MATH3027: Optimization (UK 21/22)

Week 10: Computer lab 7

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Disciplined convex optimization: CVXR

The theory for convex optimization is now sufficiently advanced that convex optimization is close to being a *solved* problem, in the sense that there are reliable and efficient algorithms that can be used to solve these problems. The difficulty, is largely in writing the problems in such a way to allow software to recognize them as conved problems. The CVXR package in R is an implementation of a number of methods that can be used to solve convex optimization problems, based on the book of Boyd and Vandenberghe. Similar packages exists in other languages: CVX (Matlab), CVXPY (Python), and Convex.jl (Julia).

Install CVXR and read the introduction at

`https://cvxr.rbind.io/cvxr_examples/cvxr_gentle-intro/`

Use CVXR to solve the following optimization problems.

1.

$$\begin{aligned} \min_{\mathbf{x}} \quad & 2x_1^2 + 3x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 - 8x_1 - 4x_2 - 2x_3 \\ \text{subject to} \quad & x_1, x_2, x_3 \geq 0\,. \end{aligned}$$

Note that you will need to write the objective function in such a way that CVXR recognizes it as a convex function. A list of functions CVXR recognizes is given at

`https://cvxr.rbind.io/cvxr_functions/`

2.

$$\begin{aligned} \min_{\mathbf{x}} \quad & 2x_1^2 + 3x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 - 8x_1 - 4x_2 - 2x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0\,. \end{aligned}$$

3.

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & 2x_1^2 + 3x_2^2 + 4x_3^2 + 2x_1x_2 - 2x_1x_3 - 8x_1 - 4x_2 - 2x_3 \\
\text{subject to} \quad & x_1 + x_2 + x_3 \leq 1 \\
& x_2 \geq 0.2 \\
& x_1, x_2, x_3 \geq 0 \,.
\end{aligned}
$$

Disguised optimization problems

Consider the following optimization problem

$$\min_{\mathbf{x}} ||A\mathbf{x} - \mathbf{b}||_p.$$

where

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Use CVXR to solve this problem for $p = 1, 2, \infty$.

Show that for $p = 1$ and $p = \infty$ this can be written as a linear programme.