Evaluation of the apparent source in laser safety

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The analysis of the apparent source is used in laser safety to evaluate the retinal hazard. This study provides an analytic method to determine the location and size of the apparent source for laser beams. This method shows that the beam cross section at the center of curvature of the wave front at the eye lens can be considered the apparent source in terms of both the location and size. The approximation given by using the beam waist as the apparent source is examined and the error introduced by this approximation is evaluated. This analysis shows that the beam waist can be considered to be the apparent source only when the distance of the eye from the beam waist is much greater than the Rayleigh length of the beam. Examples showing the location and size of the apparent source are given for different beam characteristics and viewing distances. Measurement methods are also specified to provide an alternative to the analytic method. © *2001 Laser Institute of America.*

Key words: laser safety, apparent source, retinal hazard

I. INTRODUCTION

The biological effects on the eye, caused by the radiation in the wavelength range from 400 to 1400 nm, regard mainly the retina. The damage depends on the retinal irradiance ($\mathrm{W\,m^{-2}}$) or radiant exposure ($\mathrm{J\,m^{-2}}$), and on the size of the area of the irradiated retinal tissue. Therefore, for a correct evaluation of the hazard, it is necessary to evaluate how the eye can focus the radiation forming the image on the retina. The hazard increases as the retinal image size decreases, because of the increase of the power or energy density of the radiation on the retinal tissue.

In the practical cases, the retinal image size depends on the accommodation state of the eye. Therefore, to determine the retinal image size produced by a laser source, besides the beam characteristics, it is necessary to know the focal length of the eye lens, the distance between the eye lens and the retina, possible defects in the eye accommodation, and other focusing characteristics that are specific of the eye under evaluation. However, in laser safety, it is often necessary to evaluate a source without any reference to a specific real eye, but considering a worst case that could be adequate to protect all people in all reasonable foreseeable conditions. For this reason, the evaluation of the retinal image size is performed assuming a ''standard'' eye, having an accommodation range large enough to include the accommodation ranges of all people (young people, myopic people, etc., excluding only rare cases), and also assuming that the eye accommodates to focus the laser source, minimizing the retinal image. This worst case analysis is largely used in the standardization to classify products, and also to evaluate the exposure, e.g., in the calculation of the nominal ocular hazard distance.

For this purpose, the concept of apparent source,[1] defined as the real or virtual object that forms the smallest retinal image by eye focusing, has been introduced in laser safety (see IEC 60825-1 standard). This study is relevant to the determination of the apparent source as defined above in terms of both size and location, and the aim is to give a theoretical basis for the safety analysis. Of course, when this study is applied to a practical safety problem, the actual focusing characteristics of the eye under analysis have to be considered to give a correct evaluation of the hazard.

The size of the apparent source of a beam is determined by the smallest retinal image size that the eye can produce by the accommodation, which allows us to change the focal length of the eye lens to reduce the retinal image size. The angular subtense of the apparent source is used as a measure of the retinal image size. This angular subtense is the plane angle subtended by the diameter of the apparent source at the lens of the eye. For this purpose, the diameter d_{63} must be used. The diameter d_{63} is defined as the diameter of the smallest circle containing 63% of total power. For a Gaussian profile, this corresponds to the measure of the diameter at the points where the irradiance falls to $1/e$ of the peak. In laser safety, the diameter d_{63} is used to measure the beam diameter and divergence (see IEC 60825-1) and, consequently, to evaluate the apparent source size.

An equivalent definition of the angular subtense of the apparent source is based on the retinal image size. In fact, when an object is focused by the eye, the angle that this object subtends at the eye lens is equal to the angle that the retinal image subtends at the eye lens. Hence the equivalent definition is the following: the angular subtense of the apparent source is the plane angle that the diameter of the smallest obtainable retinal image subtends at the lens. This is shown by the following equation:

$$\alpha = 2\,\arctan\left(\frac{d_r}{2L_e}\right), \tag{1}$$

where α is the angular subtense of the apparent source, d_r is the diameter of the retinal image evaluated at $1/e$ points, and L_e is the distance between the lens and the retina.

For small angles, as it normally occurs in the calculation to evaluate the retinal hazard, Eq. (1) can be simplified as

$$\alpha \approx d_r/L_e. \tag{2}$$

Regarding the definition of the distance of the apparent source from the eye, a suitable criterion should take into account how the eye focuses the radiation. Since the focal length of the lens to be used to focus an object (i.e., to form the minimum spot size on the retina) depends on the distance of the object from the lens, it is reasonable to consider that the distance of the apparent source from the eye is determined by the focal length necessary to obtain the minimum retinal image size. In particular, we can say that the source appears to the eye to be at a certain distance in base of the accommodation needed for focusing. The objects are focused on the retina, forming the minimum retinal image, when their distance from the eye lens is given by the following equation (valid for a thin lens):

$$r = \frac{1}{\frac{1}{f} - \frac{1}{L_e}}, \tag{3}$$

where r is the distance of the object from the lens, L_e is the distance between the lens and the retina, and f is the focal length of the lens. Using this criterion, when the focal length f necessary to form the minimum retinal image of the beam source is known, the distance of the apparent source can be considered equal to the distance r calculated by Eq. (3).

In the determination of the apparent source size, the focusing characteristics of the human eye have to be considered. For a normal eye, L_e is 17 mm. When a normal eye is relaxed, the focal length of the lens is 17 mm, so only objects at large distances (ideally at infinity) are focused on the retina. However, by the accommodation, the eye can vary the focal length of the lens in a range that depends on individual characteristics (e.g., this range is maximum for young people and reduces by the age). The eye accommodation allows us to focus objects at different distances from the eye, up to a minimum distance depending on the individual eye. In IEC 60825-1, this minimum distance is considered to be 100 mm (corresponding to a minimum focal length of 14.53 mm for a normal eye). In fact, in only a few cases (young people and very myopic people) can objects at shorter distances be focused by the unaided eye.

