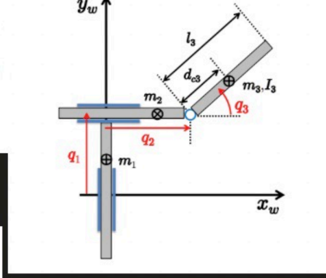
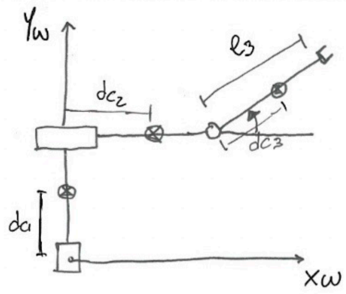
Provide the inertia matrix $\boldsymbol{M}_p(\boldsymbol{p})$ of the robot considered in Question **#7** when using for the Lagrangian dynamic modeling the new set of coordinates $\boldsymbol{p} = \begin{pmatrix} x & y & \alpha \end{pmatrix}^T \in \mathbb{R}^3$, where (x, y) are the components of the Cartesian position of the robot end-effector in world coordinates and α is the angle of the last link w.r.t. the $\boldsymbol{x}_w$ axis of the world frame.





$m_1, m_2, m_3 > 0$

we have to use as coordinates

$p = \begin{pmatrix} x \\ y \\ \alpha \end{pmatrix}$ where $x = P_{ex}$, $y = P_{ey}$

α angle of l_3 from X_w

We don't use the standard set of coordinates. Insted we use $r = (x \quad y \quad \alpha)^T$ $\quad \begin{pmatrix} x \\ y \end{pmatrix}$ coordinates of cartesian position

$$p = \begin{pmatrix} q_2 + l_3 \cos q_3 \\ q_1 + l_3 \operatorname{sen} q_3 \\ q_3 \end{pmatrix} = \alpha$$

Inertia matrix: Starting from the standard formula:

$$\left(J^{-T}(q)\, M(q)\, J^{-1}(q)\right) \cdot \ddot{p} + J^{-T}(q)\left(m(q,\dot{q}) - M(q)\, J^{-1}(q)\, \dot{J}(q)\, J^{-1}(q)\, \dot{p}\right) = u_p$$

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} q_2 + l_3 \cos q_3 \\ q_1 + l_3 \operatorname{sen} q_3 \\ q_3 \end{pmatrix} \longrightarrow \text{We need to evaluate the change of coordinates}$$

$$\begin{cases} p_1 = q_2 + l_3 \cos q_3 \\ p_2 = q_1 + l_3 \operatorname{sen} q_3 \\ p_3 = q_3 \end{cases} \qquad \begin{cases} q_2 = p_1 - l_3 s q_3 \\ q_1 = p_2 - l_3 c q_3 \\ q_3 = p_3 \end{cases} \Rightarrow \qquad p = f(q) \rightarrow q = f^{-1}(p)$$

$\downarrow$ $J(q)$ $\qquad \downarrow$ $f^{-1}(p)$

$\partial f(q) / \partial q$

$$p = \begin{pmatrix} 0 & 1 & -l_3 s_3 \\ 1 & 0 & l_3 c_3 \\ 0 & 0 & 1 \end{pmatrix} = J \quad \Rightarrow p = J \cdot q$$

, then we compute the inverse and we get the transform. matrix needed in order to have p instead of q

$$q \rightarrow p = f(q) \qquad p \rightarrow q = f^{-1}(p)$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$p = J \cdot q \qquad \underbrace{q = J^{-1} \cdot p}$$

The J^{-1} matrix allows us to define the transformation between the two coordinates