**Project 2, WP2 – first steps in Houston**

*Numerical comparison of PNLSS and NM models*

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This document shall help you getting started with the main part of the research project in Houston. We considered both the good progress you have already made during the homework preparations, and collected some specific hints towards the tasks in the initial project description. Your first task will be to read this and to find a suitable way to distribute the work among the different participants.

**Definition of benchmark models** (This task should NOT take more than a few days.)

* Benchmark models in order of increasing complexity
  1. Flat clamped-clamped beam model, truncated to single linear mode (= Duffing oscillator)
  2. Flat clamped-clamped beam model, truncated to Nm>1 modes
  3. Curved clamped-clamped beam model, truncated to Nm>1 modes
  4. Clamped-free beam with elastic dry friction element
  5. Clamped-free beam with unilateral spring element
* For each, specify the target frequency range. It should include the first 1-3 lowest natural frequencies of the underlying linear model.
* Specify representative (~5-10) levels of harmonic excitation. These should lead to responses well-spaced along the backbones.
* Benchmarks 4+5: Specify gap + stiffness of unilateral spring, limit friction force + stiffness of friction element. Specify linear viscous damping such that all modes of the underlying linear model have equal damping ratio. In these benchmarks, we want to avoid strongly nonlinear modal interactions (at least at first), causing turning points in the backbone and secondary peaks in the frequency response. To this end, the stiffness of the nonlinear elements should not be too large, and the linear damping ratio should be sufficiently high.
* Note: The directly simulated frequency responses are needed as reference.

**Consistency of PNLSS Model Identification**

Note that benchmarks 1-3 can be exactly represented by a PNLSS model. Hence, for properly identified coefficient matrices, the deviation between the input data and simulation data of the PNLSS model should be (numerically) exactly zero. This is ideal, because it allows you to tune the parameters of the identification algorithm. A properly tuned algorithm should be able to robustly identify the coefficient matrices of the linear and nonlinear terms. Ensure robustness with respect to the excitation level, perhaps also (sensor/exciter) noise (which should not be an issue with this algorithm).

The exact representation implies that you can construct directly a PNLSS model by casting the equation of motion into (time-discrete) state space. At least for the Duffing oscillator, it might then be reasonable to compare also the coefficient matrices of this manually constructed model to the identified one. However, a coordinate transform might be needed to recover the same coefficient matrices.

**Consistency of NMs and Frequency Response**

* Run a numerical nonlinear modal analysis using NLvib’s ‘NMA’ (EPMC, i.e. with mass-proportional negative viscous damping term).
* Synthesize the Frequency Response and compare with the direct computations (reference). For response ranges, where strong nonlinear modal interactions are absent, the agreement should be very good. Far away from a resonance, the residual compliance of the off-resonant modes (neglected in the single-NM model) becomes important, so that the NM model will not be accurate either. Here, the response is usually relatively low so that it is helpful to use logarithmic scaling of the amplitude axis to see the error. Consider 2 variants to handle the linear viscous damping: (a) consider directly in NMA, (b) remove during NMA and superimpose in the synthesis.

**Effect of imperfect excitation on NMs**

* Simulate Phase Resonance Tests with pure harmonic excitation (w/o shaker + stinger model). Compare identified NM with reference.
* Add shaker + stinger model. Compare again.
* Add noise. Compare again.
* Note: You might have to tune the controller parameters to achieve robust behavior (with marginal invasiveness). On this subject, discuss also with the WP1 group.

**Extension of the PNLSS model for friction and unilateral contacts**

* Augment polynomial nonlinear terms by appropriate non-polynomial ones. Note that one could introduce the same nonlinearity that is used in the benchmark model. In this case, the deviation between input data and simulation data of the identified model should go exactly to zero (if the numerical algorithm is sufficiently robust to deal with the more severe nonlinearities). In reality, however, one does not have the luxury of knowing exactly the type of nonlinearity. It is therefore interesting to use a (slightly) different model (e.g. regularized instead of non-smooth friction law; or even a more generic form, e.g. pseudo-polynomials), thus deliberately introducing a deterministic error, in order to analyze the performance of the method in the presence of epistemic uncertainty.
* Analyze if the additional terms reduce the model error and decrease the sensitivity with respect to the input level.

**Comparison of PNLSS and NM model identification**

* Analyze the ability of the identified models to accurately predict the frequency response for different excitation levels.
* Assess results. Develop hypotheses regarding strengths and limitations.
* Run additional simulations to falsify/verify these hypotheses (e.g. vary noise level or intensity of nonlinearity or load level or …).

**IDEAS FOR WORK DISTRIBUTION**

As mentioned above, we would like you to make a suggestion on how you would like to distribute the work among the participants. This way, you have the chance to combine your individual interests and strengths.

One strategy to distribute the work is to let different people work on different benchmarks. Another one is to work on different methods (PNLSS vs. NM).