Precalculus

{(1,2),(2,4),(3,6),(4,8),(5,10)}

{1,2,3,4,5}.{2,4,6,8,10}. x. y. f {1,2,3,4,5},{2,4,6,8,10}.

{ (odd,1),(even,2),(odd,3),(even,4),(odd,5) }

{even,odd}{1,2,3,4,5}.{1,3,5}{2,4}. q r n. q ha f, g, h x, y,zA, B,C

h is f of a We name the function f; height is a function of age. h=f(a)

We use parentheses to indicate the function input. f(a)

We name the function f; the expression is read as “f of a.”

h(a) h a. a h f(a+b) y=f(x) f. “y x.” x y, f(x), f,days=f(month)d=f(m). f(March)=31, d=f(m) d m N=f(y) N, y. f(2005)=300 f(2005)=300, (N), N=f(y). f(2005)=300 d.w=f(d) y=f(x), y=y(x), f, y f x. y=f(x),P=W(d), f D=f(m) m D Q=g(n). g n Q .nQ a h

f(2)=1,f(5)=3,and f(8)=6

g(−3)=5, g(0)=1,and g(4)=5

f(x)=5−3 x 2 f(x)= x 2 +3x−4 2aa+hf(a+h)−f(a) h x

f(2)= 2 2 +3(2)−4 =4+6−4 =6

f(a)= a 2 +3a−4

a+h,

f(a+h)= (a+h) 2 +3(a+h)−4 = a 2 +2ah+ h 2 +3a+3h−4

f(a+h)= a 2 +2ah+ h 2 +3a+3h−4

f(a)= a 2 +3a−4

f(a+h)−f(a) h = (a 2 +2ah+ h 2 +3a+3h−4)−(a 2 +3a−4) h = 2ah+ h 2 +3h h

= h(2a+h+3) h Factor out h. =2a+h+3 Simplify.

h(p)= p 2 +2p, h(4). h(4), p

h(p)= p 2 +2p h(4)= (4) 2 +2(4) =16+8 =24

g(m)= m−4 , g(5). g(5)=1 h(p)= p 2 +2p, h(p)=3.

h(p)=3 p 2 +2p=3 Substitute the original function h(p)= p 2 +2p. p 2 +2p−3=0

Subtract 3 from each side. (p+3)(p−1)=0 Factor.

(p+3)(p−1)=0, (p+3)=0 (p−1)=0 p

(p+3)=0, p=−3 (p−1)=0, p=1

h(p)=3 p=1 p=−3. h(1)=h(−3)=3 h(4)=24. g(m)= m−4 , g(m)=2. m=8 2n+6p=12 n p. p n. 2n+6p=12 p=f(n), p n, p=[expression involving n].

2n+6p=12 6p=12−2n Subtract 2n from both sides. p= 12−2n 6

Divide both sides by 6 and simplify. p= 12 6 − 2n 6 p=2− 1 3 n

p n

p=f(n)=2− 1 3 n

x 2 + y 2 =1 x y y=f(x). x 2

y 2 =1− x 2

y

y=± 1− x 2 =+ 1− x 2 and − 1− x 2

y=f(x). x−8 y 3 =0, y x.y=f(x)= x 3 2 x=y+ 2 y , y x, x y. x y, y y x, P. P P(goldfish)=2160. P g(3). g(n)=6.ng(n)g(3)gn=3.n=3g(3)=7.g(n)=6n,24.ng(n) g, g, g(1).g(1)=8 f(2). f(x)=4. f(2), x=2, (2,1), f(2)=1. f(x)=4, 4 y=4, 4:(−1,4) (3,4). f(x)=4:−1 3. f(−1)=4 f(3)=4, −1 3, 4. f(x)=1. x=0 x=2 q r n. r A=π r 2 , r, A. r. A A=π r 2 . A π . x y, y x, y=f(x) f. (x,y) y=f(x). f(0)=2 f(6)=1. (x,y) y=f(x) (0,2) (6,1) y=f(x)? x y=f(x) f(x)=c, cf(x)=xf(x)=| x |f(x)= x 2f(x)= x 3 f(x)= 1 xf(x)= 1 x 2f(x)= xf(x)= x 3f(x)=c, c f(x)=xf(x)=| x |f(x)= x 2f(x)= x 3f(x)= 1 x f(x)= 1 x 2f(x)= xf(x)= x 3 y=f(x). { (a,b), (c,d), (a,c) }{(a,b),(b,c),(c,c)} y x. 5x+2y=10y= x 2 x= y 23 x 2 +y=142x+ y 2 =6y=−2 x 2 +40xy= 1 xx= 3y+5 7y−1x= 1− y 2y= 3x+5 7x−1x 2 + y 2 =9 2xy=1x= y 3y= x 3y= 1− x 2x=± 1−yy=± 1−xy 2 = x 2y 3 = x 2 f f(−3),f(2),f(−a),−f(a),f(a+h).f(x)=2x−5

f(−3)=−11; f(2)=−1; f(−a)=−2a−5; −f(a)=−2a+5; f(a+h)=2a+2h−5f(x)=−5 x 2 +2x−1f(x)= 2−x +5
f(−3)= 5 +5; f(2)=5; f(−a)= 2+a +5; −f(a)=− 2−a −5; f(a+h)=2−a−h +5f(x)= 6x−1 5x+2
f(x)=| x−1 |−| x+1 |
f(−3)=2; f(2)=1−3=−2; f(−a)=| −a−1 |−| −a+1 |; −f(a)=−| a−1 | +| a+1 |; f(a+h)= | a+h−1 |−| a+h+1 |
g(x)=5− x 2 , g(x+h)−g(x) h , h≠0. g(x)= x 2 +2x, g(x)−g(a) x−a , x≠a. g(x)−g(a) x−a =x+a+2, x≠a
k(t)=2t−1: k(2). k(t)=7. f(x)=8−3x: f(−2). f(x)=−1. f(−2)=14; x=3 p(c)= c 2 +c: p(−3). p(c)=2.
f(x)= x 2 −3x: f(5). f(x)=4. f(5)=10; x=−1 x=4 f(x)= x+2 : f(7). f(x)=4. 3r+2t=18. r=f(t). f(−3). f(t)=2.
f(t)=6− 2 3 t; f(−3)=8; t=6 f(−1). f(x)=3. f(0). f(x)=−3. f(0)=1; f(x)=−3, x=−2 x=2 f(4). f(x)=1.
{ (−1,−1),(−2,−2),(−3,−3) }{ (3,4),(4,5),(5,6) }{ (2,5),(7,11),(15,8),(7,9) } y x.xyxyxy f xf(x)
f(3). f(x)=1. f(x)=1, x=2 f f(−2), f(−1), f(0), f(1), f(2). f(x)=4−2xf(x)=8−3x
f(−2)=14; f(−1)=11; f(0)=8; f(1)=5; f(2)=2f(x)=8 x 2 −7x+3f(x)=3+ x+3
f(−2)=4; f(−1)=4.414; f(0)=4.732; f(1)=4.5; f(2)=5.236f(x)= x−2 x+3f(x)= 3 x
f(−2)= 1 9 ; f(−1)= 1 3 ; f(0)=1; f(1)=3; f(2)=9f, g, h:f(x)=3x−2g(x)=5− x 2h(x)=−2 x 2 +3x−1
3f(1)−4g(−2)f(7 3)−h(−2) y= x 2 [−0.1, 0.1][−10, 10][0, 100][−100,100] y= x 3 [−0.1, 0.1]
[−0.001, 0.001][−10, 10][−100, 100][−1,000,000, 1,000,000] y= x [0, 0.01][0, 100][0, 10][0, 10,000]
y= x 3[−0.001,0.001][−0.1,0.1][−1000,1000][−1,000,000,1,000,000][−100, 100] G, p G=f(p). G p f.
f(5)=2. D, a D=g(a). g. g(100)=1. g(5000)=50; f(t) t f(5)=30f(10)=40 h(t) t h(1)=200h(2)=350
f(x)=3 (x−5) 2 +7 (0, 100]. { (2, 10),(3, 10),(4, 20),(5, 30),(6, 40) }

{2,3,4,5,6}

{ (−5,4),(0,0),(5,−4),(10,−8),(15,−12) }{−5, 0, 5, 10, 15} f(x)= x 2 −1. x f (−∞,∞). f(x)=5−x+ x 3 .
(−∞,∞) x f(x)= x+1 2−x . x.

2−x=0 −x=−2 x=2

x<2 x>2. ∪,(−∞,2)∪(2,∞). f (−∞,2)∪(2,∞). f(x)= 1+4x 2x−1 .(−∞, 1 2)∪(1 2 ,∞) x. f(x)= 7−x . x.

7−x≥0 −x≥−7 x≤7

7, (−∞,7]. f(x)= 5+2x .[− 5 2 ,∞) f(x)=− 1 x { x|10≤x<30 } x {} { x|10≤x<30 } x, x ∪,{2,3,5} {4,6}
{2,3,4,5,6}.

{ x| | x |≥3 }=(−∞,−3]∪[3,∞)

{x| statement about x} x x

{ x|4<x≤12 }

(4,12]

∪ x, x 1≤x≤3 or x>5{ x|1≤x≤3 or x>5 }[1,3]∪(5,∞){ x|x≤−2 or −1≤x<3 }(−∞,−2]∪[−1,3) −5 [−5,∞). 5
(−∞,5]. f f (−3,1]. [−4 , 0). f t b 1973≤t≤2008 180≤b≤2010. f(x)=c, c, {c} [c,c], c. f(x)=x, x.
f(x)=| x |, x. f(x)= x 2 , f(x)= x 3 , f(x)= 1 x , {x| x≠0}, f(x)= 1 x 2 ,0,0x f(x)= x , x − x x. f(x)= x 3 ,
f(x)=2 x 3 −x. (−∞,∞) (−∞,∞). f(x)= 2 x+1 . −1 (−∞,−1)∪(−1,∞). (−∞,0)∪(0,∞). f(x)=2 x+4 .

x+4≥0 when x≥−4

f(x) [−4,∞). f(−4)=0, x f [0, ∞) . f. f(x)=− 2−x . (−∞,2]; (−∞,0] f(x)=| x |.

f(x)=x if x≥0

f(x)=−x if x<0

S 0.1S S≤$10,000 $1000+0.2(S−$10,000) S>$10,000.

f(x)={ formula 1 if x is in domain 1 formula 2 if x is in domain 2 formula 3 if x is in domain 3

| x |={ x if x≥0 −x if x<0

n, C. C=5n. n C=50.

C(n)={ 5n if 0<n<10 50 if n≥10

n=0 n=10 n=10, C, g

C(g)={ 25 if 0<g<2 25+10(g−2) if g≥2

C(1.5),

C(1.5)=$25

C(4),

C(4)=25+10(4−2)=$45

g=2.

f(x)={ x 2 if x≤1 3 if 1<x≤2 x if x>2

f(x)= x 2 if x≤1; f(x)=3 if 1< x≤2; f(x)=x if x>2 x=1 x=2 (1,3)(2,2)(1,1)(2, 3)

f(x)={ x 3 if x<−1 −2 if −1<x<4 x if x>4

f(x)= x 3 f(x)= x . x f(x)= x 3 (−∞,∞). x f(x)= x [0,∞). x y −∞ ∞. f(x)=−2x(x−1)(x−2)f(x)=5−2 x 2 (−∞,∞)f(x)=3 x−2f(x)=3− 6−2x(−∞,3]f(x)= 4−3xf(x)= x 2 +4(−∞,∞)f(x)= 1−2x 3f(x)= x−1 3(−∞,∞) f(x)= 9 x−6f(x)= 3x+1 4x+2(−∞,− 1 2)∪(− 1 2 ,∞)f(x)= x+4 x−4f(x)= x−3 x 2 +9x−22 (−∞,−11)∪(−11,2)∪(2,∞)f(x)= 1 x 2 −x−6f(x)= 2 x 3 −250 x 2 −2x−15(−∞,−3)∪(−3,5)∪(5,∞)5 x−3 2x+1 5−x(−∞,5)f(x)= x−4 x−6f(x)= x−6 x−4[6,∞)f(x)= x xf(x)= x 2 −9x x 2 −81 (−∞,−9)∪(−9,9)∪(9,∞) f(x)= 2 x 3 −50x (2,8], [6,8) [−4, 4], [0, 2] [−5, 3), [0,2] (−∞,1], [0,∞) [−6,− 1 6]∪[1 6 ,6]; [−6,− 1 6]∪[1 6 ,6] [−3, ∞); [0,∞) f(x)={ x+1 if x<−2 −2x−3 if x≥−2 f(x)={ 2x−1 if x<1 1+x if x≥1 (−∞,∞)f(x)={ x+1 if x<0 x−1 if x>0f(x)={ 3 if x<0 x if x≥0 (−∞,∞)

f(x)={ x 2 if x<0 1−x if x>0

f(x)={ x 2 x+2 if x<0 if x≥0 (−∞,∞)f(x)={ x+1 if x<1 x 3 if x≥1f(x)={ |x| 1 if x<2 if x≥2 (−∞,∞)f,f(−3), f(−2), f(−1),f(0).f(x)={ x+1 if x<−2 −2x−3 if x≥−2f(x)={ 1 if x≤−3 0 if x>−3 f(−3)=1; f(−2)=0; f(−1)=0; f(0)=0f(x)={ −2 x 2 +3 if x≤−1 5x−7 if x>−1 f, f(−1), f(0), f(2), f(4). f(x)={ 7x+3 if x<0 7x+6 if x≥0f(−1)=−4; f(0)=6; f(2)=20; f(4)=34f(x)={ x 2 −2 if x<2 4+| x−5 | if x≥2 f(x)={ 5x if x<0 3 if 0≤x≤3 x 2 if x>3f(−1)=−5; f(0)=3; f(2)=3; f(4)=16 f(x)={ x+1 if x<−2 −2x−3 if x≥−2f(x)={ x 2 −2 if x<1 − x 2 +2 if x>1 (−∞,1)∪(1,∞) f(x)={ 2x−3 −3 x 2 if x<0 if x≥2 y= 1 x 2 [−0.5,−0.1] [0.1,0.5]. [−0.5,−0.1]; [4, 100] [0.1, 0.5]; [4, 100] y= 1 x [−0.5,−0.1] [0.1, 0.5]. f [−5, 8]. |f(x)|?[0, 8] x>2. f(x)= 1 x−2 . h t t h(t)=−16 t 2 +96t. [0, 6]; x C(x)=10x+500. C(x)? {x| statement about x}yC(y)

Average rate of change= Change in output Change in input = Δy Δx

= y 2 − y 1 x 2 − x 1 = f(x 2)−f(x 1) x 2 − x 1

Δ y x. Δf Δy,

Δy Δx = $1.37 7 years ≈0.196 dollars per year

Δy Δx = f(x 2)−f(x 1) x 2 − x 1

x 1 x 2 .y 2 − y 1 =Δy.x 2 − x 1 =Δx. Δy Δx .

Δy Δx = y 2 − y 1 x 2 − x 1 = $2.41−$2.84 2009−2007 = −$0.43 2 years =−$0.22 per year $2.84−$2.31 5 years = $0.53 5 years =$0.106 g(t) [−1,2].t=−1,g(−1)=4. t=2,g(2)=1. Δt=3 Δg(t)=−3

1−4 2−(−1) = −3 3 =−1

y 2 − y 1 x 1 − x 2 , (x 1 , y 1) (x 2 , y 2).

292−10 6−0 = 282 6 =47

f(x)= x 2 − 1 x[2, 4].

f(2)= 2 2 − 1 2 f(4)= 4 2 − 1 4 =4− 1 2 =16− 1 4 = 7 2 = 63 4

Average rate of change= f(4)−f(2) 4−2 = 63 4 − 7 2 4−2 = 49 4 2 = 49 8

f(x)=x−2 x[1, 9].1 2 F, d, F(d)= 2 d 2 . F(d)= 2 d 2 [2,6].

Average rate of change = F(6)−F(2) 6−2 = 2 6 2 − 2 2 2 6−2 Simplify.

= 2 36 − 2 4 4 = − 16 36 4

Combine numerator terms. =− 1 9 Simplify

− 1 9 g(t)= t 2 +3t+1[0, a].a.

Average rate of change= g(a)−g(0) a−0 Evaluate. = (a 2 +3a+1)−(0 2 +3(0)+1) a −0 Simplify. = a 2 +3a+1−1 a Simplify and factor. = a(a+3) a Divide by the common factor a. =a+3

a t=0 t=a. [0,5], 5+3=8.f(x)= x 2 +2x−8[5, a]. a+7 f(x)= x 3 −12x (−∞, −2) ∪ (2, ∞) (−2, 2). x=−2. −16 x=2. f f(b)>f(a) ab b>a. f f(b)<f(a) abb>a.fb(a,c)f(b)f(x)xxbfb(a,c)f(b)f(x)xxb p(t) t=1 t=3 t=4 (4,∞). t=1 t=3 t=4 f(x)= 2 x + x 3 . x=2 x=3, x=−3 x=−2. (−∞,−2.449) (2.449,∞). ± 6 , f(x)= x 3 −6 x 2 −15x+20 (−1,28), (5,−80). (−∞,−1)∪(5,∞) (−1,5). f f. x=1 x=1. y x=1, 2. x=−1 x=−1. x=−1, −2. y- f(x)= x 3 f x=c f(c) f(c)≥f(x) x f. f x=d f(d) f(d)≤f(x) x f. f f. x=−2 x=2, x=−2 x=2, 16. x=3, x=3,−10. Δy Δx = f(x 2)−f(x 1) x 2 − x 1 f (a,b) (b,c), f (a,c)? y= x 2 , b h. f(x)=4 x 2 −7 [1, b]4(b+1)g(x)=2 x 2 −9 [4, b]p(x)=3x+4 [2, 2+h]k(x)=4x−2 [3, 3+h] f(x)=2 x 2 +1 [x,x+h]4x+2hg(x)=3 x 2 −2 [x,x+h]a(t)= 1 t+4 [9,9+h]−1 13(13+h)b(x)= 1 x+3 [1,1+h]j(x)=3 x 3 [1,1+h]3 h 2 +9h+9r(t)=4 t 3 [2,2+h]f(x+h)−f(x) h f(x)=2 x 2 −3x [x,x+h] 4x+2h−3 f x=1 x=4. x=2 x=5.4 3 (−∞,−2.5)∪(1,∞), (−2.5, 1) (−∞,1)∪(3,4), (1,3)∪(4,∞) f (−3, 60), (3, −60) (7, 150), (−7.5, −220)f(x)= x 2 [1, 5]h(x)=5−2 x 2 [−2,4]q(x)= x 3 [−4,2] g(x)=3 x 3 −1 [−3,3]y= 1 x [1, 3]p(t)= (t 2 −4)(t+1) t 2 +3 [−3,1]k(t)=6 t 2 + 4 t 3 [−1,3] f(x)= x 4 −4 x 3 +5 (3,−22), (−∞, 3), (3, ∞) h(x)= x 5 +5 x 4 +10 x 3 +10 x 2 −1g(t)=t t+3 (−2,−2), (−3,−2), (−2, ∞)k(t)=3 t 2 3 −tm(x)= x 4 +2 x 3 −12 x 2 −10x+4 (−0.5, 6), (−3.25,−47) (2.1,−32), (−∞,−3.25) (−0.5, 2.1), (−3.25, −0.5) (2.1, ∞) n(x)= x 4 −8 x 3 +18 x 2 −6x+2 f (1.333, 5.185) f(x)= 1 x . c f (1,c) − 1 4 . f(x)= 1 x b f (2,b) − 1 10 .b=5 d(t)=2.6667 t 2 , t d(t) t=1 t=2. t t=5 t=15. f(b)<f(a) a b b>a f(b)>f(a) a b b>a C(T) C T T(d) d Cost=C(T(d)) T(d). T(5) C(T(5)). w(y) h(y) y, T

T(y)=h(y)+w(y)
T=h+w

f(x) g(x) f+g, f−g, fg, f g

(f+g)(x)=f(x)+g(x) (f−g)(x)=f(x)−g(x) (fg)(x)=f(x)g(x) (f g)(x)= f(x) g(x)

(g−f)(x) (g f)(x), f(x)=x−1 g(x)= x 2 −1.

(g−f)(x)=g(x)−f(x) (g−f)(x)= x 2 −1−(x−1) = x 2 −x =x(x−1) (g f)(x)= g(x) f(x) (g f)(x)= x 2 −1 x−1 = (x+1)(x−1) x−1 where x≠1 =x+1

(g f)(x), x≠1 x=1, (fg)(x) (f−g)(x).

f(x)=x−1 and g(x)= x 2 −1

(fg)(x)=f(x)g(x)=(x−1)(x 2 −1)= x 3 − x 2 −x+1 (f−g)(x)=f(x)−g(x)=(x−1)−(x 2 −1)=x− x 2

(f∘g)(x)=f(g(x))

“f g x,”“f g x.” ∘ f(g(x))≠f(x)g(x). g x g(x). f g(x) f(g(x)). f∘g g∘f f(g(x))≠g(f(x)) x. f(x)= x 2 g(x)=x+2,

f(g(x))=f(x+2) = (x+2) 2 = x 2 +4x+4
g(f(x))=g(x 2) = x 2 +2

x, x=− 1 2 . x f g, f∘g

(f∘g)(x)=f(g(x))

f∘g x x g g(x) f. fg f(g(x)), f(x)g(x)≠f(g(x)). f(g(x)) g(f(x)).

f(x)=2x+1 g(x)=3−x

g(x) f(x).

f(g(x))=2(3−x)+1 =6−2x+1 =7−2x

f(x) g(x).

g(f(x))=3−(2x+1) =3−2x−1 =−2x+2

g(f(x))≠f(g(x)), c(s) s s(t) t c(s(3)). s(3). t=3 s(3) s(3) c(s) f(x) x g(y) y f(g(y)) g(f(x))? y=f(x)

number of miles =f (number of hours)

g(y)

number of gallons =g (number of miles)

g(y) f(x) f(g(y)) f(x) g(y) f(x) g(y), g(f(x)) g, f(x), x f(g(y)) g(f(x)) G(r).Fa(F).a(G(r))G(a(F)) f(g(3)) g(f(3)).xf(x)g(x) f(g(3)), g(3) g: g(3)=2. f, g(3) f(2). f, f(2)=8.

g(3)=2 f(g(3))=f(2)=8

g(f(3)), f(3) f(3)=3. g,

g(f(3))=g(3)=2

f∘g g∘f xg(x)f(g(x))f(x)g(f(x)) f(g(1)) g(f(4)).f(g(1))=f(3)=3 g(f(4))=g(1)=3 x-y- x- y- x- y- f(g(1)). f(g(1)), g(1) g(x), x- g(1)=3. f.

f(g(1))=f(3)

f(x), x- f(3)=6, f(g(1))=6. g(f(2)).g(f(2))=g(5)=3 f(g(x)). f(t)= t 2 −t, f(t)= t 2 −t h(x)=3x+2, f(h(1)). h(1), h(x)

h(1)=3(1)+2 h(1)=5

f(h(1))=f(5), f(t)

f(h(1))=f(5) f(h(1))= 5 2 −5 f(h(1))=20

t x f(t)= t 2 −t h(x)=3x+2, h(f(2))h(f(−2)) f∘g g f. f∘g. f g x f(g(x)), x g g(x) f, f(g(x)) f∘g g g f. f g x x g g(x) f. f(g(x)) x g g(x) f. f(g(x)), g. f. x g g(x) f. x g g(x) f. f∘g.

(f∘g)(x) where f(x)= 5 x−1 and g(x)= 4 3x−2

g(x) x= 2 3 , f g(x) x g(x)=1.

4 3x−2 =1 4=3x−2 6=3x x=2

f∘g 2 3 2.

x≠ 2 3 or x≠2

(−∞, 2 3)∪(2 3 ,2)∪(2,∞)

(f∘g)(x) where f(x)= x+2 and g(x)= 3−x

g (−∞,3].

(f∘g)(x)= 3−x+2 or (f∘g)(x)= 5−x

(−∞,5]. f∘g, (−∞,3] f∘g. f∘g g, (−∞,3]. f∘g f, g.

(f∘g)(x) where f(x)= 1 x−2 and g(x)= x+4

[−4,0)∪(0,∞) f(x)= 5− x 2 g h, f(x)=g(h(x)). f(x). 5− x 2

h(x)=5− x 2 and g(x)= x

g(h(x))=g(5− x 2)= 5− x 2

f(x)= 4 3− 4+ x 2 g(x)= 4+ x 2h(x)= 4 3−xf=h∘g(f∘g)(x)=f(g(x)) f g ? g f g, f∘g?
f(x)=x+1 and g(x)=x−1. f(g(x))=f(x−1)=(x−1)+1=x g(f(x))=g(x+1)=(x+1)−1=x. f∘g=g∘f. f∘g?
f(x)= x 2 +2x g(x)=6− x 2 , f+g, f−g, fg, f g .(f+g)(x)=2x+6, (−∞,∞)(f−g)(x)=2 x 2 +2x−6, (−∞,∞)
(fg)(x)=− x 4 −2 x 3 +6 x 2 +12x, (−∞,∞)(f g)(x)= x 2 +2x 6− x 2 , (−∞,− 6)∪(− 6 , 6)∪(6 ,∞)
f(x)=−3 x 2 +x g(x)=5, f+g, f−g, fg, f g . f(x)=2 x 2 +4x g(x)= 1 2x , f+g, f−g, fg, f g .
(f+g)(x)= 4 x 3 +8 x 2 +1 2x , (−∞,0)∪(0,∞)(f−g)(x)= 4 x 3 +8 x 2 −1 2x , (−∞,0)∪(0,∞)
(fg)(x)=x+2, (−∞,0)∪(0,∞)(f g)(x)=4 x 3 +8 x 2 , (−∞,0)∪(0,∞) f(x)= 1 x−4 g(x)= 1 6−x ,
f+g, f−g, fg, f g . f(x)=3 x 2 g(x)= x−5 , f+g, f−g, fg, f g .(f+g)(x)=3 x 2 + x−5 , [5,∞)
(f−g)(x)=3 x 2 − x−5 , [5,∞)(fg)(x)=3 x 2 x−5 , [5,∞)(f g)(x)= 3 x 2 x−5 , (5,∞) f(x)= x g(x)=|x−3|,
g f . f(x)=2 x 2 +1 g(x)=3x−5, f(g(2))f(g(x))g(f(x))(g∘g)(x)(f∘f)(−2) f(g(x))=2 (3x−5) 2 +1;
f(g(x))=6 x 2 −2; (g∘g)(x)=3(3x−5)−5=9x−20; (f∘f)(−2)=163 f(g(x)) g(f(x)).
f(x)= x 2 +1, g(x)= x+2f(x)= x +2, g(x)= x 2 +3f(g(x))= x 2 +3 +2, g(f(x))=x+4 x +7f(x)=| x |, g(x)=5x+1
f(x)= x 3 , g(x)= x+1 x 3f(g(x))= x+1 x 3 3 = x+1 3 x , g(f(x))= x 3 +1 xf(x)= 1 x−6 , g(x)= 7 x +6
f(x)= 1 x−4 , g(x)= 2 x +4(f∘g)(x)= 1 2 x +4−4 = x 2 , (g∘f)(x)=2x−4 f(g(h(x))). f(x)= x 4 +6,
g(x)=x−6, h(x)= xf(x)= x 2 +1, g(x)= 1 x , h(x)=x+3f(g(h(x)))= (1 x+3) 2 +1 f(x)= 1 x g(x)=x−3,
(f∘g)(x) (f∘g)(x) (g∘f)(x) (g∘f)(x) (f g)x f(x)= 2−4x g(x)=− 3 x , (g∘f)(x) (g∘f)(x) (g∘f)(x)=− 3 2−4x ;
(−∞, 1 2) f(x)= 1−x x and g(x)= 1 1+ x 2 , (g∘f)(x)(g∘f)(2) p(x)= 1 x m(x)= x 2 −4, p(x) m(x)p(m(x))
m(p(x)) (0,2)∪(2,∞); (−∞,−2)∪(2,∞); (0,∞) q(x)= 1 x h(x)= x 2 −9, q(x) h(x)q(h(x))h(q(x)) f(x)= 1 x
g(x)= x−1 , (f∘g)(x) (1,∞) f(x) g(x) h(x)=f(g(x)).h(x)= (x+2) 2h(x)= (x−5) 3f(x)= x 3 g(x)=x−5
h(x)= 3 x−5h(x)= 4 (x+2) 2f(x)= 4 x g(x)= (x+2) 2h(x)=4+ x 3h(x)= 1 2x−3 3f(x)= x 3 g(x)= 1 2x−3
h(x)= 1 (3 x 2 −4) −3h(x)= 3x−2 x+5 4f(x)= x 4 g(x)= 3x−2 x+5h(x)= (8+ x 3 8− x 3) 4h(x)= 2x+6
f(x)= xg(x)=2x+6h(x)= (5x−1) 3h(x)= x−1 3f(x)= x 3g(x)=(x−1)h(x)=| x 2 +7 |h(x)= 1 (x−2) 3f(x)= x 3
g(x)= 1 x−2h(x)= (1 2x−3) 2h(x)= 2x−1 3x+4f(x)= xg(x)= 2x−1 3x+4 f, g, f(g(3))f(g(1))g(f(1))

g(f(0))f(f(5))f(f(4))g(g(2))g(g(0)) f(x), g(x), h(x),g(f(1))g(f(2))f(g(4))f(g(1))f(h(2))
h(f(2))f(g(h(4)))f(g(f(−2))) f and g xf(x)g(x)f(g(8))f(g(5))g(f(5))g(f(3))f(f(4))
f(f(1))g(g(2))g(g(6)) f and g xf(x)g(x)(f∘g)(1)(f∘g)(2)(g∘f)(2)(g∘f)(3)(g∘g)(1)(f∘f)(3) f(g(0))
g(f(0)).f(x)=4x+8, g(x)=7− x 2f(x)=5x+7, g(x)=4−2 x 2f(g(0))=27, g(f(0))=−94
f(x)= x+4 , g(x)=12− x 3f(x)= 1 x+2 , g(x)=4x+3f(g(0))= 1 5 , g(f(0))=5 f(x)=2 x 2 +1 g(x)=3x+5 f(g(2))
f(g(x))18 x 2 +60x+51g(f(−3))(g∘g)(x)g∘g(x)=9x+20 f(x)= x 3 +1 g(x)= x−1 3 . (f∘g)(x) (g∘f)(x).
(f∘g)(2) (g∘f)(2). (g∘f)(x)? (f∘g)(x)?(−∞,∞) f(x)= 1 x . (f∘f)(x). (f∘f)(x) f F(x)= (x+1) 5 , f(x)= x 5 ,
g(x)=x+1. (g∘f)(x)=F(x). (f∘g)(x)=F(x). f(x)= x 2 +2 x≥0 g(x)= x−2 .(f∘g)(6); (g∘f)(6)(f∘g)(6)=6
(g∘f)(6)=6(g∘f)(a); (f∘g)(a)(f∘g)(11); (g∘f)(11)(f∘g)(11)=11 , (g∘f)(11)=11 D(p) p. C(x) x D(C(6)).
C(D(6)). D(C(x))=6. C(D(p))=6. A(d) d t m(t). A(m(4)). m(A(4)). A(m(t))=4. m(A(d))=4. x
P(x) x. r(t)=25 t+2 , t=2.A(t)=π (25 t+2) 2 A(2)=π (25 4) 2 =2500π r(t)=2t+1, t
A(5)=π (2(5)+1) 2 =121π r, V, r(V)= 3V 4π 3 . t V(t)=10+20t. r(V(t)).N(T)=23 T 2 −56T+1,
3<T<33, TT(t)=5t+1.5,t N(T(t)). N(T(t))=23 (5t+1.5) 2 −56(5t+1.5)+1; x y g(x)=f(x)+k, f(x) k k=1
f(x)= x 3 . y=f(x). f(x)+k y+k. y y+k, y- k. f(x),g(x)=f(x)+k, kf(x).kkk V t. S(t),

S(t)=V(t)+20

t,S(t) V S V, tV(t)S(t) f(x) g(x)=f(x)−3.xf(x) g(x)=f(x)−3 g f.

f(2)=1 Given g(x)=f(x)−3 Given transformation g(2)=f(2)−3 =1−3 =−2

f(x) g(x) xf(x)g(x) h(t)=−4.9 t 2 +30t h t b(t) h(t), b(t).

b(t)=h(t)+10=−4.9 t 2 +30t+10

f(x)= x 3 . h=+1 x. f(x)= x 2 , g(x)= (x−2) 2 f. f, g(x)=f(x−h), h f. h h V(t) F(t)

V(t)= the original venting plan F(t)=starting 2 hrs sooner

V V, F, V(8)=F(6). 220 ft 2 220 ft 2 V(10)=F(8). F(t) h=−2. F(t)=V(t−(−2))=V(t+2). V(t+2)
F(t) V(t), V(t). V F:F(t)=V(t+2). f(x) g(x)=f(x−3).xf(x) g(x)=f(x−3) g f f(2)=1. g, g(x) f.

g(5)=f(5−3) =f(2) =1

xx−3f(x)g(x) g(x) g(x) f(x), x, f(x)= x 2 . g(x) f(x), g(x). f(x)= x 2

g(x)=f(x−2)

x=2 y=0; f(x) g(x) f(x−2).

f(x)= x 2 g(x)=f(x−2) g(x)=f(x−2)= (x−2) 2

+2 −2 f(0)=0. g(2)=0. f, g(2)=f(x−2)=f(0)=0. G(m) m G(m)+10 G(m+10).G(m)+10 m G(m+10) m
f(x)= x , f(x) g(x)=f(x+2) f(x) g(x) y-x- f(x)=| x |, h(x)=f(x+1)−3. f h f f(x+1) f(x+1)−3 f(x)=| x |.(0,0)
(0,0)→(−1,0)(−1,0)(−1,0)→(−1,−3) h. f(x)=| x |, h(x)=f(x−2)+4.

h(x)=f(x−1)+2

h(x)= x−1 +2

[1,∞) [2,∞). f(x)= 1 x g(x)= 1 x-1 +1 f(x), g(x)=−f(x) f(x), f(x), g(x)=f(−x) f(x), s(t)= t

V(t)=−s(t) or V(t)=− t

s(t)

H(t)=s(−t) or H(t)= −t

[0,∞) [0,∞), V(t) (−∞, 0] H(t) (−∞, 0]. f(x)=|x−1| f(x) g(x)=−f(x) h(x)=f(−x)xf(x) g(x), x g(x) h(x),
h(x) f(x) xh(x) f(x) g(x)=−f(x)h(x)=f(−x)xf(x)g(x)=−f(x)xg(x)−5−10−15−20h(x)=f(−x)xh(x)
k(t)=− 2 −t +1, ktf(t)= 2 tk(t).f(−t)= 2 −t −f(−t)=− 2 −t −f(−t)+1=− 2 −t +1 t≥0, [0,1). f(x)= x 2 ,
g(x)=−f(x) h(x)=f(−x). g(x)=f(−x) f(x)f(x)= x 2f(x)=| x | f(x)= x 3 f(x)= 1 x f(x)= 2 x f(x)=0. x

f(x)=f(−x)

y- x

f(x)=−f(−x)

f(x)=f(−x). f(x)=−f(−x). f(x)= x 3 +2x

f(−x)= (−x) 3 +2(−x)=− x 3 −2x

−f(−x)=−(− x 3 −2x)= x 3 +2x

−f(−x)=f(x), f (x,y) (−x,−y) f, (−1,−3) f(s)= s 4 +3 s 2 +7 f(x), g(x)=af(x), a f(x). a>1, 0<a<1, a<0,

a. a. a>1, a. 0<a<1, a. a<0, P(t) Q,

(0, 1)→(0, 2) (3, 3)→(3, 6) (6, 2)→(6, 4) (7, 0)→(7, 0)

Q(t)=2P(t)

t, Q P. t, a. a. f g(x)= 1 2 f(x).xf(x) g(x)= 1 2 f(x) g f f(4)=3.

g(4)= 1 2 f(4)= 1 2 (3)= 3 2

x2468g(x)1 23 27 211 2 g(x) 1 2 . f g(x)= 3 4 f(x).xf(x)xg(x) f(x)= x 3 . g(x) f(x), g(x). g(2)=2.
f(2)= 2 3 =8. g 1 4 f g(2)= 1 4 f(2). g(x)= 1 4 f(x). g f.

g(x)= 1 4 f(x)= 1 4 x 3

g(x)=3x-2 y=f(x), y=f(bx) y= x 2 . y= (0.5x) 2 y= x 2 y= (2x) 2 y= x 2 f(x), g(x)=f(bx), b f(x).
b>1, 1 b . 0<b<1, 1 b . b<0, g(x)=f(bx) b>1 0<b<1 R,

R(1)=P(2), R(2)=P(4), and in general, R(t)=P(2t).

f(x) g(x)=f(1 2 x).xf(x) g(x)=f(1 2 x) g f g(2) g(2)=f(1 2 ·2)=f(1), f(1) g f g(4).

g(4)=f(1 2 ·4)=f(2)=1

xg(x) g(x) g(x) f(x) g(x) f(x) f(x) (6,4) g(x) (2,4), x- 1 3 , 6(1 3)=2. g(2)=f(6) g(1)=f(3).
g(x)=f(3x). 1 3 . 1 4 f(1 4 x). g(x)=f(1 3 x) g(x)= 1 3 x 2f(x)+3, f(x), g(x)=f(2x+3), g f. f(7)=12.
g x g(x)=f(2x+3)=12? 2x+3=7. x,

f(bx+p)=f(b(x+ p b))

f(x)= (2x+4) 2

f(x)= (2(x+2)) 2

af(x)+k, a k. f(bx+h), h 1 b . f(b(x+h)), 1 b h. f(x), g(x)=2f(3x)+1.xf(x) f(3x) 1 3 , x- 1 3 .xf(3x)x
2f(3x)xg(x)=2f(3x)+1 f(x) k(x)=f(1 2 x+1)−3.

f(1 2 x+1)−3=f(1 2 (x+2))−3

1 2 x+2. −3 g(x)=f(x)+k k>0g(x)=f(x−h) h>0g(x)=−f(x)g(x)=f(−x)g(x)=af(x) a>0g(x)=af(x) (0<a<1)
g(x)=f(bx)(0<b<1)g(x)=f(bx) b>1 x-y-y- f(x)=f(−x). f(x)=−f(−x). f, (−x) (x) f(x). f(−x)=f(x),
f(−x)=−f(x), f(x)= x f(x)=| x | g(x)=|x-1|−3 f(x)= 1 x f(x)= 1 x 2 g(x)= 1 (x+4) 2 +2 f.y=f(x−49)
y=f(x+43) f(x+43) f.y=f(x+3)y=f(x−4) f(x-4) f.y=f(x)+5y=f(x)+8 f(x)+8 f.y=f(x)−2y=f(x)−7 f(x)−7 f.
y=f(x−2)+3y=f(x+4)−1f(x+4)−1f.f(x)=4 (x+1) 2 −5g(x)=5 (x+3) 2 −2 (−∞,−3) (−3,∞)a(x)= −x+4
k(x)=−3 x −1(0, ∞) f(x)= 2 x f(x).g(x)= 2 x +1h(x)= 2 x −3w(x)= 2 x−1f(t)= (t+1) 2 −3h(x)=|x−1|+4
k(x)= (x−2) 3 −1m(t)=3+ t+2 f, g, h g(x) h(x) f(x).xf(x)xg(x)xh(x)g(x)=f(x-1), h(x)=f(x)+1 f, g, h g(x)
h(x) f(x).xf(x)xg(x)xh(x)f(x)=|x-3|−2f(x)= x+3 −1f(x)= (x-2) 2f(x)=|x+3|−2f(x)=− xf(x)=− (x+1) 2 +2
f(x)= −x +1f(x)=3 x 4g(x)= xh(x)= 1 x +3xf(x)= (x−2) 2g(x)=2 x 4h(x)=2x− x 3 f.g(x)=−f(x) g x f.
g(x)=f(−x)g(x)=4f(x) g f.g(x)=6f(x)g(x)=f(5x) g 1 5 f.g(x)=f(2x)g(x)=f(1 3 x) g f.g(x)=f(1 5 x)
g(x)=3f(−x) g y f.g(x)=−f(3x) g f(x)=|x| y 1 4g(x)=|−4x| f(x)= x x f(x)= 1 x 2 1 3 ,
g(x)= 1 3 (x+2) 2 −3 f(x)= 1 x f(x)= x 2 1 2 , g(x)= 1 2 (x-5) 2 +1 f(x)= x 2 g(x)=4 (x+1) 2 −5
f(x)= x 2 g(x)=5 (x+3) 2 −2h(x)=−2|x−4|+3 f(x)=|x| k(x)=−3 x −1m(x)= 1 2 x 3 f(x)= x 3 1 2 .
n(x)= 1 3 |x−2|p(x)= (1 3 x) 3 −3q(x)= (1 4 x) 3 +1a(x)= −x+4 f(x)= x x=4.g(x)=f(x)−2g(x)=−f(x)
g(x)=f(x+1)g(x)=f(x−2) f(x)=f(−x), y- b>1 −1 0<b<1 f(x)=−f(−x), 0<a<1 −1 a>1 f(x)=| x |,

f(x)=| x |={ x if x≥0 −x if x<0

x x x | x−5 |. x | x−5 |≤4.

