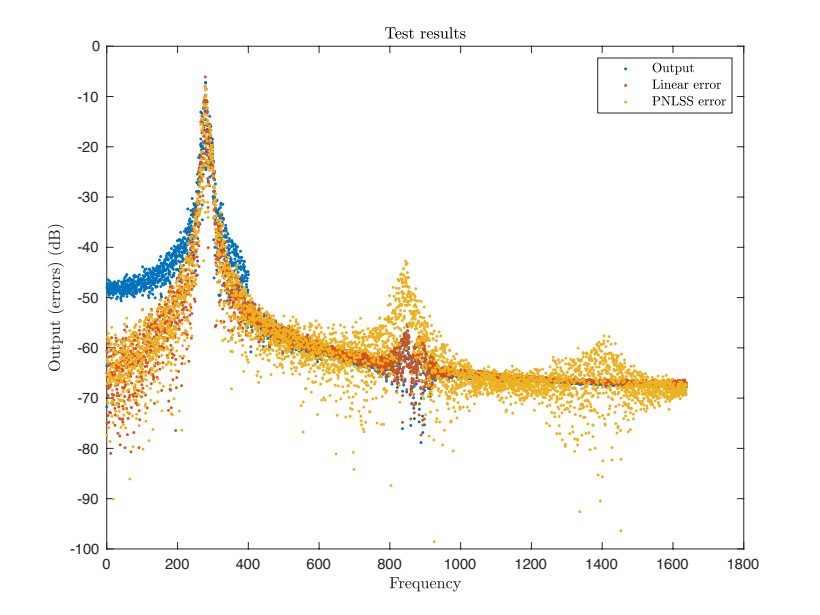
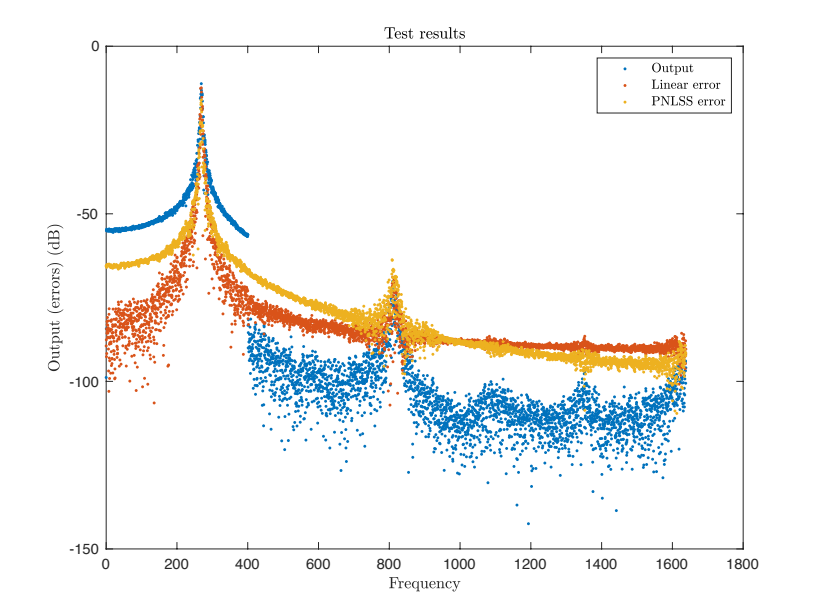
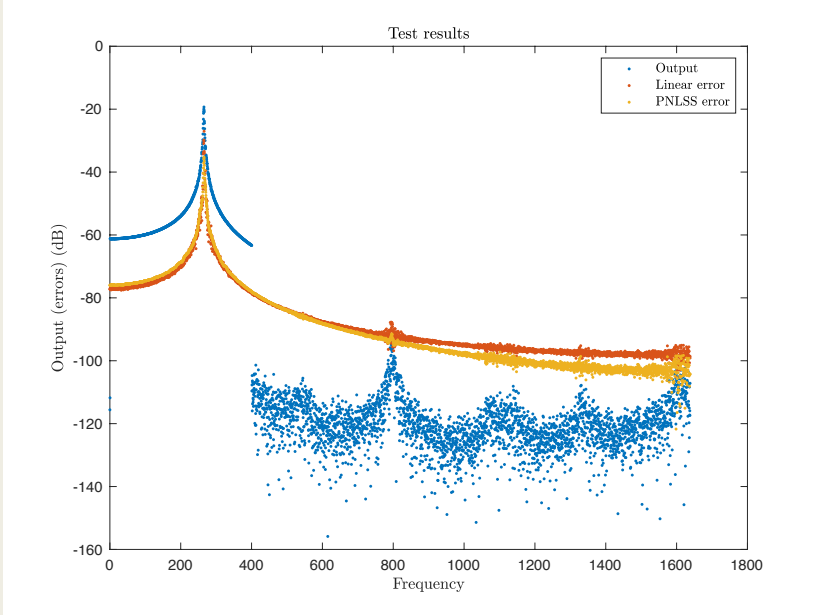
Benchmark Model I: Flat clamped-clamped beam model (Duffing Oscillator)



Amplitude = 10



Amplitude = 20



Amplitude = 50

PROJECT II GROUP 2 MEETING 1

27 Jun 2019

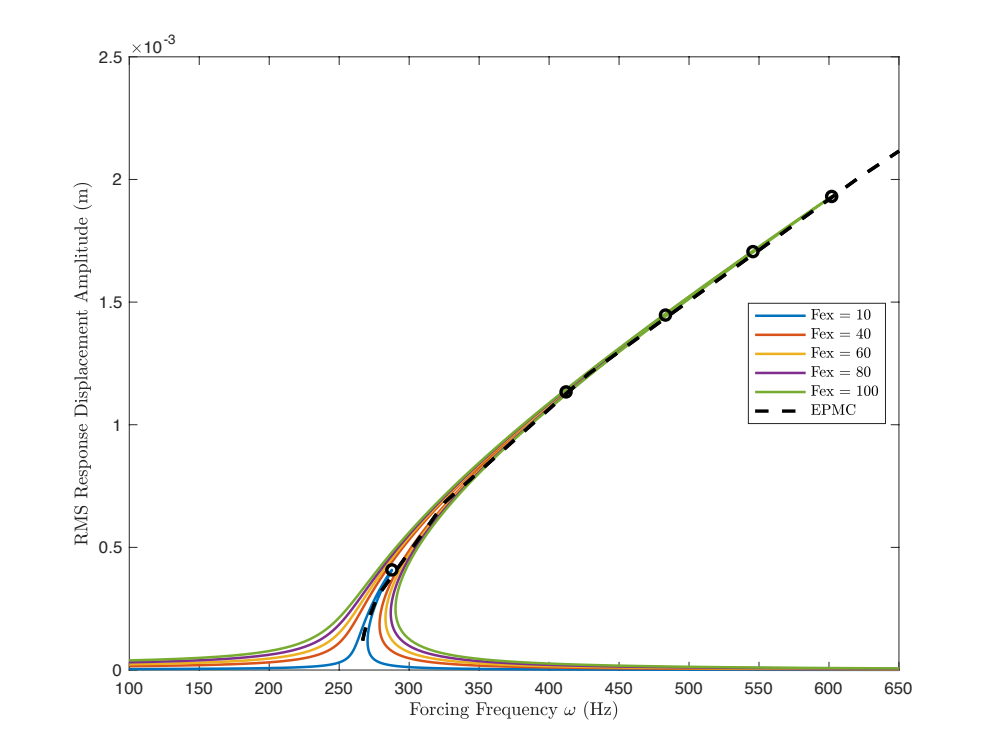
Topics:

- Definition of benchmark models Specify representative levels of harmonic excitation
- Consistency of PNLSS Model Identification

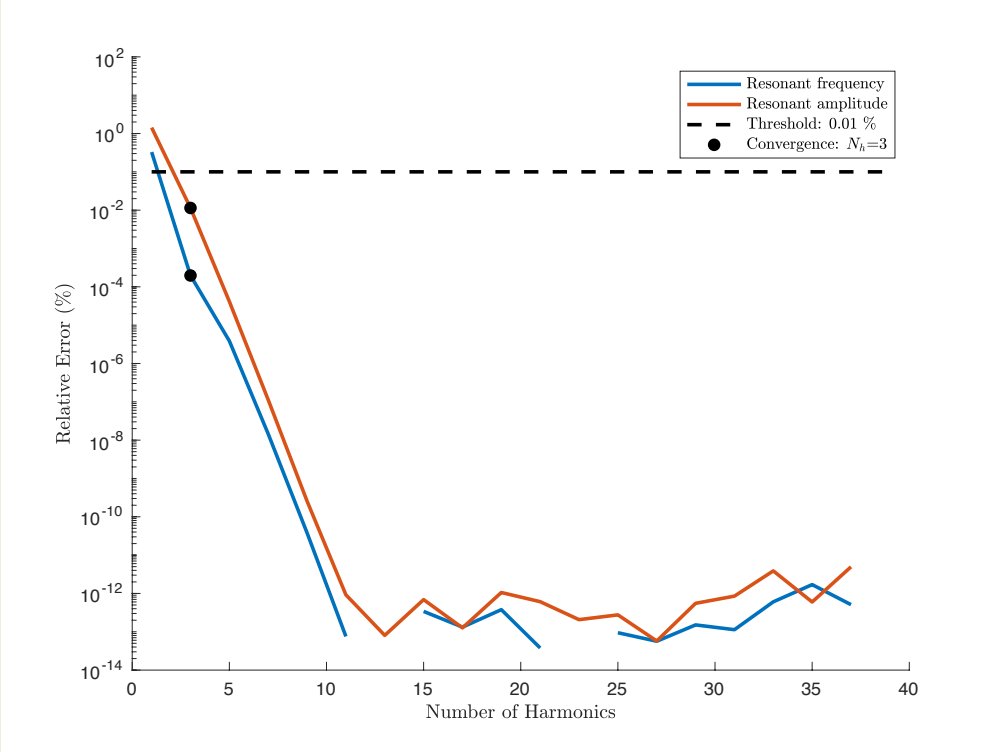
I. Definition of benchmark models

- Specify the target frequency range
- Specify representative levels of harmonic excitation
- Phase diagram study
- Damping factor study
- Study Frequency–Amplitude dependence of NNMs

Benchmark Model I: Flat clamped-clamped beam model (Duffing Oscillator)

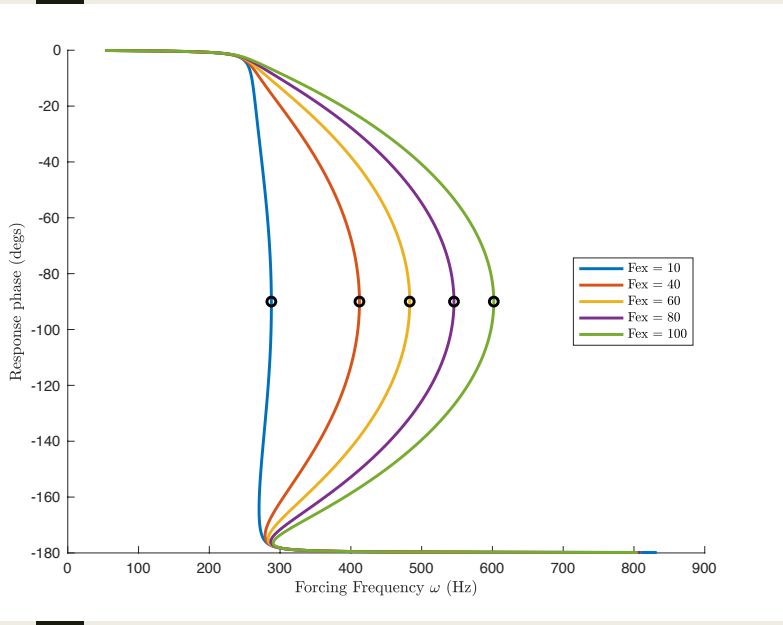


Specify frequency range - FRF

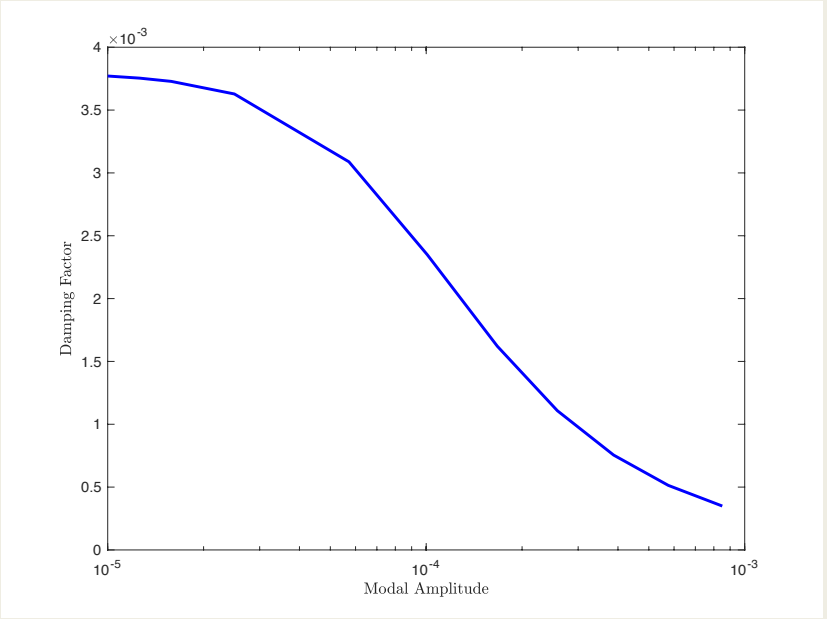


Specify number of excitation harmonics

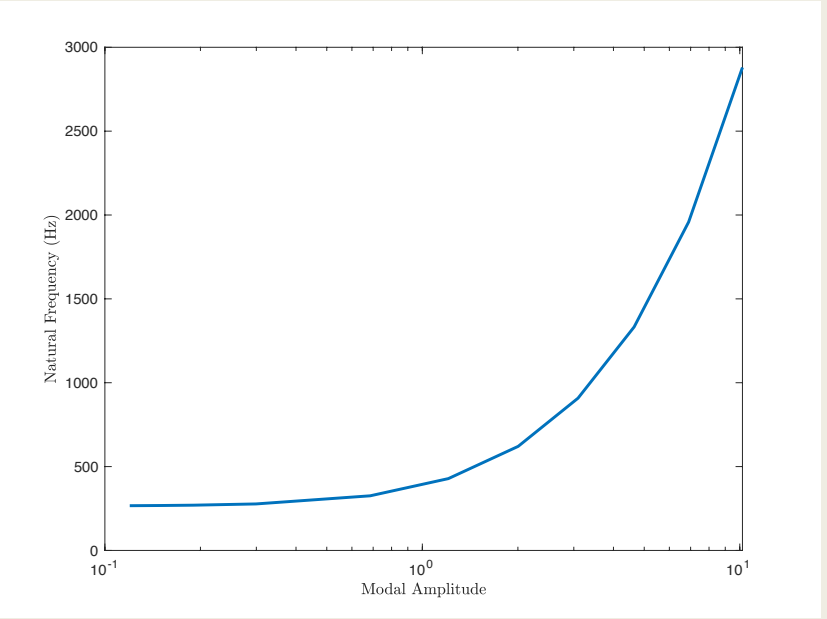
Benchmark Model I: Flat clamped-clamped beam model (Duffing Oscillator)



