In this report we will explore the results derived from the application of the financial concept of the Put-Call parity. The Put-Call parity states some properites of the pricing of the options related to a particular costruction of the portfolio. In detail, if both the options have the same strike price, holding a long call option and a short put option together forms a portfolio that has the same value as single forward contract at the specified strike price and expiration date. Practically speaking this is related to the possibility, at maturity, to exercise the profitable option, depending on the behaviour of the market. In any case, one unit of the final asset is acquired at the defined strike price. We are therefore replicating the results of a forward contract removing the general risk, at the cost of the price of the options.

1) choose a dividend paying asset (use the google,docs)

The choosen asset is Nvidia (NVDA). This is a paying dividends asset, the interest aspect is that the options are not European but American. Even though the model we will apply is appropriate particularly for the European options it might be interesting to see how it performs over an American option.

2) fix a maturity T and consider Call a Put with K1 and K2 (K1<<K2)

Maturity(months): 1 T= 0,083333

Here we choose the CALL and PUT options in order to have some strike prices K1, K2 with K1<<K2. In particular the strike prices I've chosen aren't the two extremes of the spectrum. That's because we want to avoid the scarce liquidity for the extremes strike prices and the extreme fluctuations that characterize these values.

		Strike	Last Price	Bid	Ask
Call with k1	k1=	70	38,35	39,15	39,55
Call with k2	k2=	170	0,02	0,03	0,04
Put with k1	k1=	70	0,15	0,14	0,16
Put with k2	k2=	170	52	60,85	61,4

3) Find the corresponding discount factor for maturity T

Following what mentioned in the introduction we can construct a way of expressing the discount factor D(0, T) as a function of the payoffs and strikes.
The whole formulation is omitted since part of it was given as a text of this exercise but we will cover the principal points.
The discount rate is related to the box spread arbitrage method, which always results in K_2 - K_1 at expiration. Since this element is deterministic it's value today can be obtained by multiplying for the discount factor. Therefore starting from:

$$(S-K1)^{+}-(S-K2)^{+}+(K2-S)^{+}-(K1-S)^{+}=K2-K1$$

And multiplying for the discount factor the right term, we end up with:

$$D(0,T)=\frac{p_0^{call}(K1)-p_0^{call}(K2)+p_0^{put}(K2)-p_0^{put}(K1)}{(K2-K1)}$$

To apply this formula we start from deriving the various payoffs for each Call/Put option.

p0call(k1)	39,35	p0put(k1)	0,15
p0call(k2)	0,035	p0put(k2)	61,125

That allows us to calculate the discount factor:

Discount factor (D)	1,0029

Having a discount factor that is higher than 1 looks unsettling at first, since it is related to the exponent of a negative number.
Nonetheless it's formulation for a paying dividend asset is exp(-r + δ) where δ is the continous dividend yield. Now we can see that when δ > r the total exponent is positive and the results are justified.

4) Find the implicit dividend for maturity T using the put-call parity and using At The Money options (that is new Call and Put with strikes ATM)

Now we are interested in obtaining the implicit dividend for a maturity T. This is obtained from the factor F(0,T). That's because the final formulation we are looking for is the following:

$$D_{\text{implicit}} = S_0 e^{\delta T} = \frac{S_0}{D(0,T)} - F(0,T)$$

This factor, F(0,T) represent the forward price of the underlying asset at time T. Its importance is related to its role in estimating the forward price of the assets, considering both the option prices and discounting effects
This forward price is computed using the expected payoffs of two new options, Call and Put, with strikes at the money.

$$F(0,T) = \frac{p_0^{call} - p_0^{put}}{D(0,T)} + K$$

	Strike	Last price	Bid	Ask
Call ATM (k)	109	6,25	6,05	6,15
Put ATM (k)	109	5,7	5,8	5,85

The approximated payoffs calculated as the average between bid and ask are therefore:

p0call(k)	6,1
p0put(k)	5,825

The forward price results to be: F(0,T) 109,2742
We also see that the result is reasonable with the asset considered.

In the end, using the true current price we obtain the dividend yield for the NVDA asset with maturity at 1 month: Current price: 109,9

Dividend: 0,308007

Comments over these results will be performed at the end.

We proceed to repeat the calculations for three different maturities:
T=3months, 6months, 12 months.