II. ANALYTIC METHOD TO EVALUATE THE APPARENT SOURCE

In case of an ideal TEM_{00} beam, the problem of determining the size and location of the apparent source is solved using the appropriate theory that describes the propagation of Gaussian beams.[2,3] The propagation of a Gaussian beam is described by the following equations:

$$d(z) = d_0\left(1 + \frac{z^2}{Z_R^2}\right)^{1/2}, \tag{4}$$

$$R(z) = z + \frac{Z_R^2}{z}, \tag{5}$$

$$Z_R = \frac{\pi d_0^2}{2\lambda}, \tag{6}$$

$$\Phi = \frac{d_0}{Z_R}, \tag{7}$$

where z is the distance from the beam waist, $d(z)$ is the beam diameter at distance z determined at $1/e$ peak intensity points, $R(z)$ is the radius of curvature of the wave front at distance z from the beam waist, λ is the wavelength of the radiation, d_0 is the diameter of the beam waist determined at $1/e$ peak intensity points, Z_R is the Rayleigh length, and Φ is the far-field full divergence angle determined at $1/e$ peak intensity points.

In the analyses of Gaussian beams, the beam diameter and divergence determined at $1/e^2$ points are often considered. However, in this study the beam diameter and divergence at $1/e$ points are considered to be according to the laser safety approach. These parameters at $1/e$ points are obtained multiplying the corresponding parameters at $1/e^2$ points by $1/\sqrt{2}$.

Many lasers do not have an ideal TEM_{00} beam. In the case of higher mode laser beams, the propagation is described by equations similar to the above ones, but it is necessary to introduce a new parameter: the beam quality factor M^2. This parameter is defined using an invariant of propagation,[4] which is not changed by the optics: the product of the beam waist diameter by the far-field divergence. M^2 is defined as the ratio of the value of this invariant for the real beam to its value for a TEM_{00} beam with the same wavelength. The value of M^2 is 1 for TEM_{00} beams and greater than 1 for higher order mode beams. The value of M^2 is given by

$$M^2 = \frac{d_0 \Phi}{d_{0,TEM_{00}} \Phi_{TEM_{00}}}, \tag{8}$$

where d_0 is the beam waist diameter of the real beam, Φ is the far-field full divergence angle of the real beam, $d_{0,TEM_{00}}$ is the beam waist diameter of a TEM_{00} beam, and $\Phi_{TEM_{00}}$ is the far-field full divergence angle of a TEM_{00} beam.

Equations (4), (5), and (7) are still applicable, but the formula for the Rayleigh length must be replaced by the following:

$$Z_R = \frac{\pi d_0^2}{2M^2\lambda}. \tag{9}$$

Therefore, Eq. (9) gives the value of the Rayleigh length for both TEM_{00} beams ($M^2 = 1$) and higher order mode beams ($M^2 > 1$).

If the laser beam propagates through a lens, a new beam waist is formed. Considering the case of a thin lens, the following equations give the diameter of this second beam waist and its distance from the lens:

$$d_{02}=\frac{d_{01}f}{\sqrt{(L_1-f)^2+Z_{R1}^2}}, \tag{10}$$

$$L_2=f+\frac{f^2(L_1-f)}{(L_1-f)^2+Z_{R1}^2}, \tag{11}$$

where d_{01} is the diameter of the first beam waist (i.e., the waist diameter of the input beam), d_{02} is the diameter of the second beam waist (i.e., the waist diameter of the output beam), f is the focal length of the lens, L_1 is the distance of the first beam waist from the lens, Z_{R1} is the Rayleigh length of the input beam (i.e., before the lens), and L_2 is the distance of the second beam waist from the lens.

The sign convention is that L_1 is taken as positive when the first beam waist is before the lens, while L_2 is taken as positive when the second beam waist is after the lens. The sign of L_1 determines an important characteristic of the beam: the input beam at the lens is divergent when $L_1>0$, whereas the input beam at the lens is convergent when $L_1<0$. The following equation gives the beam diameter after the lens at a fixed distance L from it:

$$d_L=d_{02}\left[1+\frac{(L-L_2)^2}{Z_{R2}^2}\right]^{1/2}, \tag{12}$$

$$Z_{R2}=\frac{\pi d_{02}^2}{2\lambda M^2}, \tag{13}$$

where d_L is the beam diameter after the lens at the distance L from the lens, and Z_{R2} is the Rayleigh length of the output beam (i.e., after the lens).

Expressing d_{02} and Z_{R2} by L_1, d_{01}, Z_{R1}, and f, the following equation for d_L can be found:

$$d_L=\frac{d_{01}}{\sqrt{L_1^2+Z_{R1}^2}}\left[L^2+\left(\frac{L_1^2+Z_{R1}^2}{Z_{R1}}\right)^2\right.$$
$$\left.\times\left(1-\frac{L}{f}+\frac{L_1L}{L_1^2+Z_{R1}^2}\right)^2\right]^{1/2}. \tag{14}$$

For the evaluation of the retinal hazard, the beam diameter after the lens has to be evaluated at a distance L_e from the eye lens. From Eq. (14), the following equations are derived:

$$d_r=\frac{d_{01}}{\sqrt{L_1^2+Z_{R1}^2}}\left[L_e^2+\left(\frac{L_1^2+Z_{R1}^2}{Z_{R1}}\right)^2\right.$$
$$\left.\times\left(1-\frac{L_e}{f}+\frac{L_1L_e}{L_1^2+Z_{R1}^2}\right)^2\right]^{1/2}, \tag{15}$$

$$\alpha=\frac{d_r}{L_e}=\frac{d_{01}}{\sqrt{L_1^2+Z_{R1}^2}}\left[1+\left(\frac{L_1^2+Z_{R1}^2}{Z_{R1}}\right)^2\right.$$
$$\left.\times\left(\frac{1}{L_e}-\frac{1}{f}+\frac{L_1}{L_1^2+Z_{R1}^2}\right)^2\right]^{1/2}, \tag{16}$$

where d_r is the beam diameter on the retina, and α is the angle that the beam diameter on the retina subtends at the eye lens.