Phase Diagram

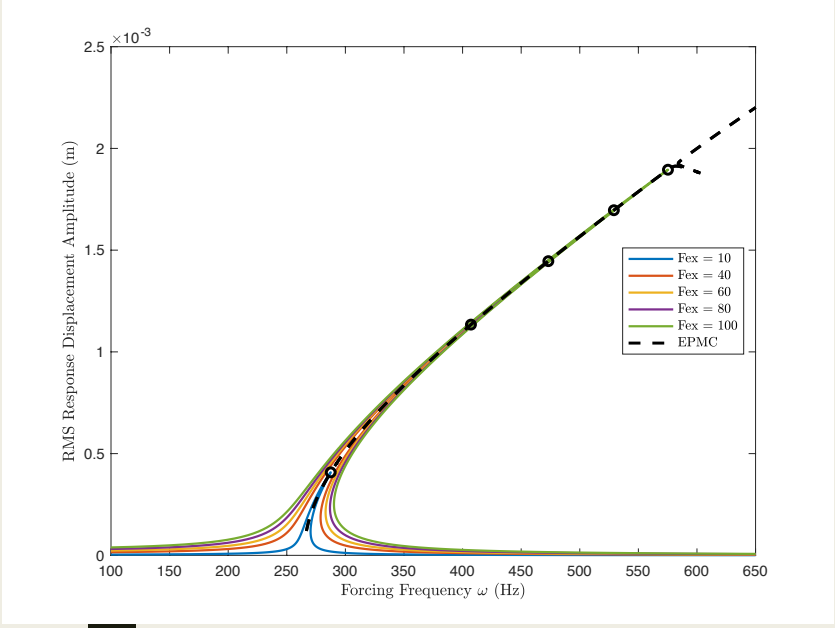


Damping Factor

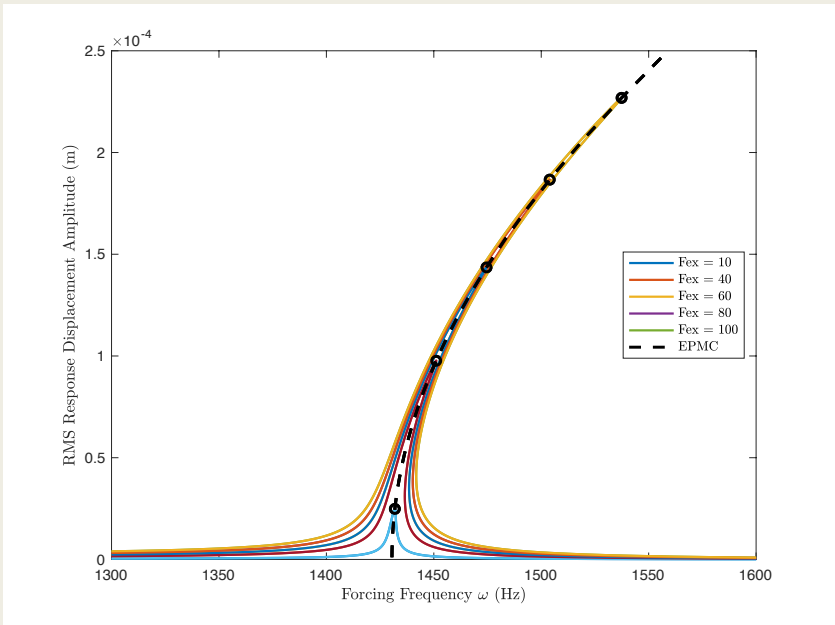


Frequency–Amplitude Dependence

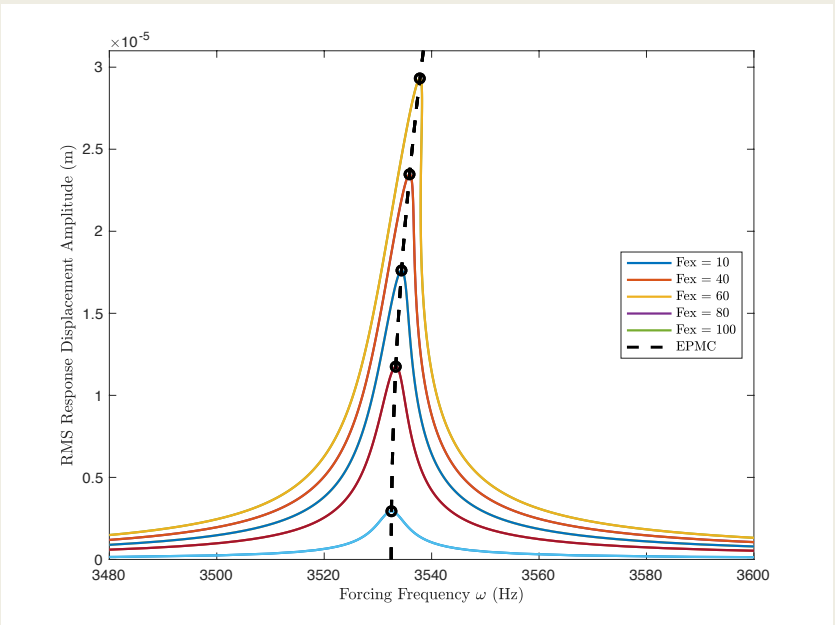
Benchmark Model II: Flat clamped-clamped beam model (Nm = 5)



Mode I

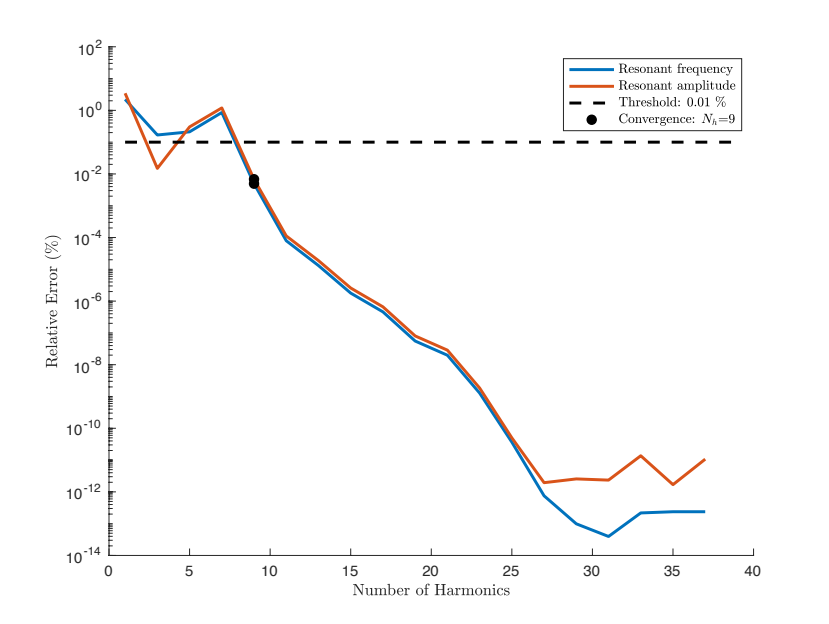


Mode III

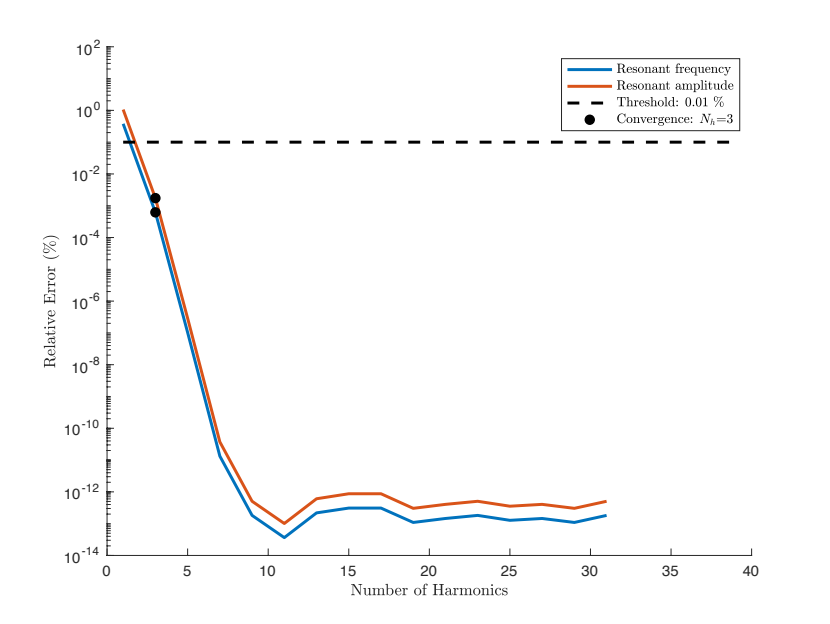


Mode V

Benchmark Model II: Flat clamped-clamped beam model (Nm = 5)



Mode I



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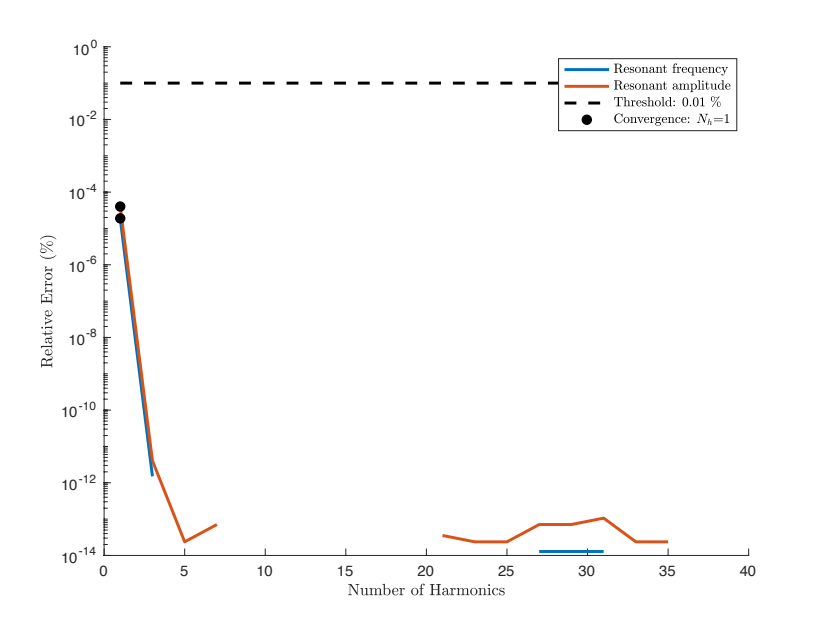
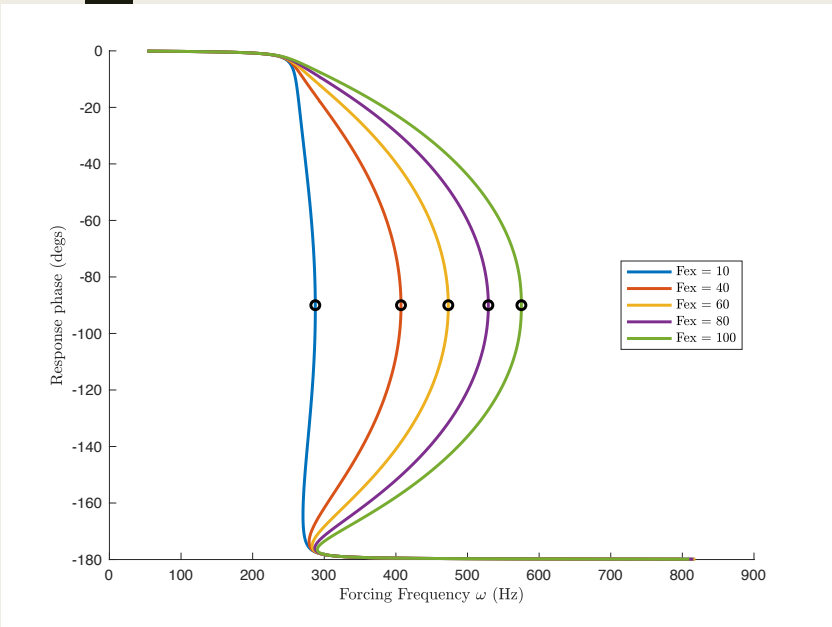
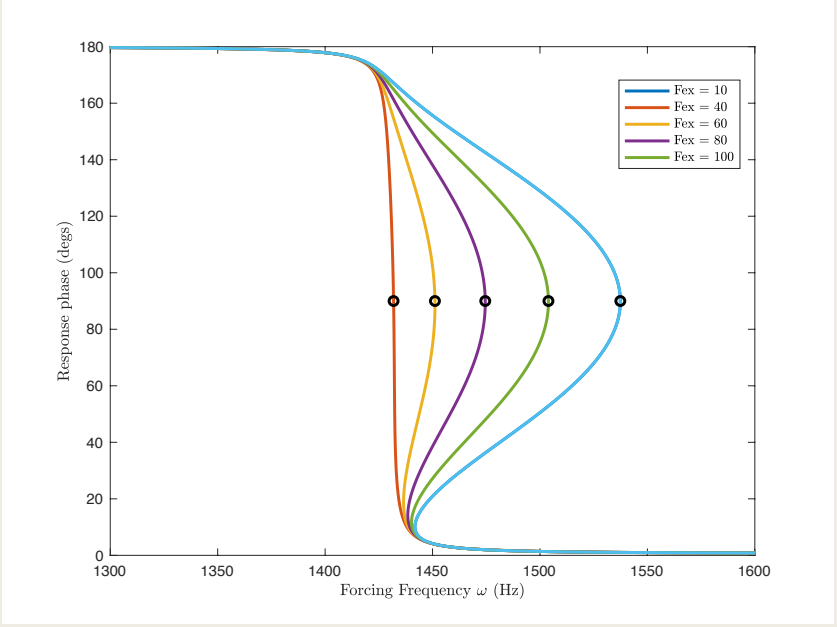


Mode V

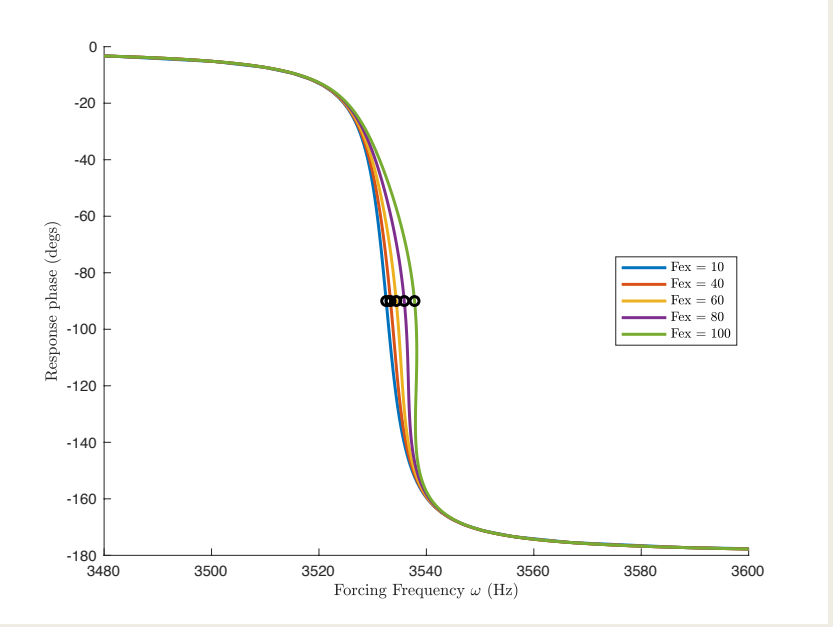
Benchmark Model II: Flat clamped-clamped beam model (Nm = 5)



Mode I

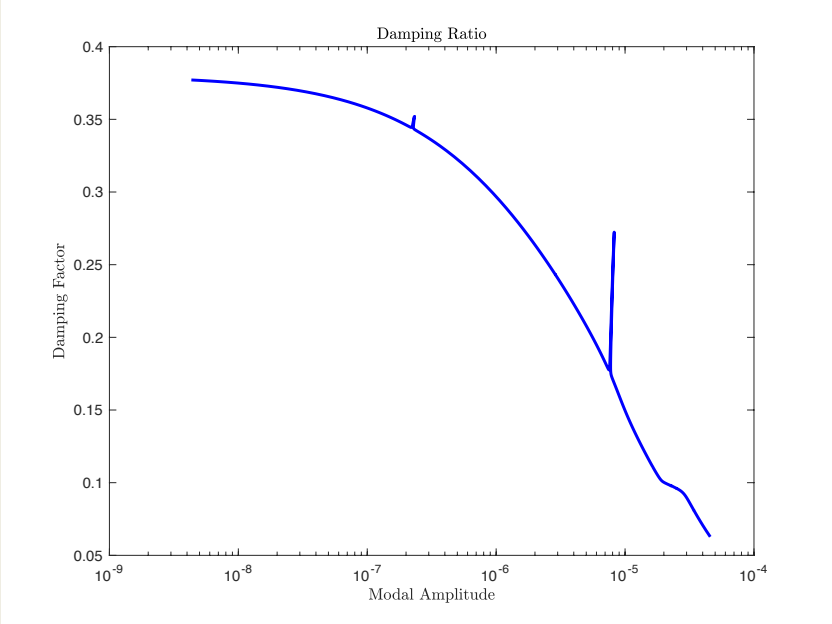


Mode III

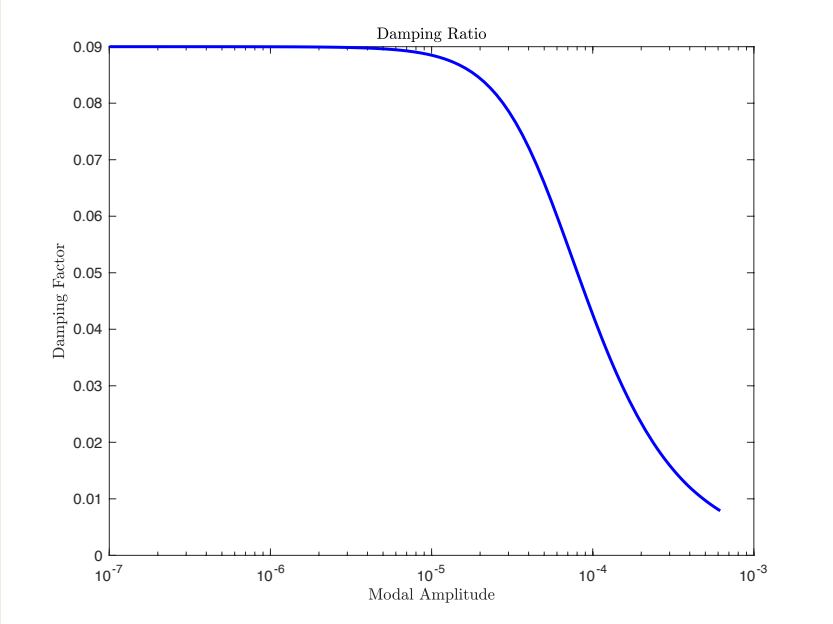


Mode V

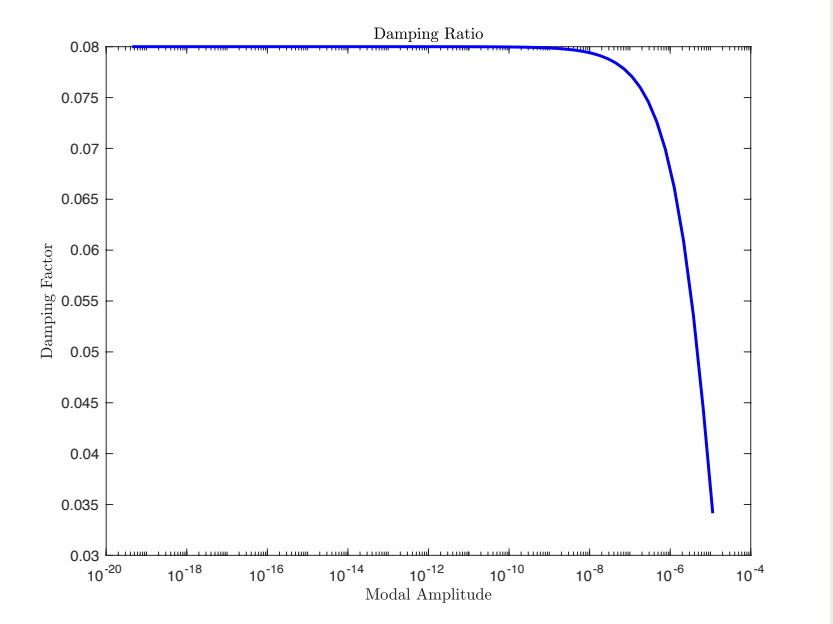
Benchmark Model II: Flat clamped-clamped beam model (Nm = 5)