2)	fix a maturity T and consider Call a Put with K1 and K2 (K1<<K2)

Maturity(months):		3		T=	0,25
		Strike	Last price	Bid	Ask
Call with k1	k1=	5	111,89	105	105,5
Call with k2	k2=	300	0,08	0,04	0,06
Put with k1	k1=	5	0,02	0	0,01
Put with k2	k2=	300	173,35	189,65	191

3)	Find the corresponding discount factor for maturity T

p0call(k1)	105,25	p0put(k1)	0,005
p0call(k2)	0,05	p0put(k2)	190,325

Discount factor (D)	1,001763

4)	Find the implicit dividend for maturity T using the put-call parity and using At The Money options (that is new Call and Put with strikes ATM)

	Strike	Last price	Bid	Ask		
Call ATM (k)	110	12,13	12	12,1	p0call(k)	12,05
Put ATM (k)	110	10,42	10,6	10,65	p0put(k)	10,625

Current price: 109,9

F(0,T)	111,4225	Dividend:	-1,71587

2)	fix a maturity T and consider Call a Put with K1 and K2 (K1<<K2)

Maturity(months):		6		T=	0,5
		Strike	Last price	Bid	Ask
Call with k1	k1=	60	50	52,3	52,55
Call with k2	k2=	160	3,45	3,35	3,4
Put with k1	k1=	60	1,06	0,98	1
Put with k2	k2=	160	51,59	50,9	51,45

3)	Find the corresponding discount factor for maturity T

p0call(k1)	52,425	p0put(k1)	0,99
p0call(k2)	3,375	p0put(k2)	51,175

Discount factor (D)	0,99235

4)	Find the implicit dividend for maturity T using the put-call parity and using At The Money options (that is new Call and Put with strikes ATM)						

	Strike	Last price	Bid	Ask		
Call ATM (k)	110	16,75	16,4	16,55	p0call(k)	16,475
Put ATM (k)	110	14,6	14,1	14,2	p0put(k)	14,15

Current price: 109,9

F(0,T) 112,3429

Dividend: -1,59571

2)	fix a maturity T and consider Call a Put with K1 and K2 (K1<<K2)

Maturity(months): 12 | T= 1

		Strike	Last price	Bid	Ask
Call with k1	k1=	60	52,75	54,9	55,3
Call with k2	k2=	170	6,75	6,6	6,7
Put with k1	k1=	60	2,15	2,14	2,17
Put with k2	k2=	170	64,1	61,65	62,25

3)	Find the corresponding discount factor for maturity T

p0call(k1)	55,1	p0put(k1)	2,155
p0call(k2)	6,65	p0put(k2)	61,95

Discount factor (D) 0,984045

4)	Find the implicit dividend for maturity T using the put-call parity and using At The Money options (that is new Call and Put with strikes ATM)

	Strike	Last price	Bid	Ask		
Call ATM (k)	110	22,75	22,65	22,8	p0call(k)	22,725
Put ATM (k)	110	18,28	18,2	18,3	p0put(k)	18,25

Current price: 109,9

F(0,T) 114,5476

Dividend: -2,86572

Summarizing all the relevant results in a single table we obtain:

Maturity (T)	Discount Factor (D)	Forward Price (F(0,T))	Implied Dividend (Div)
1 Month	1,0029	109,27	0,3080
3 months	1,0017	111,42	-1,7159
6 months	0,9924	112,34	-1,5957
12 months	0,9840	114,55	-2,8657

Observing the results we encounter some unexpected values. In particular, the negative dividends strikes as an incorrect results, that is not financially acceptable.
Obtaining negative dividends from the previous analysis can be nonetheless justified.
From a general perspective, one of the reason why the results might be negative could be related to the illiquidity of the asset. This would imply unreliability of the values of the market such as current option prices. Nonetheless the asset we have considered does not have a liquidity problem since is one of the most actively traded in the world.
Another aspect to take into consideration are the direct implications of a negative result. A negative dividends could be signaling the presence of risk-free profit opportunity and an unfair market. Price incosistencies between the prices and mispricing in the contracts implies that the dividends might end up being be negative. This is significantly evident for the case of our analysis, which exploits the put-call parity. This method is based on the assumption that calls and puts with the same strike and maturity are consistently priced, if there is a mismatch in the price of one of the two, the dividend extracted will reflect this discrepancy.
Ultimately it is important to mention that the formulas were formulated for European option, since the asset we are working on have American option, where early exercise is possible, this may introduce errors in the extracted values, especially for options with significant dividend payments.
In conclusion the analysis has been satisfactory, the application of the Put-Call parity model has resulted in some contrasting results that reflected the environment we are working in.