Report 4

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Contents

1 Introduction

In the context of the Black-Scholes pricing of options, the Greeks are important to understand the behavior of the option. The Greeks indicates a set of values, which corresponds to partial derivatives of the option price with respect to a specific variable. They are used to understand how option prices interact with changes in the market parameters. Being the derivatives, the Greeks are related to the differential equation of the price $V(S,t)$ of a European option in the Black Scholes framework.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0,$$

where S is the underlying asset price, σ is the volatility, r is the risk-free interest rate, and t is time.

2 Theoretical Approach

2.1 Delta

The Delta Greek (Δ) represents the rate of change of the option's value with respect to changes in the price of the underlying asset. It is defined as the first partial derivative with respect to the underlying asset price S:

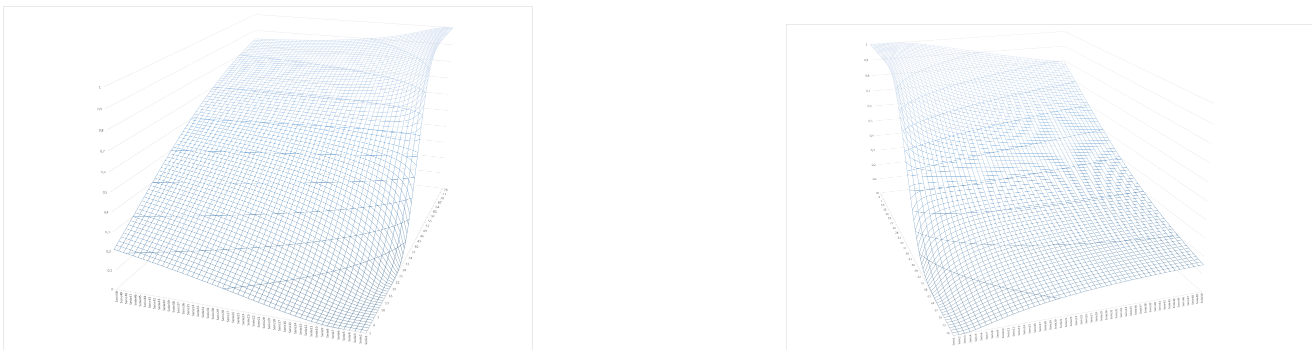
$$\Delta = \frac{\partial V}{\partial S}$$

For a European call option under the Black-Scholes model, the analytical expression for Delta depends on the type of option considered:

$$\Delta_{\text{call}} = N(d_1) \qquad\qquad \Delta_{\text{put}} = N(d_1) - 1$$

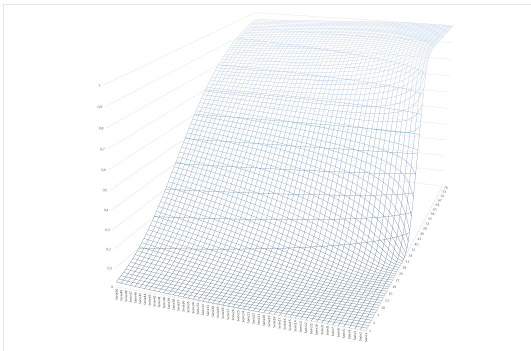
where $N(\cdot)$ denotes the standard normal cumulative distribution function.

In the following we see the 3D plot of the Delta as a function of: the price of the underlying, maturity and with shocks of volatility of magnitude of $\pm 50\%$.

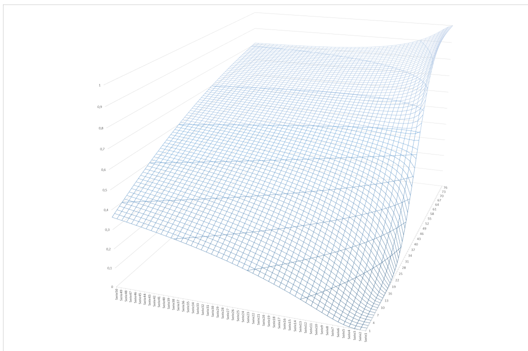


(a) Theoretical Δ 3D Call visualization

(b) Theoretical Δ 3D Put visualization



(a) Shock of -50% in volatility σ



(b) Shock of $+50\%$ in volatility σ

From a theoretical standpoint Delta should approach 1 when the option is deep in-the-money, and it approaches 0 when the option is deep out-of-the-money. In particular we expect that around at-the-money, Delta has a value close to 0.5. For what concerns maturity we expect Delta to become less sensitive to the present changes in the underlying which reflects in smoother curves. An increase volatility serves as a reduction in the sensitivity around the ATM region, flattening the curve and making it less steep. In the case of put options, Delta approaches -1 when deep in-the-money and short time to maturity, and 0 when out-of-the-money.

