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**DATA.STAT.770 Dimensionality Reduction and Visualization, Spring 2025**  
**Exercise set 2**

**Problem A1:**

a)

```
import numpy as np
import matplotlib.pyplot as plt

data = np.loadtxt('/Users/vudinhthi2304/Desktop/DATASTAT770/DATA.STAT.770-
Dimensionality-Reduction-and-
Visualization/Exercise2/week02/noisy_sculpt_faces.txt')

# Extract features and angles
features = data[:, :-3]
angles = data[:, -3:]

# Part a)
def nearest_neighbor_predictor(features, angles):
    n = features.shape[0]
    predictions = np.zeros_like(angles)

    for i in range(n):
        # Compute the sum of squared differences
        distances = np.sum((features - features[i]) ** 2, axis=1)
        distances[i] = np.inf # Exclude the current face

        # Find the nearest neighbor
        nearest_neighbor = np.argmin(distances)
        predictions[i] = angles[nearest_neighbor]

    return predictions
```

Baseline leave-one-out error: 507129.623964493

b)

```
# Part b)
def leave_one_out_error(features, angles):
    predictions = nearest_neighbor_predictor(features, angles)
    errors = np.sum((predictions - angles) ** 2, axis=1)
    total_error = np.sum(errors)
    return total_error

def forward_selection(features, angles):
    n_features = features.shape[1]
    selected_features = []
    min_error = float('inf')
    best_features = None
    errors = []

    while True:
```

```

        best_feature = None
        for feature in range(n_features):
            if feature in selected_features:
                continue
            current_features = selected_features + [feature]
            current_error = leave_one_out_error(features[:, current_features],
angles)
            if current_error < min_error:
                min_error = current_error
                best_feature = feature
                best_features = current_features

        if best_feature is None:
            break

        selected_features.append(best_feature)
        errors.append(min_error)
        print(f'Selected feature: {best_feature}, Current error: {min_error}')

    return best_features, min_error, errors

best_features, best_error, errors_list = forward_selection(features, angles)
print(f'Best leave-one-out error: {best_error}')
print(f'Number of features selected: {len(best_features)}')
print(f'Selected features: {best_features}')

```

Selected feature: 189, Current error: 310117.257272852  
 Selected feature: 53, Current error: 186791.79094106288  
 Selected feature: 137, Current error: 125693.66783699277  
 Selected feature: 224, Current error: 118213.72795439238  
 Selected feature: 32, Current error: 108774.43934657286  
 Best leave-one-out error: 108774.43934657286  
 Number of features selected: 5  
 Selected features: [189, 53, 137, 224, 32]

c)

```

# Part c)
def forward_selection_variant(features, angles):
    n_features = features.shape[1]
    selected_features = []
    errors = []

    for _ in range(n_features):
        best_feature = None
        min_error = float('inf')

        for feature in range(n_features):
            if feature in selected_features:
                continue
            current_features = selected_features + [feature]

```

```

        current_error = leave_one_out_error(features[:, current_features],
angles)
        if current_error < min_error:
            min_error = current_error
            best_feature = feature

    if best_feature is not None:
        selected_features.append(best_feature)
        errors.append(min_error)
        print(f'Selected feature: {best_feature}, Current error: {min_error}')

    return selected_features, errors

# Run the variant of forward selection
selected_features, errors = forward_selection_variant(features, angles)
print(f'Order of features added: {selected_features}')
print(f'Errors with each number of features: {errors}')

