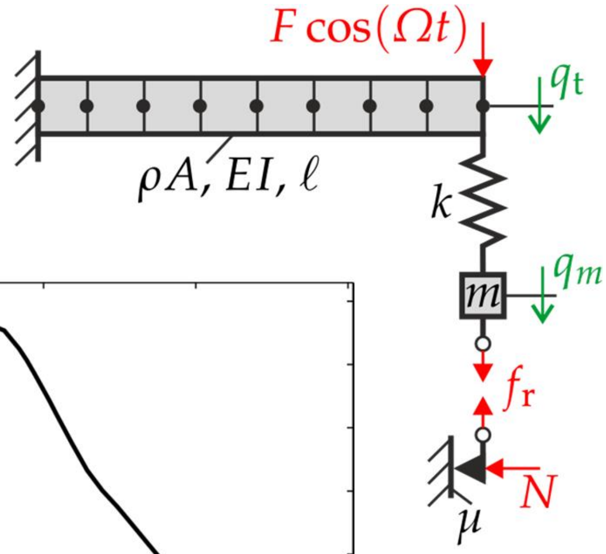
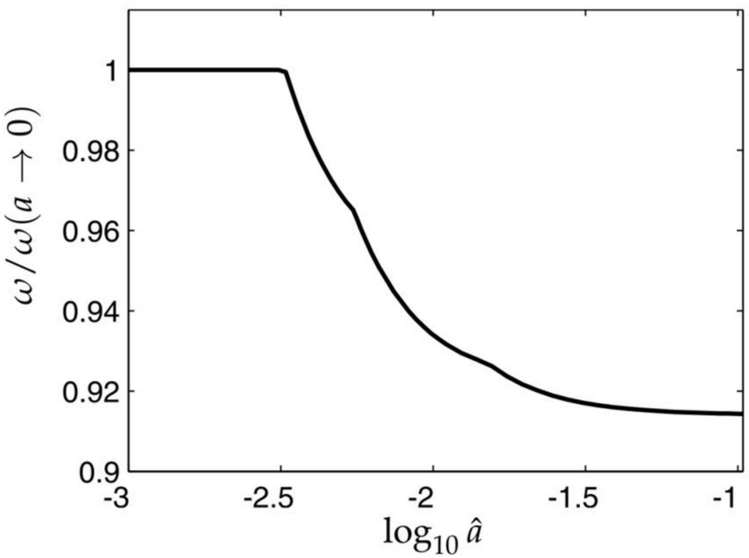
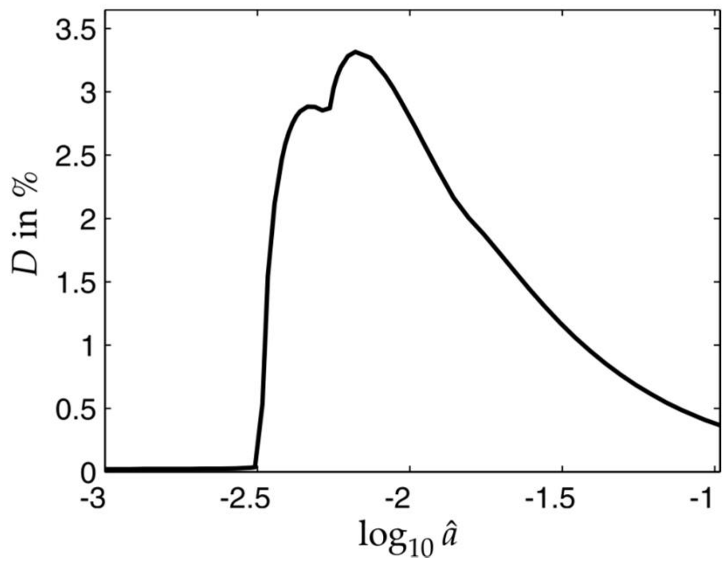
Example: Friction-damped system



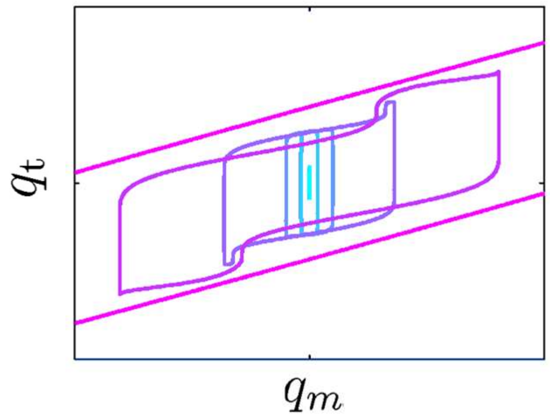
Modal characteristics of mode 1





The mode shape undergoes dramatic local changes.

