Corte Riccardo, Physics of Data, 2160845.

NVIDIA Corporation (NASDAQ: NVDA) is a leading semiconductor company specializing in GPUs, which have been largely utilized in gaming and AI. It was founded in 1993, its technological innovations have allowed it to be the most important industry in the GPU market, it is also leading in the research for the creation of Quantum computers. Due to the youth of the company and the recent applications, in the last five years, NVIDIA has exhibited extraordinary growth, with a 5-year stock appreciation of over 2,196%, driven by AI and cloud computing expansion. Despite a recent YTD decline (-12.35%), it has significantly outperformed the broader market over the long term, reflecting strong investor confidence that is confirmed by the quadrimestral paying of the dividends.
With a market capitalization of $2.87 trillion and minimal debt, NVIDIA maintains a strong financial position, enabling substantial investments in research and development. Major institutional investors, including Vanguard and BlackRock, collectively own over 16% of its shares, underscoring significant confidence from leading financial institutions.
While NVIDIA dominates the GPU market, it faces competition from AMD, Intel, and AI-focused chipmakers like Google (TPUs). Nonetheless egulatory challenges, such as U.S. export restrictions on AI chips to China, pose potential risks.
In the last five years NVIDIA hav become a keay element of S&P 500, playinga crucial role in the stock market and the broader tech industry. Its influence in AI and semiconductor innovation cements its position as a critical company in modern computing, ensuring a promising future despite short-term market volatility.

We now proceed to perform the analysis for a CALL option with maturity T=3 months

Price	Adj Close	R_t
Ticker	NVDA	
Date		
2024-12-24	140,2071	
2024-12-26	139,9171	-0,00207
2024-12-27	136,9974	-0,02087
2024-12-30	137,4774	0,003503
2024-12-31	134,2776	-0,02327
2025-01-02	138,2973	0,029935
2025-01-03	144,4567	0,044538
2025-01-06	149,4162	0,034332
2025-01-07	140,1271	-0,06217
2025-01-08	140,0971	-0,00021
2025-01-10	135,8975	-0,02998
2025-01-13	133,2177	-0,01972
2025-01-14	131,7479	-0,01103
2025-01-15	136,2275	0,034001
2025-01-16	133,5577	-0,0196
2025-01-17	137,6973	0,030995
2025-01-21	140,817	0,022656
2025-01-22	147,0565	0,044309
2025-01-23	147,2065	0,00102
2025-01-24	142,6069	-0,03125
2025-01-27	118,4091	-0,16968

I've chosen a CALL option with maturity at 20/06/2025, and Strike k=121, since our current value of the stock is S0=120,6.
The Mid price calculated as the average between bid (11,7) and ask (11,85) is: 11,77.

Secondly I've proceded to calculate the return t using:

$$R_t = \frac{(P_t - P_{t-1})}{P_{t-1}}$$

and the stardard deviation for the daily and yearly returns

б_d = 0,0414

б_y = б_d * $\sqrt{252}$ = 0,6581

The final goal is to calculate the price of a CALL option starting from the daily returns. The

2025-01-28	128,9781	0,089259
2025-01-29	123,6886	-0,04101
2025-01-30	124,6385	0,00768
2025-01-31	120,059	-0,03674
2025-02-03	116,6493	-0,0284
2025-02-04	118,6391	0,017058
2025-02-05	124,8185	0,052086
2025-02-06	128,6682	0,030842
2025-02-07	129,828	0,009015
2025-02-10	133,5577	0,028728
2025-02-11	132,7878	-0,00576
2025-02-12	131,1279	-0,0125
2025-02-13	135,2776	0,031646
2025-02-14	138,8372	0,026314
2025-02-18	139,3872	0,003961
2025-02-19	139,2172	-0,00122
2025-02-20	140,0971	0,006321
2025-02-21	134,4176	-0,04054
2025-02-24	130,268	-0,03087
2025-02-25	126,6184	-0,02802
2025-02-26	131,2679	0,036721
2025-02-27	120,139	-0,08478
2025-02-28	124,9085	0,0397
2025-03-03	114,0495	-0,08694
2025-03-04	115,9793	0,016921
2025-03-05	117,2892	0,011294
2025-03-06	110,5598	-0,05737
2025-03-07	112,6796	0,019173
2025-03-10	106,9702	-0,05067
2025-03-11	108,75	0,016639
2025-03-12	115,74	0,064276
2025-03-13	115,58	-0,00138
2025-03-14	121,67	0,052691
2025-03-17	119,53	-0,01759
2025-03-18	115,43	-0,0343
2025-03-19	117,52	0,018106
2025-03-20	118,53	0,008594
2025-03-21	117,7	-0,007
2025-03-24	121,4193	0,0316

standard way to perform this derivation starts by obtaining the parameters u,d of the binomial model that in this first derivation are related to the maturity T=3/12 years.

$$u = e^{\frac{3}{12}\cdot\sigma_y}$$

$$d = e^{-\frac{3}{12}\cdot\sigma_y}$$

u = 1,1788, d = 0,8482.

