Project Description

2019 Tribomechadynamics Research Camp

Project 2: *Jointed Structures with Geometric Nonlinearities*

April 3, 2019

1 Motivation and background

Lightweight design drives structures into vibration regimes where geometric nonlinearities can no longer be neglected. At the same time, the primary cause of mechanical damping in assembled structures is usually the nonlinear dry frictional interactions in joints. This project addresses systems which are driven into dynamic regimes where either nonlinear contact interactions or geometric nonlinearities, or both are important.

Experimental vibration analysis can be useful to

- understand and characterize the dynamics,
- identify a predictive model, and
- generate a data basis for model updating.

Most available methods are limited to linear vibration regimes. The Swiss Army Knife of nonlinear experimental vibration analysis has not been invented yet. The overall *goal of this project* is to gain further insight into the individual *opportunities and limitations of available methods.*

An important challenge of experimental vibration analysis is the *intrusiveness* of the testing procedure and the excitation system. By attaching a stinger and an electrodynamic exciter to a specimen, one inevitably introduces additional stiffness and mass, and via the electro-mechanic transducer also an electrical circuit to the system. In some cases, the effect of the excitation system on the identified vibration characteristics (natural frequencies, damping ratios, etc.) might be negligible, in other cases certainly not. Nonlinearity can magnify these effects, (a) because we need strong and therefore more invasive excitation systems to drive specimens in the nonlinear regime, (b) because nonlinearity opens the door to interactions not present in the linear case. Moreover, many testing procedures require a feedback-controlled excitation. Closing the control loop brings additional potential for intrusive testing. Applying a purely harmonic external forcing without altering the underlying natural vibration characteristics is trivial in a simulation, but almost impossible in reality.

Control Based Continuation

Control Based Continuation (CBC) is a versatile tool for experimental vibration analysis. The method can be viewed as a procedure with an outer and an inner loop. In the inner loop, a feedback controller attempts to minimize the deviation between measured and targeted periodic vibration state. This works also for states that are unstable in the absence of the controller, and thus the controller has a *stabilizing* effect. Once the system reaches a steady state, the time signals are analyzed and the next outer iteration can start.

In general, there will be a residual deviation between targeted and measured state and thus the control action is nonzero. The purpose of the outer loop is to iteratively adjust the target state, in such a way that the control action gets (sufficiently close) to zero, indicating that targeted and measured state are (close to) equal. The requirement that the (essentially periodic) control action vanishes, is commonly formulated in terms of the first few Fourier coefficients. The unknowns for the next iteration of the inner loop are the Fourier coefficients of the targeted vibration state. This corresponds to an algebraic equation system, which is usually solved by a Newton-type method or a fixed-point algorithm.

CBC can be used for a wide range of tasks. For instance, it can be used to track frequency response curves, backbone curves, or limit point bifurcations. An interesting task is to generate forced response points by fixing the frequency and incrementally increasing the response amplitude (the forcing level is hereby allowed to vary). If this procedure is repeated at different frequencies, frequency response curves for fixed forcing level can be re-constructed afterwards from the generated data points. Besides overcoming turning points, this procedure is able to systematically detect isolated frequency response curves. When the target and measured state are the applied force rather than the vibration response, one can use this general methodology to do vibration testing with purely harmonic forcing (and compensate zeroth and higher harmonics). A potential challenge of CBC could be its relatively high testing effort, and to design robust controllers for certain types of nonlinearities.

Phase Control

Phase Control using phase-locked loop controllers is an alternative to CBC, but limited to rather specific tasks. Here, the feedback controller attempts to minimize the deviation between measured and targeted phase (of a certain harmonic) of an essentially periodic vibration response, by adjusting the frequency of the excitation. *Phase*

Controllers can be used to track frequency response curves and backbone curves. For the former, turning points are usually not a problem since the phase is typically unique near a particular resonance. The stabilization of unstable response regimes is an important advantage over conventional open-loop testing with prescribed frequency.