2.2 Rho

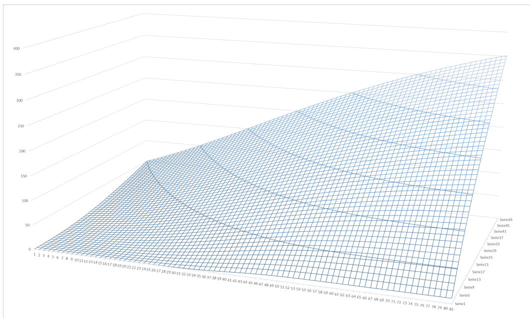
The Rho Greek (ρ) measures the sensitivity of the option value to changes in the risk-free interest rate r. It is defined as the partial derivative of the option price with respect to r:

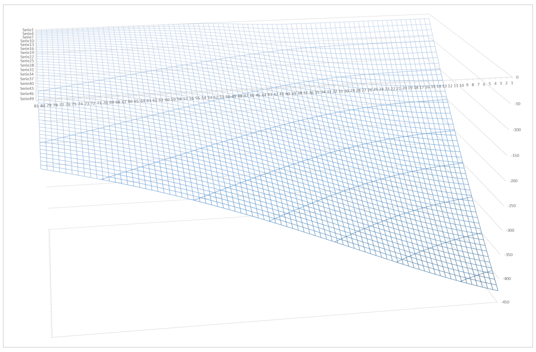
$$\rho = \frac{\partial V}{\partial r}$$

Under the Black-Scholes framework, the analytical expressions for Rho differ between calls and puts:

$$\rho_{\text{call}} = TKe^{-rT}N(d_2) \qquad\qquad \rho_{\text{put}} = -TKe^{-rT}N(-d_2)$$

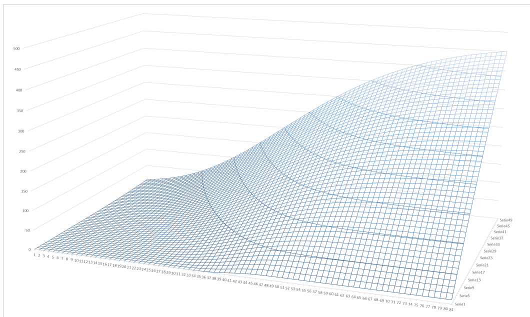
In the following we see the 3D plot of the Rho as a function of: the price of the underlying, maturity, and with shocks in volatility of magnitude $\pm 50\%$.

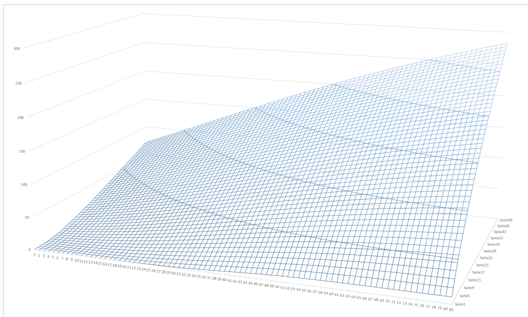




(a) Theoretical ρ 3D Call visualization

(b) Theoretical ρ 3D Put visualization





(a) Shock of -50% in volatility σ

(b) Shock of $+50\%$ in volatility σ

Rho increases with time to maturity, as the effect of changes in the interest rate is amplified over longer time intervals. For call options, Rho is positive, meaning the value increases with r, while for puts it is negative. When volatility increases, Rho is supposed to flatten, whereas lower volatility concentrates sensitivity around options that are in-the-money and with longer maturity.

2.3 Gamma

The Gamma Greek (Γ) measures the rate of change of Delta with respect to changes in the price of the underlying asset. It is defined as the second partial derivative of the option value with respect to the underlying asset price S:

$$\Gamma = \frac{\partial^2 V}{\partial S^2}$$

For a European option, Gamma is the same for both calls and puts, and is given by:

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T}}$$

where $N'(\cdot)$ is the standard normal probability density function and σ is the volatility of the underlying asset.

In the following we see the 3D plot of the Gamma as a function of: the price of the underlying, maturity, and with shocks in volatility of magnitude $\pm 50\%$.

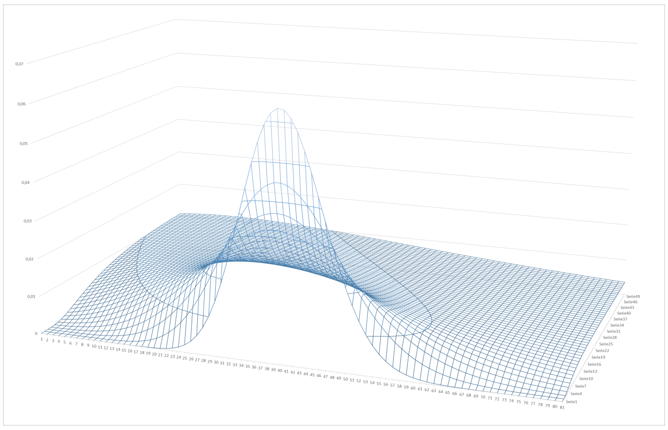
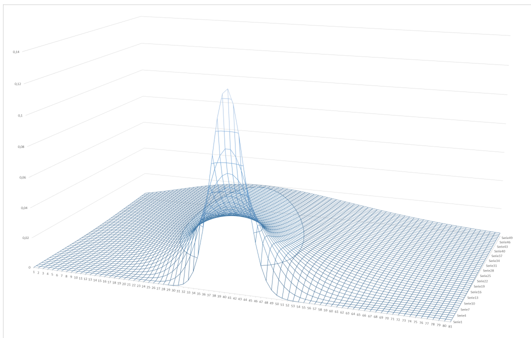
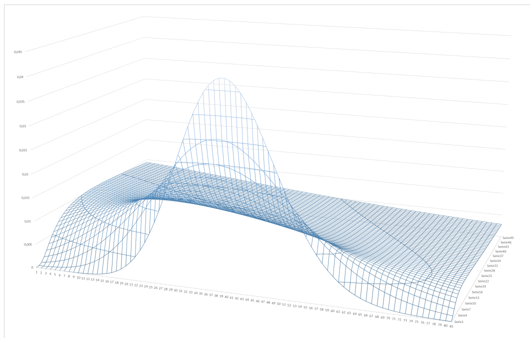


Figure 5: Theoretical Γ 3D visualization



(a) Shock of -50% in volatility σ



(b) Shock of $+50\%$ in volatility σ

From a theoretical point of view, Gamma reaches its peak when the option is at-the-money and close to maturity. This is because the Delta is more sensitive to changes in the underlying in that region. As volatility increases, Gamma flattens, indicating a more gradual change in Delta. A decrease in volatility makes the Gamma surface narrower and more concentrated around the ATM region.

