

OPEN PIT STRATEGIC MINE PLANNING UNDER UNCERTAINTY – ROBUST AND FLEXIBLE PLANS

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ABSTRACT

A strategic mine plan is a fundamental part of any mining operation. It must provide a realistic extraction and processing schedule, as well as an accurate measure of the value of the project. There are several challenges to generating a successful strategic mine plan. Most importantly, key input information required for mine planning, such as actual ore grades and market prices, is uncertain in nature, thus a mine plan generated with deterministic data has a very low probability of performing as expected.

Traditional mine planning approaches and existing software packages have failed to address uncertainty, which ultimately leads to underperforming and misvalued operations. In this paper we present the first software application that successfully takes into consideration market price uncertainty and generates feasible mine plans, in a timeframe competitive with commercially available software. The methodology presented provides a platform to consider market price uncertainty without an exponential increase in computational cost. By including uncertainty, our solution is able to create robust and flexible mine plans. A robust plan is one that best performs under unstable economic conditions. A flexible plan incorporates Real Options Analysis, allowing to postpone decisions such as expanding capacity, until the uncertainty has been reduced. The software has been tested for this paper in a real 12 million block model, with a mining operation lasting over 50 years, delivering competitive results for deterministic mine plans, increasing expected NPV in 2.4% with robust plans and the expected NPV of a fictitious expansion project in 68% with flexible plans.

INTRODUCTION

One of the key steps in any mining operation, at its early stages and during all the life of the mine, is to develop a strategic mining plan. A strategic mine plan consists of a strategy of extraction and processing that should attempt to maximize project value, while meeting the existing operative constraints. A strategic mine plan accounts for the complete mining process, and determines, among other things, the life of the mine, the yearly extraction and processing volumes, required mine and plant capacities, *etc.* But its most important attribute is that it must form an accurate estimation of the value of the operation, and must serve as a blueprint for more detailed, mid and short term planning.

Developing a strategic mine plan for an open pit mine is a very complex problem. It usually starts with a tridimensional representation of the deposit in the form of orthogonal blocks with specific characteristics, such as density, geological rock type and ore grade. The amount of blocks depends on the size of the deposit and the granularity required for a sufficiently accurate geological representation of the actual ore body. In practice, models can have from a few thousands to several million blocks. For each block, it must be determined whether to extract it, when to extract it, and where to send it. The possible destinations are processing plants, stockpiles or dumps. Of course, blocks can only be extracted according to the precedence constraints imposed by the open pit and its maximum slope angles and capacity restrictions for each period of operation. The first important challenge is then to determine an extraction program that maximizes project net present value. This problem can be proved to be equivalent to the Precedence Constrained Knapsack Problem (Gleixner, 2008), which makes it of NP-Hard computational complexity and therefore unknown if solvable in polynomial time.

Another important set of constraints has to do with the operative feasibility of the plan. Any solution found must comply with the capabilities and restrictions of an open pit mining operation. For example, mining equipment is very expensive to move, so a solution should take this cost into account and define sequences in which extracted blocks are not unrealistically spread apart, but instead form clusters wide enough for machinery and equipment to operate. A strategic mine plan that fails to be translated into a more detailed mid- or short-term plans defeats its purpose, because the modifications introduced in the extraction sequence will affect production estimates and project value. As an added difficulty to the objective of an appropriate strategic mine plan, key input data, such as ore grades and commodity market prices are uncertain in nature. A plan generated and evaluated with deterministic data has a very low probability of performing as expected, and delivers no information on how it would perform under different conditions.

Traditionally, mine planners have tackled these difficulties by resorting to the Ultimate Pit Limit problem (UPL), a relaxation of the open pit mine strategic planning problem, which does not consider time. By assuming that every block can be extracted instantly, UPL applies no value discounts and cannot by itself create groups of blocks that can be reasonably extracted – it just creates one big group. The UPL problem can be solved with the maximum cut algorithm (Lerchs and Grossman, 1965) or with more recent network based formulations such as Pseudoflow or Push-Relabel for the Maximum Flow Problem (Hochbaum and Chen, 2000), all of which deliver solutions in a reasonable time, even for large mines.

