

Agenda

- ▶ General pnlss settings
- ▶ Selecting multisine amplitude, ex: benchmark 4
- ▶ Validate PNLSS model using NFRC obtained by NLvib
- ▶ Use multiple amplitude data for estimation
- ▶ Can we trust NFRC obtained from discrete state space model?

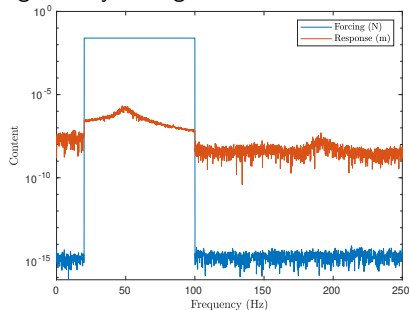
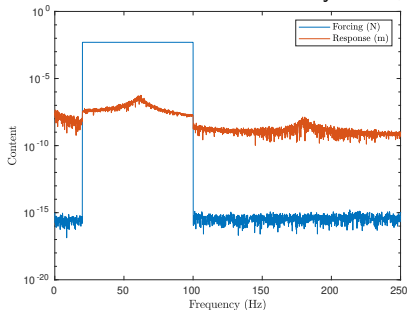
general PNLSS settings around 1 mode

```
% benchmark 1-3
n = 2;
whichtermsx = 'statesonly';
whichtermsy = 'empty';
nx = [3]      % benchmark 1/2
nx = [2,3]    % benchmark 3
% Benchmark 4
n = 3;
whichtermsx = 'full';
whichtermsy = 'full';
nx = [2,3];
ny = [2,3];
% benchmark 5
n = 2;
whichtermsx = 'full';
whichtermsy = 'empty';
nx = [2,3];
```

Selecting MS amplitude - benchmark 4 example

- ▶ Simulate with different amplitudes
- ▶ Inspect FRF and select amplitudes corresponding to fully stuck or fully sliding
- ▶ The expected frequencies are found from linear modal analysis of the two extremes.

Left: fully stuck; right: fully sliding

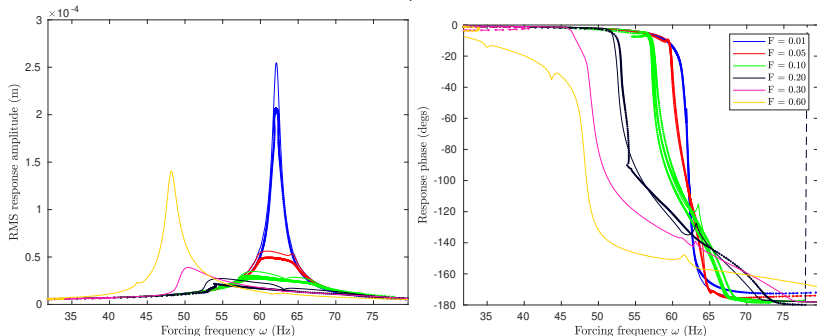


Comparing NFRC: NLvib vs PNLSS

PNLSS estimated at $A = 0.1$ (low amplitude)

- ▶ Match well for lower amplitude (but damping overestimated)
- ▶ For increasing amplitude, the model 'blow up'
- ▶ As expected. Model is only valid within the range it was estimated

full line: PNLSS; thin line: NLvib

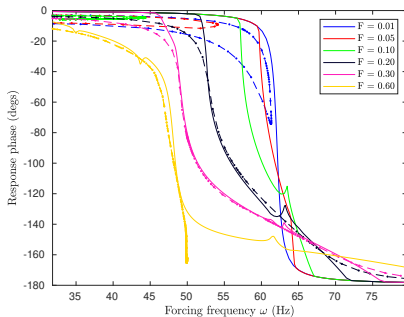
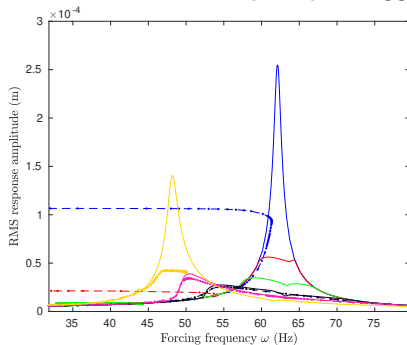