2.4 Theta

The Theta Greek (Θ) measures the sensitivity of the option price with respect to time. It is defined as the partial derivative of the option value with respect to the time to maturity T:

$$\Theta = \frac{\partial V}{\partial T}$$

For a European call option Theta is given by:

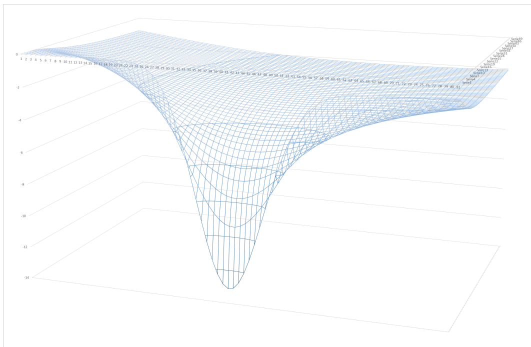
$$\Theta_{\text{call}} = -\frac{SN'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

For a European put option Theta is:

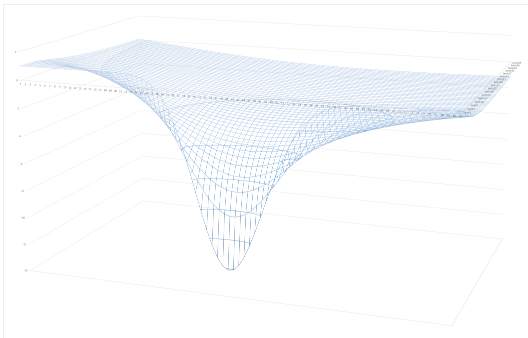
$$\Theta_{\text{put}} = -\frac{SN'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

where r is the risk-free rate.

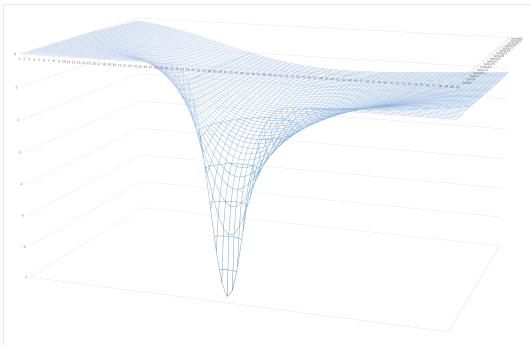
In the following we see the 3D plot of the Theta as a function of: the price of the underlying, maturity, and with shocks in volatility of magnitude $\pm 50\%$.



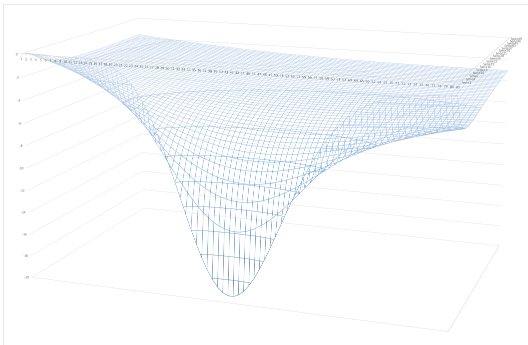
(a) Theoretical Θ 3D Call visualization



(b) Theoretical Θ 3D Put visualization



(a) Shock of -50% in volatility σ



(b) Shock of $+50\%$ in volatility σ

Theta is negative for long positions since the option loses value with time. Its absolute value increases as the option approaches maturity, and it has a peak when the option is at-the-money. Higher volatility increases the option's time value, reducing the steepness of Theta decay. Conversely, a drop in volatility accelerates the decay, intensifying the negative Theta around the ATM region. Even though the general aspect of the Call and Put results are similar, the results obtained for the put options are characterized by some initial positive values.

2.5 Vega

The Vega Greek (ν) measures the sensitivity of the option value to changes in the volatility of the underlying asset. It is defined as the partial derivative of the option price with respect to volatility σ:

$$\nu = \frac{\partial V}{\partial \sigma}$$

For both European call and put options Vega has the same analytical expression:

