# Report 2
# Stochastic method for finance

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## 1 Introduction

This report aims to calculate dividends from call and put options using two methodologies: Box-spread and call-put Parity. In particular, we are considering stocks from the investment bank Morgan Stanley. First, we are going to briefly describe the company, then we show theoretically the two methods that are useful for this exercise, and finally we discuss methodology and results.

## 2 Morgan Stanley

Morgan Stanley is a prominent American multinational investment bank and financial services company headquartered in Midtown Manhattan, New York City. Established on September 16, 1935, by J.P. Morgan & Co. partners Henry Sturgis Morgan and Harold Stanley, the company originated in response to the Glass–Steagall Act, which mandated the separation of commercial and investment banking activities. Morgan Stanley ranked No. 61 in the 2023 Fortune 500 list of the largest United States corporations by total revenue and in the same year ranked n.30 in Forbes Global 2000.

Over the years, Morgan Stanley has evolved into a leading player in the financial services industry, boasting a global presence with offices in 41 countries and a workforce of over 75,000 employees. The firm caters to a diverse clientele, including corporations, governments, institutions, and individuals.

The company operates across three primary business segments: Institutional Securities, Wealth Management, and Investment Management. In Institutional Securities, Morgan Stanley provides investment banking services such as capital raising, financial advisory, and trading to institutional clients. The Wealth Management division offers brokerage and investment advisory services to high-net-worth individuals, while Investment Management focuses on asset management products and services for both institutional and retail clients.

As of 2024, Morgan Stanley's net worth stood at $153.98 billion, reflecting its strong financial position and stability in the market.

Morgan Stanley's history is marked by significant milestones, including its merger with Dean Witter Discover $ Co. in 1997, which led to the formation of the current entity. Despite facing challenges such as the 2008 financial crisis, the company has persevered and expanded its operations globally.

Today, Morgan Stanley is recognized as a systemically important financial institution and continues to play a pivotal role in the global financial markets. With a rich legacy of innovation and excellence, the firm remains committed to providing exceptional financial solutions and driving sustainable growth for its clients worldwide.

## 3 Boxspread

In options trading, a box spread represents a combination of positions designed to achieve a specific payoff, often referred to as a "delta-neutral interest rate position." For instance, it can be constructed by pairing a bull spread made from calls (long K1 call, short K2 call) with a bear spread made from puts (long K2 put, short K1 put). This arrangement results in a constant payoff equal to the difference in exercise prices ($(K1-K2)^+$), provided the underlying stock does not go ex-dividend before options' expiration. In the presence of a dividend X, the settled value of the box becomes $(K1-K2)^+ + \mathrm{X}$. According to the no-arbitrage assumption, the net premium required to acquire this position should align with the present value of the payoff.

The term "box spreads" originates from the rectangular arrangement of prices for the underlying options, resembling two columns in a quotation. Alternatively known as "alligator spreads," this name stems from the extensive number of trades involved in opening and closing them, which may "eat" into one's profits through commission fees.

Box spreads are typically executed with European options, as their exercise is restricted until expiration. Other options styles, such as American options, are less conducive to box spreads due to the risk of premature exercise, potentially exposing traders to unintended risks associated with individual legs of the spread.

# 4 Call-Put Parity

In financial mathematics, the put-call parity establishes a connection between the pricing of a European call option and a European put option. Both options have the same strike price and expiration date. Essentially, it states that holding a long call option and a short put option together forms a portfolio equivalent to (and thus carries the same value as) a single forward contract at the specified strike price and expiration date. This equivalence arises because if the price at expiration exceeds the strike price, the call option will be exercised, whereas if it falls below, the put option will be exercised. In either scenario, one unit of the asset is acquired at the strike price, mirroring the outcome of a forward contract.

## 4.1 mathematical description

We define as the payoff call $(S_T-K)^+$, with $S_T$ the price of the Stock at time T, and K the strike price, and payoff call $(K-S_T)^+$. The latter can be expressed also as $(S_T-K)^-$. In the same way as one can reconstruct a function from its positive and negative parts, $x = x^+ - x^-$, the payoff of a call and a punt:

$$(S_T-K)^+ - (S_T-K)^- = S_T - K \tag{1}$$

Now taking the conditional expectation at time 0 under $\mathbf{Q}$ and recalling that

$$price_0(payoff_T) = e^{-RT}K \tag{2}$$

we find

$$p_0^{put} = p_0^{call} - S_0 + e^{-RT}K \tag{3}$$

In the case of dividends the put-call parity readas

$$p_0^{put} = p_0^{call} - S_0 + e^{-RT}K + e^{-RT}S_0e^{\delta T} \tag{4}$$

where the last added term is the market value at time 0 of future dividends.