Comparing NFRC: NLvib vs PNLSS

PNLSS estimated at $A = 0.5$ (high amplitude)

- ▶ Match for forcing resulting in microslip, $F = 0.2$ & $F = 0.3$
- ▶ For all other forcing levels, the behavior is unpredictable

full line: PNLSS; thin line: NLvib

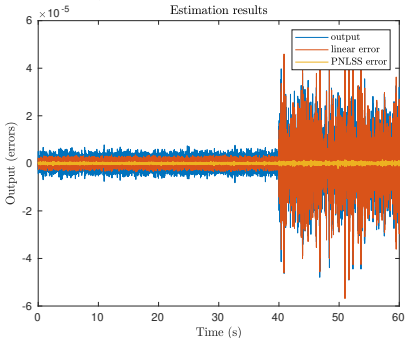
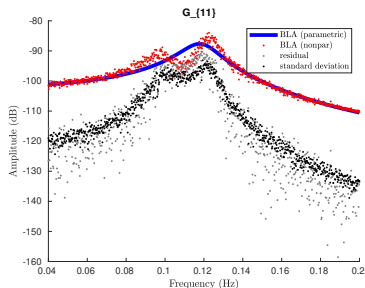


Estimate model from combined data

Combine low and high amplitude data for estimating

- ▶ Linear model gives a poor fit.
- ▶ Keep 3 state model. Misfit to BLA is due to nonlinearity.
- ▶ Nonlinear model seems to perform well on this particular data

left: Subspace; right: Estimation, time domain

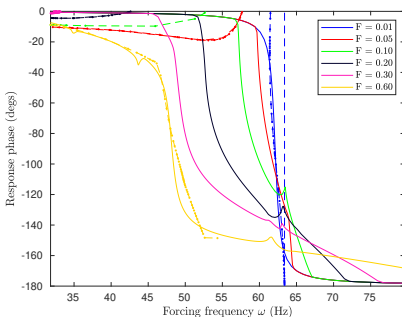
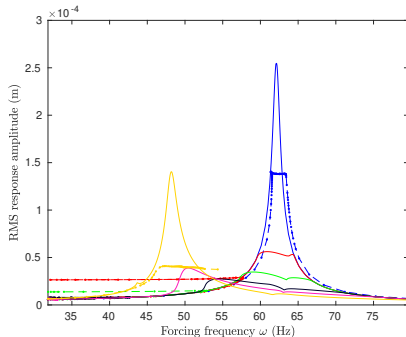


Comparing NFRC: NLvib vs PNLSS

Combine low and high amplitude data for estimating

- Poor performance for all excitation levels

full line: PNLSS; thin line: NLvib

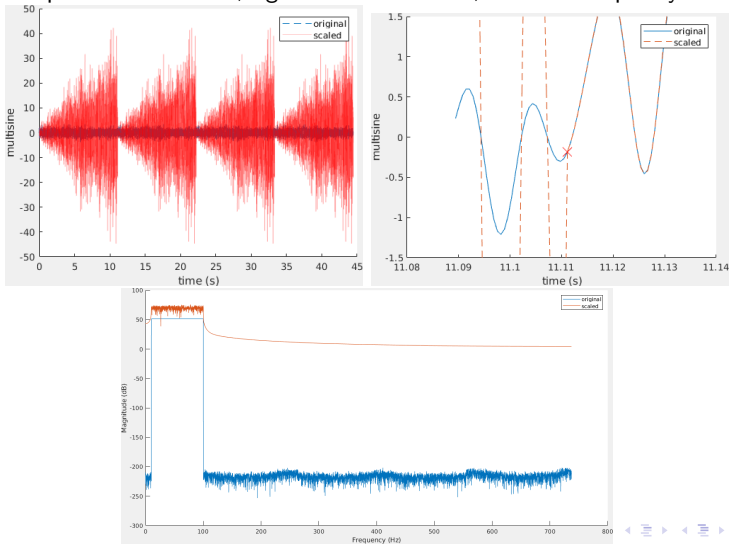


Alternative data covering all amplitudes

Let amplitude vary linearly from low to high - Arrowhead.

- ▶ But frequency content destroyed when signal is not periodic
- ▶ Thus (we assume) not suitable for training. Maybe for model selecting.

