$$\dot{q}_{TP} = \begin{pmatrix} -1.607 \\ 0 \\ -1.607 \end{pmatrix} \text{rad/s} \qquad \dot{q}_{PS} = \begin{pmatrix} -0.8873 \\ -0.1111 \\ 0.6650 \end{pmatrix}$$

$$e_{TP} = \begin{pmatrix} 0 \\ 0.5 \\ 0.5 \end{pmatrix} \Rightarrow \|e_{TP}\| = 0.7071 \qquad e_{PS} = \begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix} \Rightarrow \|e_{PS}\| = 0.5574$$

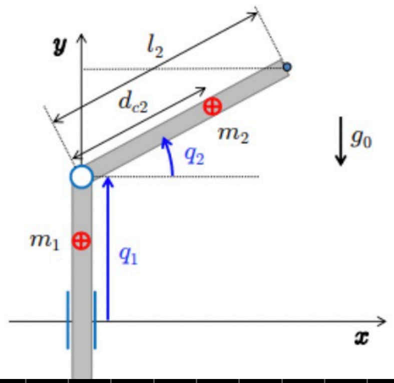
We'll have a task error since the matrix is <u>NOT</u> full ran

Exercise #5

Figure 2 shows a PR robot and its inertia matrix, already expressed in terms of three dynamic coefficients a, b and c. The robot moves in a vertical plane. A task trajectory $y_d(t) \in \mathbb{R}$ is assigned to the coordinate y of the end-effector position. With the robot being at rest in the configuration $\bar{q} = \begin{pmatrix} 1 & \pi/2 \end{pmatrix}^T$, provide the joint force/torque inputs $\tau_A \in \mathbb{R}^2$ and $\tau_B \in \mathbb{R}^2$ executing the desired task that instantaneously minimize, respectively,

$$H_A = \frac{1}{2}\|\tau\|^2 \qquad \text{or} \qquad H_B = \frac{1}{2}\|\tau\|^2_{M^{-2}(\bar{q})}.$$

Which of the two solutions τ_A and τ_B has the largest first component in absolute value?



$$M(q) = \begin{pmatrix} a & b\cos q_2 \\ b\cos q_2 & c \end{pmatrix} > 0$$

We have a task trajectory only for the y-component of the e-e.

→ The Robot is at rest at $\bar{q} = (1 \quad \pi/2)^T$. Provide the torque

$r = q_1 + l_2 s_2$, position of the end effector in Space

$\dot{r} = (1 \quad -l_2 c_2)\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix}$, $\dot{q}_1 - \dot{q}_2 l_2 c_2$

We have a problem constrained by two functions

Starting from the general Dynamic Model

→ $M(q) + c(q, \dot{q}) + g(q) = \tau$

This is a quadratic problem

$M(q) \cdot \ddot{q} + m(q, \dot{q}) + g(q)$

Also we need to minimize the two objective functions H_1 and H_2.

→ These two functions are:

$$\begin{cases} H_1(\ddot{q}) = \frac{1}{2}\|\tau\|^2 = \frac{1}{2}\ddot{q}^T M^2(q)\ddot{q} + m^T(q,\dot{q})M(q)\dot{q} + \frac{1}{2}n^T(q,\dot{q})n(q,\dot{q}) \\ H_2(\ddot{q}) = \frac{1}{2}\|\tau\|^2_{M^{-2}} = \frac{1}{2}\ddot{q}^T\ddot{q} + m^T(q,\dot{q})M^{-1}(q)\ddot{q} + \frac{1}{2}m^T(q,\dot{q})M^{-2}(q)n(q,\dot{q}) \end{cases}$$

→ Both these functions hield a closed form solution:

$$\begin{cases} \tau_1 = \left(J(q)M^{-1}\right)^{\#}\left(\ddot{r} - \dot{J}(q)\dot{q} + J(q)M^{-1}(q)\, m(q,\dot{q})\right) \\ \tau_2 = M(q)J^{\#}(q)\left(\ddot{r} - \dot{J}(q)\dot{q} + J(q)M^{-1}(q)m(q,\dot{q})\right) \end{cases}$$

We first need to verify if the Jacobian is full rank

our task vector reflects only the position along the y direction

→ It would've been a 2-D vector if the task was to position the e-e in the space

$r = q_1 + l_2 \sin q_2 \Rightarrow J = (1 \quad -l_2 c_2)$, then evaluating it at the specified configuration

$\bar{q} = (1 \quad \pi/2) \Rightarrow (1 \quad 0)$, Task Jacobian evaluated at $\bar{q}$

$$\tau_1 = \left((1 \quad 0)\, M^{-1}(q)\right)^{\#} \cdot \left(\ddot{r} - \cancel{\dot{J}(q)}\dot{q} + J(q)M^{-1}(q)\, m(q,\dot{q})\right)$$

$$M = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}$$

Constant Jacobian matrix

$$\tau_2 = M(q)J^{\#}(q)\left(\ddot{r} - \cancel{\dot{J}(q)}\dot{q} + J(q)M^{-1}(q)m(q,\dot{q})\right)$$

In this case, after solving the problem, we have two equal torques for both functions