To evaluate the apparent source, it is necessary to determine the eye focal length that gives the minimum retinal spot size. As can be easily deduced by Eqs. (15) and (16), to minimize the beam diameter after the lens at the distance L_e from it and the corresponding angle subtended at the lens, the value of the focal length must satisfy the following equation:

$$1-\frac{L_e}{f_m}+\frac{L_1L_e}{L_1^2+Z_{R1}^2}=0, \tag{17}$$

where f_m is the value of the focal length that minimizes d_r and α. Equation (17) gives

$$f_m=\frac{L_e(L_1^2+Z_{R1}^2)}{L_1^2+Z_{R1}^2+L_1L_e}=\frac{1}{\dfrac{1}{L_e}+\dfrac{1}{R_1(L_1)}}, \tag{18}$$

where $R_1(L_1)$ is the radius of curvature of the input beam at a distance L_1 from the beam waist (i.e., at the lens). Equation (18) can be modified to evaluate the possibility to consider L_1 as the distance of the apparent source from the eye lens

$$\frac{1}{f_m}=\frac{1}{L_e}+\frac{1}{L_1\left(1+\dfrac{Z_{R1}^2}{L_1^2}\right)}. \tag{19}$$

Equation (19) shows that if L_1 is much greater than Z_{R1} (see Sec. III), the focal length can be determined using L_1 as the distance of the apparent source from the eye lens.

If f_m is within the eye accommodation range, this value of the focal length can be used in Eqs. (15) and (16) to derive simple equations giving the smallest retinal image diameter $d_{r,m}$ obtainable by eye focusing and the angular subtense α_m subtended by this image at the lens (i.e., the angular subtense of the apparent source) as follows:

$$d_{r,m}=\frac{d_{01}L_e}{\sqrt{L_1^2+Z_{R1}^2}}, \tag{20}$$

$$\alpha_m=\frac{d_{01}}{\sqrt{L_1^2+Z_{R1}^2}}. \tag{21}$$

Equations (20) and (21) show that $d_{r,m}$ and α_m decrease as $|L_1|$ and Z_{R1} increase. In particular, the dependence of $d_{r,m}$ and α_m on the distance L_1 is given by the factor $(L_1^2+Z_{R1}^2)^{-1/2}$. If $|L_1|\gg Z_{R1}$, Z_{R1} can be neglected in Eqs. (20) and (21). In this case, $d_{r,m}$ and α_m decrease linearly as $|L_1|$ increases, according to the geometric optics, provided that the apparent source is considered located at the beam waist (see Sec. III). Moreover, the analysis of the relation between the retinal image diameter given by Eq. (20) and the far-field divergence [see Eq. (7)] gives other interesting results. If $|L_1|\gg Z_{R1}$, the image diameter decreases as the divergence increases. On the contrary, if $|L_1|\ll Z_{R1}$, the image diameter increases as the divergence increases.

It is important to determine in which cases f_m is within the eye accommodation range, because only in such cases can the eye obtain the minimum retinal image as indicated by Eqs. (20) and (21).

Regarding the maximum limit $f_{e,\max}$ of the eye accommodation range, it can be easily shown that if $L_1 \geqslant 0$, f_m is never greater than L_e, which is equal to $f_{e,\max}$ in a normal eye. This result can be shown in the following way:

$$f_m = \frac{L_e}{1 + \dfrac{L_1 L_e}{L_1^2 + Z_{R1}^2}}. \tag{22}$$

Therefore, if $L_1 \geqslant 0$, only the minimum limit of the accommodation range has to be considered to evaluate the retinal image size.

Regarding the possibility to have f_m less than the minimum limit $f_{e,\min}$ of the accommodation range, it can be shown that this situation does not occur if L_1 is not less than the conjugate point of L_e obtained using $f_{e,\min}$, i.e., if L_1 is not less than the minimum limit for the focusing distance (according to IEC 60825-1, this minimum distance is 100 mm, so $f_{e,\min}$ is about 14.53 mm). In fact, from Eq. (19), $f_m \geqslant f_{e,\min}$ if

$$\frac{1}{L_1 + \dfrac{Z_{R1}^2}{L_1}} \leqslant \frac{1}{f_{e,\min}} - \frac{1}{L_e}. \tag{23}$$

A sufficient condition for the validity of the above inequality is

$$\frac{1}{L_1} \leqslant \frac{1}{f_{e,\min}} - \frac{1}{L_e}. \tag{24}$$

This last inequality corresponds to have L_1 not less than the conjugate point of L_e, using $f_{e,\min}$ as focal length. It should be noted that this condition for L_1 is sufficient but not necessary. In fact, when L_1 is also smaller than the conjugate point of L_e, f_m can be greater than $f_{e,\min}$, making Eqs. (20) and (21) applicable and indicating that the eye is still able to focus the beam. This often occurs when the beam divergence and M^2 are low.