```

Selected feature: 189, Current error: 310117.257272852  
 Selected feature: 53, Current error: 186791.79094106288  
 Selected feature: 137, Current error: 125693.66783699277  
 Selected feature: 224, Current error: 118213.72795439238  
 Selected feature: 32, Current error: 108774.43934657286  
 Selected feature: 24, Current error: 113215.58440938324  
 Selected feature: 147, Current error: 92182.74733004993  
 Selected feature: 102, Current error: 83145.94150030859  
 Selected feature: 13, Current error: 76545.49962277396  
 Selected feature: 192, Current error: 68555.7720831847  
 Selected feature: 229, Current error: 67249.20883899964  
 Selected feature: 138, Current error: 58086.88470231136  
 Selected feature: 84, Current error: 53196.35343814999  
 Selected feature: 207, Current error: 49986.69828281485  
 Selected feature: 206, Current error: 51445.59659133378  
 Selected feature: 107, Current error: 48746.53651537712  
 Selected feature: 200, Current error: 50259.34443514915  
 Selected feature: 180, Current error: 48704.941505726616  
 Selected feature: 26, Current error: 48156.73466100886  
 Selected feature: 167, Current error: 46925.03475732019  
 Selected feature: 116, Current error: 46245.04197879646  
 Selected feature: 245, Current error: 48130.861018028845  
 Selected feature: 8, Current error: 48193.85971002209  
 Selected feature: 132, Current error: 53729.723594547126  
 Selected feature: 205, Current error: 50890.70962294393  
 Selected feature: 105, Current error: 51050.84242275881  
 Selected feature: 80, Current error: 47578.335252495584  
 Selected feature: 54, Current error: 45523.50821726195  
 Selected feature: 135, Current error: 44799.7653716227  
 Selected feature: 125, Current error: 45075.12179828019  
 Selected feature: 120, Current error: 43192.54833695477  
 Selected feature: 250, Current error: 44033.613751460674

Selected feature: 174, Current error: 43887.2520916703  
Selected feature: 12, Current error: 42049.2783239452  
Selected feature: 218, Current error: 41477.711772163224  
Selected feature: 46, Current error: 40838.02229442647  
Selected feature: 153, Current error: 43306.855498781675  
Selected feature: 113, Current error: 41764.96641387828  
Selected feature: 56, Current error: 42033.745454403004  
Selected feature: 9, Current error: 43568.035766330126  
Selected feature: 124, Current error: 43552.35642854261  
Selected feature: 208, Current error: 44299.949772008564  
Selected feature: 194, Current error: 42597.93211310777  
Selected feature: 59, Current error: 42326.068515265084  
Selected feature: 228, Current error: 43911.86248493137  
Selected feature: 104, Current error: 40814.16862300696  
Selected feature: 98, Current error: 40839.95626412244  
Selected feature: 89, Current error: 40590.842454710924  
Selected feature: 165, Current error: 41833.25715588732  
Selected feature: 17, Current error: 40635.943777072556  
Selected feature: 166, Current error: 41718.645969139405  
Selected feature: 73, Current error: 41618.54961586933  
Selected feature: 96, Current error: 39266.60138015826  
Selected feature: 18, Current error: 39160.97482155963  
Selected feature: 246, Current error: 40692.351181376835  
Selected feature: 253, Current error: 41338.94614268295  
Selected feature: 142, Current error: 40420.41014823809  
Selected feature: 114, Current error: 40769.55147591676  
Selected feature: 146, Current error: 41039.54935481189  
Selected feature: 81, Current error: 42745.43256443976  
Selected feature: 190, Current error: 42267.19069356858  
Selected feature: 197, Current error: 42094.67507033936  
Selected feature: 214, Current error: 42830.891521044236  
Selected feature: 122, Current error: 41858.185701503375  
Selected feature: 112, Current error: 43472.93101780308  
Selected feature: 71, Current error: 43834.59372763995  
Selected feature: 182, Current error: 43923.2140430634  
Selected feature: 222, Current error: 47488.811010813595  
Selected feature: 101, Current error: 48187.008656656784  
Selected feature: 62, Current error: 47318.59504161139  
Selected feature: 99, Current error: 47518.989285784664  
Selected feature: 184, Current error: 49408.386665803155  
Selected feature: 170, Current error: 48591.84066430311  
Selected feature: 219, Current error: 48098.428288332776  
Selected feature: 179, Current error: 47229.0401529183  
Selected feature: 128, Current error: 47229.0401529183  
Selected feature: 37, Current error: 49604.56074508418  
Selected feature: 57, Current error: 51187.89246598157  
Selected feature: 226, Current error: 48849.01319489871  
Selected feature: 162, Current error: 48598.107944180985  
Selected feature: 87, Current error: 48877.08178097935  
Selected feature: 171, Current error: 50375.699931474526

