MATH3027: Optimization (UK 21/22)

Week 8: Computer lab 5

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Now that we are into the computational material, we will run these computer sessions as genuine computer labs. I will be on hand to answer questions, but won't spend the entire session talking. The coursework will involve similar calculations, so I highly recommend you spend time doing these problems for yourself.

There are also computational problems for you to tackle in the notes.

Nonlinear Regression

Consider the nonlinear model in $\theta \in \mathbb{R}$

$$f(\theta; \mathbf{x}) = x_1 e^{x_2 \theta} \cos(x_3 \theta + x_4)\,,$$

with parameters $\mathbf{x} \in \mathbb{R}^4$ for which we want to find the optimal value $\mathbf{x}^*$ minimizing the norm of the ℓ_2-error with respect to m observations of the *true* model

$$f_i := f(\theta_i)\,, \qquad i = 1, \dots, m\,.$$

We formulate this problem as a nonlinear least squares problem

$$\min_{\mathbf{x} \in \mathbb{R}^4} g(\mathbf{x}) := \sum_{i=1}^{m} (f(\theta_i; \mathbf{x}) - \hat{f}_i)^2\,. \tag{NLS}$$

- Write code to implement the Gauss-Newton method for this problem.
- Using the true parameters $\mathbf{x} = (1, 2, \pi, 0)^\top$, generate 200 samples of the model in $[-1, 1]$, contaminate them with Gaussian noise with mean 0 and variance 0.025. Use this data as input for your NLS problem, using as initial guess $\mathbf{x}^0 = (5, 5, 5, 5)^\top$ and a tolerance of $\|\nabla g(\mathbf{x}^k)\| \le 10^{-4}$.
- Implement a damped Newton method for the problem and compare the performance to Gauss-Newton.