Left: repeated arrow-head; right: zoom at $t = T$; bottom: frequency content



Can we trust NFRC and phase from discrete time PNLSS?

We evaluate PNLSS performance based on NFRC and phase plot. But can we trust these plots?

Next slides show NFRC and phases computed from discrete time state space model for the duffing oscillator. The discretisation is done at different fs

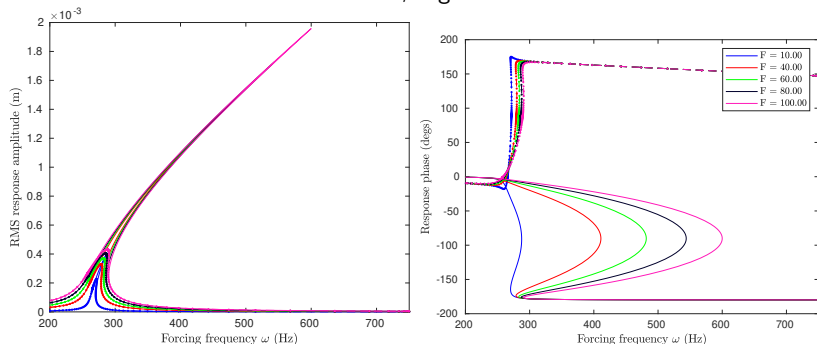
The same behavior is seen for a linear ss model, where the conversion at a given fs is exact; ie. it can not only be attributed to the euler discretization of the nonlinearity.

Thin line is 'analytical model', dotted is discrete time state space.

- ▶ poor agreement

$$fs = 2^{12} = 4096Hz$$

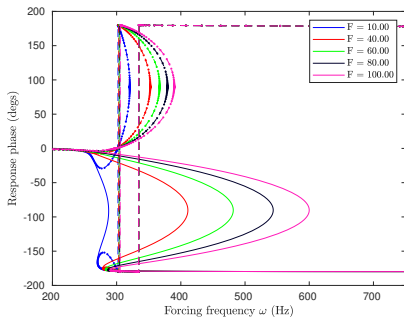
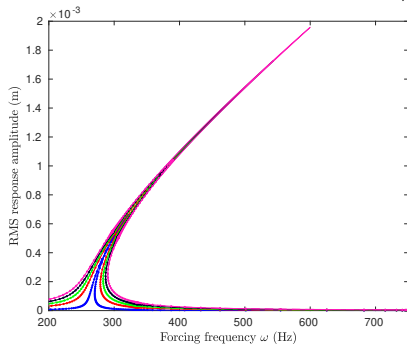
Left: NFRC; Right: Phase