−4≤x−5 x−5≤4 1≤x x≤9

| x−5 |≤4 1≤x≤9. x | x−2 |≤3 ±1%, ±5%, ±10%. ±5%. R

| R−680 |≤34

p | p−80 |≤20 y=2| x−3 |+4. y=| x | (3,4)

f(x)=2|x−3|−2, treating the stretch as a vertical stretch, or f(x)=|2(x−3)|−2,
treating the stretch as a horizontal compression.

x f(x).

f(x)=a|x−3|−2

2=a| 1−3 |−2 4=2a a=2

f(x)=−| x+2 |+3 8=| 2x−6 |,

2x−6=8 or 2x−6=−8 2x=14 2x=−2 x=7 x=−1

| x |=4, | 2x−1 |=3 | 5x+2 |−4=9

A B, | A |=B, B≥0, A=B A=−B. B<0, | A |=B | A |=B A=B −A=B, B>0. x. f(x)=| 4x+1 |−7x f(x)=0 0=|4x+1|−7 Substitute 0 for f(x). 7=|4x+1| Isolate the absolute value on one side of the equation. 7=4x+1 or −7=4x+1 Break into two separate equations and solve. 6=4x −8=4x x= 6 4 =1.5 x= −8 4 =−2 x=1.5 x=−2. f(x)=| 2x−1 |−3, x f(x)=0. x=−1 x=2 | A |=B? 2+| 3x−5 |=1. | A |=B A=B A=−B. x. 1=4| x−2 |+2.

1=4| x−2 |+2 −1=4| x−2 | − 1 4 =| x−2 |

f(x)=1 g(x)=4| x−2 |+2 f g 1=4| x−2 |+2 f(x)=−| x+2 |+3 f(0)=1, (0,1). f(x)=0 x=−5 x=1 (−5,0) (1,0).

|A|<B,|A|≤B,|A|>B, or |A|≥B,

A B x. x

| x |<200 or −200<x<200

x x x | x−600 |.

|x−600|<200 or −200<x−600<200 −200+600<x−600+600<200+600 400<x<800

| x−A |≤B a b b | x−A |=B. | x−A |≤B. |x −5|≤4. | x−5 |=4. | x−5 |=4.

x−5=4 x=9 or x−5=−4 x=1

x=1 x=9,

x<1, 1<x<9, and x>9.

xf(x)<4 >4?x<1| 0−5 |=51<x<9| 6−5 |=1x>9| 11−5 |=6 1≤x≤9 | x−5 |≤4 1≤x≤9, [1,9]. f(x)=| x−5 |. g(x)=4 x=1 x=9. f g 1<x<9. f(x) g(x). 1≤x≤9. [1,9].

|x−A|<C, |x−A|>C, −C<x−A<C, x−A<−C or x−A>C.

< > ≤ or ≥.

| x−5|≤4 −4≤x−5≤4 Rewrite by removing the absolute value bars. −4+5≤x−5+5≤4+5 Isolate the x.

1≤x≤9

| x+2 |≤6.4≤x≤8f(x)=− 1 2 | 4x−5 |+3, x- f(x)<0, − 1 2 |4x−5|+3<0.

− 1 2 |4x−5|<−3 Multiply both sides by −2, and reverse the inequality. |4x−5|>6

| 4x−5 |=6.

4x−5=6 4x−5=−6 4x−5=6 or 4x=−1 x= 11 4 x=− 1 4

f x=− 1 4 x= 11 4 x=− 1 4 x= 11 4 . x=− 1 4 , x= 11 4 .

x<− 1 4 or x> 11 4

(−∞,−0.25)∪(2.75,∞). −2| k−4 |≤−6.k≤1 k≥7; (−∞,1]∪[7,∞) | A |<B, | A |≤B, | A |>B, or | A |≥B.
|A|=B. A, B. A −B. x x x x 1 2 | x+4 |= 1 2 x f(x) f(x) |f(x)−8|<0.03|x+3|=9|6−x|=5{ 1,11 }
|5x−2|=11|4x−2|=11{ 9 4 , 13 4 }2|4−x|=73|5−x|=5{ 10 3 , 20 3 }3|x+1|−4=55| x−4 |−7=2
{ 11 5 , 29 5 }0=−| x−3 |+22| x−3 |+1=2{ 5 2 , 7 2 }| 3x−2 |=7| 3x−2 |=−7| 1 2 x−5 |=11| 1 3 x+5 |=14
{ −57,27 }−| 1 3 x+5 |+14=0f(x)=2| x+1 |−10(0,−8); (−6,0), (4,0)f(x)=4| x−3 |+4f(x)=−3| x−2 |−1
(0,−7); xf(x)=−2| x+1 |+6| x−2 |>10(−∞,−8)∪(12,∞)2| v−7 |−4≥42| 3x−4 |≤8−4 3 ≤x≤4| x−4 |≥8
| 3x−5 |≥13(−∞,− 8 3]∪[6,∞)| 3x−5 |≥−13| 3 4 x−5 |≥7(−∞,− 8 3]∪[16,∞)| 3 4 x−5 |+1≤16y=|x−1|
y=|x+1|y=|x|+1y=| x |−2y=−| x |y=−| x |−2y=−| x−3 |−2f(x)=−|x−1|−2f(x)=−|x+3|+4f(x)=2|x+3|+1
f(x)=3| x−2 |+3f(x)=| 2x−4 |−3f(x)=| 3x+9 |+2f(x)=−| x−1 |−3f(x)=−| x+4 |−3f(x)= 1 2 | x+4 |−3
f(x)=10|x−2|[0,4]. [0,20] f(x)=−100|x|+100 [−5,5]. f(x)=−0.1| 0.1(0.2−x) |+0.3x-
f(x)=4× 10 9 | x−(5× 10 9) |+2× 10 9|−2x− 2 3 (x+1)|+3>−1(−∞,∞)ax-f(x)=2| x+1 |+a. a y
f(x)=2| x+1 |+a. a y y x=0. x p | p−0.08 |≤0.015 x x | x−5.0 |≤0.01 x | A |=B, B≥0; A=B A=−B
| A |<B, | A |≤B, | A |>B, or | A |≥B

C= 5 9 (F−32)

F

5 9 (75−32)≈24°C.

F C.

26= 5 9 (F−32) 26· 9 5 =F−32 F=26· 9 5 +32≈79 f(x), f −1 (x), “f x.” −1 −1 f −1 (x) 1 f(x) 1 f(x) f a −1 a=1 a, f −1 ◦f

(f −1 ◦f)(x)= f −1 (f(x))= f −1 (y)=x

x f. f(x), g(x) f(x) g(f(x))=x f(g(x))=x y=4x y= 1 4 x

(f −1 ◦f)(x)= f −1 (4x)= 1 4 (4x)=x

(f ◦ f −1)(x)=f(1 4 x)=4(1 4 x)=x

y=4x y= 1 4 x f(x)=y, f −1 (x) f f −1 (y)=x. f −1 (f(x))=x x f. f(f −1 (x))=x x f −1 f −1 f. f −1“ f f −1 , f −1 (x),“fx.”

f −1 (x)≠ 1 f(x)

f(2)=4 f(5)=12,

f(2)=4, then f −1 (4)=2; f(5)=12, then f −1 (12)=5.

g, g(4)=2 g(12)=5.(x,f(x))(x,g(x))(2,4)(4,2)(5,12)(12,5) h −1 (6)=2, h? h(2)=6 f(x) g(x), f(g(x))=x g(f(x))=x.g= f −1f= g −1 . g≠ f −1 f≠ g −1 . f(x)= 1 x+2 g(x)= 1 x −2, g= f −1 ?

g(f(x))= 1 (1 x+2) −2 =x+2−2 =x

g= f −1 and f= g −1

f(g(x))= 1 1 x −2+2 = 1 1 x =x

f(x)= x 3 −4 g(x)= x+4 3 , g= f −1 ? f(x)= x 3 g(x)= 1 3 x, g= f −1 ?

f(g(x))= x 3 27 ≠x

x 3 = x 1 3 , f(x)= (x−1) 3 and g(x)= x 3 +1, g= f −1 ? f f −1 , f f −1 . f f −1 , f f −1 . f(x)= x f −1 (x)= x 2 , [0,∞), f(x)= x . f(x)= x 2 . f(x)= x 2 [0,∞), f(x)= (x−1) 2 [1,∞), f −1 (x)= x +1. f f −1 [1,∞). f −1 f [0,∞). g h, f, g=h. f(x) f −1 (x). f(x) f −1 (x).f(x)=cf(x)=xf(x)= x 2f(x)= x 3 f(x)= 1 xf(x)= 1 x 2f(x)= x 3f(x)= xf(x)=| x | [0,∞), (0,∞). f (1,∞) f (−∞,−2). f −1 (−∞,−2) f −1 (1,∞). f(t) t f −1 (70).t (minutes)f(t) (miles) f f. f −1 (70), f, f −1 (70)=90. f(a)=b, f −1 (b)=a. f −1 (70)=a, a f(a)=70. t f(t)=70, t=90. f(60), f −1 (60).t (minutes)f(t) (miles)f(60)=50. f −1 (60)=70. g(x) g(3) g −1 (3).g(3), (3,1) g(3)=1. g −1 (3), g −1 (3) g(x)=3. (5,3) g(5)=3, g −1 (3)=5. g −1 (1), g −1 (4). y x— x y. f x. x y.

C= 5 9 (F−32)

C= 5 9 (F−32) C· 9 5 =F−32 F= 9 5 C+32

C=h(F)= 5 9 (F−32),

F= h −1 (C)= 9 5 C+32.

h C −1 x y y= 1 3 (x−5)x=3y+5 f(x)= 2 x−3 +4.

y= 2 x−3 +4 Set up an equation. y−4= 2 x−3 Subtract 4 from both sides. x−3= 2 y−4 Multiply both sides by x−3 and divide by y−4. x= 2 y−4 +3 Add 3 to both sides.

f −1 (y)= 2 y−4 +3 f −1 (x)= 2 x−4 +3. f f f −1 xf −1 (y)f(x)y f(x)=2+ x−4 .

y=2+ x−4 (y−2) 2 =x−4 x= (y−2) 2 +4

f −1 (x)= (x−2) 2 +4. f [4,∞). f [2,∞), f −1 [2,∞). f −1 (x) x. f −1 f f −1 [2,∞) f −1 f −1 f. f(x)=2− x ?f −1 (x)= (2−x) 2 ; domain of f: [0,∞); domain of f −1 : (−∞,2] f(x)= x 2 [0,∞), [0,∞) f −1 (x)= x .f f −1x-f and f −1 ? f −1 (x) f(x) y=x, f(x) f −1 (x). (0,∞) (−∞,∞), (−∞,∞) (0,∞). y=x, (1,0) (0,1) (4,2) (2,4). f f −1 f= f −1 , f(f(x))=x,

1 1 x =x

f(x)=c−x, c g(x) f(x), g(f(x))=f(g(x))=x. y=f(x) x y. x y. y=x. y y f(x)= x 2 f(x)= 1 x y=f(x), x y. x y. y. y y= f −1 (x). f(x)=a−x a. f −1 (x) f(x)=x+3f −1 (x)=x−3f(x)=x+5f(x)=2−xf −1 (x)=2−x f(x)=3−xf(x)= x x+2f −1 (x)= −2x x−1f(x)= 2x+3 5x+4 f f f(x)= (x+7) 2f(x): [−7,∞); f −1 (x)= x −7 f(x)= (x−6) 2f(x)= x 2 −5 f(x): [0,∞); f −1 (x)= x+5 f(x)= x 2 +x g(x)= 2x 1−x : f(g(x)) g(f(x)). f(x) g(x)? f(g(x))=x g(f(x))=x. f g f(x) g(x) f(x)= x−1 3 g(x)= x 3 +1 f(g(x))=x, g(f(x))=xf(x)=−3x+5 g(x)= x−5 −3f(x)= xf(x)= 3x+1 3f(x)=−5x+1f(x)= x 3 −27 f f(0).3 f(x)=0. f −1 (0).2 f −1 (x)=0. f −1 . f(6) and f −1 (2). f f. [2,10] f f. f f(6)=7, f −1 (7).6 f(3)=2, f −1 (2). f −1 (−4)=−8, f(−8).−4 f −1 (−2)=−1, f(−1).xf(x) f(1).0 f(x)=3. f −1 (0). 1 f −1 (x)=7. f f −1 (x).xf(x)xf −1 (x) f(x)= 3 x−2f(x)= x 3 −1f −1 (x)= (1+x) 1/3 f(x)= 1 x−1 . x y f(x)= 9 5 x+32. f −1 (x)= 5 9 (x−32). x, C

C(r)=2πr. r(C). r(36π) t, d(t)=50t. t(d). t(180) t(d)= d 50 , t(180)= 180 50 . { (a,b),(c,d),(e,d) }
{ (5,2),(6,1),(6,2),(4,8) }y 2 +4=x, x y f(−3); f(2); f(−a); −f(a); f(a+h).f(x)=−2 x 2 +3xf(−3)=−27;
f(2)=−2;f(−a)=−2 a 2 −3a;−f(a)=2 a 2 −3a;f(a+h)=−2 a 2 +3a−4ah+3h−2 h 2f(x)=2| 3x−1 |f(x)=−3x+5
f(x)=| x−3 |f(x)=| x+1 |f(x)= x 2 −2f(2)f(−2)2 f(x)=−2, x. f(x)=1, x.x=−1.8 or x=1.8 h(t)=−16 t 2 +80t
h(2)−h(1) 2−1h(a)−h(1) a−1−64+80a−16 a 2 −1+a =−16a+64f(x)= 2 3x+2f(x)= x−3 x 2 −4x−12
(−∞,−2)∪(−2,6)∪(6,∞)f(x)= x−6 x−4f(x)={ x+1 x<−2 −2x−3 x≥−2 x=1 to x=2.f(x)=4x−3
f(x)=10 x 2 +x31f(x)=− 2 x 2 (2,∞); (−∞,2)(−3,1); (−∞,−3)∪(1,∞) (−2,−3); (1,3) [−3,3].
[−10,10]. (−1.8,10) (f∘g)(x) (g∘f)(x) f(x)=4−x, g(x)=−4xf(x)=3x+2, g(x)=5−6x
(f∘g)(x)=17−18x; (g∘f)(x)=−7−18xf(x)= x 2 +2x, g(x)=5x+1f(x)= x+2 , g(x)= 1 x
(f∘g)(x)= 1 x +2 ; (g∘f)(x)= 1 x+2 f(x)= x+3 2 , g(x)= 1−x (f∘g) (f∘g)(x) f(x)= x+1 x+4 , g(x)= 1 x
(f∘g)(x)= 1+x 1+4x , x≠0, x≠− 1 4f(x)= 1 x+3 , g(x)= 1 x−9f(x)= 1 x , g(x)= x(f∘g)(x)= 1 x , x>0
f(x)= 1 x 2 −1 , g(x)= x+1 H f g H(x)=(f∘g)(x).H(x)= 2x−1 3x+4 g(x)= 2x−1 3x+4 ; f(x)= x
H(x)= 1 (3 x 2 −4) −3f(x)= (x−3) 2f(x)= (x+4) 3f(x)= x +5f(x)=− x 3f(x)= −x 3f(x)=5 −x −4
f(x)=4[| x−2 |−6]f(x)=− (x+2) 2 −1 g f g(x)=f(x−1)g(x)=3f(x)f(x)=| x−3 |f(x)=3 x 4g(x)= xh(x)= 1 x +3x
f(x)=| x |.f(x)= 1 2 | x+2 |+1f(x)=−3| x−3 |+3f(x)=| x−5 |f(x)=−| x−3 |f(x)=| 2x−4 || x+4 |=18x=−22, x=14
| 1 3 x+5 |=| 3 4 x−2 || 3x−2 |<7(− 5 3 ,3)| 1 3 x−2 |≤7 f −1 (x) f(x)=9+10xf(x)= x x+2f −1 (x)= −2x x−1
f f f(x)= x 2 +1f(x)= x 3 −5g(x)= x+5 3 : f(g(x))g(f(x)).f(x)g(x)? f(g(x))=xg(f(x))=x.fgf(x)= 1 x
f(x)=−3 x 2 +xf(5)=2,f −1 (2).5f(1)=4,f −1 (4).y=2x+8{ (2,1),(3,2),(−1,1),(0,−2) } f(x)=−3 x 2 +2x
f(−2) f(a) f(x)=−2 (x−1) 2 +3 f(x)= 3−x f(x)=2 x 2 −5x, f(a+1)−f(1).2 a 2 −a
f(x)={ x+1 if −2<x<3 −x if x≥3 f(x)=3−2 x 2 +x f(b)−f(a) b−a .−2(a+b)+1
f(x)=3−2 x 2 +x and g(x)= x (g∘f)(x)(g∘f)(1)2 H(x)= 5 x 2 −3x 3 f g, (f∘g)(x)=H(x).f(x)= x+6 −1
f(x)= 1 x+2 −1f(x)=− 5 x 2 +9 x 6evenf(x)=− 5 x 3 +9 x 5f(x)= 1 xodd f(x)=−2| x−1 |+3.| 2x−3 |=17.x=−7
x=10 −| 1 3 x−3 |≥17. f(x)=3x−5f −1 (x)= x+5 3f(x)= 4 x+7 g (−∞,−1.1) and (1.1,∞)(1.1,−0.9) f(2).
f(2)=2 f(−2).f(x)={ | x | if x≤2 3 if x>2xF(x) F(6). F(x)=5.x=2 F −1 (15). f(x)=−2x+11, f −1 (x).
f −1 (x)=− x−11 2 f(x), f −1 (x) f −1 (f(x))=x x f; f(f −1 (x))=x x f −1xmb

Equation form y=mx+b Equation notation f(x)=mx+b

D(t)Dt.m,b

D(t)=83t+250

D(t)=83t+250,(0,250)D(t)=83t+250.f(x)=2 x +1.

f(x)=mx+b

bx=0m(0,b).P,d,P(d)=0.434d+14.696.f(x)=mx+b is an increasing function if m>0.
f(x)=mx+b is an decreasing function if m<0.f(x)=mx+b is a constant function if m=0.f(x)=60xx
f(x)=500−60xxxf(x)=50x 1x 2 ,y 1y 2(x 1 , y 1)(x 2 , y 2)m,

m= change in output (rise) change in input (run) = Δy Δx = y 2 − y 1 x 2 − x 1

ΔyΔxy1y2f,y1=f(x1)y2=f(x2),

m= f(x 2)−f(x 1) x 2 − x 1

(x 1, y 1)(x 2, y 2),yx.(x 2, y 2)(x 1 , y 1),units for the output units for the inputm

m= change in output (rise) change in input (run) = Δy Δx = y 2 − y 1 x 2 − x 1

x 1x 2y 1y 2f(x)(3,−2)(8,1)(3,−2)(8,1).

m= change in output change in input = 1−(−2) 8−3 = 3 5

m=0.6.m>0.f(x)(2, 3)(0, 4)m= 4−3 0−2 = 1 −2 =− 1 2m<0.27,800−23,400=4400

4,400 people 4 years =1,100 people year

m= 1,868−1,442 2,012−2,009 = 426 3 =142 people per year

y− y 1 =m(x− x 1)

m= y− y 1 x− x 1 assuming x≠ x 1 m(x− x 1)= y− y 1 x− x 1 (x− x 1) Multiply both sides by (x− x 1).

m(x− x 1)=y− y 1 Simplify. y− y 1 =m(x− x 1) Rearrange.

y−4=− 1 2 (x−6)

y−4=− 1 2 (x−6) y−4=− 1 2 x+3 Distribute the − 1 2 . y=− 1 2 x+7 Add 4 to each side.

y=− 1 2 x+7.

y− y 1 =m(x− x 1)

mx 1 and y 1x and y(4,1).m=2x 1 =4y 1 =1.

y− y 1 =m(x− x 1) y−1=2(x−4)

y−1=2(x−4) y−1=2x−8 Distribute the 2. y=2x−7 Add 1 to each side.

y−1=2(x−4)y=2x−7,(6,–1).m=3.x1=6y1 =−1.

y− y 1 =m(x− x 1) y−(−1)=3(x−6) Substitute known values. y+1=3(x−6) Distribute −1 to find point-slope form.

y+1=3(x−6) y+1=3x−18 Distribute 3. y=3x−19 Simplify to slope-intercept form.

–2(–2, 2).y−2=−2(x+2)y=−2x−2(0, 1)(3, 2).

m= y 2 − y 1 x 2 − x 1 = 2−1 3−0 = 1 3

y− y 1 =m(x− x 1) y−1= 1 3 (x−0)

y−1= 1 3 (x−0) y−1= 1 3 x Distribute the 1 3 . y= 1 3 x+1 Add 1 to each side.

m= y 2 − y 1 x 2 − x 1 = 7−1 8−5 = 6 3 =2

m=2.(5,1).

y− y 1 =m(x− x 1) y−1=2(x−5)

y 2 −1=2(x 2 −5).

y−1=2(x−5) y−1=2x−10 y=2x−9

y=2x−9.(–1,3) (0,0).y−0=−3(x−0)y=−3xf(0, 7)(4, 4).

m= y 2 − y 1 x 2 − x 1 = 4−7 4−0 =− 3 4

y− y 1 =m(x− x 1) y−4=− 3 4 (x−4)

y−4=− 3 4 (x−4) y−4=− 3 4 x+3 y=− 3 4 x+7

b=7.bmmbf(x)=− 3 4 x+7,y=− 3 4 x+7.f(0, 2)(−2,−4).

m= y 2 − y 1 x 2 − x 1 = −4−2 −2−0 = −6 −2 =3

y− y 1 =m(x− x 1) y−(−4)=3(x−(−2)) y+4=3(x+2)

y+4=3(x+2) y+4=3x+6 y=3x+2

(0, 2)b=2.CC(x)x37.5.C(x)=1250+37.5x.

C(100)=1250+37.5(100) =5000

ff(3)=−2f(8)=1

f(3)=−2→(3,–2) f(8)=1→(8,1)

m= y 2 − y 1 x 2 − x 1 = 1−(−2) 8−3 = 3 5

y− y 1 =m(x− x 1) y−(−2)= 3 5 (x−3)

y+2= 3 5 (x−3) y+2= 3 5 x− 9 5 y= 3 5 x− 19 5

f(x)f(2)=–11,f(4)=−25,y=−7x+3ff(c).f(x)=mx+b.x=c.N,t,N(0)=200,b=200.m=15.N(t)=15t+200.t=12.

N(12)=15(12)+200 =180+200 =380

I,n,I(n),(3,760)(5,920).

m= 920−760 5−3 = $160 2 policies =$80 per policy

I(n)=80n+b 760=80(3)+b when n=3, I(3)=760 760−80(3)=b 520=b

bn=0,

I(n)=80n+520

b=1000.m,

P(w)=40w+1000

(2,1080)(6,1240)

m= 1240−1080 6−2 = 160 4 =40

f(x)=mx+b,b.xH(x),xxH(x)H(x)=0.5x+12.5f(x)=mx+b

m= change in output (rise) change in input (run) = Δy Δx = y 2 − y 1 x 2 − x 1y− y 1 =m(x− x 1)mbE(t), tE(t)=3000−70t.E(t),tE(t)=1200+40t.d(t)=100−10tny= 1 4 x+6y=3x−5y=3 x 2 −23x+5y=15 3 x 2 +5y=153x+5 y 2 =15−2 x 2 +3 y 2 =6− x−3 5 =2yf(x)=4x+3g(x)=5x+6a(x)=5−2xb(x)=8−3x h(x)=−2x+4k(x)=−4x+1j(x)=12x−3p(x)=14x−5n(x)=−13x−2m(x)=−38x+3(2, 4)(4, 10)(1, 5)(4, 11)(−1,4) (5,2)−13(8,−2)(4,6)(6, 11)(−4, 3)45f(−5)=−4,f(5)=2f(−1)=4f(5)=1f(x)=−12x+72(2,4)(4,10)(1,5)(4,11) y=2x+3(−1, 4)(5, 2)(−2, 8)(4, 6)y=−13x+223(−2, 0)(0,–3)(−5, 0)(0, 4)y=45x+4−54y= 2 3 x+1y=−2x+3 y=3

x
g(x)
g(x)=−3x+5xh(x)xf(x)f(x)=5x−5
x
k(x)
x
g(x)
g(x)=−252x+6
x
f(x)
x
f(x)
f(x)=10x−24
x
k(x)
ff(0.1)=11.5, and f(0.4)=–5.9,f(x)=−58x+17.3f[–10, 10]: f(x)=0.02x−0.01.x−10x10.f
[–10, 10]:fx)=2,500x+4,000w,k,k.k,wkp,q,q.k.pqa=11,900b=1001.1q(p)=1000p−100f[−10,10]183116
−1010.f[−0.1,0.1]−22.5−0.10.1.ff(x)=ax+b[−4,4]ab.a=2; b=3a=2; b=4a=2; b=–4a=2; b=–5x
(x,2),(−4,6), m=3x=−163(10,y),(25,100), m=−5(a, b)(a, b+1)x=a(2a, b)(a, b+1)(a, 0)(c, d)
y=dc−ax−adc−ann,p,$30p(n)=mn+bpnC(n)=24+0.1n,nC(n)n,y,y=mn+bn−400.P(t),tI(x)=1054x+23,286,x
C,F(C).F(28).F(–40).f(x)=mx+b, then m<0.f(x)=mx+b, then m>0.y−y1=m(x−x1)f(x)=mx+bf(x)=x.
f(x)=2x,(1,2).(2,4).f(x)=−23x+5
x=0 f(0)=− 2 3 (0)+5=5⇒(0,5) x=3 f(3)=− 2 3 (3)+5=3⇒(3,3) x=6 f(6)=− 2 3 (6)+5=1⇒(6,1)
f(x)=−23x+5.f(x)=−23x+5.f(x)=−34x+6x=0m,
f(x)=12x+1
12.x=0.(0,1).(0,1)m=riserun.m=12,(0,1),f(x)=mx+bb(0,b)m
m= change in output (rise) change in input (run) = Δy Δx = y 2 − y 1 x 2 − x 1
riserunf(x)=−23x+5x=0x=0(0,5).–23.–2(0,5)(−3,7),(−6,9),(−9,11).f(x)=xf(x)=mx,mmf(x)=xmfmm>1fm
0<m<1.m,f(x)=x.f(x)=mx+b,bbf(x)=xfbb|b|bf(x)=x.f(x)=mx+b.f(x)=x.m.bf(x)=12x−3m=1212.b=−3y=x,
12.y=12x,f(x)=4+2x,
f(2)= 1 2 (2)−3 =1−3 =−2
(0, 4)(−2,0).
m= rise run = 4 2 =2
y=2x+4
f(x)=2x+3g(x)=2x−3h(x)=−2x+3j(x)=12x+3g(0, 3)f−3.(0,–3)12(0, 3),jfjf(x)x.
f(x)=3x−6
x.
0=3x−6 6=3x 2=x x=2
(2, 0).y=c,cy=5xf(x)=0.0=mx+b.f(x)=12x−3.x.
0= 1 2 x−3 3= 1 2 x 6=x x=6
(6, 0).(6, 0)f(x)=12x−3.f(x)=14x−4.(16, 0)m=0f(x)=mx+b,f(x)=b.f(x)=2.f(x)=2.x=0.x=2.x=2,f(x)=b.x=a.
−4,y=−4.7,x=7.1.m1 and m2–1.
m1m2=−1
18,18
f(x)= 1 4 x+2 negative reciprocal of 1 4 is −4 f(x)=−4x+3 negative reciprocal of−4 is 1 4
−4(14)=−1
f(x)=m1x + b1 and g(x)=m2x + b2 are parallel if m1 = m2.
b1=b2m1=m2,
f(x)= m 1 x+ b 1 and g(x)= m 2 x+ b 2 are perpendicular if m 1 m 2 =−1, and so m 2 =− 1 m 1 .
f(x)=2x+3 h(x)=−2x+2 g(x)= 1 2 x−4 j(x)=2x−6

f(x)=2x+3j(x)=2x−612g(x)=12x−4h(x)=−2x+2f(x)=2x+3j(x)=2x–6g(x)=12x–4h(x)=–2x+2

f(x)=3x+1

(0,1).f(x).f(x).

g(x)=3x+6 h(x)=3x+1 p(x)=3x+ 2 3

f(1, 7).b

y− y 1 =m(x− x 1) y−7=3(x−1) y−7=3x−3 y=3x+4

g(x)=3x+4f(x)=3x+1(1, 7).f(x)=3x+6(3, 0).m=3,x=3,f(x)=0

g(x)=3x+b 0=3(3)+b b=–9

f(x)(3, 0)g(x)=3x−9.

f(x)=2x+4

−12.–12f(x).f(x).

g(x)=− 1 2 x+4 h(x)=− 1 2 x+2 p(x)=− 1 2 x− 1 2

f(x)(4, 0).–12.b.

g(x)=mx+b 0=− 1 2 (4)+b 0=−2+b 2=b b=2

−12

g(x)=−12x+2.

g(x)=−12x+2f(x)=2x+4(4, 0).xyg(x)=mx+b.b.f(x)=3x+3(3, 0).m=3,–13.

g(x)=– 1 3 x+b 0=– 1 3 (3)+b 1=b b=1

f(x)(3, 0)g(x)=−13x+1.h(x)=2x–4,(0,0)h(x)h(x)f(x)=2xg(x)=− 1 2 x(−2, 6)(4,5).(4,5).

m 1 = 5−6 4−(−2) = −1 6 =− 1 6

m 2 = −1 − 1 6 =−1(− 6 1) =6

(4,5).

g(x)=6x+b 5=6(4)+b 5=24+b −19=b b=−19

(4,5)

y=6x−19

(−2,–15)(2,–3).(6,4).y=–13x+6f(x)=g(x).h(t)=3t−4j(t)=5−t.h(t)=j(t).

3t−4=5−t 4t=9 t= 9 4

94.

j(9 4)=5− 9 4 = 11 4

(94,114).j(t):j(t)h(t)j(t)j(t)f(x)=x.(0,5)(5, 0)C,xR,xC and R.

C(x)=120x+250,000

xR(x)=140x.x.

C(x)=R(x) 250,000+120x=140x 250,000=20x 12,500=x

x=12,500

y,

R(20)=140(12,500) =$1,750,000

(12,500,1,750,000). f(x)=b.x=b.f(x)=mx+b,bx.f(x)=ax=a,(a,a).yaxa.y=mx+bb.y=mx+bmb.
4x−7y=10 7x+4y=13y+x=12 −y=8x+13y+4x=12 −6y=8x+16x−9y=10 3x+2y=1y= 2 3 x+1 3x+2y=1
y= 3 4 x+1 −3x+4y=1f(x)=−x+2g(x)=2x+4(−2, 0)(0, 4)h(x)=3x−5k(x)=−5x+1(15, 0)(0, 1)−2x+5y=20
7x+2y=56(8, 0)(0, 28)(0,6)(3,−24)(−1,19)(8,−71)(−8,−55)(10, 89)(9,−44)(4,−14)
Line 1: m=8 Line 2: m=–6 Neither(2,3)(4,−1)(6,3)(8,5)(1,7)(5,5)(−1,−3)(1,1)
Line 1: m=– 1 2 Line 2: m=2 Perpendicular(0,5)(3,3)(1,−5)(3,−2)(2,5)(5,−1)(−3,7)(3,−5)
Line 1: m=–2 Line 2: m=–2 Parallelf(x)=−5x−3(2, −12).g(x)=3x−1(4,9).g(x)=3x−3h(t)=−2t+4(-4, –1).
p(t)=3t+4(3,1).p(t)=−13t+2f(x)=−2x−1g(x)=−x.f(x)=2x+5g(x)=−3x−5.(−2,1)f(x)= −45x +27425
h(x)=94x + 7310.f(x)=74x + 45760g(x)=43x + 315.(−175,53)f(x)=−x−1f(x)=−2x−1f(x)=−12x−1
f(x)=2f(x)=2+xf(x)=3x+2(–4, 0)(0, –2)(–2, 0)(0, 4)(0, 7)−32(0, 3)25(−6, –2)(6, –6)(−3, –4)(3, 0)
f(x)=−2x−1g(x)=−3x+2h(x)=13x+2k(x)=23x−3f(t)=3+2tp(t)=−2 + 3tx=3x=−2r(x)=4q(x)=34x=−9y+36
x3−y4=13x−5y=153x=153y=12g(x)f(x)=x3 4 ,g(x).g(x)=0.75x−5.5(0,−5.5)g(x)f(x)=x13,g(x).y=3x=−3
y= 3 4 x+1 −3x+4y=122x−3y=12 5y+x=302x=y−3y+4x=15(2, 7)x−2y+2=3 x−y=35x+3y=−65 x−y=−5
(−10, −5)g(x)=−0.01x + 2.01(1, 2).g(x)=−0.01x+2.01(1, 2).y=100x−98f(x)=−0.1x+200 and g(x)=20x+0.1.

fg.f(x)g(x)?g(x)f(x)?x< 1999 201 x> 1999 201f(x)=b,bx=a,a M, t, M(t) M. M(t)=mt+b. x-

$$0=-400t+3500 \quad t= 3500 400 \quad =8.75$$

x- x- 0≤t≤8.75.y-y-x-y-

$$f(x)=mx+b \quad =-250x+1000$$

xx-

$$0=-250x+1000 \quad 1000=250x \quad 4=x \quad x=4$$

x- x-x-x-y- y-t,P(t),(t=9),

$$m= \text{change in output change in input}$$

t=0,(0,6200).t=5,(5,8100). (0,6200) (5,8100).

$$m= 8100-6200 \; 5-0 \quad = 1900 \; 5 \quad =380 \text{ people per year}$$

$$P(t)=380t+6200$$

t=9.

$$P(9)=380(9)+6,200 \quad =9,620$$

P(t)=15000 t.

$$15000=380t+6200 \quad 8800=380t \quad t\approx 23.158$$

C x, C(x)=0.25x+25,000(0,25,000). t, A(t), E(t), t=0,AE

$$A(t)=4tE(t)=3t$$

A,E,D, D, t, A(t),E(t), D(t)

$$D (t) 2 =A (t) 2 +E (t) 2 \quad = (4t) 2 + (3t) 2 \quad =16 t 2 +9 t 2 \quad =25 t 2 \quad D(t)=\pm 25 t 2$$

$$\text{Solve for } D(t) \text{ using the square root} \quad =\pm 5|t|$$

t, D(t) D(t)=5t. DD(t)=2 t.

$$D(t)=2 \quad 5t=2 \quad t= 2 \; 5 =0.4$$

(30, 10), (20,0).

$$m= 10-0 \; 30-0 = 1 \; 3$$

$$W(x)= 1 \; 3 \; x$$

m=–3.

$$E(x)=-3x+b \quad 0=-3(20)+b \text{ Substitute in } (20, 0) \quad b=60 \; E(x)=-3x+60$$

$$1 \; 3 \; x=-3x+60 \; 10 \; 3 \; x=60 \; 10x=180 \quad x=18 \text{ Substituting this back into } W(x) \quad y=W(18) \quad = 1 \; 3 \; (18) \quad =6$$

$$\text{distance}= (x \; 2 - x \; 1) \; 2 + (y \; 2 - y \; 1) \; 2 \quad = (18-0) \; 2 + (6-0) \; 2 \quad \approx 18.974 \text{ miles}$$

x,d,K(d):M(d)K(0)=20M(0)=16 K(d)=\$0.59 P(d)=\$0.63 f(x)=mx+b.

$$K(d)=0.59d+20 \; M(d)=0.63d+16$$

K(d)<M(d). K(d) K(d)

$$K(d)=M(d) \; 0.59d+20=0.63d+16 \quad 4=0.04d \quad 100=d \quad d=100$$

K(d)d>100. y=0, mx+b x=3, f(x)=1+2x, f(x) (2, 7). f(x)=12– 1 3 x, f(x) f(x)=9– 6 7 x, f(x) g(x)=2, f(x)=3x, f(x) (6,1). P t, t P(t)=75,000+2500t P. P P W t.W(t)=7.5t+0.5WW(−15,0) (0, 7.5) C t. C(t)=12,025−205tC. C (58.7, 0)(0,12,025) C y, t, t y, y t, y=−2t+180 y, t, t y, y t, y=30t−300P, P(t)=305+174tPtC(x)=0.15x+10xP(t)=190t+4360P.R,t,R(t)=16−2.1tR,t,riserun.

$$m=6050=1.2$$

$$T(c)=1.2c+30$$

cT(c)x=50,

$$T(30)=30+1.2(30) \quad =66 \text{ degrees}$$

$$40=30+1.2c \; 10=1.2c \quad c\approx 8.33$$

54°F

$$T(c)=30.281+1.143c$$

$$T(30)=30.281+1.143(30) \quad =64.571 \quad \approx 64.6 \text{ degrees}$$

r.rr,r.r=0.9509.t,

$$C(t)=113.318+2.209t$$

(t=14),

$$C(14)=113.318+2.209(14) \quad =144.244$$

r,(x)(y).

$$y=ax+b \ a=-1.341 \ b=32.234 \ r=-0.896$$

x,y,

$$y=ax+b \ a=6.301 \ b=-1.044 \ r=-0.970$$

60° F.r=0.95r=−0.89r=0.26r=−0.39r=0.985xyy=1.640x+13.800r=0.987xyxyxy

$$y=-0.962x+26.86, \quad r=-0.965$$

xyxyy=−1.981x+60.197r=−0.998xyxyy=0.121x−38.841, r=0.998f(x)=0.5x+10x=−2, 1, 5, 6, 9
f(x)=−2x−10x=−2, 1, 5, 6, 9(−2,−6),(1,−12),(5,−20),(6,−22),(9,−28)y=−2x−10
(46, 1,600), (48, 1,550), (50, 1,505), (52, 1,540), (54, 1,495)P(189.8,0)
(2500, 2000), (2650, 2001), (3000, 2003), (3500, 2006), (4200, 2010)y,y=0.00587x+1985.41
(46, 250), (48, 305), (50, 350), (52, 390), (54, 410)y=20.25x−671.5
(46, 250), (48, 225), (50, 205), (52, 180), (54, 165).y=−10.75x+742.502x+3y=76x2−y=5f(x)=7x−2
g(x)=−x+2(7,5)(3,17)y=−3x+26(6,0)(0,10)y=2x−2xg(x)xg(x)2x−6y=12 −x+3y=1y= 1 3 x−2 3x+y=−9
7x+9y=−63(−9,0);(0,−7)f(x)=2x−1(5,11)(10,1)(−1,3)(−5,11)m=−2;m=−2;(8,−10)(0,−26)(2,5)(4,4)
f(x)=5x−1y=−0.2x+21(0, 2)−12f(t)=2t−5x=y+62x−y=13f(x)=10−2xft.y,x,xx,y=−300x + 11,500
P(t)=100t + 1700P.xyxyy=−1.294x+49.412; r=−0.974
(3,600, 2000); (4,000, 2001); (4,700, 2003); (6,000, 2006)y,2x + 3y=7f(x)=−2x + 5f(x)=7x + 9y =−1.5x −6
y=−2x − 1xg(x)xg(x)ny= 3 4 x−9 −4x−3y=8−2x+y=3 3x+ 3 2 y=52x + 7y=−14.(−7,0)(0,−2)(−2,−6)(3,14)
(2,6)(4,14)f(x)=4x+3(8,10).y=−0.25x+12(0,5)−52f(x)=−x+6x=y+22x−3y=−1f(x)=12−4xfCt.yxxyyx
y=875x + 10,675y=−46.875t+1250xy(4,500, 2000); (4,700, 2001); (5,200, 2003); (5,800, 2006)
y=0.00455x + 1979.5r=0.999r,

$$x^2+4=0$$

i

$$\sqrt{-1}=i$$

$$i^2=(\sqrt{-1})^2=-1$$

i.

$$\sqrt{-25}=\sqrt{25\cdot(-1)} \quad =\sqrt{25}\sqrt{-1} \quad =5i$$

5i −5i 25 a+bi a bi 5+2i 3+4 3 i . a+bi a bi b=0, a+bi a=0 b −a a −1 . −1 i. a ·i −9

$$\sqrt{-9}=\sqrt{9}\sqrt{-1}=3i$$

0+3i. −24 −24 =0+2i 6 (a,b), a b −2+3i. −2 3i. (−2,3) −2+3i 3−4i 3, −4i. (3,−4) −4−i

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

3−4i 2+5i.

$$(a+bi)+(c+di)=(a+c)+(b+d)i \quad (3-4i)+(2+5i)=(3+2)+(-4+5)i \quad =5+i$$

2+5i 3−4i. (3−4i)−(2+5i)=1−9i 4(2+5i).

$$4(2+5i)=(4\cdot 2)+(4\cdot 5i) \quad =8+20i$$

−4(2+6i).−8−24i

$$(a+bi)(c+di)=ac+adi+bci+bdi^2$$

$i^2=-1$,

$$(a+bi)(c+di)=ac+adi+bci-bd$$

$$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$$

(4+3i)(2−5i). (a+bi)(c+di)=(ac−bd)+(ad+bc)i

$$(4+3i)(2-5i)=(4\cdot 2-3\cdot(-5))+(4\cdot(-5)+3\cdot 2)i \quad =(8+15)+(-20+6)i \quad =23-14i$$

(3−4i)(2+3i).18+i a+bi a−bi. a+bi a−bi, a−bi a+bi. c+di a+bi, a b

$$\frac{c+di}{a+bi} \text{ where } a\neq 0 \text{ and } b\neq 0$$

$$\frac{(c+di)}{(a+bi)}\cdot\frac{(a-bi)}{(a-bi)}=\frac{(c+di)(a-bi)}{(a+bi)(a-bi)}$$

$$=\frac{ca-cbi+adi-bdi^2}{a^2-abi+abi-b^2i^2}$$

$i^2=-1$.