It is also necessary to know which is the eye focal length that minimizes the retinal image size when f_m is outside the accommodation range. This problem can be solved considering that the left side of Eq. (17) is an increasing function of the focal length, and using this result in the analysis of Eqs. (15) and (16) to evaluate the dependence of d_r and α on the focal length. This analysis shows that for $f > f_m$, d_r and α decrease as f decreases; therefore, if $f_m < f_{e,\min}$, the minimum value of d_r and α given by eye focusing is obtained when $f = f_{e,\min}$. On the contrary, for $f < f_m$, d_r and α decrease as f increases; therefore, if $f_m > f_{e,\max}$, the minimum values of d_r and α given by eye focusing are obtained when $f = f_{e,\max}$. Hence, if f_m is outside the eye accommodation range, the value of the eye focal length that minimizes d_r and α is $f_{\lim}$, defined as the extreme of the eye accommodation range that is closer to f_m (i.e., $f_{\lim}$ is equal to $f_{e,\min}$ or $f_{e,\max}$, whichever is closer to f_m). In this case, the retinal image diameter $d_{r,\lim}$ and the corresponding angular subtense $\alpha_{\lim}$ are given by

$$d_{r,\lim} = \frac{d_{01}}{\sqrt{L_1^2 + Z_{R1}^2}} \left[L_e^2 + \left(\frac{L_1^2 + Z_{R1}^2}{Z_{R1}} \right)^2 \times \left(1 - \frac{L_e}{f_{\lim}} + \frac{L_1 L_e}{L_1^2 + Z_{R1}^2} \right)^2 \right]^{1/2}, \tag{25}$$

$$\alpha_{\lim} = \frac{d_{\lim}}{L_e} = \frac{d_{01}}{\sqrt{L_1^2 + Z_{R1}^2}} \left[1 + \left(\frac{L_1^2 + Z_{R1}^2}{Z_{R1}} \right)^2 \times \left(\frac{1}{L_e} - \frac{1}{f_{\lim}} + \frac{L_1}{L_1^2 + Z_{R1}^2} \right)^2 \right]^{1/2}. \tag{26}$$

Therefore, if f_m is outside the eye accommodation range, the angular subtense of the apparent source is $\alpha_{\lim}$.

It is also useful to consider the beam diameter at the lens, obtained by Eq. (4) with $z = L_1$, as follows:

$$d_{\text{lens}} = d_{01} \left(1 + \frac{L_1^2}{Z_{R1}^2} \right)^{1/2}, \tag{27}$$

where d_{lens} is the beam diameter at the lens.

Using d_{lens}, Eqs. (20) and (21) can be replaced by the following new equations:

$$d_{r,m} = \frac{d_{01}^2 L_e}{Z_{R1} d_{\text{lens}}} \tag{28}$$

$$\alpha_m = \frac{d_{01}^2}{Z_{R1} d_{\text{lens}}}. \tag{29}$$

Equations (28) and (29) show that, for given values of the input beam waist and wavelength, the minimum retinal image diameter and the corresponding angular subtense depend on the beam size at the lens only. Therefore, if the wavelength and the beam waist radius are known, by the measure of the beam diameter at the lens it is possible to determine the minimum retinal image diameter and the angular subtense of the apparent source, even if the location of the beam waist and the focal length f_m are unknown. However, if f_m is outside the eye accommodation range, Eqs. (28) and (29) cannot be used because they are derived from Eqs. (20) and (21), which are valid only if f_m is within the accommodation range. In this case, the retinal image diameter and the angular subtense of the apparent source are given by Eqs. (25) and (26), respectively.

According to the criterion described in Sec. I, the distance r_s of the apparent source from the lens can be determined using Eq. (3) as follows:

$$r_s = \frac{1}{\dfrac{1}{f_m} - \dfrac{1}{L_e}} = L_1 + \frac{Z_{R1}^2}{L_1} = R_1(L_1). \tag{30}$$

The important result of Eq. (30) is that the center of curvature of the wave front at the lens is the location of the apparent source. It is interesting to note that since r_s depends on L_1 [see Eq. (30)] the location of the apparent source is not a fixed point, but it varies with the distance of the lens from the beam waist.

To determine the location of the apparent source, the value of f_m given by Eq. (18) has to be used in Eq. (30), even if f_m is outside the eye accommodation range. In fact, the distance of the apparent source must be evaluated independently from the possibility for the eye to obtain the minimum image on the retina. Of course, if f_m is outside the eye accommodation range, the retinal image diameter is greater than $d_{r,m}$, while the angular subtense of the apparent source is greater than α_m [see Eqs. (25) and (26)].

With regards to the apparent source size, there is another interesting result: if f_m is within the eye accommodation range, the angular subtense of the apparent source is determined by the diameter of the beam evaluated at the center of curvature of the wave front incident on the lens. In particular, the angular subtense α_m is equal to the angle that this diameter subtends at the lens. This result can be demonstrated as follows:

$$d[L_1-R_1(L_1)]=d_0\sqrt{1+\frac{[L_1-R_1(L_1)]^2}{Z_{R1}^2}}=d_0\sqrt{\frac{L_1^2+Z_{R1}^2}{L_1^2}}, \tag{31}$$

$$\frac{d[L_1-R_1(L_1)]}{R_1(L_1)}=d_0\sqrt{\frac{L_1^2+Z_{R1}^2}{L_1^2}}\frac{L_1}{L_1^2+Z_{R1}^2}=\frac{d_0}{\sqrt{L_1^2+Z_{R1}^2}}=\alpha_m, \tag{32}$$

where $d[L_1-R_1(L_1)]$ is the input beam diameter at the center of curvature of the wave front incident on the lens. These results show that if f_m is within the eye accommodation range, the apparent source can be considered as the cross section (containing 63% of the power) of the beam at the center of curvature of the wave front at the lens, in terms of both the location and size. In this case, the eye focal length needed for focusing and the retinal image size can be determined by the application of the geometric optics, considering this cross section as the virtual object to be focused by the eye. However, unlike the virtual objects in the geometric optics, the apparent source has location and size that depend on the location of the eye (in particular, they vary with the distance of the eye from the beam waist).

