MAS6003(2)/MAS473 Extended Linear Models

Exercises 1

Consider the following model

$$\begin{aligned} Y_{ij} &= \begin{cases} \mu + b_i + \epsilon_{i1}, & i = 1, ..., n, \text{ if } j = 1 \\ \mu + \beta + b_i + \epsilon_{i2}, & i = 1, ..., n, \text{ if } j = 2 \end{cases} \\ & b_i \sim N(0, \sigma_b^2), \quad \epsilon_{ij} \sim N(0, \sigma^2), \end{aligned} \tag{1}$$

where Y_{ij} is the response of the ith subject to the jth treatment and n is the number of pairs of observations (see §2.5.1 in the notes and the model for the `ergoStool` dataset).

1. Show that $d_i = y_{i1} - y_{i2}$ satisfies the usual assumptions for the within group differences in the paired t-test; i.e., Gaussian with mean given by the true between-treatment difference with constant variance (independent of i).

$$d_i \sim N(-\beta, 2\sigma^2)$$

2. Box, Hunter & Hunter collected a famous dataset on shoe wear . They asked 10 subjects to wear shoes made from two different materials, and then measured the shoe wear after a set period. The data are available as `BHHshoes` in `MAS473.RData`. Conduct a classical paired t-test to test for difference in mean wear between the two material types.

3. In R, create a single plot to visualise both the difference in wear between the two material types, and the differences between the subjects.

4. Fit a mixed effects model in R, with `material` as a fixed effect and `Subject` as a random effect. Explain why the specific choices of fixed and random effects are appropriate. Use a treatment contrast parameterisation for `material` (i.e. parameterize your model as given above). From the R output, give the estimated parameter values for your model.

5. Now repeat the calculation for the fixed effect parameter estimates, but without using the `lmer` command. In otherwords, derive the values yourself (you can still use R to do the arithmetic).

   Show how the estimated standard errors for the fixed effects have been calculated.

6. Compare the `t value` for the `material` effect parameter with the test-statistic from the paired t-test, and comment briefly on the result.