The current strategy to solve the open pit strategic mine planning problem is to generate several ultimate pits (nested pits) using different commodity prices and, based on the generated

sequence, design operationally extractable sections called phases. There are several disadvantages to this approach. First, the difference in weight and volume between two pits is not regular, so, in order to design phases, the pits have to be divided in some way, which is currently done based on the mine planner's expert criteria and experience. In some deposits, the difference between two pits can be large enough to make the pit information not a useful guide. A second disadvantage is that the sequence generated is based on fixed ascending prices, since this is the only way to get increasingly larger pits with this approach. However, the mine plan generated can be subject to any price behavior, and not necessarily one in which prices increase with time, making the extraction sequence highly unlikely to be near optimal for other price trends.

The consideration of price uncertainty is a great opportunity for strategic mine planning, since it would allow the evaluation of a mine plan's performance under different scenarios. Even more, it would open the opportunity to generate a plan that has the best performance considering the existing uncertainty, which we call a *best robust plan* (Alvarado et al, 2011).

The inclusion of uncertainty also opens the door to more advanced financial analysis, like the use of Real Options. In any mining operation there are several investment choices that must be made and not all are compromised at its conception. For example, it is usual to decide the closure date of the mine when it has been in operation for several years. Also, modifying the capacity of processing plants is an option all mining operations can consider and is sometimes taken, depending on commodity prices. If the strategic mine plan doesn't consider the existence of such options and is only a static compromised plan from start to the life of the mine (LOM), then it doesn't reflect the actual value of the operation. By incorporating Real Options, it is possible to generate *flexible plans* that adequately react to different economic scenarios.

In this paper we present an algorithm to solve the open pit strategic mine planning problem that includes basic operational constraints. This algorithm generates an extraction sequence that can be easily translated into mid- and short-term pushbacks, taking into consideration market price uncertainty, by generating robust and flexible plans. Its implementation is a new strategic mine planning software called *DeepMine*. We have tested DeepMine for this paper using a real large size mining operation, with more than 12 million blocks. The results for deterministic plans are highly competitive with existing software applications in terms of project NPV. A *best robust plan* increases the overall project expected NPV in 2.4%. Testing with a fictitious sulfide plant expansion, a *flexible plan* increases the expansion project NPV in 68%.

METHODOLOGY

The new algorithm we present in this paper is primarily based on *approximate dynamic programming* and a *branch and bound* strategy. The inputs for the algorithm include the 3D block model of the deposit, with its terrain topography and maximum slope angles, the ultimate pit limit based on an estimated long term price of the commodities involved –for which the pseudoflow algorithm was implemented based on its original description (Hochbaum, 2008)–, and upper limits for mine and processing plant capacities.

The main goal of the algorithm is to obtain the sequence of extraction and processing decisions that create the most value from the deposit, subject to basic operational restrictions. The algorithm should be able to perform both under deterministic price conditions based on fixed commodity prices per year, and under uncertain price conditions based on stochastic simulation.

A key design consideration is to base all extraction and processing decisions on economic criteria. There are two important reasons for this. First, it allows to solve the *dynamic cutoff grade* problem (Osanloo et al, 2008) as part of the scheduling process. Second, it makes it easier to decouple the problem of scheduling from the commodity price input. By achieving this decoupling it is possible to include commodity price uncertainty, without an exponential increase in computing resources.

The first step of the algorithm is to generate a solution to the deterministic open pit mine planning problem. For this purpose, a maximum life of mine is established, which is divided in a certain number of equal duration time periods. For each period, the algorithm requires a set of commodity prices, an upper limit for processing plants and mine capacity, processing plants costs, mining costs, transportation costs, stock handling costs, the discount rate at which to bring forward future cash flows, and the amount of metal that can be recovered at each processing plant from each of the geologic rock types found in the deposit.

Dynamic programming attempts to solve an optimization problem by exploring a solution space in discrete states, where an initial state exists, and any modification of the initial state can produce a new state, which can be further modified into several new states. The algorithm rests in the principle of optimality, by which, if there is an optimal solution to go from an initial state to a desired final state— in this case, the extraction and processing sequence that maximizes the mining operation's NPV—, then any sub-path of states is also optimal (Bertsekas, 2005). In the most strict sense, to apply this approach in the open pit strategic mine planning problem, the algorithm should start at the original site topography and remove one block at a time, producing, for each block, as many states as different possible destinations. This would produce a solution space too large even for the smallest deposits. A more appropriate approach is to reduce the exploration space by building states that consist of several blocks mined at once. This allows the reduction of the state space and also the enforcement of basic operative constraints. We call these groups of blocks *working zones*.