= ca−cbi+adi−bd(−1) a 2 −abi+abi− b 2 (−1) = (ca+bd)+(ad−cb)i a 2 + b 2

a+bi a−bi. 2+i 5− 1 2 i a+bi. a−bi, 2−i 5 . a+bi 0− 1 2 i. a−bi, 0+ 1 2 i. 1 2 i. i. (2+5i) (4−i).

(2+5i) (4−i)

(2+5i) (4−i) · (4+i) (4+i)

(2+5i) (4−i) · (4+i) (4+i) = 8+2i+20i+5 i 2 16+4i−4i− i 2 = 8+2i+20i+5(−1) 16+4i−4i −(−1) Because i 2 =−1 = 3+22i 17 = 3 17 + 22 17 i

Separate real and imaginary parts.

f(x)= x 2 −5x+2. f(3+i). x=3+i f(x)= x 2 −5x+2 f(3+i)=−5+i. 3+i −5+i. f(x)=2 x 2 −3x. f(8−i). 102−29i f(x)= 2+x x+3 . f(10i). x=10i

2+10i 10i+3 Substitute 10i for x. 2+10i 3+10i Rewrite the denominator in standard form. 2+10i 3+10i · 3–10i 3–10i Prepare to multiply the numerator and denominator by the complex conjugate of the denominator. 6–20i+30i–100 i 2 9–30i+30i–100 i 2

Multiply using the distributive property or the FOIL method. 6–20i+30i–100(–1) 9–30i+30i–100(–1) Substitute –1 for i 2 . 106+10i 109 Simplify. 106 109 + 10 109 i Separate the real and imaginary parts.

f(x)= x+1 x−4 . f(−i).− 3 17 + 5i 17 i i

i 1 =i i 2 =−1 i 3 = i 2 ·i=−1·i=−i i 4 = i 3 ·i=−i·i=− i 2 =−(−1)=1 i 5 = i 4 ·i=1·i=i

i, i i.

i 6 = i 5 ·i=i·i= i 2 =−1 i 7 = i 6 ·i= i 2 ·i= i 3 =−i i 8 = i 7 ·i= i 3 ·i= i 4 =1 i 9 = i 8 ·i= i 4 ·i= i 5 =i

i i 35 . i 4 =1, i 4 35=4·8+3.

i 35 = i 4·8+3 = i 4·8 · i 3 = (i 4) 8 · i 3 = 1 8 · i 3 = i 3 =−i

i 35 i 35 i 3 i 35 i 35i 34 ·ii 33 · i 2i 31 · i 4i 19 · i 16(i 2) 17 ·ii 33 ·(−1)i 31 ·1i 19 · (i 4) 4 (−1) 17 ·i− i 33i 31i 19 i. i i i If f(x)= x 2 +x−4, f(2i).−8+2iIf f(x)= x 3 −2, f(i).If f(x)= x 2 +3x+5, f(2+i).14+7iIf f(x)=2 x 2 +x−3, f(2−3i).If f(x)= x+1 2−x , f(5i).− 23 29 + 15 29 iIf f(x)= 1+2x x+3 , f(4i).1−2i−2+3ii−3−4i(3+2i)+(5−3i)8−i(−2−4i)+(1+6i)(−5+3i)−(6−i)−11+4i(2−3i)−(3+2i) (−4+4i)−(−6+9i)2−5i(2+3i)(4i)(5−2i)(3i)6+15i(6−2i)(5)(−2+4i)(8)−16+32i(2+3i)(4−i) (−1+2i)(−2+3i)−4−7i(4−2i)(4+2i)(3+4i)(3−4i)3+4i 26−2i 32− 2 3 i−5+3i 2i6+4i i4−6i2−3i 4+3i 3+4i 2−i2 5 + 11 5 i2+3i 2−3i−9 +3 −1615i− −4 −4 −252+ −12 21+i 34+ −20 2i 81i 15i 22−1 (1+i) k k=4, 8, and 12. k=16. (1−i) k k=2, 6, and 10. k=14. (1+i)k − (1−i) kk=4, 8, and 12 k=16. x 6 +1=0 3 2 + 1 2 i.(3 2 + 1 2 i) 6 =−1 x 8 −1=0 2 2 + 2 2 i.1 i + 4 i 33i1 i 11 − 1 i 21i 7 (1+ i 2)i −3 +5 i 7 (2+i)(4−2i) (1+i)(1+3i)(2−4i) (1+2i)(3+i) 2 (1+2i) 2−2i3+2i 2+i +(4+3i)4+i i + 3−4i 1−i 9 2 − 9 2 i3+2i 1+2i − 2−3i 3+i

a+bi,

a bi bi i= −1 x y=0. y- (3,1). x=3. x- y- (0,7)

f(x)=a x 2 +bx+c

a,b, c a≠0. a>0, a<0, x=− b 2a . x= −b± b 2 −4ac 2a , a x 2 +bx+c=0 x- x x=− b 2a , y= x 2 +4x+3. a=1,b=4, c=3. a>0, x=− 4 2(1) =−2. x=−2 (−2,−1). x- x- (−3,0) (−1,0).

f(x)=a (x−h) 2 +k

(h, k) a>0, a<0, y=−3 (x+2) 2 +4. x−h=x+2 h=–2. a=−3,h=−2, k=4. a<0, (−2, 4). y= x 2 . k>0, k<0, k>0, h>0, h<0, h<0, a | a |>1, x- | a |<1, x- | a |>1,

a (x−h) 2 +k=a x 2 +bx+c a x 2 −2ahx+(a h 2 +k)=a x 2 +bx+c

−2ah=b, so h=− b 2a .

a h 2 +k=c k=c−a h 2 =c−a− (b 2a) 2 =c− b 2 4a

h, f(h)=k. f(x)=a x 2 +bx+c a,b, c a≠0. f(x)=a (x−h) 2 +k. (h,k)

h=− b 2a , k=f(h)=f(−b 2a).

h. k. h k. f(x)=a (x–h) 2 +k. x f(x). | a |. a>0. a<0 x- g f(x)= x 2 , f(x)= x 2 g(x)=a (x+2) 2 –3. (0,–1),

−1=a (0+2) 2 −3 2=4a a= 1 2

(g)x= 1 2 (x+2) 2 –3.

g(x)= 1 2 (x+2) 2 −3 = 1 2 (x+2)(x+2)−3 = 1 2 (x 2 +4x+4)−3 = 1 2 x 2 +2x+2−3 = 1 2 x 2 +2x−1

Y1= 1 2 (x+2) 2 −3. TBLSET, TblStart=–6 ΔTbl = 2, TABLE. xy (−4, 7), (h)x=– 7 16 (x+4) 2 +7. h(−7.5) h(−7.5)≈1.64; a, b, and c. h, a b h=– b 2a . k, k=f(h)=f(− b 2a). f(x)=2 x 2 –6x+7.

h=– b 2a =– –6 2(2) = 6 4 = 3 2

k=f(h) =f(3 2) =2 (3 2) 2 −6(3 2)+7 = 5 2

a

f(x)=a x 2 +bx+c f(x)=2 x 2 −6x+7

f(x)=2 (x– 3 2) 2 + 5 2

(k), (x). g(x)=13+ x 2 −6x, g(x)= x 2 −6x+13 g(x)= (x−3) 2 +4 f(x)=a x 2 +bx+c a f(x)≥f(− b 2a), [f(− b 2a),∞); a f(x)≤f(− b 2a), (−∞,f(− b 2a)]. f(x)=a (x−h) 2 +k a f(x)≥k; a f(x)≤k. a a a k. f(x)≥k, [k,∞). f(x)≤k, (−∞,k]. f(x)=−5 x 2 +9x−1. a x-

h=– b 2a =– 9 2(−5) = 9 10

f(h).

f(9 10)=5 (9 10) 2 +9(9 10)−1 = 61 20

f(x)≤ 61 20 , (−∞, 61 20]. f(x)=2 (x– 4 7) 2 + 8 11 . f(x)≥ 8 11 , [8 11 ,∞). L. W, L+W+L=80, 2L+W=80. W, L.

W=80−2L

A=LW=L(80−2L) A(L)=80L−2 L 2

L.

A(L)=−2 L 2 +80L.

a a=−2,b=80, c=0.

h=− 80 2(−2) k=A(20) =20 and =80(20)−2 (20) 2 =800

L=20 p Q Revenue=pQ. p=30 Q=84,000. p=32 Q=79,000.

m= 79,000−84,000 32−30 = −5,000 2 =−2,500

Q=−2500p+b Substitute in the point Q=84,000 and p=30 84,000=−2500(30)+b Solve for b

b=159,000

Q=−2,500p+159,000

Revenue=pQ Revenue=p(−2,500p+159,000) Revenue=−2,500 p 2 +159,000p

h=− 159,000 2(−2,500) =31.8

maximum revenue=−2,500 (31.8) 2 +159,000(31.8) =2,528,100

y- x- x- f(x), y- f(0) y- f(x)=0 f(x)=3 x 2 +5x−2. f(0).

f(0)=3 (0) 2 +5(0)−2 =−2

(0,−2). f(x)=0.

0=3 x 2 +5x−2

0=(3x−1)(x+2)

0=3x−1 0=x+2 x= 1 3 or x=−2

(1 3 ,0) (−2,0). (0,−2). (1 3 ,0) (−2,0). x- a b h=− b 2a . x=h k. h k. x- x- x- f(x)=2 x 2 +4x−4.

0=2 x 2 +4x−4

f(x)=a (x−h) 2 +k

a=2. h k.

h=− b 2a k=f(−1) =− 4 2(2) =2 (−1) 2 +4(−1)−4 =−1 =−6

f(x)=2 (x+1) 2 −6

0=2 (x+1) 2 −6 6=2 (x+1) 2 3= (x+1) 2 x+1=± 3 x=−1± 3

x- (−1− 3 ,0) (−1+ 3 ,0). x- g(x)=13+ x 2 −6x. x- x- x 2 +x+2=0. x= −b± b 2 −4ac 2a . a, b and c. x 2 +x+2=0, a=1, b=1, and c=2.

x= −b± b 2 −4ac 2a = −1± 1 2 −4·1·(2) 2·1 = −1± 1−8 2 = −1± −7 2 = −1±i 7 2

x= −1+i 7 2 x= −1−i 7 2 x= −1 2 + i 7 2 x= −1 2 − i 7 2 . H(t)=−16 t 2 +80t+40.

h=− 80 2(−16) = 80 32 = 5 2 =2.5

y-

k=H(− b 2a) =H(2.5) =−16 (2.5) 2 +80(2.5)+40 =140

H(t)=0.

t= −80± 80 2 −4(−16)(40) 2(−16) = −80± 8960 −32
t= −80− 8960 −32 ≈5.458 or t= −80+ 8960 −32 ≈−0.458

H(t)=−16 t 2 +96t+112.

f(x)=a x 2 +bx+c
x= −b± b 2 −4ac 2a
f(x)=a (x−h) 2 +k

x- x- y- y- y- a≠0 a=0 f(x)= x 2 −12x+32g(x)= x 2 +2x−3f(x)= (x+1) 2 −2, (−1,−4)f(x)= x 2 −x
f(x)= x 2 +5x−2f(x)= (x+ 5 2) 2 − 33 4 , (− 5 2 ,− 33 4)h(x)=2 x 2 +8x−10k(x)=3 x 2 −6x−9
f(x)=3 (x−1) 2 −12, (1,−12)f(x)=2 x 2 −6xf(x)=3 x 2 −5x−1f(x)=3 (x− 5 6) 2 − 37 12 , (5 6 ,− 37 12)
y(x)=2 x 2 +10x+12f(x)=2 x 2 −10x+4 − 17 2 5 2 . x= 5 2 .f(x)=− x 2 +4x+3f(x)=4 x 2 +x−1
− 17 16 − 1 8 . x=− 1 8 .h(t)=−4 t 2 +6t−1f(x)= 1 2 x 2 +3x+1 − 7 2 −3. x=−3.f(x)=− 1 3 x 2 −2x+3
f(x)= (x−3) 2 +2 (−∞,∞). [2,∞).f(x)=−2 (x+3) 2 −6f(x)= x 2 +6x+4 (−∞,∞). [−5,∞).f(x)=2 x 2 −4x+2
k(x)=3 x 2 −6x−9 (−∞,∞). [−12,∞).x 2 =−25x 2 =−8{ 2i 2 ,−2i 2 }x 2 +36=0x 2 +27=0{ 3i 3 ,−3i 3 }
x 2 +2x+5=0x 2 −4x+5=0{2+i,2−i}x 2 +8x+25=0x 2 −4x+13=0{2+3i,2−3i}x 2 +6x+25=0x 2 −10x+26=0
{5+i,5−i}x 2 −6x+10=0x(x−4)=20{2+2 6 , 2−2 6 }x(x−2)=102 x 2 +2x+5=0{ − 1 2 + 3 2 i, − 1 2 − 3 2 i }
5 x 2 −8x+5=05 x 2 +6x+2=0{ − 3 5 + 1 5 i, − 3 5 − 1 5 i }2 x 2 −6x+5=0x 2 +x+2=0
{ − 1 2 + 1 2 i 7 , − 1 2 − 1 2 i 7 }x 2 −2x+4=0 (h,k) (x,y) (h,k)=(2,0),(x,y)=(4,4)f(x)= x 2 −4x+4
(h,k)=(−2,−1),(x,y)=(−4,3)(h,k)=(0,1),(x,y)=(2,5)f(x)= x 2 +1(h,k)=(2,3),(x,y)=(5,12)
(h,k)=(−5,3),(x,y)=(2,9)f(x)= 6 49 x 2 + 60 49 x+ 297 49(h,k)=(3,2),(x,y)=(10,1)(h,k)=(0,1),(x,y)=(1,0)
f(x)=− x 2 +1(h,k)=(1,0),(x,y)=(0,1)f(x)= x 2 −2x(1, −1), x=1. (0,0), (2,0).f(x)= x 2 −6x−1
f(x)= x 2 −5x−6 (5 2 , −49 4), (0,−6),(−1,0),(6,0).f(x)= x 2 −7x+3f(x)=−2 x 2 +5x−8 (5 4 , − 39 8),
x= 5 4 . (0, −8).f(x)=4 x 2 −12x−3f(x)= x 2 −4x+1f(x)=−2 x 2 +8x−1f(x)= 1 2 x 2 −3x+ 7 2xy
f(x)= x 2 +1xyxyf(x)=2− x 2xyxyf(x)=2 x 2 f(x)= x 2 ,f(x)=2 x 2 , and f(x)= 1 3 x 2 .
f(x)= x 2 ,f(x)= x 2 +2 f(x)= x 2 ,f(x)= x 2 +5 f(x)= x 2 −3.
f(x)= x 2 ,f(x)= (x−2) 2 ,f (x−3) 2 , and f(x)= (x+4) 2 . h(x)= −32 (80) 2 x 2 +x x h(x) h(x)=.0001 x 2
−2000≤x≤2000 | x | h(x) (1,−2), (−∞,∞). [−2,∞). (−1,2) (−5,11), (−∞,∞) (−∞,11]. (−100,100), (1,1)
f(x)=2 x 2 . y-f(x)=2 x 2 −1 (−1,4) f(x)=2 x 2 . y- (2,3) f(x)=3 x 2 . y-f(x)=3 x 2 −9 (1,−3) f(x)=− x 2 .
y- (4,3) f(x)=5 x 2 . y-f(x)=5 x 2 −77 (1,−6) f(x)=3 x 2 . −1. f(x)=− x 2 +100x. f(x)=−2 x 2 +250x.6 −6;
f(x)= x 2 +12x. p=$45−0.0125x, x R=x·p. h(t)=−4.9 t 2 +229t+234. h(t)=−4.9 t 2 +24t+8. x=− b 2a .
f(x)=a x 2 +bx+c, a,b, c a≠0. f(x)=a (x−h) 2 +k, (h, k) x y=0,

2009
2010
2011
2012
2013
800
897
992
1,083
1,169

P(t)=−0.3 t 3 +97t+800, P(t) t r

A(r)=π r 2

r

V(r)= 4 3 π r 3

π 4 3 π, r

f(x)=k x p

k p k f(x)= 2 x

f(x)=1 Constant function f(x)=x Identify function f(x)= x 2 Quadratic function f(x)= x 3 Cubic function f(x)= 1 x Reciprocal function f(x)= 1 x 2 Reciprocal squared function f(x)= x Square root function f(x)= x 3 Cube root function

f(x)= x 0 f(x)= x 1 f(x)= x 2 f(x)= x 3 . f(x)= x −1 f(x)= x −2 . f(x)= x 1/2 f(x)= x 1/3 . f(x)=2 x 2 ·4 x 3 g(x)=− x 5 +5 x 3 −4x h(x)= 2 x 5 −1 3 x 2 +4f(x) f(x)=8 x 5 . f(x)= x 2 , g(x)= x 4 and h(x)= x 6 , ∞ −∞ x x→∞, x x f(x)

as x→±∞, f(x)→∞

f(x)= x 3 , g(x)= x 5 , and h(x)= x 7 , f(x)= x n f(x)= x n , n even, y- f(x)= x n , n odd, x f(x) x f(x)

as x→−∞, f(x)→−∞ as x→∞, f(x)→∞

x→−∞ x→∞ f(x)=k x n n f(x)=k x n n f(x)= x 8 . x f(x) x→∞, f(x)→∞. x x→−∞, f(x)→∞. f(x)=− x 9 . −1 x- f(x)= x 9 . x x

as x→−∞, f(x)→∞ as x→∞, f(x)→−∞

xf(x) x, x, f(x)=−5 x 4 . x f(x) x→±∞, f(x)→−∞ r w

r(w)=24+8w

A

A(r)=π r 2

A(w)=A(r(w)) =A(24+8w) =π (24+8w) 2

A(w)=576π+384πw+64π w 2

n

f(x)= a n x n +...+ a 2 x 2 + a 1 x+ a 0

a i a i x i

f(x)=2 x 3 ·3x+4 g(x)=−x(x 2 −4) h(x)=5 x +2

f(x)= a n x n +...+ a 2 x 2 + a 1 x+ a 0 , f(x) f(x)=6 x 4 +4.g(x) g(x)=− x 3 +4x.h(x) x x

f(x)=3+2 x 2 −4 x 3 g(t)=5 t 5 −2 t 3 +7t h(p)=6p− p 3 −2

f(x), x −4 x 3 . −4. g(t), t 5, 5. 5 t 5 . 5. h(p), p 3, 3. −p 3 ; −1. f(x)=4 x 2 − x 6 +2x−6. − x 6 . −1. x f(x)=5 x 4 +2 x 3 −x−45 x 4f(x)=−2 x 6 − x 5 +3 x 4 + x 3−2 x 6f(x)=3 x 5 −4 x 4 +2 x 2 +13 x 5 f(x)=−6 x 3 +7 x 2 +3x+1−6 x 3 x f(x) x f(x)

as x→−∞, f(x)→−∞ as x→∞, f(x)→∞

x x x→∞, f(x)→−∞; as x→−∞, f(x)→−∞. f(x)=−3 x 2 (x−1)(x+4), f(x).

f(x)=−3 x 2 (x−1)(x+4) =−3 x 2 (x 2 +3x−4) =−3 x 4 −9 x 3 +12 x 2

f(x)=−3 x 4 −9 x 3 +12 x 2 . −3 x 4 ;

as x→−∞, f(x)→−∞ as x→∞, f(x)→−∞

f(x)=0.2(x−2)(x+1)(x−5), 0.2 x 3 , x f(x) x f(x) (0, a 0). x- x=0 x- f(x)=(x−2)(x+1)(x−4), y- x- x.

f(0)=(0−2)(0+1)(0−4) =(−2)(1)(−4) =8

0=(x−2)(x+1)(x−4) x−2=0 or x+1=0 or x−4=0 x=2 or x=−1 or x=4

x- (2,0),(–1,0), (4,0). f(x)= x 4 −4 x 2 −45, y- x-

f(0)= (0) 4 −4 (0) 2 −45 =−45

(0,–45).

f(x)= x 4 −4 x 2 −45 =(x 2 −9)(x 2 +5) =(x−3)(x+3)(x 2 +5)

0=(x−3)(x+3)(x 2 +5)

x−3=0 or x+3=0 or x 2 +5=0 x=3 or x=−3 or (no real solution)

(3,0) (–3,0). f(x)=f(−x). f(x)=2 x 3 −6 x 2 −20x, y- x- (0,0); (0,0),(–2,0),(5,0) x- nth n n x- n n–1 n n n–1 x- f(x)=−3 x 10 +4 x 7 − x 4 +2 x 3 . 10, n n−1 x- f(x)=108−13 x 9 −8 x 4 +14 x 12 +2 x 3 x- x- x- f(x)=−4x(x+3)(x−4), y- f(0).

f(0)=−4(0)(0+3)(0−4) =0

y- (0,0). x-

0=−4x(x+3)(x−4) x=0 or x+3=0 or x−4=0 x=0 or x=−3 or x=4 x- (0,0),(–3,0), (4,0). f(x)=0.2(x−2)(x+1)(x−5), x- (2,0),(−1,0), (5,0), (0,2), x-

f(x)= a n x n +...+ a 2 x 2 + a 1 x+ a 0

n n n−1 x f(x). x f(x). x→−∞, f(x)→−∞ x→∞, f(x)→−∞. f(x)= x 5f(x)= (x 2) 3f(x)=x− x 4
f(x)= x 2 x 2 −1f(x)=2x(x+2) (x−1) 2f(x)= 3 x+1−3x 47−2 x 2−2 x 2 − 3 x 5 + x−6 x(4− x 2)(2x+1)
x 2 (2x−3) 2f(x)= x 4As x→∞, f(x)→∞, as x→−∞, f(x)→∞f(x)= x 3f(x)=− x 4
As x→−∞, f(x)→−∞, as x→∞, f(x)→−∞f(x)=− x 9f(x)=−2 x 4 − 3 x 2 + x−1
As x→−∞, f(x)→−∞, as x→∞, f(x)→−∞f(x)=3 x 2 + x−2f(x)= x 2 (2 x 3 −x+1)
As x→∞, f(x)→∞, as x→−∞, f(x)→−∞f(x)= (2−x) 7f(t)=2(t−1)(t+2)(t−3) (0,12),
(1,0);(−2,0);and (3,0).g(n)=−2(3n−1)(2n+1)f(x)= x 4 −16 (0,−16). (2,0) (−2,0).f(x)= x 3 +27
f(x)=x(x 2 −2x−8) (0,0). (0,0),(4,0), (−2, 0).f(x)=(x+3)(4 x 2 −1)f(x)=− x 3f(x)= x 4 −5 x 2xf(x)
as x→−∞, f(x)→∞, as x→∞, f(x)→∞f(x)= x 2 (1−x) 2f(x)=(x−1)(x−2)(3−x)xf(x)
as x→−∞, f(x)→∞, as x→∞, f(x)→−∞f(x)= x 5 10 − x 4f(x)= x 3 (x−2) y- (0, 0). x- (0, 0), (2, 0).
As x→−∞, f(x)→∞, as x→∞, f(x)→∞f(x)=x(x−3)(x+3)f(x)=x(14−2x)(10−2x) y- (0,0) x-
(0, 0), (5, 0), (7, 0). As x→−∞, f(x)→−∞, as x→∞, f(x)→∞f(x)=x(14−2x) (10−2x) 2f(x)= x 3 −16x y-
(0, 0). x- (−4, 0), (0, 0), (4, 0). As x→−∞, f(x)→−∞, as x→∞, f(x)→∞f(x)= x 3 −27f(x)= x 4 −81 y-
(0, −81). x- (3, 0), (−3, 0). As x→−∞, f(x)→∞, as x→∞, f(x)→∞f(x)=− x 3 + x 2 +2x
f(x)= x 3 −2 x 2 −15x y- (0, 0). x- (−3, 0), (0, 0), (5, 0). As x→−∞, f(x)→−∞, as x→∞, f(x)→∞
f(x)= x 3 −0.01x y- (0,−4). x- (−2,0), (2,0). as x→−∞, f(x)→∞, as x→∞, f(x)→∞.f(x)= x 2 −4 y- (0,9). x-
(−3,0), (3,0). as x→−∞, f(x)→−∞, as x→∞, f(x)→−∞. y- (0,0). x- (0,0), (2,0).
as x→−∞, f(x)→−∞, as x→∞, f(x)→∞.f(x)= x 3 −4 x 2 +4x y- (0,1). x- (1,0).
as x→−∞, f(x)→∞, as x→∞, f(x)→−∞. y- (0,1). x- as x→−∞, f(x)→∞, as x→∞, f(x)→∞.f(x)= x 4 +1 d,
m, V(m)=8 m 3 +36 m 2 +54m+27 x x. x- x.V(x)=4 x 3 −32 x 2 +64xx f(x)=k x p k p, a i x i
f(x)= a n x n +...+ a 2 x 2 + a 1 x+ a 0

R(t)=−0.037 t 4 +1.414 t 3 −19.777 t 2 +118.696t−205.332

R t t=6 f h g k g k f x f(x)=0 f. x- x- f, f(x)=0. x- f(x)= x 6 −3 x 4 +2 x 2 . f(x)=0.

x 6 −3 x 4 +2 x 2 =0 Factor out the greatest common factor. x 2 (x 4 −3 x 2 +2)=0

Factor the trinomial. x 2 (x 2 −1)(x 2 −2)=0 Set each factor equal to zero.

(x 2 −1)=0 (x 2 −2)=0 x 2 =0 or x 2 =1 or x 2 =2 x=0 x=±1 x=± 2

x- (0,0),(1,0),(−1,0),(2 ,0), (− 2 ,0). x- f(x)= x 3 −5 x 2 −x+5. f(x)=0

x 3 −5 x 2 −x+5=0 Factor by grouping. x 2 (x−5)−(x−5)=0 Factor out the common factor. (x 2 −1)(x−5)=0 Factor the difference of squares. (x+1)(x−1)(x−5)=0 Set each factor equal to zero.

x+1=0 or x−1=0 or x−5=0 x=−1 x=1 x=5

x- (−1,0),(1,0), (5,0). y- g(x)= (x−2) 2 (2x+3). g(0).

g(0)= (0−2) 2 (2(0)+3) =12

(0,12). g(x)=0.

(x−2) 2 (2x+3)=0

(x−2) 2 =0 (2x+3)=0 x−2=0 or x=− 3 2 x=2

x- (2,0) (− 3 2 ,0). x- h(x)= x 3 +4 x 2 +x−6. x=−3,−2, 1. x

h(−3)=h(−2)=h(1)=0.

h(x)= x 3 +4 x 2 +x−6,

h(−3)= (−3) 3 +4 (−3) 2 +(−3)−6=−27+36−3−6=0 h(−2)= (−2) 3 +4 (−2) 2 +(−2)−6=−8+16−2−6=0

h(1)= (1) 3 +4 (1) 2 +(1)−6=1+4+1−6=0

x-

h(x)= x 3 +4 x 2 +x−6 =(x+3)(x+2)(x−1)

y- f(x)= x 4 −19 x 2 +30x. (0,0); (0,0),(−5,0),(2,0), (3,0) x-

f(x)=(x+3) (x−2) 2 (x+1) 3 .

x- x- −3 (x+3)=0. x- x=−3. x- 2 (x−2) 2 =0.

(x−2) 2 =(x−2)(x−2)

(x−2) x=2, (x−2) x- −1 (x+1) 3 =0. f(x)= x 3 . x- x- x- x- x- (x−h) p , x- h p. x=h p. x- n, n. n. n. x=−3. −3 2. x=−1. x=4.

f(x)= a n x n + a n−1 x n−1 +...+ a 1 x+ a 0

x x a n x n , x f(x) x f(x) x f(x) f(x)= x 4 − x 3 −4 x 2 +4x f n n−1 f(x)=− x 3 +4 x 5 −3 x 2 ++1 f(x)=− (x−1) 2 (1+2 x 2) f(x)=−x + 3 4 x 5 −3 x 2 ++1 f(x)=4 x 5 − x 3 −3 x 2 ++1 5−1=4. f(x)=− (x−1) 2 (1+2 x 2) 4−1=3. y- f(−x)=f(x). f(−x)=−f(x). x- f(x)=−2 (x+3) 2 (x−5). x- x=−3, x- x=5, f(0).

f(0)=−2 (0+3) 2 (0−5) =−2·9·(−5) =90

y- (0,90). −2 x 3 , x→−∞ f(x)→∞, x- f(−x)=−2 (−x+3) 2 (−x−5) f(x), (−3,0), x- (0,90), y- y- (5,0). x→∞ f(x)→−∞, f(x)=−2 (x+3) 2 (x−5) f(x)= 1 4 x (x−1) 4 (x+3) 3 . f a b f, a<b f(a)≠f(b), f f(a) f(b). f x=a x- x=b x- x=a x=b x- (c, f(c)). c f(c)=0. x- a b. f f(a) f(b) c a b f(c)=0. f(x)= x 3 −5 x 2 +3x+6 x=1 x=4. f(x) x=1,2,3,and4. xf(x) x=2. f(3) f(4) x=1 x=4. x=1 x=4. f(x)=7 x 5 −9 x 4 − x 2 x=1 x=2. f f(1) f(2) x=1 x=2. x- x- p x= x 1 , x 2 ,..., x n ,
f(x)=a (x− x 1) p 1 (x− x 2) p 2 ⋯ (x− x n) p n p i a x- x=−3,2, 5. y- (0,2). x=−3 x=5, x=2,

f(x)=a(x+3) (x−2) 2 (x−5)

y- (0,−2), a.

f(0)=a(0+3) (0−2) 2 (0−5) −2=a(0+3) (0−2) 2 (0−5) −2=−60a a= 1 30

f(x)= 1 30 (x+3) (x−2) 2 (x−5).f(x)=− 1 8 (x−2) 3 (x+1) 2 (x−4) x=a x=a. a, f(a)≥f(x) x x=a. a, f(a)≤f(x) x x=a. a, f(a)≥f(x) x. a, f(a)≤f(x) x. f(x)=x w. (14−2w) (20−2w) w

V(w)=(20−2w)(14−2w)w =280w−68 w 2 +4 w 3

w, 20−2w 14−2w, w 0<w<7. V(w) w [0, 7]. [−1,4] f(x)=−0.2 (x−2) 3 (x+1) 2 (x−4). (0,−6.5), (3.5,7). x- x- x- n n−1 n−1 f(a) and f(b) c a b f(c)=0. x- f? x- x- f(x)=0. n n a b a b. x- x- C(t)=2(t−4)(t+1)(t−6) C(t)=3(t+2)(t−3)(t+5)(−2,0),(3,0),(−5,0) C(t)=4t (t−2) 2 (t+1) C(t)=2t(t−3) (t+1) 2 (3,0),(−1,0),(0,0) C(t)=2 t 4 −8 t 3 +6 t 2 C(t)=4 t 4 +12 t 3 −40 t 2 (0,0), (−5,0), (2,0) f(x)= x 4 − x 2 f(x)= x 3 + x 2 −20x(0,0), (−5,0), (4,0)f(x)= x 3 +6 x 2 −7x f(x)= x 3 + x 2 −4x−4(2,0), (−2,0), (−1,0)f(x)= x 3 +2 x 2 −9x−18f(x)=2 x 3 − x 2 −8x+4 (−2,0), (2,0), (1 2 ,0)f(x)= x 6 −7 x 3 −8f(x)=2 x 4 +6 x 2 −8(1,0), (−1,0)f(x)= x 3 −3 x 2 −x+3 f(x)= x 6 −2 x 4 −3 x 2(0,0), (3 ,0), (− 3 ,0)f(x)= x 6 −3 x 4 −4 x 2f(x)= x 5 −5 x 3 +4x (0,0), (1,0), (−1,0), (2,0), (−2,0)f(x)= x 3 −9x, x=−4 x=−2.f(x)= x 3 −9x, x=2 x=4.f(2)=−10 f(4)=28.f(x)= x 5 −2x, x=1 x=2.f(x)=− x 4 +4, x=1 x=3f(1)=3 f(3)=−77. f(x)=−2 x 3 −x, x=−1 x=1. f(x)= x 3 −100x+2, x=0.01 x=0.1f(0.01)=1.000001 f(0.1)=−7.999. f(x)= (x+2) 3 (x−3) 2 f(x)= x 2 (2x+3) 5 (x−4) 2 − 3 2 f(x)= x 3 (x−1) 3 (x+2)f(x)= x 2 (x 2 +4x+4) f(x)= (2x+1) 3 (9 x 2 −6x+1)f(x)= (3x+2) 5 (x 2 −10x+25) − 2 3 with multiplicity 5, 5 with multiplicity 2f(x)=x(4 x 2 −12x+9)(x 2 +8x+16)f(x)= x 6 − x 5 −2 x 4 0 with multiplicity 4, 2 with multiplicity 1, −1 with multiplicity 1f(x)=3 x 4 +6 x 3 +3 x 2 f(x)=4 x 5 −12 x 4 +9 x 33 2 f(x)=2 x 4 (x 3 −4 x 2 +4x)f(x)=4 x 4 (9 x 4 −12 x 3 +4 x 2) 0 with multiplicity 6, 2 3 with multiplicity 2 x- y-f(x)= (x+3) 2 (x−2)g(x)=(x+4) (x−1) 2(1, 0) (−4, 0)y-(0, 4)x→−∞f(x)→−∞x→∞f(x)→∞h(x)= (x−1) 3 (x+3) 2k(x)= (x−3) 3 (x−2) 2(3,0) (2,0)y-(0,−108)x→−∞f(x)→−∞x→∞f(x)→∞.m(x)=−2x(x−1)(x+3)n(x)=−3x(x+2)(x−4) (0, 0), (−2, 0), (4, 0)y(0, 0).x→−∞f(x)→∞x→∞f(x)→−∞.f(x)=− 2 9 (x−3)(x+1)(x+3) f(x)= 1 4 (x+2) 2 (x−3) x=−2, x=1, x=3. (0,−4).f(x)=− 2 3 (x+2)(x−1)(x−3) x=−5, x=−2, x=1. (0,6) x=3 x=1 x=−3. (0,9)f(x)= 1 3 (x−3) 2 (x−1) 2 (x+3) x=4, x=1 x=−2. (0,−3). x=1, x=3. (2,15). f(x)=−15 (x−1) 2 (x−3) 3 x=4, x=3, x=2. (0,−24). x=−3, x=−2 x=1. (0,12).f(x)=−2(x+3)(x+2)(x−1) x=−3 x=2 x=−2. (0, 4). x= 1 2 x=6 x=−2. (0,18).f(x)=− 3 2 (2x−1) 2 (x−6)(x+2) x=−3 x=0. (1,32).f(x)= x 3 −x−1 (−.58, −.62), (.58, −1.38) f(x)=2 x 3 −3x−1f(x)= x 4 +x (−.63, −.47) f(x)=− x 4 +3x−2f(x)= x 4 − x 3 +1 (.75, .89)f(x)= (x−500) 2 (x+200) x x x.f(x)=4 x 3 −36 x 2 +80x 2x 2x x. x +1 x +1 x.f(x)=4 x 3 −36 x 2 +60x+100 x+2 3x+6 V= 1 3 π r 2 h r h. f(x)=π(9 x 3 +45 x 2 +72x+36) f(a) f(a)≥f(x) x. f(a) f(a)≤f(x) x. a b f, a<b f(a)≠f(b), f f(a) f(b); x-

(x−h) p , x=h p.

V=l·w·h =61.5·40·30 =73,800

(m³).

h= V l·w = 73,800 61.5·40 =30

3 x 4 −3 x 3 −33 x 2 +54x. 3x; x−2.

dividend = (divisor · quotient) + remainder 178=(3·59)+1 =177+1 =178

2 x 3 −3 x 2 +4x+5 x+2

2 x 3 −3 x 2 +4x+5 x+2 =2 x 2 −7x+18− 31 x+2

2 x 3 −3 x 2 +4x+5=(x+2)(2 x 2 −7x+18)−31

f(x) d(x) d(x) f(x), q(x) r(x)

f(x)=d(x)q(x)+r(x)

q(x) r(x) d(x). r(x)=0, d(x) f(x). d(x) q(x) f(x). 5 x 2 +3x−2 x+1. 5x−2.

5 x 2 +3x−2 x+1 =5x−2

5 x 2 +3x−2=(x+1)(5x−2)

6 x 3 +11 x 2 −31x+15 3x−2.

6 x 3 +11 x 2 −31x+15 3x−2 =2 x 2 +5x−7+ 1 3x−2

(3x−2)(2 x 2 +5x−7)+1=6 x 3 +11 x 2 −31x+15

6 x 3 +11 x 2 −31x+15 3x−2 2 x 2 +5x−7 1 16 x 3 −12 x 2 +20x−3 4x+5.

4 x 2 −8x+15− 78 4x+5

2 x 3 −3 x 2 +4x+5 x+2 2x²−7x+18 −31. x−k. k k. k. 5 x 2 −3x−36 x−3. k k. k. 5x+12. x−3

(x−3)(5x+12)+0=5 x 2 −3x−36

4 x 3 +10 x 2 −6x−20 x+2. x+2 k=−2. 4 x 2 +2x−10. x+2 4 x 3 +10 x 2 −6x−20.

f(x)=4 x 3 +10 x 2 −6x−20 x=k=−2. x+2 4 x 3 +10 x 2 −6x−20. −9 x 4 +10 x 3 +7 x 2 −6 x−1.

