TRC 2019 Project 2 WP 2 Meeting 4

Eve, Nidish, Paw

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Benchmark 0 (SDOF Linear Harmonic Oscillator) I

Comparison of frequency responses around resonance

- Linear system simulated for different amplitudes used to train PNLSS model (with non-linear order fixed to 3)
- The linear part of identified model is compared with the original model using the analytical transfer function
- The identified model is also investigated to gain insight into issues related to over-fitting
- Matches are perfect

Benchmark 0 (SDOF Linear Harmonic Oscillator) II

Comparison of frequency responses around resonance

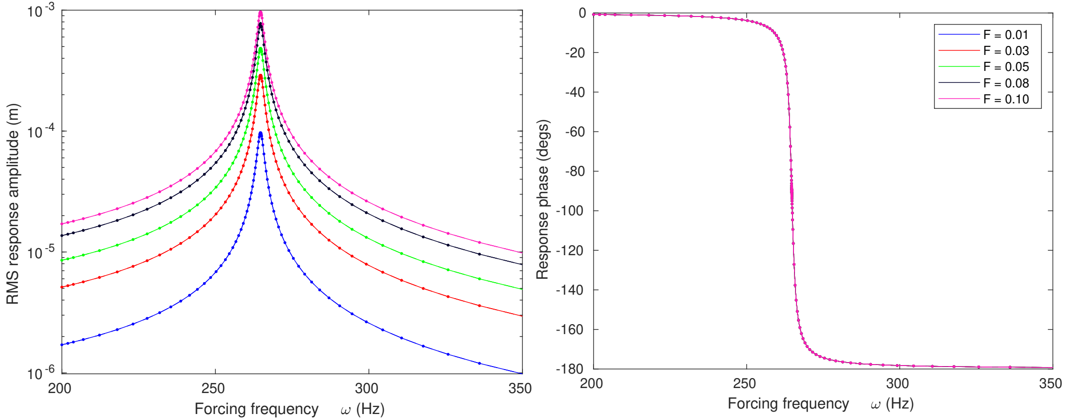
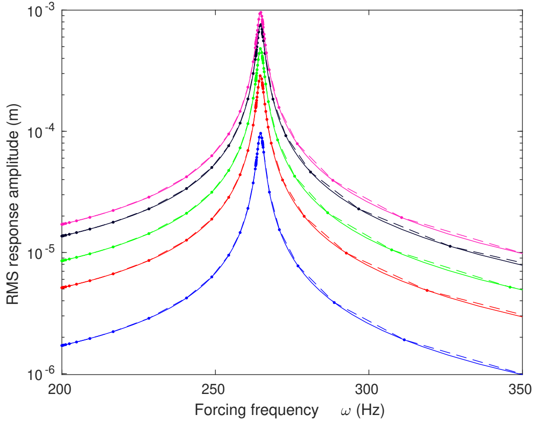


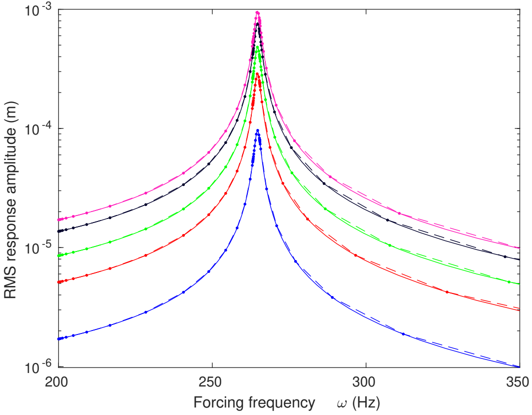
Figure: Comparison of analytical FRFs of linear parts (cont. from HB; dotted from identified model). Model from data trained at 0.01N rms multisine (150Hz, 400Hz). No amplitude dependence observed (as expected)

Benchmark 0 (SDOF Linear Harmonic Oscillator) III

Comparison of frequency responses around resonance



(a) $||u||_2 = 0.01\text{N}$



(b) $||u||_2 = 0.25\text{N}$

Figure: Comparison of simulated FRFs.

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) I

Overview and setup

- There was a serious scaling issue we debugged a couple of days back, which showed us that the amplitudes we were using were unrealistically huge
- This is fixed now and we have results that are more expectable
- We also wanted to compare if both the models estimate the stability regimes identically so we just added corresponding expressions to the post-processing functions for nlvib

Stability exponents for continuous time HB (Hill's method)

- This is obtained by solving the quadratic eigenvalue problem,

$$\mathbf{J}_{\tilde{\mathbf{NL}}} + \tilde{\mathbf{C}}\lambda + \tilde{\mathbf{M}}\lambda^2 = 0. \tag{1}$$

- $\mathbf{J}_{\tilde{\mathbf{NL}}}$ is the non-linear frequency-domain Jacobian; $\tilde{\mathbf{C}}$ & $\tilde{\mathbf{M}}$ are $(eye(2N_h+1)\otimes\mathbf{C})$ & $(eye(2N_h+1)\otimes\mathbf{M})$ respectively
- The first N_d exponents closes to the origin are retained.