The coordinates have a phase lag ('complex mode').

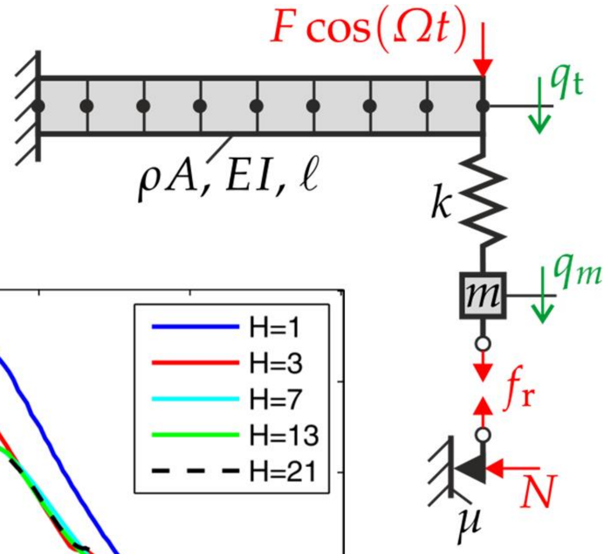


$$\rho A = 235, EI = 4.625 \cdot 10^6, \ell = 2, k = 3\frac{EI}{\ell^3},$$

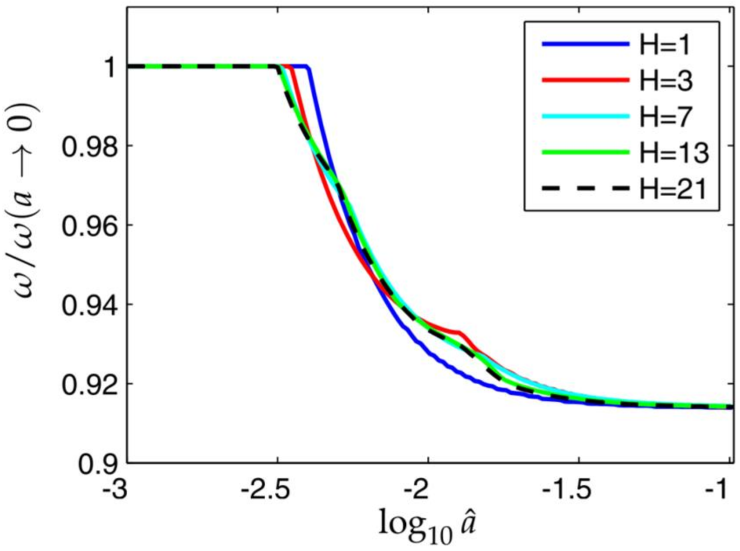
$$m = 0.001 m_{\mathrm{beam}}, \mu N = 100, \varepsilon = 10^{-4}$$

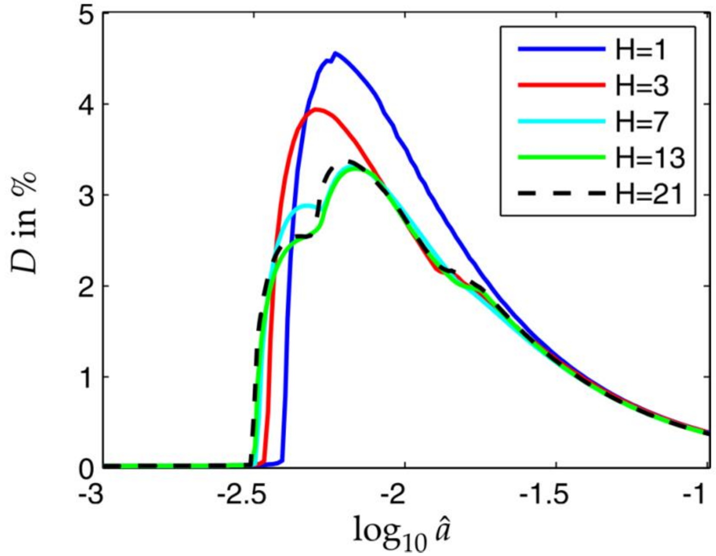
$$f_r = \mu N \tanh \frac{\dot{q}_m}{\varepsilon}$$