If we now rewrite:

$$D(0,T) = e^{-RT}, \tag{5}$$

$$F_0(T) = (\frac{S_0}{e^{-RT}} - S_0e) = e^{RT}S_0' \tag{6}$$

considering $S_0'$ the new stock price taking into account the dividends, $C_0$ and $P_0$ price of call and put, we end up with an equation from where $F_0(T)$ can be deduced.

$$C_0 - P_0 = D_0(T)(F_0(T) - K) \tag{7}$$

# 5 Analysis and methodology

In this report, we analyze options from Morgan Stanley (MS), in particular in this case we use American options.
In this section, we explore the determination of dividends for different maturities $T$, specifically 1, 3, 6 months, and 1 year. For each maturity, we initiate the process by selecting four options - two calls and two puts. Within each group, we designate a strike price $K_1$ and $K_2$, resulting in one put and one call with strike $K_1$, and one put and one call with strike $K_2$. We now employ the box spread method to ascertain the discount rate $D(0,T)$. Since we choose Call and Put with strikes K1 and K2 such that K1<<K2, the discount factor can be deduced as follows:

$$(S-K1)^+ - (S-K2)^+ + (K2-S)^+ - (K1-S)^+ = K2 - K1 \tag{8}$$

using previous arguments we end up with,

$$D(0,T) = \frac{p_0^{call}(K1) - p_0^{call}(K2) + p_0^{put}(K2) - p_0^{put}(K1)}{(K2-K1)}. \tag{9}$$

The outcomes for all four maturities are consolidated in the subsequent table.

Table 1: 26 April 2024

| Type | Strike | Bid | Ask | Mid | T |
|---|---|---|---|---|---|
| Call | 75 | 18.15 | 20.25 | 19.2 | 0.083 |
| | 96 | 1.65 | 1.76 | 1.705 | |
| Put | 75 | 0.01 | 0.12 | 0.065 | |
| | 96 | 3.55 | 3.85 | 3.7 | |

Table 2: 21 June 2024

| Type | Strike | Bid | Ask | Mid | T |
|---|---|---|---|---|---|
| Call | 75 | 19.35 | 20 | 19.675 | 0.25 |
| | 100 | 1.87 | 1.92 | 1.895 | |
| Put | 75 | 0.29 | 0.31 | 0.3 | |
| | 100 | 7.75 | 7.95 | 7.85 | |

Table 3: 20 September 2024

| Type | Strike | Bid | Ask | Mid | T |
|---|---|---|---|---|---|
| Call | 75 | 19.9 | 20.6 | 20.25 | 0.5 |
| | 100 | 3.7 | 3.8 | 3.75 | |
| Put | 75 | 0.99 | 1.03 | 1.01 | |
| | 100 | 9.35 | 9.55 | 9.45 | |

Table 4: June 2025

| Type | Strike | Bid | Ask | Mid | T |
|---|---|---|---|---|---|
| Call | 75 | 22.6 | 23.2 | 22.9 | 1.167 |
| | 100 | 8.15 | 8.45 | 8.3 | |
| Put | 75 | 2.93 | 3.75 | 3.34 | |
| | 100 | 15.5 | 16.7 | 16.1 | |

| | 1M | 3M | 6M | 1Y |
|---|---|---|---|---|
| D | 1.0 | 1.01 | 0.99 | 1.09 |

Subsequently, we opt for new options with strikes at the money for each maturity, consisting of one call and one put option. We compute the factor $F(0,T)$ as follows:

$$F(0,T) = \frac{p_0^{call} - p_0^{put}}{D(0,T)} + K. \tag{10}$$

With both $F(0,T)$ and $D(0,T)$ determined, we derive the implicit dividend using the formula:

$$D_{\text{implicit}} = S_0 e^{\delta T} = \frac{S_0}{D(0,T)} - F(0,T). \tag{11}$$

The results of these calculations are summarized in the table below.

Table 5: Options Data

| Maturity | Type | Bid | Ask | Mid | Strike | F | Div |
|---|---|---|---|---|---|---|---|
| 1M | Call | 3.3 | 3.4 | 3.35 | 93 | 94.287 | -0.895 |
| | put | 2.02 | 2.09 | 2.06 | | | |
| 3M | Call | 5 | 5.15 | 5.075 | 92.5 | 94.104 | -1.358 |
| | put | 3.4 | 3.5 | 3.45 | | | |
| 6M | Call | 7.3 | 7.4 | 7.35 | 92.5 | 94.655 | -0.459 |
| | put | 5.15 | 5.25 | 5.2 | | | |
| 1Y | Call | 11.8 | 12.6 | 12.2 | 92.5 | 95.355 | -9.491 |
| | put | 8.85 | 9.3 | 9.08 | | | |

# 6 Conclution

It appears that the results obtained from the data analysis are unexpected, particularly regarding the values of D(0,T) for almost all maturities, where they should ideally be less than 1. Additionally, negative implicit dividends have been observed, which can be unsettling.

Upon investigation, one plausible explanation for these unexpected outcomes is outlined in Chapter 12.3 of Hull's book. It's crucial to acknowledge that a box-spread arbitrage strategy is viable only with European options. In fact parity in put-call option is true just for European options. However, in this scenario, American options were utilized, leading to the observed discrepancies. Consequently, it is reasonable to conclude that the strategy didn't yield the expected results, resulting in negative dividends.

In essence, it's essential to consider the nature of the options used and the suitability of the chosen strategy, as different types of options and trading strategies can produce varying outcomes.