The size and the distance of the beam waist of the input beam are often present in the equations of this section, because those parameters are known in most cases, making their use convenient in describing the beam. However, sometimes the waist of the input beam is not accessible, or is ''virtual'' (i.e., the beam waist is formed behind an optical system, giving a beam divergent at all distances after that optical system). In these cases, the beam waist of the input beam can always be obtained characterizing the accessible part of the input beam. A method is to plot the diameter of the input beam versus the distance from the optical system, and to find the best fit curve for the collected data. As an alternative, an appropriate lens can be used to form an accessible beam waist; then, the location and size of the waist of the input beam can be obtained applying the propagation theory to the beam through the lens [see Eqs. (10) and (11)].

III. USE OF THE BEAM WAIST AS THE APPARENT SOURCE

If the distance of the eye lens from the beam waist is much greater than the Rayleigh length, the center of curvature of the wave front at the lens is very close to the beam waist. Therefore, if $L_1 \gg Z_{R1}$, both the eye focal length needed for focusing and the retinal image can be determined by the geometric optics considering the input beam waist as the virtual object to be focused by the eye. Therefore, using this approximation, the beam waist can be considered to be the apparent source, in terms of both the location and size. In this case, the apparent source has location and size that are both fixed (in particular, independent from the distance L_1) like the virtual objects in the geometric optics. This result is consistent with the fact that, at distances from the beam waist much greater than the Rayleigh length, the laser beam can be treated using the geometric optics. In this case, by applying the geometric optics, the retinal image diameter can be considered to be the diameter of the image of the beam waist, and can be calculated as follows:

$$d_{r,w}=\frac{d_{01}L_e}{L_1}, \tag{33}$$

where $d_{r,w}$ is the retinal image diameter calculated using the beam waist as a virtual object.

It is interesting to evaluate the error Δ_d given by the use of the beam waist as the apparent source in the determination of the retinal image diameter. For this purpose, it is necessary to compare $d_{r,w}$ with the exact value obtained by the method of Sec. II. Since $d_{r,w}$ is valid only if the eye can focus the object, the comparison has to be done only in the case where the image can be focused on the retina, i.e., when the focal length f_m is within the eye accommodation range. As shown by Eq. (22), this implies that $L_1 \geqslant 0$ [if $L_1<0$, the denominator of the right side of Eq. (22) is less than 1, giving $f_m>L_e=f_{e,\max}$]. In this case, the exact value of the retinal image diameter is $d_{r,m}$, given by Eq. (20). This error can be evaluated as follows:

$$\Delta_d=\frac{d_{r,w}-d_{r,m}}{d_{r,m}}. \tag{34}$$

The error Δ_α referred to the angle α subtended by the retinal image at the lens is equal to Δ_d, as shown below:

$$\Delta_\alpha=\frac{\alpha_w-\alpha_m}{\alpha_m}=\frac{\frac{d_{r,w}}{L_e}-\frac{d_{r,m}}{L_e}}{\frac{d_{r,m}}{L_e}}=\frac{d_{r,w}-d_{r,m}}{d_{r,m}}=\Delta_d, \tag{35}$$

where α_w is the angle subtended by $d_{r,w}$ at the lens. Replacing $d_{r,m}$ by its expression given by Eq. (20), from Eq. (34) the following equation is derived:

$$\Delta_d=\frac{\frac{d_{01}L_e}{L_1}-\frac{d_{01}L_e}{\sqrt{L_1^2+Z_{R1}^2}}}{\frac{d_{01}L_e}{\sqrt{L_1^2+Z_{R1}^2}}}=\frac{\sqrt{L_1^2+Z_{R1}^2}}{L_1}-1. \tag{36}$$

TABLE I. Errors Δ_d and Δ_α for some values of k.

k	$\Delta_d = \Delta_\alpha$
1	0.4142=41.42%
2	0.1180=11.80%
3	0.0541=5.41%
4	0.0308=3.08%
5	0.0198=1.98%
10	0.0050=0.50%
50	0.0002=0.02%

TABLE II. Error Δ_r for some values of k.

k	Δ_r
1	−0.5=−50%
2	−0.2=−20%
3	−0.1=−10%
4	−0.0588=−5.88%
5	−0.0385=−3.85%
10	−0.0099=−0.99%
50	−0.0004=−0.04%

It is useful to evaluate Δ_d and Δ_α considering the distance of the beam from the lens L_1 in units of Z_{R1}, i.e., considering $L_1 = kZ_{R1}$, where k is the coefficient that determines how much greater L_1 is compared with Z_{R1}. Hence, the following equation can be used to determine Δ_d and Δ_α as functions of k:

$$\Delta_d(k) = \Delta_\alpha(k) = \frac{\sqrt{k^2 Z_{R1}^2 + Z_{R1}^2}}{kZ_{R1}} - 1 = \frac{\sqrt{k^2+1}}{k} - 1. \quad (37)$$

As shown by Eq. (37), the only parameter needed to calculate the errors Δ_d and Δ_α is the distance of the beam waist expressed in units of Rayleigh length. Therefore, if L_1 and Z_{R1} are known, the errors are easily calculated. Some examples, giving the value of Δ_d as function of k, are shown in Table I. It is also interesting to evaluate the error Δ_r given by the use of the beam waist as the apparent source in determining the location of the apparent source. Using this approximation, the distance of the apparent source can be considered to be equal to the distance of the beam waist from the eye lens, i.e., L_1. For the calculation of the error in the evaluation of distance of the apparent source, it is necessary to compare L_1 with $r_s = R_1(L_1)$ given by Eq. (30). This error is given by

$$\Delta_r = \frac{L_1 - R_1(L_1)}{R_1(L_1)} = \frac{L_1 - L_1 - \dfrac{Z_{R1}^2}{L_1}}{L_1 + \dfrac{Z_{R1}^2}{L_1}} = -\frac{Z_{R1}^2}{L_1^2 + Z_{R1}^2}. \quad (38)$$

Considering the distance L_1 in units of Z_{R1} (i.e., $L_1 = kZ_{R1}$), from Eq. (38) the following equation is derived:

$$\Delta_r(k) = -\frac{Z_{R1}^2}{k^2 Z_{R1}^2 + Z_{R1}^2} = -\frac{1}{k^2+1}. \quad (39)$$

If L_1 and Z_{R1} are known, the error Δ_r is easily calculated as in the case of Δ_d. Some examples, giving the value of Δ_r as a function of k, are shown in Table II.

