CS2100 COMPUTER ORGANISATION

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Lecture #16

Quine-McCluskey

NUS National University of Singapore | School of Computing

Lecture #16: Quine-McCluskey

This topic is only for your own reading only.

- A tabulation method similar in concept to K-map
- Applicable for functions with any number of variables
  - K-maps are useful for functions with at most 5 or 6 variables
- Tedious on paper, but can be automated (programmed)
- Non-examinable
  - But knowing it may enhance your understanding of K-maps

PIs and EPIs

- To find the simplest SOP expression from a K-map, you need to obtain:
  - Minimum number of literals per product term; and
  - Minimum number of product terms.
- Achieved through K-map using
  - *Biggest groupings* of minterms (prime implicants) where possible; and
  - *No redundant groupings* (look for essential prime implicants)

Eg: $F(A,B,C,D) = \Sigma m(2,3,4,5,7,8,10,13,15)$

Step 1: List out all minterms in groups with same number of 1s in their binary codes.

1st column

2: 0010
4: 0100
8: 1000

Codes with one 1

3: 0011
5: 0101
10: 1010

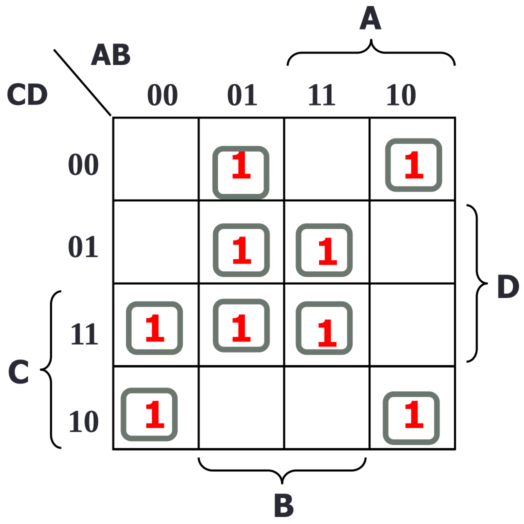
Codes with two 1s

7: 0111
13: 1101

Codes with three 1s

15: 1111

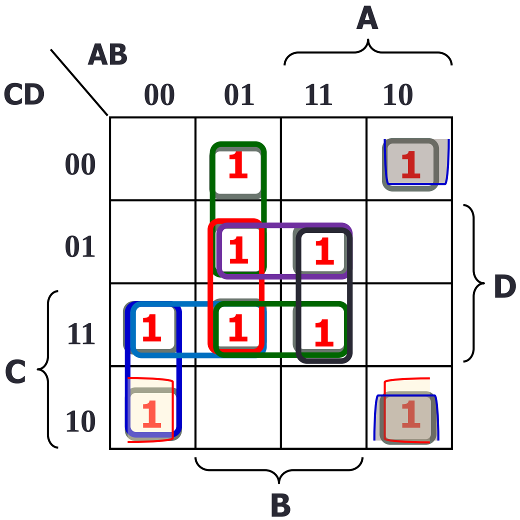
Codes with four 1s



Eg: F(A,B,C,D) = Σm(2,3,4,5,7,8,10,13,15)

Step 2: Combine codes that differ by 1 bit into bigger group, write the combined code in next column.

1st column	2nd column	
✓ 2: 0010	2,3: 001-	Codes with one 1
✓ 4: 0100	2,10: -010	
✓ 8: 1000	4,5: 010-	
	8,10: 10-0	
✓ 3: 0011	3,7: 0-11	Codes with two 1s
✓ 5: 0101	5,7: 01-1	
✓ 10: 1010	5,13: -101	
✓ 7: 0111	7,15: -111	Codes with three 1s
✓ 13: 1101	13,15: 11-1	
✓ 15: 1111		



Eg: F(A,B,C,D) = Σm(2,3,4,5,7,8,10,13,15)

Step 3: Repeat step 2 – Combine codes that differ by 1 bit into bigger group, write the combined code in next column.