Mode I

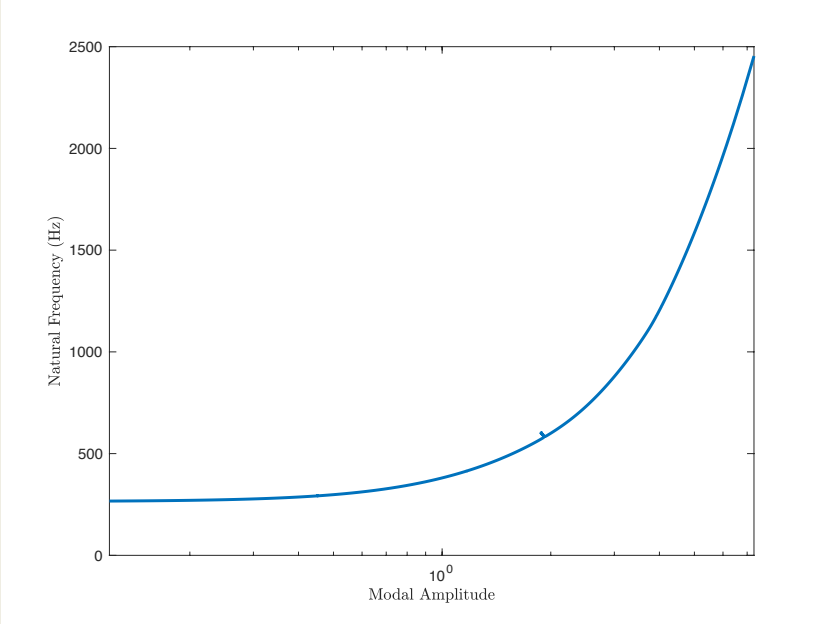


Mode III

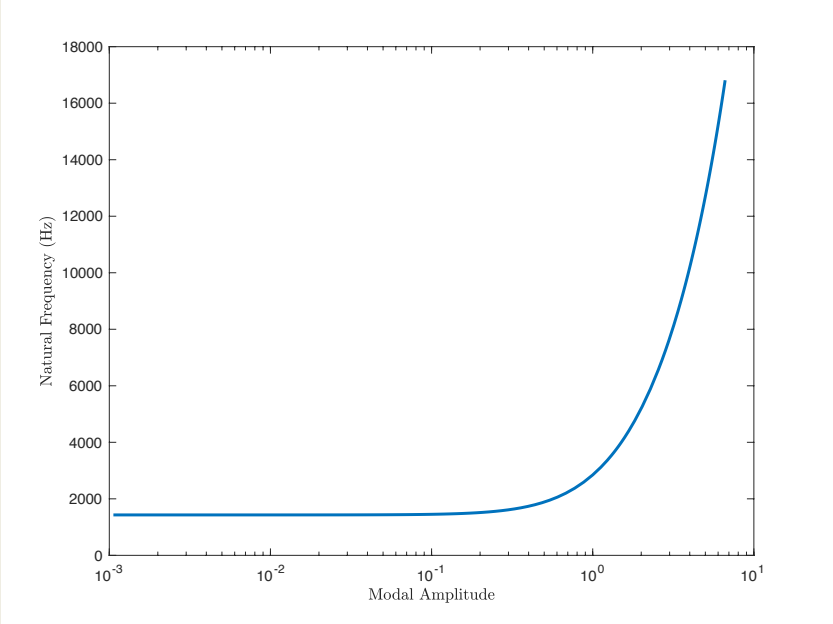


Mode V

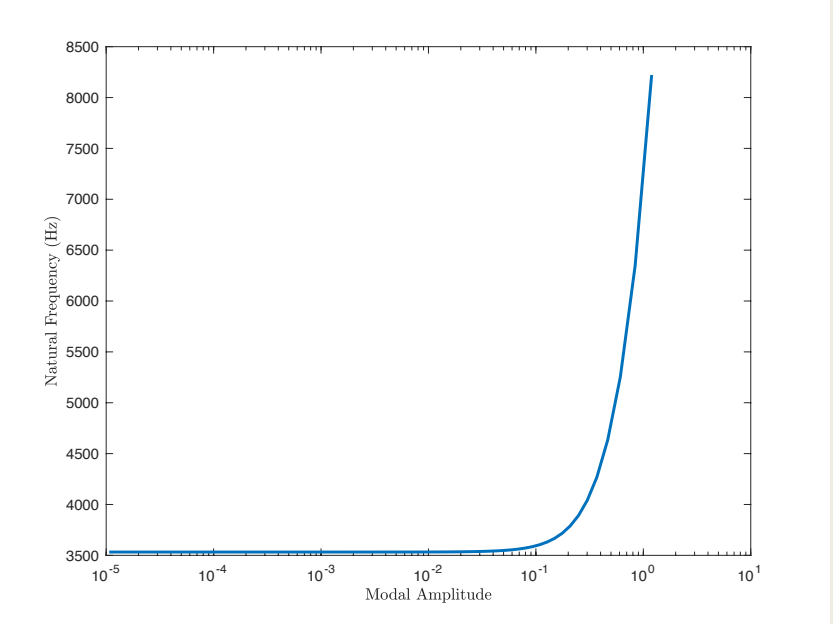
Benchmark Model II: Flat clamped-clamped beam model (Nm = 5)



Mode I

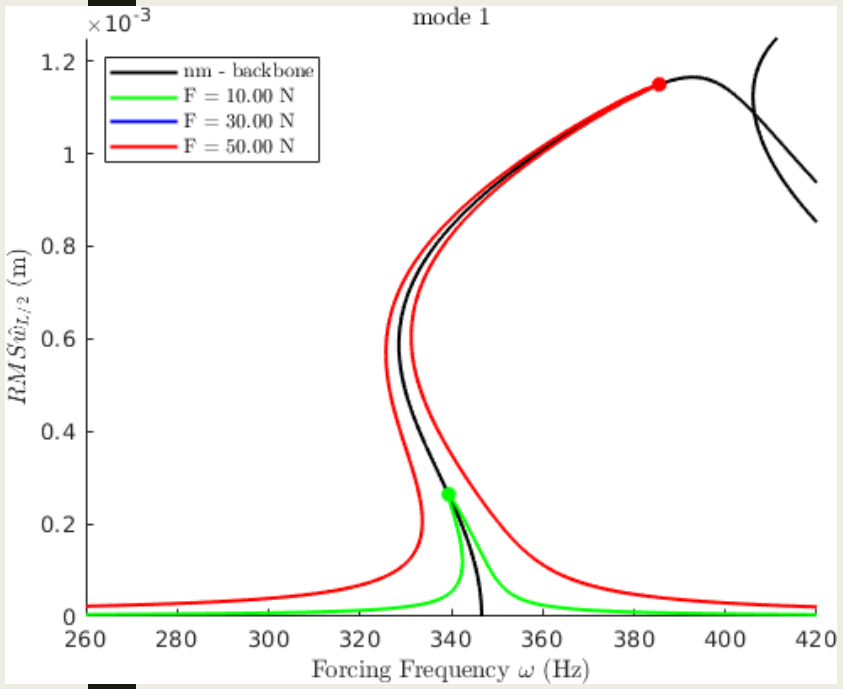


Mode III

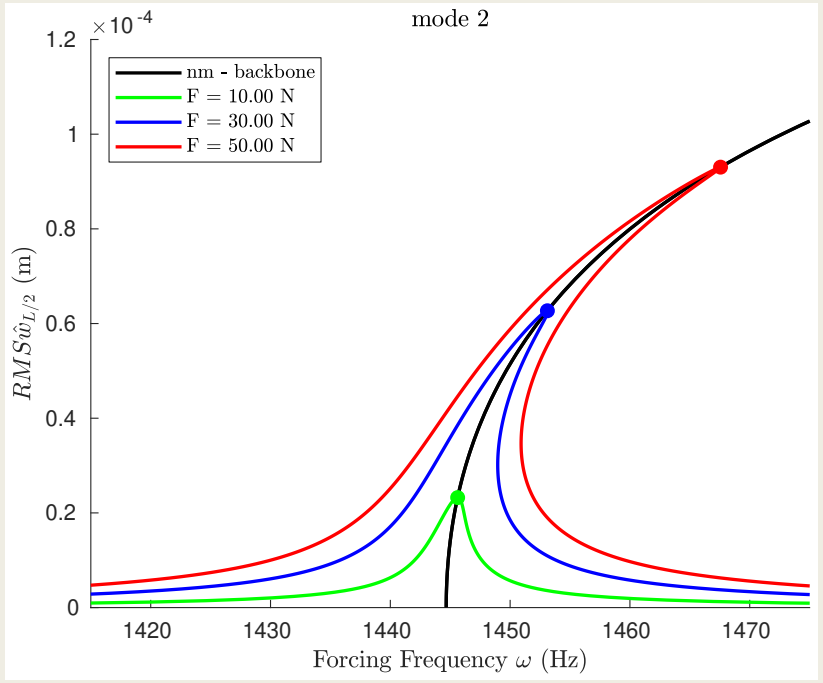


Mode V

Benchmark Model III: Curved clamped-clamped beam model (Nm = 3)



Mode I



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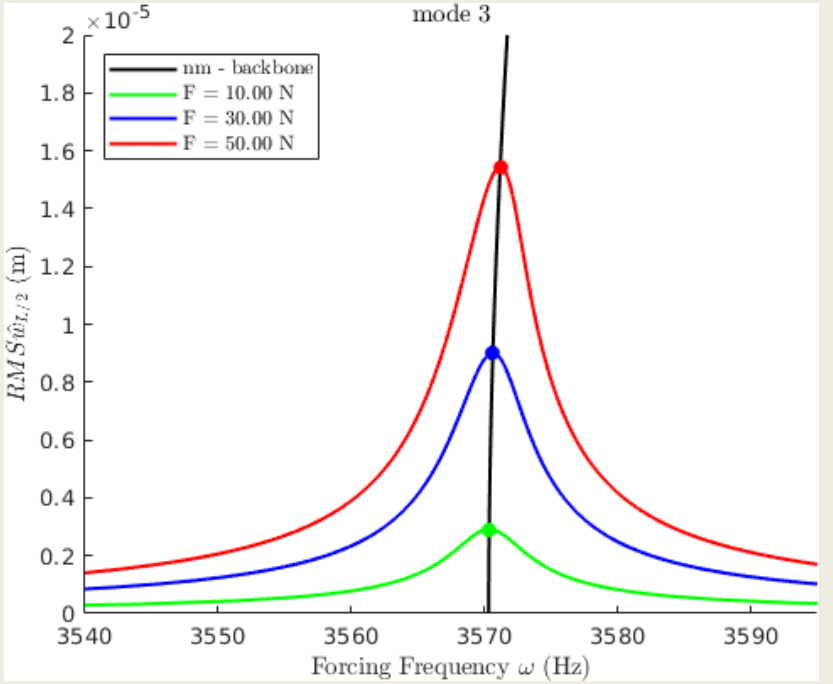
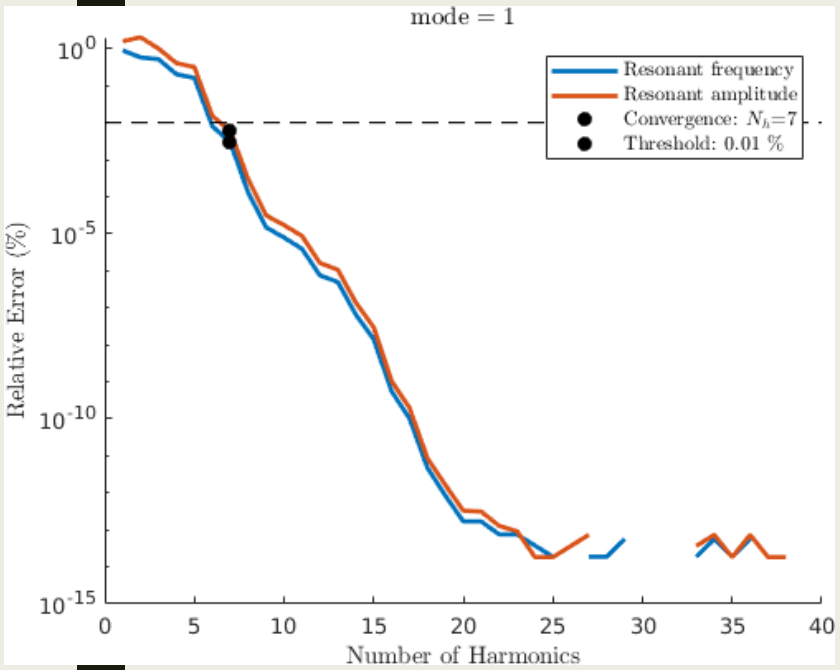
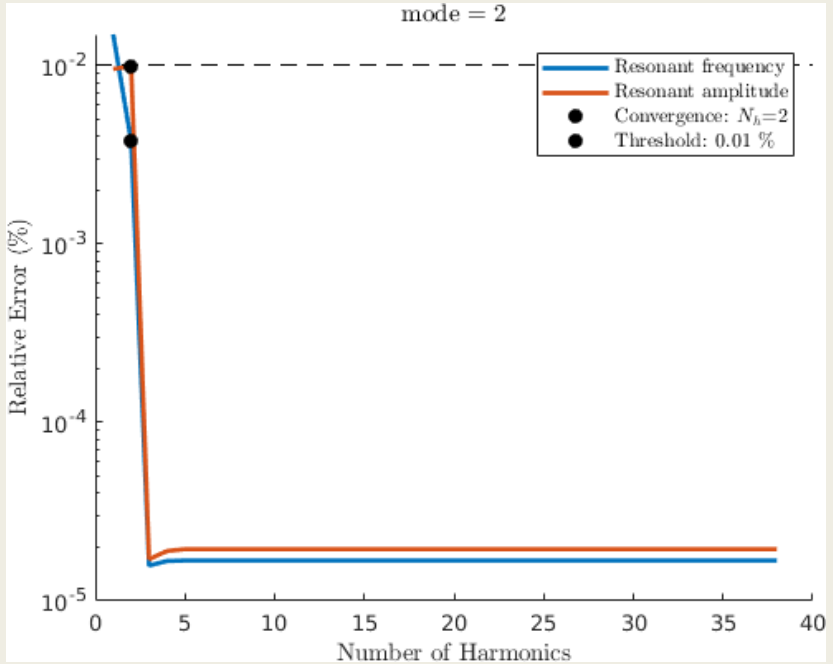


Mode III

Benchmark Model III: Curved clamped-clamped beam model (Nm = 3)