Example: Friction-damped system



Modal characteristics of mode 1

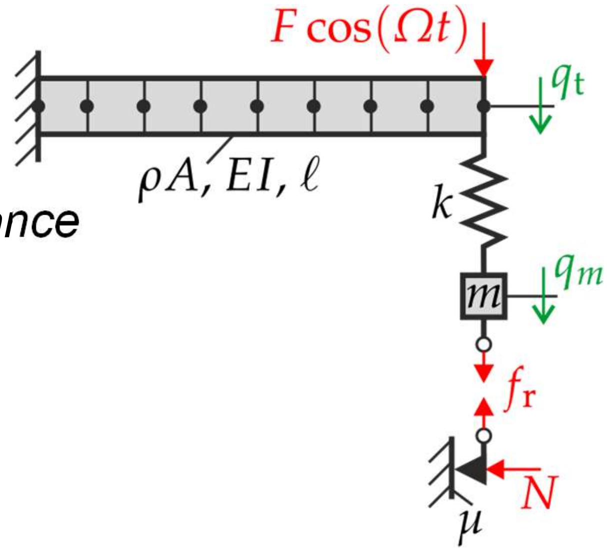




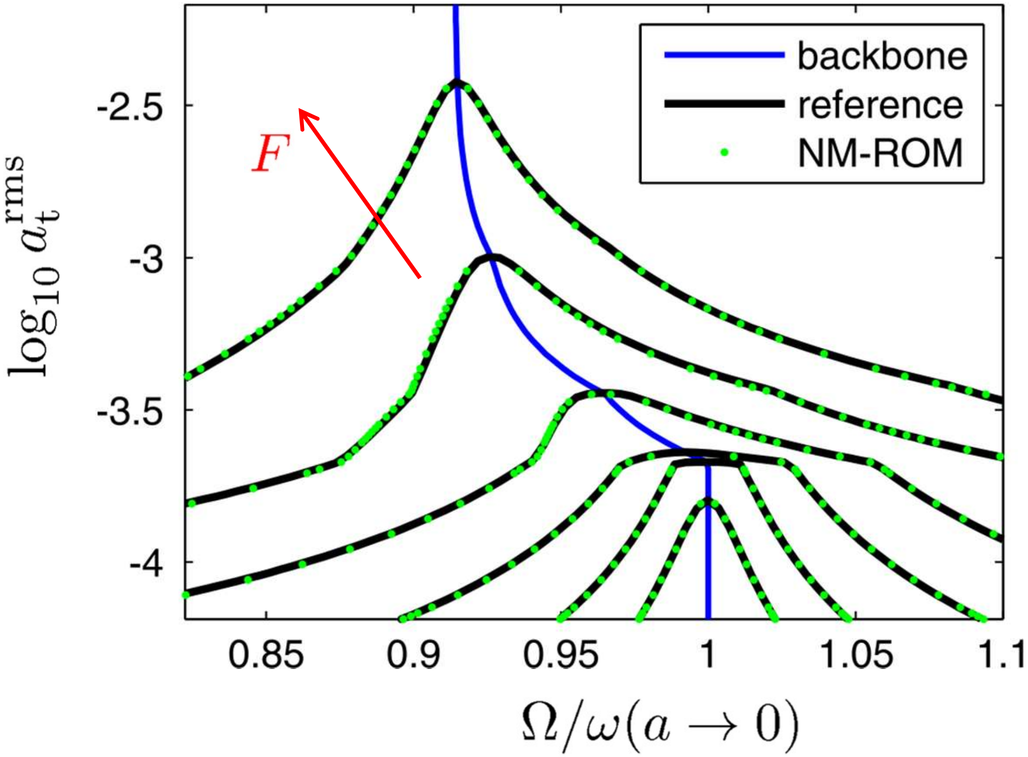
Several harmonics are needed to accurately capture the dynamics of the nonlinear mode.

$$\rho A = 235, EI = 4.625 \cdot 10^6, \ell = 2, k = 3\frac{EI}{\ell^3},$$
$$m = 0.001 m_{\mathrm{beam}}, \mu N = 100, \varepsilon = 10^{-4}$$
$$f_{\mathrm{r}} = \mu N \tanh\frac{\dot{q}_m}{\varepsilon}$$

Example: Friction-damped system



Frequency response near the 1st mode's primary resonance



A closed-form solution can be derived for the modal reduction (NM-ROM):

$$\Omega_{1,2}^2(a) = p_2 \pm \sqrt{p_2^2 - \omega^4 + \frac{\left|\psi_1^{\mathrm{H}} \boldsymbol{F}_{\mathrm{ex},1}\right|^2}{a^2}} \quad \text{with} \quad p_2 = \omega^2 - \frac{(2D\omega + \delta_D)^2}{2}$$

(exists if radicand >0)