1st column

✓ 2: 0010
✓ 4: 0100
✓ 8: 1000

✓ 3: 0011
✓ 5: 0101
✓10: 1010

✓ 7: 0111
✓13: 1101

✓15: 1111

2nd column

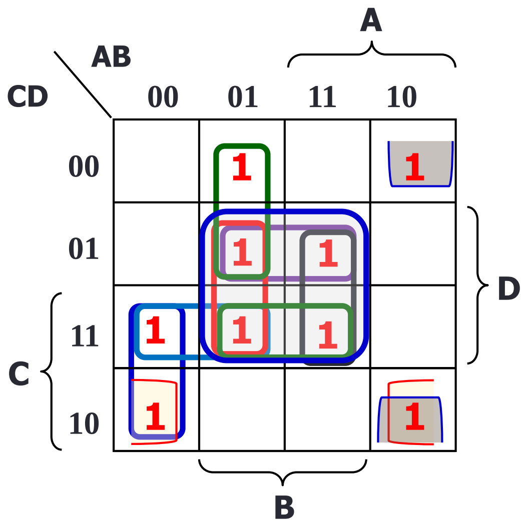
2,3: 001-
2,10: -010
4,5: 010-
8,10: 10-0

3,7: 0-11
✓ 5,7: 01-1
✓ 5,13: -101

✓ 7,15: -111
✓13,15: 11-1

3rd column

5,7,13,15: -1-1
~~5,7,13,15: -1-1~~



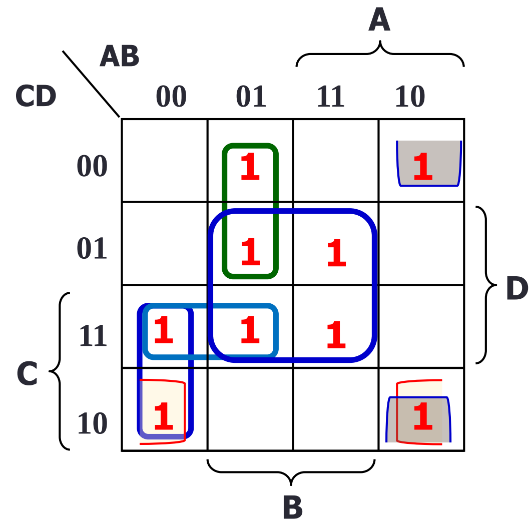
We have completed Phase 1: Identifying all the Prime Implicants (PIs)!

Eg: F(A,B,C,D) = Σm(2,3,4,5,7,8,10,13,15)

Phase 2: Identify the Essential Prime Implicants (EPIs)

- Draw the PI chart

		2	3	4	5	7	8	10	13	15
	2,3: 001- (A'.B'.C)	✓	✓							
	2,10: -101 (B.C'.D)	✓						✓		
EPI	4,5: 010- (A'.B.C')			✓	✓					
EPI	8,10: 10-0 (A.B'.D')						✓	✓		
	3,7: 0-11 (A'.C.D)		✓			✓				
EPI	5,7,13,15: -1-1 (B.D)				✓	✓			✓	✓



Where are the EPIs? Look for columns containing a single tick.

EPIs are: A'.B.C', A.B'.D', and B.D

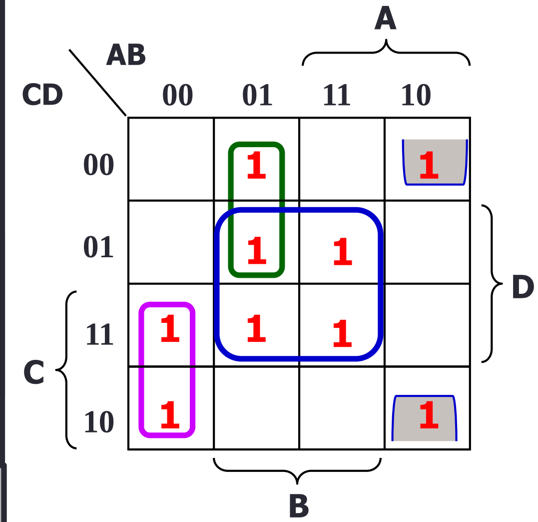
But we are not done yet. There are still minterms not covered by the EPIs!

Eg: F(A,B,C,D) = Σm(2,3,4,5,7,8,10,13,15)

Phase 2: After identifying the EPIs

- Draw the reduced PI chart if there are minterms not covered

	2	3	4	5	7	8	10	13	15
2,3: 001- (A'.B'.C)	✓	✓							
2,10: -101 (B.C'.D)	✓						✓		
EPI 4,5: 010- (A'.B.C')			✓	✓					
EPI 8,10: 10-0 (A.B'.D')						✓	✓		
3,7: 0-11 (A'.C.D)		✓			✓				
EPI 5,7,13,15: -1-1 (B.D)				✓	✓			✓	✓



1. Find out what are the minterms covered by the EPIs.
2. Remove the EPIs and minterms they cover from the chart → reduced PI chart.
3. Find the minimum number of remaining PIs to cover the remaining minterms.

Answer:
B.D + A'.B.C' + A.B'.D' + A'.B'.C

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