$$\nu = S\sqrt{T}N'(d_1)$$

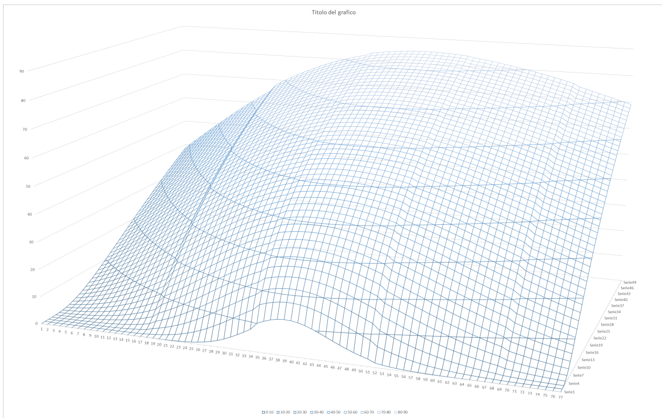
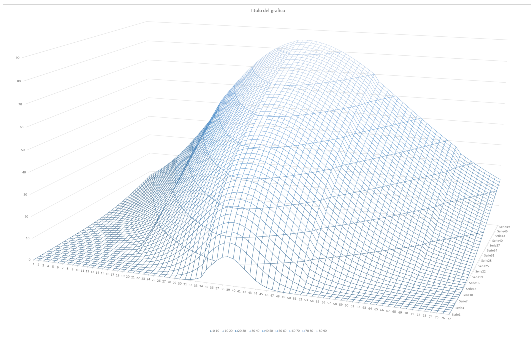
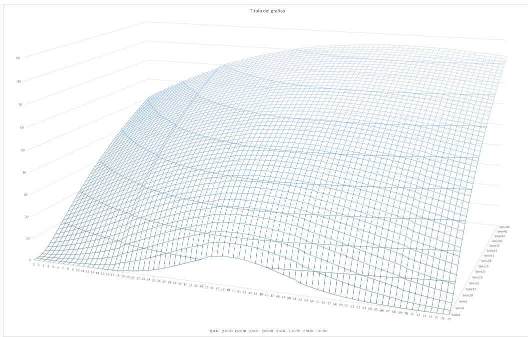


Figure 9: Theoretical ν 3D visualization



(a) Shock of -50% in volatility σ



(b) Shock of $+50\%$ in volatility σ

Vega is highest when the option is at-the-money and decreases as the option becomes deep in- or out-of-the-money. It also increases with time to maturity, this is because having a larger time implies a larger dependance and sensitivity to changes that depends on volatility. When volatility increases, the Vega surface tends to flatten, reducing the relative sensitivity, while lower volatility makes Vega sharper around the ATM region.

3 Empirical

The next step consisted in an hands on session with the advanced instrument Refnitiv (LSEG). Refinitiv is a real-time provider of high level financial market data. We exploited it's functionalities to download and analyze the data of NASDAQ, stock exchange group concentrated on technology. The NASDAQ assets does not pay dividends and is characterized by having european options.

3.1 Volatility Surface

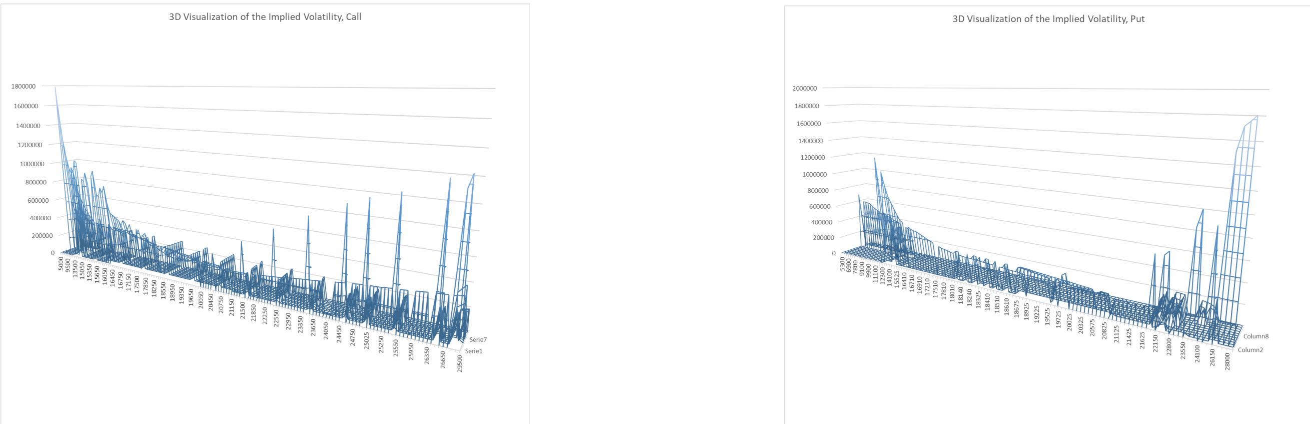
It is our interest to visualize the volatility surface as a function of the strikes and the maturities for a Call option. To perform this task vary methods have been exploited:

- Refinitiv plot:
  Refinitiv allows for directly visualizing the 3D plot of the volatility surface directly from it's web page. This functionality has been largely searched but probably due to the limited access granted to the students it hasn't been found.

- Download the Data:
  Another option was to directly download the data in excel and visualize the 3D plot in excel. This is what will be shown. To download the data in excel the refinitiv add-in has been installed and using the "Wizard", a function was used.

  ```
  =@RDP.Data("0#NDX*.U";"EXPIR_DATE;STRIKE_PRC;CF_BID;CF_ASK;IMP_VOLT;DELTA;GAMMA";"CH=Fd RH=IN";C6)
  ```