Identification of nonlinear modal models

From the measured backbone curve, a nonlinear modal (NM) model can be identified (nonlinear experimental modal analysis). In accordance with the Extended Periodic Motion Concept (EPMC), a nonlinear mode is viewed as periodic oscillations of the autonomous system. It corresponds to an associated linear mode at zero amplitude and continues this mode to the nonlinear regime. Many systems have natural dissipation. To make the motions periodic in this case, the EPMC introduces an artificial negative damping term that compensates the natural dissipation over one vibration cycle. Commonly a mass proportional viscous damping term is used, which makes the EPMC consistent with the linear case under modal damping.

The nonlinear modes describe the vibration behavior in terms of amplitude (or energy) dependent natural frequency, modal damping ratio and modal deflection shape (including zeroth and higher harmonics). As long as the vibration energy is confined in an isolated nonlinear mode, the dynamics can be well-approximated by a single NM oscillator. This permits to efficiently predict the near-resonant forced response (under various load patterns and load levels), frequency sweeps through resonance, the autonomous free decay response etc.. The reduced model representing a single nonlinear mode is abbreviated NM-ROM.

Identification of polynomial nonlinear state space models

A more generic approach is to build a polynomial nonlinear state space (PNLSS) model. This approach is not a priori limited to a certain dynamic regime (e. g. where a single mode dominates), but can generally be used to model the input-output behavior of dynamical (not necessarily mechanical) systems. Transient random inputs can be applied to generate the data basis for the identification of the model. A benefit of this approach is that no feedback controller is required for the excitation. A challenge of this approach is that inevitable model errors (wrong model order, finite polynomial truncation order, non-smooth nonlinearities) make the model dependent on the input. As we have seen in the past, e. g. for a magnetic cantilever beam or a free-free beam with a lap joint, different PNLSS models are then identified for different levels of the input.

2 Purpose of project and work program

The purpose of this project is to gain further insight into the individual opportunities and limitations of the aforementioned methods, for the experimental vibration analysis of structures subjected to geometric nonlinearities and nonlinear contact interactions.

The project is split into two main work packages (WPs). Each participant is assigned to a specific WP:

- group WP1: *Florian, Gaetan, Erhan.*
- group WP2: *Nidish, Paw, Eve.*

WP1 is primarily *experimental* and compares CBC vs. Phase Control.
WP2 is primarily *numerical* and compares PNLSS vs. NM models.

There are definitely synergies so that it makes sense for the participants to exchange. However, each main WP is designed in such a way that it can be worked on *completely independently*.

WP0: Preparation of Specimens for the Experimental Investigations

This WP shall be completed pre-Houston in Stuttgart.

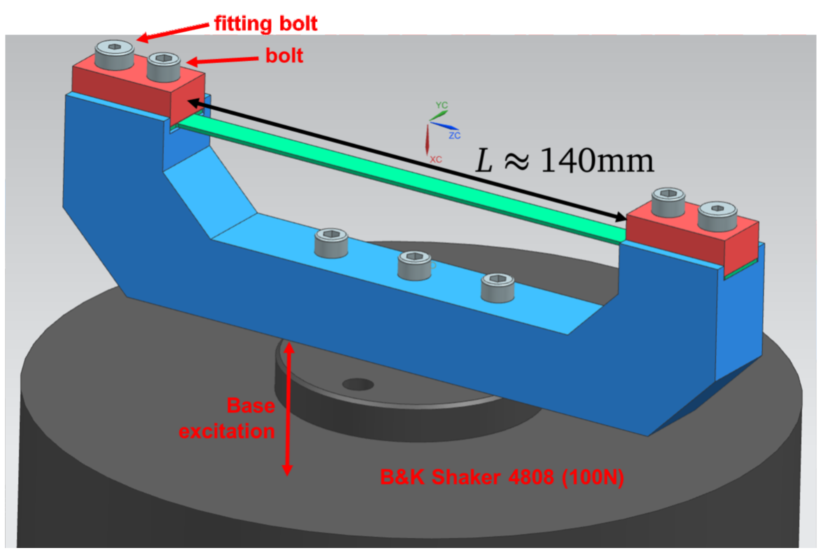

Figure 1: Setup of the benchmark system

The general setup consists of a thin beam that is bolted at both ends to a stiff support frame, which in turn can be mounted on a shaker (operating vertically), cf. Fig. 1. Two different beam specimens are considered:

- a straight beam (*hardening* effect), and
- a slightly curved beam (*softening-hardening* effect thanks to snap-through).

Preliminary theoretical investigations confirmed the indicated hardening and softening-hardening effects, respectively. These investigations were based on a FE model of the setup. A reduced order model was derived by projection onto the first few linear vibration modes. Geometrically nonlinear effects were modeled by quadratic and cubic stiffness terms in the modal coordinate system. The associated coefficients were estimated with the Implicit Condensation method (using as input nonlinear static responses to imposed forcing in the form of linear combinations of modes). The results for the backbone curve and the frequency response curve are depicted in Fig. 2.

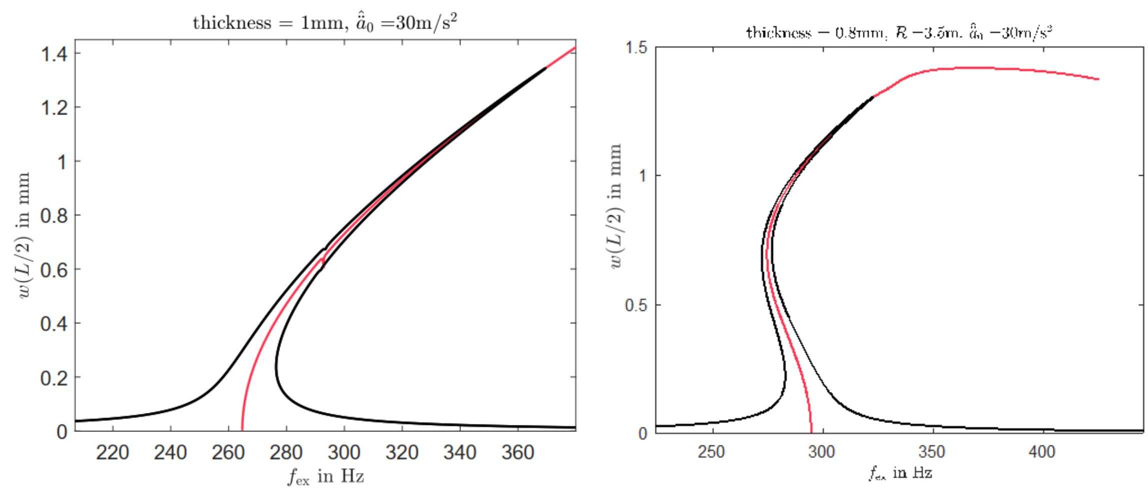

Figure 2: Backbone and frequency response curve (for one excitation level) for the flat beam (left) and the curved beam (right)

Note that the predicted softening-hardening behavior of the curved beam is particularly interesting: If damping is sufficiently low, the frequency response curve has 4 turning points, giving rise to as much as 5 co-existing periodic responses for a single frequency. Moreover, the uppermost and stable branch cannot be expected to be reached during frequency sweep (neither forward nor backward).
Once the specimens are designed and manufactured, some initial tests will be carried out on *one* of the specimens, too:

- linear experimental modal analysis
- phase control (some initial nonlinear frequency response and backbone measurements)
- CBC (ensure hard- and software are available, compatible and do some initial tests)

WP1: CBC vs. Phase Control (*experimental*)