As shown by Eqs. (36) and (38), since $L_1 \geqslant 0$ (because f_m is assumed to be within the eye accommodation range), $d_{r,w}$ and α_w are always greater than $d_{r,m}$ and α_m, respectively, while L_1 is always less than $R_1(L_1)$. These results are important in the safety evaluation. In particular, the overestimation of the retinal image size leads to an underestimation of the retinal hazard, while the error in determining the distance of the apparent source from the eye affects the position of the measurement aperture for the classification (see IEC 60825-1) and, consequently, can modify the amount of radiation passing through the aperture (and thus collected by the detector).

Of course, as already stated at the beginning of this section, the values of Δ_d, Δ_α, and Δ_r calculated using Eqs. (34)–(39) are valid only if f_m is within the eye accommodation range.

IV. PROCEDURE FOR DETERMINING THE APPARENT SOURCE

The following procedure can be used to determine the diameter of the apparent source, and the distance of the apparent source from the eye, when the wavelength λ, the beam waist diameter d_0 and its distance L_1 from the eye are known.

(1) Z_{R1} is calculated by Eq. (6).

(2) The value of f_m is calculated by Eq. (17).

(3) (a) If f_m is within the eye accommodation range, it must be considered the focal length of the eye that minimizes the retinal image size, so the diameter of the minimum retinal image is $d_{r,m}$, given by Eq. (20). The angular subtense of the apparent source is α_m, given by Eq. (21). (b) If f_m is outside the eye accommodation range, $f_{\lim}$ must be considered the eye focal length that minimizes the retinal image size. Therefore, the diameter of the minimum retinal image is $d_{r,\lim}$, given by Eq. (25). The angular subtense of the source is $\alpha_{\lim}$, given by Eq. (26).

(4) The distance of the apparent source from the eye is r_s, given by Eq. (30). This value is equal to the radius of curvature of the wave front at the eye lens. Therefore, the center of curvature at the lens must be considered as the location of the apparent source.

If the wavelength λ and the input beam waist diameter d_{01} are known, but the distance L_1 is unknown, it is still possible to determine the size of the apparent source and its distance from the eye by measuring the beam diameter d_{lens} at the lens. In fact, if d_{lens} is known, the value of L_1 is determined by the following equation [derived from Eq. (27)]:

$$L_1 = Z_{R1}\sqrt{\left(\frac{d_{\text{lens}}}{d_{01}}\right)^2 - 1}. \quad (40)$$

Then, once L_1 is determined, the procedure described above can be used to determine the size and location of the apparent source. If f_m is within the eye accommodation range, Eqs. (28) and (29) can also be used to calculate $d_{r,m}$ and α_m.

TABLE III. Values of Φ_1, Z_{R1}, f_m, r_s, and α for λ=632.8 nm and different values of d_{01}, M^2, and L_1.

			He–Ne laser λ=632.8 nm				
d_{01} (mm)	M^2	L_1 (m)	Φ_1 (mrad)	Z_{R1} (m)	F_m (mm)	r_s (m)	α (mrad)
0.1	1	0.1	4.03	0.025	14.65	0.11	0.97
0.1	1	2.0	4.03	0.025	16.86	2.00	0.05
0.1	10	0.1	40.29	0.002	14.53	0.10	1.00
0.1	10	2.0	40.29	0.002	16.86	2.00	0.05
1.0	1	0.1	0.40	2.482	17.00	61.72	0.40
1.0	1	2.0	0.40	2.482	16.94	5.08	0.31
1.0	10	0.1	4.03	0.248	16.61	0.72	3.74
1.0	10	2.0	4.03	0.248	16.86	2.03	0.50
10.0	1	0.1	0.04	248.230	17.00	616 181.07	0.04
10.0	1	2.0	0.04	248.230	17.00	30 811.05	0.04
10.0	10	0.1	0.40	24.823	17.00	6161.91	0.40
10.0	10	2.0	0.40	24.823	17.00	310.09	0.40

V. EXAMPLES

In this section there are some examples showing the values of the distance and size of the apparent source for beams with different characteristics (d_{01} and M^2) and viewing distances (L_1). These results are given in Tables III and IV, where there are also indicated Z_{r1}, f_m, and the input beam far-field divergence Φ_1.

For the evaluation of the retinal hazard, it is particularly important to compare the angular subtense α of the apparent source with the angle subtended at the lens by the minimum retinal image obtainable by the focusing system of the eye. According to IEC 60825-1, this minimum angle is 1.5 mrad. Since $L_c = 17$ mm, this angle corresponds to a minimal retinal image diameter of about 25 μm. For this purpose, it should be noted that α is always less than 1.5 mrad for $M^2 = 1$ in both Tables III and IV. This result has a general validity with few exceptions, provided that the distance L_1 is not less than 100 mm. In fact, if $M^2=1$ and $L_1 \geqslant 100$ mm, α can have values greater than 1.5 mrad only if the wavelength is greater than 706 nm and the input beam waist d_{01} is in an appropriate range depending on λ and L_1. This range is from 0.16 to 0.57 mm for $\lambda=1400$ nm and $L_1=100$ mm, and reduces for lower wavelengths or greater values of L_1. Moreover, another necessary condition for having α greater than 1.5 mrad is to have L_1 not too much greater than 100 mm. In fact, for $M^2=1$ and $L_1 > 198$ mm, α is always less than 1.5 mrad. For $M^2=1$, if L_1 is not less than 100 mm and λ is in the range from 400 to 1400 nm (the spectral region for the retinal hazard), the maximum value that α can reach is about 2.1 mrad (this value is obtained by using $L_1=100$ mm, $\lambda=1400$ nm, and $d_{01}=0.30$ mm).

