

# MATH3027: Optimization 2022

## Week 8: Computer lab 5

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Now that we are into the computational material, we will run these computer sessions as genuine computer labs. I will be on hand to answer questions, but won't spend the entire session talking. The coursework will involve similar calculations, so I highly recommend you spend time doing these problems for yourself.

There are also computational problems for you to tackle in the notes.

### Nonlinear Regression

Consider the nonlinear model in  $\theta \in \mathbb{R}$

$$f(\theta; \mathbf{x}) = x_1 e^{x_2 \theta} \cos(x_3 \theta + x_4),$$

with parameters  $\mathbf{x} \in \mathbb{R}^4$  for which we want to find the optimal value  $\mathbf{x}^*$  minimizing the norm of the  $\ell_2$ -error with respect to  $m$  observations of the *true* model

$$f_i := f(\theta_i), \quad i = 1, \dots, m.$$

We formulate this problem as a nonlinear least squares problem

$$\min_{\mathbf{x} \in \mathbb{R}^4} g(\mathbf{x}) := \sum_{i=1}^m (f(\theta_i; \mathbf{x}) - \hat{f}_i)^2. \quad (\text{NLS})$$

- Write code to implement the Gauss-Newton method for this problem.
- Using the true parameters  $\mathbf{x} = (1, 2, \pi, 0)^\top$ , generate 200 samples of the model in  $[-1, 1]$ , contaminate them with Gaussian noise with mean 0 and variance 0.025. Use this data as input for your NLS problem, using as initial guess  $\mathbf{x}^0 = (5, 5, 5, 5)^\top$  and a tolerance of  $\|\nabla g(\mathbf{x}^k)\| \leq 10^{-4}$ .
- Implement a damped Newton method for the problem and compare the performance to Gauss-Newton.