Selected feature: 70, Current error: 50580.904132735704  
Selected feature: 188, Current error: 50655.593665749235  
Selected feature: 244, Current error: 50246.641874261564  
Selected feature: 198, Current error: 48822.33276538652  
Selected feature: 49, Current error: 48190.59729954385  
Selected feature: 215, Current error: 48793.51233719832  
Selected feature: 35, Current error: 48747.7147318507  
Selected feature: 15, Current error: 48958.345749860004  
Selected feature: 0, Current error: 49727.96256230792  
Selected feature: 52, Current error: 51855.077462542264  
Selected feature: 151, Current error: 50888.00203703632  
Selected feature: 2, Current error: 49351.64265645672  
Selected feature: 6, Current error: 50272.658701273525  
Selected feature: 100, Current error: 51484.043771094584  
Selected feature: 201, Current error: 49294.70832360552  
Selected feature: 199, Current error: 49007.74659196625  
Selected feature: 181, Current error: 51293.730262507  
Selected feature: 115, Current error: 52092.942294369  
Selected feature: 133, Current error: 54305.55542715681  
Selected feature: 33, Current error: 51782.66409165066  
Selected feature: 220, Current error: 53490.961915217435  
Selected feature: 251, Current error: 50876.04720105699  
Selected feature: 159, Current error: 52820.09360951372  
Selected feature: 238, Current error: 51938.20593967722  
Selected feature: 230, Current error: 54584.32607301758  
Selected feature: 187, Current error: 56147.584059504006  
Selected feature: 126, Current error: 57913.49997836994  
Selected feature: 106, Current error: 55352.74893485998  
Selected feature: 169, Current error: 54158.19204005362  
Selected feature: 148, Current error: 53543.058226428744  
Selected feature: 68, Current error: 54699.79421140733  
Selected feature: 240, Current error: 56077.39575881372  
Selected feature: 67, Current error: 54191.93650195313  
Selected feature: 58, Current error: 52757.955756556985  
Selected feature: 79, Current error: 48824.01348221154  
Selected feature: 51, Current error: 52964.96950585049  
Selected feature: 93, Current error: 56215.763378311254  
Selected feature: 38, Current error: 58890.00855278306  
Selected feature: 212, Current error: 66331.47548986146  
Selected feature: 97, Current error: 68134.1219821682  
Selected feature: 185, Current error: 65108.716556801104  
Selected feature: 39, Current error: 63800.53180512514  
Selected feature: 186, Current error: 65594.02174033133  
Selected feature: 252, Current error: 69836.83233356822  
Selected feature: 48, Current error: 69601.5348118147  
Selected feature: 191, Current error: 67476.05500370351  
Selected feature: 110, Current error: 67555.06321023473  
Selected feature: 41, Current error: 67456.41282526619  
Selected feature: 195, Current error: 66199.19406495277  
Selected feature: 1, Current error: 68337.12435411895