- run thorough tests, starting with *straight beam* specimen
  (a) phase control:
    - tune the controller
    - frequency response for different excitation levels ($\approx$ 5 different ones; preferably including one with 4 turning points in case of the curved beam)
    - backbone
    - analyze repeatability
    - optional***: extend controller to phase resonance of (some) higher harmonics

  (b) CBC:
    - tune the controller, and perhaps other algorithm settings
    - define suitable frequency and amplitude discretization
    - define number of controlled harmonics
    - run amplitude 'sweeps' at fixed frequencies
    - track backbone (actually we want phase resonance for all considered harmonics)
    - analyze different strategies for ensuring non-invasiveness (if possible, compensate 0-th and higher harmonics in applied excitation)
    - analyze repeatability

  (c) vary bolt torque ($\approx$ 3 different settings); start from highest where geometric nonlinearity should be dominant; for weaker clamping, the geometric nonlinearity should be reduced and contact interactions should become more important (dry friction, unilateral contacts)

  (d) repeat (a)-(c) for *curved beam* specimen

- compare frequency responses
  - from CBC
  - from phase control
  - from nonlinear mode (identified from phase-controlled-backbone vs. CBC-backbone)

- assess results
  - explain differences (frequency response; accuracy of NM model/characteristics)
  - discuss influence and relevance of 0-th and higher harmonics (sensitivity of frequency response; sensitivity of nonlinear mode isolation quality)
  - discuss effort, robustness, quality
  - derive guidelines when and how to apply which method

WP2: PNLSS vs. NM Models (*numerical*)

- definition of benchmark models (models are provided)
  1. clamped-free beam with local nonlinear element (point-wise excitation via shaker and stinger):
     (a) elastic dry friction element
     (b) unilateral spring
  2. clamped-clamped thin beam with geometric nonlinearities (distributed base excitation via shaker and stiff support structure)
     (a) straight beam (simplified model is available)
     (b) curved beam (simplified model is available)

     optional***: augment model at the clamping by linear spring + elastic dry friction element in axial direction, linear spring + unilateral spring element in rotational direction

- PNLSS
  - construct and apply multi-sine excitation, different levels
  - identify polynomial nonlinear state space model
  - analyze the dependency of the identified model on the excitation level
  - augment nonlinear terms by appropriate non-polynomial ones (in JP's discretion)
  - analyze the robustness against sensor (and exciter?) noise
  - assess the quality of the identified model

- NM
  - run nonlinear modal analysis
  - analyze influence of imperfect isolation (single controlled frequency; including model of shaker and stinger/support structure); reference: EPMC with mass-proportional negative viscous damping term
  - analyze the robustness against sensor (and exciter?) noise
  - assess the quality of the identified model

- strengths and limitations of PNLSS and NM
  - assess results, develop hypotheses regarding strengths and limitations
  - run additional simulations to falsify/verify hypotheses (e.g. vary noise level or intensity of nonlinearity or load level or ...)

The quality of the models will be assessed by their ability to accurately predict the behavior of the reference model. A focus will be placed on the nonlinear vibration behavior under harmonic excitation near the primary resonance of a particular mode, either in the point-wise form (1) or as base excitation (2). We may look both at the steady-state, typically periodic response, as well as the response to sine sweeps.

3 Homework assignments and meetings

To have a productive summer, it is essential that the participants are well prepared. This shall be achieved by the following homework tasks. In accordance with the WPs, the homework tasks are split by group. Recall that the group WP1 consists of *Florian, Gaetan, Erhan*, group WP2 is *Nidish, Paw, Eve*.

Group WP1 (experimental, CBC vs. Phase Control)

All participants of this group are assigned the following tasks 1 and 2. Florian's homework is also WP0. Gaetan and Erhan are assigned also the following task 3.

1. Read literature on CBC: [3, 2].
2. Read literature on phase controlled backbone and frequency response testing; nonlinear modes: [4].
3. Model-based analysis of the methods, as detailed in the following.

