CIS3900-002: Mathematics of Machine Learning Fall 2023

Homework 2

Release Date: September 13, 2023 *Due Date: September 20, 2023*

1 Written Questions

- **6.3** from the textbook.
- **6.12** from the textbook.
- **MGF/Chernoff.** Let $X_1, \ldots, X_N$ be i.i.d. random variables, with moment generating function $m_X(t) = \mathbb{E}[e^{tX}]$ and mean $\mu = \mathbb{E}[X]$. Consider the average of all N random variables: $Y = \frac{1}{N}\sum_{i=1}^{N} X_i$. Derive the form of $m_Y(t)$, the moment generating function for Y, and use the Chernoff method to derive a tail bound.
- **Bonus**: Use the Chernoff method to derive a tail bound $P(X - 1 \geq u)$ for the χ^2 random variable and for $u \leq 1$. A χ^2 random variable is $X = Z^2$ where $Z \sim N(0, 1)$ is a standard normal, and note that χ^2 has mean 1.

  Hint: consider the MGF when $t < 1/2$, and use the fact that $\frac{e^{-t}}{\sqrt{1-2t}} \leq e^{2t^2}$ for $|t| \leq 1/4$.