Selected feature: 140, Current error: 68278.45478622302  
Selected feature: 233, Current error: 63454.769460520605  
Selected feature: 78, Current error: 68780.43454492364  
Selected feature: 210, Current error: 67530.66050090811  
Selected feature: 241, Current error: 64689.236158483734  
Selected feature: 172, Current error: 65668.65074709404  
Selected feature: 145, Current error: 66866.5593939598  
Selected feature: 163, Current error: 64199.68868961926  
Selected feature: 111, Current error: 68724.58989240401  
Selected feature: 27, Current error: 70677.6427399893  
Selected feature: 16, Current error: 68313.61355255781  
Selected feature: 134, Current error: 71314.15829165293  
Selected feature: 20, Current error: 68710.76657507585  
Selected feature: 247, Current error: 65962.55641056989  
Selected feature: 254, Current error: 66910.24044605372  
Selected feature: 40, Current error: 71928.37697470676  
Selected feature: 85, Current error: 76251.37549565396  
Selected feature: 176, Current error: 74970.30845199381  
Selected feature: 143, Current error: 75716.56762112003  
Selected feature: 150, Current error: 86149.54401667719  
Selected feature: 45, Current error: 82592.2650983391  
Selected feature: 164, Current error: 87813.49720668049  
Selected feature: 5, Current error: 92505.19931515558  
Selected feature: 22, Current error: 96546.20746708573  
Selected feature: 64, Current error: 92772.79824217819  
Selected feature: 248, Current error: 98492.69646381836  
Selected feature: 60, Current error: 93686.33589415981  
Selected feature: 213, Current error: 82616.4259424722  
Selected feature: 3, Current error: 77112.27670186381  
Selected feature: 211, Current error: 84335.6281062005  
Selected feature: 202, Current error: 88425.73779130157  
Selected feature: 155, Current error: 94180.0171529628  
Selected feature: 141, Current error: 89906.42350343725  
Selected feature: 177, Current error: 90613.61920218173  
Selected feature: 42, Current error: 94570.1578621839  
Selected feature: 152, Current error: 91843.21011379073  
Selected feature: 118, Current error: 95558.97776845784  
Selected feature: 77, Current error: 95077.01268124278  
Selected feature: 91, Current error: 97663.31280506164  
Selected feature: 204, Current error: 99691.30505184202  
Selected feature: 76, Current error: 105309.73375109186  
Selected feature: 94, Current error: 110224.42774098563  
Selected feature: 50, Current error: 110771.03427662884  
Selected feature: 61, Current error: 107978.36054004799  
Selected feature: 158, Current error: 106803.23494758146  
Selected feature: 23, Current error: 111084.84103656132  
Selected feature: 29, Current error: 114451.9986984646  
Selected feature: 144, Current error: 113117.8530550324  
Selected feature: 175, Current error: 116850.73259458723  
Selected feature: 65, Current error: 117543.54404876671

Selected feature: 221, Current error: 124738.22935765768  
Selected feature: 129, Current error: 121634.87422939415  
Selected feature: 117, Current error: 126566.89251714273  
Selected feature: 83, Current error: 131824.56598524185  
Selected feature: 234, Current error: 137473.89654208883  
Selected feature: 131, Current error: 133830.8202857773  
Selected feature: 249, Current error: 128896.52949655015  
Selected feature: 136, Current error: 128162.37322573506  
Selected feature: 44, Current error: 135671.97812138402  
Selected feature: 154, Current error: 128358.13232427265  
Selected feature: 255, Current error: 128965.40868401145  
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Selected feature: 31, Current error: 124413.81909116427  
Selected feature: 236, Current error: 125220.2474963744  
Selected feature: 47, Current error: 134204.14473212676  
Selected feature: 149, Current error: 124611.54410527286  
Selected feature: 92, Current error: 124732.45319292939  
Selected feature: 217, Current error: 125243.22315049694  
Selected feature: 157, Current error: 139574.866724721  
Selected feature: 63, Current error: 144952.2629091502  
Selected feature: 75, Current error: 146970.80882307419  
Selected feature: 7, Current error: 143561.53941140656  
Selected feature: 86, Current error: 162956.7659631501  
Selected feature: 237, Current error: 156836.32589350542  
Selected feature: 103, Current error: 168620.4031482979  
Selected feature: 95, Current error: 181974.36228295846  
Selected feature: 235, Current error: 164657.3729108529  
Selected feature: 183, Current error: 181123.98279589022  
Selected feature: 232, Current error: 177089.70828688494  
Selected feature: 34, Current error: 175078.80027517723  
Selected feature: 231, Current error: 167954.40435708186  
Selected feature: 4, Current error: 167675.31042921747  
Selected feature: 74, Current error: 174780.44511851098  
Selected feature: 139, Current error: 179613.86949156676  
Selected feature: 11, Current error: 180444.0327999745  
Selected feature: 127, Current error: 184891.75296261997  
Selected feature: 223, Current error: 187546.71131189555  
Selected feature: 242, Current error: 204751.47374597212  
Selected feature: 21, Current error: 203357.27576579488  
Selected feature: 225, Current error: 190337.43435331123  
Selected feature: 55, Current error: 201762.43782711483  
Selected feature: 36, Current error: 198356.04799781484  
Selected feature: 178, Current error: 196374.01673301647  
Selected feature: 108, Current error: 198555.1105544803  
Selected feature: 19, Current error: 208166.69802702375  
Selected feature: 90, Current error: 209797.94069751757  
Selected feature: 227, Current error: 204442.1077987515  
Selected feature: 119, Current error: 227039.85960609082  
Selected feature: 243, Current error: 236823.17266327483  
Selected feature: 28, Current error: 246208.91291236517