−9 x 3 + x 2 +8x+8+ 2 x−1 . 3 x 4 +18 x 3 −3x+40 x+7. 3 x 3 −3 x 2 +21x−150+ 1,090 x+7

3 x 4 −3 x 3 −33 x 2 +54x. 3x x−2.

V=l·w·h 3 x 4 −3 x 3 −33 x 2 +54x=3x·(x−2)·h

h, 3x.

3x·(x−2)·h 3x = 3 x 4 −3 x 3 −33 x 2 +54x 3x (x−2)h= x 3 − x 2 −11x+18

h

h= x 3 − x 2 −11x+18 x−2

2 1 −1 −11 18 2 2 −18 1 1 −9 0

x 2 +x−9 x 2 +x−9. 3 x 3 +14 x 2 −23x+6. x+6. 3 x 2 −4x+1f(x)=d(x)q(x)+r(x)q(x)≠0 x−k. n

(x 2 +5x−1)÷(x−1)x+6+ 5 x-1 , quotient: x+6, remainder: 5(2 x 2 −9x−5)÷(x−5)

(3 x 2 +23x+14)÷(x+7)3x+2, quotient: 3x+2, remainder: 0(4 x 2 −10x+6)÷(4x+2)

(6 x 2 −25x−25)÷(6x+5)x−5, quotient: x−5, remainder: 0(− x 2 −1)÷(x+1)(2 x 2 −3x+2)÷(x+2)

2x−7+ 16 x+2 , quotient: 2x−7, remainder: 16(x 3 −126)÷(x−5)(3 x 2 −5x+4)÷(3x+1)

x−2+ 6 3x+1 , quotient: x−2, remainder: 6(x 3 −3 x 2 +5x−6)÷(x−2)(2 x 3 +3 x 2 −4x+15)÷(x+3)

2 x 2 −3x+5, quotient: 2 x 2 −3x+5, remainder: 0(3 x 3 −2 x 2 +x−4)÷(x+3)

(2 x 3 −6 x 2 −7x+6)÷(x−4)2 x 2 +2x+1+ 10 x−4(6 x 3 −10 x 2 −7x−15)÷(x+1)

(4 x 3 −12 x 2 −5x−1)÷(2x+1)2 x 2 −7x+1− 2 2x+1(9 x 3 −9 x 2 +18x+5)÷(3x−1)

(3 x 3 −2 x 2 +x−4)÷(x+3)3 x 2 −11x+34− 106 x+3(−6 x 3 + x 2 −4)÷(2x−3)

(2 x 3 +7 x 2 −13x−3)÷(2x−3)x 2 +5x+1(3 x 3 −5 x 2 +2x+3)÷(x+2)(4 x 3 −5 x 2 +13)÷(x+4)

4 x 2 −21x+84− 323 x+4(x 3 −3x+2)÷(x+2)(x 3 −21 x 2 +147x−343)÷(x−7)x 2 −14x+49

(x 3 −15 x 2 +75x−125)÷(x−5)(9 x 3 −x+2)÷(3x−1)3 x 2 +x+ 2 3x−1(6 x 3 − x 2 +5x+2)÷(3x+1)

(x 4 + x 3 −3 x 2 −2x+1)÷(x+1)x 3 −3x+1(x 4 −3 x 2 +1)÷(x−1)(x 4 +2 x 3 −3 x 2 +2x+6)÷(x+3)

x 3 − x 2 +2(x 4 −10 x 3 +37 x 2 −60x+36)÷(x−2)(x 4 −8 x 3 +24 x 2 −32x+16)÷(x−2)

x 3 −6 x 2 +12x−8(x 4 +5 x 3 −3 x 2 −13x+10)÷(x+5)(x 4 −12 x 3 +54 x 2 −108x+81)÷(x−3)

x 3 −9 x 2 +27x−27(4 x 4 −2 x 3 −4x+2)÷(2x−1)(4 x 4 +2 x 3 −4 x 2 +2x+2)÷(2x+1)2 x 3 −2x+2

x−2, 4 x 3 −3 x 2 −8x+4x−2, 3 x 4 −6 x 3 −5x+10 (x−2)(3 x 3 −5)x+3, −4 x 3 +5 x 2 +8

x−2, 4 x 4 −15 x 2 −4 (x−2)(4 x 3 +8 x 2 +x+2)x− 1 2 , 2 x 4 − x 3 +2x−1x+ 1 3 , 3 x 4 + x 3 −3x+1
x 2 −x+3 (x 2 +2x+4)(x−1)(x 2 +2x+4) x 2 +2x+5 x 2 +x+1(x−5)(x 2 +x+1)x 2 +2x+24 x 3 −33 x−2
Quotient: 4 x 2 +8x+16, remainder: −12 x 3 +25 x+33 x 3 +2x−5 x−1Quotient: 3 x 2 +3x+5, remainder: 0
−4 x 3 − x 2 −12 x+4x 4 −22 x+2Quotient: x 3 −2 x 2 +4x−8, remainder: −6 x k −1 x−1 k=1, 2, 3. k=4?
x k +1 x+1 k=1, 3, 5. k=7?x 6 − x 5 + x 4 − x 3 + x 2 −x+1 x 4 − k 4 x−k k=1, 2, 3. k=4? x k x+1
k=1, 2, 3. k=4?x 3 − x 2 +x−1+ 1 x+1 x k x−1 k=1, 2, 3. k=4?x+1 x−i1+ 1+i x−ix 2 +1 x−ix+1 x+i
1+ 1−i x+ix 2 +1 x+ix 3 +1 x−ix 2 −ix−1+ 1−i x−i x+5, 2 x 2 +9x−5. 2x + 5, 4 x 3 +10 x 2 +6x+15
2 x 2 +3 3x–4, 6 x 4 −8 x 3 +9 x 2 −9x−4 12 x 3 +20 x 2 −21x−36, 2x+3, 3x−4.2x+3
18 x 3 −21 x 2 −40x+48, 3x–4, 3x–4. 10 x 3 +27 x 2 +2x−24, 5x–4, 2x+3.x+2 10 x 3 +30 x 2 −8x−24,
2, x+3. π(25 x 3 −65 x 2 −29x−3), 5x+1.x−3 π(4 x 3 +12 x 2 −15x−50), 2x+5.
π(3 x 4 +24 x 3 +46 x 2 −16x−32), x+4.3 x 2 −2 f(x) d(x) d(x) f(x), q(x) r(x) f(x)=d(x)q(x)+r(x) q(x)
r(x) d(x). x−k x–k, k, f(k) f(x) d(x) d(x) f(x), q(x) r(x)

f(x)=d(x)q(x)+r(x)

d(x), x−k,

f(x)=(x−k)q(x)+r

x−k r. x=k,

f(k)=(k−k)q(k)+r =0·q(k)+r =r

f(k) f(x) x−k. f(x) x−k, f(k). f, f(x) x=k x−k. f(k). f(x)=6 x 4 − x 3 −15 x 2 +2x−7 x=2. x−2.

2 6 −1 −15 2 −7 12 22 14 32 6 11 7 16 25

f(2)=25. f(2).

f(x)=6 x 4 − x 3 −15 x 2 +2x−7 f(2)=6 (2) 4 − (2) 3 −15 (2) 2 +2(2)−7 =25

f(x)=2 x 5 −3 x 4 −9 x 3 +8 x 2 +2 x=−3. f(−3)=−412

f(x)=(x−k)q(x)+r.

k r f(k)=0 f(x)=(x−k)q(x)+0 f(x)=(x−k)q(x). x−k f(x). k f(x), x−k f(x). x−k f(x), f(x)=(x−k)q(x)+r k
n n n k f(x) (x−k) f(x). (x−k). (x−k) (x+2) x 3 −6 x 2 −x+30. (x+2)

−2 1 −6 −1 30 −2 16 −30 1 −8 15 0

(x+2)

(x+2)(x 2 −8x+15)
(x+2)(x−3)(x−5)

x 3 −6 x 2 −x+30 f(x)= x 3 +4 x 2 −4x−16 (x−2) x= 2 5 x= 3 4 .
f(x)= a n x n + a n−1 x n−1 +...+ a 1 x+ a 0 f(x) p q p a 0 q a n . f(x), p q , p q f(p q).
f(x)=2 x 4 −5 x 3 + x 2 −4. f(x) p=±1,±2,±4. q=±1,±2.

p q =± 1 1 , ± 1 2 p q =± 2 1 ,± 2 2 p q = ± 4 1 ,± 4 2

2 2 =1 4 2 =2,

p q = Factors of the last Factors of the first =±1,±2,±4,± 1 2

f(x)=2 x 3 + x 2 −4x+1. p q f(x), p q

p q = factor of constant term factor of leading coefficient = factor of 1 factor of 2

±1 ±1 ±2. p q ±1 ± 1 2 . x f(x).

f(−1)=2 (−1) 3 + (−1) 2 −4(−1)+1=4 f(1)=2 (1) 3 + (1) 2 −4(1)+1=0 f(− 1 2)=2 (− 1 2) 3 + (− 1 2) 2 −4(− 1 2)+1=3 f(1 2)=2 (1 2) 3 + (1 2) 2 −4(1 2)+1=− 1 2

−1,− 1 2 , and 1 2 f(x). f(x). f(x)= x 3 −5 x 2 +2x+1. f, f(x)=4 x 3 −3x−1. p q f(x), p q

p q = factor of constant term factor of leading coefficient = factor of –1 factor of 4

−1 ±1 4 ±1,±2, ±4. p q ±1, ± 1 2 , ± 1 4 .

1 4 0 −3 −1 4 4 1 4 4 1 0

(x−1)

(x−1)(4 x 2 +4x+1).

f(x)

(x−1) (2x+1) 2 .
2x+1=0 x=− 1 2

− 1 2 f x=−0.5, −0.5. x=1, x=1. f f(x)=0. c 1 . f(x) x− c 1 x− c 1 c 2 . x− c 2 f(x). f(x) n>0, a f(x) n

f(x)=a(x− c 1)(x− c 2)...(x− c n)

c 1 , c 2 ,..., c n f(x) n f(x)=3 x 3 +9 x 2 +x+3. p q f(x), p q

p q = factor of constant term factor of leading coefficient = factor of 3 factor of 3

±1 ±3. p q , ±3,±1, and ± 1 3 .

−3 3 9 1 3 −9 0 −3 3 0 1 0

(x+3)

(x+3)(3 x 2 +1)

3 x 2 +1=0 x 2 =− 1 3 x=± − 1 3 =± i 3 3

f(x) ± i 3 3 . f x=−3, x=−3. x=−3 x=−3 f(x)=2 x 3 +5 x 2 −11x+4. −4, 1 2 , and 1. n n n (x−c), c f a+bi, b≠0, f(x). x−(a+bi) f(x). f x−(a−bi) f(x). x−(a−bi), x−(a+bi), f a+bi, a−bi f(x). (x−c), c f a+bi, a−bi, f f, (c,f(c)) i, f(−2)=100. x=i x=−i (x+3), (x−2), (x−i), (x+i).

f(x)=a(x+3)(x−2)(x−i)(x+i) f(x)=a(x 2 +x−6)(x 2 +1) f(x)=a(x 4 + x 3 −5 x 2 +x−6)

f(−2)=100. x=−2 f(2)=100 f(x).

100=a((−2) 4 + (−2) 3 −5 (−2) 2 +(−2)−6) 100=a(−20) −5=a

f(x)=−5(x 4 + x 3 −5 x 2 +x−6)

f(x)=−5 x 4 −5 x 3 +25 x 2 −5x+30

i −i i −i −i i. 2+3i 2−3i −2i f(1)=10. f(x)=− 1 2 x 3 + 5 2 x 2 −2x+10 f(x) f(−x) f(−x) f(x)

f(x)= a n x n + a n−1 x n−1 +...+ a 1 x+ a 0 f(x) f(−x) f(x)=− x 4 −3 x 3 +6 x 2 −4x−12. f(−x)

f(−x)=− (−x) 4 −3 (−x) 3 +6 (−x) 2 −4(−x)−12 f(−x)=− x 4 +3 x 3 +6 x 2 +4x−12

f(x)=2 x 4 −10 x 3 +11 x 2 −15x+12. V=lwh. l=w+4. h= 1 3 w.

V=(w+4)(w)(1 3 w) V= 1 3 w 3 + 4 3 w 2

351= 1 3 w 3 + 4 3 w 2 Substitute 351 for V. 1053= w 3 +4 w 2 Multiply both sides by 3. 0= w 3 +4 w 2 −1053 Subtract 1053 from both sides.

±3, ±9, ±13, ±27, ±39, ±81, ±117, ±351, ±1053. x=1.

1 1 4 0 −1053 1 5 5 1 5 5 −1048

x=3. x=9.

l=w+4=9+4=13 and h= 1 3 w= 1 3 (9)=3

f(k), f(x) x−k. k f(x) (x−k) f(x). (x−c), c f(−x) (x 4 −9 x 2 +14)÷(x−2)(3 x 3 −2 x 2 +x−4)÷(x+3) −106(x 4 +5 x 3 −4x−17)÷(x+1)(−3 x 2 +6x+24)÷(x−4) 0
(5 x 5 −4 x 4 +3 x 3 −2 x 2 +x−1)÷(x+6)(x 4 −1)÷(x−4)255(3 x 3 +4 x 2 −8x+2)÷(x−3)
(4 x 3 +5 x 2 −2x+7)÷(x+2)−1f(x)=2 x 3 −9 x 2 +13x−6; x−1f(x)=2 x 3 + x 2 −5x+2; x+2−2, 1, 1 2
f(x)=3 x 3 + x 2 −20x+12; x+3f(x)=2 x 3 +3 x 2 +x+6; x+2−2f(x)=−5 x 3 +16 x 2 −9; x−3
x 3 +3 x 2 +4x+12; x+3−34 x 3 −7x+3; x−12 x 3 +5 x 2 −12x−30, 2x+5− 5 2 , 6 , − 6
x 3 −3 x 2 −10x+24=02 x 3 +7 x 2 −10x−24=02, −4, − 3 2x 3 +2 x 2 −9x−18=0x 3 +5 x 2 −16x−80=0
4, −4, −5x 3 −3 x 2 −25x+75=02 x 3 −3 x 2 −32x−15=05, −3, − 1 22 x 3 + x 2 −7x−6=0
2 x 3 −3 x 2 −x+1=01 2 , 1+ 5 2 , 1− 5 23 x 3 − x 2 −11x−6=02 x 3 −5 x 2 +9x−9=03 2
2 x 3 −3 x 2 +4x+3=0x 4 −2 x 3 −7 x 2 +8x+12=02, 3, −1, −2x 4 +2 x 3 −9 x 2 −2x+8=0
4 x 4 +4 x 3 −25 x 2 −x+6=01 2 , − 1 2 , 2, −32 x 4 −3 x 3 −15 x 2 +32x−12=0
x 4 +2 x 3 −4 x 2 −10x−5=0−1, −1, 5 , − 54 x 3 −3x+1=08 x 4 +26 x 3 +39 x 2 +26x+6− 3 4 , − 1 2
x 3 + x 2 +x+1=0x 3 −8 x 2 +25x−26=02, 3+2i, 3−2ix 3 +13 x 2 +57x+85=03 x 3 −4 x 2 +11x+10=0
− 2 3 , 1+2i, 1−2ix 4 +2 x 3 +22 x 2 +50x−75=02 x 3 −3 x 2 +32x+17=0− 1 2 , 1+4i, 1−4if(x)= x 3 −1
f(x)= x 4 − x 2 −1f(x)= x 3 −2 x 2 −5x+6f(x)= x 3 −2 x 2 +x−1f(x)= x 4 +2 x 3 −12 x 2 +14x−5
f(x)=2 x 3 +37 x 2 +200x+300f(x)= x 3 −2 x 2 −16x+32f(x)=2 x 4 −5 x 3 −5 x 2 +5x+3
f(x)=2 x 4 −5 x 3 −14 x 2 +20x+8f(x)=10 x 4 −21 x 2 +11f(x)= x 4 +3 x 3 −4x+4f(x)=2 x 3 +3 x 2 −8x+5
±5, ±1, ± 5 2f(x)=3 x 3 +5 x 2 −5x+4f(x)=6 x 4 −10 x 2 +13x+1±1, ± 1 2 , ± 1 3 , ± 1 6
f(x)=4 x 5 −10 x 4 +8 x 3 + x 2 −8f(x)=6 x 3 −7 x 2 +11, 1 2 , − 1 3f(x)=4 x 3 −4 x 2 −13x−5
f(x)=8 x 3 −6 x 2 −23x+62, 1 4 , − 3 2f(x)=12 x 4 +55 x 3 +12 x 2 −117x+54
f(x)=16 x 4 −24 x 3 + x 2 −15x+255 4 (2,f(2))=(2,4) (2,f(2))=(2,4)f(x)= 4 9 (x 3 + x 2 −x−1)

1 2 (−3,f(−3))=(−3,5) − 1 2 1 2 (−2,f(−2))=(−2,6)f(x)=− 1 5 (4 x 3 −x) (−2,f(−2))=(−2,10) 16π 72π 48π 28.125π 1 3 98 9 π f(x) f(−x) k f(x) (x−k) f(x) (x−c), c p q p q f(x) x−k, f(k) x, C(x)=15,000x−0.1 x 2 +1000. x x. x

f(x)= 15,000x−0.1 x 2 +1000 x

f(x)= 1 x . (y=0) as x→−∞. x=0 (y=0) as x→∞. x f(x) x→ a −x a x<a ax→ a +x a x>a ax→∞x x x→−∞ x x f(x)→∞f(x)→−∞f(x)→a a f(x)= 1 x f(x)= 1 x . x=0; xf(x)= 1 x

as x→ 0 − ,f(x)→−∞

xf(x)= 1 x

As x→ 0 + , f(x)→∞.

x=0 x=a a.

As x→a,f(x)→∞, or as x→a,f(x)→−∞.

f(x)= 1 x x x

As x→∞,f(x)→0,and as x→−∞,f(x)→0.

y=0. y=b

As x→∞ or x→−∞, f(x)→b.

x=2, x=2.

As x→ 2 − ,f(x)→−∞, and as x→ 2 + , f(x)→∞.

y=4.

As x→∞, f(x)→4 and as x→−∞, f(x)→4.

x→±∞, f(x)→0; x→0, f(x)→∞

f(x)= 1 x+2 +3

f(x)= 3x+7 x+2

x=−2, x=−2.

As x→− 2 − , f(x)→−∞,and as x→− 2 + , f(x)→∞.

y=3.

As x→±∞, f(x)→3.

x→3,f(x)→∞, x→±∞,f(x)→−4. f(x)= 1 (x−3) 2 −4. f(x)= 3x+7 x+2 . P(x) and Q(x).

f(x)= P(x) Q(x) = a p x p + a p−1 x p−1 +...+ a 1 x+ a 0 b q x q + b q−1 x q−1 +...+ b 1 x+ b 0 ,Q(x)≠0 t

water: W(t)=100+10t in gallons sugar: S(t)=5+1t in pounds

C,

C(t)= 5+t 100+10t

C(t) t= 12.

C(12)= 5+12 100+10(12) = 17 220

C(0)= 5+0 100+10(0) = 1 20

17 220 ≈0.08> 1 20 =0.05,

1 10 =0.1

y= 0.1. C, 12 11 f(x)= x+3 x 2 −9 .

x 2 −9=0 x 2 =9 x=±3

x=±3. x=±3. x=±3. x=3 x=−3. f(x)= 4x 5(x−1)(x−5) . x=1 x=5. k(x)= 5+2 x 2 2−x− x 2 .

k(x)= 5+2 x 2 2−x− x 2 = 5+2 x 2 (2+x)(1−x)

(2+x)(1−x)=0 x=−2,1

x=–2 x=1 f(x)= x 2 −1 x 2 −2x−3

f(x)= (x+1)(x−1) (x+1)(x−3)

x+1 x=−1, x–3 x=3, x=a a k(x)= x−2 x 2 −4 .

k(x)= x−2 (x−2)(x+2)

x–2. x=2. x+2. x=−2. x=−2. x=−2, x=2 f(x)= x 2 −25 x 3 −6 x 2 +5x . x=5. x=0, x=1. y=0.

Example: f(x)= 4x+2 x 2 +4x−5

f(x)≈ 4x x 2 = 4 x . g(x)= 4 x , y=0. y=0 f(x)= p(x) q(x) , q(x)≠0 where degree of p<degree of q.

Example: f(x)= 3 x 2 −2x+1 x−1

f(x)≈ 3 x 2 x =3x. g(x)=3x. g(x)=3x f g, y=3x. 3 x 2 −2x+1 x−1 . 3x+1, g(x)=3x+1.
f(x)= p(x) q(x) ,q(x)≠0 p>degree of q by 1. y= a n b n , a n b n p(x) q(x) f(x)= p(x) q(x) ,q(x)≠0.

Example: f(x)= 3 x 2 +2 x 2 +4x−5

f(x)≈ 3 x 2 x 2 =3. g(x)=3, x→±∞,f(x)→3, y=3. f(x)= p(x) q(x) ,q(x)≠0 where degree of p=degree of q.

f(x)= 3 x 5 − x 2 x+3

f(x)≈ 3 x 5 x =3 x 4 ,

x→±∞, f(x)→∞

y=0.g(x)= 6 x 3 −10x 2 x 3 +5 x 2h(x)= x 2 −4x+1 x+2k(x)= x 2 +4x x 3 −8 f(x)= p(x) q(x) , q(x)≠0.
g(x)= 6 x 3 −10x 2 x 3 +5 x 2 : p=degree of q=3, y= 6 2 y=3.h(x)= x 2 −4x+1 x+2 : p=2 q=1. p>q
x 2 −4x+1 x+2 .

2 1 −4 1 −2 12 1 −6 13

x–2 y=–x–2.k(x)= x 2 +4x x 3 −8 : p=2 < q=3, y=0. C(t)= 5+t 100+10t . t, 10t,

t→∞, C(t)→ 1 10

y= 1 10 . C 1 10 . 1 10

f(x)= (x−2)(x+3) (x−1)(x+2)(x−5)

x=1,–2,and 5, x→±∞, f(x)→0. y=0. f(x)= (2x−1)(2x+1) (x−2)(x+3) x=2 x=–3; y=4.
f(x)= (x−2)(x+3) (x−1)(x+2)(x−5) .

f(0)= (0−2)(0+3) (0−1)(0+2)(0−5) = −6 10 =− 3 5 =−0.6

0= (x−2)(x+3) (x−1)(x+2)(x−5) This is zero when the numerator is zero. 0=(x−2)(x+3) x=2, −3
(0,–0.6), (2,0) (–3,0).
f(x)= 1 (x−3) 2 −4= 1−4 (x−3) 2 (x−3) 2 = 1−4(x 2 −6x+9) (x−3)(x−3) = −4 x 2 +24x−35 x 2 −6x+9
x→±∞, f(x)→−4; so y=–4 x=3, x→3,f(x)→∞. (2.5,0) (3.5,0). (0, −35 9).
f(x)= (x+1) 2 (x−3) (x+3) 2 (x−2) x=−1 (x+1) 2 x=3 (x−3) x=−3 (x+3) 2 f(x)= 1 x 2 . x=2, (x−2)
f(x)= 1 x . f(x)= (x+2)(x−3) (x+1) 2 (x−2) .

f(0)= (0+2)(0−3) (0+1) 2 (0−2) =3

x=–2 x=3. (0,3) (–2,0) (3,0). x+1=0 x–2=0, x=–1 x=2. y=0. x=–1 x=2,
f(x)= (x+2) 2 (x−2) 2 (x−1) 2 (x−3) , y= 1 2 . x=1 and x=3. (0, 4 3 .) (2,0) and (–2,0). (–2,0) (2,0)
x= x 1 , x 2 , ..., x n , x= v 1 , v 2 ,…, v m , x i =any v j ,

f(x)=a (x− x 1) p 1 (x− x 2) p 2 ⋯ (x− x n) p n (x− v 1) q 1 (x− v 2) q 2 ⋯ (x− v m) q n

p i q i a x=–2 x=3. x=–1 1 x , x=2 1 x 2 ,

f(x)=a (x+2)(x−3) (x+1) (x−2) 2 .

(0,–2).

−2=a (0+2)(0−3) (0+1) (0−2) 2 −2=a −6 4 a= −8 −6 = 4 3

f(x)= 4(x+2)(x−3) 3(x+1) (x−2) 2 .
f(x)= P(x) Q(x) = a p x p + a p−1 x p−1 +...+ a 1 x+ a 0 b q x q + b q−1 x q−1 +...+ b 1 x+ b 0 , Q(x)≠0
f(x)= 1 x f(x)= 1 x 2 . x= x 1 , x 2 ,…, x n , x= v 1 , v 2 ,…, v m , x i =any v j ,

f(x)=a (x− x 1) p 1 (x− x 2) p 2 ⋯ (x− x n) p n (x− v 1) q 1 (x− v 2) q 2 ⋯ (x− v m) q n

f(x)= x−1 x+2f(x)= x+1 x 2 −1All reals x≠–1, 1f(x)= x 2 +4 x 2 −2x−8f(x)= x 2 +4x−3 x 4 −5 x 2 +4
All reals x≠–1, –2, 1, 2f(x)= 4 x−1f(x)= 2 5x+2 x=− 2 5 ; y=0; x≠− 2 5f(x)= x x 2 −9f(x)= x x 2 +5x−36
x=4, –9; y=0; x≠4, –9f(x)= 3+x x 3 −27f(x)= 3x−4 x 3 −16x x=0, 4, −4; y=0; x≠0,4, −4
f(x)= x 2 −1 x 3 +9 x 2 +14xf(x)= x+5 x 2 −25 x=−5; y=0; x≠5,−5f(x)= x−4 x−6f(x)= 4−2x 3x−1
x= 1 3 ; y=− 2 3 ; x≠ 1 3 .f(x)= x+5 x 2 +4f(x)= x x 2 −xf(x)= x 2 +8x+7 x 2 +11x+30
f(x)= x 2 +x+6 x 2 −10x+24x-intercepts none, y-intercept (0, 1 4)f(x)= 94−2 x 2 3 x 2 −12f(x)= x 2x+1
x→− 1 2 + ,f(x)→−∞,x→− 1 2 − ,f(x)→∞ x→±∞,f(x)→ 1 2f(x)= 2x x−6f(x)= −2x x−6
x→ 6 + ,f(x)→−∞,x→ 6 − ,f(x)→∞, x→±∞,f(x)→−2f(x)= x 2 −4x+3 x 2 −4x−5
f(x)= 2 x 2 −32 6 x 2 +13x−5 x→− 1 3 + ,f(x)→∞,x→− 1 3 − , f(x)→−∞,x→ 5 2 − ,f(x)→∞,x→ 5 2 +
f(x)→−∞x→±∞,f(x)→ 1 3f(x)= 24 x 2 +6x 2x+1f(x)= 4 x 2 −10 2x−4y=2x+4f(x)= 81 x 2 −18 3x−2

f(x)= 6 x 3 −5x 3 x 2 +4y=2xf(x)= x 2 +5x+4 x−1V.A. x=0,H.A. y=2V.A. x=2, H.A. y=0
p(x)= 2x−3 x+4V.A. x=−4, H.A. y=2;(3 2 ,0);(0,− 3 4)q(x)= x−5 3x−1s(x)= 4 (x−2) 2
V.A. x=2, H.A. y=0, (0,1)r(x)= 5 (x+1) 2f(x)= 3 x 2 −14x−5 3 x 2 +8x−16
V.A. x=−4, x= 4 3 , H.A. y=1;(5,0);(− 1 3 ,0);(0, 5 16)g(x)= 2 x 2 +7x−15 3 x 2 −14+15
a(x)= x 2 +2x−3 x 2 −1V.A. x=−1, H.A. y=1;(−3,0);(0,3)b(x)= x 2 −x−6 x 2 −4
h(x)= 2 x 2 + x−1 x−4V.A. x=4, S.A. y=2x+9;(−1,0);(1 2 ,0);(0, 1 4)k(x)= 2 x 2 −3x−20 x−5
w(x)= (x−1)(x+3)(x−5) (x+2) 2 (x−4)V.A. x=−2, x=4, H.A. y=1,(1,0);(5,0);(−3,0);(0,− 15 16)
z(x)= (x+2) 2 (x−5) (x−3)(x+1)(x+4) x=5 x=−5, (2,0) (−1,0), (0,4)y=50 x 2 −x−2 x 2 −25 x=−4
x=−1, (1,0) (5,0), (0,7) x=−4 x=−5, (4,0) (−6,0), y=7y=7 x 2 +2x−24 x 2 +9x+20 x=−3 x=6,
(−2,0) (1,0), y=−2 x=−1, x=2, (0,2)y= 1 2 x 2 −4x+4 x+1 x=3, x=1, (0,4)y=4 x−3 x 2 −x−12
y=−9 x−2 x 2 −9y= 1 3 x 2 +x−6 x−1y=−6 (x−1) 2 (x+3) (x−2) 2f(x)= 1 x−2xyxy x=2, y=0f(x)= x x−3
f(x)= 2x x+4xyxy x=−4, y=2f(x)= 2x (x−3) 2f(x)= x 2 x 2 +2x+1xyxy x=−1, y=1 f(x). f(x)>0.
f(x)= 2 x+1f(x)= 4 2x−3(3 2 ,∞)f(x)= 2 (x−1)(x+2)f(x)= x+2 (x−1)(x−4)(−2,1)∪(4,∞)
f(x)= (x+3) 2 (x−1) 2 (x+1)f(x)= x 2 −4 x−2(2,4)f(x)= x 3 +1 x+1f(x)= x 2 +x−6 x−2(2,5)
f(x)= 2 x 2 +5x−3 x+3f(x)= x 3 + x 2 x+1(−1,1) t t C(t)= 8+2t 300+20t C t C(t)= 2t 3+ t 2 . t C t
C(t)= 100t 2 t 2 +75 . x x A(x)=50 x 2 + 800 x . x x A(x)=π x 2 + 100 x . x y=b x=a a

V= 1 3 π r 2 h = 1 3 π r 2 (2r) = 2 3 π r 3

V r.

r= 3V 2π 3

V r. f g f, (a, b), g,(b, a). x y y(x)=a x 2 . a.

18=a 6 2 a= 18 36 = 1 2

y(x)= 1 2 x 2

y 2x, x x

y= 1 2 x 2 2y= x 2 x=± 2y

x

y= x 2 2 , x>0

x 2x.

Area=l·w =36·2x =72x =72 2y

f −1 (x). f −1 (x) f(x). f(x), (f(x)) −1 = 1 f(x) . f −1 f, f f −1 . f x, f −1

f −1 (f(x))=x, for all x in the domain of f

f(f −1 (x))=x, for all x in the domain of f −1

f g, x f g.

g(f(x))=f(g(x))=x

f(x) y. x y. y, f −1 (x). f(x)= 1 x+1 f −1 (x)= 1 x −1 x≠0,−1 f −1 (f(x))=x f(f −1 (x))=x.
f −1 (f(x))= f −1 (1 x+1) = 1 1 x+1 −1 =(x+1)−1 =x f(f −1 (x))=f(1 x −1)
= 1 (1 x −1)+1 = 1 1 x =x

f(x)= 1 x+1 f −1 (x)= 1 x −1 f(x)= x+5 3 f −1 (x)=3x−5
f −1 (f(x))= f −1 (x+5 3)=3(x+5 3)−5=(x−5)+5=x f(f −1 (x))=f(3x−5)= (3x−5)+5 3 = 3x 3 =x
f(x)=5 x 3 +1. x.

y=5 x 3 +1 x=5 y 3 +1 x−1=5 y 3 x−1 5 = y 3 f −1 (x)= x−1 5 3

f f −1 . y=x. x y, (a,b) f, (b,a) f −1 . (0,1) f, (1,0) f −1 . (1,6) f, (6,1) f −1 . f(x)= x+4 3 .
f −1 (x)= x 3 −4 f(x) with y. x and y. y, f −1 (x). f −1 (x) f:f(x)= (x−4) 2 , x≥4f(x)= (x−4) 2 , x≤4
f(x)= (x−4) 2 x≥4 x≤4 f(x) y.

y= (x−4) 2 Interchange x and y. x= (y−4) 2 Take the square root. ± x =y−4
Add 4 to both sides. 4± x =y

x y f(x), x x y, y x≥4, y≥4, x≥4, f(x)≥4,

f −1 (x)=4+ x

x≤4, f(x)≤4,

f −1 (x)=4− x

y=x. (4, 0) f (0, 4) f −1 . (a, b) f, (b, a) f −1 . f f −1 y=x. f f −1 y=x.

f(x)= (x−2) 2 −3.

(2, –3) x≥2. f(x) y, x.

y= (x−2) 2 −3 Interchange x and y. x= (y−2) 2 −3 Add 3 to both sides. x+3= (y−2) 2 Take the square root. ± x+3 =y−2 Add 2 to both sides. 2± x+3 =y Rename the function. f −1 (x)=2± x+3

x≥2,

f −1 (x)=2+ x+3

x≥2. x≤2, f −1 (x)=2− x+3 . y=x. (2, −3) f (−3, 2) f −1 .

domain of f=range of f −1 =[2,∞)
domain of f −1 =range of f=[−3,∞)

f f −1 y=x. f(x)= x 2 +1, x≥0. f −1 (x)= x−1 f(x) y, x. f(x)= x−4 . f(x)≥0. f(x) y, x.

y = x−4 Replace f(x) with y. x = y−4 Interchange x and y. x = y−4 Square each side. x 2 =y−4 Add 4. x 2 +4 =y Rename the function f −1 (x). f −1 (x) = x 2 +4

f −1 (x)= x 2 +4,x≥0

y=x. f(x)= 2x+3 . f −1 (x)= x 2 −3 2 ,x≥0

V= 2 3 π r 3

V= 2 3 π r 3 V r. π=3.14. V. r≥0 V≥0. r V,

V= 2 3 π r 3 r 3 = 3V 2π Solve for r 3 . r= 3V 2π 3 Solve for r.

V=100 π=3.14.

r= 3V 2π 3 = 3·100 2·3.14 3 ≈ 47.7707 3 ≈3.63

f(x)= (x+2)(x−3) (x−1) . (x+2)(x−3) (x−1) ≥0. x (0, 6). x=−2, f(x) f(x) −2≤x<1 or x≥3, [−2,1)∪[3,∞).
C= 20+0.4n 100+n C n n C. n C.

C= 20+0.4n 100+n C(100+n)=20+0.4n 100C+Cn=20+0.4n 100C−20=0.4n−Cn 100C−20=(0.4−C)n n= 100C−20 0.4−C

C=0.35 (35%).

n= 100(0.35)−20 0.4−0.35 = 15 0.05 =300

f(x)= x+3 x−2 . f −1 (x)= 2x+3 x−1 f −1 f, f f −1 . x y. f(x)= (x−4) 2 , [4,∞) f −1 (x)= x +4
f(x)= (x+2) 2 , [−2,∞)f(x)= (x+1) 2 −3, [−1,∞) f −1 (x)= x+3 −1f(x)=2− 3+x
f(x)=3 x 2 +5, (− ∞,0],[0,∞)f −1 (x)=− x−5 3f(x)=12− x 2 , [0,∞)f(x)=9− x 2 , [0,∞)f(x)= 9−x
f(x)=2 x 2 +4, [0,∞)f(x)= x 3 +5 f −1 (x)= x−5 3f(x)=3 x 3 +1f(x)=4− x 3 f −1 (x)= 4−x 3
f(x)=4−2 x 3f(x)= 2x+1f −1 (x)= x 2 −1 2 , [0,∞)f(x)= 3−4xf(x)=9+ 4x−4
f −1 (x)= (x−9) 2 +4 4 , [9,∞)f(x)= 6x−8 +5f(x)=9+2 x 3f −1 (x)= (x−9 2) 3f(x)=3− x 3
f(x)= 2 x+8f −1 (x)= 2−8x xf(x)= 3 x−4f(x)= x+3 x+7 f −1 (x)= 7x−3 1−xf(x)= x−2 x+7
f(x)= 3x+4 5−4x f −1 (x)= 5x−4 4x+3f(x)= 5x+1 2−5xf(x)= x 2 +2x, [−1,∞)f −1 (x)= x+1 −1
f(x)= x 2 +4x+1, [−2,∞)f(x)= x 2 −6x+3, [3,∞)f −1 (x)= x+6 +3f(x)= x 2 +2, x≥0f(x)=4− x 2 , x≥0
f −1 (x)= 4−xf(x)= (x+3) 2 , x≥−3f(x)= (x−4) 2 , x≥4f −1 (x)= x +4f(x)= x 3 +3f(x)=1− x 3
f −1 (x)= 1−x 3f(x)= x 2 +4x, x≥−2f(x)= x 2 −6x+1, x≥3f −1 (x)= x+8 +3f(x)= 2 xf(x)= 1 x 2 , x≥0
f −1 (x)= 1 xf(x)= (x+1)(x−1) xf(x)= (x+2)(x−3) x−1[−2,1)∪[3,∞)f(x)= x(x+3) x−4f(x)= x 2 −x−20 x−2
[−4,2)∪[5,∞)f(x)= 9− x 2 x+4f(x)= x 3 −x−2, y=1, 2, 3(–2, 0); (4, 2); (22, 3)f(x)= x 3 +x−2, y=0, 1, 2
f(x)= x 3 +3x−4, y=0, 1, 2(–4, 0); (0, 1); (10, 2)f(x)= x 3 +8x−4, y=−1, 0, 1f(x)= x 4 +5x+1, y=−1, 0, 1
(–3, –1); (1, 0); (7, 1) a,b,c f(x)=a x 3 +bf(x)= x 2 +bxf −1 (x)= x+ b 2 4 − b 2f(x)= a x 2 +bf(x)= ax+b 3
f −1 (x)= x 3 −b af(x)= ax+b x+c h(t), t h(t)=200−4.9 t 2 . t h, t(h)= 200−h 4.9 , h(t), t
h(t)=600−16 t 2 . t h, V, r, V(r)= 4 3 π r 3 . r V, r(V)= 3V 4π 3 , A, r, A(r)=4π r 2 . r V, n
C(n)= 25+.6n 100+n C, n. n C n(C)= 100C−25 .6−C , T, l, T(l)=2π l 32.2 l T V, r, h, V=π r 2 h. V
r(V)= V 6π , A, r, h, A=2π r 2 +2πrh. V V, r, h, V= 1 3 π r 2 h. r h r(V)= V 4π , r, V, e=0.16s e, s
e=0.16se=0.16(4,600)=736e=0.16(9,200)=1,472e=0.16(18,400)=2,944 y=k x n k k=0.16 n=1.
x and y

y=k x n

y nth x. k= y x n , k x, y. y x y x. y=25 x=2, y x y=k x 3 . y x.

k= y x 3 = 25 2 3 = 25 8

y= 25 8 x 3

x=6 y.

y= 25 8 (6) 3 =675

(0,0). y x. y=24 x=3, y x 128 3 T= 14,000 d d, T= 14,000 d14,000 500 =2814,000 350 =40 14,000 250 =56 y= k x k=14,000. x y

y= k x n

k y nth x. k= x n y. t v vt=distance. vt=100.

t(v)= 100 v =100 v −1

x, y. y x y x. y=25 x=2, y x y= k x 3 . y x.

k= x 3 y = 2 3 ·25 =200

y= k x 3 , k=200 y= 200 x 3

x=6 y.

y= 200 6 3 = 25 27

y x. y=8 x=3, y x 9 2 c, n, d. x y z, x=kyz. x y z, x= ky z . x y z. x=6 y=2 z=8, x y=1 z=27.

x= k y 2 z 3

x=6, y=2, z=8 k.

