Exercises: Computer Lab 1

I highly recommend that you use Rstudio as your R GUI. I also recommend that you learn to use R Markdown as a way to document your code and to write your coursework solutions. You can get started with R Markdown by clicking File → New File → R Markdown and following the instructions.

1. Suppose $X_1, \ldots, X_{10}$ are iid $U[0,1]$ random variables. Find the distribution of

$$R(X) = \max_i\{X_i\} - \min_i\{X_i\}.$$

   What is $\mathbb{P}(R(X) > 0.99)$?

   If you are struggling, note that the code from the problems discussed in lectures is available on the module webpage.

2. Estimate the integral

$$I = \int_0^{10} \frac{1}{(1+x^2)} dx$$

   using at least two different choices for the proposal density $g(\cdot)$.

   Find 95% confidence intervals for your estimates using the central limit theorem.

3. Let

$$I_1 = \int_0^1 e^{-x^2} \mathrm{d}x \quad \text{and} \quad I_2 = \int_0^1 (\cos(50x) + \sin(20x))^2 \mathrm{d}x$$

   Estimate both of these integrals using Monte Carlo (using an estimator of your choice). Show that the root mean square error of your estimator scales as $O(n^{-1/2})$. To do this, you will need to repeat the analysis multiple times for a range of values of n (i.e., for $n = 10, 50, 100, 500, 1000, 5000, 10000$ estimate the integral 100 times and calculate the standard error).

   Calculate these integrals using the mid-ordinate rule. Show that the error now scales as $O(n^{-2})$. Note that because the mid-ordinate rule is very accurate for 1d intervals, you should only consider values of n between (2 and 100 say).