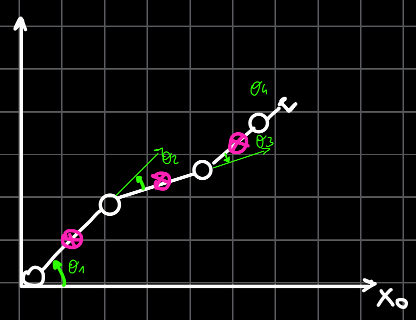
Coordinate change from DH coordinates to generalized coordinates



We want to use generalized coordinates in place of DH parameters

↓ We need to apply a suitable transformation

$q_1 = \theta_1 \qquad q_3 = \theta_1+\theta_2+\theta_3$

$q_2 = \theta_1+\theta_2 \qquad q_4 = \theta_1+\theta_2+\theta_3+\theta_4$

DH coordinates are defined in terms of θ_i:

$$\boxed{\alpha \quad a \quad d \quad \theta}$$

Exercise 1

Consider the 4R planar robot in Fig. 1, with generic lengths, masses and inertias of the links but with the center of mass of each link placed on its kinematic axis. As shown in the figure, the absolute angles of the links with respect to the axis $\boldsymbol{x}_0$ must be used as generalized coordinates $\boldsymbol{q}$.

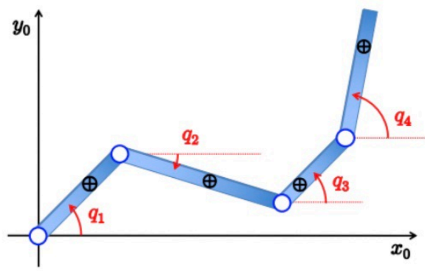


Figure 1: A 4R planar robot.

- Compute the inertia matrix $\boldsymbol{M}(\boldsymbol{q})$ of this robot.
- From the elements of $\boldsymbol{M}(\boldsymbol{q})$, derive the expression of the robot inertia matrix when using instead the Denavit-Hartenberg joint angles $\boldsymbol{\theta}$ as generalized coordinates.
- With the experience gained for the case $n = 4$, provide the general expression of the kinetic energy $T_i(\boldsymbol{q}, \dot{\boldsymbol{q}})$ of link i in a nR planar robot using the generalized coordinates $\boldsymbol{q}$ and under similar assumptions.

Let's first compute the inertia matrix of the Robot

↳ We need first to compute the posit. of the center of mass.

$$p_{c1} = \begin{pmatrix} l_{d1}\, c q_1 \\ l_{d1}\, s q_1 \end{pmatrix} \rightarrow \dot{p}_{c1} = V_1 = \begin{pmatrix} -l_{d1} S_1 \\ l_{d1} C_1 \end{pmatrix}$$

$$p_{c2} = \begin{pmatrix} l_1 C_1 + l_{d2}\, c q_2 \\ l_1 S_1 + l_{d2}\, S q_2 \end{pmatrix} \rightarrow \dot{p}_{c2} = V_2 = \begin{pmatrix} -l_1 S_1 - l_{d2} S_2 \\ +l_1 C_1 + l_{d2} C_2 \end{pmatrix}$$

$$p_{c3} = \begin{pmatrix} l_1 C_1 + l_2 C_2 + l_{d3} C_3 \\ l_1 S_1 + l_2 S_2 + l_{d3} S_3 \end{pmatrix} \rightarrow \dot{p}_{c3} = V_3 = \begin{pmatrix} -l_1 S_1 - l_2 S_2 - l_{d3} S_3 \\ l_1 C_1 + l_2 C_2 + l_{d3} C_3 \end{pmatrix}$$

$$p_{c4} = \begin{pmatrix} l_1 C_1 + l_2 C_2 + l_3 C_3 + l_{d4}\, C q_4 \\ l_1 S_1 + l_2 S_2 + l_3 S_3 + l_{d4}\, S q_4 \end{pmatrix} \rightarrow \dot{p}_{c4} = V_4 = \begin{pmatrix} -l_1 S_1 - l_2 S_2 - l_3 S_3 - l_{d4} S_4 \\ l_1 C_1 + l_2 C_2 + l_3 C_3 + l_{d4} C_4 \end{pmatrix}$$

$T_{TOT} = T_1 + T_2 + T_3 + T_4$ → We then procede by computing : $\sum_{i=1}^{N} \frac{1}{2} V_i^T m_i V_i + \frac{1}{2} \omega_i^T I_{zz} \omega_i$

↓ And then we get the inertia matrix

$U_{TOT} = U_1 + U_2 + U_3 + U_4$

Then starting from the DH variables, we need to compute the coordinates transformation.

↳ The transformation that link DH to generalized coordinates:

$\theta_1 = q_1 \qquad \theta_3 = q_1+q_2+q_3$

$\theta_2 = q_1+q_2 \qquad \theta_4 = q_1+q_2+q_3+q_4$

$q_1 = \theta_1$
$q_2 = \theta_2+\theta_1$
$q_3 = \theta_1+\theta_2+\theta_3$
$q_4 = \theta_1+\theta_2+\theta_3+\theta_4$

$q_1 = \theta_1 \qquad \theta_3 = q_3 - q_2$

$\theta_2 = q_2 - q_1 \qquad \theta_4 = q_4 - q_3$

$$\theta = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}}_{T^{-1}} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$$

$$q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix}$$

Thus the new inertia matrix will be: $\theta \rightarrow q$

$\cos\theta_1 = \cos(q_1)$
$\cos\theta_2 = \cos(q_2 - q_1)$
$\cos\theta_3 = \cos(q_3 - q_2)$
$\cos\theta_4 = \cos(q_4 - q_3)$