6= k 2 2 8 3 6= 4k 2 3=k

x= 3 y 2 z 3

x y=1 z=27, y z

x= 3 (1) 2 27 3 =1

x y z. x=40 y=4 z=2, x y=10 z=25. x=20

y=k x n ,k is a nonzero constant.

y= k x n ,k is a nonzero constant.

y x x=6, y=12. y x x=4, y=80. y=5 x 2 y x x=36, y=24. y x x=36, y=24. y=10 x 3 y x x=27, y=15. y x x=1, y=6. y=6 x 4 y x x=4, y=2. y x x=3, y=2. y= 18 x 2 y x x=2, y=5. y x x=3, y=1. y= 81 x 4 y x x=25, y=3. y x x=64, y=5. y= 20 x 3 y x z x=2 and z=3, y=36. y x, z, and w x=1, z=2, w=5, then y=100. y=10xzw y x z x=3 and z=4, then y=72. y x z x=2 and z=25, then y=100. y=10x z y x z W. x=1, z=2, and w=36, then y=48. y x and z w. x=3, z=5, and w=6, then y=10. y=4 xz w y x z w. x=3, z=4, and w=3, then y=6. y x z w t . x=3, z=1, w=25, and t=2, then y=6. y=40 xz w t 2 y x. x=3, then y=12. Find y when x=20. y x. x=2, then y=16. Find y when x=8. y=256 y x. x=3, then y=5. Find y when x=4. y x. x=16, then y=4. Find y when x=36. y=6 y x. x=125, then y=15. Find y when x=1,000. y x. x=3, then y=2. Find y when x=1. y=6 y x. x=4, then y=3. Find y when x=2. y x. x=3, then y=1. Find y when x=1. y=27 y x. x=64, y=12. y x=36. y x. x=27, y=5. y x=125. y=3 y x and z. x=4 z=2, y=16. y x=3 z=3. y x, z, and w. x=2, z=1, w=12, y=72. y x=1, z=2, w=3. y=18 y x z. x=2 z=4, y=144. y x=4 z=5. y x z. x=2 z=9, y=24. y x=3 z=25. y=90 y x z w. x=5, z=2, w=20, y=4. y x=3 z=8, w=48. y x z w. x=2, z=2, w=64, y=12. y x=1, z=3, w=4. y= 81 2 y x z w t . x=2, z=3, w=16, t=3, y=1. y x=3, z=2, w=36, t=5. y x x=2, y=3. y= 3 4 x 2 y x x=2, y=4. y x x=36, y=2. y= 1 3 x y x x=6, y=2. y x x=1, y=4. y= 4 x 2 T, a, T a. T a. s t, v t, K m v. (4+3i)+(−2−5i)2−2i(6−5i)−(10+3i)(2−3i)(3+6i)24+3i2−i 2+ix 2 −4x+5=0 {2+i, 2−i}x 2 +2x+10=0f(x)= x 2 −4x−5f(x)= (x−2) 2 −9 vertex (2,–9), intercepts (5,0); (–1,0); (0,–5) f(x)=−2 x 2 −4x(−2,3) (3,6). f(x)= 3 25 (x+2) 2 +3 (−3,6.5) (2,6). h, x, h(x)= −32 (120) 2 x 2 +x. f(x)=4 x 5 −3 x 3 +2x−1f(x)= 5 x+1 − x 2f(x)= x 2 (3−6x+ x 2)f(x)=2 x 4 +3 x 3 −5 x 2 +7 f(x)=4 x 3 −6 x 2 +2As x→−∞, f(x)→−∞, as x→∞, f(x)→∞f(x)=2 x 2 (1+3x− x 2) f(x)= (x+3) 2 (2x−1) (x+1) 3− 1 2f(x)= x 5 +4 x 4 +4 x 3f(x)= x 3 −4 x 2 +x−41 2 f(x)= x 3 −5x+1 x 3 −2 x 2 +4x+4 x−2 x 2 +4 3 x 4 −4 x 2 +4x+8 x+1x 3 −2 x 2 +5x−1 x+3x 2 −5x+20− 61 x+3 x 3 +4x+10 x−32 x 3 +6 x 2 −11x−12 x+42 x 2 −2x−3(x+4)(2 x 2 −2x−3)3 x 4 +3 x 3 +2x+2 x+1

2 x 3 −3 x 2 −18x−8=0{ −2, 4, − 1 2 }3 x 3 +11 x 2 +8x−4=02 x 4 −17 x 3 +46 x 2 −43x+12=0
{ 1, 3, 4, 1 2 }4 x 4 +8 x 3 +19 x 2 +32x+12=0x 3 −3 x 2 −2x+4=02 x 4 − x 3 +4 x 2 −5x+1=0
f(x)= x+2 x−5(−2,0)and(0,− 2 5) x=5 y=1.f(x)= x 2 +1 x 2 −4f(x)= 3 x 2 −27 x 2 +x−2 (0, 27 2)
x=1, x=−2, y=3.f(x)= x+2 x 2 −9f(x)= x 2 −1 x+2y= x−2f(x)= 2 x 3 − x 2 +4 x 2 +1f(x)= (x−2) 2 , x≥2
f −1 (x)= x +2f(x)= (x+4) 2 −3, x≥−4f(x)= x 2 +6x−2, x≥−3f −1 (x)= x+11 −3f(x)=2 x 3 −3f(x)= 4x+5 −3
f −1 (x)= (x+3) 2 −5 4 , x≥−3f(x)= x−3 2x+1 y x. x=3, y=36, y x=4.y=64 y x x=25, y=2, y x=4. y x z.
x=1 z=2, y=6, y x=2 z=3.y = 72 y x z w. x=3, z=4, w=2, y=48, y x=4, z=5, w=3. V T (3−4i)(4+2i)
20−10i1−4i 3+4ix 2 −4x+13=0{2+3i, 2−3i}f(x)= x 3 (3−6 x 2 −2 x 2)f(x)=8 x 3 −3 x 2 +2x−4
As x→−∞, f(x)→−∞, as x→∞, f(x)→∞f(x)=−2 x 2 (4−3x−5 x 2)f(x)= x 2 +2x−8f(x)= (x+1) 2 −9
(−1,−9) (2,0);(−4,0); (0,−8) (2,0) (4,12).f(x)= (x−3) 3 (3x−1) (x−1) 2f(x)=2 x 6 −6 x 5 +18 x 4
2 x 3 +3x−4 x+22 x 2 −4x+11− 26 x+2x 4 +3 x 2 −4 x−22 x 3 +5 x 2 −7x−12 x+32 x 2 −x−4
(x+3)(2 x 2 −x−4)f(x)=2 x 3 +5 x 2 −6x−9f(x)=4 x 4 +8 x 3 +21 x 2 +17x+4− 1 2 −1±i 15 2
f(x)=4 x 4 +16 x 3 +13 x 2 −15x−18f(x)= x 5 +6 x 4 +13 x 3 +14 x 2 +12x+8 −2 ±i x=3 x=1 x=−2
(0,12). x= 1 2 x=−3 (1,8).f(x)=2 (2x−1) 3 (x+3)8 x 3 −21 x 2 +6=0f(x)= x+4 x 2 −2x−3
(−4,0), (0,− 4 3) x=3, x =−1, y=0f(x)= x 2 +2x−3 x 2 −4f(x)= x 2 +3x−3 x−1y=x+4f(x)= x−2 +4
f(x)=3 x 3 −4f −1 (x)= x+4 3 3f(x)= 2x+3 3x−1 y x x=3, y=2. y x=1.y=18 y x z. x=2 z=27, y=12, y
x=5 z=8. k 1.25 1.2% 2031. f(x)=3x+4,xf(x)= 2 xg(x)=2x f(x)=a b x , a b b>1, 0<b<1, f(x)= 2 x −3 3.
x−3−2−10123f(x)= 2 x2 −3 = 1 82 −2 = 1 42 −1 = 1 22 0 =12 1 =22 2 =42 3 =8 f f(x)= 2 x (−∞,∞),
(0,∞), x→∞,f(x)→∞, x→−∞,f(x)→0, f(x) f(x) y=0 x,

f(x)=a b x

a b b≠1. f f a>0. f a<0. (0,a), y=0.f(x)= 4 3(x−2)g(x)= x 3h(x)= (1 3) xj(x)= (−2) x g(x)= x 3
g(x)= x 3 b≠1. j(x)= (−2) x −2, 0.f(x)=2 x 2 −3x+1g(x)= 0.875 xh(x)=1.75x+2j(x)= 1095.6 −2x
g(x)= 0.875 x j(x)= 1095.6 −2x 1.b b=−9 x= 1 2 . f(x)=f(1 2)= (−9) 1 2 = −9 ,1?1 1: b=1.
f(x)= 1 x =1 x. f(x)= b x ,x f(x)= 2 x . f(3)?

f(x) = 2 x f(3) = 2 3 Substitute x=3. =8 Evaluate the power.

f(x)=30 (2) x . f(3)?

f(x) =30 (2) x f(3) =30 (2) 3 Substitute x=3. =30(8) Simplify the power first. =240 Multiply.

f(3)=30 (2) 3 ≠ 60 3 =216,000

f(x)=5 (3) x+1 . f(2)

f(x) =5 (3) x+1 f(2) =5 (3) 2+1 Substitute x=2. =5 (3) 3 Add the exponents. =5(27) Simplify the power. =135 Multiply.

f(x)=8 (1.2) x−5 . f(3) 5.5556 x a b b≠1,

f(x)=a b x

a b x A(x)=100+50x. B(x)=100 (1+0.5) x .x0100+50(0)=100100 (1+0.5) 0 =1001100+50(1)=150
100 (1+0.5) 1 =1502100+50(2)=200100 (1+0.5) 2 =2253100+50(3)=250100 (1+0.5) 3 =337.5x
A(x)=100+50xB(x)=100 (1+0.5) x [0,∞), [100,∞). B(x)=100 (1+0.5) x . 1+0.5=1.5
B(x)=100 (1.5) x , 1.5 x 1.25 1.2%. P(t)=1.25 (1.012) t , t 2013. 2031? t=18, 18

P(18)=1.25 (1.012) 18 ≈1.549

0.6%. P(t)=1.39 (1.006) t , t 2013. 1.548 a b, (0,a), a a, f(x)=a (b) x , b. (0,a), f(x)=a (b) x . a b. a
b f(x)=a (b) x . N(t) (N) t. t a=80. N(t)=80 b t b:

N(t) =80 b t 180 =80 b 6 Substitute using point (6, 180). 9 4 = b 6 Divide and write in lowest terms. b = (9 4) 1 6 Isolate b using properties of exponents. b ≈1.1447 Round to 4 decimal places.

N(t)=80 (1.1447) t . (0, 80) (6, 180). [0,∞), [80,∞). N(t)=80 (1.1447) t ,t 129 2013, N t.(0,129)
(2,236); N(t)=129 (1.3526) t (−2,6) (2,1). f(x)=a b x , a b. (−2,6) 6=a b −2 (2,1) 1=a b 2 a b: a
b: b a: f(x)=2.4492 (0.6389) x . (−2, 6) (2, 1). f(x)=2.4492 (0.6389) x (1,3) (2,4.5), f(x)=2 (1.5) x
x, (0,a) a a, f(x)=a (b) x , b. (0,a), f(x)=a (b) x . a b. f(x)=a (b) x . (0,3), a=3. (0,3) (2,12).

y=a b x Write the general form of an exponential equation. y=3 b x Substitute the initial value 3 for a. 12=3 b 2 Substitute in 12 for y and 2 for x. 4= b 2 Divide by 3. b=±2 Take the square root.

b, b=2. a b f(x)=3 (2) x .f(x)= 2 (2) x . 1.4142 (1.4142) x . y=a· b x (2,24.8) (5,198.4). a=6.2 b=2 y=6.2· 2 x .y≈12· 1.85 x t, P, r, n:

A(t)=P (1+ r n) nt

A(t)=P (1+ r n) nt

A(t) t P r n P=3000. r = 0.03. n=4. A(10), t = 10.

A(t) =P (1+ r n) nt Use the compound interest formula. A(10) =3000 (1+ 0.03 4) 4·10

Substitute using given values. ≈$4045.05 Round to two decimal places.

r=0.06. k=2. P, 18 P.

A(t) =P (1+ r n) nt Use the compound interest formula. 40,000 =P (1+ 0.06 2) 2(18)

Substitute using given values A, r, n, and t. 40,000 =P (1.03) 36 Simplify. 40,000 (1.03) 36 =P Isolate P.

P ≈$13,801 Divide and round to the nearest dollar.

A(t)= (1+ 1 n) n(1+ 1 1) 1(1+ 1 2) 2(1+ 1 4) 4(1+ 1 12) 12(1+ 1 365) 365(1+ 1 8766) 8766 (1+ 1 525960) 525960(1+131557600) 31557600 n n (1+ 1 n) n e.

(1+ 1 n) n ,as n increases without bound

e≈2.718282. e 3.14 . [e x]. [e^( ]. 3.14 [ )]. 5 e 3.14 ≈23.10387. e. e −0.5 . e −0.5 ≈0.60653 e t, a r,

A(t)=a e rt

a r t r>0 r<0

A(t)=P e rt

P r t t, a r. r>0. r<0. t. A(t). r=0.10. P=1000. t=1

A(t) =P e rt Use the continuous compounding formula. =1000 (e) 0.1

Substitute known values for P, r, and t. ≈1105.17 Use a calculator to approximate.

17.3% r = −0.173. 100 a=100. t=3

A(t) =a e rt Use the continuous growth formula. =100 e −0.173(3) Substitute known values for a, r, and t.

≈59.5115 Use a calculator to approximate.

3.77× 10 −26 f(x)= b x , where b>0, b≠1f(x)=a b x , where a>0, b>0, b≠1

A(t)=P (1+ r n) nt , where A(t) is the account value at time t t is the number of years

P is the initial investment, often called the principal r is the annual percentage rate (APR), or nominal rate

n is the number of compounding periods in one year

A(t)=a e rt , wheretae e≈2.718282 1 t e e≈2.718282. [e x] [exp(x)] e. e 1 8 24 3.25% 20 $5 t h(t)=−4.9 t 2 +18t+40. t, A(t)=115 (1.025) t . B(t)=82 (1.029) t . B(t)=82 (1.029) t . 20 t=20 43 100 y=300 (1−t) 5y=220 (1.06) x 1.06, 1.y=16.5 (1.025) 1 xy=11,701 (0.97) t 0.97, 0 1.(0,6) (3,750) (0,2000) (2,20)f(x)=2000 (0.1) x(−1, 3 2) (3,24)(−2,6) (3,1)

f(x)= (1 6) − 3 5 (1 6) x 5 ≈2.93 (0.699) x(3,1) (5,4)xf(x)xh(x)xm(x)xf(x)xg(x) A(t)=P (1+ r n) nt . 10,250 (1+ 0.04 12) 120 . $10,250 3.6% 20 $13,268.58 PP=A(t)· (1+ r n) −nt $14,472.74 5.5% 5 5 $4,572.56 r.4%y=3742 (e) 0.75t 0.y=150 (e) 3.25 ty=2.25 (e) −2t 0. $12,000 7.2% 30 30 $669.42 f(x)=2 (5) x , f(−3)f(x)=− 4 2x+3 , f(−1)f(−1)=−4f(x)= e x , f(3)f(x)=−2 e x−1 , f(−1)f(−1)≈−0.2707 f(x)=2.7 (4) −x+1 +1.5,f(−2)f(x)=1.2 e 2x −0.3, f(3)f(3)≈483.8146f(x)=− 3 2 (3) −x + 3 2 , f(2) (0,3) (3,375)y=3· 5 x(3,222.62) (10,77.456)(20,29.495) (150,730.89)y≈18· 1.025 x(5,2.909) (13,0.005) (11,310.035) (25,356.3652)y≈0.2· 1.95 x APY= (1+ r 12) 12 −1. I(n) n

APY= A(t)−a a = a (1+ r 365) 365(1) −a a = a[(1+ r 365) 365 −1] a = (1+ r 365) 365 −1;

I(n)= (1+ r n) n −1 f(x)=a· b x a b b≠1. b b= e n n e b>1, f(x)=a· (1 b) x . b= e n , f(x)=a (e) −nx n f f(x)=a· (1 b) x b>1. n>0,f(x)=a· (1 b) x =a (b −1) x =a ((e n) −1) x =a (e −n) x =a (e) −nx . A r t A(t)=a (e) rt , a t I(t)= e rt −1.47,622 1.39%; $155,368.09$35,838.76$82,247.78; $449.75 f(x)= b x f(x)= 2 x . 1.x−3−2−10123f(x)= 2 x1 81 41 21248 2. 2 f(x)=a b x , b a.x; x x f(x)= 2 x . f(x)= 2 x (0,∞), y=0. f(x)= b x g(x)= (1 2) x . 1.x-3-2-10123g (x) =(1 2) x84211 21 41 8 1 2 . x; x x g(x)= (1 2) x . g(x)= (1 2) x (0,∞), y=0. f(x)= b x , b>0, b≠1, y=0 (−∞, ∞) (0,∞) (0,1) b>1 b<1 f(x)= b x , 3 (0,1). (−∞,∞), (0,∞), y=0. f(x)= 0.25 x . b=0.25 y=0.x−3−2−10123f(x)= 0.25 x641641 0.250.06250.015625 (0,1), (−1,4) (1,0.25). (−∞,∞); (0,∞); y=0. f(x)= 4 x . (−∞,∞); (0,∞); y=0.

f(x)= b x d f(x)= b x , d f(x)= 2 x , d=3: g(x)= 2 x +3 h(x)= 2 x −3. f(x)= 2 x (−∞,∞) 3 g(x)= 2 x +3: 3 (0,4). 3 y=3. (3,∞). 3 h(x)= 2 x −3: 3 (0,−2). 3 y=−3. (−3,∞). c f(x)= b x , c f(x)= 2 x , c=3: g(x)= 2 x+3 , h(x)= 2 x−3 . f(x)= 2 x (−∞,∞), y=0, 3 g(x)= 2 x+3 , (0,8). 2 x+3 =(8) 2 x , 8. 3 h(x)= 2 x−3 , (0, 1 8). 2 x−3 =(1 8) 2 x , 1 8 . c d, f(x)= b x+c +d f(x)= b x d d. c c. (0, b c +d). y=d. (d,∞). (−∞,∞), f(x)= b x+c +d, y=d. (−c,d). f(x)= b x c c c c f(x)= b x d d d d (−∞,∞), (d,∞), y=d. f(x)= 2 x+1 −3. f(x)= b x+c +d, b=2, c=1, d=−3. y=d y=−3. (−c,d), (−1,−3). f(x)= b x (−∞,∞); (−3,∞); y=−3. f(x)= 2 x−1 +3. (−∞,∞); (3,∞); y=3. f(x)= b x+c +d x, f(x) f(x). x, 42=1.2 (5) x +2.8 1.2 (5) x +2.8 x y. x=2. x≈2.166. 4=7.85 (1.15) x −2.27 x≈−1.608 f(x)= b x |a|>0. f(x)= 2 x , a=3, g(x)=3 (2) x a= 1 3 , h(x)= 1 3 (2) x g(x)=3 (2) x f(x)= 2 x 3. h(x)= 1 3 (2) x f(x)= 2 x 1 3 . a>0, f(x)=a (b) x a |a|>1. a |a|<1. (0,a). y=0, (0,∞), (−∞,∞), f(x)=4 (1 2) x . b= 1 2 x x a=4, f(x)= (1 2) x 4.x−3−2−10123

f(x)=4(1 2) x

321684210.5 (0,4), (−1,8) (1,2). (−∞,∞); (0,∞); y=0. f(x)= 1 2 (4) x . (−∞,∞); (0,∞); y=0. f(x)= b x −1, −1, f(x)= 2 x , g(x)= −2 x , h(x)= 2 −x , g(x)=− 2 x f(x)= 2 x g(x)= 2 −x f(x)= 2 x f(x)=− b x f(x)= b x (0,−1). (−∞,0) y=0 (−∞,∞), f(x)= b −x f(x)= b x (0,1), y=0, (0,∞), (−∞,∞), g(x), f(x)= (1 4) x f(x)= (1 4) x f(x) −1 g(x)=− (1 4) x . x−3−2−10123

g(x)=−(1 4) x

−64−16−4−1−0.25−0.0625−0.0156 (0,−1), (−1,−4) (1,−0.25). (−∞,∞); (−∞,0); y=0. g(x), f(x)= 1.25 x (−∞,∞); (0,∞); y=0. f(x)= b x c d

f(x)= b x+c +d

| a |>1 0<| a |<1

f(x)=a b x

f(x)=− b x

f(x)= b −x = (1 b) x

f(x)=a b x+c +d

f(x)=a b x+c +d

y= b x , b>1, c | a | | a |>0. | a | 0<| a |<1. d a<0. f(x)= e x 2 4 f(x)=a b x+c +d. a,b,c, d. f(x)= e x , b=e. 2 a=2. x −x e −x . d=4.

f(x) =a b x+c +d =2 e −x+0 +4 =2 e −x +4

(−∞,∞); (4,∞); y=4. f(x)= e x 1 3 , 2f(x)=− 1 3 e x −2; (−∞,∞); (−∞,2); y=2. f(x)= b x f(x)=a b x+c +d f(x)= b x (0, 1), (−∞, ∞), (0, ∞), y=0. b>1, y=0, 0<b<1, y=0. f(x)= b x +d f(x)= b x . f(x)= b x+c f(x)= b x . f(x)= b x+c +d f(x)=a b x , a>0, | a |>1 0<| a |<1 f(x)= b x . f(x)= b x −1, f(x)=− b x , −1, f(x)= b −x , f(x)=a b x+c +d. f(x)=a b x+c +d, x f(x)= 3 x 4. g(x)? g(x)=4 (3) −x ; (0,4); 0. f(x)= (1 2) −x 1 5 . g(x)? f(x)= 10 x 7 g(x)? g(x)=− 10 x +7; (0,6); 7. f(x)= (1.68) x 3 2, 3 g(x)? f(x)=− 1 2 (1 4) x−2 +4 2 4, 4 g(x)? g(x)=2 (1 4) x ; (0, 2); 0. f(x)=3 (1 2) x g(x)=−2 (0.25) x (0,−2)h(x)=6 (1.75) −xf(x)=3 (1 4) x ,g(x)=3 (2) x , h(x)=3 (4) xf(x)= 1 4 (3) x , g(x)=2 (3) x , h(x)=4 (3) xf(x)=2 (0.69) xf(x)=2 (1.28) xf(x)=2 (0.81) xf(x)=4 (1.28) x f(x)=2 (1.59) xf(x)=4 (0.69) x f(x)=a b x . b? b? a? a?f(x)= 1 2 (4) xf(x)=3 (0.75) x −1 f(x)=−4 (2) x +2 f(x)= 2 x . f(x)= 2 −xh(x)= 2 x +3 h(x)=3; 3.f(x)= 2 x−2f(x)=−5 (4) x −1x→∞ f(x)→−∞x→−∞f(x)→−1f(x)=3 (1 2) x −2f(x)=3 (4) −x +2x→∞f(x)→2x→−∞f(x)→∞ f(x)= 4 x . f(x) f(x) f(x)= 4 x −3 f(x) f(x) f(x)= 4 x−5 f(x) f(x) f(x)= 4 −x y= 2 x . y=− 2 x +3 y=−2 (3) x +7 x.g(x)= 1 3 (7) x−2 g(6).g(6)=800+ 1 3 ≈800.3333f(x)=4 (2) x−1 −2 f(5). h(x)=− 1 2 (1 2) x +6 h(−7).h(−7)=−58 f(x)=a b x +d.−50=− (1 2) −x116= 1 4 (1 8) xx≈−2.953 12=2 (3) x +15=3 (1 2) x−1 −2x≈−0.222−30=−4 (2) x+2 +2 F(x)= (b) x G(x)= (1 b) x . b x (1 b) x b>0. G(x)= (1 b) x F(x)= b x ; b>0 f(x)= b x , (1 b) x F(−x). f(x)= 4 x , g(x)= 4 x−2 , h(x)=(1 16) 4 x . b x (1 b n) b x b>0. g(x) h(x) f(x); b>0, f(x)= b x , (1 b n) b x f(x−n). 10 8−4 = 10 4 =10,000 10 x =500, x x? 10 x =500. 10 2 =100 10 3 =1000, x y= 10 x y= b x x= b y x y y y x, y= log b (x). b b b x y, b x y.b y x, 2 5 =32, log 2 32=5.

log b (x)=y⇔ b y =x, b>0,b≠1

b log b (x), f(x). log b x. x. y= log b (x) y= b x b x x>0,b>0,b≠1,

y= log b (x) is equivalent to b y =x

log b (x) b x b x." y b x. x y b is (0,∞). b is (−∞,∞). log b (x)=y, y= log b x b,y,andx. log b x=y b y =x.log 6 (6)= 1 2log 3 (9)=2 b,y,and x. b y =x.log 6 (6)= 1 2 b=6,y= 1 2 ,and x= 6.

log 6 (6)= 1 2

6 1 2 = 6 .

log 3 (9)=2 b=3,y=2,and x=9. log 3 (9)=2 3 2 =9. log 10 (1,000,000)=6log 5 (25)=2 log 10 (1,000,000)=6 10 6 =1,000,000log 5 (25)=2 5 2 =25 b, x, y. x= log b (y).2 3 =85 2 =25 10 −4 = 1 10,000 b,y,andx. x= log b (y).2 3 =8 b=2, x=3, y=8. 2 3 =8 log 2 (8)=3.5 2 =25 b=5, x=2, y=25. 5 2 =25 log 5 (25)=2.10 −4 = 1 10,000 b=10, x=−4, y= 1 10,000 . 10 −4 = 1 10,000 log 10 (1 10,000)=−4.3 2 =95 3 =1252 −1 = 1 23 2 =9 log 3 (9)=25 3 =125 log 5 (125)=32 −1 = 1 2 log 2 (1 2)=−1 log 2 8. 2 2 3 =8, log 2 8=3. log 7 49 log 3 27 7 2 =49. log 7 49=2 3 3 =27. log 3 27=3 log 2 3 4 9 2 3 4 9 ? 2 2 =4 3 2 =9, (2 3) 2 = 4 9 . log 2 3 (4 9)=2. y= log b (x), x b: b y =x. b y b x? y= log 4 (64) 4 y =64.

4 3 =64

log (64) 4 =3

y= log 121 (11) log 121 (11)= 1 2 121 = (121) 1 2 =11 y= log 3 (1 27) 3 y = 1 27 . 1 27 ? 3 3 =27, 1 27 ? b −a = 1 b a .

3 −3 = 1 3 3 = 1 27

log 3 (1 27)=−3. y= log 2 (1 32) log 2 (1 32)=−5 log(x) log 10 (x). 10. log 10 (x) log(x). x x>0,

y=log(x) is equivalent to 10 y =x

log(x) 10 x x. y 10 x. y=log(x), x 10: 10 y =x. 10 y 10 x? y=log(1000) 10 y =1000. 10

10 3 =1000

log(1000)=3. y=log(1,000,000).log(1,000,000)=6 y=log(x), x, y=log(321) log(321)≈2.5065. 10 2 =100 10 3 =1000. log(321) log(100) log(1000).

100 < 321 < 1000 2 < 2.5065 < 3

y=log(123) log(123)≈2.0899 10 x =500 x

10 x =500 log(500) =x Use the definition of the common log.

500, log(500)≈2.699. 2.699. 8,500 10 x =8500 x 3.929. e. e e log e (x), ln(x). ln(x) ln1=0. ln e e. log e (x)ln(x).x x>0,

y=ln(x) is equivalent to e y =x

ln(x) e x x. y e x. y=e x y=ln(x) ln(e x)=x x e = ln(x) x x>0. y=ln(x), x, y=ln(500) 500, ln(500)≈6.2146 ln(−500). x>0,b>0,b≠1,y= log b (x) b y =x. x>0,y=log(x) 10 y =x. x>0,y=ln(x) e y =x. b b. 10. 10 b b y =x log b x=y b>0,b≠1. b b y, b y log b x, x b y. f(x)= log b x g(x)= b x ? log b x=y x b y =x, x. b, b, b e. log e (x), ln(x).log 4 (q)=mlog a (b)=ca c =blog 16 (y)=x log x (64)=yx y =64log y (x)=−11log 15 (a)=b15 b =alog y (137)=xlog 13 (142)=a13 a =142 log(v)=tln(w)=ne n =w4 x =yc d =klog c (k)=dm −7 =n19 x =ylog 19 y=xx − 10 13 =yn 4 =103 log n (103)=4(7 5) m =ny x = 39 100log y (39 100)=x10 a =be k =hln(h)=k x log 3 (x)=2log 2 (x)=−3 x= 2 −3 = 1 8log 5 (x)=2log 3 (x)=3x= 3 3 =27log 2 (x)=6log 9 (x)= 1 2x= 9 1 2 =3log 18 (x)=2 log 6 (x)=−3x= 6 −3 = 1 216log(x)=3ln(x)=2x= e 2log(100 8)10 log(32)322log(.0001)e ln(1.06)1.06 ln(e −5.03)e ln(10.125) +414.125 b log 3 (1 27)log 6 (6)1 2log 2 (1 8)+46 log 8 (4)4log(10,000) log(0.001)−3log(1)+72log(100 −3)−12ln(e 1 3)ln(1)0ln(e −0.225)−325ln(e 2 5)10log(0.04)ln(15) 2.708ln(4 5)log(2)0.151ln(2) x=0 f(x)=log(x)? x=0? x=0. x=0 f(x)=log(x). n n=log(0). 10 n =0, n x=0 f(x)=log(x). f(x)=0 f(x)=log(x)? x? x lnx=2? x lnx=2. x= e 2 , x= e 2 . ln(x)=ln(e 2)=2. log 3 (27) log 4 (1 64) =−1? ln(e 1.725) ln(1) =1.725? ln(1)=0, ln(e 1.725) ln(1) EI EI= log 2 (f 2 t), f t 8 2 EI −2,2 log I 1 I 2 = M 1 − M 2 M x; log 10 (x) log(x). b x; y= log b (x) e x; log e (x) ln(x). $2500 5%, t A=2500 e 0.05t . y= b x x b>0,b≠1, y (−∞,∞). y (0,∞).

y= log b (x) y= b x . y= log b (x) y= b x : (0,∞). y= log b (x) y= b x : (−∞,∞). y= log b (x) y= b x . y= log b (x) f(x)= log 4 (2x−3). x 2x−3, x:

2x−3>0 Show the argument greater than zero. 2x>3 Add 3. x>1.5 Divide by 2.

f(x)= log 4 (2x−3) (1.5,∞). x. f(x)= log 2 (x+3)? x+3>0.

x+3>0 The input must be positive. x>−3 Subtract 3.

f(x)= log 2 (x+3) (−3,∞). f(x)= log 5 (x−2)+1?(2,∞) f(x)=log(5−2x)? 5−2x>0.

5−2x>0 The input must be positive. −2x>−5 Subtract 5. x< 5 2

Divide by −2 and switch the inequality.

f(x)=log(5−2x) (−∞, 5 2). f(x)=log(x−5)+2?(5,∞) y= log b (x) y= log b (x). y= b x , y=x. y= 2 x x= log 2 (y) x−3−2−101232 x =y1 81 41 21248log 2 (y)=x−3−2−10123 f(x)= 2 x g(x)= log 2 (x). f(x)= 2 x(−3, 1 8)(−2, 1 4)(−1, 1 2)(0,1)(1,2)(2,4)(3,8)g(x)= log 2 (x)(1 8 ,−3)(1 4 ,−2) (1 2 ,−1)(1,0)(2,1)(4,2)(8,3) f g. f(x)= 2 x g(x)= log 2 (x) y=x.f(x)= 2 x (0,1) g(x)= log 2 (x) (1,0). f(x)= 2 x ,(−∞,∞), g(x)= log 2 (x). f(x)= 2 x ,(0,∞), g(x)= log 2 (x). x b>0, b≠1, f(x)= log b (x): x=0 (0,∞) (−∞,∞) (1,0) (b,1) b>1 0<b<1 b f(x)= log b (x) ln(x) e≈2.718.) f(x)= log b (x), x=0. (1,0). (b,1). (0,∞), (−∞,∞), x=0. f(x)= log 5 (x). b=5 x=0, (1,0). (5,1) (0,∞), (−∞,∞), x=0. f(x)= log 1 5 (x). (0,∞), (−∞,∞), x=0. y= log b (x) c f(x)=lo g b (x), c c. f(x)= log b (x) c>0 g(x)= log b (x+c), h(x)= log b (x−c). c, f(x)= log b (x+c) y= log b (x) c c>0. y= log b (x) c c<0. x=−c. (−c,∞). (−∞,∞). f(x)= log b (x+c), c>0, f(x)= log b (x) c c<0, f(x)= log b (x) c x=−c. c x (−c,∞), (−∞,∞), x=−c. f(x)= log 3 (x−2) f(x)= log 3 (x−2), x+(−2)=x−2. c=−2, c<0. f(x)= log 3 (x) x=−(−2) x=2. (1 3 ,−1),(1,0), (3,1). x (7 3 ,−1),(3,0), (5,1). (2,∞), (−∞,∞), x=2. f(x)= log 3 (x+4) (−4,∞), (−∞,∞), x=–4. d f(x)= log b (x), d d. f(x)= log b (x) g(x)= log b (x)+d h(x)= log b (x)−d. d, f(x)= log b (x)+d y= log b (x) d d>0. y= log b (x) d d<0. x=0. (0,∞). (−∞,∞). f(x)= log b (x)+d, d>0, f(x)= log b (x) d d<0, f(x)= log b (x) d x=0. d y (0,∞), (−∞,∞), x=0. f(x)= log 3 (x)−2 f(x)= log 3 (x)−2, d=–2. d<0. f(x)= log 3 (x) x=0. (1 3 ,−1), (1,0), (3,1). (1 3 ,−3),(1,−2), (3,−1). (0,∞), (−∞,∞), x=0. (0,∞), (−∞,∞), x=0. f(x)= log 2 (x)+2 (0,∞), (−∞,∞), x=0. f(x)= log b (x) a>0, a>1 f(x)= log b (x) g(x)=a log b (x) h(x)= 1 a log b (x). a>1, f(x)=a log b (x) y= log b (x) a a>1. y= log b (x) a 0<a<1. x=0. (1,0). (0,∞). (−∞,∞). f(x)=a log b (x),a>0, |a|>1, f(x)= log b (x) a |a|<1, f(x)= log b (x) a x=0. y a. (0,∞), (−∞,∞), x=0. f(x)=2 log 4 (x) f(x)=2 log 4 (x), a=2. f(x)= log 4 (x) x=0. (1 4 ,−1),(1,0), (4,1). y (1 4 ,−2),(1,0) , (4,2). (0, ∞), (−∞,∞), x=0. (0,∞), (−∞,∞), x=0. f(x)= 1 2 log 4 (x) (0,∞), (−∞,∞), x=0. f(x)=5log(x+2). x=−2. (−1,0). (−2,∞). (−1,0) (8,5). x=8 x=8, x+2=10, (−2,∞), (−∞,∞), x=−2. f(x)=3log(x−2)+1. (2,∞), (−∞,∞), x=2. f(x)= log b (x) −1, −1, b>1, f(x)= log b (x) g(x)= −log b (x) h(x)= log b (−x). f(x)= −log b (x) y= log b (x) (0,∞), (−∞,∞), x=0, f(x)= log b (−x) y= log b (x) (−∞,0). (−∞,∞), x=0, f(x)= log b (x),If f(x)=− log b (x) If f(x)= log b (−x) x=0. x=0. (1,0). (1,0). f(x)= log b (x) f(x)= log b (x) (0,∞), (−∞,∞), x=0. (−∞,0), (−∞,∞), x=0. f(x)=log(−x) f(x)=log(−x), b=10 −1,f f(x)=log(−x) x x=0. (−1,0). (−∞,0), (−∞,∞), x=0. f(x)=−log(−x). (−∞,0), (−∞,∞), x=0. x, x, 4ln(x)+1=−2ln(x−1) 4ln(x)+1 −2ln(x−1) x y. x=1. x≈1.339. 5log(x+2)=4−log(x) x≈3.049 y= log b (x) c d y= log b (x+c)+d | a |>1 | a |<1y=a log b (x)y=− log b (x)y= log b (−x)y=a log b (x+c)+d y= log b (x),

f(x)=a log b (x+c)+d

y= log b (x),b>1, d c | a | | a |>0. | a | 0<| a |<1. a<0. f(x)=log(−x), f(x)=−2 log 3 (x+4)+5? x=−4. x=−4. f(x)=3+ln(x−1)?x=1 x=−2

f(x)=−alog(x+2)+k

(−1,1) (2,–1). (−1,1),

1=−alog(−1+2)+k Substitute (−1,1). 1=−alog(1)+k Arithmetic. 1=k log(1)=0.

