Comparing The Estimated Discrete-Time Model From PNLSS With The Original Continuous-Time Model

Project 2 WP2: TRC 2019

Meeting 1

Fri Jun 28, 2019

Linear Terms I

- PNLSS is implemented in discrete time and hence, finding physical meanings for the obtained coefficients is not trivial
- For the linear parts of the system, the conversion may be carried out using the state transition matrix

Continuous Time Model

$$\dot{X} = \mathbf{A}X + \mathbf{B}u$$
$$Y = \mathbf{C}X + \mathbf{D}u$$

Discrete Time Model

$$\dot{X}_{k+1} = \mathbf{A_d}X_k + \mathbf{B_d}u_k$$
$$Y_k = \mathbf{C_d}X_k + \mathbf{D_d}u_k$$

Conversion Formulae:

$$\mathbf{A_d} = e^{\mathbf{A}\Delta t} \qquad \mathbf{B_d} = \mathbf{A}^{-1}\left(\mathbf{A_d} - \mathbf{I}\right)\mathbf{B} \qquad \mathbf{C_d} = \mathbf{C} \qquad \mathbf{D_d} = \mathbf{D}$$

$$\mathbf{A} = \log\left(\mathbf{A}\right) f_{samp} \qquad \mathbf{B} = \left(\mathbf{A_d} - \mathbf{I}\right)^{-1}\mathbf{A}\mathbf{B_d}\mathbf{B} \qquad \mathbf{C} = \mathbf{C_d} \qquad \mathbf{D} = \mathbf{D_d}$$

$$f_{samp} = \frac{1}{\Delta t}$$

Linear Terms II

- The model identified from PNLSS need not have states identical to the physical ones used for simulations (displacements, velocities).
- So these matrices must be transformed to a canonical form so that the coefficients may directly be compared
- The physical transformation based on (Etienne Gourc, JP Noel, et.al "Obtaining Nonlinear Frequency Responses from Broadband Testing" `https://orbi.uliege.be/bitstream/2268/190671/1/294_gou.pdf`)

$$\text{Original Model} \begin{bmatrix} 0 & 1 \\ -2.7665 \times 10^6 & -12.6409 \end{bmatrix}, \begin{bmatrix} 0 \\ 1.3195 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}$$

$$\text{PNLSS:} F_{RMS} = 15 \begin{bmatrix} 0 & 1 \\ -2.7676 \times 10^6 & -12.7202 \end{bmatrix}, \begin{bmatrix} 2.0402 \times 10^{-4} \\ 1.2888 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}$$

$$\text{PNLSS:} F_{RMS} = 150 \begin{bmatrix} 0 & 1 \\ -2.7676 \times 10^6 & -12.7251 \end{bmatrix}, \begin{bmatrix} 2.0394 \times 10^{-4} \\ 1.2886 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}$$

Non-Linear Terms

- Things are a little more complicated for the nonlinear terms
- We're unable to find a proper way to transform the discrete time coefficient matrix (**E** in the code) to a continuous time equivalent.
- Maybe integrals are involved? Since we have time history data and also the nonlinearities are smooth, we may be able to evaluate the integrals accurately.
- We have tried to convert the coefficients matrices to the physical domain (as in the previous slide), but are unable to proceed further in order to truly compare the coefficients.