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) II

- A solution is stable if $\angle\lambda > 90^\circ - 1^\circ$

Stability exponents for discrete time HB

- Starting from the same idea as Hill's method, the perturbed solution is taken as

$$X_n = X_n^* + s_n e^{\lambda t_n} \tag{2}$$

- Subtituting this into the EOM,

$$\begin{aligned} X_{n+1} =& \mathbf{A} X_n + \mathbf{B} u_n + \mathbf{E} g(X_n) \\ \implies \qquad X_{n+1}^* + =& \mathbf{A} X_n^* + \mathbf{B} u_n + \mathbf{E} g(X_n^*) \\ s_{n+1} e^{\lambda(t_n + \Delta t)} \qquad & + \left[\mathbf{A} + \frac{\partial}{\partial X} [\mathbf{E} g(X)] \Big|_{X_n^*} \right] s_n e^{\lambda t_n} \end{aligned} \tag{3}$$

- Applying force balance and using Fourier Ansatz,

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) III

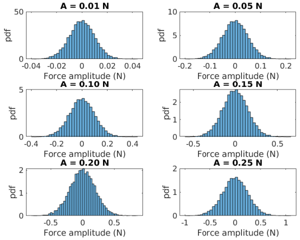
Overview and setup

$$\begin{bmatrix}1 & cos(i\omega t_n) & sin(i\omega t_n)\end{bmatrix} \left(\begin{bmatrix}\mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{cos(i\omega\Delta t)} & \boldsymbol{sin(i\omega\Delta t)} \\ \mathbf{0} & \boldsymbol{-sin(i\omega\Delta t)} & \boldsymbol{cos(i\omega\Delta t)}\end{bmatrix} e^{\lambda\Delta t} - \begin{bmatrix}\boldsymbol{A_{nl}} & & \mathbf{0} \\ & \boldsymbol{A_{nl}} & \\ \mathbf{0} & & \boldsymbol{A_{nl}}\end{bmatrix} \right) \begin{Bmatrix} S_{a^0} \\ S_{a^i} \\ S_{b^i} \end{Bmatrix} = \{0\} \tag{4}$$

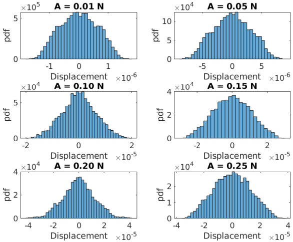
- The exponents λ are obtained as $log(< eigs >) \times fs$.
- Once again, only the first d λ's are chosen which are closest to the origin and interpreted identically to Hill's exponents.

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) I

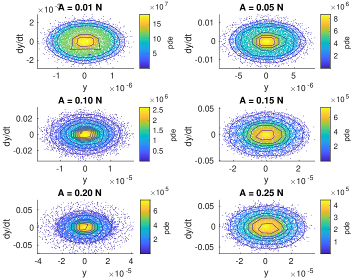
Looking at the signals & responses



(a) Excitation



(b) Response

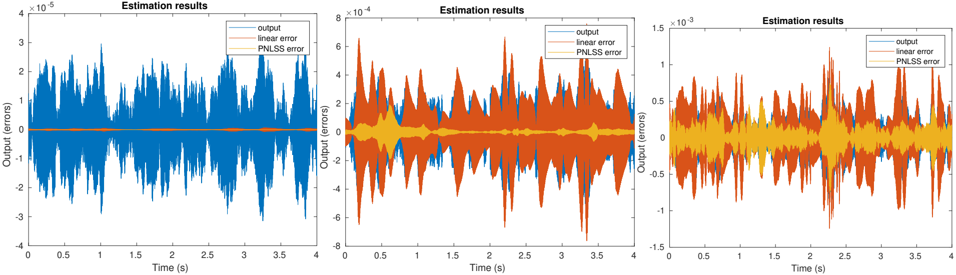


(c) State-plane Response

- In the following we plot the responses of the continuous time model with stable-continuous lines and unstable-dashed lines
- And, the responses of the discrete time model with stable-dotted lines and unstable-crossed lines