Mode I



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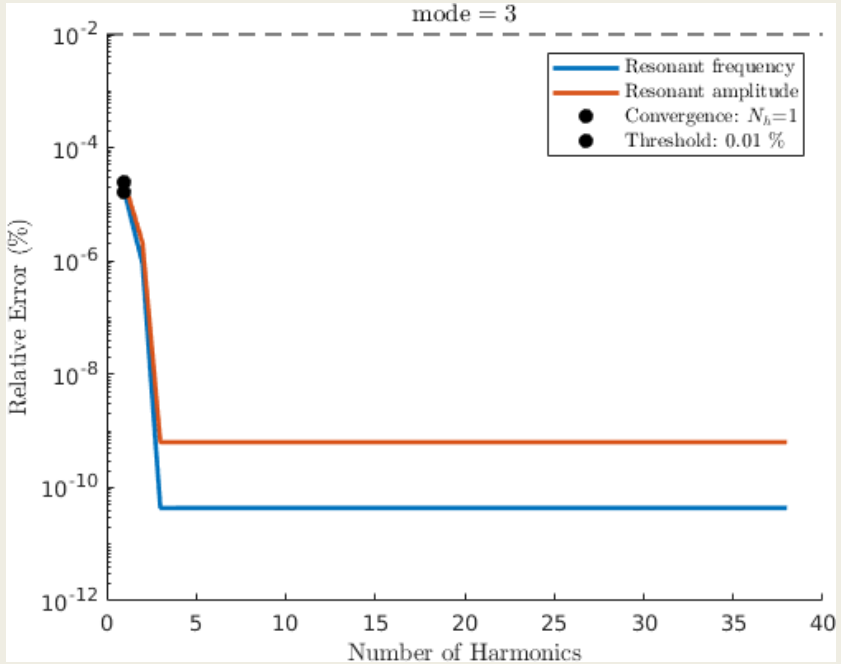
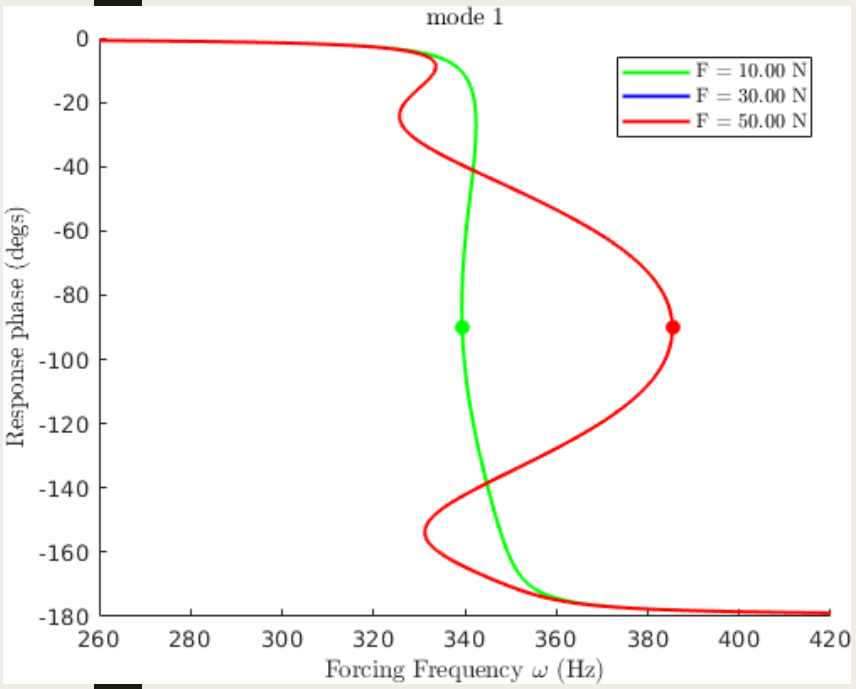
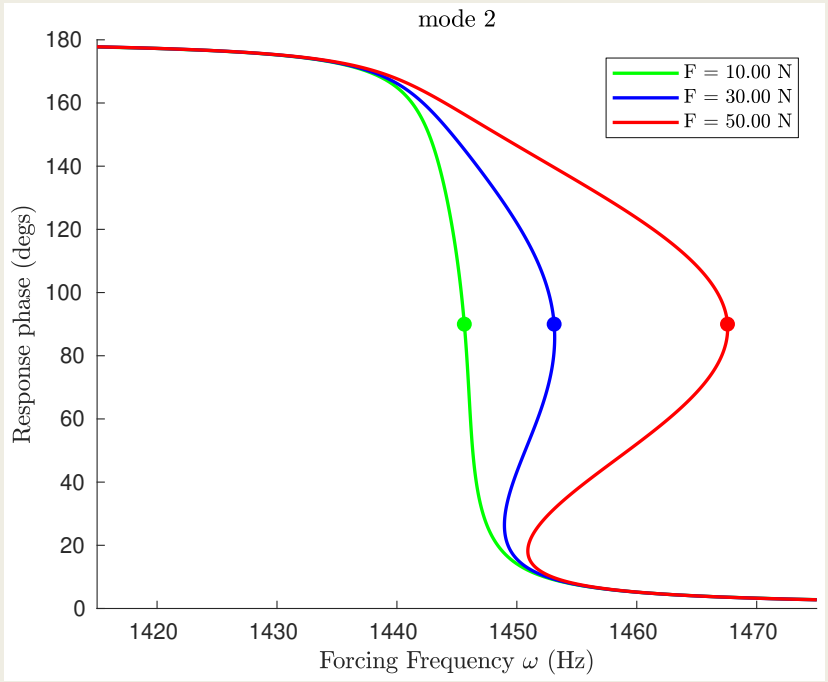


Mode III

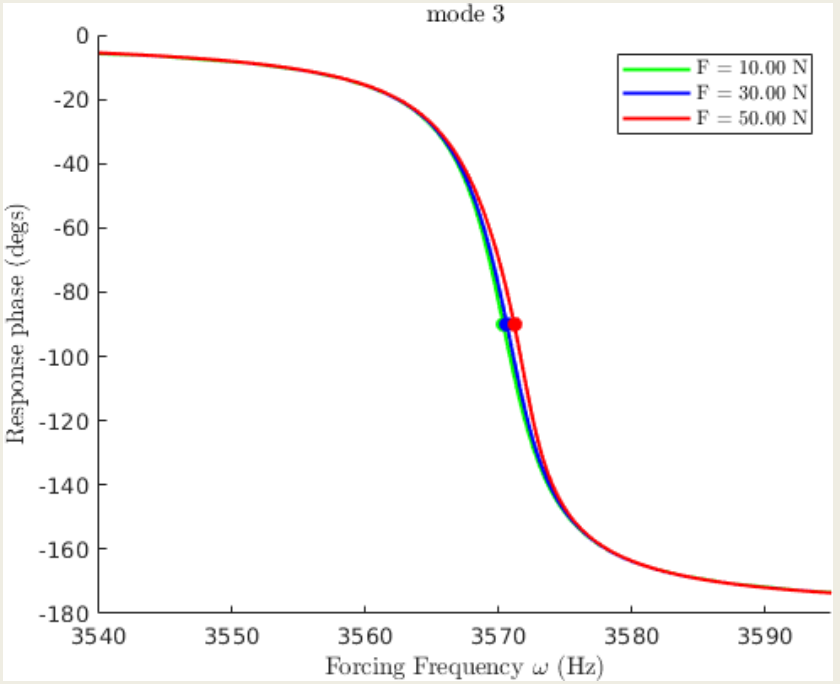
Benchmark Model III: Curved clamped-clamped beam model (Nm = 3)



Mode I

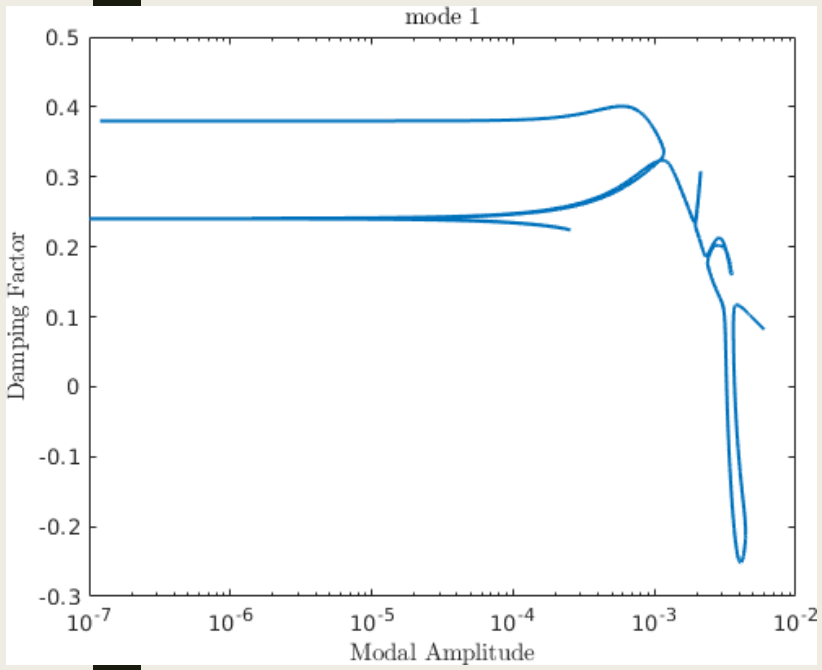


Mode II

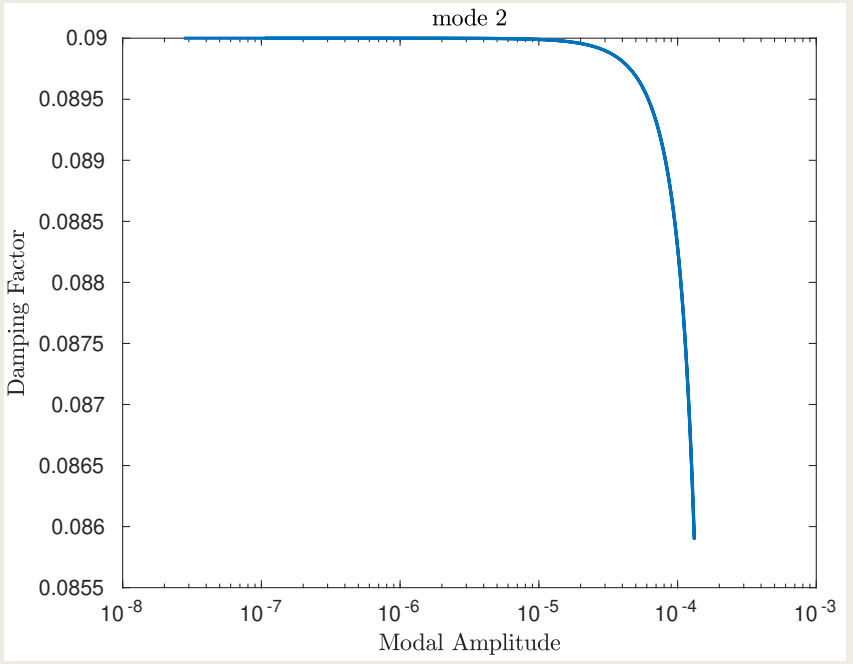


Mode III

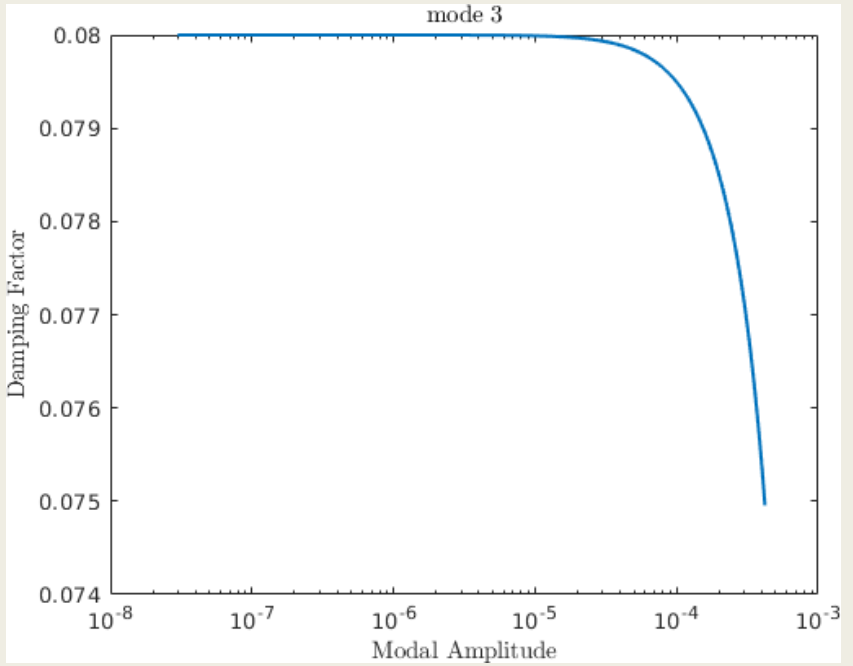
Benchmark Model III: Curved clamped-clamped beam model (Nm = 3)



Mode I

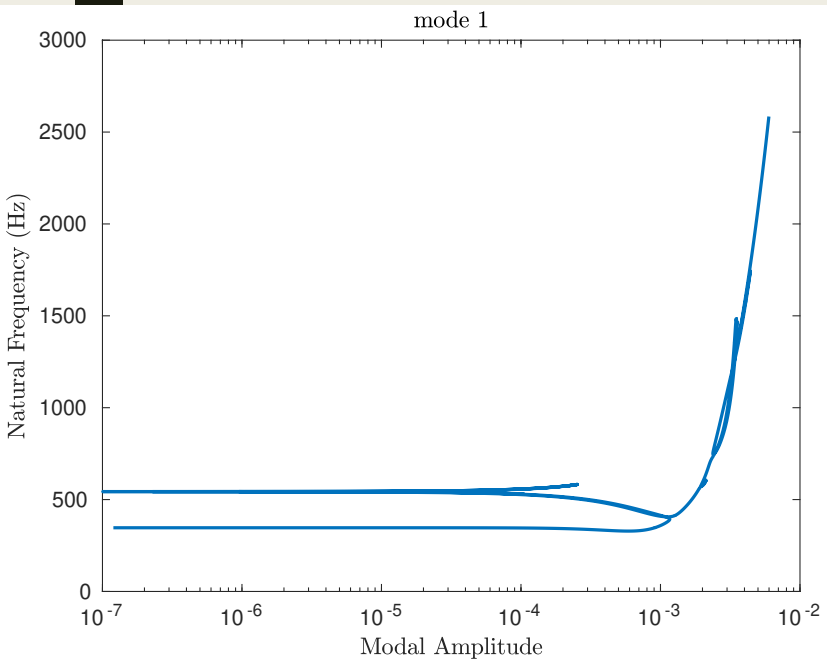


Mode II

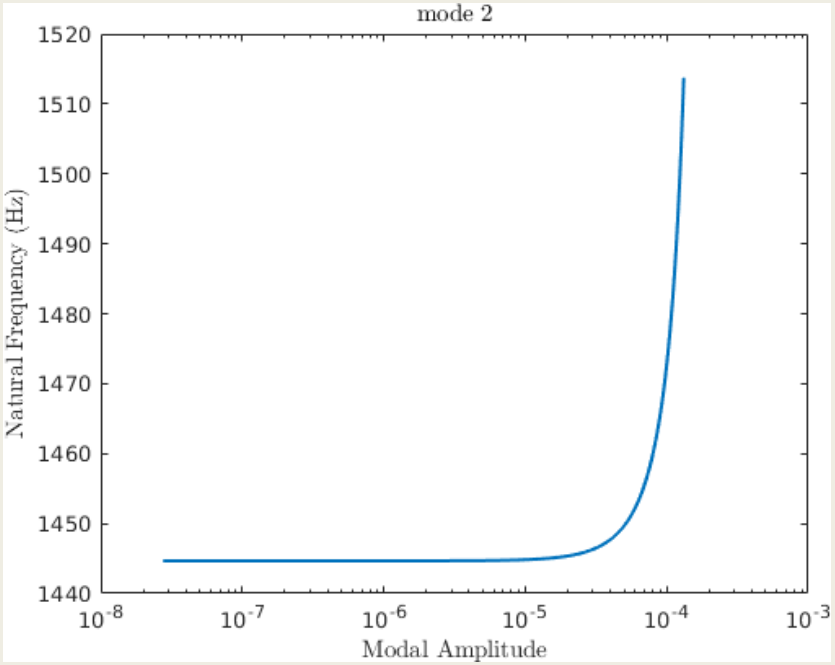


Mode III

Benchmark Model III: Curved clamped-clamped beam model (Nm = 3)