(2,–1)

−1=−alog(2+2)+1 Plug in (2,−1). −2=−alog(4) Arithmetic. a= 2 log(4) Solve for a.

f(x)=− 2 log(4) log(x+2)+1.xf(x)xf(x)f(x)=2ln(x+3)−1 x=−3 x=−3 {x | x>−3}. x→− 3 + ,f(x)→−∞

x→∞,f(x)→∞. f(x)= log b (x) f(x)=a log b (x+c)+d x. f(x)= log b (x) (1,0), (0,∞), (−∞,∞), x=0, b>1, 0<b<1, f(x)= log b (x+c) y= log b (x) c c>0. c c<0. f(x)= log b (x)+d y= log b (x) d d>0. d d<0. a>0, f(x)=a log b (x) y= log b (x) a |a|>1. y= log b (x) a |a|<1. y= log b (x) −1, −1, f(x)=− log b (x) f(x)= log b (−x) f(x)=a log b (x+c)+d. f(x)=a log b (x+c)+d, x=−c f(x)=a log b (x+c)+d, y=x. (a,b) (b,a) f(x)= log b (x). x f(x)= log 3 (x+4)h(x)=ln(1 2 −x) (−∞, 1 2); (−∞,∞)g(x)= log 5 (2x+9)−2h(x)=ln(4x+17)−5 (− 17 4 ,∞); (−∞,∞) f(x)= log 2 (12−3x)−3 f(x)= log b (x−5) (5,∞); x=5 g(x)=ln(3−x) f(x)=log(3x+1) (− 1 3 ,∞); x=− 1 3 f(x)=3log(−x)+2 g(x)=−ln(3x+9)−7 (−3,∞); x=−3f(x)=ln(2−x)f(x)=log(x− 3 7)(3 7 , ∞)x= 3 7 x→ (3 7) + ,f(x)→−∞x→∞,f(x)→∞h(x)=−log(3x−4)+3g(x)=ln(2x+6)−5(−3,∞)x=−3x→− 3 + f(x)→−∞x→∞f(x)→∞f(x)= log 3 (15−5x)+6h(x)= log 4 (x−1)+1 (1,∞); (−∞,∞); x=1; (5 4 ,0); f(x)=log(5x+10)+3g(x)=ln(−x)−2 (−∞,0); (−∞,∞); x=0; (− e 2 ,0); f(x)= log 2 (x+2)−5 h(x)=3ln(x)−9 (0,∞); (−∞,∞); x=0; (e 3 ,0); d(x)=log(x)f(x)=ln(x)g(x)= log 2 (x)h(x)= log 5 (x) j(x)= log 25 (x)f(x)= log 1 3 (x)g(x)= log 2 (x)h(x)= log 3 4 (x)f(x)=log(x) g(x)= 10 xf(x)=log(x) g(x)= log 1 2 (x)f(x)= log 4 (x) g(x)=ln(x)f(x)= e x g(x)=ln(x)f(x)= log 4 (−x+2)g(x)=− log 4 (x+2) h(x)= log 4 (x+2) f(x)= log 2 (x+2) f(x)=2log(x) f(x)=ln(−x)g(x)=log(4x+16)+4g(x)=log(6−3x)+1 h(x)=− 1 2 ln(x+1)−3 y= log 2 (x) f(x)= log 2 (−(x−1)) f(x)= log 3 (x) f(x)= log 4 (x) f(x)=3 log 4 (x+2) f(x)= log 5 (x) log(x−1)+2=ln(x−1)+2x=2log(2x−3)+2=−log(2x−3)+5ln(x−2)=−ln(x+1)x≈2.303 2ln(5x+1)= 1 2 ln(−5x)+11 3 log(1−x)=log(x+1)+ 1 3x≈−0.472 b b≠1. log b 1 f(x)= log 1 2 (x) g(x)=− log 2 (x). f(x)= log 1 2 (x) g(x)=− log 2 (x) b≠1, log b (x)=− log 1 b (x). f(x)=ln(x+2 x−4)? x+2 x−4 >0 f(x)= x+2 x−4 , (−∞,−2) (4,∞). (−∞,−2)∪(4,∞). f(x)=log(x 2 +4x+4) a

pH=−log([H +]) =log(1 [H +])

−log([H +]) log(1 [H +])

log b 1=0 log b b=1

log 5 1=0 5 0 =1. log 5 5=1 5 1 =5.

log b (b x)=x b log b x =x,x>0

log(100), log 10 (10 2), log b (b x)=x log 10 (10 2)=2. e ln(7) , e log e 7 , b log b x =x e log e 7 =7.

log b M= log b N if and only if M=N

log 3 (3x)= log 3 (2x+5) x. x:

3x=2x+5 Set the arguments equal. x=5 Subtract 2x.

log 3 (3x)+ log 3 (2x+5)=2? x a x b = x a+b . x M,N, b, b≠1,

log b (MN)= log b (M)+ log b (N).

m= log b M n= log b N. b m =M b n =N.

log b (MN) = log b (b m b n) Substitute for M and N. = log b (b m+n) Apply the product rule for exponents. =m+n Apply the inverse property of logs. = log b (M)+ log b (N) Substitute for m and n.

log b (wxyz).

log b (wxyz)= log b w+ log b x+ log b y+ log b z

log b (MN)= log b (M)+ log b (N) for b>0

log 3 (30x(3x+4)). 30

log 3 (30x(3x+4))= log 3 (2·3·5·x·(3x+4))

log 3 (30x(3x+4))= log 3 (2)+ log 3 (3)+ log 3 (5)+ log 3 (x)+ log 3 (3x+4)

log b (8k).log b 2+ log b 2+ log b 2+ log b k=3 log b 2+ log b k x a b = x a−b . x M,N, b, b≠1,

log b (M N)= log b (M)− log b (N).

m= log b M n= log b N. b m =M b n =N.

log b (M N) = log b (b m b n) Substitute for M and N. = log b (b m−n) Apply the quotient rule for exponents. =m−n Apply the inverse property of logs. = log b (M)− log b (N) Substitute for m and n.

log(2 x 2 +6x 3x+9),

log(2 x 2 +6x 3x+9)=log(2x(x+3) 3(x+3)) Factor the numerator and denominator. =log(2x 3) Cancel the common factors.

log(2x 3)=log(2x)−log(3) =log(2)+log(x)−log(3)

log b (M N)= log b M− log b N

log 2 (15x(x−1) (3x+4)(2−x)).

log 2 (15x(x−1) (3x+4)(2−x))= log 2 (15x(x−1))− log 2 ((3x+4)(2−x))

log 2 (15x(x−1))− log 2 ((3x+4)(2−x))=[log 2 (3)+ log 2 (5)+ log 2 (x)+ log 2 (x−1)]−[log 2 (3x+4)+ log 2 (2−x)] = log 2 (3)+ log 2 (5)+ log 2 (x)+ log 2 (x−1)− log 2 (3x+4)− log 2 (2−x)

x=− 4 3 x=2. x>0,x>1,x>− 4 3 , x<2. log 3 (7 x 2 +21x 7x(x−1)(x−2)).

log 3 (x+3)− log 3 (x−1)− log 3 (x−2) x 2 ?

log b (x 2) = log b (x·x) = log b x+ log b x =2 log b x

100= 10 2 3 = 3 1 2 1 e = e −1

log b (M n)=n log b M

log 2 x 5 . x,

log 2 (x 5)=5 log 2 x

ln x 2 . 2lnx log 3 (25) log 3 (25)= log 3 (5 2).

log 3 (5 2)=2 log 3 (5)

ln(1 x 2).−2ln(x) 4ln(x) 4ln(x), x, 4ln(x)=ln(x 4). 2 log 3 4 log 3 16

log b (6x y) = log b (6x)− log b y = log b 6+ log b x− log b y

log b (A C) = log b (A C −1) = log b (A)+ log b (C −1) = log b A+(−1) log b C = log b A− log b C

ln(x 4 y 7)

ln(x 4 y 7)=ln(x 4 y)−ln(7)

ln(x 4 y)−ln(7)=ln(x 4)+ln(y)−ln(7)

ln(x 4)+ln(y)−ln(7)=4ln(x)+ln(y)−ln(7)

log(x 2 y 3 z 4).2logx+3logy−4logz log(x).

log(x) =log x (1 2) = 1 2 logx

ln(x 2 3).2 3 lnx ln(x 2 + y 2)? log 6 (64 x 3 (4x+1) (2x−1)).

log 6 (64 x 3 (4x+1) (2x−1)) = log 6 64+ log 6 x 3 + log 6 (4x+1)− log 6 (2x−1)

Apply the Quotient Rule. = log 6 2 6 + log 6 x 3 + log 6 (4x+1)− log 6 (2x−1)

Simplify by writing 64 as 2 6 . =6 log 6 2+3 log 6 x+ log 6 (4x+1)− log 6 (2x−1) Apply the Power Rule.

ln((x−1) (2x+1) 2 (x 2 −9)).1 2 ln(x−1)+ln(2x+1)−ln(x+3)−ln(x−3)

log 3 (5)+ log 3 (8)− log 3 (2)

log 3 (5)+ log 3 (8)= log 3 (5·8)= log 3 (40)

log 3 (40)− log 3 (2)

log 3 (40)− log 3 (2)= log 3 (40 2)= log 3 (20)

log3−log4+log5−log6.log(3·5 4·6); log(5 8) log 2 (x 2)+ 1 2 log 2 (x−1)−3 log 2 ((x+3) 2).

log 2 (x 2)+ 1 2 log 2 (x−1)−3 log 2 ((x+3) 2)= log 2 (x 2)+ log 2 (x−1)− log 2 ((x+3) 6)

log 2 (x 2)+ log 2 (x−1)− log 2 ((x+3) 6)= log 2 (x 2 x−1)− log 2 ((x+3) 6)

log 2 (x 2 x−1)− log 2 ((x+3) 6)= log 2 x 2 x−1 (x+3) 6

2logx−4log(x+5)+ 1 x log(3x+5)

2logx−4log(x+5)+ 1 x log(3x+5)=log(x 2)−log((x+5) 4)+log((3x+5) x −1)

log(x 2)−log((x+5) 4)+log((3x+5) x −1)=log(x 2)−log((x+5) 4 (3x+5) x −1)

log(x 2)−log((x+5) 4 (3x+5) x −1)=log(x 2 (x+5) 4 ((3x+5) x −1))

log(5)+0.5log(x)−log(7x−1)+3log(x−1) log(5 (x−1) 3 x (7x−1))

4(3log(x)+log(x+5)−log(2x+3)).log x 12 (x+5) 4 (2x+3) 4 ; log (x 3 (x+5) (2x+3)) 4 .
pH=−log[H +]. C P P=–log(C). 2C.

pH=−log(2C)

pH=−log(2C)=−(log(2)+log(C))=−log(2)−log(C)

P=–log(C),

pH=P−log(2)≈P−0.301

e, M,b, n, n≠1 b≠1,

log b M= log n M log n b

y= log b M. n b y =M.

log n (b y) = log n M Apply the one-to-one property. y log n b = log n M Apply the power rule for logarithms. y = log n M log n b Isolate y. log b M = log n M log n b Substitute for y.

log 5 36

log 5 36 = log(36) log(5) Apply the change of base formula using base 10. ≈2.2266 Use a calculator to evaluate to 4 decimal places.

M,b, n, n≠1 b≠1,

log b M= log n M log n b .

log b M= lnM lnb

log b M= logM logb

log b M, n, n≠1. n, log(x), ln(x), e. n M. n b. log 5 3 log 5 3 n=e.

log b M = lnM lnb log 5 3 = ln3 ln5

log 0.5 8 ln8 ln0.5 log9 log 10 9. log9= ln9 ln10 . log 2 (10) e.

log 2 10= ln10 ln2 Apply the change of base formula using base e. ≈3.3219 Use a calculator to evaluate to 4 decimal places.

log 5 (100) ln100 ln5 ≈ 4.6051 1.6094 =2.861log b (MN)= log b (M)+ log b (N)
log b (M N)= log b M− log b Nlog b (M n)=n log b Mlog b M= log n M log n b n>0,n≠1,b≠1 e
log b (x n)? log b (x 1 n)= 1 n log b (x).log b (7x·2y)log b (2)+ log b (7)+ log b (x)+ log b (y)
ln(3ab·5c)log b (13 17)log b (13)− log b (17)log 4 (x z w)ln(1 4 k)−kln(4)log 2 (y x)
ln(7)+ln(x)+ln(y)ln(7xy)log 3 (2)+ log 3 (a)+ log 3 (11)+ log 3 (b)log b (28)− log b (7)log b (4)
ln(a)−ln(d)−ln(c)− log b (1 7)log b (7)1 3 ln(8)log(x 15 y 13 z 19)15log(x)+13log(y)−19log(z)
ln(a −2 b −4 c 5)log(x 3 y −4)3 2 log(x)−2log(y)ln(y y 1−y)log(x 2 y 3 x 2 y 5 3)
8 3 log(x)+ 14 3 log(y)log(2 x 4)+log(3 x 5)ln(6 x 9)−ln(3 x 2)ln(2 x 7)2log(x)+3log(x+1)
log(x)− 1 2 log(y)+3log(z)log(x z 3 y)4 log 7 (c)+ log 7 (a) 3 + log 7 (b) 3log 7 (15) e
log 7 (15)= ln(15) ln(7)log 14 (55.875) 10 log 5 (6)=a log 5 (11)=b. a b. log 11 (5)
log 11 (5)= log 5 (5) log 5 (11) = 1 blog 6 (55)log 11 (6 11)
log 11 (6 11)= log 5 (6 11) log 5 (11) = log 5 (6)− log 5 (11) log 5 (11) = a−b b = a b −1
log 3 (1 9)−3 log 3 (3)6 log 8 (2)+ log 8 (64) 3 log 8 (4)32 log 9 (3)−4 log 9 (3)+ log 9 (1 729)
log 3 (22)2.81359log 8 (65)log 6 (5.38)0.93913log 4 (15 2)log 1 2 (4.7)−2.23266 x
log 12 (2x+6)+ log 12 (x+2)=2. x log 6 (x+2)− log 6 (x−3)=1. x=4;
log 6 (x+2)− log 6 (x−3)= log 6 (x+2 x−3)=1. x:
6 1 = x+2 x−3 0 = x+2 x−3 −6 0 = x+2 x−3 − 6(x−3) (x−3) 0 = x+2−6x+18 x−3 0 = x−4 x−3 x =4
log 6 (4+2)− log 6 (4−3)= log 6 (6)− log 6 (1) x=4. b x =m? log b (n)= 1 log n (b) b>1 n>1. b
n 1. log b (n)= log n (n) log n (b) = 1 log n (b) . log 81 (2401)= log 3 (7)? b,S, T, b>0, b≠1,
b S = b T S=T. 3 4x−7 = 3 2x 3 . x, 3. x:

3 4x−7 = 3 2x 3 3 4x−7 = 3 2x 3 1 Rewrite 3 as 3 1 . 3 4x−7 = 3 2x−1 Use the division property of exponents. 4x−7 =2x−1 Apply the one-to-one property of exponents. 2x =6 Subtract 2x and add 7 to both sides. x =3 Divide by 3.

S and T, b≠1,

b S = b T if and only if S=T

b S = b T , S T b S = b T . S=T, 2 x−1 = 2 2x−4 .

2 x−1 = 2 2x−4 The common base is 2. x−1=2x−4 By the one-to-one property the exponents must be equal. x=3 Solve for x.

5 2x = 5 3x+2 .x=−2 256= 4 x−5 . 2. x:

256= 4 x−5 2 8 = (2 2) x−5 Rewrite each side as a power with base 2. 2 8 = 2 2x−10 Use the one-to-one property of exponents. 8=2x−10 Apply the one-to-one property of exponents. 18=2x Add 10 to both sides. x=9 Divide by 2.

b S = b T . S=T, 8 x+2 = 16 x+1 .

8 x+2 = 16 x+1 (2 3) x+2 = (2 4) x+1 Write 8 and 16 as powers of 2. 2 3x+6 = 2 4x+4 To take a power of a power, multiply exponents. 3x+6=4x+4 Use the one-to-one property to set the exponents equal. x=2 Solve for x.

5 2x = 25 3x+2 .x=−1 2 5x = 2 .

2 5x = 2 1 2 Write the square root of 2 as a power of 2. 5x= 1 2 Use the one-to-one property. x= 1 10 Solve for x.

5 x = 5 .x= 1 2 3 x+1 =−2. x 2 x =−100. log(a)=log(b) a=b, 5 x+2 = 4 x .

5 x+2 = 4 x There is no easy way to get the powers to have the same base. ln 5 x+2 =ln 4 x Take ln of both sides. (x+2)ln5=xln4 Use laws of logs. xln5+2ln5=xln4 Use the distributive law. xln5−xln4=−2ln5 Get terms containing x on one side, terms without x on the other. x(ln5−ln4)=−2ln5 On the left hand side, factor out an x. xln(5 4)=ln(1 25) Use the laws of logs. x= ln(1 25) ln(5 4) Divide by the coefficient of x.

2 x = 3 x+1 .x= ln3 ln(2 3) 2 x = 3 x ?0. e. e y=A e kt , t. A. k. 100=20 e 2t .

100 =20 e 2t 5 = e 2t Divide by the coefficient of the power. ln5 =2t Take ln of both sides. Use the fact that ln(x) and e x are inverse functions. t = ln5 2 Divide by the coefficient of t.

t=ln 5 . 3 e 0.5t =11.t=2ln(11 3) ln (11 3) 2 y=A e kt k≠0, y A 2=−3 e t . 4 e 2x +5=12.

4 e 2x +5=12 4 e 2x =7 Combine like terms. e 2x = 7 4 Divide by the coefficient of the power. 2x=ln(7 4) Take ln of both sides. x= 1 2 ln(7 4) Solve for x.

3+ e 2t =7 e 2t .t=ln(1 2)=− 1 2 ln(2) e 2x − e x =56.

e 2x − e x =56 e 2x − e x −56 =0 Get one side of the equation equal to zero. (e x +7)(e x −8) =0 Factor by the FOIL method. e x +7 =0 or e x −8=0 If a product is zero, then one factor must be zero. e x =−7 or e x =8 Isolate the exponentials. e x =8 Reject the equation in which the power equals a negative number. x =ln8 Solve the equation in which the power equals a positive number.

e x =−7 ln(−7) e 2x = e x +2.x=ln2 log b (x)=y b y =x. log 2 (2)+ log 2 (3x−5)=3. x:

log 2 (2)+ log 2 (3x−5)=3 log 2 (2(3x−5))=3 Apply the product rule of logarithms. log 2 (6x−10)=3 Distribute. 2 3 =6x−10 Apply the definition of a logarithm. 8=6x−10 Calculate 2 3 . 18=6x Add 10 to both sides. x=3 Divide by 6.

S b c, b>0, b≠1,

log b (S)=c if and only if b c =S

2lnx+3=7.

2lnx+3=7 2lnx=4 Subtract 3. lnx=2 Divide by 2. x= e 2 Rewrite in exponential form.

6+lnx=10.x= e 4 2ln(6x)=7.

2ln(6x)=7 ln(6x)= 7 2 Divide by 2. 6x= e (7 2) Use the definition of ln. x= 1 6 e (7 2) Divide by 6.

2ln(x+1)=10.x= e 5 −1 lnx=3.

lnx=3 x= e 3 Use the definition of the natural logarithm.

e 3 ≈20. e 3 ≈20.0855. y=lnx y=3 (e 3 ,3), 2 x =1000 x≈9.97 x>0,S>0,T>0 b, b≠1,

log b S= log b T if and only if S=T.

If log 2 (x−1)= log 2 (8),then x−1=8.

x−1=8, x, x=9. x=9 log 2 (9−1)= log 2 (8)=3. log(3x−2)−log(2)=log(x+4). x:

log(3x−2)−log(2)=log(x+4) log(3x−2 2)=log(x+4) Apply the quotient rule of logarithms.

3x−2 2 =x+4 Apply the one to one property of a logarithm. 3x−2=2x+8

Multiply both sides of the equation by 2. x=10 Subtract 2x and add 2.

x=10 log(3x−2)−log(2)=log(x+4).

log(3(10)−2)−log(2)=log((10)+4) log(28)−log(2)=log(14) log(28 2)=log(14)

The solution checks.

S T b, b≠1,

log b S= log b T if and only if S=T

log b S= log b T. S=T, ln(x 2)=ln(2x+3).

ln(x 2)=ln(2x+3) x 2 =2x+3 Use the one-to-one property of the logarithm. x 2 −2x−3=0 Get zero on one side before factoring. (x−3)(x+1)=0 Factor using FOIL. x −3=0 or x+1=0 If a product is zero, one of the factors must be zero. x=3 or x=−1 Solve for x.

3 −1. −1 ln(x 2)=ln1. x=1 x=−1

A(t)= A 0 e ln(0.5) T t A(t)= A 0 e ln(0.5) t T A(t)= A 0 (e ln(0.5)) t T A(t)= A 0 (1 2) t T

A 0 T t y t

y=1000e ln(0.5) 703,800,000 t 900=1000 e ln(0.5) 703,800,000 t

After 10% decays, 900 grams are left. 0.9= e ln(0.5) 703,800,000 t Divide by 1000. ln(0.9)=ln(e ln(0.5) 703,800,000 t) Take ln of both sides. ln(0.9)= ln(0.5) 703,800,000 t ln(e M)=M

t=703,800,000× ln(0.9) ln(0.5) years Solve for t. t≈106,979,777 years

t=703,800,000× ln(0.8) ln(0.5) years ≈ 226,572,993 years. S T b, b S = b T S=T. b c, b≠1,
log b (S)=c b c =S. b, b≠1, log b S= log b T S=T. e, log b (S)=c, S b c =S, log b (S)=c. y= log b (S)
y=c log b S= log b T, S T S=T 4 −3v−2 = 4 −v64· 4 3x =16x=− 1 33 2x+1 · 3 x =243
2 −3n · 1 4 = 2 n+2n=−1625· 5 3x+3 =12536 3b 36 2b = 216 2−bb= 6 5(1 64) 3n ·8= 2 69 x−10 =1x=10
2 e 6x =13e r+10 −10=−422· 10 9a =29−8· 10 p+7 −7=−24p=log(17 8)−77 e 3n−5 +5=−89e −3k +6=44
k=− ln(38) 3−5 e 9x−8 −8=−62−6 e 9x+8 +2=−74x= ln(38 3)−8 92 x+1 = 5 2x−1e 2x − e x −132=0
x=ln12 7 e 8x+8 −5=−9510 e 8x+3 +2=8x= ln(3 5)−3 84 e 3x+3 −7=538 e −5x−2 −4=−90
3 2x+1 = 7 x−2e 2x − e x −6=0x=ln(3)3 e 3−3x +6=−31log(1 100)=−210 −2 = 1 100log 324 (18)= 1 2
5 log 7 n=10n=49−8 log 9 x=164+ log 2 (9k)=2k= 1 362log(8n+4)+6=1010−4ln(9−8x)=6x= 9−e 8
ln(10−3x)=ln(−4x)log 13 (5n−2)= log 13 (8−5n)n=1log(x+3)−log(x)=log(74)
ln(−3x)=ln(x 2 −6x)log 4 (6−m)= log 4 3mln(x−2)−ln(x)=ln(54)
log 9 (2 n 2 −14n)= log 9 (−45+ n 2)ln(x 2 −10)+ln(9)=ln(10)x=± 10 3 x.log(x+12)=log(x)+log(12)
ln(x)+ln(x−3)=ln(7x)x=10log 2 (7x+6)=3ln(7)+ln(2−4 x 2)=ln(14)x=0
log 8 (x+6)− log 8 (x)= log 8 (58)ln(3)−ln(3−3x)=ln(4)x= 3 4log 3 (3x)− log 3 (6)= log 3 (77)
x,log 9 (x)−5=−4x=9log 3 (x)+3=2ln(3x)=2x= e 2 3 ≈2.5ln(x−5)=1log(4)+log(−5x)=2x=−5
−7+ log 3 (4−x)=−6ln(4x−10)−6=−5x= e+10 4 ≈3.2log(4−2x)=log(−4x)
log 11 (−2 x 2 −7x)= log 11 (x−2)ln(2x+9)=ln(−5x)log 9 (3−x)= log 9 (4x−8)x= 11 5 ≈2.2
log(x 2 +13)=log(7x+3)3 log 2 (10) −log(x−9)=log(44)x= 101 11 ≈9.2ln(x)−ln(x+3)=ln(6)
$6,500 7.25% $27,710.24 D D=10log(I I 0), I I 0 = 10 −12 8.3· 10 2 P=1650 e 0.5t t 20,000? x
1000 (1.03) t =5000 e 5x =17 ln(17) 5 ≈0.5673 (1.04) 3t =8 3 4x−5 =38
x= log(38)+5log(3) 4log(3) ≈2.07850 e −0.12t =10 7 e 3x−5 +7.9=47x≈2.2401
ln(3)+ln(4.4x+6.8)=2log(−0.7x−9)=1+5log(5)x≈−44655.7143 P P=14.7 e −0.21x ,x 8.369
M= 2 3 log(E E 0) E E 0 = 10 4.4 1.4· 10 13 5.83 b log b x =x. y=A e kt . t t t=ln((y A) 1 k)
A=a (1+ r k) kt . t. T T= T s +(T 0 − T s) e −kt , T s T 0 k t t t=ln((T− T s T 0 − T s) − 1 k) e

y=A0ekt

A0eky=A0ektA0eky=2e3x.y=3e−2x.4.01134972 × 1013.1013.y=A0ekty=0(−∞, ∞)(0,∞)(0,A0)k>0k<0 k>0k<0.A0A0A0=10.k,(t=1)1020.

20=10 e k·1 2= e k Divide by 10 ln2=k Take the natural logarithm

k=ln(2). y=10 e (ln2)t =10 (e ln2) t =10· 2 t . y=10e(ln2)t104.107,

12A0=Aoekt

kA0.

1 2 A 0 = A o e kt 1 2 = e kt Divide by A 0 . ln(1 2)=kt Take the natural log. −ln(2)=kt Apply laws of logarithms. − ln(2) k =t Divide by k.

t,k

t=−ln(2)k

A=Aoekt.A12A0tk.kk=−ln(2)t.t.

A= A 0 e kt The continuous growth formula. 0.5 A 0 = A 0 e k·5730 Substitute the half-life for t and 0.5 A 0 for f(t). 0.5= e 5730k Divide by A 0 . ln(0.5)=5730k Take the natural log of both sides. k= ln(0.5) 5730 Divide by the coefficient of k. A= A 0 e (ln(0.5) 5730)t Substitute for r in the continuous growth formula.

f(t)=A0e(ln(0.5)5730)t.t,ln(0.5)5730≈−1.2097f(t)=A0e−0.0000000087tt

A≈ A 0 e (ln(0.5) 5730)t

AA0

A= A 0 e kt The continuous growth formula. 0.5 A 0 = A 0 e k·5730 Substitute the half-life for t and 0.5 A 0 for f(t). 0.5= e 5730k Divide by A 0 . ln(0.5)=5730k Take the natural log of both sides. k= ln(0.5) 5730 Divide by the coefficient of k. A= A 0 e (ln(0.5) 5730)t Substitute for r in the continuous growth formula.

t:

t=ln(AA0)−0.000121

rA≈A0e−0.000121tr=AA0≈e−0.000121t.t,

t=ln(r)−0.000121

k. t= ln(r) −0.000121 t. 20%=0.20 k t:

t= ln(r) −0.000121 Use the general form of the equation. = ln(0.20) −0.000121 Substitute for r. ≈13301 Round to the nearest year.

13,301 years±1% or 13,301 years±133 years. A= A 0 e kt , 2 A 0 = A 0 e kt .

2 A 0 = A 0 e kt 2= e kt Divide by A 0 . ln2=kt Take the natural logarithm. t= ln2 k Divide by the coefficient of t.

t= ln2 k

t= ln2 k The doubling time formula. 2= ln2 k Use a doubling time of two years. k= ln2 2 Multiply by k and divide by 2. A = A 0 e ln2 2 t Substitute k into the continuous growth formula.

A= A 0 e ln2 2 t .f(t)= A 0 e ln2 3 t

T(t)=a e kt + T s

T(t)=A b ct + T s T(t)=A e ln(b ct) + T s Laws of logarithms. T(t)=A e ctlnb + T s Laws of logarithms. T(t)=A e kt + T s Rename the constant c ln b, calling it k.

T, T s

T(t)=A e kt + T s

t A k T s T(t)=A e k t + T s A k. 165°F, 35°F 150°F. 70°F

T(t)=A e kt +35

T(0)=165.

165=A e k0 +35 Substitute (0,165). A=130 Solve for A.

T(10)=150, k.

150=130 e k10 +35 Substitute (10, 150). 115=130 e k10 Subtract 35. 115 130 = e 10k Divide by 130. ln(115 130)=10k Take the natural log of both sides. k= ln(115 130) 10 =−0.0123 Divide by the coefficient of k.

T(t)=130 e −0.0123t +35.

70=130 e −0.0123t +35 Substitute in 70 for T(t). 35=130 e −0.0123t Subtract 35. 35 130 = e −0.0123t Divide by 130. ln(35 130)=−0.0123t Take the natural log of both sides t= ln(35 130) −0.0123 ≈106.68 Divide by the coefficient of t.

70°F.a, b,c, x

f(x)= c 1+a e −bx

f(x)= c 1+a e −bx

c 1+a c b t=0 b=0.6030.

f(x)= c 1+a e −bx

c=1000. a, t=0 c 1+a =1, a=999. f(x)= 1000 1+999 e −0.6030x ≈293.8. c=1000.

f(x)= 1000 1+999 e −0.6030xxy y=aln(bx). (1,0), 0=alnb. a=0 ln(b)=0. b=1 y=aln(x). (9,4.394) a:

y=aln(x) 4.394=aln(9) a= 4.394 ln(9)

a= 4.394 ln(9) ≈2, y=2ln(x). y=2lnx. y=ln(x 2) y=ln(x 2) x>0. y=ln(x 2)=2ln(x) x>0. x<0, y=ln(x 2) y=2ln(x) y=ln(x 2) xy y=2 e 0.5x . 10 e. e e. y=a b x , y= A 0 e kx . y=a b x y=a e ln(b x) . y=a e xln(b) =a e ln(b)x . a= A 0 k=ln(b) y= A 0 e kx . y=2.5 (3.1) x y= A 0 e kx . y=2.5 (3.1) x =2.5 e ln(3.1 x) Insert exponential and its inverse. =2.5 e xln3.1 Laws of logs. =2.5 e (ln3.1) x Commutative law of multiplication

y=3 (0.5) x e y=3 e (ln0.5)x A= A 0 e kt ,k<0, t=− ln(2) k .t= ln(A A 0) −0.000121 .A 0 A t A= A 0 e kt ,k>0, t= ln2 kT(t)=A e kt + T s , T s A=T(0)− T s , k f(x)=a b x . b>1, 0<b<1, A= A 0 e kx , A 0 A 0 k>0 k<0. t, k, t= ln(k) −0.000121 t. y=a b x y= A 0 e kx k=lnb. 10 2 T(t)=68 e −0.0174t +72. f(x)= 150 1+8 e −2x . f(0). f(0)≈16.7; f(4). xf(x) f(x)= 1.2 x f(x)=1.68 (0.65) x e xf(x)xf(x)xf(x)xf(x) t P(t)= 1000 1+9 e −0.6t . 1.4 2 900? 7.3 P.4 8.18 P(t)= P 0 e rt P 0 r>0. M= 2 3 log(S S 0). S.

M= 2 3 log(S S 0) log(S S 0)= 3 2 M S S 0 = 10 3M 2 S= S 0 10 3M 2

y= c 1+a e −rx ? b x = e xln(b) b≠1. y= b x b b≠1. ln(y)=ln(b x) ln(y)=xln(b) e ln(y) = e xln(b) y= e xln(b) t A=125 e (−0.3567t) ;A≈43 f(10) 0.5 1.15% 60 t 250 250 32 f(t)=250 e (−0.00914t) ; 76 1590 10.4 r≈−0.0667, 6.67% 5730 1350 3 f(t)=1350 e (0.03466t) ; P(180)≈691,200 360 5 1000 20 f(t)=256 e (0.068110t) ; 10 100° 69° F 95° F. 80° F? 88 2 165°F 75°F 145°F.T(t)=90 e (−0.008377t) +75, t 110° F? 113 log(x)=1.5; x≈31.623 10 −10 W m 2 , 10 −4 W m 2 , 10 2 W m 2 M= 2 3 log(S S 0). 3.9 750 5.82 N(t)= 500 1+49 e −0.7t N(3)≈71 t N(t) 13 12 4.75 f(t)=13 (0.0805) tf(t)=13 e 0.9195t f(t)=13 e (−0.0839t)f(t)= 4.75 1+13 e −0.83925t f(x)= c 1+a e −bx c 1+a c b y=a b x y= A 0 e kx . y=a b x . y=a b x a>0):b y=a. b>1, x 0<b<1, x r, r 2 . r 2 r,

y=a b x

b b>1, 0<b<1, y=a b x . x y 0.16.

y=0.58304829 (2.20720213E10) x

y=0.58304829 (22,072,021,300) x

r 2 ≈0.97 0.16. 0.16 x y.

y =0.58304829 (22,072,021,300) x Use the regression model found in part (a). =0.58304829 (22,072,021,300) 0.16 Substitute 0.16 for x. ≈26.35 Round to the nearest hundredth.

y=522.88585984 (1.19645256) x . y=a+bln(x). x, (1,a) b>0, b<0,

y=a+bln(x)

x, b>0, b<0, y=a+bln(x). x x=1 x=2 y

y=42.52722583+13.85752327ln(x)

x=14 y:

y =42.52722583+13.85752327ln(x) Use the regression model found in part (a). =42.52722583+13.85752327ln(14) Substitute 14 for x. ≈79.1 Round to the nearest tenth.

x x=1 y y=141.91242949+10.45366573ln(x)

y= c 1+a e −bx

c 1+a b>0, (ln(a) b , c 2). y=c.c
y= c 1+a e −bx
c 1+a . y=c a, b, c y= c 1+a e −bx . x x=0 y c.
y= 105.7379526 1+6.88328979 e −0.2595440013x
x=18 y:
y = 105.7379526 1+6.88328979 e −0.2595440013x Use the regression model found in part (a). = 105.7379526 1+6.88328979 e −0.2595440013(18) Substitute 18 for x. ≈99.3 Round to the nearest tenth
c=100 x x=0 y y= 25.65665979 1+6.113686306 e −0.3852149008x . 25,634 y=a b x y=a+bln(x)
y= c 1+a e −bx y=10.209 e −0.294xy=5.598−1.912ln(x)y=2.104 (1.479) xy=4.607+2.733ln(x)
y= 14.005 1+2.79 e −0.812x P(t)= 175 1+6.995 e −0.68t ? P(0)=22 A(t)=1550 (1.085) x e.
h(p)=67.682−5.792ln(p). p h(p)=62?p≈2.67 P(t)= 90 1+5 e −0.42t . P(t)=45? (0,15) P x
P(x)= 68 1+16 e −0.28x . 3 4 20 6.8 P P(x)= 558 1+54.8 e −0.462x , x 10 10 3 100 e.
f(x)=776.682 e 0.3549x x f(x)=4000. f(x)=4000,x≈4.6.f(x)=731.92 (0.738) x e. x f(x)=250. y=a+bln(x)
x=10. f(10)≈9.5 x f(x)=7. f(x)=7,x≈2.7. y=a+bln(x) f(x)=7.544−2.268ln(x) x=10. x f(x)=8.
y= c 1+a e −bx x f(x)=12.5,x≈2.1.xf(x) y= c 1+a e −bx f(x)= 136.068 1+10.324 e −0.480x136 x
P(t)= c 1+a e −bt , t=0 P(0)= P 0 . c−P(t) P(t) = c− P 0 P 0 e −bt . a e −bt :
c−P(t) P(t) = c− c 1+a e −bt c 1+a e −bt = c(1+a e −bt)−c 1+a e −bt c 1+a e −bt = c(1+a e −bt −1) 1+a e −bt c 1+a e −bt =1+a e −bt −1=a e −bt
a e −bt . t=0, P 0 = c 1+a e −b(0) = c 1+a .
c− P 0 P 0 e −bt = c− c 1+a c 1+a e −bt = c(1+a)−c 1+a c 1+a e −bt = c(1+a−1) 1+a c 1+a e −bt =(1+a −1) e −bt =a e −bt
c−P(t) P(t) = c− P 0 P 0 e −bt . f(x) g(x) (1.5,1.5) (8.5, 8.5). y=x f(x)=1.034341 e 0.247800x .
f(g(x))=x,
g(f(x)) =4.035510ln(1.034341 e 0.247800x)−0.136259 =4.03551(ln(1.034341)+ln(e 0.2478x)) −0.136259 =4.03551(ln(1.034341)+0.2478x)−0.136259 =0.136257+0.999999x−0.136259 = −0.000002+0.999999x ≈0+x =x
f −1 (x) f(x)= c 1+a e −bx . P(t)= 20 1+4 e −0.5t P(t) P −1 (t) y=156 (0.825) t 0.825, 0 1.
A(t)=205 (1.13) t , t 6 (2, 2.25) (5,60.75).y=0.25 (3) x 8.12% 20 $42,888.18 7.5% 3
y=2.294 e −0.654t $10,500 6.25% 25 f(x)=3.5 (2) x . f(x)=4 (1 8) x f(x)= 6.5 x 7. g(x)?
g(x)=7 (6.5) −x ; (0, 7); 0. f(x)= 2 x . log 17 (4913)=x 17 x =4913 ln(s)=t a − 2 5 =b log a b=− 2 5
e −3.5 =h x log 64 (x)= 1 3 x= 64 1 3 =4 log 5 (1 125) log(0.000001) log(0.000001)=−6
log(4.005) ln(e −0.8648) ln(e −0.8648)=−0.8648 ln(18 3) g(x)=log(7x+21)−4. h(x)=2ln(9−3x)+1.
g(x)=ln(4x+20)−17. x>−5; x=−5; x→− 5 + ,f(x)→−∞ x→∞,f(x)→∞. ln(7r·11st)
log 8 (x)+ log 8 (5)+ log 8 (y)+ log 8 (13) log 8 (65xy) log m (67 83) ln(z)−ln(x)−ln(y)
ln(z xy) ln(1 x 5) − log y (1 12) log y (12) log(r 2 s 11 t 14). ln(2b b+1 b−1).
ln(2)+ln(b)+ ln(b+1)−ln(b−1) 2 5ln(b)+ln(c)+ ln(4−a) 2 3 log 7 v+6 log 7 w− log 7 u 3
log 7 (v 3 w 6 u 3) log 3 (12.75) e. 5 12x−17 =125 x x= log(125) log(5) +17 12 = 5 3
216 3x · 216 x = 36 3x+2 125 (1 625) −x−3 = 5 3 x=−3 7· 17 −9x −7=49. 3 e 6n−2 +1=−60.
5 e 3x −4=6 2 e 5x−2 −9=−56. 5 2x−3 = 7 x+1 . e 2x − e x −110=0. x=ln(11) −5 log 7 (10n)=5.
9+6ln(a+3)=33.a= e 4 −3 log 8 (7)+ log 8 (−4x)= log 8 (5). ln(5)+ln(5 x 2 −5)=ln(56). x=± 9 5
D D=10log(I I 0), I I 0 = 10 −12 6.3· 10 −3 P(t)=256,114 e 0.25t t 5.45 f −1 f(x)=2· e x+1 −5.
f −1 f(x)=0.25· log 2 (x 3 +1).f −1 (x)= 2 4x −1 3 300 17% t 24 f(t)=300 (0.83) t ;f(24)≈3.43 g
350° 71°F 175°F. 85°F? 45 N(t)= 1200 1+199 e −0.625t t 8.5 (−2,100) (0,4). y=4 (0.2) x ;
y=4 e -1.609438x P(t)= 250,000 1 + 499 e −0.45t ? P(t)= 14,250 1 + 29 e −0.62t , t 75% 7.2
y=16.68718−9.71860ln(x) A(t)=8 (1.17) t , t 3 13 (0, 4) (2, 9). 6.25% 4 $1,947 7.4% 15
f(x)=5 (0.5) −x (0, 5) f(x)= (1 2) x . log 8.5 (614.125)=a 8.5 a =614.125 e 1 2 =m x lo g 1 7 (x)=2
x= (1 7) 2 = 1 49 log(10,000,000) ln(0.716) ln(0.716)≈−0.334 g(x)=log(12−6x)+3.
f(x)= log 5 (39−13x)+7. x<3; x=3; x→ 3 − ,f(x)→−∞ x→−∞,f(x)→∞ log(17a·2b)

log t (96)− log t (8) log t (12) log 8 (a 1 b) ln(y 3 z 2 · x−4 3).3 ln(y)+2ln(z)+ ln(x−4) 3
4ln(c)+ln(d)+ ln(a) 3 + ln(b+3) 3 16 3x−5 =1000 x x= ln(1000) ln(16) +5 3 ≈2.497
(1 81) x · 1 243 = (1 9) −3x−1 −9 e 10a−8 −5=−41a= ln(4)+8 10 10 e 4x+2 +5=56.
−5 e −4x−1 −4=64. 2 x−3 = 6 2x−1 . e 2x − e x −72=0. x=ln(9) 4log(2n)−7=−11
log(4 x 2 −10)+log(3)=log(51) x=± 3 3 2 D D=10log(I I 0), I I 0 = 10 −12 4.7· 10 −1 112 17 80
f(t)=112 e −.019792t ; 35 e. 71° 35° F. 63° F. T(t)=36 e −0.025131t +35;T(60)≈ 43 o F
P(t)= 360 1+6.2 e −0.35t , t P(t)= 16,120 1+25 e −0.75t , t 80% y=15.10062 (1.24621) x
y= 18.41659 1+7.54644 e −0.68375xEF→. ED → EF → . ∠DEF.θϕorφαβγ ∠θ 1 360
90° 360° = 1 4 . 360° 360° =1.

30° 360° = 1 12
1 12 = 1 3 (1 4)
−135° 360° =− 3 8
− 3 8 =− 3 2 (1 4)

C=2πr. r, 2π 2π≈6.28 2π 2π

2π radians= 360 ∘ π radians= 360 ∘ 2 = 180 ∘ 1 radian= 180 ∘ π ≈ 57.3 ∘

t s s r.

s=rθ θ= s r

s=r, θ= r r = 1 radian.s r.s=2r.2πC=2πr,r2π(2)=4π2π(3)=6π.

Smaller circle: 1 2 π 2 = 1 4 π Larger circle: 3 4 π 3 = 1 4 π

1 4 π, 2π π s r s r 2π 1 360° 2π ≈57.3°. C=2πr, C=2π.

1 rotation =360° =2π radians 1 2 rotation=180° =π radians 1 4 rotation=90° = π 2 radians

1 rotation=2πr
s= 1 3 (2πr) = 2πr 3

radian measure= 2πr 3 r = 2πr 3r = 2π 3

3π 2
θ 180 = θ R π

θ θ π. π.

Degrees 180 = Radians π
θ 180 = θ R π

π 6

θ 180 = θ R π θ 180 = π 6 π θ= 180 6 θ= 30 ∘
θ 180 = θ R π θ 180 = 3 π θ= 3(180) π θ≈ 172 ∘

− 3π 4 15

θ 180 = θ R π 15 180 = θ R π 15π 180 = θ R π 12 = θ R

30 ∘ = π 6 . 15 ∘ = 1 2 (30 ∘), 1 2 (π 6) π 12 . 7π 10 2π. t t t ′ , t θ0°≤θ<360°. θ=80° α
0°≤α<360°.α=150°α

−45°+360°=315°

β 0°≤β<360°. β=60° 2π, 2π. 2π 2π, 2π 0 2π. β 19π 4 , 0≤β<2π. 2π

19π 4 −2π= 19π 4 − 8π 4 = 11π 4

11π 4 2π,

11π 4 −2π= 11π 4 − 8π 4 = 3π 4

3π 4 19π 4 , θ − 17π 6 0≤θ<2π. 7π 6 θ s r θ= s r . s θ

s=rθ

r,sθ. θ r θ:s=rθ.