The final formula, to calculate the current CALL option price, that we want to reconduct to is:

$$C_0 = (1 - rT)\left[qf^u + (1-q)f^d\right]$$

where:

"r" is the trimestral interest rate.
"q" is the risk neutral probability
"fu , fd" are the payoffs in the Up/Down states.

We choose the Libor rate as the interest rate. From this we proceed to calculate the Capitalisation Factor both for simple compunding and simple discounting. These elements that can be found both in the risk neutral probability and in the price of the option.

$$CF_c = 1 + rT, \quad CF_d = \frac{1}{1 + rT}$$

The next step consist in calculating the risk neutral probability. Since Nvidia pays the dividends we must take into account these benefits. The whole analysis can be resumed in the adjoint of a related term in the calculation of the risk neutral probability.

$$q = \frac{(1+rT) - (d+1+\delta T)}{u-d}$$

Where δ is the dividend yield for the Nvidia company.
Now the result obtained by applying the formulation we found in class ends up having a risk neutral probability which is intrinsically wrong, since it is negative: q= -2,5597.
We therefore proceed by considering other options such as: looking at the literature for different formulations, or simply ignoring the correction related to the dividends.
We proceed first by applying the formula which is mentioned in: "Hall: Options, Futures, and Other Derivatives, 11/e, Global Edition" (Page 304), that is here applied by computing the taylor expansion of the exponent.

$$q = \frac{(1+rT-\delta T) - d}{u-d}$$

The value obtained for the risk neutral probability in this case is: q = 0,4652
If we instead ignore the correction related to dividends by removing the corrective term, we obtain: q=0,4956.
We will proceed to calculate the final option price with both these formulations.
To compute the CALL price the only thing missing is to derive the payoffs starting from:

$$f^u = \max(S_u - K, 0), \quad f^d = \max(S_d - K, 0)$$

Where Su and Sd are the underlying asset prices in the up and down states, and are easily computed multiplying the current stock price by the parameters u,d of the binomial model.
Since we are taking a CALL with strike close to the current value of the stock it is almost sure that the payoff for the down state will be 0, since Sd will be surely less than the strike price. We can therefore calculate the prices of the option, which ends up being:

Hall formula:	C = 10,4268
Ignoring dividends factor:	C = 11,1045

We can now compare this result with the expected result, corresponding to the Mid price between bid and ask, that we already calculated and that is equal to 11,77.
The results are pretty similar, and since the binomial model is a really simple model that constitutes an approximation, therefore the results are satisfactory.
Unexpectedly the results obtained by ignoring the payoff of the dividends ends up being more accurate than their more sophisticated counterpart, which is, as expected, lower. This is probably related to the fact that the volatility is higher than what the binomial model assumes, therefore the theoretical price is be lower than the observed market price, making a "not discounted by dividends price", more accurate.

We can now repeat the analysis for the case of a CALL option with maturity at T=6 months.
The main difference between the previous computation and this one is that we need to update the maturity T, which implies a change in the values of: u, d, q, and C.
It is also necessary to update the interest rate by using the Libor at six months.
Similarly as before we compute three different risk neutral probabilities and use the two meaningful ones to calculate the price of the option.

The results that we obtain for this case are:
Hall formula: C = 18,0912
Ignoring dividends factor: C = 19,52528
For this option the Mid price for the bid (17,7) and ask (17,9) is: 17,8.
In this case the price is more accurate if the formula indicated by the Hall tome, is used.
Compared to the previous case we have a minor error, around 2%, which is lower than the one for a maturity at three months which was around 11%.
This result is not coherent with our expectations, we expect our binomial model to have more inaccurate predictions for options with further expiration dates. This is also related to the approximation after which we considered the exponential as its Taylor expansion, which is a valid assumption under small time intervals.

The justification for this accuracy might be that the market prices for short-term options are more sensitive to supply/demand fluctuations, involving a major inaccuracy for short-term option with respect to long term ones.
Another aspect could be that for this case, the bid and ask price simply aligns better with the binomial model.

Analysis for maturity at three monts:

bid	ask	Mid price	sigma_d	sigma_y	u	d
11,7	11,85	11,775	0,041459	0,658145	1,178846	0,848287

Libor_3M	CF_comp	CF_disc	del dividend yield	q risk-neutral probability	K strike price
4,85372%	1,012134	0,988011	0,04	-2,55976	121
				0,465415	
				0,495667	

S0 current price	Su	Sd	fu	fd	C
121,419283	143,1347	102,9984	22,13468	0	10,42682
					11,10456

Analysis for maturity at six months:

bid	ask	Mid price	sigma_d	sigma_y	u	d
17,7	17,9	17,8	0,03606	0,572438	1,331384	0,751098

Libor_6M	CF_comp	CF_disc	del dividend yield	q risk-neutral probability	K strike price
4,68213%	1,023411	0,977125	0,04	-1,28848	121
				0,434807	
				0,469273	

S0 current price	Su	Sd	fu	fd	C
121,4193	161,6557	91,19778	40,65572	0	18,09124
					19,52528

With the following colours are indicated the calculations that used the formulas:

Formula for risk neutral probability derived at lesson.

"Hall: Options, Futures, and Other Derivatives, 11/e, Global Edition" (Page 304)

Formula obtained in class, ignoring the dividend term