Mode I



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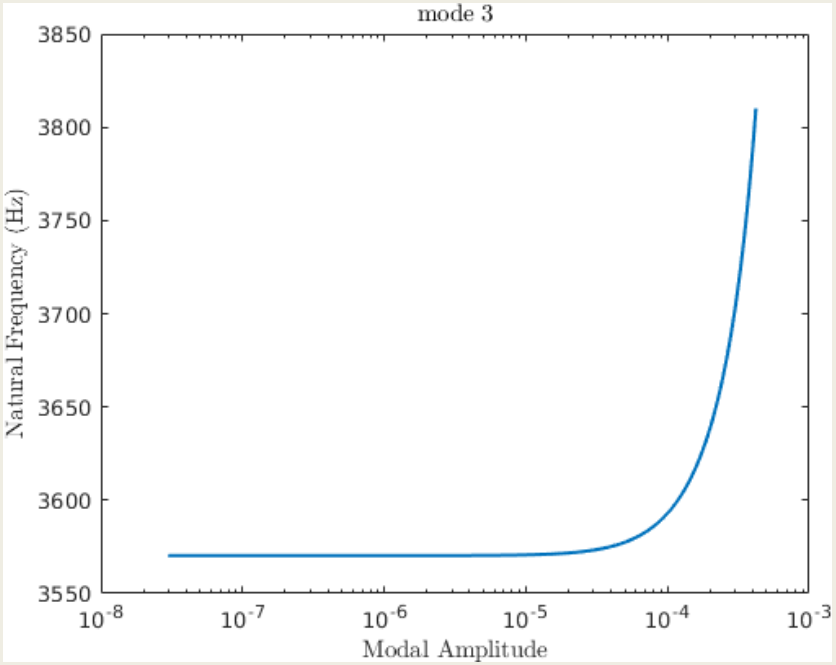
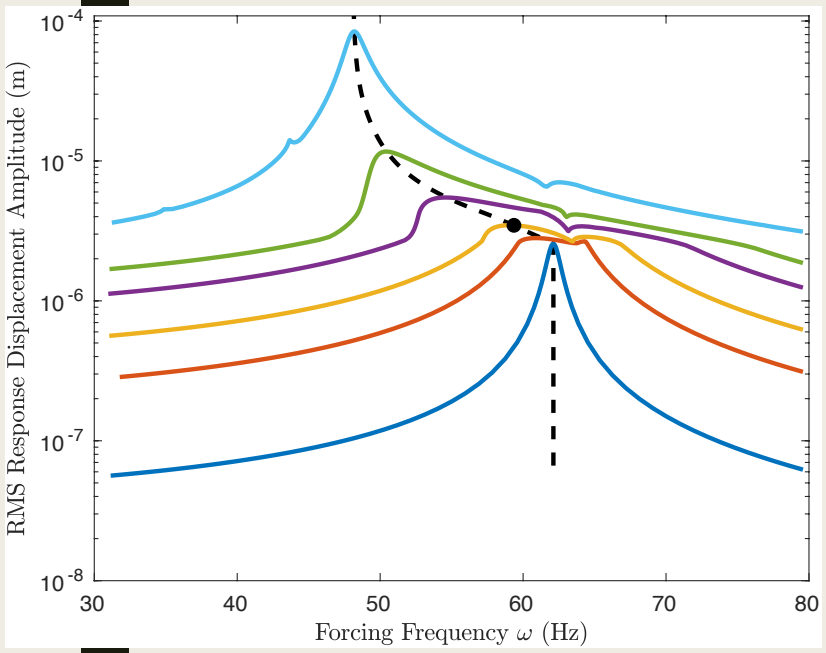
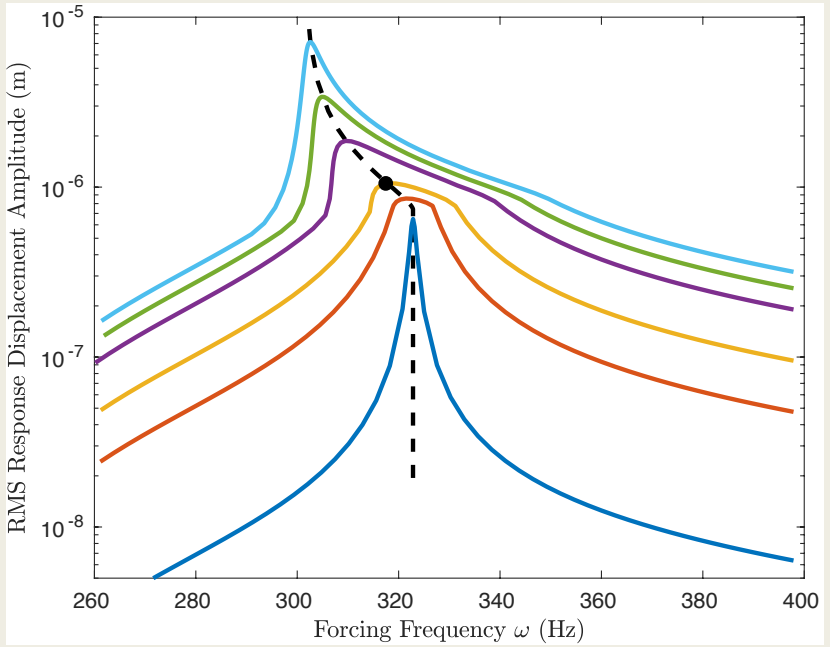


Mode III

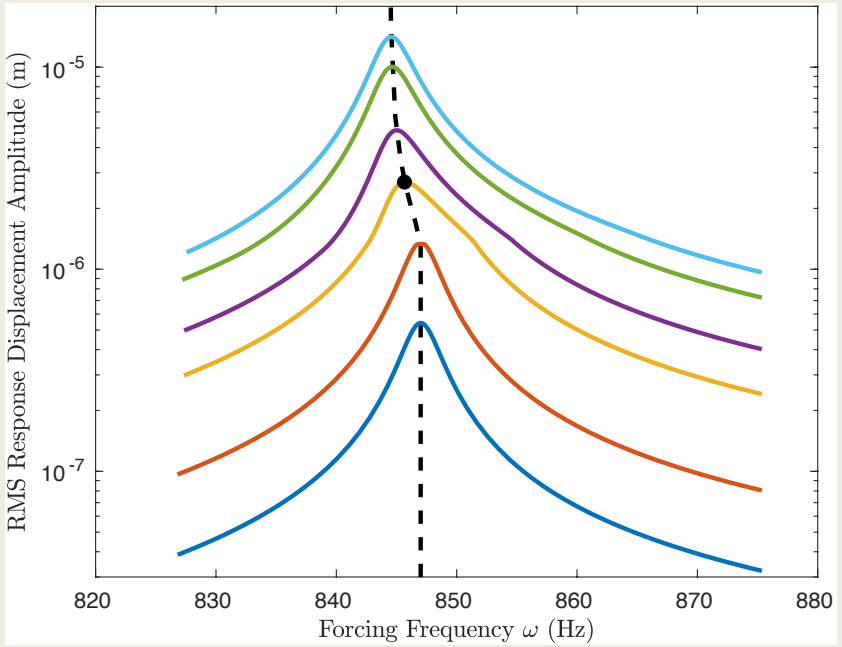
Benchmark Model IV: Cantilever beam with elastic dry friction element



Mode I

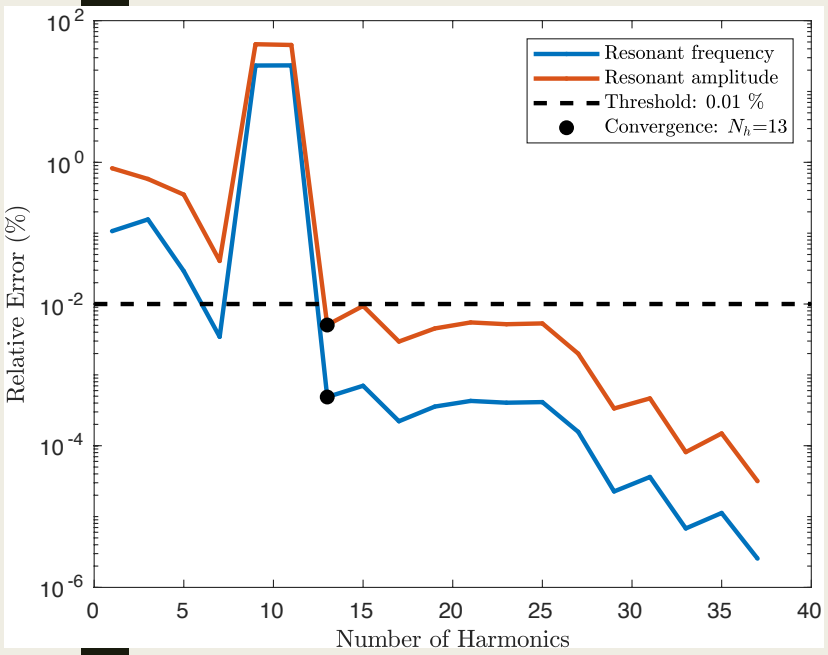


Mode II

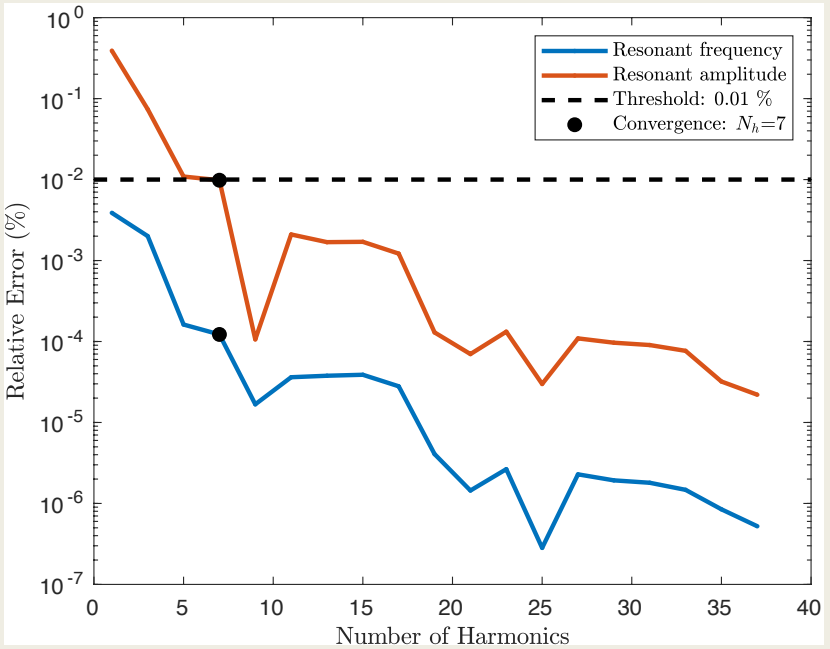


Mode III

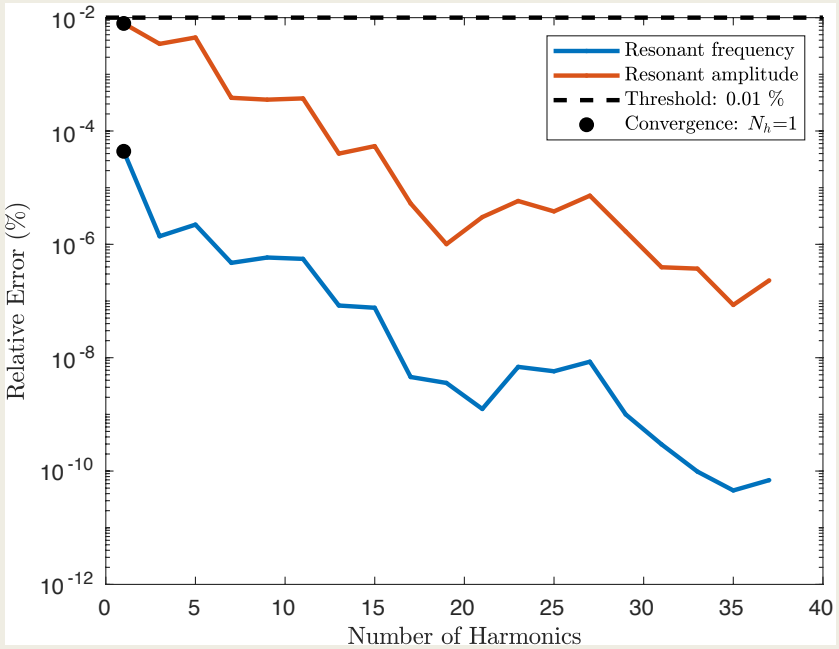
Benchmark Model IV: Cantilever beam with elastic dry friction element



Mode I

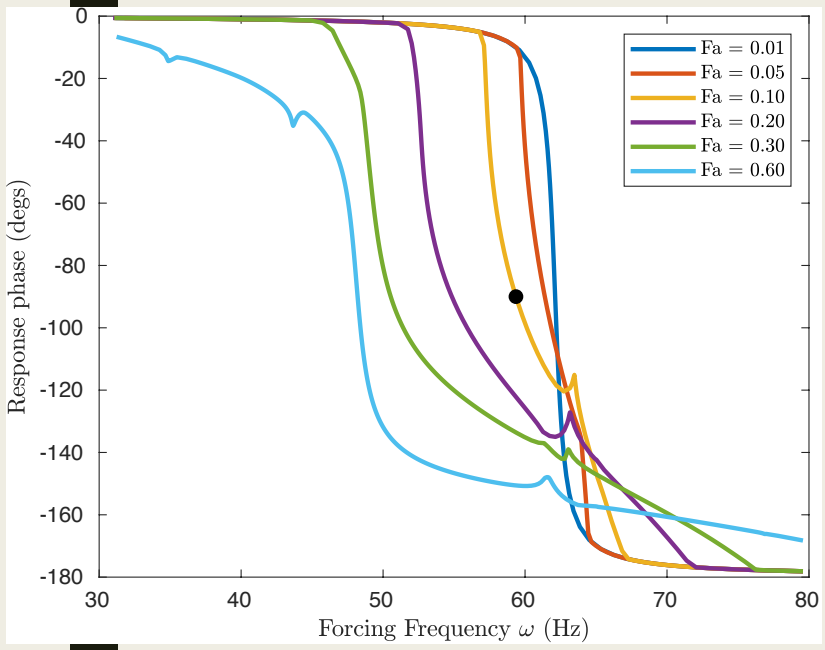


Mode II

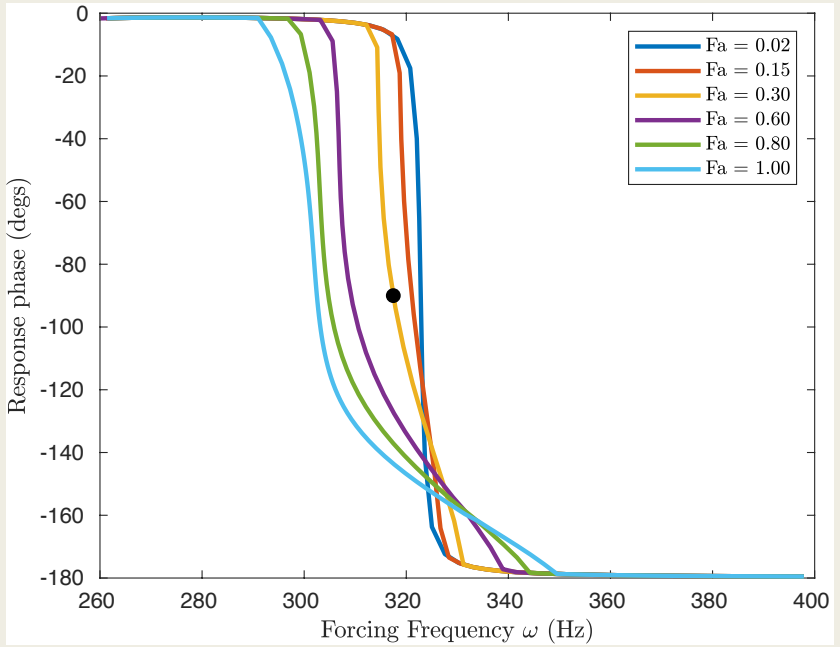


Mode III

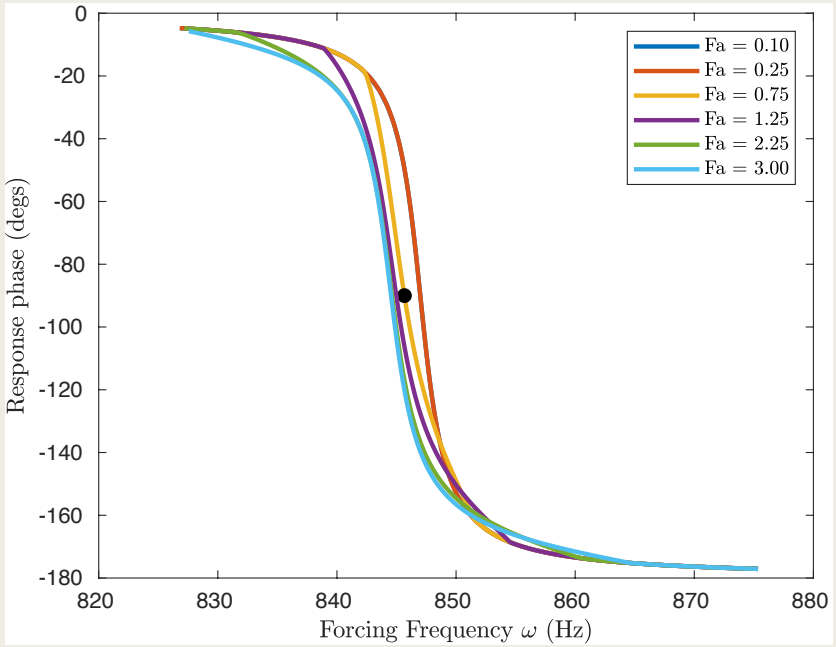
Benchmark Model IV: Cantilever beam with elastic dry friction element