C=2πr =2π(36 million miles) ≈226 million miles
(0.0114)226 million miles = 2.58 million miles
radian= arclength radius = 2.58 million miles 36 million miles =0.0717

215π 18 =37.525 units r A=π r 2 . θ, θ 2π θ 2π θ

Area of sector=(θ 2π)π r 2 = θπ r 2 2π = 1 2 θ r 2

r θ,

A= 1 2 θ r 2

r, θ. θ θ r: A= 1 2 θ r 2 .

30 degrees=30· π 180 = π 6 radians

Area = 1 2 (π 6) (20) 2 ≈104.72

104.72 ft 2 . 10π 10π v s t

v= s t

360 degrees 4 seconds = ω θ t

ω= θ t

s=rθ, θ, θ=ωt.

s=rθ =rωt

v= s t = rωt t =rω

r, ω, θ t.

ω= θ t

v, s, t.

v= s t

v=rω

ω, r ω= θ t 2π ω= 2π 5 ≈1.257 −3π 2 ω= θ t . v=rω.

180 rotations minute · 2π radians rotation =360π radians minute

v=(14 inches)(360π radians minute) =5040π inches minute

5040π inches minute · 1 feet 12 inches · 1 mile 5280 feet · 60 minutes 1 hour

≈14.99 miles per hour (mph)

s=rθA= 1 2 θ r 2ω= θ tv= s tv=rω θ 180 = θ R π . 2π. 2π 37π 45π 6π 2− π 1022π 34π 3− π 6− 4π 32π 3 7π 2 ≈11.00 in 281π 20 ≈12.72 cm 23π 4π 9 − 5π 4π 3− 7π 3− 5π 1211π 6π 2 −3π π 5π 6 π 4 π 3 . 5.02π 3 ≈5.265π 6 .25π 9 ≈8.7321π 10 ≈6.60 π 2 π 2π − π 910π 34π 313π 644π 98π 9 v,v, 1,809,557.37 mm/min=30.16 m/s(1 minute= 1 60 degree)5.76 (1 minute= 1 60 degree). 3960 20 120° θ 2π 30° or (π 6), 45° or (π 4) 60° or (π 3). t s. s=rt,r=1, s=t. t, (x,y). x y f(t)=cos t f(t)=sin t, x=cos t y=sin t. t (0,0) 1 1. (x,y) s. (x,y) (x,y) t t t. y. t t. x. sin tsin(t)cos tcos(t). cos 2 t (cos(t)) 2 . t (x,y) t,

cos t=x

sin t=y

(x,y) t,tP:sin t=y.tP: cos t=x. P t, cos(t) sin(t). cos t sin t

x=cos t= 1 2 y=sin t= 3 2

t (− 2 2 , 2 2) cos tsin t.cos(t)=− 2 2 ,sin(t)= 2 2 x y. cos(90°) sin(90°). 90° (x,y)

x=cos t=cos(90°)=0 y=sin t=sin(90°)=1

π. cos(π)=−1,sin(π)=0 x 2 + y 2 =1. x=cos t y=sin t, x y cos 2 t+ sin 2 t=1. cos 2 t+ sin 2 t=1, t,

cos 2 t+ sin 2 t=1

t t. sin(t) cos(t). t sin(t)= 3 7 t cos(t). t,

cos 2 (t)+ sin 2 (t)=1 cos 2 (t)+ 9 49 =1 cos 2 (t)= 40 49 cos(t)=± 40 49 = ± 40 7 =± 2 10 7

cos(t)=− 2 10 7

cos(t)= 24 25tsin(t).sin(t)=− 7 25 45° π 4 , 45°–45°–90° t= π 4 y=x. y=x x y (y=x),

x 2 + y 2 =1

y=x,

x 2 + x 2 =1

2 x 2 =1

x,

x 2 = 1 2 x=± 1 2

x= 1 2 . t= π 4

(x,y)=(x,x)=(1 2 , 1 2) x= 1 2 ,y= 1 2 cos t= 1 2 ,sin t= 1 2

cos t= 1 2 2 2 = 2 2 sin t= 1 2 2 2 = 2 2

(x,y) 1 45° (2 2 , 2 2). 30°, π 6 30°, −30°, 60°, 2y, r=2y y= 1 2 r. sin t=y

sin(π 6)= 1 2 r

r=1

sin(π 6)= 1 2 (1) = 1 2

cos 2 π 6 + sin 2 (π 6)=1 cos 2 (π 6)+ (1 2) 2 =1 cos 2 (π 6)= 3 4

Use the square root property. cos(π 6)= ± 3 ± 4 = 3 2

Since y is positive, choose the positive root.

(x,y) 1 30° (3 2 , 1 2). t= π 3 BAD, A 60°. B, ABC 60°. 180°, C 60°. ABC ABD ABC BD AC, AC AD 1 2 1 2 . AD B, BAD x 1 2 .

x 2 + y 2 =1

x= 1 2 ,

(1 2) 2 + y 2 =1

y,

1 4 + y 2 =1 y 2 =1− 1 4 y 2 = 3 4 y=± 3 2

t= π 3 y= 3 2 , t= π 3 (x,y) 1 60° (1 2 , 3 2),

(x,y)=(1 2 , 3 2) x= 1 2 ,y= 3 2 cos t= 1 2 ,sin t= 3 2

π 6 , π 4 , π 3 , π 2 ,3 22 2 1 2 1 2 2 2 3 2 cos(30) cos(5π 3) COS(5 × π ÷ 3) ENTER

cos(5π 3)=0.5

20°,

SIN(20 × π ÷ 180) ENTER

sin(π 3). 2π x, y, r, [−1,1]. [−1,1]. [−1,1]. α t; β t;

sin(t)= sin(α) and cos(t)=−cos(α) sin(t)=−sin(β) and cos(t)= cos(β)

t, t 0 90°, 0 π 2 0 2π, | π−t | | 180°−t |. 2π−t 360°−t. 0 2π, 2π 0 2π. 225° 225°

| (180°−225°) |=| −45° |=45°

5π 3 . π 3 (x,y) cos(150°) sin(150°). cos 5π 4 sin 5π 4 .

cos(30°)= 3 2 and sin(30°)= 1 2 .

cos(150°)=− 3 2 and sin(150°)= 1 2

5π 4 5π 4 −π= π 4 . π 4 2 2 . x y

cos 5π 4 =− 2 2 and sin 5π 4 =− 2 2

315° cos(315°) sin(315°). − π 6 cos(− π 6) sin(− π 6). cos(315°)= 2 2 , sin(315°)= − 2 2 cos(− π 6)= 3 2 , sin(− π 6)=− 1 2 (x,y) (x,y)

x=cos t y=sin t

(x,y) x y 7π 6 . 7π 6 π.

7π 6 −π= π 6

cos(π 6)= 3 2 sin(π 6)= 1 2

x y

cos(7π 6)=− 3 2 sin(7π 6)=− 1 2

(x,y) x=cos θy=sin θ. (− 3 2 ,− 1 2) 5π 3 . (1 2 ,− 3 2) cos t=xsin t=ycos 2 t+ sin 2 t=1 t t t [−1,1]. t, t t, t tsin(t)<0 cos(t)<0 sin(t)>0 cos(t)>0sin(t)>0 cos(t)<0sin(t)<0cos(t)>0sin π 2sin π 33 2cos π 2 cos π 31 2sin π 4cos π 42 2sin π 6sin πsin 3π 2cos πcos 0cos π 63 2sin 0240°60°−170°100°80°−315° 135°45°5π 42π 3π 35π 6−11π 3π 3− 7π 4−π 8π 8225°300°60°,sin(300°)=− 3 2 ,cos(300°)= 1 2 320°135° 45°, sin(135°)= 2 2 , cos(135°)=− 2 2210°120°60°, sin(120°)= 3 2 , cos(120°)=− 1 2250°150° 30°, sin(150°)= 1 2 , cos(150°)=− 3 25π 47π 6π 6 , sin(7π 6)=− 1 2 ,cos(7π 6)=− 3 25π 33π 4π 4 , sin(3π 4)= 2 2 , cos(4π 3)=− 2 24π 32π 3π 3 , sin(2π 3)= 3 2 , cos(2π 3)=− 1 25π 67π 4π 4 , sin(7π 4)=− 2 2 ,cos(7π 4)= 2 2 cos(t)= 1 7 t sin(t). cos(t)= 2 9 t sin(t). 77 9 sin(t)= 3 8 t cos(t).

sin(t)=− 1 4 t cos(t).− 15 4 220°. 120°. (−10,10 3) 7π 4 . 5π 9 . (−2.778,15.757) [−1,1] t .
sint= 1 2 ,cost=− 3 2sin t=− 2 2 ,cos t=− 2 2sin t= 3 2 ,cos t=− 1 2sin t=− 2 2 ,cos t= 2 2sin t=0,cos t=−1
sin t=−0.596,cos t=0.803sin t= 1 2 ,cos t= 3 2sin t=− 1 2 ,cos t= 3 2sin t=0.761,cos t=−0.649sin t=1,cos t=0
sin 5π 9cos 5π 9sin π 10cos π 10sin 3π 4cos 3π 4sin 98°cos 98°cos 310°sin 310°
sin(11π 3)cos(−5π 6)sin(3π 4)cos(5π 3)2 4sin(− 4π 3)cos(π 2)sin(−9π 4)cos(−π 6)− 6 4
sin(π 6)cos(−π 3)sin(7π 4)cos(−2π 3)2 4cos(5π 6)cos(2π 3)cos(−π 3)cos(π 4)2 4
sin(−5π 4)sin(11π 6)sin(π)sin(π 6) (0,1),(0,−1) (0.707,−0.707) (−0.866,−0.5) (0,0) π 3 , π 4 ,
π 6 . 1 12 (x,y) t, (x,y) t y x ,x≠0. t, t, t sin t cos t ,cos t≠0. tan. t 1 cos t = 1 x ,x≠0. sec. t
cos t sin t = x y , y≠0. cot. t 1 sin t = 1 y ,y≠0. csc. t (x,y) t

tan t= y x ,x≠0 sec t= 1 x ,x≠0 csc t= 1 y ,y≠0 cot t= x y ,y≠0

(− 3 2 , 1 2) sin t,cos t,tan t,sec t,csc t, cot t. (x,y) t,
sin t=y= 1 2 cos t=x=− 3 2 tan t= y x = 1 2 − 3 2 = 1 2 (− 2 3)=− 1 3 =− 3 3 sec t= 1 x = 1 − 3 2 =− 2 3 =
− 2 3 3 csc t= 1 y = 1 1 2 =2 cot t= x y = − 3 2 1 2 =− 3 2 (2 1)=− 3
(2 2 ,− 2 2) sin t,cos t,tan t,sec t,csc t, cot t.sin t=− 2 2 ,cos t= 2 2 ,tan t=−1,sec t= 2 ,csc t=− 2 ,cot t=−1
sin t,cos t,tan t,sec t,csc t, cot t t= π 6 . sin π 6 = 1 2 cos π 6 = 3 2 .

tan π 6 = sin π 6 cos π 6 = 1 2 3 2 = 1 3 = 3 3
sec π 6 = 1 cos π 6 = 1 3 2 = 2 3 = 2 3 3
csc π 6 = 1 sin π 6 = 1 1 2 =2
cot π 6 = cos π 6 sin π 6 = 3 2 1 2 = 3

sin t,cos t,tan t,sec t,csc t, cot t t= π 3 .
sin π 3 = 3 2 cos π 3 = 1 2 tan π 3 = 3 sec π 3 =2 csc π 3 = 2 3 3 cot π 3 = 3 3 x y 0 π 6 , or 30°
π 4 , or 45°π 3 , or 60°π 2 , or 90°3 22 21 21 22 23 23 332 3 3222 3 333 3 − 5π 6 . π 6 , − 5π 6 x y
cos(− 5π 6)=− 3 2 ,sin(− 5π 6)=− 1 2 ,tan(− 5π 6)= 3 3 sec(− 5π 6)=− 2 3 3 ,csc(− 5π 6)=−2,cot(−
5π 6)= 3
− 7π 4 . sin(−7π 4)= 2 2 ,cos(−7π 4)= 2 2 ,tan(−7π 4)=1,
sec(−7π 4)= 2 ,csc(−7π 4)= 2 ,cot(−7π 4)=1 f(x)= x 2 , (4) 2 = (−4) 2 ,(−5) 2 = (5) 2 , f(x)= x 2
f(−x)=f(x). f(x)= x 2 f(x)= x 3 , f(x)= x 3 f(−x)=−f(x). f(x)= x 3 y.
sin t=y sin(−t)=−y sin t≠sin(−t) cos t=x cos(−t)=x cos t=cos(−t)
tan(t)= y x tan(−t)=− y x tan t≠tan(−t) sec t= 1 x sec(−t)= 1 x sec t=sec(−t)
csc t= 1 y csc(−t)= 1 −y csc t≠csc(−t) cot t= x y cot(−t)= x −y cot t≠cot(−t) f(−x)=f(x).
f(−x)=−f(x).

cos(−t)=cos t sec(−t)=sec t
sin(−t)=−sin t tan(−t)=−tan t csc(−t)=−csc t cot(−t)=−cot t

t −t? −t t 3 , −t? − 3

tan t= sin tcos t
sec t= 1 cos t
csc t= 1 sin t
cot t= 1 tan t = cos t sin t

sin(45°)= 2 2 ,cos(45°)= 2 2 ,tan(45°). sin(5π 6)= 1 2 ,cos(5π 6)=− 3 2 ,evaluate sec(5π 6).

tan(45°)= sin(45°) cos(45°) = 2 2 2 2 =1
sec(5π 6)= 1 cos(5π 6) = 1 − 3 2 = −2 3 1 = −2 3 =− 2 3 3

csc(7π 6).−2sec t tan t .
sec t tan t = 1 cos t sin t cos t To divide the functions, we multiply by the reciprocal. = 1 cos t cos t
sin t Divide out the cosines. = 1 sin t Simplify and use the identity. =csc t
sec t tan t csc t,

sec t tan t =csc t

tan t(cos t).sin t cos 2 t+ sin 2 t=1. cos 2 t:

cos 2 t cos 2 t + sin 2 t cos 2 t = 1 cos 2 t 1+ tan 2 t= sec 2 t sin 2 t:

cos 2 t sin 2 t + sin 2 t sin 2 t = 1 sin 2 t cot 2 t+1= csc 2 t

1+ tan 2 t= sec 2 t

cot 2 t+1= csc 2 t

cos(t)= 12 13 t cos 2 t+ sin 2 t=1,

(12 13) 2 + sin 2 t=1 sin 2 t=1− (12 13) 2 sin 2 t=1− 144 169 sin 2 t= 25 169

sin t=± 25 169 sin t=± 25 169 sin t=± 5 13

− 5 13 .

tan t= sin t cos t = − 5 13 12 13 =− 5 12 sec t= 1 cos t = 1 12 13 = 13 12 csc t= 1 sin t = 1 − 5 13 =− 13 5

cot t= 1 tan t = 1 − 5 12 =− 12 5

sec(t)=− 17 8 0<t<π,cost=− 8 17 ,sint= 15 17 ,tant=− 15 8csct= 17 15 ,cott=− 8 15 2π, x f(x)

f(x+4)=f(x). P f f(x+P)=f(x) x. 2π. π. t

sin t=y=− 3 2 cos t=x=− 1 2 tan t= sint cost = − 3 2 − 1 2 = 3 sec t= 1 cost = 1 − 1 2 =−2 csc t= 1 sint = 1 − 3 2 =− 2 3 3 cot t= 1 tant = 1 3 = 3 3

t sin t=−1,cos t=0,tan t=Undefined sec t= Undefined,csc t=−1,cot t=0 sin(t)=− 3 2 cos(t)= 1 2 , sec(t),csc(t),tan(t), cot(t).

sec t= 1 cos t = 1 1 2 =2 csc t= 1 sin t = 1 − 3 2 − 2 3 3 tan t= sin t cos t = − 3 2 1 2 =− 3 cot t= 1 tan t = 1 − 3 =− 3 3

sin(t)= 2 2 cos(t)= 2 2 , sec(t),csc(t),tan(t), and cot(t).sec t= 2 ,csc t= 2 ,tan t=1,cot t=1 π 180 30°,

(for a scientific calculator): 1 30× π 180 COS

(for a graphing calculator): 1 cos(30π 180)

1 / 1 / 5π 7 .

1 / (5 × π / 7) SIN =

csc(5π 7)≈1.279

− π 8 . ≈−2.414tan t= sint costsec t= 1 costcsc t= 1 sintcot t= 1 tan t = cos t sin t f(−x)=f(x) f(−x)=−f(x). P f f(x+P)=f(x) x. [0,2π), π 4 x= π 4 , 5π 4 , π y x 2 + y 2 =1. x π. π tan π 6sec π 62 3 3csc π 6 cot π 63tan π 4sec π 42csc π 4cot π 4tan π 3sec π 3csc π 3cot π 33 3tan 5π 6sec 7π 6− 2 3 3 csc 11π 6cot 13π 63tan 7π 4sec 3π 4− 2csc 5π 4cot 11π 4tan 8π 3sec 4π 3csc 2π 3cot 5π 3− 3 3 tan 225°sec 300°csc 150°cot 240°3 3tan 330°sec 120°csc 210°cot 315° sin t= 3 4 , t

cos t,sec t,csc t,tan t,cot t. cos t=− 1 3 , t sin t,sec t,csc t,tan t,cot t.

sin t=− 2 2 3 ,sec t=−3,csc t=− 3 2 4 ,tan t=2 2 ,cot t= 2 4 tan t= 12 5 , 0≤t< π 2 , sin t,cos t,sec t,csc t, cot t.

sin t= 3 2 cos t= 1 2 , sec t,csc t,tan t, cot t.sec t=2,csc t= 2 3 3 , tan t= 3 , cot t= 3 3

sin 40°≈0.643 cos 40°≈0.766 sec 40°,csc 40°,tan 40°,and cot 40°. sin t= 2 2 , sin(−t)? − 2 2 cos t= 1 2 , cos(−t)? sec t=3.1, sec(−t)? csc t=0.34, csc(−t)? tan t=−1.4, tan(−t)? cot t=9.23, cot(−t)?

sin t= 2 2 ,cos t= 2 2 ,tan t=1,cot t=1,sec t= 2 ,csc t= 2

sin t=− 3 2 ,cos t=− 1 2 ,tan t= 3 ,cot t= 3 3 ,sec t=−2,csc t=− 2 3 3csc 5π 9cot 4π 7sec π 10tan 5π 8 sec 3π 4csc π 4tan 98°cot 33°cot 140°sec 310° tan(t)≈2.7, sin(t)≈0.94, cos(t). tan(t)≈1.3, cos(t)≈0.61, sin(t). sin(t)≈0.79 csc(t)≈3.2, cos(t)≈0.95, tan(t). cot(t)≈0.58, cos(t)≈0.5, csc(t). csct≈1.16 f(x)=2sin x cos x f(x)=3 sin 2 x cos x + sec xf(x)=sin x −2 cos 2 x f(x)= csc 2 x+sec x csc t tan t sec t csc tsin t cos t =tan t h=15cos(1 600 d), h d h=16cos(1 500 d), h d 1000π P=20sin(2πt)+100 P, t h, y=2cos x+6, x 55°. h, y=2cos x+5, x 55°. csc t= 1 y ,y≠0 cot t= x y ,y≠0 P f f(x+P)=f(x) sec t= 1 x ,x≠0 tan t= y x ,x≠0 30°(π 6), 45°(π 4), 60°(π 3).

cos t=x sin t=y

t t (x,y) y x.

cos t= x 1 =x

sin t= y 1 =y

(x,y) x, t. y,t. 1,t. t,

sin(t)= opposite hypotenuse cos(t)= adjacent hypotenuse tan(t)= opposite adjacent cos α.

cos(α)= adjacent hypotenuse = 15 17

sin t.7 25sin α,cos α,tan α,sec α,csc α, cot α.

sin α= opposite α hypotenuse = 4 5 cos α= adjacent to α hypotenuse = 3 5 tan α= opposite α adjacent to α = 4 3 sec α= hypotenuse adjacent to α = 5 3 csc α= hypotenuse opposite α = 5 4 cot α= adjacent to α opposite α = 3 4

sin t,cos t,tan t,sec t,csc t, cot t.

sin t= 33 65 ,cos t= 56 65 ,tan t= 33 56 , sec t= 65 56 ,csc t= 65 33 ,cot t= 56 33 30°,60°, 45°, 0° and 90°.

30°,60°,90° π 6 , π 3 , π 2 s, 3 s,2s. 45°,45°,90° π 4 , π 4 , π 2 s,s, 2 s. π 3 ,

sin(π 3)= opp hyp = 3 s 2s = 3 2 cos(π 3)= adj hyp = s 2s = 1 2 tan(π 3)= opp adj = 3 s s = 3 sec(π 3)= hyp adj = 2s s =2 csc(π 3)= hyp opp = 2s 3 s = 2 3 = 2 3 3 cot(π 3)= adj opp = s 3 s = 1 3 = 3 3

π 4 ,sin(π 4)= 2 2 ,cos(π 4)= 2 2 ,tan(π 4)=1,sec(π 4)= 2 ,csc(π 4)= 2 ,cot(π 4)=1 π 6 π 3 , π 3 , 3 2 , π 6 , π 6 , 1 2 , π 3 .

sin π 3 =cos π 6 = 3 s 2s = 3 2 sin π 6 =cos π 3 = s 2s = 1 2

π 3 π 6 π 3 π 6 , sin(π 3) cos(π 6) 3 s 2s. cos(π 3) sin(π 6) s 2s. π 6 π 3 π, π 2 , π 2 .

π 2 π 12 5π 12 , 5π 12 π 12 . t,cos t= 5 13 , sin(π 2 −t)= 5 13

cos t=sin(π 2 −t)
sin t=cos(π 2 −t)
tan t=cot(π 2 −t)
cot t=tan(π 2 −t)
sec t=csc(π 2 −t)
csc t=sec(π 2 −t)

sin t= 5 12 , (cos π 2 −t).

sin t=cos(π 2 −t).
cos(π 2 −t)= 5 12 .

csc(π 6)=2, sec(π 3).

tan(30°)= 7 a

a.

a= 7 tan(30°) =12.1
sin(30°)= 7 c

c.

c= 7 sin(30°) =14

π 3 adjacent=10; opposite=10 3 π 657° 57° 57°, h

tan θ= opposite adjacent tan(57°)= h 30 Solve for h. h=30tan(57°) Multiply. h≈46.2 Use a calculator.

5π 12

cos t=sin(π 2 −t) sin t=cos(π 2 −t) tan t=cot(π 2 −t) cot t=tan(π 2 −t) sec t=csc(π 2 −t) csc t=sec(π 2 −t)

cos(34°)=sin(__°)cos(π 3)=sin(___)π 6csc(21°)=sec(___°)tan(π 4)=cot(__)π 4 a A, b B, c
cos B= 4 5 ,a=10sin B= 1 2 , a=20b= 20 3 3 ,c= 40 3 3tan A= 5 12 ,b=6tan A=100,b=100
a=10,000,c=10,000.5sin B= 1 3 , a=2a=5, ∡ A= 60 ∘b= 5 3 3 ,c= 10 3 3c=12, ∡ A= 45 ∘ A.sin A5 29 29
cos Atan A5 2csc Asec A29 2cot A A.sin A5 41 41cos Atan A5 4csc Asec A41 4cot Ac=14, b=7 3
a=15, b=15b=9.9970, c=12.2041a=2.0838, b=11.8177b=15, ∡B= 15 ∘a=55.9808,c=57.9555
c=200, ∡B= 5 ∘c=50, ∡B= 21 ∘a=46.6790,b=17.9184a=30, ∡A= 27 ∘b=3.5, ∡A= 78 ∘
a=16.4662,c=16.8341 x. x. x. x. 36°, 23°. 43°, 31°. 15°, 2°. 18°, 3°. 40°. 43°. 36°. 38°. 80°. 80°. 60°,
π 4 45°− 5π 3− 7π 6 3π 5 420°60°−80° 2π− 20π 112π 1114π 55π 4− π 3 sin π 3 .3 2 cos π 4 . cos π.
300°. 3π 4 .π 4 330°. 5π 4 .− 2 2[−1,1]cos π 6tan π 4csc π 3sec π 42sec 11π 3sec 315°2 sec(t)=−2.5
sec(−t)? tan(t)=−0.6, tan(−t)? tan(t)= 1 3 , tan(t−π). cos(t)= 2 2 , sin(t+2π).2 2 − 2 2cos π 4cot π 33 3

tan π 6cos(π 2)=sin(__°)csc(18°)=sec(__°)cos B= 3 5 ,a=6b=8,c=10tan A= 5 9 ,b=6sin A11 157 157
tan Ba=4, b=4 70°. 5π 6 150° −620° 30°. 5π 4 0° 360° 375°.15° 2π − 4π 7 . 315° − π 6 2π 75
sin π 6 . 240°.− 3 2[−1,1] cot π 4 . tan π 3 .3 csc 7π 4 . tan 210°.3 3 csc t=0.68, csc(−t)? cos t= 3 2 ,
cos(t−2π).3 2 cos(π 6)=sin(___)π 3 ABC:sin B= 3 4 ,c=12a= 9 2 ,b= 9 3 2x0π 6π 4π 3π 22π 33π 45π 6
πsin(x)01 2

2 2

3 2

1

3 2

2 2

1 20 π, π 2π, x0π 6π 4π 3π 22π 33π 45π 6πcos(x)1

3 2

2 2

1 2

0

− 1 2

− 2 2

− 3 2

−1 [−1,1]. 2π, 2π. f(x+P)=f(x) x f. P>0 sin(−x)=−sin x. cos(−x)=cos x. 2π. (−∞,∞) [−1,1].
y=sin x y=cos x y-

y=Asin(Bx−C)+D and y=Acos(Bx−C)+D

B P= 2π | B | . | B |>1, 2π | B |<1, 2π f(x)=sin(x), B=1, 2π, f(x)=sin(2x), B=2, π f(x)=sin(x 2),
B= 1 2 , 4π | B |. C=0 D=0

y=Asin(Bx)

y=Acos(Bx)

2π | B | . f(x)=sin(π 6 x). y=Asin(Bx). B= π 6 ,

P= 2π |B| = 2π π 6 =2π· 6 π =12

g(x)=cos(x 3). 6π B A A | A | | A | x=D; D=0 | A |>1, f(x)=4 sin x

f(x)=2 sin x.

| A |<1, C=0 D=0

y=Asin(Bx) and y=Acos(Bx)

A, | A |.

| A | = amplitude = 1 2 | maximum − minimum |

f(x)=−4sin(x)? y=Asin(Bx). A=−4, | A |=| −4 |=4. A f(x)= 1 2 sin(x)? 1 2 A B C D.
y=Asin(Bx−C)+D and y=Acos(Bx−C)+D or y=Asin(B(x− C B))+D and y=Acos(B(x− C B))+D
C B C>0, C<0, | C |, f(x)=sin(x−π) π f(x)=sin(x− π 4), π 4 C D y=cos(x)+D y=D. D f(x)=sin x
f(x)=sin x+2, f(x)=Asin(Bx−C)+D f(x)=Acos(Bx−C)+D, C B D f(x)=sin(x+ π 6)−2.
y=Asin(Bx−C)+D. B=1 C=− π 6 .

C B =− π 6 1 =− π 6

π 6 C. f(x)=sin(x+ π 6)−2 f(x)=sin(x−(− π 6))−2. C f(x)=3cos(x− π 2).π 2 ; f(x)=cos(x)−3.
y=Acos(Bx−C)+D. D=−3 f(x)=3sin(x)+2. f(x)=Asin(Bx−C)+D, | A |. P= 2π | B | . C B . y=D.
y=3sin(2x)+1. y=Asin(Bx−C)+D.A=3, | A |=3. B=2, P= 2π | B | = 2π 2 =π. C=0 C B = 0 2 =0. D=1,
y=1. π, y=1, y= 1 2 cos(x 3 − π 3). y=0; | A |= 1 2 ; P= 2π | B | =6π; C B =π

y=Asin(Bx−C)+D y=Acos(Bx−C)+D

x=0, (0,0). x=0, y=0.5. D D=0.5. | A |=0.5. | A |= 1 2 =0.5. A=−0.5. B=1; C=0.

g(x)=−0.5cos(x)+0.5

f(x)=sin(x)+2 −5, −2. D=−2. | A |=3. x=1 x=7,P= 2π | B | =6. B,

B= 2π P = 2π 6 = π 3

y=3sin(π 3 x−C)−2 y=3cos(π 3 x−C)−2.

y=3cos(π 3 x− π 3)−2 or y=−3cos(π 3 x+ 2π 3)−2

y=4sin(π 5 x− π 5)+4 y=−4sin(π 5 x+ 4π 5)+4

y=Asin(Bx−C)+D and y=Acos(Bx−C)+D,

C=0 D=0 y=Asin(Bx), | A |. P= 2π | B | . A A x= π 2| B | A>0 A<0, y=A. x= π | B | . A>0 A<0 x= 3π 2| B | y=−A. x= π 2| B | . f(x)=−2sin(πx 2). y=Asin(Bx). A=−2,

| A |=2

B= π 2 ,

P= 2π π 2 =2π· 2 π =4

A x=0, x=2 x=4. x=1 x=3. x=1, x=3. g(x)=−0.8cos(2x). y=0; | A |=0.8; P= 2π | B | =π; C B =0 y=Asin(Bx−C)+D or y=Acos(Bx−C)+D. | A |. P= 2π | B | . C B . f(x)=Asin(Bx) C B D. f(x)=3sin(π 4 x− π 4). f(x)=3sin(π 4 x− π 4). | A |=| 3 |=3. | B |=| π 4 |= π 4 ,

P= 2π | B | = 2π π 4 =2π· 4 π =8

C= π 4 ,

C B = π 4 π 4 =1.

g(x)=−2cos(π 3 x+ π 6). y=0; | A |=2; P= 2π | B | =6; C B =− 1 2 y=−2cos(π 2 x+π)+3,

y=Acos(Bx−C)+D

A=−2, | A |=2. | B |= π 2 , P= 2π | B | = 2π π 2 =2π· 2 π =4. C=−π, C B = −π, π 2 =−π· 2 π =−2. −2. D=3, y=3, A y=r sin(x), y(x)=3 sin(x). 2π; (3,0) x=2π,4π,6π,.... –3 3, 3. f(x)=7cos(x)? P

y=−3cos(x)+4

y –1 x=0) –7 x=π) y x. y x.y=3cos(x)−4 67.5+2=69.5 67.5, A=67.5 69.5, D=69.5 30, B= 2π 30 = π 15 −cos(t)

y=−67.5cos(π 15 t)+69.5

t y f(x)=Asin(Bx−C)+D f(x)=Acos(Bx−C)+D 2π.sin x cos x P= 2π | B | . | A | | A |>1, | A |<1, C B D f(x+P)=f(x) P. P y=sin x y=cos x? y=sin x y=cos x. A cos(Bx+C)+D, A D y=A sin(Bx+C)+D? f(t)=sin t? t sin t x>0. f(x)=2sin xf(x)= 2 3 cos x 2 3 ; 2π; y=0; y= 2 3 x=0; y=− 2 3 x=π; 2πf(x)=−3sin xf(x)=4sin x 2π; y=0; y=4 x= π 2 ; y=−4 x= 3π 2 ; x=0 x=2πf(x)=2cos x f(x)=cos(2x) π; y=0; y=1 x=π; y=−1 x= π 2 ; x=0 x=πf(x)=2 sin(1 2 x)f(x)=4 cos(πx) y=0; y=4 x=0; y=−4 x=1f(x)=3 cos(6 5 x)y=3 sin(8(x+4))+5 π 4 ; y=5; y=8 x=0.12; y=2 x=0.516; −4; x=0 x= π 4y=2 sin(3x−21)+4y=5 sin(5x+20)−2 2π 5 ; y=−2; y=3 x=0.08; y=−7 x=0.71; −4; −2; x=0 x= 2π 5 x=0. x>0. f(t)=2sin(t− 5π 6)f(t)=−cos(t+ π 3)+1 2π; y=1; y=2 x=2.09; y=2 t=2.09; y=0 t=5.24; − π 3 ; t=0 t=2πf(t)=4cos(2(t+ π 4))−3f(t)=−sin(1 2 t+ 5π 3) 4π; y=0; y=1 t=11.52; y=−1 t=5.24; − 10π 3 ; f(x)=4sin(π 2 (x−3))+7 y=−3; f(x)=2sin(π 2 x)−3 y=3; f(x)=−2cos(2π 5 x)+3 y=0; f(x)=−4cos(π(x− π 2)) y=1; f(x)=2cos(πx)+1 f(x)=sin x. [0,2π), f(x)=0. [0,2π), f(x)= 1 2 . π 6 , 5π 6 f(π 2). [0,2π),f(x)= 2 2 . x.π 4 , 3π 4 [0,2π), [0,2π),3π 2 f(−x)=−f(x). f(x)=sin x f(x)=cos x. [0,2π), f(x)=cos x=0.π 2 , 3π 2 [0,2π), f(x)= 1 2 . [0,2π), f(x)=cos x.π 2 , 3π 2 [0,2π), [0,2π), f(x)= 3 2 .π 6 , 11π 6 h(x)=x+sin x [0,2π]. h(x)=x+sin x [−100,100]. x. f(x)=x sin x [0,2π] f(x)=sin x. f(x)=x sin x [−10,10] f(x)= sin x x [−5π,5π] h(t) h(t). h(t). y=13.5; h(t)=12.5sin(π 5 (t−2.5))+13.5; A y=D, D f(x) f(x+P)=f(x) P x C B f(x)=Asin(Bx−C)+D f(x)=Acos(Bx−C)+D

tan x= sin x cos x

π kπ k − π 2 π 2 ,

tan(−x)= sin(−x) cos(−x) Definition of tangent. = −sin x cos x

Sine is an odd function, cosine is even. =− sin x cos x

The quotient of an odd and an even function is odd. =−tan x Definition of tangent.

x− π 2− π 3− π 4− π 6π 6π 4π 3π 2tan(x)− 3− 3 33 33 π 3 <x< π 2 , π 3 ≈1.05 π 2 ≈1.57, x 1.05<x<1.57 xtan x x π 2 , y=tan x xtan x x − π 2 , x cos x=0. cos(π 2)=0 cos(3π 2)=0. y=tan x x= π 2 and 3π 2 . y=tan x. π 2 π 3π 2 ,

y=Atan(Bx)

A B. A. | A |. P= π | B | . x, x≠ π 2| B | + π | B | k k (−∞,∞). x= π 2| B | + π | B | k, k y=Atan(Bx)
f(x)=Atan(Bx). (− P 2 , P 2) ± P 2 P= π B . (− π 2 , π 2), x=− π 2 , x= π 2 .

f(P 4)=Atan(B P 4)=Atan(B π 4B)=A

tan(π 4)=1. f(x)=Atan(Bx), | A |. B P= π | B | . x=− P 2 x= P 2 . A>0, A<0 (P 4 ,A), (0,0),
(− P 4 ,−A), y=0.5tan(π 2 x). A B. A=0.5 B= π 2 , π π 2 =2, x=±1.

f(0.5)=0.5tan(0.5π 2) =0.5tan(π 4) =0.5

(0.5,0.5),(0,0), (−0.5,−0.5). f(x)=3tan(π 6 x). C D

f(x)=Atan(Bx−C)+D

tan x | A |. π | B | . x≠ C B + π | B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π 2| B | k, k y=A tan(Bx)
y=Atan(Bx−C)+D, y=Atan(Bx−C)+D. | A |. B P= π | B | . C C B . y=Atan(Bx) C B D.
x= C B + π 2| B | k, k y=−2tan(πx+π) −1. y=Atan(Bx−C)+D. A=−2, | A |=2. B=π, P= π | B | = π π =1.
C=−π, C B = −π π =−1. x=− 3 2 x=− 1 2 (−1.25,1), (−1,−1), (−0.75,−3). A<0. A=2 −2? y=−1, P
f(x)=Atan(π P x). (x,f(x)) A. P=8. P= π | B | , B= π P = π 8 .f(x)=Atan(π 8 x). A, (2,2).

2=Atan(π 8 ⋅2)=Atan(π 4)

tan(π 4)=1, A=2. f(x)=2tan(π 8 x).g(x)=4tan(2x) sec x= 1 cos x . π 2 , 3π 2 , y=sec x x
sec(−x)=sec x. f(x)=secx= 1 cosx | A | | A |. 2π | B | . x≠ π 2| B | k, k (−∞,−| A |]∪[| A |,∞).
x= π 2| B | k, k y=Asec(Bx) csc x= 1 sin x . 0, π, y=csc x x csc(−x)=−cscx. f(x)=cscx= 1 sinx | A |.
2π | B | . x≠ π | B | k, k (−∞,−| A |]∪[| A |,∞). x= π | B | k, k y=Acsc(Bx)

y=Asec(Bx−C)+D

y=Acsc(Bx−C)+D

| A |. 2π | B | . x≠ C B + π 2| B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π 2| B | k, k y=Asec(Bx) | A |.
2π | B | . x≠ C B + π 2| B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π |B| k, k y=Acsc(Bx) y=Asec(Bx),
y=Asec(Bx). | A |. B P= 2π | B | . y=Acos(Bx). y=cos x y=sec x y=Asec(Bx). f(x)=2.5sec(0.4x).
y=Asec(Bx). A=2.5 2.5. B=0.4 P= 2π 0.4 =5π. 5π g(x)=2.5cos(0.4x). x=1.25π x=3.75π. (0,2.5)
(2.5π,−2.5). f(x)=−2.5sec(0.4x). A f(x)=Asec(Bx−C)+D (−∞,−| A |+D]∪[| A |+D,∞).
f(x)=Asec(Bx−C)+D, y=A sec(Bx−C)+D. | A |. B 2π | B | . C C B . y=A sec(Bx) . C B D.
x= C B + π 2| B | k, k y=4sec(π 3 x− π 2)+1. y=4sec(π 3 x− π 2)+1. | A |=4.