Selected feature: 82, Current error: 235280.33004409206  
Selected feature: 160, Current error: 224008.030337183  
Selected feature: 30, Current error: 224572.49902487322  
Selected feature: 121, Current error: 238773.3572516557  
Selected feature: 72, Current error: 248951.13007420246  
Selected feature: 66, Current error: 247070.01970181565  
Selected feature: 43, Current error: 241934.72054576746  
Selected feature: 14, Current error: 252313.5312138403  
Selected feature: 203, Current error: 258758.0809408015  
Selected feature: 130, Current error: 262349.6134041262  
Selected feature: 239, Current error: 287322.8306262382  
Selected feature: 123, Current error: 296284.73997282493  
Selected feature: 193, Current error: 294167.7421557834  
Selected feature: 173, Current error: 312811.26293239585  
Selected feature: 25, Current error: 305573.0479343124  
Selected feature: 196, Current error: 349832.39081952884  
Selected feature: 10, Current error: 361580.46963596524  
Selected feature: 168, Current error: 344152.41367836774  
Selected feature: 156, Current error: 398915.60100867564  
Selected feature: 109, Current error: 441962.20120538597  
Selected feature: 216, Current error: 440858.23224591726  
Selected feature: 88, Current error: 451606.12871276453  
Selected feature: 161, Current error: 442018.16643308965  
Selected feature: 209, Current error: 507129.623964493  
Order of features added: [189, 53, 137, 224, 32, 24, 147, 102, 13, 192, 229, 138, 84, 207, 206, 107, 200, 180, 26, 167, 116, 245, 8, 132, 205, 105, 80, 54, 135, 125, 120, 250, 174, 12, 218, 46, 153, 113, 56, 9, 124, 208, 194, 59, 228, 104, 98, 89, 165, 17, 166, 73, 96, 18, 246, 253, 142, 114, 146, 81, 190, 197, 214, 122, 112, 71, 182, 222, 101, 62, 99, 184, 170, 219, 179, 128, 37, 57, 226, 162, 87, 171, 70, 188, 244, 198, 49, 215, 35, 15, 0, 52, 151, 2, 6, 100, 201, 199, 181, 115, 133, 33, 220, 251, 159, 238, 230, 187, 126, 106, 169, 148, 68, 240, 67, 58, 79, 51, 93, 38, 212, 97, 185, 39, 186, 252, 48, 191, 110, 41, 195, 1, 140, 233, 78, 210, 241, 172, 145, 163, 111, 27, 16, 134, 20, 247, 254, 40, 85, 176, 143, 150, 45, 164, 5, 22, 64, 248, 60, 213, 3, 211, 202, 155, 141, 177, 42, 152, 118, 77, 91, 204, 76, 94, 50, 61, 158, 23, 29, 144, 175, 65, 221, 129, 117, 83, 234, 131, 249, 136, 44, 154, 255, 69, 31, 236, 47, 149, 92, 217, 157, 63, 75, 7, 86, 237, 103, 95, 235, 183, 232, 34, 231, 4, 74, 139, 11, 127, 223, 242, 21, 225, 55, 36, 178, 108, 19, 90, 227, 119, 243, 28, 82, 160, 30, 121, 72, 66, 43, 14, 203, 130, 239, 123, 193, 173, 25, 196, 10, 168, 156, 109, 216, 88, 161, 209]  
Errors with each number of features: [310117.257272852, 186791.79094106288, 125693.66783699277, 118213.72795439238, 108774.43934657286, 113215.58440938324, 92182.74733004993, 83145.94150030859, 76545.49962277396, 68555.7720831847, 67249.20883899964, 58086.88470231136, 53196.35343814999, 49986.69828281485, 51445.59659133378, 48746.53651537712, 50259.34443514915, 48704.941505726616, 48156.73466100886, 46925.03475732019, 46245.04197879646, 48130.861018028845, 48193.85971002209, 53729.723594547126, 50890.70962294393, 51050.84242275881, 47578.335252495584, 45523.50821726195, 44799.7653716227, 45075.12179828019, 43192.54833695477, 44033.613751460674, 43887.2520916703, 42049.2783239452, 41477.711772163224, 40838.02229442647, 43306.855498781675, 41764.96641387828, 42033.745454403004, 43568.035766330126, 43552.35642854261, 44299.949772008564, 42597.93211310777, 42326.068515265084, 43911.86248493137, 40814.16862300696, 40839.95626412244, 40590.842454710924, 41833.25715588732, 40635.943777072556,