Mode I

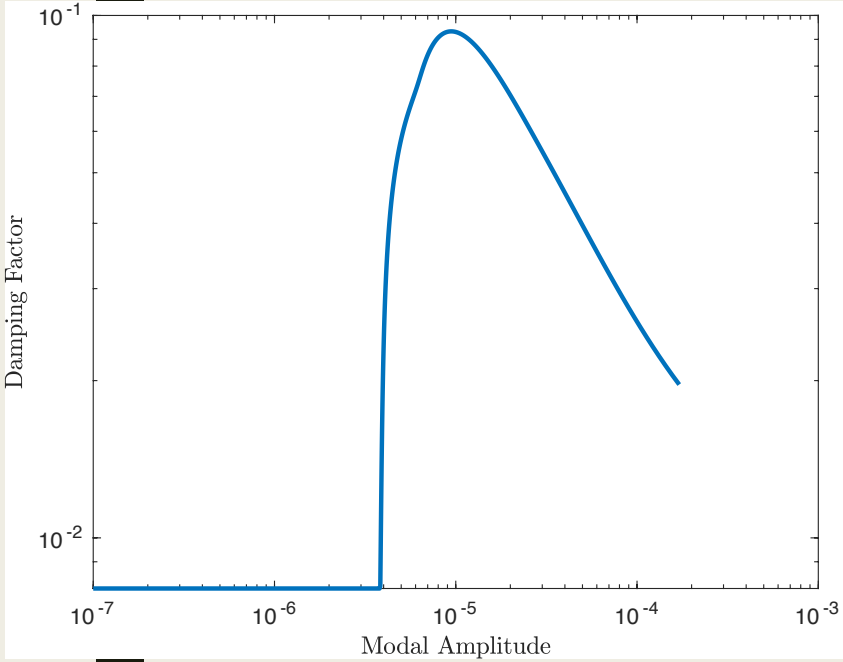


Mode II

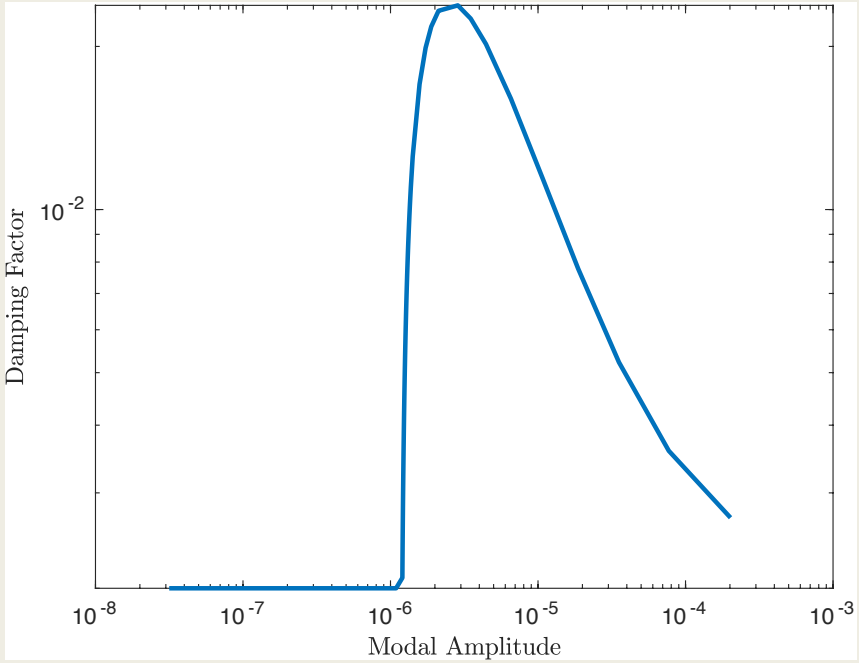


Mode III

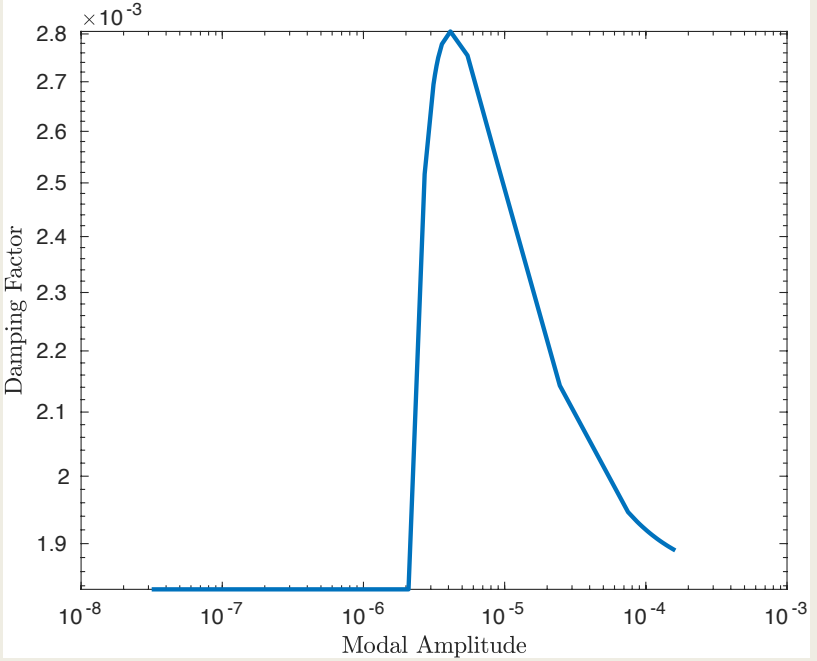
Benchmark Model IV: Cantilever beam with elastic dry friction element



Mode I

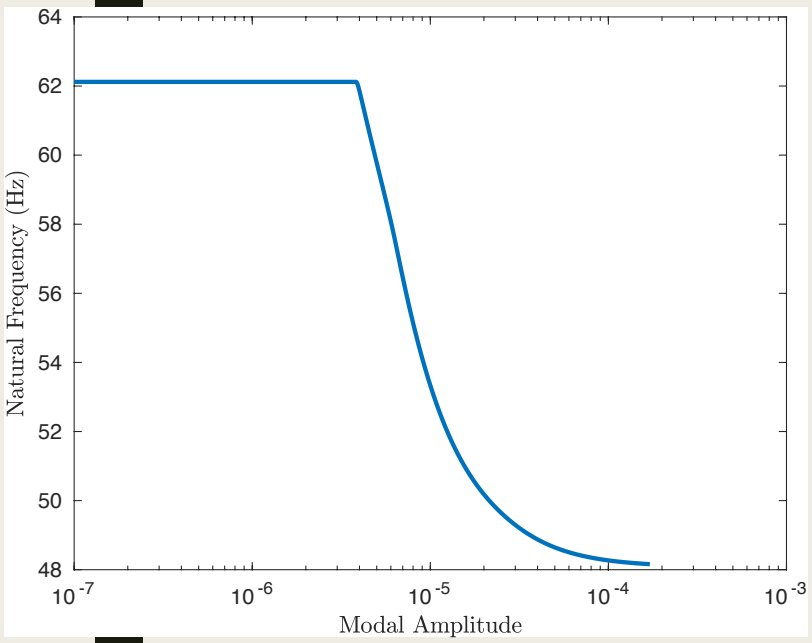


Mode II

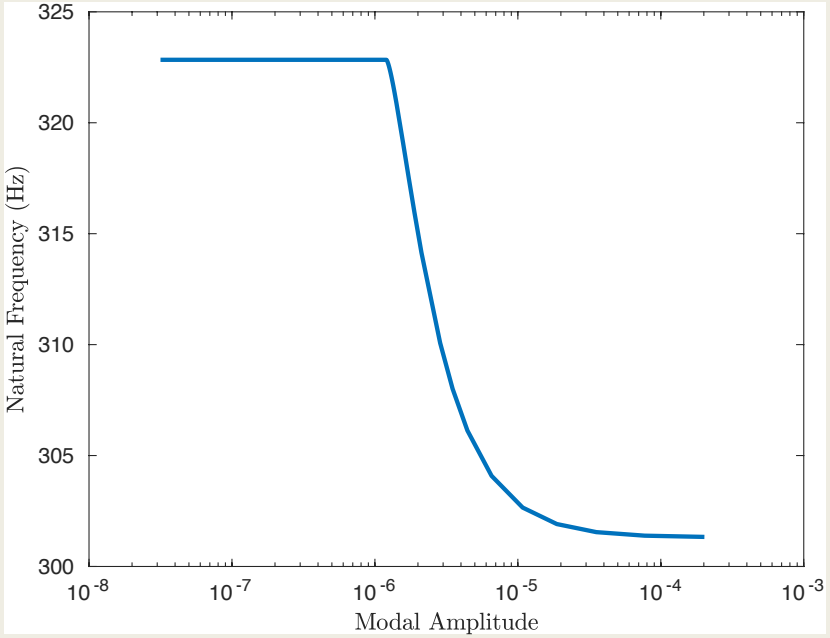


Mode III

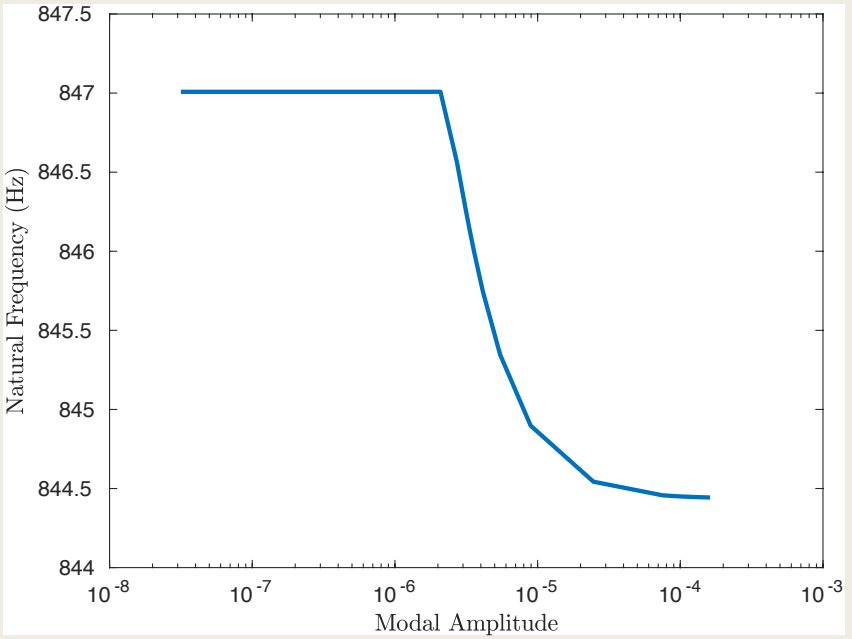
Benchmark Model IV: Cantilever beam with elastic dry friction element



Mode I

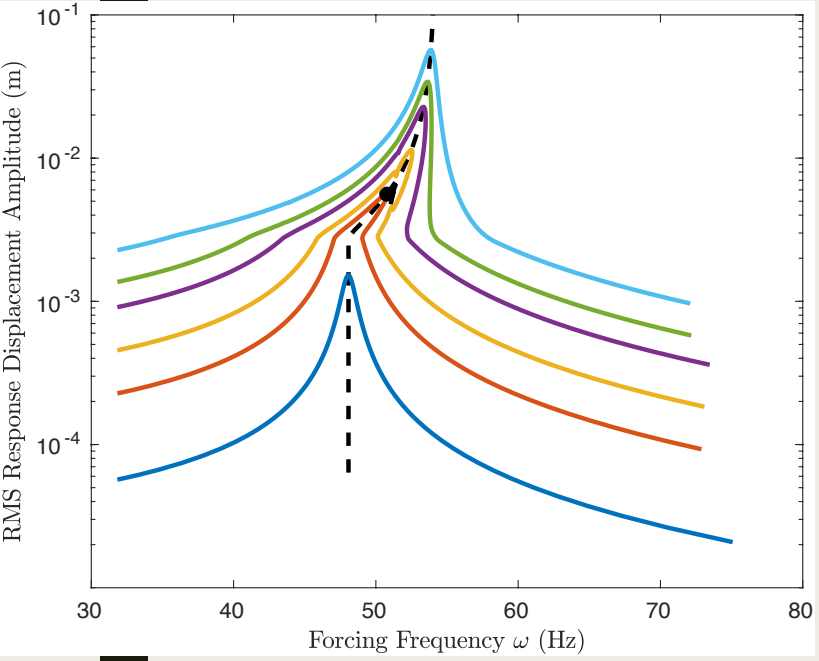


Mode II

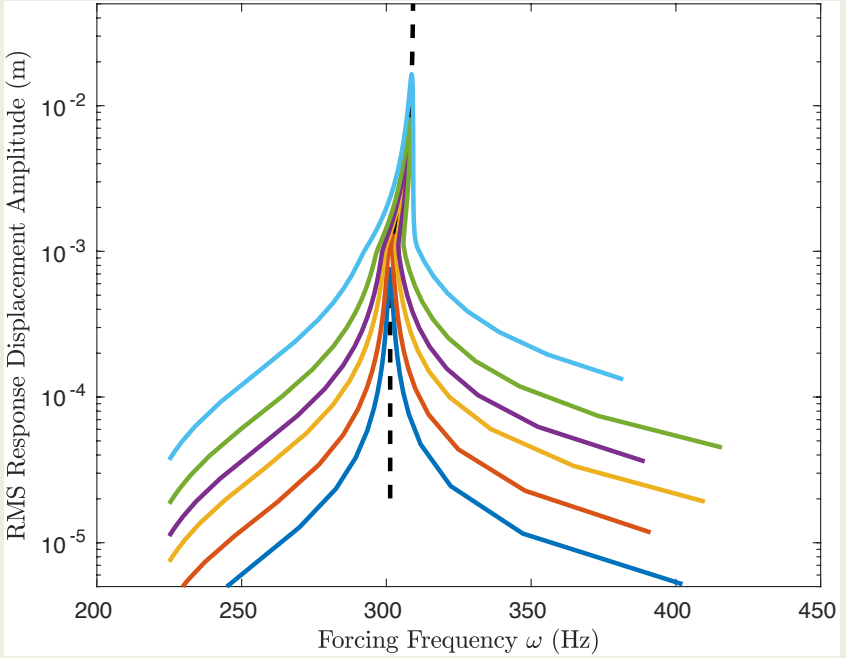


Mode III

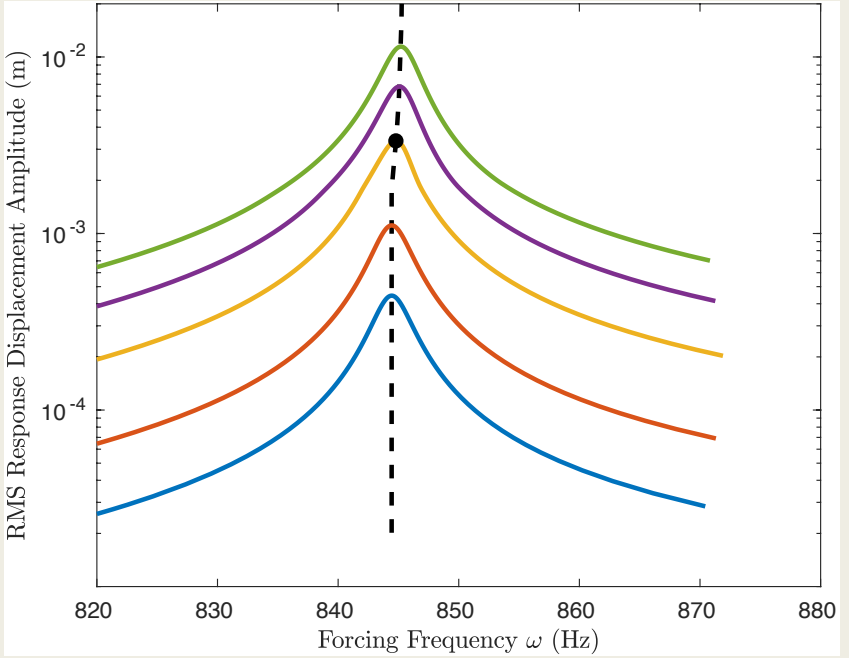
Benchmark Model V: Cantilever beam with unilateral spring element



