hw03

Victor Huang

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Q1. (Exercise 6.4)

$f(x;\theta) = \frac{x^3 e^{\frac{-x}{\theta}}}{6\theta^4}$ for $x \geq 0$, find maximum likelihood estimate of θ

$$L(\theta) = \Pi_{i=1}^{n} f(x;\theta)$$

$$L(\theta) = \Pi_{i=1}^{n} \frac{x^3 e^{\frac{-x}{\theta}}}{6\theta^4}$$

$$L(\theta) = \frac{(\Pi x)^3 e^{\frac{-\sum x_i}{\theta}}}{6^n \theta^{4n}}$$

We can log both sides and set derivative equal to 0 to get MLE

$$log(L(\theta)) = 3log(\Pi x_i) - \frac{\sum x_i}{\theta} - nlog(6) - 4nlog(\theta)$$

$$\frac{d}{d\theta} log(L(\theta)) = 0$$

$$0 + \frac{\sum_{i=1}^{n} x_i}{\theta^2} - 0 - \frac{4n}{\theta} = 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta^2} = \frac{4n}{\theta}$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = 4n$$

$$\sum_{i=1}^{n} x_i = 4n\theta$$

$$\hat{\theta} = \frac{1}{4n} \sum_{i=1}^{n} x_i$$

$$\hat{\theta} = \frac{1}{4n} \bar{x}$$

check

$$l''(\theta) = -2\frac{(4n)^3}{(\sum_{i=1}^{n} x_i)^2} + \frac{(4n)^3}{(\sum_{i=1}^{n} x_i)^2} = -\frac{(4n)^3}{(\sum_{i=1}^{n} x_i)^2} < 0$$

Q2. (Exercise 6.6)

a). $f(x;\theta) = e^{\theta - x}$ for $x > \theta > 0$. Find the MLE of θ

$$L(\theta) = \Pi_{i=1}^{n} f(x;\theta)$$

$$L(\theta) = \Pi_{i=1}^{n} e^{\theta - x}$$

$$L(\theta) = \Pi_{i=1}^{n} e^{-(x-\theta)}$$

$$L(\theta) = e^{-\sum_{i=1}^{n}(x_i - \theta)}$$

$$L(\theta) = e^{-\sum_{i=1}^{n} x_i} e^{n\theta}$$

$$log(L(\theta)) = -nlog(e^{n\theta})$$

$$L(\theta) = \theta n - \sum_{i=1}^{n} x_i$$

$$L'(\theta) = n - 0 = n$$

$$n \neq 0$$

Therefore, MLE is not possible

b). MLE is still not possible and does not exist.

Q3. (Exercise 6.12)

$$X_1, X_2 = N(\mu, 3^2)$$

$$Y_1, Y_2 = N(2\mu, 3^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

$$L(p) = P(X_1 = 3, X_2 = -1, Y_1 = 3, Y_2 = 2) = P(X_1 = 3)P(X_2 = -1)P(Y_1 = 3)P(Y_2 = 2)$$

$$L(p) = \frac{1}{3\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{3-\mu}{\sigma})^2} . \frac{1}{3\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{-1-\mu}{\sigma})^2} . \frac{1}{3\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{3-2\mu}{\sigma})^2} . \frac{1}{3\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{3-2\mu}{\sigma})^2}$$

$$L(p) = (\frac{1}{3\sqrt{2\pi}})^4 e^{\frac{-1}{2}(\frac{3-\mu}{\sigma})^2 + \frac{-1}{2}(\frac{-1-\mu}{\sigma})^2 + \frac{-1}{2}(\frac{3-2\mu}{\sigma})^2 + \frac{-1}{2}(\frac{3-2\mu}{\sigma})^2}$$

$$l(p) = log(\frac{1}{324\pi^2}) + \frac{-1}{2}((\frac{3-\mu}{3})^2 + (\frac{-1-\mu}{3})^2 + (\frac{3-2\mu}{3})^2 + (\frac{2-2\mu}{3})^2)$$

$$l(p) = log(\frac{1}{324\pi^2}) - \frac{1}{18}(23 - 24\mu + 10\mu^2)$$

$$l'(p) = \frac{1}{18}(20\mu - 24) = \frac{-1}{18}(20\mu - 24)$$

$$\frac{-1}{18}(20\mu - 24) = 0$$

$$\mu = \frac{6}{5}$$

check

$$l'(p) = \frac{-20}{18}\mu + \frac{24}{18}$$

$$l''(p) = \frac{-20}{18} < 0$$