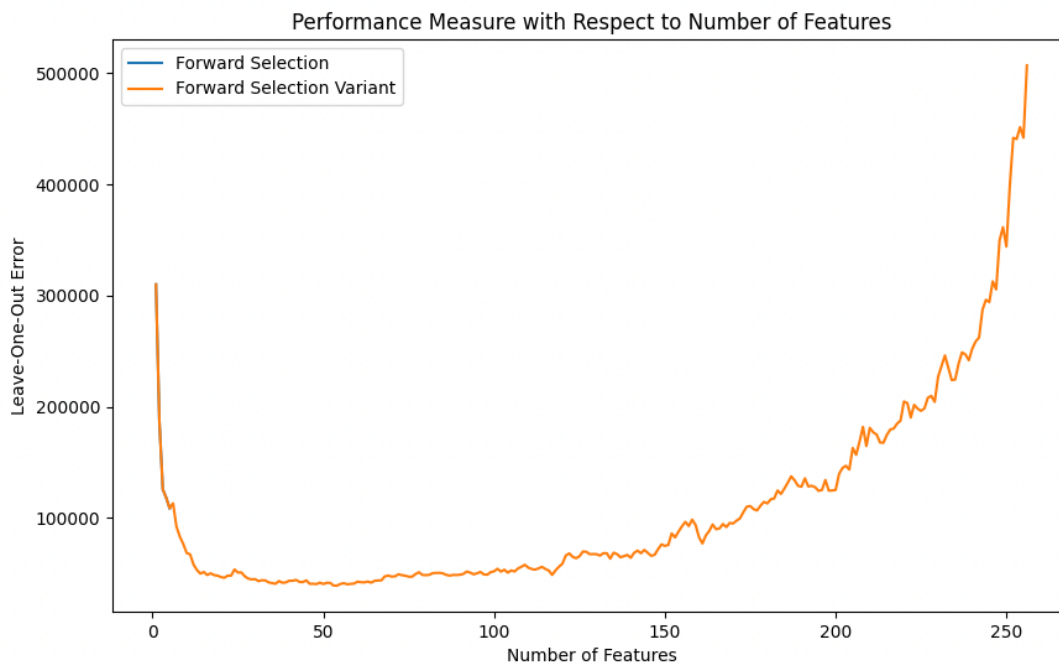
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398915.60100867564, 441962.20120538597, 440858.23224591726, 451606.12871276453, 442018.16643308965, 507129.623964493]

d)

- At some point in the algorithm, from the 6th added feature to approximately the third quarter of the features that the algorithm reach a better solution than in step b). Because the variant of forward selection continues to add features even if they do not improve the performance, which can sometimes lead to better solutions as more features are considered. Therefore the algorithm might find a combination of features that work well together, even if individual features do not show immediate improvement.

- Plot a curve of the performance measure with respect to the number of features.



## Problem A2:

a) Discussion on Simple Variable Ranking Methods:

We can use correlation methods such as Pearson correlation which measures the linear relationship between two variables. For the Sculpt Faces data set, we can compute the Pearson correlation coefficient between each feature and each of the target angles (pose and lighting angles). For each feature (pixel), we compute the Pearson correlation coefficient with each of the three target angles, then we can calculate the average the absolute values of these correlations to get a single ranking score for each feature. We can finally rank the features based on their average Pearson correlation scores.

b) In step c) of problem A1, we iteratively added the most useful features using forward feature selection, the order of variable selection is different from the ranking obtained by the leave-one-out error method. Forward selection typically chooses features based on their incremental contribution to reducing prediction error, considering interactions

between features. Meanwhile, ranking by correlation treats each feature independently, which may result in a different order.