The way in which the states are created is a key concept in the commercial software DeepMine, and therefore details about how the process works cannot be exposed, but a qualitative description should suffice to explain the basics of the algorithm. The key concepts involved are explained below.

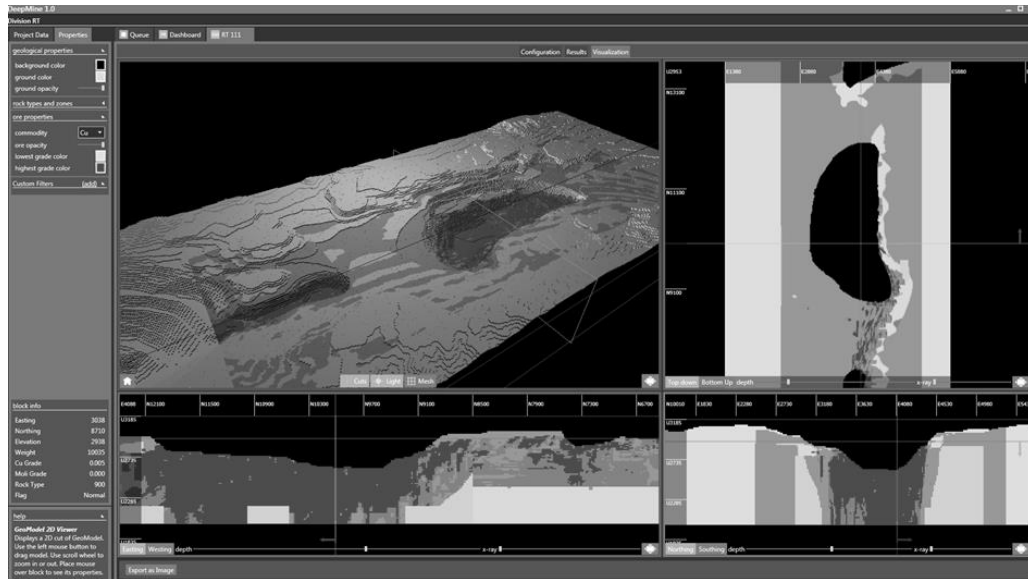


Figure 1 – DeepMine's user interface.

State Generation

A mine plan can be represented as a collection of states $S = \{S1, S2, S3... Sm\}$, one for each period, up to a previously set maximum, which should overestimate the possible LOM. For large operations, this value may reach 50 years or more. Each state is considered as a set of extraction and processing decisions up to a certain point in time: for each block, it can be known if it was extracted or not, and if it was, where it was sent. For each period, the algorithm generates different possible states by selecting different candidate working zones. The state space corresponds to all the possible combinations of candidate working zones, which is considerably small compared with a block-per-block solution space. Once the working zones to be extracted have been decided for a specific state, all blocks within them are evaluated and assigned the destination based on the economic value of each alternative.

Pruning

In large mining operations, the exponential nature of the state expansion makes it necessary to cut the exploration tree before reaching the maximum number of periods. Therefore, after a given number of periods, the different options are ranked using a fast evaluation heuristic, the best are selected to generate new states in the next iteration and the remaining states are deleted. This pruning process ensures that memory and processing time are constrained.

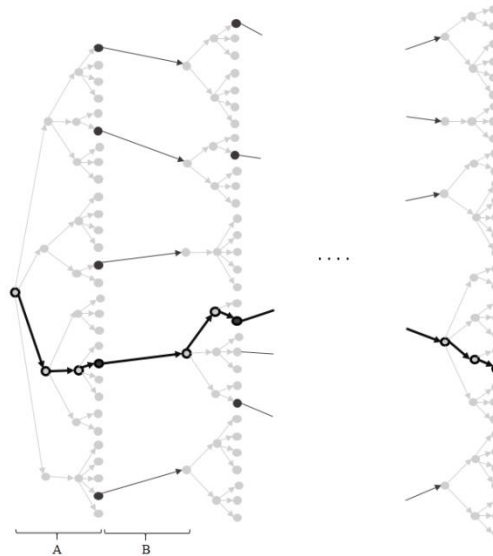


Figure 2 – State generation tree. From the initial state (original topography) several possible mine states are developed within expansion cycles (A). After a given number of periods (three in this figure), only the best states are expanded for another cycle (B). At the end of the last cycle, the best state will determine the chosen mine plan (outlined path).