As described in IEC 60825-1, the retinal hazard due to the thermal effect depends on the ratio of α to 1.5 mrad. According to the above considerations, for a TEM_{00} Gaussian beam ($M^2=1$) this ratio is almost always less than 1 and the few exceptions above indicated give, in any case, values not greater than 1.4 (i.e., the ratio of 2.1 to 1.5 mrad). These results justify the common practice of considering this ratio not greater than 1 for direct viewing (including viewing of specular reflections) of TEM_{00} Gaussian beams.

TABLE IV. Values of Φ_1, Z_{R1}, f_m, r_s, and α for λ=1064 nm and different values of d_{01}, M^2, and L_1.

			Nd:YAG laser λ=1064 nm				
d_{01} (mm)	M^2	L_1 (m)	Φ_1 (mrad)	Z_{R1} (m)	f_m (mm)	r_s (m)	α (mrad)
0.1	1	0.1	6.77	0.0148	14.58	0.10	0.99
0.1	1	2.0	6.77	0.0148	16.86	2.00	0.05
0.1	20	0.1	135.47	0.0007	14.53	0.10	1.00
0.1	20	2.0	135.47	0.0007	16.86	2.00	0.05
1.0	1	0.1	0.68	1.4763	16.99	21.90	0.68
1.0	1	2.0	0.68	1.4763	16.91	3.09	0.40
1.0	20	0.1	13.55	0.0738	15.31	0.15	8.05
1.0	20	2.0	13.55	0.0738	16.86	2.00	0.50
10.0	1	0.1	0.07	147.6312	17.00	217 950.65	0.07
10.0	1	2.0	0.07	147.6312	17.00	10 899.53	0.07
10.0	20	0.1	1.35	7.3816	17.00	544.98	1.35
10.0	20	2.0	1.35	7.3816	16.99	29.24	1.31

VI. APPARENT SOURCE DETERMINATION BY MEASUREMENTS

In the previous sections, the determination of the apparent source by an analytic method is described. However, it is also necessary to have an alternative method based on measurements. For instance, the equations of the previous sections are not applicable in the case of lack of information on the input beam characteristics. Moreover, the analytic method does not consider the truncation[5] of the beam caused by the eye pupil aperture. The truncation effect, influencing the characteristics of the beam after the aperture, could modify the location and size of the apparent source. To solve these problems, appropriate measurement methods are provided in this section to evaluate the apparent source, in terms of both the location and size. These measurement methods are based on the approach described in the previous sections with regards to the definitions of the distance and size of the apparent source.

The first measurement method described below simulates the human eye. The measurement apparatus consists of a lens, an appropriate circular aperture on the lens to simulate the eye pupil (according to IEC 60825-1, a circular aperture with 7 mm diameter), and a detector placed at a distance of 17 mm from the lens. The spot size on the detector must be measured to determine the location and size of the apparent source, as indicated in the procedures below described.

(a) The focal length of the lens must be varied to find the value that minimizes the spot size on the detector. Using this value of the focal length, the location of the apparent source is the conjugate point of the detector.

(b) The focal length of the lens must be varied within the eye accommodation range (from 14.53 to 17 mm) to find the value that minimizes the spot size on the detector. Using this value of the focal length, the plane angle that the diameter of the spot on the detector subtends at the lens is the angular subtense of the apparent source.

It is important to point out that the value of the focal length to be determined in procedure (b) has to be looked for within the eye accommodation range, while this limitation is not present in (a). The reason is that, as already stated in Sec. II, the distance of the apparent source from the eye depends only on the location of the eye and does not depend on the accommodation range, whereas the size of the apparent source, which is determined by the retinal image size, depends on the eye focusing characteristics, including the accommodation limits. Consequently, the value of the focal length found in procedure (b) can be different from the one found in procedure (a).

If an optical system consisting of two or more lenses is used to vary the focal length, an approximation is introduced, because the use of a single lens gives a better simulation of the eye focusing system. Nevertheless, this approximation is normally acceptable, taking into account that other approximations are however present (e.g., the aberration of the eye is not reproduced by the measurement system). However, particular attention must be paid in evaluating the distances to be used in determining the conjugate point of the detector. In particular, the distance of the beam waist from the objective to the front principal plane of the optical system has to be used, while at the same time the distance of the detector from the optical system to its rear principal plane also has to be used.

The above basic method gives a good simulation of the eye focusing system. However, since there is the need to also have easily applicable methods, two alternative measurement methods are described below.