Mode I

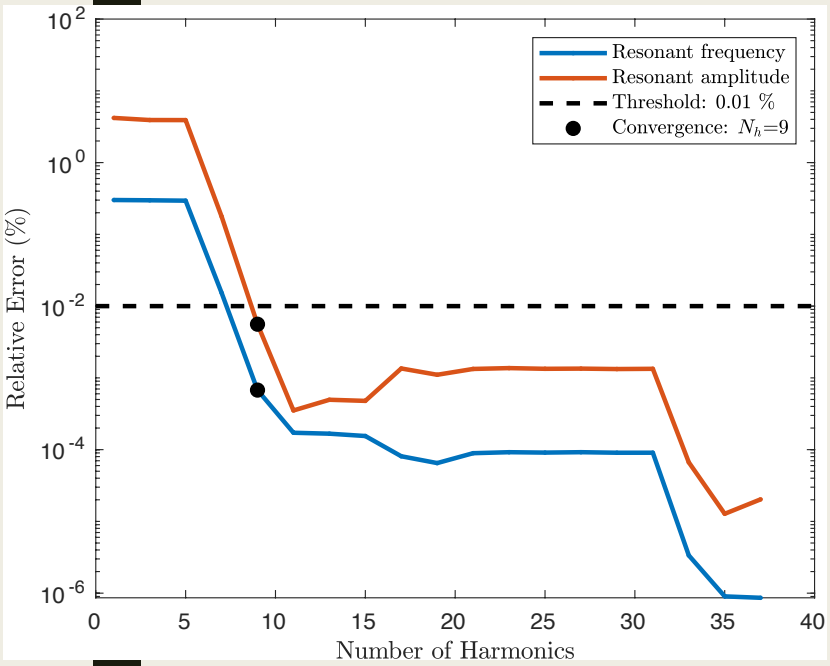


Mode II

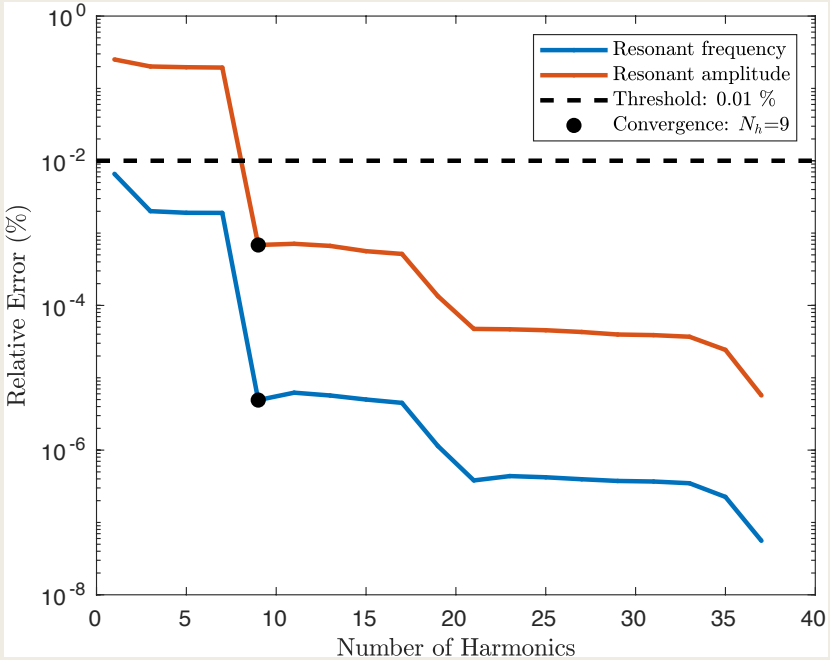


Mode III

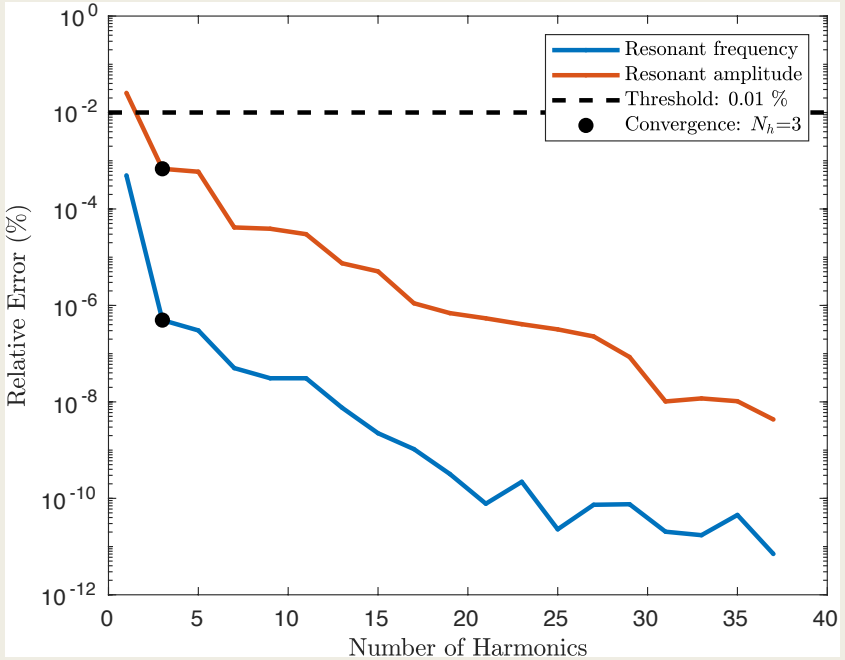
Benchmark Model V: Cantilever beam with unilateral spring element



Mode I

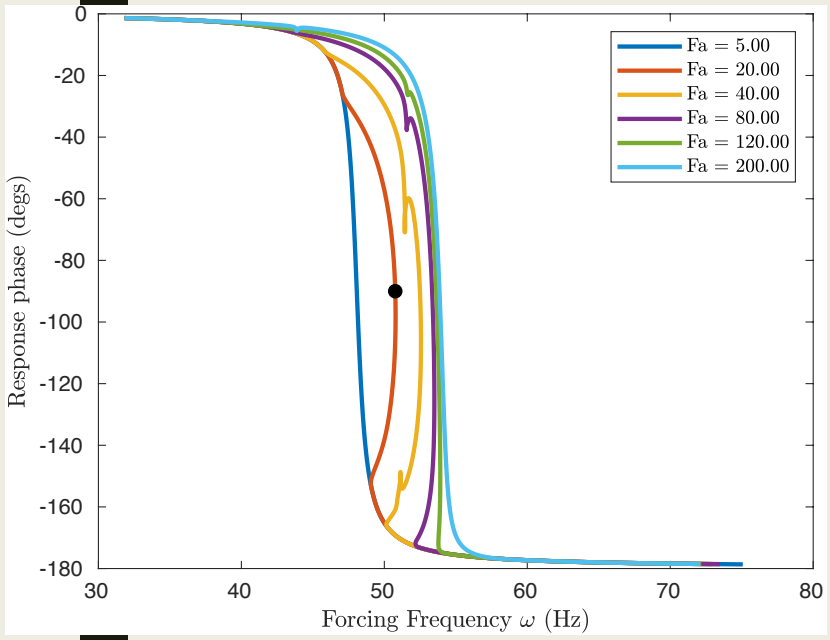


Mode II

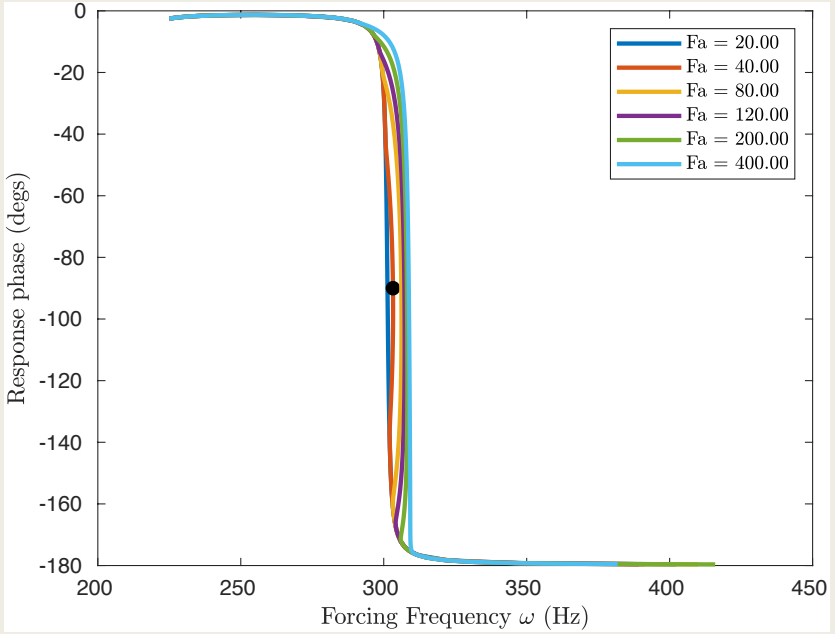


Mode III

Benchmark Model V: Cantilever beam with unilateral spring element



Mode I



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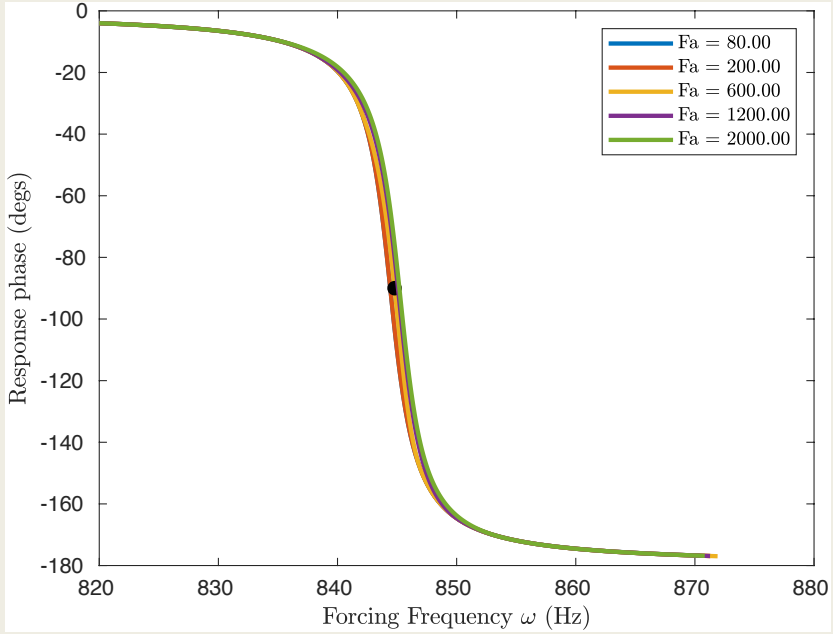
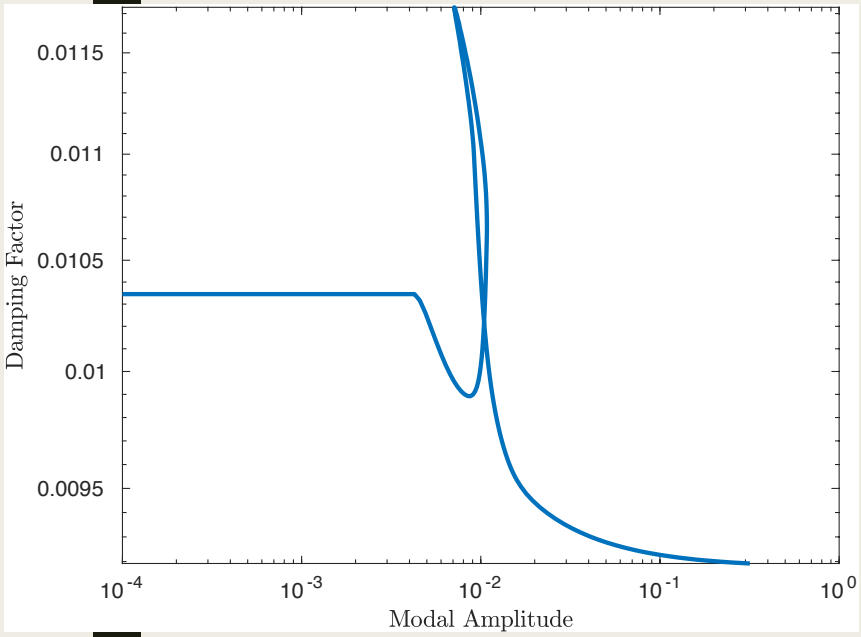
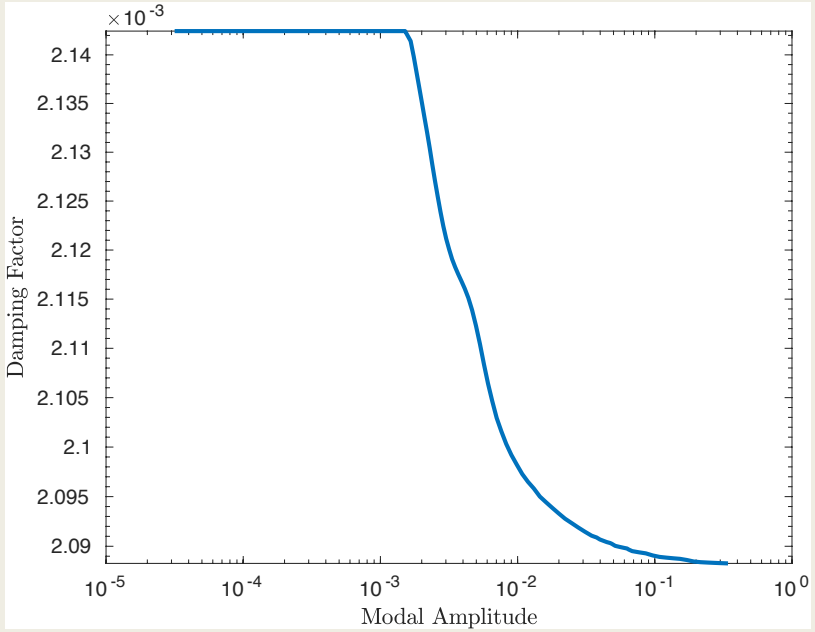


Mode III

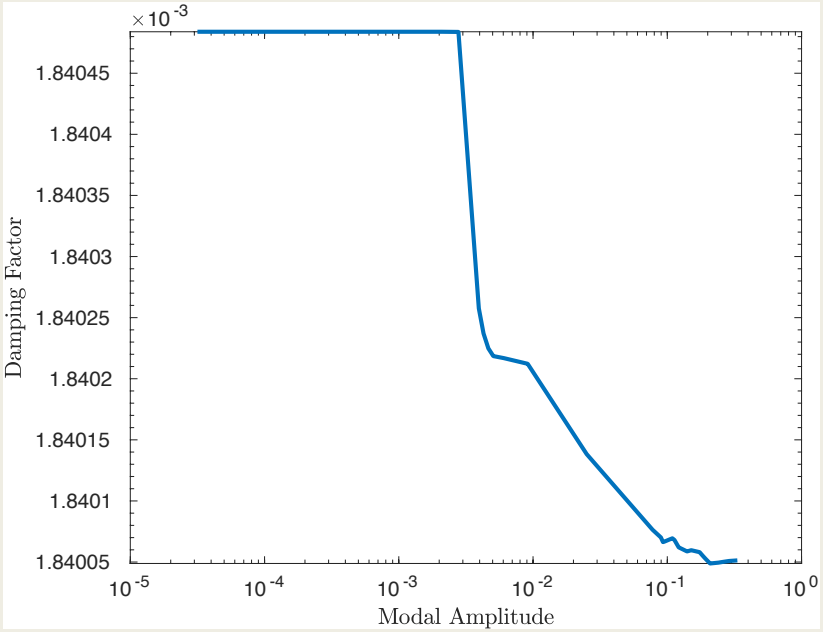
Benchmark Model V: Cantilever beam with unilateral spring element



Mode I

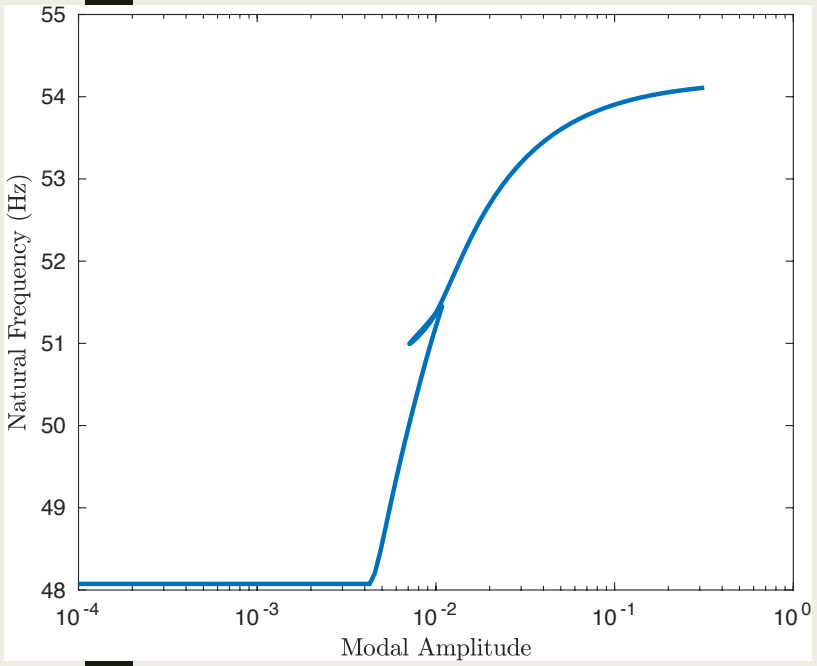


Mode II

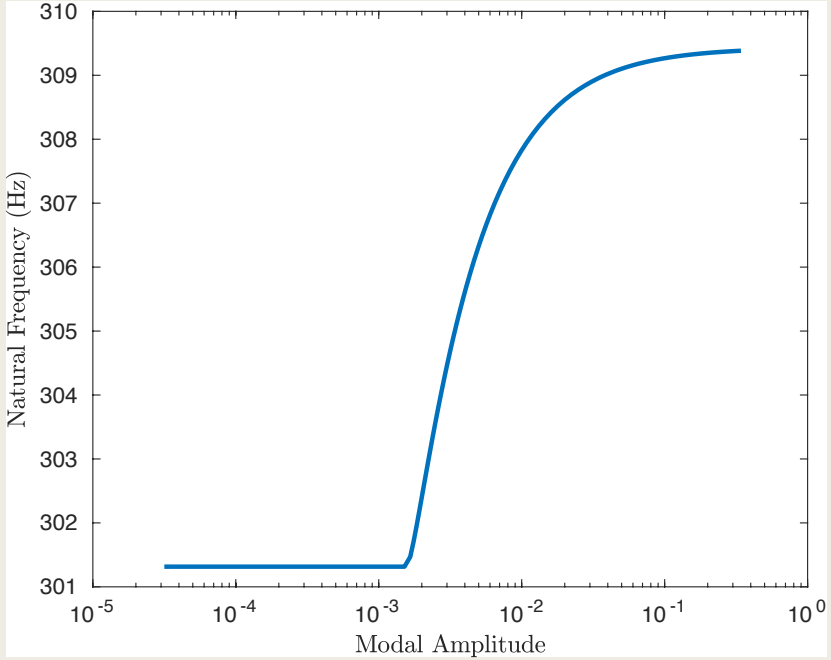


Mode III

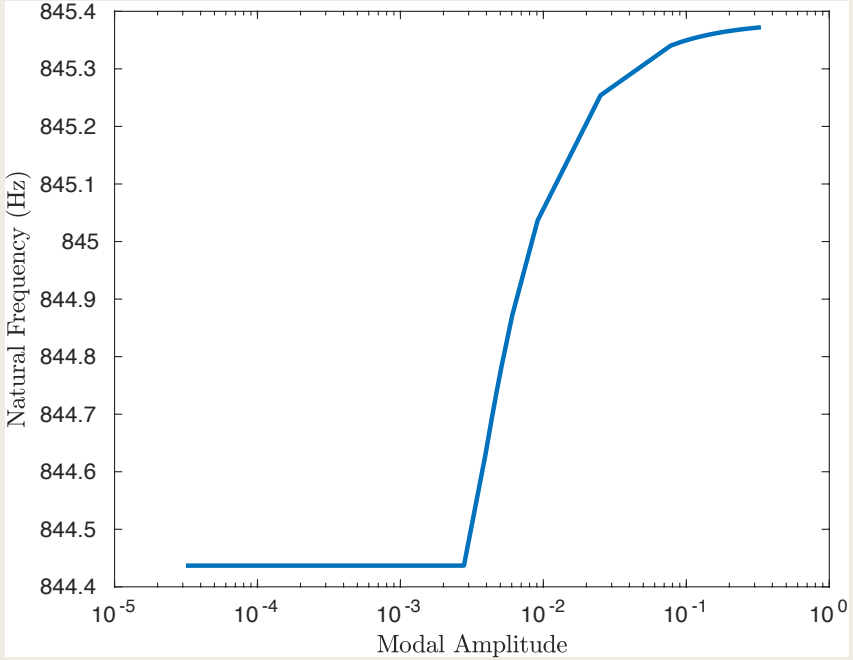
Benchmark Model V: Cantilever beam with unilateral spring element



