MATH3027: Optimization (UK 21/22)

Week 6: Computer lab 4

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Now that we are into the computational material, we will run these computer sessions as genuine computer labs. I will be on hand to answer questions, but won't spend the entire session talking. The coursework will involve similar calculations, so I highly recommend you spend time doing these problems for yourself.

Before you attempt the question below, please do the computational exercises in the notes.

Quadratic Optimization Benchmark

Consider the quadratic minimization problem

$$\min_{\mathbf{x}} \left\{ \mathbf{x}^\top \mathbf{A} \mathbf{x} : \mathbf{x} \in \mathbb{R}^5 \right\}$$

where $\mathbf{A}$ is the 5×5 Hilbert matrix defined by

$$\mathbf{A}_{i,j} = \frac{1}{i+j-1}, \quad i, j = 1, 2, 3, 4, 5$$

Write an R function to create the Hilbert matrix of a given size.

Run the following methods and compare the number of iterations required by each of the methods when the initial vector is $\mathbf{x}^0 = (1, 2, 3, 4, 5)^\top$ to obtain a solution $\mathbf{x}^*$ with $\|\nabla f(\mathbf{x})\| \leq 10^{-4}$:

- Gradient method with backtracking stepsize rule and parameters $\alpha = 0.5, \beta = 0.5, s = 1$
- Gradient method with backtracking stepsize rule and parameters $\alpha = 0.1, \beta = 0.5, s = 1$
- Diagonally scaled gradient method with diagonal elements $\mathbf{D}_{i,i} = \frac{1}{\mathbf{A}_{i,i}}, i = 1,2,3,4,5$ and exact line search;

- Diagonally scaled gradient method with diagonal elements $\mathbf{D}_{i,i} = \frac{1}{\mathbf{A}_{i,i}}, i = 1,2,3,4,5$ and backtracking line search with parameters $\alpha = 0.1, \beta = 0.5\ s = 1$.