$$f_s = 2^{16} = 65536 \text{ Hz}$$

► poor agreement

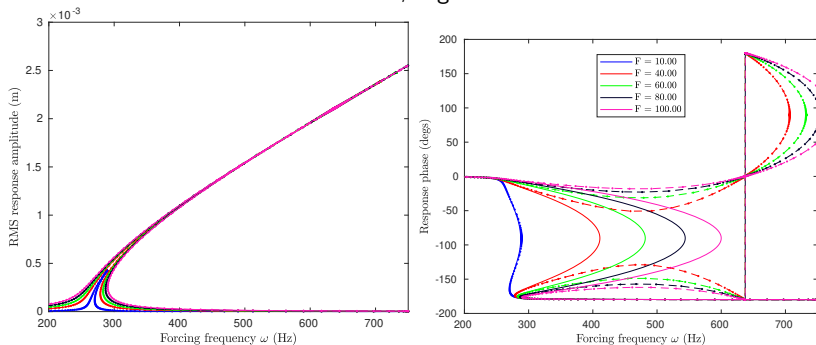
Left: NFRC; Right: Phase



$$f_s = 2^{20} = 1048576 \text{ Hz}$$

- Match for lowest force

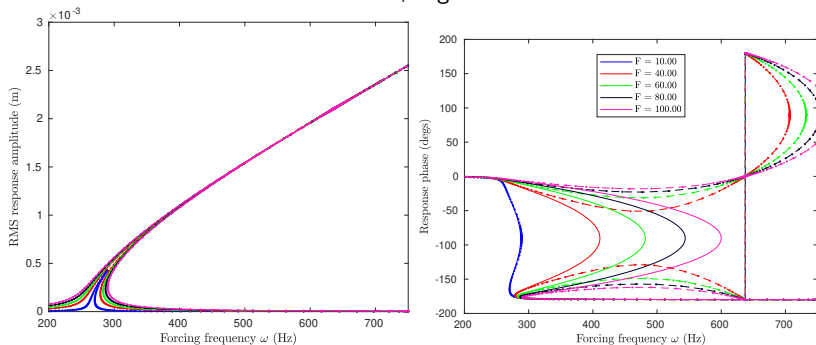
Left: NFRC; Right: Phase



$$f_s = 2^{20} = 1048576 \text{ Hz}$$

- Match for lowest force

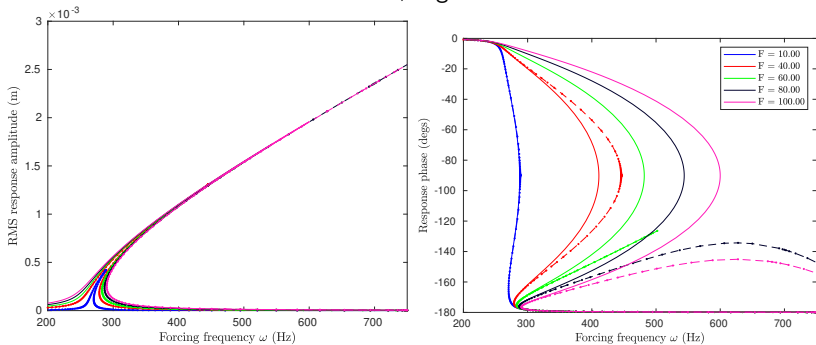
Left: NFRC; Right: Phase



$$f_s = 2^{21} = 2097152 \text{ Hz}$$

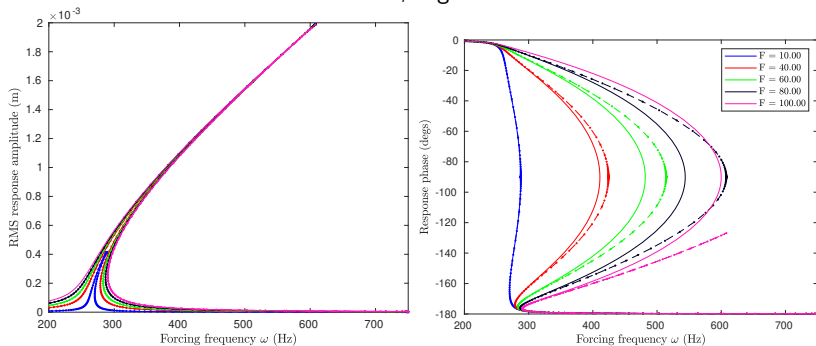
- Starting to get better

Left: NFRC; Right: Phase



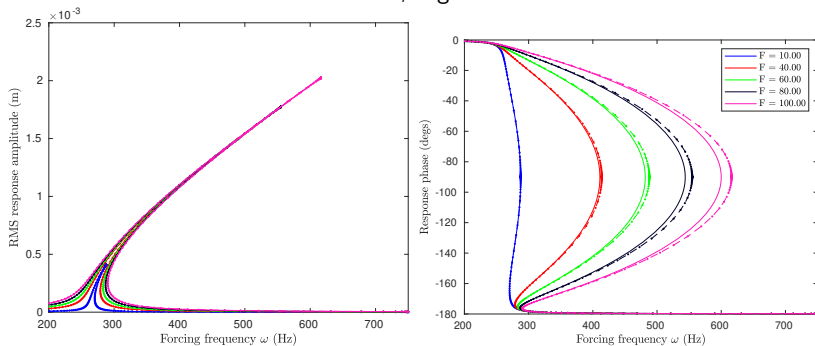
$$f_s = 2^{22} = 4194304 \text{ Hz}$$

Left: NFRC; Right: Phase



$$f_s = 2^{24} = 16777216 \text{ Hz}$$

Left: NFRC; Right: Phase



$$f_s = 2^{28} = 268435456 \text{ Hz}$$

At a sampling frequency of 268 million Hz, there is good agreement.
This raises the question: At which sampling frequency shall we identify the PNLSS model?

Left: NFRC; Right: Phase