For the model-based analysis, Stuttgart will provide a model for the geometrically nonlinear beam, the support structure and the shaker. In the homework, the nonlinear frictional-unilateral contact interactions between beam and support structure are neglected, a perfect clamping will be assumed. The first sub-task is to determine the frequency response of only the geometrically nonlinear beam with harmonic base excitation (no model of support structure or shaker considered here), for about 5 different excitation levels, in the frequency range of the first primary resonance. Then, a numerical nonlinear modal analysis in accordance with the EPMC is done, and the associated backbone curve is compared with the frequency response curves. For both frequency response and nonlinear modal analysis, we recommend to use the MATLAB tool NLvib, see Appendix A. Using the closed-form solution in Appendix B, analyze how well the identified NM-ROM can approximate the harmonically excited and damped behavior.

To estimate whether phase control can be used to trace out the entire frequency response, analyze if each response point has a unique phase, for the frequency range around the resonance peak. As sensor coordinate, use the elastic deformation velocity at the center of the beam. To estimate whether higher harmonics might become relevant for the controlled excitation, analyze the energy contained in the different harmonic components.

If progress permits this, the next step is to apply phase control and CBC (in the numerical simulation). For this sub-task, the whole model including the support structure and the shaker is considered. For the numerical simulations, we highly recommend using Simulink, this allows us to reuse the implementations for the experiments (Simulink models can be uploaded on our dSPACE system). Here, we again split the tasks: As Gaetan has already experience with CBC, he shall focus on CBC, while Erhan shall focus on phase control. To this end, Gaetan shall work closely with Ludovic (whom he is planning to visit), and Erhan shall get support from Stuttgart (Maren can provide a simple implementation of a phase controller in Simulink, along with some data processing tools for the identification of the nonlinear modal properties from the steady-state signals). In general, it is interesting to look both at frequency responses and backbones, but it makes sense to place a focus on backbones for this sub-task. The identified results are compared against those from direct simulation of the idealized model (without support structure or shaker).

Group WP2 (numerical, PNLSS vs. NM models)

The following tasks are assigned to all members of group WP2.

1. Familiarize with some simple benchmark models (specified below) and their dynamic behavior.

2. Familiarize with NM model identification and run some first tests on these benchmarks.

3. Familiarize with PNLSS model identification and run some first tests on these benchmarks.

For (1), Stuttgart will provide two models: First, a clamped-free beam with elastic dry friction element, under single-point harmonic forcing. Second, a clamped-clamped beam with geometric nonlinearities, with ideal boundary conditions (no friction, no unilateral contacts), under harmonic base excitation. You shall analyze these numerically with the MATLAB tool NLvib, see Appendix A. For both models, focus on the first primary resonance, and determine the frequency response for different excitation levels.

For (2), a numerical nonlinear modal analysis in accordance with the EPMC is done, and the associated backbone curve is compared with the frequency response curves. Using the closed-form solution in Appendix B, analyze how well the identified NM-ROM can approximate the harmonically excited and damped behavior. To estimate whether higher harmonics might be important, analyze the energy contained in the different harmonic components. If progress permits this, focus on the clamped-free beam with elastic dry friction element, and augment the model by shaker, stinger and phase controller (implemented in Simulink, provided by Stuttgart). Simulate the phase-controlled backbone curve extraction and identification of the NM model. The code for the identification will be provided by Stuttgart. Assess the quality of this NM-ROM as above.

For (3), JP will provide you with hints on literature and software. It can here be useful to apply a modal truncation to just a single linear mode. For the geometrically nonlinear beam, this effectively yields a single-degree-of-freedom Duffing oscillator (which should not pose a severe problem to PNLSS model identification). Do the identification for different excitation levels. Analyze if the model depends on the excitation level. Assess the prediction quality for the steady-state frequency response around the first primary resonance (different excitation levels). If progress permits this, focus on the clamped-free beam with elastic dry friction element, and augment the model by shaker, stinger (no controller needed in this case). Then run the identification for this model (multi-sine is now input as voltage to the shaker, not directly as forcing). Assess the quality of this PNLSS model as above.