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) I

Comparison of frequency responses around resonance

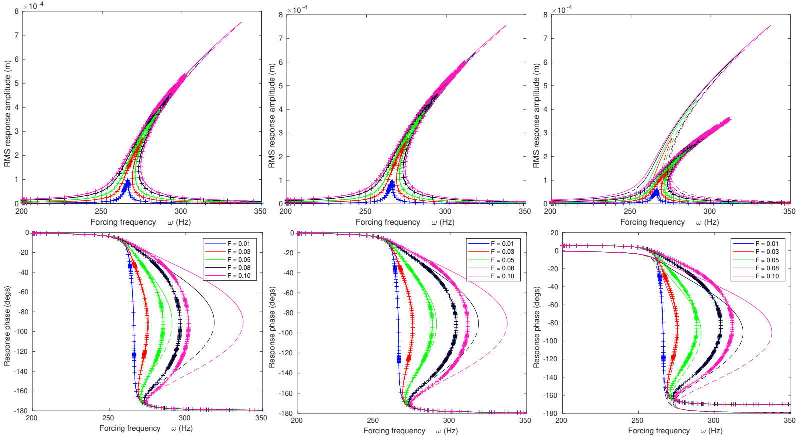


(a) $u_{rms} = 0.01N$ (b) $u_{rms} = 0.15N$ (c) $u_{rms} = 0.25N$ (non-periodic)

Figure: Time-domain residues ($n_x = [3]$)

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) II

Comparison of frequency responses around resonance



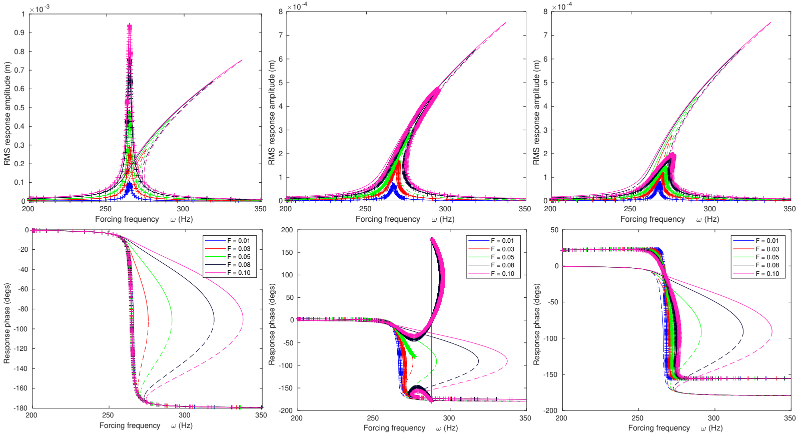
(a) $u_{rms} = 0.01N$ (b) $u_{rms} = 0.15N$ (c) $u_{rms} = 0.25N$ (non-periodic response)

Figure: $n_x = [3]$

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) III

Comparison of frequency responses around resonance

Sufficiency of non-linear terms



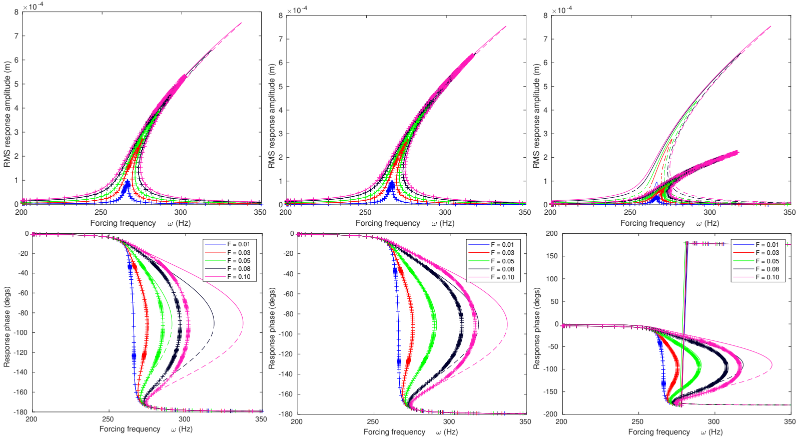
(a) $u_{rms} = 0.01N$ (b) $u_{rms} = 0.15N$ (c) $u_{rms} = 0.25N$ (non-periodic response)

