possible, using the *Revolve* transformation to translate the 2D model analysis in its 3D version.

Looking at the eigenfrequency solutions on the 2D model of the glass, it is visible how only one of them is comparable to those of the first 3D study. In fact, looking at the tables in Fig.3,6, it is clear that only the first 2D value is approximately similar to those of the 3D model.

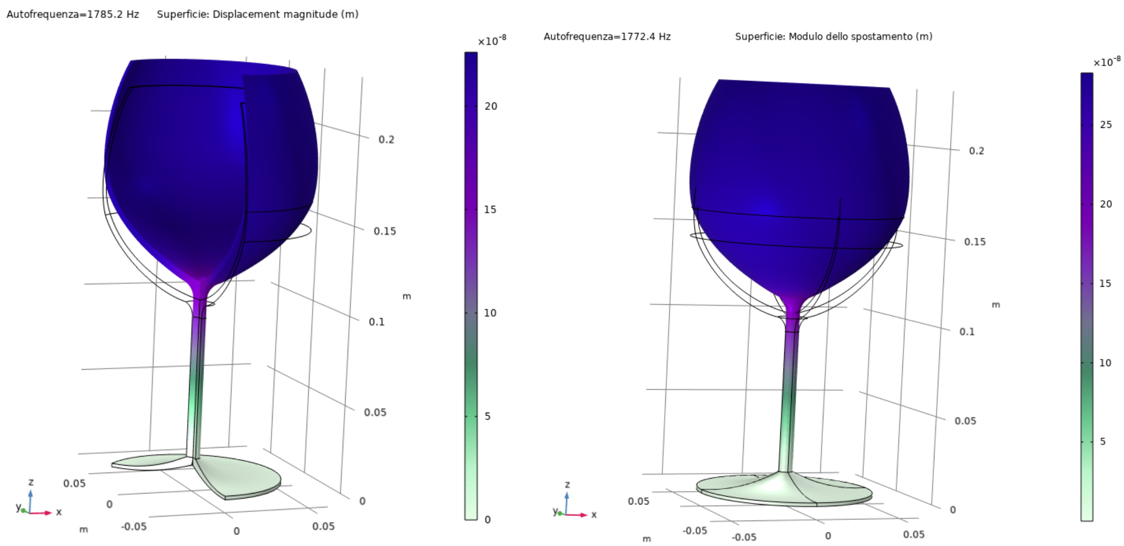


Figure 8: Symmetrical mode around frequency 1780Hz in 2D axisymmetric approximation (left) and 3D study evaluation (right)

The study does not show any rigid mode.
The other frequencies are only representative of the symmetric modes and are at best an approximation.

4.2 Circumferential mode extension

As seen before, the 2D axisymmetric analysis is an approximation that allows to study symmetric solution taking into account the axial and radial displacement. In order to have a more accurate approximation of the 3D real case for the torsional modes, the out-of-plane displacements must be considered: this allows to introduce torsion with respect to the symmetry axis.

Frequency (Hz)
0.0041967i
288.96
1785.2
11920
14691
15760
16990
17680
18859
19364
21186
21835
24161
25471
27698
31856
32985
35674
37038
40852

Figure 9: Eigenfrequency values according to the 2D model

Considering the torsional eigenmodes we will also have an additional rigid body mode that corresponds to the pure rotation around the revolution axis.

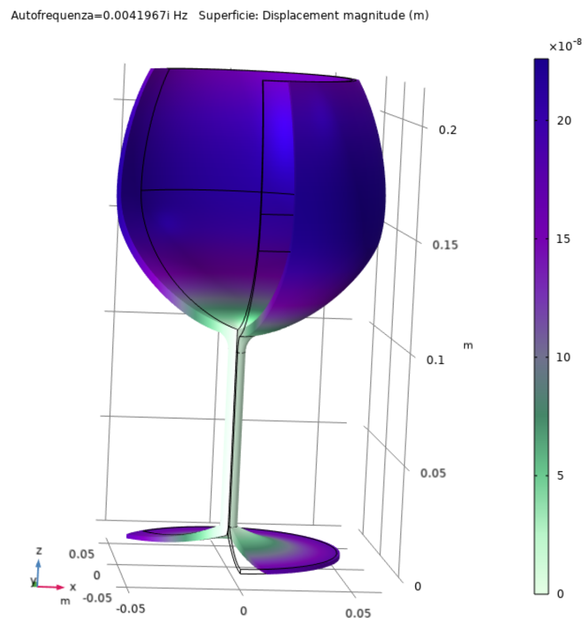


Figure 10: First rigid mode in the 2D axisymmetric study considering circumferential mode extension

COMSOL allows to select the value of the azimuthal mode m in the axialsymmetry approximation, which can assume integer values. In the case of $m = 0$, allows to express torsional modes, which the 2D approximation lacks.
Taking into account the torsional modes, the study seems to be more precise: the rigid mode in this case appears to be just one and describes a purely rotational deformation, as shown in Fig.10 As for the other modes, the extension adds a second mode similar to those describes by the 3D mode:

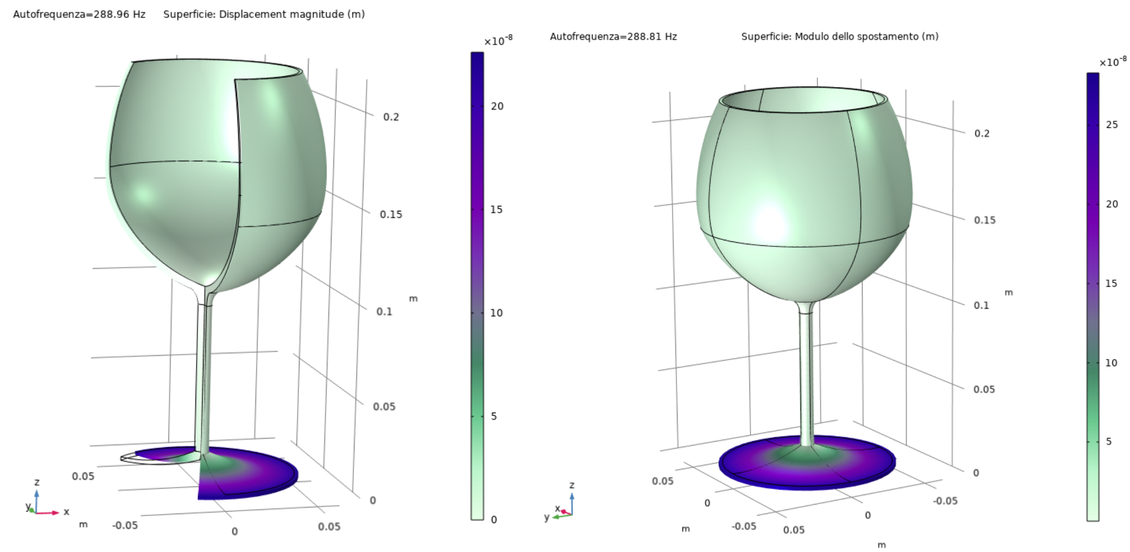


Figure 11: Symmetrical mode around frequency 288Hz in 2D axisymmetric circumeferential mode approximation (left) and 3D study evaluation (right)