Meetings

To get to know each other better and facilitate the preparation, we will have video meetings. The participants prepare presentations on their progress, questions and encountered difficulties during these meetings. Details on the meeting platform and dates is sent in a separate email.

A NLvib: A MATLAB tool for nonlinear vibration problems

NLvib is a free MATLAB tool for the computational analysis of nonlinear vibrations using Harmonic Balance, the shooting method and numerical path continuation. It is available via `http://www.ila.uni-stuttgart.de/nlvib`, including the MATLAB source code, examples and documentation. A Springer book on Harmonic Balance is also available [1]. Among others, it contains a guide to get started with NLvib (documentation of the tool, solved exercises etc.).

B Closed-form expression of frequency response for NM-ROM

Once the nonlinear natural frequency $\tilde{\omega}(a)$, damping ratio $\tilde{\zeta}(a)$ and fundamental harmonic of the modal deflection shape $\tilde{\boldsymbol{\varphi}}_1(a)$ (mass-normalized) are known as functions of the modal amplitude a, frequency responses can be *synthesized*. Under the assumption that the vibrations are dominated by the considered nonlinear mode, the system behaves like a single degree of freedom oscillator, such that an explicit approximation can be derived as

$$\Omega^2 = p_2 \pm \sqrt{p_2^2 - \tilde{\omega}^4 + \frac{|\tilde{\boldsymbol{\varphi}}_1^{\mathrm{H}} \boldsymbol{F}_{\mathrm{exc},1}|^2}{a^2}} \tag{1}$$

with $p_2 = \tilde{\omega}^2 - 2(\tilde{\zeta}\tilde{\omega})^2$ and $\boldsymbol{F}_{\mathrm{exc},1}$ is the fundamental harmonic of the excitation force. Eqn. (1) is straight-forward to apply if the $\boldsymbol{F}_{\mathrm{exc},1}$ remains the same for the whole frequency response. Then Eqn. (1) gives two frequencies for every amplitude a. By solving for Ω instead of the modal amplitude a, turning points in the frequency responses (and even isola) do not require special attention. If the radicand in Eqn. (1) is negative, the corresponding amplitude is not reached for the given force level. Details can be found in [4].
The situation becomes slightly more complicated if the excitation force depends (in a known way) on the frequency Ω (as in the case of base excitation or unbalance excitation). If the considered frequency range is narrow, one might achieve good

results by setting $\boldsymbol{F}_{\mathrm{exc},1}$ to a constant obtained for some nominal frequency. Otherwise one can substitute the known expression for $\boldsymbol{F}_{\mathrm{exc},1}(\Omega)$, and try to explicitly solve for Ω (or solve the implicit equation, or ...).

References

[1] Krack, M., Gross, J.: Harmonic Balance for Nonlinear Vibration Problems. Springer (2019). DOI 10.1007/978-3-030-14023-6

[2] Renson, L., Barton, D.A.W., Neild, S.A.: Experimental tracking of limit-point bifurcations and backbone curves using control-based continuation. Int. J. Bifurcation Chaos **27**(01), 1730,002 (2017). DOI 10.1142/S0218127417300026

[3] Renson, L., Gonzalez-Buelga, A., Barton, D., Neild, S.A.: Robust identification of backbone curves using control-based continuation. Journal of Sound and Vibration **367**, 145–158 (2016). DOI 10.1016/j.jsv.2015.12.035

[4] Scheel, M., Peter, S., Leine, R.I., Krack, M.: A phase resonance approach for modal testing of structures with nonlinear dissipation. Journal of Sound and Vibration **435**, 56–73 (2018). DOI 10.1016/j.jsv.2018.07.010