MATH3027: Optimization (UK 21/22)

Week 9: Computer lab 6

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There was no computational material in this week's notes, so it is an easy computer lab this week. We will look at solving linear programmes (LPs) using the 'lpSolve' R package. A linear programme is an optimization problem that can be written in the form

$$\begin{aligned} & & \min & \quad \mathbf{c}^\top \mathbf{x} \\ & (\mathbf{LP}): & \text{s.t.} & \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & & & \quad \mathbf{B}\mathbf{x} = \mathbf{g} \\ & & & \quad \mathbf{x} \geq 0. \end{aligned}$$

In the notes, we don't study the specific algorithms designed so solve LPs, instead we just show that if a linear programme has a solution, then it has at least one solution that is a basic feasible solution, or equivalently, at least one solution which is an extreme point. In large problems, there can be too many of these to enumerate by hand, but thankfully there are efficient algorithms that can be used such as the simplex algorithm[1], which you can read about here.

Install the `lpSolve` package in R, and read the tutorial at https://towardsdatascience.com/linear-programming-in-r-444e9c199280.

```
install.packages('lpSolve')
```

Solve the LPs given in the notes using the `lp` command.

[1] But the material is beyond the scope of the module.

Wordy example

A company manufactures two products (A and B) and the profit per unit sold is £3 and £5 respectively. Each product has to be assembled on a particular machine, each unit of product A taking 12 minutes of assembly time and each unit of product B 25 minutes of assembly time. The company estimates that the machine used for assembly has an effective working week of only 30 hours (due to maintenance/breakdown).

Technological constraints mean that for every five units of product A produced at least two units of product B must be produced.

- Formulate the problem of how much of each product to produce to maximize profits as a linear program .
- Solve this linear program graphically.
- Check your solution using `lp`.
- The company has been offered the chance to hire an extra machine, thereby doubling the effective assembly time available. What is the maximum amount you would be prepared to pay (per week) for the hire of this machine and why?