The first alternative to the basic method makes use of the distance of the detector from the lens equal to a generic value L_0 instead of L_e (17 mm). Also in this case, the location of the apparent source is determined as specified in procedure (a). In fact, the value of the distance of the apparent source from the lens is $R_1(L_1)$, which does not depend on the distance of the detector from the lens. Regarding the size of the apparent source, the procedure specified in procedure (b) is still applicable. In fact, as shown by Eq. (16), the value of α is not changed if L_e is replaced by L_0, provided that the focal length f (the focal length to focus on the retina) is replaced by a new focal length f_L that satisfies the following equation:

$$\frac{1}{L_e}-\frac{1}{f}=\frac{1}{L_0}-\frac{1}{f_L} \tag{41}$$

giving

$$f_L=\frac{1}{\frac{1}{L_0}+\frac{1}{f}-\frac{1}{L_e}}. \tag{42}$$

Therefore, since Eq. (41) is satisfied, because the condition of minimizing the spot on the detector requires that both members of Eq. (41) be equal to $1/R_1(L_1)$, procedure (b) is applicable even if $L_0 \neq L_e$. However, the variation range for the focal length specified in procedure (b) has to be changed as follows:

$$f_{L,\min}=\frac{1}{\frac{1}{L_0}+\frac{1}{f_{e,\min}}-\frac{1}{L_e}}=\frac{1}{\frac{1}{L_0}+\frac{1}{100\,\mathrm{mm}}}, \tag{43}$$

$$f_{L,\max}=\frac{1}{\frac{1}{L_0}+\frac{1}{f_{e,\max}}-\frac{1}{L_e}}=L_0, \tag{44}$$

where $f_{L,\min}$ is the minimum limit of the variation range for f to be used in procedure (b) and $f_{L,\max}$ is the maximum limit of the variation range for f to be used in procedure (b). Equations (43) and (44) are derived from Eq. (42) replacing f by the limits of the eye accommodation range, and taking into account that $f_{e,\min}$ is the focal length to focus objects at a distance of 100 mm from the eye and $f_{e,\max}$ is the focal length to focus objects at infinity ($f_{e,\max}=L_e=17\,\mathrm{mm}$).

The second alternative is to vary the distance of the detector from the lens, using a fixed focal length f_0 (the use of a fixed focal length makes this method much easier to apply than the previous ones). This new method is as follows.

(A) The distance of the detector from the lens must be varied to find the value that minimizes the angle that the spot on the detector subtends at the lens. Using this value of the distance, the location of the apparent source is the conjugate point of the detector.

(B) The distance of the detector from the lens must be varied within an appropriate range to find the value that minimizes the angle that the spot on the detector subtends at the lens. Using this value of the distance, this minimized plane angle that the diameter of the spot on the detector subtends at the lens is the angular subtense of the apparent source.

It should be noted that, for the same reasons indicated in the description of the basic method, the appropriate variation range is used in procedure (B) only. Therefore, the value of the distance found in procedure (B) can be different from the one found in procedure (A). The limits of the variation range for the distance to be used in procedure (B) are as follows:

$$L_{\min}=\frac{1}{\frac{1}{f_0}+\frac{1}{L_e}-\frac{1}{f_{e,\max}}}=f_0, \tag{45}$$

$$L_{\max}=\frac{1}{\frac{1}{f_0}+\frac{1}{L_e}-\frac{1}{f_{e,\min}}}=\frac{1}{\frac{1}{f_0}-\frac{1}{100\,\mathrm{mm}}}, \tag{46}$$

where $L_{\min}$ is the minimum limit of the variation range for L to be used in procedure (B) and $L_{\max}$ is the maximum limit of the variation range for L to be used in procedure (B). Equations (45) and (46) are derived the same as Eqs. (43) and (44). As shown by Eq. (46), $L_{\max}$ is greater than zero (i.e., the location of the detector is after the lens) only if $f_0 < 100\,\mathrm{mm}$. Therefore, for the determination of the size of the apparent source by this alternative method, it is necessary to use a fixed focal length less than 100 mm (the minimum focusing distance of the eye).

Procedure (A) requires us to minimize the angle subtended by the spot, not the spot size. The reason is the following: the conjugate point of the detector results in being the center of curvature at the lens if Eq. (17) is still satisfied replacing f_m by f_0 and L_e by L, so that the angle subtended by the spot diameter [see Eq. (16)] is minimized; however, since the distance L is not a constant, minimizing the spot on the detector [see Eq. (15)] does not give the same result as minimizing the angle subtended by the spot [see Eq. (16)]. The same considerations apply to procedure (b), where there is also the requirement of the variation range for L. The limits of this range [see Eqs. (45) and (46)] are derived from Eq. (41), replacing f_L by f_0 and f by the limits of the eye accommodation range. In fact, as already explained in the description of the first alternative and shown by Eq. (16), the value of angle that the spot size subtends at the lens does not change if the focal length and the distance of the lens from the detector are different from f and L_e, provided that they satisfy Eq. (41) when they replace f_L and L_0, respectively.

The equivalence of the two alternative methods to the basic one is proved using the equations of the analytic method described in Sec. II. Therefore, the validity of this equivalence cannot be ensured when other effects (e.g., the truncation of the measurement aperture) not considered by the analytic method are important in determining the spot size on the detector.

VII. CONCLUSIONS

The effects of the laser radiation on the retina depend on the optical properties of the eye which determine the retinal image size. For this reason, it is convenient to have the concept of ''apparent source'' based on the characteristics of the eye focusing.

This study provides an analytic method to determine the location and size of the apparent source. This method, which describes the propagation of the beam through a thin lens, gives the following results: the location of the apparent source is the center of curvature of the wave front at the lens, while the diameter of the apparent source is the diameter of the beam cross section at this center of curvature.

The use of appropriate measurements to determine the apparent source is also described. The first measurement method shown in this study is based on the simulation of the eye focusing system (variable focal length and distance of the detector from the lens equal to L_e) and is applicable in any situation. Two alternative methods are also provided to make the measurements easy to perform (especially the second alternative, allowing the use of a fixed focal length, is easily applicable).

Another topic of this study is the possibility of identifying the apparent source with the beam waist. For this purpose, it is important to note that the location and size of the apparent source normally do not coincide with the location and size of the beam waist. Only if the distance of the eye lens from the beam waist is much greater than the Rayleigh length can the beam waist be considered the apparent source in terms of both the location and size.

The results of this study can also be a guide to be used in a practical safety problem, but in this case the focusing characteristics of the examined eye, and in particular the actual accommodation range, have to be considered in using the appropriate equations.

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