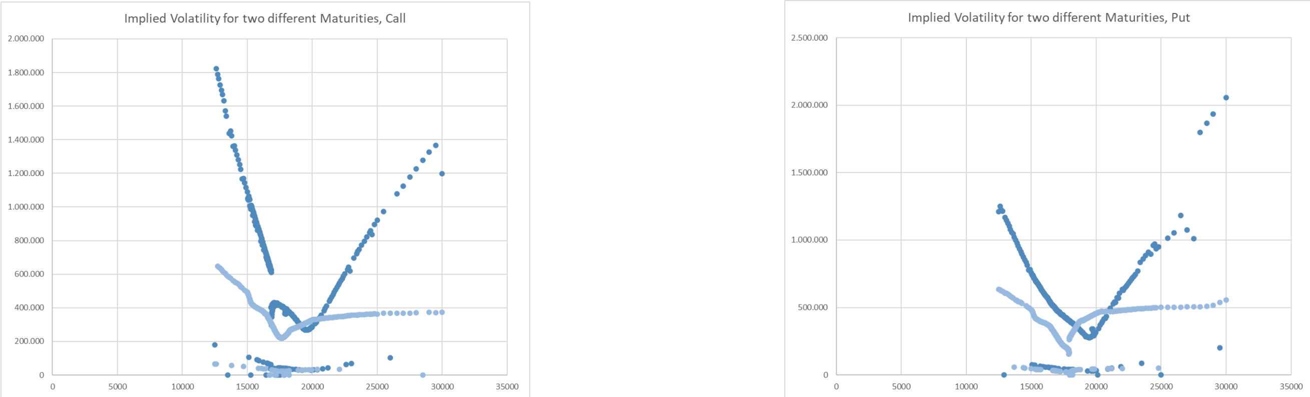
It is nonetheless necessary to point that due to a lack of data for many of the maturities, the 3D plot in excel has resulted unsatisfactory, in particular the problem was that excel is coded to always represent the missing "#N/D" values for a 3D matrix. This resulted in a poorly significative plot.



(a) Call Option (b) Put Option

Figure 11: 3D Visualization of the Volatility Using Excel

To better understand this plot, we proceded to visualize two of the maturities having the most points. This very meaningful plot will allows us to proceed with our analysis and understand how the volatility behaves as maturity increases.



(a) Call Option (b) Put Option

Figure 12: 2D Visualization of the Volatility Using Excel

In the plot, the light blue corresponds to a 1-month maturity, while the dark blue corresponds to a maturity of 2 days. We can see that the implied volatility for a Call option with only 2 days to maturity results in a clear "smile" shape. This is because short-term options are highly sensitive to extreme moves in the underlying, which increases the implied volatility for both deep in-the-money and out-of-the-money strikes. Meanwhile, the light blue plot shows the transition from a "smile" configuration to a "skew" configuration, which becomes more pronounced for longer maturities.

Both call and put implied volatility surfaces start with a clear smile, which gradually turns into a skew as maturity increases. The reduced number of points for the Put option reflects an higher liquidity for the Call market.

To complete these 3D visualization charts we also insert the 3D plot of the implied volatility obtained using the Refinitiv-Python workframe. Even though the point of these labs is to utilize other methods, just for the sake of the visualization, the plot obtained using the data downloaded using the Python API for Refinitiv will be here reported.

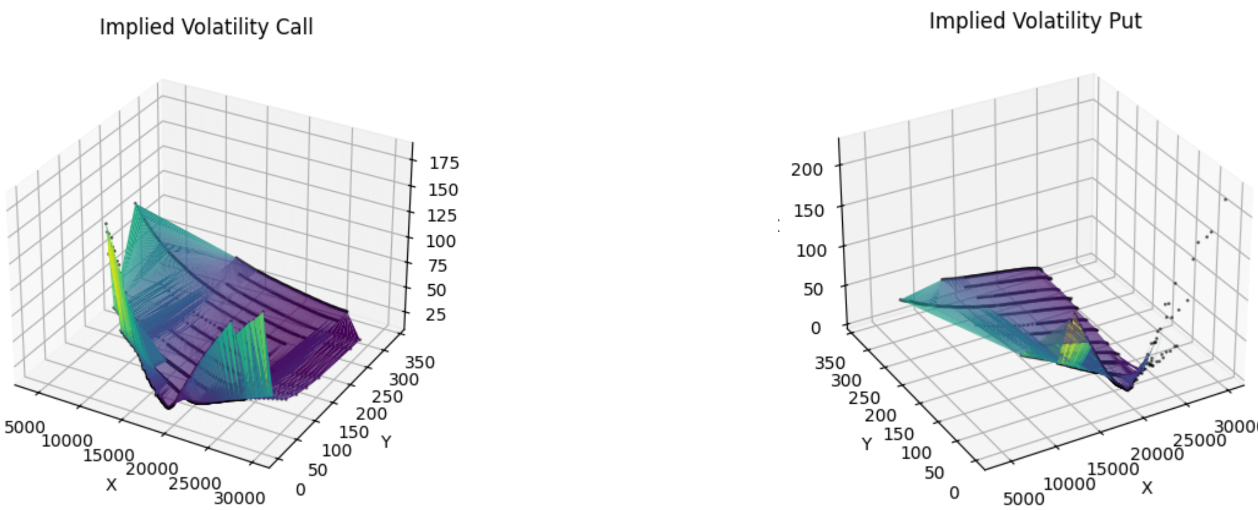


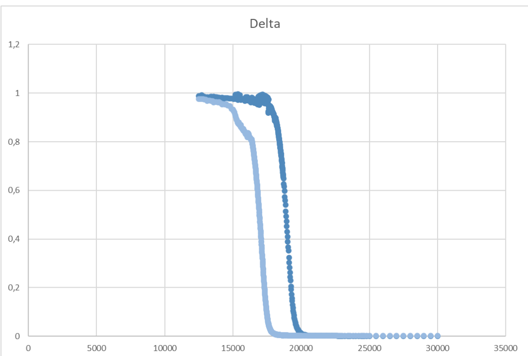
Figure 13: X axis: Strike Price, Y axis: Time to Maturity (d), Z axis: Implied Volatility

