Assessed project released tomorrow on MOLE

Oats data Full 3×4 factorial arrangement

3 varieties of oats, 4 treatments (concentration of Nitrogen fertilizer)

Different location

Block 1

Variety				
'Victory'	0	0.2	0.4	0.6
'Golden Wonder'	0.2	0.4	0.6	0
'Marvellous'	0.4	0.6	0	0.2

3 plots

subplots

nitrogen concentration in the subplot

Block 2 . . . .

Block 6

Y_{ijk} = yield (of oats) in block i, plot j, subplot k

x_{ijk} = Nitrogen concentration in block i, plot j, subplot k — continuous covariate

$v(i,j) = 1, 2, 3$ depending on the variety of oat planted in block i, plot j. — discrete factor

block, plot & subplot are also discrete factors.

Fixed effect: Effect of Nitrogen & variety of oat – are of primary interest.

Random effect: block, plot, subplot effects are not of direct interest

$$Y_{ijk} = \mu + \underbrace{\tau_{v(i,j)}}_{\text{variety}} + \underbrace{\beta x_{ijk}}_{\text{Nitrogen}} + \underbrace{b_i}_{\text{block}} + \underbrace{b_{ij}}_{\text{plot within block}} + \underbrace{\varepsilon_{ijk}}_{\text{subplot within plot within block}}$$

$i = 1, \dots, 6$ blocks, $j = 1, \dots, 3$ plots, $k = 1, \dots, 4$ subplot within plot

Set $\tau_1 = 0$ so $\mu = \mathbb{E}$ yield variety 1 when 0% Nitrogen used.

NB Y_{11k} & Y_{21k} refers to different plots – plot is not a factor.

Model is $\quad b_i \sim N(0, \sigma_1^2) \quad b_{ij} \sim N(0, \sigma_2^2) \quad \varepsilon_{ijk} \sim N(0, \sigma^2)$

$$Y_{ijk} \sim N\Big(\underbrace{\mu + \tau_{v(i,j)} + \beta x_{ijk}}, \underbrace{\sigma_1^2 + \sigma_2^2 + \sigma^2}\Big)$$

mean structure comes from fixed effects

variance structure comes from the random effects

Check

$$\mathrm{Cor}(Y_{ijk}, Y_{ij'k'}) = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2 + \sigma^2}$$

within block correlation

$$\mathrm{Cor}(Y_{ijk}, Y_{ijk'}) = \frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2 + \sigma_2^2 + \sigma^2}$$

within plot correlation

Random interactions Machine example

Y_{ijk} = productivity of worker i on machine j, replicate k

		Machine (j) A	B	C
Worker (i)	1	$Y_{111}, Y_{112}, Y_{113}$	$Y_{121}, \dots, Y_{123}$	$Y_{131}, \dots, Y_{133}$
	2			
	$\vdots$			
	6	....		..... Y_{633}

Model 1 $\quad Y_{ijk} = \beta_j + b_i + \varepsilon_{ijk}$

$$b_i \sim N(0, \sigma_b^2) \qquad \varepsilon_{ijk} \sim N(0, \sigma^2)$$

Problem – effect of switching between machines is the same for all workers.

Lets add interactions

Model 2 $\quad Y_{ijk} = \beta_j + b_i + b_{ij} + \varepsilon_{ijk}$

interaction between worker j & machine i.

$$b_i \sim N(0, \sigma_1^2) \qquad b_{ij} \sim N(0, \sigma_2^2)$$

$$\varepsilon_{ijk} \sim N(0, \sigma^2)$$

Problem - same variance for each worker on each machine
- simple variance structure.

Model 3 $\quad Y_{ijk} = \beta_j + b_{ij} + \varepsilon_{ijk}$

$$\underline{b}_{i\cdot} = \begin{pmatrix} b_{i1} \\ b_{i2} \\ b_{i3} \end{pmatrix} \sim N\left(\underline{0}, \Sigma\right)$$