When the expansion process reaches the previously defined upper bound on the life of the mine, all resulting paths in the state tree, from the initial state (the mine's original topography) to the last, form a candidate strategic mine plan. Every path is traversed and shortened to the point in time where the NPV reached its peak value, so that negative cash flows in the last few years of a mine plan do not offset previously good results; this also enables the algorithm to deliver the best LOM as an output. Finally, the candidate plan with the best NPV is chosen as the resulting strategic mine plan.

Working Zones

One of the key operative constraints that must be enforced to have a feasible strategic mine plan is to generate an extraction sequence that can be actually carried out in a mining operation. Modern mining operations use heavy equipment and large excavators that are very slow and expensive to move. To enforce this restriction, the group of blocks to be extracted must form a cohesive union, since the machinery involved should stay for some time in a given area, and mine large volumes of material, represented by continuous blocks. Also, there is a practical limit in how many benches such an area may encompass. For most operations this area cannot exceed 10 or 12 benches a year. The algorithm makes use of the concept of a *working zone*, as one of such groupings of blocks. Using heuristics that take into consideration the value of blocks, it is possible to form a set of working zones that, taken together, are likely to reach the upper limit of plant capacity and make good use of remaining mining capacity in a given period. Each group of working zones defines a state at a given period. Since one working zone will actually represent a volume of material that will need to be extracted from the mine at a given point in time, the maximum number of working points N at a given period is an input of the algorithm, and, therefore, it may represent the maximum number of excavators that will be

working actively in the mine at any one time. The maximum height of a working zone is defined by the limit of the maximum benches the mine operation can advance each year.

Uncertain Mine Plans and Best Robust Plan

In mine plans that are constructed based on economic criteria, or attempting to maximize a project's NPV, a key input factor is the mineral commodity market price, since it is the key to determining if a block should be mined or not, and where it should be sent. Therefore, if all other parameters are fixed, the set of prices used in the mine plan generation will determine the nature of the plan. This set of prices is known as the *planning price vector*.

Actual market prices, however, are not known beforehand, so any mine plan is subject to be evaluated in a different price vector. A mine plan can be considered as a function of a geological block model, a set of mining-related parameters, a *planning price vector*, and an *evaluation price vector*. The output value of such a function would be the extraction and processing sequence and project NPV. In a deterministic setting, there is no need to distinguish between planning and evaluation prices, but mineral commodity market prices are uncertain in nature, therefore it makes sense to separate the two vectors in order to assess the value of the mining project under different conditions. Even more, evaluation prices could actually be determined by a stochastic process simulation, in order to analyze the performance of a given mine plan in a more rich spectrum of possible price scenarios. Therefore, an uncertain mine plan should not provide the project NPV as a key output, but rather a distribution of possible values, taken from the evaluation of the mine plan in each iteration of an appropriate stochastic simulation.

In real mining operations, mid and short term planning occurs within the constraints of detailed designs of extraction phases, and even though the mine plan is usually redone periodically based on the last observed prices and updated geological data, the agility with which an extraction plan can react to commodity price fluctuations is reduced. In contrast, the processing decisions can be easily updated. This translates into updated cutoff-grade strategies for phases that are already compromised.

In order to account for this variability, and better represent the performance of a mine plan under uncertainty, the process of mine plan evaluation under different price scenarios must recalculate the destination decisions for each of the extracted blocks. Therefore, an uncertain mine plan is fixed in the sense that the blocks to be extracted follow an unchanging sequence, no matter what happens with commodity prices, but it's flexible because it accounts for the actual short and mid-term reaction a mining operation can have to price changes. This conceptualization allows to compromise an extraction sequence and define phases, without compromising production, and, most importantly, with full knowledge of the variation in production that can be observed each year based on the different price scenarios.

Using this information, an uncertain mine plan can be defined as one that is constructed from a stochastic simulation of commodity prices, instead of a fixed vector. The planification price is determined for each period taking into consideration a previously defined risk profile for the investors, using a mixture of *value at risk* and *expected prices*. Then, when different alternatives are evaluated, each one is executed in every one of the scenarios in the stochastic simulation, in order to choose the one that delivers the best expected NPV, instead of the best NPV.

An interesting exercise that can be done given the formulation provided above is to generate several uncertain mine plans using different risk profiles -therefore exploring the space of possible planification prices-, in order to find out how the mineral deposit responds to risk. A mining operation can be classified as risk-lover or risk-adverse, depending if it best responds to high risk or low risk profiles. When comparing the different results, one will outstand as the one that best responds to price fluctuations, delivering the highest expected NPV, which is called the *best robust plan*.