This plot, which gets colorated by means of an iterpolations, where the real data is represented by the black lines; shows clearly how the initial "smile" for small maturities gets progressively trasformed into a "skew" as the time to maturity (Y axis) increase. To explain more clearly why as time to maturity increases we obtain a "skew" we could imagine that as maturity increases, investors become more concerned about potential large downward moves in the underlying asset over time. This leads to higher demand for out-of-the-money put options, pushing their implied volatility up more than that of calls. The skew reflects the market's perception of asymmetric risk and preference for protection against losses.

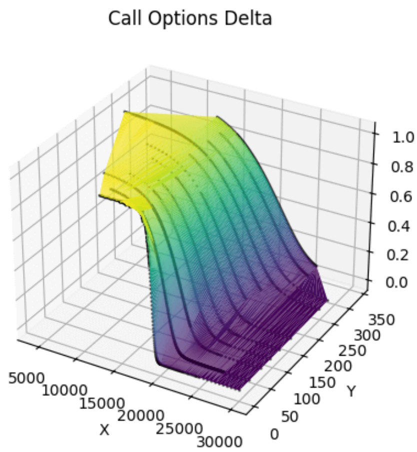
Another interesting result shown in the 2D plot Fig:(12a) is the presence of a "carpet" a set of smaller points that mirrors the main volatility smile, but at a lower magnitude. The lower magnitude in implied volatility could be due to reduced demand that ends up creating a similar plot that reflect the same structural behavior of implied volatility, just with attenuated market conditions.

3.2 Greeks

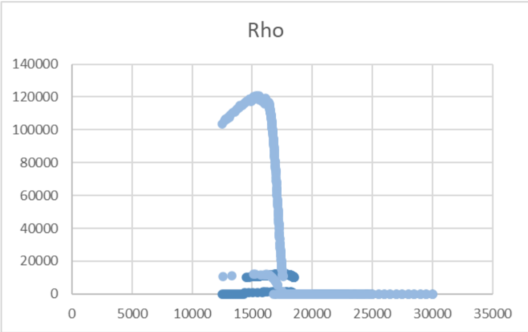
It is now of our interest to also verify how the real greeks behaves as time to maturity increses. In particular, we expect the greeks to smoothen. The analysis is performed like before and the results will be showed starting from a 2D plot. A 3D Refinitiv-Python plot will also be included to complete the analysis. Just like before, for the 2D plots lighter colors will represent a maturity of 1 month while dark color will correspond to a maturity of 1 day. In the 3D plots the X axis will represent the Strike Price, the Y axis the Time to Maturity and the Z axis the value of the Greeks.



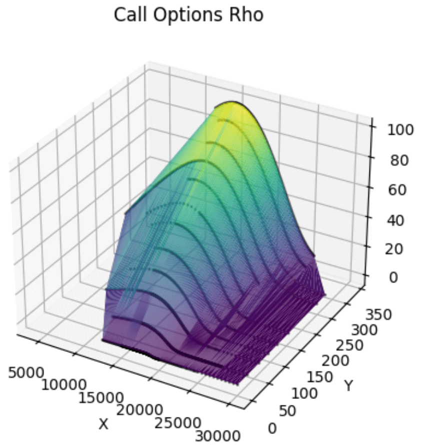
(a) 2D Δ Visualization for different Maturities



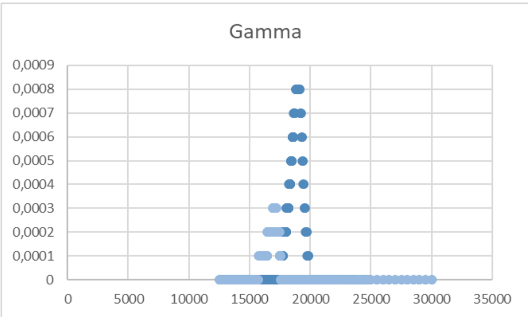
(b) 3D Δ Visualization



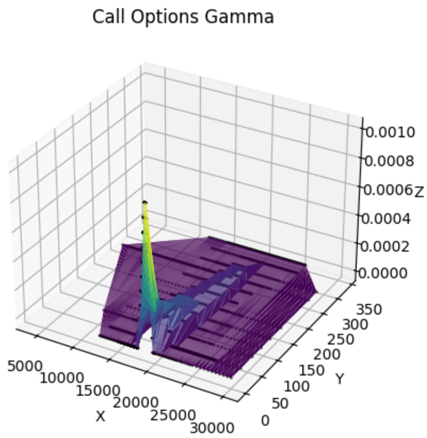
(a) 2D ρ Visualization for different Maturities