Figure: $n_x = [2]$

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) IV

Comparison of frequency responses around resonance

Sufficiency of non-linear terms



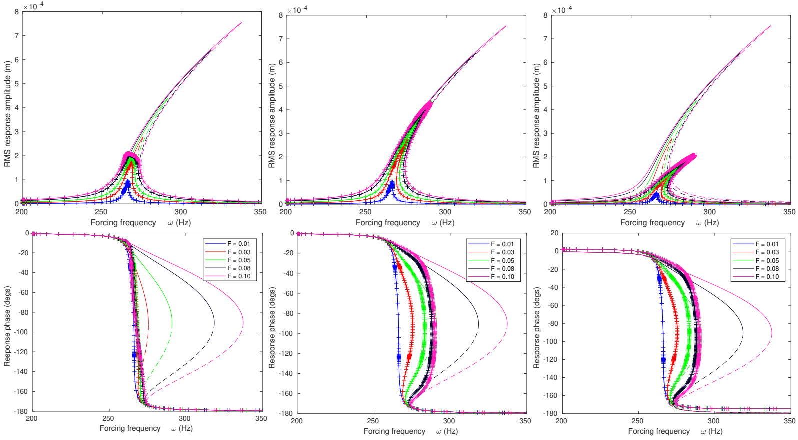
(a) $u_{rms} = 0.01N$ (b) $u_{rms} = 0.15N$ (c) $u_{rms} = 0.25N$ (non-periodic response)

Figure: $n_x = [2, 3]$

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) V

Comparison of frequency responses around resonance

Sufficiency of non-linear terms



(a) $u_{rms} = 0.01N$

(b) $u_{rms} = 0.15N$

(c) $u_{rms} = 0.25N$ (non-periodic response)

Figure: $n_x = [3, 5]$

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) VI

Comparison of frequency responses around resonance

- The continuous time model stability estimates are extremely unreliable
- Possibly something wrong with my implementation - any pointers?

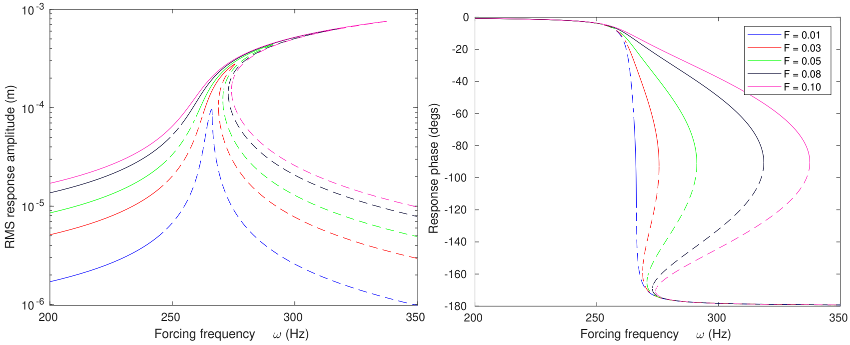


Figure: Frequency response of continuous time model

Benchmark 1 (SDOF Nonlinear Beam (Duffing oscillator)) VII

Comparison of frequency responses around resonance

- The discrete time model stability estimates are even more unreliable - shows that no solution is stable!
- Something definitely wrong with my implementation - pointers?

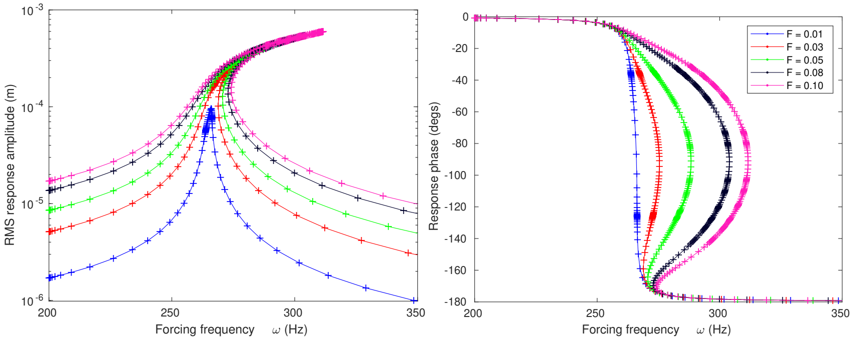


Figure: Frequency response of continuous time model

Queries

- Dealing with non-periodic data (periodic input leading to non-periodic responses).
  - How to go about transient handling? Will it be good to try and fit transients too?
  - Since for non-periodic responses (say, quasi-periodic) the starting phase (initial conditions) are very crucial, do we use the corresponding routine? We had difficulty in getting this to work very well..
- Stability exponents in the frequency domain.
  - Tips on interpreting hill's coefficients (threshold complex angle for instability, etc)
  - Tips on the exponents for the discrete time models

Steps forward

1. Conducting EPMC backbone estimates for the imperfect model (with shaker)
2. Conducting the PNLSS studies for the MDOF benchmarks
3. Conducting PNLSS with imperfect model (shaker force as input; response as output)
4. Comparing stability