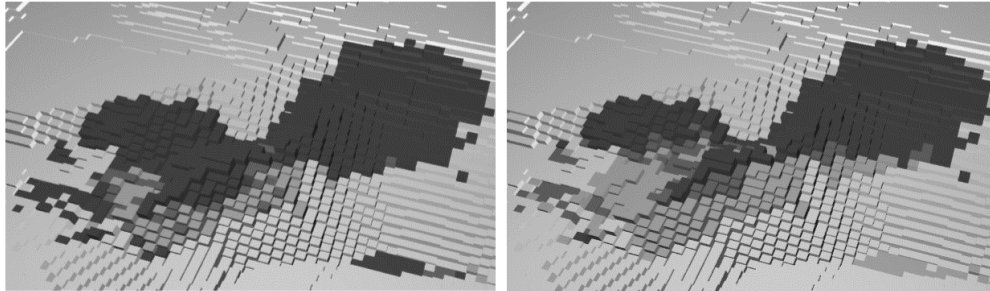


Figure 3 – Variability of destinations for the same extraction group. The darkest blocks are sent to dump and the grey ones are sent to processing plants, the shade of grey in between is for blocks sent to stockpile. There is a clear difference between a low (left) and a high (right) price scenario.

Flexible Mine Plans

Once uncertainty has been introduced, it is possible to leverage techniques that have been available for a long time in financial applications. One that is of particular interest to better achieve the objective of actually representing the value of a mining project, is the concept of *Real Options*. Introducing this concept into the confection of mine plans is not a novel idea (Davis and Newman, 2008), but it has not been included in any commercial strategic mine planning software.

The main idea of *Real Options* is to include in the evaluation of a project the fact that investors don't need to compromise today a complete plan of action for several years to come. The methodology recognizes that some decisions will not be taken now, but in a few years in the future. These decisions are called *options*, which at the moment of the decision can be taken or not. For example, a copper oxide mine might consider the option of installing a concentrator plant and thus process the sulfides existent in the deposit. Since the price uncertainty is too high to make a decision at present time, there is an inherent value in leaving the option open to decide once the uncertainty has been reduced. The value associated with the option comes from the difference between a mine plan that leaves this option open until a given period in time and the better of the two possible compromised plans, in which each course of action is taken and can't be changed. When the decision period of the option comes, there will be a commodity price called the *trigger price*, at which the expected net present value of each side of the option is the same. Then, for prices above or below the trigger price –depending on the nature of the option-, there will be one path that has higher value. A *flexible mine plan* is generated incorporating a Real Option, which captures the choice and therefore has greater expected value.

A *flexible mine plan* is composed, therefore, of three different uncertain mine plans. The first one is constructed from the initial mine state until the period of the option. At the option period a decision must be taken to do something differently or continue with the same configuration. The trigger price is used to determine, for each iteration of the commodity price stochastic simulation if the option should be taken or not. For the remaining portion of the plan, two new uncertain mine plans must be generated, one in which the option was taken, and another one in which the option was not taken. These three plans can also be considered as two plans that have a common section before the option period.

An important difficulty in implementing flexibility in uncertain mine plans is the determination of the trigger price for a given option. The flexible plan is a result of the trigger price used, but the optimal trigger price for each option depends on the flexible plan. Therefore, the only option is to use a search strategy and try different values for the trigger price until one is found that is sufficiently close to making each alternative path equally valuable. For big operations –millions of blocks- the amount of computational resources required to find a solution may be too much to be practical.

An alternative solution to the problem is to leave the trigger price as an input. Typically, expert miners will have a very good idea of what commodity prices would make the different options significant. The downside of this approach is that the value returned by the option is suboptimal, and it could even be negative, which should never happen by definition, since taking one path or another is an option and not an obligation.

RESULTS AND DISCUSSION

The proposed algorithm has been tried and tested using several different mines, from a few thousand blocks up to several millions and improved based on input from experienced mine planning professionals.

In this paper we present the results of testing the software with the copper mine Radomiro Tomic, a large size operation in Chile, of more than 12 million blocks and a life of mine of over 50 years, with copper as its main commodity and molybdenum as a byproduct. For this tests, the operation of this mine considers 4 main processing plants, 2 stockpiles, and one dump.

The algorithms were implemented using the C++ programming language, and all tests were executed on Intel Core i7-2670QM CPUs at 2.20 GHz, with 8 GB of RAM memory, running Microsoft Windows 7 SP1 64 bits.