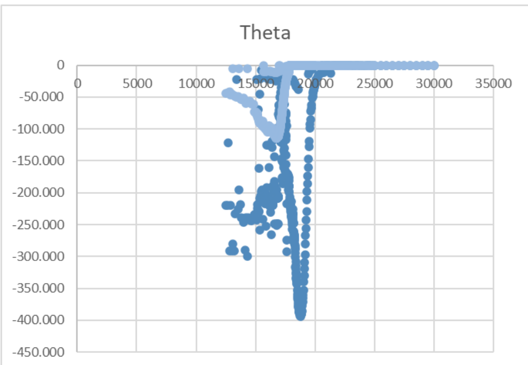
(b) 3D ρ Visualization



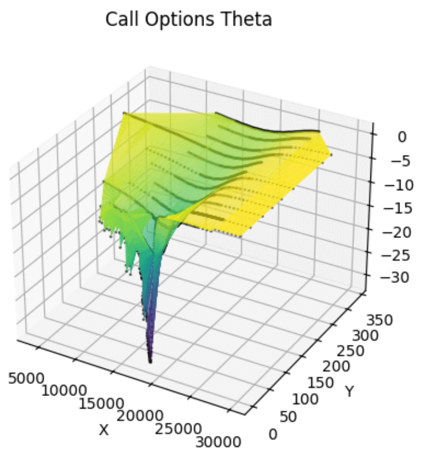
(a) 2D Γ Visualization for different Maturities



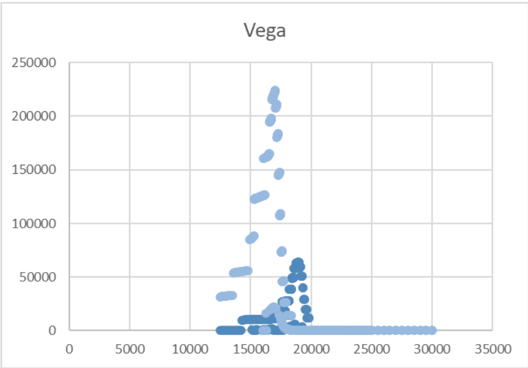
(b) 3D Γ Visualization



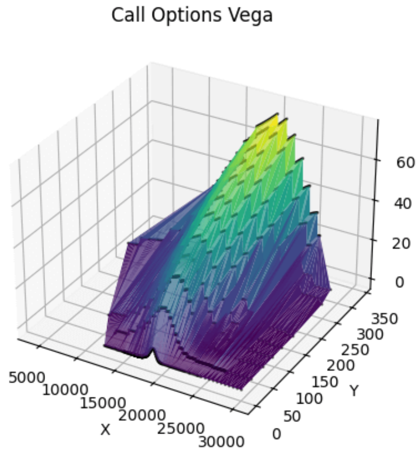
(a) 2D Θ Visualization for different Maturities



(b) 3D Θ Visualization



(a) 2D ν Visualization for different Maturities



(b) 3D ν Visualization

The results respects our expectations. It is also interesting to verify that there is an higher unreliability for the greeks obtained close to maturity. This inaccuracy results in sloppier and imprecise plots, these dicrepacies gets nicely canceled as we the time to maturity increases.

In conclusion the strong similarity between the hereby represented greeks and the theoretical results obtained in the first session confirms the accuracy of our calculations.

4 Merging the Approaches

4.1 Greeks Comparison

We are now interested in verifying whether the results obtained using the two methods actually coincide. To do so, the VBA code implemented in the first part of this report is applied to the specific case of the asset considered in the empirical section. This requires adjusting various parameters, such as the strike price (set at-the-money), the price of the underlying, volatility (proxied by the 3-Month LIBOR rate), and the maturity, which is set to three months.

The theoretical results obtained for the Nasdaq option with $T = 3;$months and strike $S = 18800$, and the values provided by Refinitiv, are summarized in the following table:

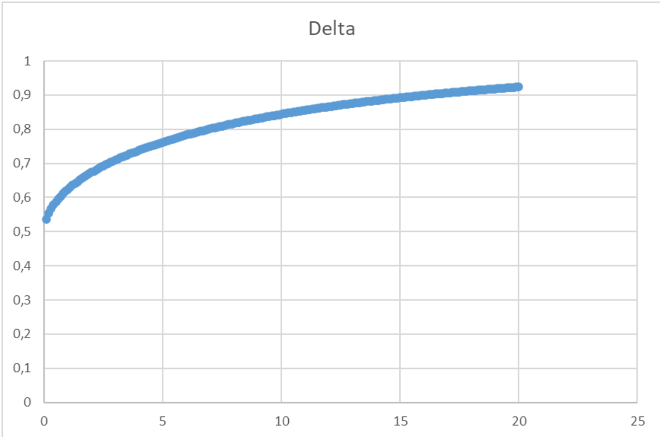
	Delta Δ	Gamma Γ	Rho ρ	Theta Θ	Vega ν
Refinitiv	$0,5535$	$0,0001$	$2400,91$	$-6476,7$	$3757,66$
Theoretical	$0,5604$	$0,0001$	$2236,68$	$-3395,02$	$3706,97$

Table 1: Comparison between Refinitiv and theoretical Greek values for a Nasdaq call option

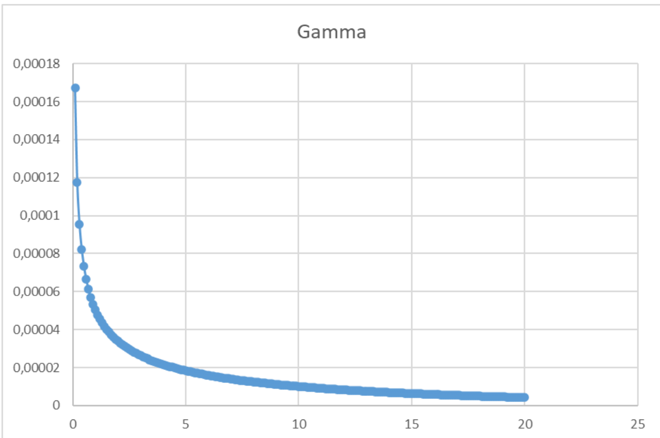
We see that all values are similar, suggesting that the theoretical model provides a good approximation of the actual market data. The main difference lies in the value of Theta, which is significantly underestimated by the theoretical model. This deviation could arise from the way Refinitiv analyze the market and select the dividend yields for its calculation. Additionally, Theta is known to vary sharply near expiration, making it more sensitive to small changes in input parameters and potentially more prone to model mismatch in that region.