Mode I



Mode II

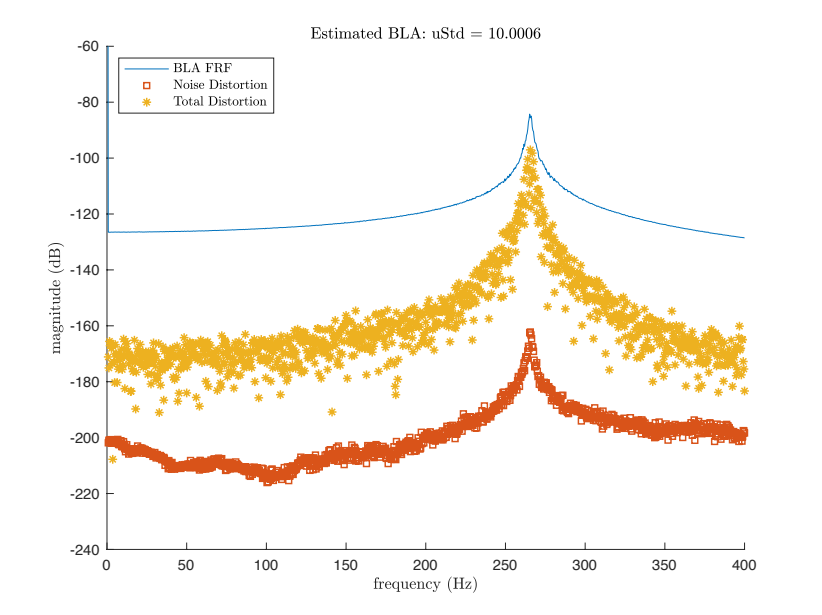


Mode III

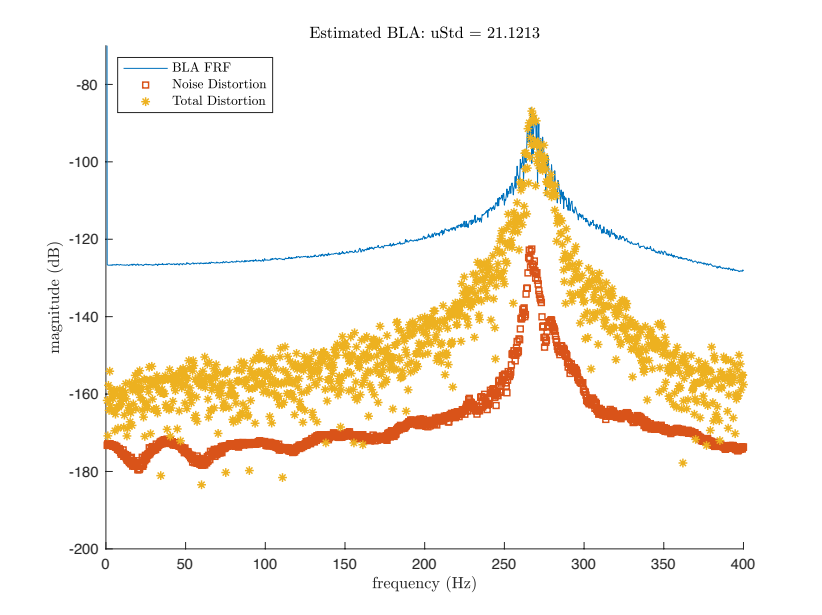
II. Consistency of PNLSS Model Identification

- Set up ODE45 multi-sine generation codes for the input and output data in time series
- Set up PNLSS model analysis codes

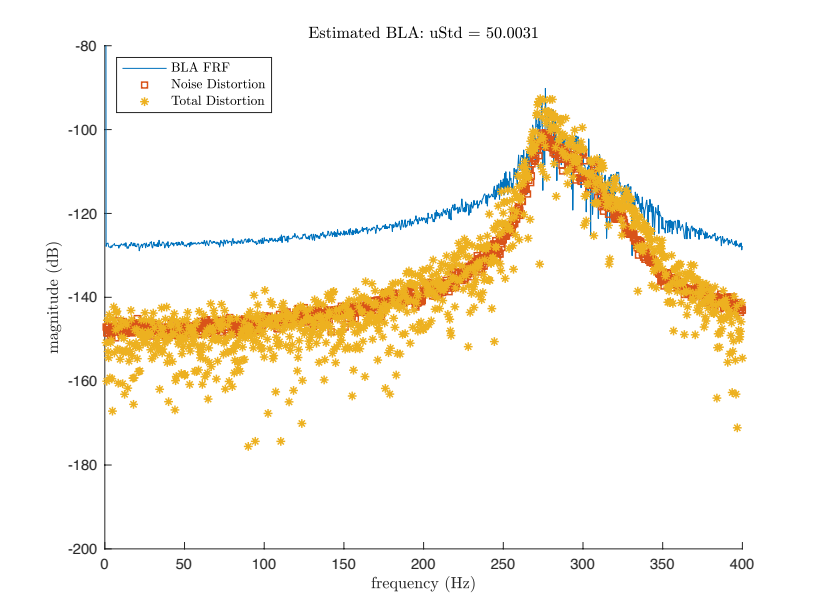
Benchmark Model I: Flat clamped-clamped beam model (Duffing Oscillator)



Amplitude = 10

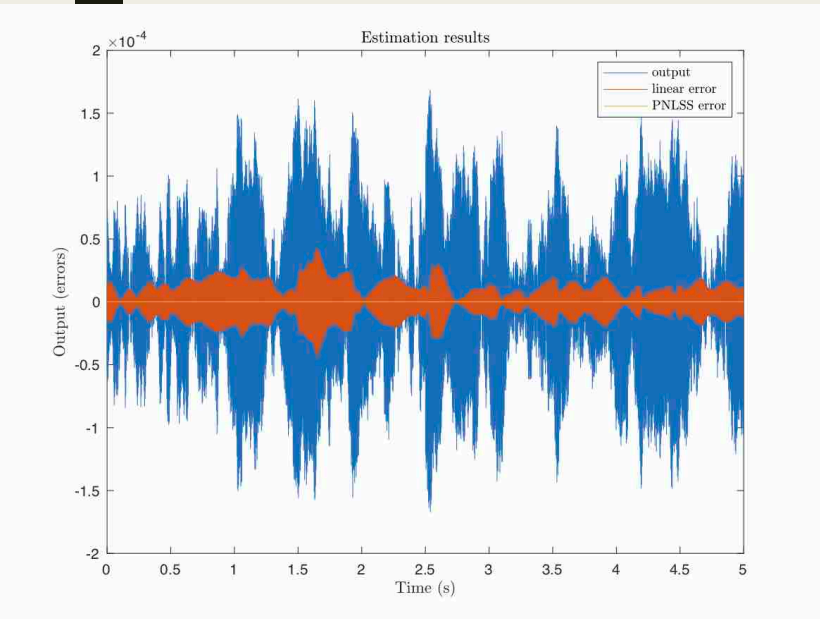


Amplitude = 20

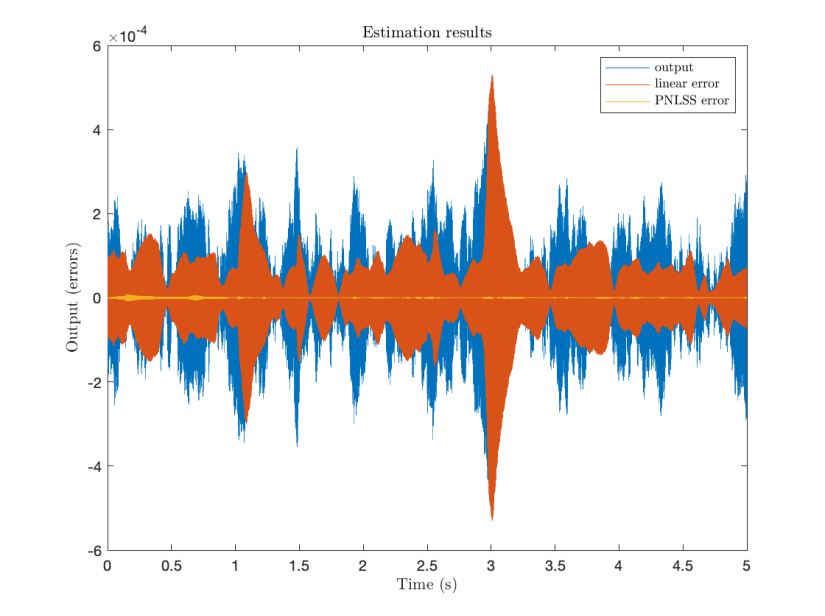


Amplitude = 50

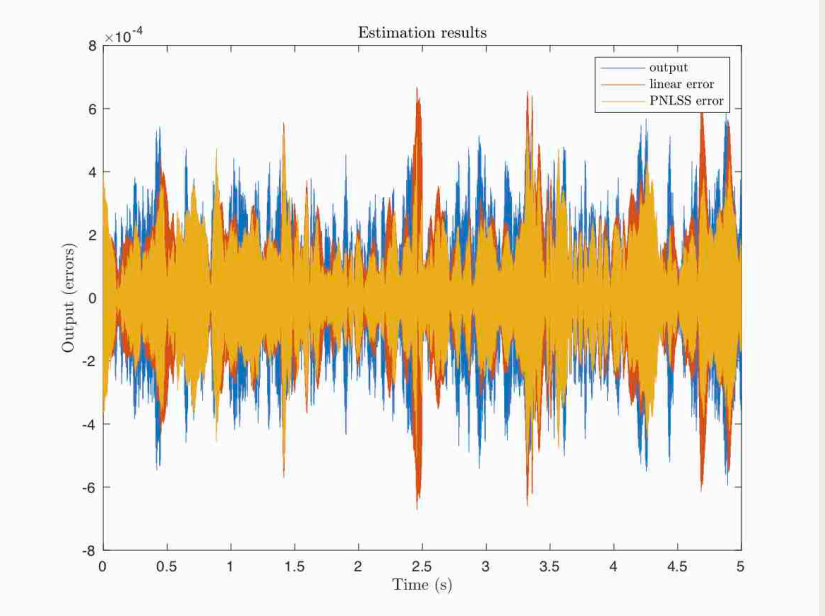
Benchmark Model I: Flat clamped-clamped beam model (Duffing Oscillator)



Amplitude = 10



Amplitude = 20



Amplitude = 50

Comparing The Estimated Discrete-Time Model From PNLSS With The Original Continuous-Time Model

Project 2 WP2: TRC 2019

Meeting 1

Fri Jun 28, 2019

Linear Terms I

- PNLSS is implemented in discrete time and hence, finding physical meanings for the obtained coefficients is not trivial
- For the linear parts of the system, the conversion may be carried out using the state transition matrix

Continuous Time Model

$$\dot{X} = \mathbf{A}X + \mathbf{B}u$$
$$Y = \mathbf{C}X + \mathbf{D}u$$

Discrete Time Model

$$\dot{X}_{k+1} = \mathbf{A_d}X_k + \mathbf{B_d}u_k$$
$$Y_k = \mathbf{C_d}X_k + \mathbf{D_d}u_k$$

Conversion Formulae:

$$\mathbf{A_d} = e^{\mathbf{A}\Delta t} \qquad \mathbf{B_d} = \mathbf{A}^{-1}\left(\mathbf{A_d} - \mathbf{I}\right)\mathbf{B} \qquad \mathbf{C_d} = \mathbf{C} \qquad \mathbf{D_d} = \mathbf{D}$$

$$\mathbf{A} = \log\left(\mathbf{A}\right) f_{samp} \qquad \mathbf{B} = \left(\mathbf{A_d} - \mathbf{I}\right)^{-1}\mathbf{A}\mathbf{B_d}\mathbf{B} \qquad \mathbf{C} = \mathbf{C_d} \qquad \mathbf{D} = \mathbf{D_d}$$

$$f_{samp} = \frac{1}{\Delta t}$$

Linear Terms II

- The model identified from PNLSS need not have states identical to the physical ones used for simulations (displacements, velocities).
- So these matrices must be transformed to a canonical form so that the coefficients may directly be compared
- The physical transformation based on (Etienne Gourc, JP Noel, et.al "Obtaining Nonlinear Frequency Responses from Broadband Testing" `https://orbi.uliege.be/bitstream/2268/190671/1/294_gou.pdf`)

$$\text{Original Model} \begin{bmatrix} 0 & 1 \\ -2.7665 \times 10^6 & -12.6409 \end{bmatrix}, \begin{bmatrix} 0 \\ 1.3195 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}$$

$$\text{PNLSS:} F_{RMS} = 15 \begin{bmatrix} 0 & 1 \\ -2.7676 \times 10^6 & -12.7202 \end{bmatrix}, \begin{bmatrix} 2.0402 \times 10^{-4} \\ 1.2888 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}$$

$$\text{PNLSS:} F_{RMS} = 150 \begin{bmatrix} 0 & 1 \\ -2.7676 \times 10^6 & -12.7251 \end{bmatrix}, \begin{bmatrix} 2.0394 \times 10^{-4} \\ 1.2886 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}$$

Non-Linear Terms

- Things are a little more complicated for the nonlinear terms
- We're unable to find a proper way to transform the discrete time coefficient matrix (**E** in the code) to a continuous time equivalent.
- Maybe integrals are involved? Since we have time history data and also the nonlinearities are smooth, we may be able to evaluate the integrals accurately.
- We have tried to convert the coefficients matrices to the physical domain (as in the previous slide), but are unable to proceed further in order to truly compare the coefficients.

Summary:

- Definition of benchmark models:
  - *The higher the mode, the less nonlinear effects on the response*
  - *The higher the mode, the less the no. of harmonics needed*
  - *Methods on selecting no. of harmonics:*
    - Run the FRF simulation on the benchmark model with different number of harmonics considered in the HBM methods (1~39).
    - The single harmonic excitation force level is chosen to be the highest possible forcing level just before the system showing the modal interaction effect (or tongue on the FRF plots).
    - Assuming 39 harmonics in the HBM is sufficient enough to produce accurate solution for models with few number of DOFs.
    - Compute the peak value of the FRF result and value where $\mathrm{phase} = \pm 90°$ for each simulation. Comparing the values to the ones computed from HBM with 39 harmonic. Calculate the relative error and chose the number of harmonics which is below 0.1%.
- Consistency of PNLSS Model Identification:
  - *As the amplitude of the multi-sine signal increases, the output data generated from the ODE45 codes becomes less periodic. This leads to the failure on the PNLSS model identification. Need to investigate more on this issue.*

To Do List for Benchmark Identification:

- Plot deflection shape for each mode on different points on the backbone curve
- Plotting mode characteristic to physical amplitude to check the negative damping factor for benchmark model III