For deterministic results we used a variable commodity price vector. For uncertain plans we used a *Brownian geometric movement with mean reversion* process simulation to generate 1,000 different price vectors, using parameters specified to make the expected prices follow the deterministic price vector, in order to make the results for uncertain and deterministic plans comparable.

The results obtained with deterministic mine plans are more than competitive compared with the strategic mine planning tool used regularly on site, even when using low search breadth and depth settings. Most indicators follow the same trend as in the latest official mine plan, which supports the thesis that the obtained mine plans are feasible to be turned into mid and short term plans.

Best Robust Plans

To rule out the influence of the randomness search, and therefore better capture the isolated value of considering uncertainty in the search of the plan that returns the highest expected net present value, each mine was run with no expansion -only one path of the search tree-. It may be possible, however, to find better results if we allow the algorithm to explore a bigger solution space.

In this case the running times are much bigger for the best robust plans since several uncertain plans are internally generated and evaluated in all 1000 scenarios, changing the chosen destinations for each block, each time. The runtime depends highly on the number of blocks and, as a second factor, in the number of iterations used in the stochastic simulation. For Radomiro Tomic, a single uncertain plan was obtained in one hour and 15 minutes, while the best robust plan had a total runtime of 13 hours and 10 minutes. Comparing both, the best robust plan had a better economic performance, with an increase of 2.41% in the expected NPV.

Flexible Plans

Radomiro Tomic is currently evaluating an expansion of its sulfide processing plant. To assess an even further expansion, a flexible mine plan was constructed with DeepMine to a fictitious expansion of an additional 50% relative to the current projected capacity, with a related investment of 3,500 MMUS\$. To start operating with the increased capacity in 2020, the decision must be made in 2018.

Separate deterministic assessments of each option of the plan show that there is potential for the project, since the mine plan with the expansion yields an additional 170MMUS\$. The flexible mine plan generated with DeepMine however, with the option of expanding plan capacity only if the copper price is above 3 US\$/lb. by 2018, increases overall NPV by 530 MMUS\$.

With the plant expansion considered in this exercise, there is profit to be made in the expected price scenario. But if one considers that future prices are uncertain, there is a price range in which the expansion is not convenient. By using a flexible plan, the plant is only expanded in the ranges that are profitable, incrementing the project expected value with an additional 360 MMUS\$, a difference of over 68%.

CONCLUSIONS

When it comes to creating strategic mine plans, it has been known by industry and academy that using deterministic market input was not the ideal way to plan mining operations or to evaluate a mining project. Nonetheless, due to the lack of software tools capable of incorporating uncertainty in the confection of strategic mine plans, developing deterministic plans is the industry de facto standard. This has constituted a historical constraint for mine planning.

In this paper, a new commercial software application for strategic mine planning of open pit mines, called DeepMine, was presented. As previously explained, DeepMine is capable of generating mine plans based on a stochastic simulation rather than a deterministic price vector. This allows the generation of several advanced types of plans: uncertain mine plans, which provides a fixed sequence of extraction, but recognizes variability for the destination of the

extracted material; the best robust plan, which explores different planning risk scenarios to find the plan with best expected NPV; and flexible mine plans, which incorporate Real Options as a way of including future decisions as part of the plan.

This new types of mine plans represent a leap forward in the evaluation of mining operations and in uncertain based planning. The experience testing the software on site at Codelco's Radomiro Tomic shows that even for a large size mining operation, DeepMine can effectively construct feasible strategic mine plans and by considering price uncertainty increase overall expected NPV. Considering the results above, by generating robust and flexible mine plans with DeepMine, it was possible to increase the overall project NPV in 2.4% and the NPV of a fictitious sulfide plant expansion project in 68%. It could be expected that for marginal mining operations this increments could be even larger.

The obtained results show the improvements that can be achieved by incorporating uncertainty and Real Options into strategic mine planning, but there is still potential for more. In its current version, DeepMine successfully generates strategic mine plans based on market price uncertainty, but there are other sources of uncertainty which can also be important to project evaluation and mine plan performance, such as ore-grade uncertainty, costs uncertainty, variability in equipment performance, risk management, *etc.*, which could be included to improve the value and richness of the generated plans.

With this first step taken a door has been opened to take strategic mine planning closer to its goal of accurately measuring project value and effectively programming the mine's exploitation.

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