4.2 At The Money Behavior

It is now interesting to visualize the theoretical behavior of the Greeks as maturity changes, focusing on the at-the-money (ATM) case. The following plots are expected to match the general behavior of the Greeks shown in the empirical results from Refinitiv. In particular, we investigate how the Greeks evolve as the time to maturity increases [1]:

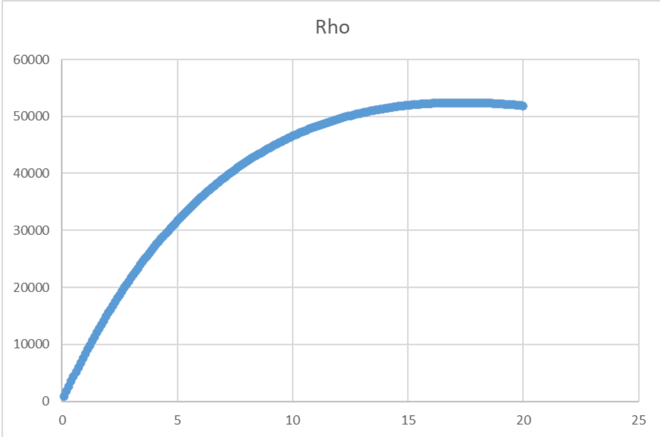


At-the-money Delta for a call option starts close to 0.5. As maturity increases, Delta becomes smoother and less reactive to short-term fluctuations in the underlying, shifting delta towards higher values.

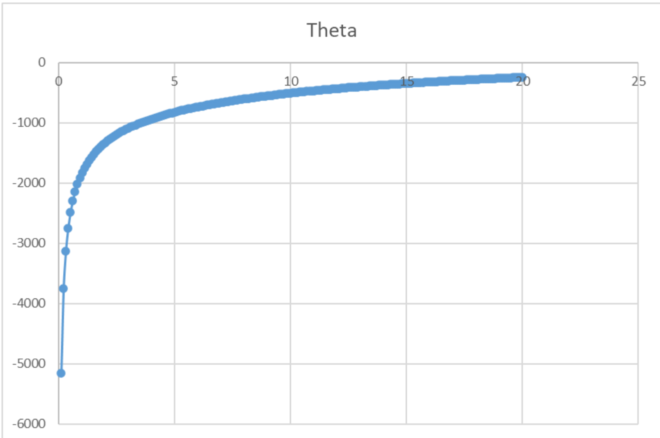


Gamma is maximized when the option is at-the-money and close to expiration. As maturity increases, Gamma flattens and spreads out, indicating that Delta becomes less sensitive to price changes. This behavior reflects the results obtained for the delta.

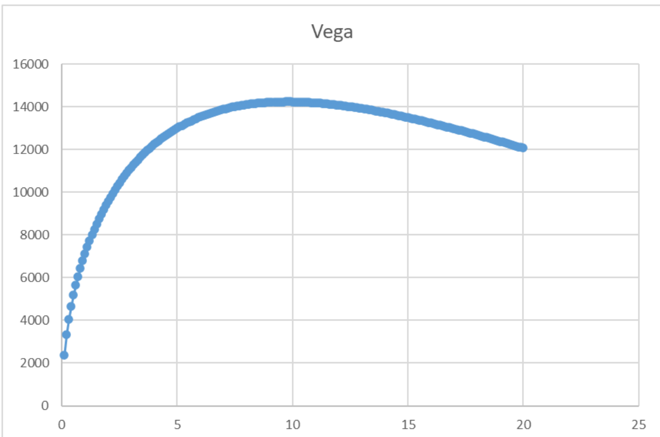
[1] In this case we reach maturities of twenty years.



Rho increases with maturity for call options. Since Rho measures sensitivity to interest rates, its effect are felt with more intensity the longer is the option's "life span". For small maturities, near expiration, changes in interest rates have little impact, leading to smaller Rho values.



Theta is more negative and "spiky" around the ATM region, reflecting the rapid time decay of option value. As maturity increases, Theta becomes less negative and the decay of time value slows down, resulting in a flatter surface.



measures the sensitivity of the option's value to changes in volatility. As we can see, it reaches its peak for at-the-money options with intermediate maturities. After this point, as maturity increases, Vega decreases gradually. This means that very short or very long maturities are less affected by changes in volatility compared to medium-term options.

5 Conclusion

In conclusion, we have gained a deep complete understanding of how the option Greeks behave and what their key characteristics are. Moreover, we also familiarized ourselves with a new professional tool: LSEG Workspace/Refinitiv, both by learning how to navigate its interface, and by extracting market data using the corresponding Excel add-in. The final comparison between the theoretical calculations and the empirical data from Refinitiv confirmed the validity and precision of the theoretical models implemented.