2π |B| = 2π π 3 = 2π 1 ⋅ 3 π =6

C B = π 2 π 3 = π 2 ⋅ 3 π =1.5

y=Asec(Bx), C B =1.5 D=6. x=0,x=3, x=6. (1.5,5) (4.5,−3). f(x)=−6sec(4x+2)−8. csc x x x≠kπ k.
y=Acsc(Bx−C)+D be x≠ C+kπ B ? y=Acsc(Bx), y=Acsc(Bx). | A |. B P= 2π | B | . y=Asin(Bx).
y=sin x y=csc x y=Acsc(Bx). f(x)=−3csc(4x). y=Acsc(Bx). | A |=| −3 |=3, B=4, P= 2π 4 = π 2 . π 2
g(x)=−3sin(4x). x=0, x= π 4 , x= π 2 . (π 8 ,−3) (3π 8 ,3). f(x)=0.5csc(2x). f(x)=Acsc(Bx−C)+D,
y=Acsc(Bx−C)+D. | A |. B 2π | B | . C C B . y=Acsc(Bx) D. x= C B + π | B | k, k y=2csc(π 2 x)+1.
y=2csc(π 2 x)+1. | A |=2. 2π | B | = 2π π 2 = 2π 1 ⋅ 2 π =4. 0 π 2 =0. y=Acsc(Bx) D=1. x=0,x=2,x=4.
f(x)=2sin(π 2 x)+1, f(x)=2cos(π 2 x)+1 g(x)=2sec(π 2 x)+1 cot x= 1 tan x . 0,π, y=cot x x tan x=0;
cot x x tan x=0, cot x=0 x tan x | A |. P= π | B | . x≠ π | B | k, k (−∞,∞). x= π | B | k, k y=Acot(Bx)

y=Acot(Bx−C)+D

| A |. π | B | . x≠ C B + π | B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π | B | k, k y=Acot(Bx)
f(x)=Acot(Bx), f(x)=Acot(Bx). | A |. P= π | B | . y=Atan(Bx). y=Acot(Bx). y=3cot(4x),
f(x)=Acot(Bx) f(x)=3cot(4x). | A |=3. P= π 4 . y=3tan(4x). (π 16 ,3) (3π 16 ,−3). y=3cot(4x).
x=0, x= π 4 . y=3tan(4x) y=3cot(4x). f(x)=Acot(Bx−C)+D, f(x)=Acot(Bx−C)+D. | A |.
P= π | B | . C B . y=Atan(Bx) C B D. x= C B + π | B | k, k f(x)=4cot(π 8 x− π 2)−2.
f(x)=Acot(Bx−C)+D. A=4, B= π 8 , P= π | B | = π π 8 =8. C= π 2 , C B = π 2 π 8 =4.
f(x)=4tan(π 8 x− π 2)−2. (6,2),(8,−2), (10,−6). f(x)=4cot(π 8 x− π 2)−2. x=4 x=12. y=5tan(π 4 t)
t y [0,5]. f(1) y=Atan(Bt) | A | π B π π 4 = π 1 ⋅ 4 π =4. t=2 f(1)=5tan(π 4 (1))=5(1)=5;
y=A tan(Bx−C)+Dy=A sec(Bx−C)+Dy=A csc(Bx−C)+Dy=A cot(Bx−C)+D π.
f(x)=Atan(Bx−C)+D 2π. f(x)=Asec(Bx−C)+D f(x)=Acsc(Bx−C)+D π 0,±π,±2π,.... (−∞,∞),

± π 2 ,± 3π 2 ,....f(x)=Acot(Bx−C)+D y=csc x. y=csc x y=sin x, y=sin x y=csc x. y=sin x y=csc x.
y=cos x y=sec x? tan x π. tan(x+π)=tan x. y=csc x? y=csc x y=sin x? 2π.f(x)=tan xf(x)=sec x
f(x)=csc xf(x)=cot xf(x)=2tan(4x−32)h(x)=2sec(π 4 (x+1))m(x)=6csc(π 3 x+π) tan x=−1.5,
tan(−x). sec x=2, sec(−x). csc x=−5, csc(−x). xsin x=2, (−x)sin(−x). x cot(−x)cos(−x)+sin(−x)
−cotxcosx−sinxcos(−x)+tan(−x)sin(−x)f(x)=2tan(4x−32) π 4 ;
x= 1 4 (π 2 +πk)+8, where k is an integer h(x)=2sec(π 4 (x+1)) m(x)=6csc(π 3 x+π)
x=3k, where k is an integerj(x)=tan(π 2 x)p(x)=tan(x− π 2) π; x=πk, where k is an integer
f(x)=4tan(x)f(x)=tan(x+ π 4) π; x= π 4 +πk, where k is an integerf(x)=πtan(πx−π)−πf(x)=2csc(x)
2π; x=πk, where k is an integerf(x)=− 1 4 csc(x)f(x)=4sec(3x) 2π 3 ;
x= π 6 k, where k is an odd integerf(x)=−3cot(2x)f(x)=7sec(5x) 2π 5 ;
x= π 10 k, where k is an odd integerf(x)= 9 10 csc(πx)f(x)=2csc(x+ π 4)−1 2π;
x=− π 4 +πk, where k is an integerf(x)=−sec(x− π 3)−2f(x)= 7 5 csc(x− π 4) 7 5 ; 2π;
x= π 4 +πk, where k is an integerf(x)=5(cot(x+ π 2)−3) | A |, A=1, π 3 ; (h, k)=(π 4 ,2)
y=tan(3(x− π 4))+2 A=−2, π 4 , (h, k)=(− π 4 , −2)f(x)=csc(2x)f(x)=csc(4x)f(x)=2cscx
f(x)= 1 2 tan(100πx) csc x 1 sin x .f(x)=| csc(x) |f(x)=| cot(x) |f(x)= 2 csc(x)f(x)= csc(x) sec(x)
f(x)=1+ sec 2 (x)− tan 2 (x). f(x)=sec(0.001x)f(x)=cot(100πx)f(x)= sin 2 x+ cos 2 x
f(x)=20tan(π 10 x) x, f(x), [0, 5]. f(1) f(2.5) x, x d(x), d(x)=1.5sec(x). d(x)? d(x) d(x).
d(− π 3). d(π 6). (− π 2 , π 2); x=− π 2 x= π 2 ; | x | π 2 x=− π 3 , x= π 6 , x=0 g(x), x x
g(x)=250,000csc(π 30 x). g(x) [0, 35]. g(5) x π 120 x. h(x), x h(x) (0, 60). h(0) h(30).
h(x) x h(x)=2tan(π 120 x);h(0)=0: h(30)=2: x h(x) f(x)=sin x, f −1 (x)= sin −1 x. sin −1 x
1 sinx . sin(π 6)= 1 2 , π 6 = sin −1 (1 2). cos(π)=−1, π= cos −1 (−1). tan(π 4)=1,
π 4 = tan −1 (1). f(a)=b, f −1 (b)=a. [− π 2 , π 2] [0,π]. [− π 2 , π 2]; [0,π] (− π 2 , π 2).
(− π 2 , π 2) y= sin −1 x x=sin y. arcsinx.

y= sin −1 x has domain [−1,1] and range [− π 2 , π 2]

y= cos −1 x x=cos y. arccos x.

y= cos −1 x has domain [−1,1] and range [0,π]

y= tan −1 x x=tan y. arctan x.

y= tan −1 x has domain (−∞,∞) and range (− π 2 , π 2)

sin −1 x [−1,1] [− π 2 , π 2],cos −1 x [−1,1] [0,π], tan −1 x (− π 2 , π 2). y=x. [− π 2 , π 2],
sin y=x, sin −1 x=y. [0,π], cos y=x, cos −1 x=y. (− π 2 , π 2), tan y=x, tan −1 x=y.
sin(5π 12)≈0.96593, sin y=x, sin −1 x=y x=0.96593, y= 5π 12 .

sin −1 (0.96593)≈ 5π 12

cos(0.5)≈0.8776,arccos(0.8776)≈0.5 π 6 π 4 π 3 x x y x,sin −1 (1 2)sin −1 (− 2 2)cos −1 (− 3 2)
tan −1 (1) sin −1 (1 2) 1 2 . x sin(x)= 1 2 ? π 6 5π 6 , [− π 2 , π 2], sin −1 (1 2)= π 6 .
sin −1 (− 2 2), 5π 4 7π 4 − 2 2 , [− π 2 , π 2]. 7π 4 :sin −1 (− 2 2)=− π 4 . cos −1 (− 3 2),
[0,π] − 3 2 . cos −1 (− 3 2)= 5π 6 . tan −1 (1), (− π 2 , π 2) tan −1 (1)= π 4 .sin −1 (−1)
tan −1 (−1)cos −1 (−1)cos −1 (1 2) − π 2 ; − π 4 ; π; π 3 θ sin −1 (0.97) sin −1 (0.97)≈1.3252.
sin −1 (0.97)≈75.93°. cos −1 (−0.4) h a θ= cos −1 (a h). h p θ= sin −1 (p h). θ= tan −1 (p a). θ.

cos θ= 9 12 θ= cos −1 (9 12) Apply definition of the inverse. θ≈0.7227 or about 41.4096°

Evaluate.

θ.sin −1 (0.6)=36.87°=0.6435 f(x) g(x) { sin(x),cos(x),tan(x) } f −1 (y) g −1 (y) f(f −1 (y))=y y f
f −1 . f −1 (f(x)).

sin(sin −1 x)=x for −1≤x≤1 cos(cos −1 x)=x for −1≤x≤1 tan(tan −1 x)=x for −∞<x<∞

sin −1 (sin x)=x only for − π 2 ≤x≤ π 2 cos −1 (cos x)=x only for 0≤x≤π tan −1 (tan x)=x only for − π 2 <x< π 2

sin −1 (sin x)=x? x [− π 2 , π 2], x [− π 2 , π 2]. sin −1 (sin(3π 4))= π 4 .
f(θ)=sin θ, cos θ, or tan θ, θ f, then f −1 (f(θ))=θ. φ f f(φ)=f(θ). f −1 (f(θ))=φ.sin −1 (sin(π 3))
sin −1 (sin(2π 3))cos −1 (cos(2π 3))cos −1 (cos(− π 3))π 3 is in [− π 2 , π 2],

sin −1 (sin(π 3))= π 3 .2π 3 is not in [− π 2 , π 2], sin(2π 3)=sin(π 3), sin −1 (sin(2π 3))= π 3 . 2π 3 is in [0,π], cos −1 (cos(2π 3))= 2π 3 .− π 3 is not in [0,π], cos(− π 3)=cos(π 3) π 3 is in [0,π], cos −1 (cos(− π 3))= π 3 . tan −1 (tan(π 8)) and tan −1 (tan(11π 9)).π 8 ; 2π 9 f −1 (g(x)). x, θ, π 2 −θ. cos θ= b c =sin(π 2 −θ), sin −1 (cos θ)= π 2 −θ 0≤θ≤π. θ θ π 2 . sin θ= a c =cos(π 2 −θ), cos −1 (sin θ)= π 2 −θ − π 2 ≤θ≤ π 2 . sin −1 (cos x) cos −1 (sin x), x is in [0,π], sin −1 (cos x)= π 2 −x. x is not in [0,π], y in [0,π] cos y=cos x.

sin −1 (cos x)= π 2 −y

x is in [− π 2 , π 2], cos −1 (sin x)= π 2 −x. x is not in [− π 2 , π 2], y in [− π 2 , π 2] sin y=sin x.

cos −1 (sin x)= π 2 −y

sin −1 (cos(13π 6))

cos(13π 6)=cos(π 6 +2π) =cos(π 6) = 3 2

sin −1 (3 2)= π 3

x= 13π 6 , y= π 6 ,

sin −1 (cos(13π 6))= π 2 − π 6 = π 3

cos −1 (sin(− 11π 4)).3π 4 f(g −1 (x)), f g x g −1 , sin(cos −1 x)= 1− x 2 . sin 2 x+ cos 2 x=1, sin(cos −1 (4 5)). θ= cos −1 (4 5), cos θ= 4 5 , sin θ.

sin 2 θ+ cos 2 θ=1 Use our known value for cosine. sin 2 θ+ (4 5) 2 =1 Solve for sine. sin 2 θ=1− 16 25 sin θ=± 9 25 =± 3 5

θ= cos −1 (4 5) sin θ 3 5 . cos θ= 4 5 , sin θ= 3 5 [0,π], sin(cos −1 (4 5))=sin θ= 3 5 . cos(tan −1 (5 12)).12 13 sin(tan −1 (7 4)). tan θ= 7 4 .

4 2 + 7 2 = hypotenuse 2 hypotenuse= 65

sin θ= 7 65

sin(tan −1 (7 4))=sin θ = 7 65 = 7 65 65

cos(sin −1 (7 9)).4 2 9 cos(sin −1 (x 3)) −3≤x≤3. θ sin θ= x 3 .

sin 2 θ+ cos 2 θ=1 Use the Pythagorean Theorem. (x 3) 2 + cos 2 θ=1 Solve for cosine. cos 2 θ=1− x 2 9 cosθ=± 9− x 2 9 =± 9− x 2 3

[− π 2 , π 2],

cos(sin −1 (x 3))= 9− x 2 3

sin(tan −1 (4x)) − 1 4 ≤x≤ 1 4 .4x 16 x 2 +1 f(x), x= f −1 (y), f(x)=y. f(x)=y x= f −1 (y) x f. π 4 = tan −1 (1) and π 6 = sin −1 (1 2). sin(cos −1 (x))= 1− x 2 . sin −1 (cos x)= π 2 −x 0≤x≤π cos −1 (sin x)= π 2 −x − π 2 ≤x≤ π 2 . f(x)= sin −1 x g(x)= cos −1 x y=sinx [− π 2 , π 2]; y=sinx, f(x)= sin −1 x. y=cosx [0,π]; y=cosx,f(x)= cos −1 x. y=cos x y= cos −1 x cos −1 (cos(− π 6)) − π 6 ? π 6 =arcsin(0.5).π 6 − π 2 π 2 sec −1 (2). sin x, [− π 2 , π 2] [− π 2 , π 2] arccos(cos x)=x x.arccos(−x)=π−arccos x. θ 1 arccos(−x) x>0 θ 2 θ 2 arccosxx>0 θ 2 θ 1 θ 2 =π− θ 1 arccos(−x) π−arccosxsin −1 (2 2)sin −1 (− 1 2)− π 6cos −1 (1 2)cos −1 (− 2 2)3π 4 tan −1 (1)tan −1 (− 3)− π 3tan −1 (−1)tan −1 (3)π 3tan −1 (−1 3)cos −1 (−0.4)arcsin(0.23) arccos(3 5)cos −1 (0.8)tan −1 (6) θ sin −1 (cos(π))tan −1 (sin(π))cos −1 (sin(π 3)) tan −1 (sin(π 3))sin −1 (cos(−π 2))tan −1 (sin(4π 3))sin −1 (sin(5π 6))tan −1 (sin(−5π 2)) − π 4cos(sin −1 (4 5))sin(cos −1 (3 5))sin(tan −1 (4 3))cos(tan −1 (12 5))5 13 cos(sin −1 (1 2)) x tan(sin −1 (x−1))x−1 − x 2 +2xsin(cos −1 (1−x))cos(sin −1 (1 x))x 2 −1 x cos(tan −1 (3x−1))tan(sin −1 (x+ 1 2))x+0.5 − x 2 −x+ 3 4 sin −1 (1 2)− cos −1 (2 2)+ sin −1 (3 2)− cos −1 (1) cos −1 (3 2)− sin −1 (2 2)+ cos −1 (1 2)− sin −1 (0)

sin t= x x+1 .cos t2x+1 x+1sec tcot t2x+1 xcos(sin −1 (x x+1))tan −1 (x 2x+1)t y= sin −1 x y=arccos x [−1,1]; [0,π] y= tan −1 x x sin x= sin −1 x? x=0.00 x cos x= cos −1 x? arctan(10,000). y= 3 5 x y= −3 7 x f(x)=−3cos x+3 2π; y=3; f(x)= 1 4 sin xf(x)=3cos(x+ π 6) 2π; y=0; f(x)=−2sin(x− 2π 3)f(x)=3sin(x− π 4)−4 2π; y=−4; f(x)=2(cos(x− 4π 3)+1) f(x)=6sin(3x− π 6)−1 2π 3 ; y=−1; f(x)=−100sin(50x−20)f(x)=tan x−4 π; y=−4; x= π 2 +πk, k f(x)=2tan(x− π 6)f(x)=−3tan(4x)−2 π 4 ; y=−2; x= π 8 + π 4 k, k f(x)=0.2cos(0.1x)+0.3

f(x)= 1 3 sec x2π; x= π 2 k, k f(x)=3cot xf(x)=4csc(5x) 2π 5 ; x= π 5 k, k f(x)=8sec(1 4 x)
f(x)= 2 3 csc(1 2 x) 4π; x=2πk, k f(x)=−csc(2x+π) y=12,000+8,000sin(0.628x), [0,40]sin −1 (1)
cos −1 (3 2)π 6tan −1 (−1)cos −1 (1 2)π 4sin −1 (− 3 2)sin −1 (cos(π 6))π 3cos −1 (tan(3π 4))
sin(sec −1 (3 5))cot(sin −1 (3 5))tan(cos −1 (5 13))12 5sin(cos −1 (x x+1)) f(x)=cos x
f(x)=sec x [0,2π) y=x. y f(x)=sin x f(x)=csc x f (x)= x 1 − x 3 3! + x 5 5! − x 7 7! [−1,1]
f(x)=sin x f(x)=0.5sin x 2π; y=0 f(x)=5cos xf(x)=5sin x 2π; y=0f(x)=sin(3x)
f(x)=−cos(x+ π 3)+1 2π; y=1f(x)=5sin(3(x− π 6))+4f(x)=3cos(1 3 x− 5π 6) 6π; y=0
f(x)=tan(4x)f(x)=−2tan(x− 7π 6)+2 π; y=0, x= 2π 3 +πk, k f(x)=πcos(3x+π)f(x)=5csc(3x)
2π 3 ; y=0, x= π 3 k, k f(x)=πsec(π 2 x)f(x)=2csc(x+ π 4)−3 2π; y=−3 y=0;f(x)=2sin(π(x−1))
y=sin(π 6 x+π)−3 −6; y=−3y=8sin(7π 6 x+ 7π 2)+6 t D, t.D(t)=68−12sin(π 12 x)
g(x)=3tan(6x+42) π 6 ; −7n(x)=4csc(5π 3 x− 20π 3)f(x)=sec(πx); tan x=3, tan(−x). sec x=4,
sec(−x).4 m(x)=sin(2x)+cos(3x) [−10,10] [−3,3]. n(x)=0.02sin(50πx) x:[0,1] [0,3]. 1 25 .
f(x)= sin x x [−0.5,0.5] f(x)= 3 5 cos(6x). f(x)?3 5 f(x)? [0,2π]?
(0.5,1),(1.6,2.1),(2.6,3.1),(3.7,4.2),(4.7,5.2),(5.6,6.28) π 3 , (h,k)=(π 4 ,2) π 6 ,
(h,k)=(− π 4 ,3)f(x)=2cos(12(x+ π 4))+3f(x)=5cos(3x)+4sin(2x)f(x)= e sint 2π.sin −1 (3 2)
tan −1 (3)π 3cos −1 (− 3 2)cos −1 (sin(π))π 2cos −1 (tan(7π 4))cos(sin −1 (1−2x))1− (1−2x) 2
cos −1 (−0.4)cos(tan −1 (x 2))1 1+ x 4 sin t= x x+1 .tan tcsc tx+1 x θ arcsin(sin(5π 6))= 5π 6
arccos(cos(5π 6))= 5π 6 arccos x= cos −1 x arcsin x= sin −1 x arctan x= tan −1 x cos −1 x, sin −1 x,
tan −1 x, sin 2 θ+ cos 2 θ=11+ cot 2 θ= csc 2 θ1+ tan 2 θ= sec 2 θ 1+ cot 2 θ= csc 2 θ
1+ cot 2 θ= csc 2 θ
1+ cot 2 θ=(1+ cos 2 θ sin 2 θ) Rewrite the left side. =(sin 2 θ sin 2 θ)+(cos 2 θ sin 2 θ)
Write both terms with the common denominator. = sin 2 θ+ cos 2 θ sin 2 θ = 1 sin 2
θ = csc 2 θ
1+ tan 2 θ= sec 2 θ
1+ tan 2 θ=1+ (sin θ cos θ) 2 Rewrite left side. = (cos θ cos θ) 2 + (sin θ cos θ) 2
Write both terms with the common denominator. = cos 2 θ+ sin 2 θ cos 2 θ = 1
cos 2 θ = sec 2 θ
tan(−θ)=−tan θ cot(−θ)=−cot θsin(−θ)=−sin θ csc(−θ)=−csc θcos(−θ)=cos θ sec(−θ)=sec θ
f(− x)= −f(x) x f. sin(−θ)=−sin θ. π 2 − π 2 . sin(π 2) sin(− π 2).
sin(π 2)=1 and sin(− π 2)=−sin(π 2) =−1
y=sin θ
f(−x)=f(x) for all x in the domain of f
cos(−θ)=cos θ. π 4 − π 4 . cos(π 4) cos(− π 4).
cos(− π 4)=cos(π 4) ≈0.707
y=cos θ θ sin(−θ)=−sin θ, cos(− θ)=cos θ, tan(− θ)=−tan θ.
tan(− θ)= sin(−θ) cos(− θ) = −sin θ cos θ =−tan θ. tan(−θ)=−tan(θ) θ cot(−θ)=−cot θ,
cot(−θ)= cos(−θ) sin(−θ) = cos θ −sin θ =−cot θ. cot(−θ)=−cot(θ) θ
csc(−θ)= 1 sin(−θ) = 1 −sin θ =−csc θ. sec(−θ)= 1 cos(−θ) = 1 cos θ =sec θ. sin θ= 1 csc θ
csc θ= 1 sin θcos θ= 1 sec θsec θ= 1 cos θtan θ= 1 cot θcot θ= 1 tan θtan θ= sin θ cos θcot θ= cos θ sin θ
cos 2 θ+ sin 2 θ=1
1+ cot 2 θ= csc 2 θ
1+ tan 2 θ= sec 2 θ
tan(−θ)=−tan θ
cot(−θ)=−cot θ
sin(−θ)=−sin θ
csc(−θ)=−csc θ
cos(−θ)=cos θ
sec(−θ)=sec θ

$\sin\theta = 1 \csc\theta$
$\cos\theta = 1 \sec\theta$
$\tan\theta = 1 \cot\theta$
$\csc\theta = 1 \sin\theta$
$\sec\theta = 1 \cos\theta$
$\cot\theta = 1 \tan\theta$
$\tan\theta = \sin\theta \cos\theta$
$\cot\theta = \cos\theta \sin\theta$

$\cot\theta = 1 \tan\theta$. $y=\cot\theta$ $y = 1 \tan\theta$. $\tan\theta\cos\theta=\sin\theta$.

$\tan\theta\cos\theta = (\sin\theta \cos\theta)\cos\theta$ $= (\sin\theta \cos\theta) \cos\theta$ $= \sin\theta$

$\tan\theta$ $\sin\theta$ $\cos\theta$. $\csc\theta\cos\theta\tan\theta=1$.

$\csc\theta\cos\theta\tan\theta = (1 \sin\theta)\cos\theta(\sin\theta \cos\theta)$ $= \cos\theta \sin\theta (\sin\theta \cos\theta)$ $= \sin\theta\cos\theta \sin\theta\cos\theta$ $=1$

$(1+\sin x)[1+\sin(-x)] = \cos 2 x$

$(1+\sin x)[1+\sin(-x)]=(1+\sin x)(1-\sin x)$ Since $\sin(-x)=-\sin x$ $=1-\sin 2 x$ Difference of squares $= \cos 2 x$ $\cos 2 x=1-\sin 2 x$

$\sec 2 \theta-1 \sec 2 \theta = \sin 2 \theta$

$\sec 2 \theta-1 \sec 2 \theta = (\tan 2 \theta+1)-1 \sec 2 \theta$ $\sec 2 \theta= \tan 2 \theta+1$ $= \tan 2 \theta \sec 2 \theta$ $= \tan 2 \theta(1 \sec 2 \theta)$ $= \tan 2 \theta(\cos 2 \theta)$ $\cos 2 \theta= 1 \sec 2 \theta$ $=(\sin 2 \theta \cos 2 \theta)(\cos 2 \theta)$ $\tan 2 \theta= \sin 2 \theta \cos 2 \theta$ $=(\sin 2 \theta \cos 2 \theta)(\cos 2 \theta)$ $= \sin 2 \theta$

$\sec 2 \theta-1 \sec 2 \theta = \sec 2 \theta \sec 2 \theta - 1 \sec 2 \theta$ $=1-\cos 2 \theta$ $= \sin 2 \theta$

$\sec 2 \theta= \tan 2 \theta+1$ $\cot\theta \csc\theta = \cos\theta$.

$\cot\theta \csc\theta = \cos\theta \sin\theta 1 \sin\theta$ $= \cos\theta \sin\theta \cdot \sin\theta 1$ $=\cos\theta$

$2\tan\theta\sec\theta$

$2 \tan\theta\sec\theta=2(\sin\theta \cos\theta)(1 \cos\theta)$ $= 2 \sin\theta \cos 2 \theta$ $= 2 \sin\theta 1-\sin 2 \theta$

Substitute $1-\sin 2 \theta$ for $\cos 2 \theta$

$2\tan\theta\sec\theta= 2 \sin\theta 1-\sin 2 \theta$

$\sin 2 (-\theta)-\cos 2 (-\theta) \sin(-\theta)-\cos(-\theta) =\cos\theta-\sin\theta$

$\sin 2 (-\theta)-\cos 2 (-\theta) \sin(-\theta)-\cos(-\theta) = [\sin(-\theta)] 2 - [\cos(-\theta)] 2 \sin(-\theta)-\cos(-\theta)$ $= (-\sin\theta) 2 - (\cos\theta) 2 -\sin\theta-\cos\theta$ $\sin(-x)=-\sin x$ and $\cos(-x)=\cos x$ $= (\sin\theta) 2 - (\cos\theta) 2 -\sin\theta-\cos\theta$ Difference of squares $= (\sin\theta-\cos\theta)(\sin\theta+\cos\theta) -(\sin\theta+\cos\theta)$ $= (\sin\theta -\cos\theta)(\sin\theta+\cos\theta) -(\sin\theta+\cos\theta)$ $=\cos\theta-\sin\theta$

$\sin 2 \theta-1 \tan\theta\sin\theta-\tan\theta = \sin\theta+1 \tan\theta$.

$\sin 2 \theta-1 \tan\theta\sin\theta-\tan\theta = (\sin\theta+1)(\sin\theta-1) \tan\theta(\sin\theta-1) = \sin\theta+1 \tan\theta$

$(1-\cos 2 x)(1+\cot 2 x)=1$.

$(1-\cos 2 x)(1+\cot 2 x)=(1-\cos 2 x)(1+\cos 2 x \sin 2 x)$ $=(1-\cos 2 x)(\sin 2 x \sin 2 x + \cos 2 x \sin 2 x)$ Find the common denominator. $=(1-\cos 2 x)(\sin 2 x+ \cos 2 x \sin 2 x)$ $=(\sin 2 x)(1 \sin 2 x)$ $=1$

$(\sin x+1)(\sin x-1)=0$ $(x+1)(x-1)=0$, $a 2 - b 2 =(a-b)(a+b)$, $2 \cos 2 \theta+\cos\theta-1$. $a x 2 +bx+c$. $\cos\theta=x$,

$2 x 2 +x-1$

$(2x+1)(x-1)$. x. x $\cos\theta$ θ. $4 \cos 2 \theta-1$.

$4 \cos 2 \theta-1= (2 \cos\theta) 2 -1$ $=(2 \cos\theta-1)(2 \cos\theta+1)$

$\cos\theta=x$, $4 x 2 -1$, $(2x-1)(2x+1)$. x $\cos\theta$ $25-9 \sin 2 \theta$. $25-9 \sin 2 \theta=(5-3 \sin\theta)(5+3 \sin\theta)$.

$\csc 2 \theta-\cot 2 \theta$

$1+\cot 2 \theta= \csc 2 \theta$

$1+\cot 2 \theta$ $\csc 2 \theta$.

$\csc 2 \theta-\cot 2 \theta=1+\cot 2 \theta-\cot 2 \theta$ $=1$

cos θ 1+sin θ = 1−sin θ cos θ . 1−sin θ.)

cos θ 1+sin θ (1−sin θ 1−sin θ)= cos θ(1−sin θ) 1− sin 2 θ = cos θ(1−sin θ) cos 2 θ = 1−sin θ cos θ

sin 2 θ+ cos 2 θ=1 1+ cot 2 θ= csc 2 θ 1+ tan 2 θ= sec 2 θ

tan(−θ)=−tan θ cot(−θ)=−cot θ sin(−θ)=−sin θ csc(−θ)=−csc θ cos(−θ)=cos θ sec(−θ)=sec θ

sin θ= 1 csc θ cos θ= 1 sec θ tan θ= 1 cot θ csc θ= 1 sin θ sec θ= 1 cos θ cot θ= 1 tan θ

tan θ= sin θ cos θ cot θ= cos θ sin θ

g(x)=cos x f(x)=sin x h(x)=tan x G(x)= cos 2 x,F(x)= sin 2 x, H(x)= tan 2 x? F,G, H,

F(−x)=sin(−x)sin(−x)=(−sin x)(−sin x)= sin 2 x=F(x),G(−x)=cos(−x)cos(−x)=cos xcos x= cos 2 x=G(x)

H(−x)=tan(−x)tan(−x)=(−tan x)(−tan x)= tan 2 x=H(x). f(x)=sec x [−π,π]. f(x)=sec x? sec t,

cos t=0, sec t= 1 0 , sin 2 t+ cos 2 t=1 sin x cos x sec xsin xsin(−x)cos(−x)csc(−x)tan xsin x+sec x cos 2 x sec xcsc x+cos xcot(−x)cot t+tan t sec(−t)csc t3 sin 3 t csc t+ cos 2 t+2 cos(−t)cos t−tan(−x)cot(−x)−1 −sin(−x)cos x sec x csc x tan x cot x1+ tan 2 θ csc 2 θ + sin 2 θ+ 1 sec 2 θsec 2 x

(tan x csc 2 x + tan x sec 2 x)(1+tan x 1+cot x)− 1 cos 2 x1− cos 2 x tan 2 x +2 sin 2 xsin 2 x+1 tan x+cot x csc x ; cos xsec x+csc x 1+tan x ; sin x1 sin xcos x 1+sin x +tan x; cos x

1 sin xcos x −cot x; cot x1 cot x1 1−cos x − cos x 1+cos x ; csc x

(sec x+csc x)(sin x+cos x)−2−cot x; tan xtan x1 csc x−sin x ; sec x and tan x

1−sin x 1+sin x − 1+sin x 1−sin x ; sec x and tan x−4sec xtan xtan x; sec xsec x; cot x± 1 cot 2 x +1 sec x; sin xcot x; sin x± 1− sin 2 x sin xcot x; csc xcos x− cos 3 x=cos x sin 2 x

cos x− cos 3 x=cos x(1− cos 2 x)=cosx sin 2 xcos x(tan x−sec(−x))=sin x−1

1+ sin 2 x cos 2 x = 1 cos 2 x + sin 2 x cos 2 x =1+2 tan 2 x

1+ sin 2 x cos 2 x = 1 cos 2 x + sin 2 x cos 2 x = sec 2 x+ tan 2 x= tan 2 x+1+ tan 2 x=1+2 tan 2 x

(sin x+cos x) 2 =1+2 sin xcos xcos 2 x− tan 2 x=2− sin 2 x− sec 2 x

cos 2 x− tan 2 x=1− sin 2 x−(sec 2 x−1)=1− sin 2 x− sec 2 x+1=2− sin 2 x− sec 2 x

1 1+cos x − 1 1−cos(−x) =−2 cot x csc xcsc 2 x(1+ sin 2 x)= cot 2 x

(sec 2 (−x)− tan 2 x tan x)(2+2 tan x 2+2 cot x)−2 sin 2 x=cos 2xtan x sec x sin(−x)= cos 2 x sec(−x) tan x+cot x =−sin(−x)1+sin x cos x = cos x 1+sin(−x)cos 2 θ− sin 2 θ 1− tan 2 θ = sin 2 θ

3 sin 2 θ+4 cos 2 θ=3+ cos 2 θ

3 sin 2 θ+4 cos 2 θ=3 sin 2 θ+3 cos 2 θ+ cos 2 θ=3(sin 2 θ+ cos 2 θ)+ cos 2 θ=3+ cos 2 θ

sec θ+tan θ cot θ+cos θ = sec 2 θ f(−x)=−f(x), f(−x)=f(x),cos(α+β)=cos α cos β−sin α sin β

cos(α−β)=cos α cos β+sin α sin β P α (cos α,sin α) Q β (cos β,sin β). POQ α−β. A (α−β) (cos(α−β),sin(α−β)); B (1,0). POQ AOB P Q A B. P Q

d PQ = (cos α−cos β) 2 + (sin α−sin β) 2 = cos 2 α−2 cos α cos β+ cos 2 β+ sin 2 α−2 sin α sin β+ sin 2 β

= (cos 2 α+ sin 2 α)+(cos 2 β+ sin 2 β)−2 cos α cos β−2 sin α sin β = 1+1−2 cos α cos β−2 sin α sin β = 2−2 cos α cos β−2 sin α sin β

A B.

d AB = (cos(α−β)−1) 2 + (sin(α−β)−0) 2 = cos 2 (α−β)−2 cos(α−β)+1+ sin 2 (α−β)

= (cos 2 (α−β)+ sin 2 (α−β))−2 cos(α−β)+1 = 1−2 cos(α−β)+1 = 2−2 cos(α−β)

2−2 cos α cos β−2 sin αsin β = 2−2 cos(α−β) 2−2 cos α cos β−2 sin αsin β=2−2 cos(α−β)

2 −2.

cos αcos β+sin αsin β=cos(α−β)

cos(α+β)=cos αcos β−sin αsin β

cos(α−β)=cos αcos β+sin αsin β

cos(5π 4 − π 6).

cos(α−β)=cos αcos β+sin αsin β cos(5π 4 − π 6)=cos(5π 4)cos(π 6)+sin(5π 4)sin(π 6)

=(− 2 2)(3 2)−(2 2)(1 2) =− 6 4 − 2 4 = − 6 − 2 4 cos(π 3 − π 4).2 + 6 4 cos(75 ∘). 75 ∘ = 45 ∘ + 30 ∘ , cos(75 ∘) cos(45 ∘ + 30 ∘).

cos(45 ∘ + 30 ∘)=cos(45 ∘)cos(30 ∘)−sin(45 ∘)sin(30 ∘) = 2 2 (3 2)− 2 2 (1 2) = 6 4 − 2 4 = 6 − 2 4

cos(105 ∘).2 − 6 4

sin(α+β)=sin α cos β+cos α sin β

sin(α−β)=sin α cos β−cos α sin β

sin(45 ∘ − 30 ∘)sin(135 ∘ − 120 ∘)

sin(α−β)=sin α cos β−cos α sin β sin(45 ∘ − 30 ∘)=sin(45 ∘)cos(30 ∘)−cos(45 ∘)sin(30 ∘)

sin(45 ∘)= 2 2 , cos(30 ∘)= 3 2 , cos(45 ∘)= 2 2 , sin(30 ∘)= 1 2

sin(45 ∘ − 30 ∘)= 2 2 (3 2)− 2 2 (1 2) = 6 − 2 4

sin(α−β)=sin α cos β−cos α sin β sin(135 ∘ − 120 ∘)=sin(135 ∘)cos(120 ∘)−cos(135 ∘)sin(120 ∘)

sin(135 ∘)= 2 2 ,cos(120 ∘)=− 1 2 ,cos(135 ∘)= 2 2 ,sin(120 ∘)= 3 2

sin(135 ∘ − 120 ∘)= 2 2 (− 1 2)−(− 2 2)(3 2) = − 2 + 6 4 = 6 − 2 4

sin(135 ∘ − 120 ∘)= 2 2 (− 1 2)−(− 2 2)(3 2) = − 2 + 6 4 = 6 − 2 4

sin(cos −1 1 2 + sin −1 3 5).

sin(α+β). α= cos −1 1 2 β= sin −1 3 5 .

cos α= 1 2 ,0≤α≤π sin β= 3 5 ,− π 2 ≤β≤ π 2

sin α cos β.

sin α= 1− cos 2 α = 1− 1 4 = 3 4 = 3 2 cos β= 1− sin 2 β = 1− 9 25 = 16 25 = 4 5

sin(cos −1 1 2 + sin −1 3 5)=sin(α+β) =sin α cos β+cos α sin β = 3 2 · 4 5 + 1 2 · 3 5 = 4 3 +3 10

tan x= sin x cos x ,cos x≠0.

tan(α+β)= sin(α+β) cos(α+β) = sin α cos β+cos α sin β cos α cos β−sin α sin β = sin α cos β+cos α sin β cos α cos β cos α cos β−sin α sin β cos α cos β

Divide the numerator and denominator by cos α cos β = sin α cos β cos α cos β + cos α sin β cos α cos β cos α cos β cos α cos β − sin α sin β cos α cos β = sin α cos α + sin β cos β 1− sin α sin β cos α cos β = tan α+tan β 1−tan α tan β

tan(α+β)= tan α+tan β 1−tan α tan β

tan(α−β)= tan α−tan β 1+tan α tan β

tan(π 6 + π 4).

tan(α+β)= tan α+tan β 1−tan α tan β tan(π 6 + π 4)= tan(π 6)+tan(π 4) 1−(tan(π 6))(tan(π 4))

tan(π 6)= 1 3 ,tan(π 4)=1

tan(π 6 + π 4)= 1 3 +1 1−(1 3)(1) = 1+ 3 3 3 −1 3 = 1+ 3 3 (3 3 −1) = 3 +1 3 −1

tan(2π 3 + π 4).1− 3 1+ 3 sin α= 3 5 ,0<α< π 2 ,cos β=− 5 13 ,π<β< 3π 2 ,sin(α+β)cos(α+β)

tan(α+β)tan(α−β) sin(α+β), sin α= 3 5 0<α< π 2 . α α a:

a 2 + 3 2 = 5 2 a 2 =16 a=4

cos β=− 5 13 π<β< 3π 2 , β −5, β

(−5) 2 + a 2 = 13 2 25+ a 2 =169 a 2 =144 a=±12

β a=−12. α β. α cos α= 4 5 . β sin β=− 12 13 . sin(α+β).

sin(α+β)=sin αcos β+cos αsin β =(3 5)(− 5 13)+(4 5)(− 12 13) =− 15 65 − 48 65 =− 63 65

cos(α+β)

cos(α+β)=cos α cos β−sin α sin β =(4 5)(− 5 13)−(3 5)(− 12 13) =− 20 65 + 36 65 = 16 65

tan(α+β), sin α= 3 5 cos α= 4 5 ,

tan α= 3 5 4 5 = 3 4

sin β=− 12 13 cos β=− 5 13 ,

tan β= −12 13 −5 13 = 12 5

tan(α+β)= tan α+tan β 1−tan α tan β = 3 4 + 12 5 1− 3 4 (12 5) = 63 20 − 16 20 =− 63 16

tan(α−β),

tan(α−β)= tan α−tan β 1+tan α tan β = 3 4 − 12 5 1+ 3 4 (12 5) = − 33 20 56 20 =− 33 56

α β

tan(α+β)= sin(α+β) cos(α+β)

π 2 , π 2 , θ, (π 2 −θ). sin θ=cos(π 2 −θ): θ θ. sin θ=cos(π 2 −θ)cos θ=sin(π 2 −θ)
tan θ=cot(π 2 −θ)cot θ=tan(π 2 −θ)sec θ=csc(π 2 −θ)csc θ=sec(π 2 −θ)

cos(α−β)=cos αcos β+sin αsin β,

cos(π 2 −θ)=cos π 2 cos θ+sin π 2 sin θ =(0)cos θ+(1)sin θ =sin θ

tan π 9 tan θ=cot(π 2 −θ).

tan(π 9)=cot(π 2 − π 9) =cot(9π 18 − 2π 18) =cot(7π 18)

sin π 7 cos(5π 14) sin(α+β)+sin(α−β)=2 sin α cos β.

sin(α+β)=sin α cos β+cos α sin β sin(α−β)=sin α cos β−cos α sin β

sin(α+β)+sin(α−β)=sin α cos β+cos α sin β+sin α cos β−cos α sin β =2 sin α cos β

sin(α−β) cos α cos β =tan α−tan β

sin(α−β) cos α cos β = sin α cos β−cos αsin β cos αcos β = sin α cos β cos α cos β − cos α sin β cos α cos β Rewrite using a common denominator. = sin α cos α − sin β cos β Cancel. =tan α−tan β Rewrite in terms of tangent.

tan(π−θ)=−tan θ.

tan(π−θ)= tan(π)−tan θ 1+tan(π)tanθ = 0−tan θ 1+0⋅tan θ =−tan θ

L 1 L 2 θ L 1 L 2 .

tan θ= m 2 − m 1 1+ m 1 m 2

m 1 m 2 L 1 L 2 tan θ 1 = m 1 tan θ 2 = m 2 .

tan θ=tan(θ 2 − θ 1) = tan θ 2 −tan θ 1 1+tan θ 1 tan θ 2 = m 2 − m 1 1+ m 1 m 2

R S α tan β= 47 50 , tan(β−α)= 40 50 = 4 5 .

tan(β−α)= tan β−tan α 1+tan βtan α

4 5 = 47 50 −tan α 1+ 47 50 tan α 4(1+ 47 50 tan α)=5(47 50 −tan α)

4(1)+4(47 50)tan α=5(47 50)−5 tan α 4+3.76 tan α=4.7−5 tan α 5 tan α+3.76 tan α=0.7

8.76 tan α=0.7 tan α≈0.07991 tan −1 (0.07991)≈.079741

α≈0.079741(180 π)≈ 4.57 ∘

cos(α+β)=cos α cos β−sin αsin βcos(α−β)=cos α cos β+sin α sin βsin(α+β)=sin α cos β+cos α sin β
sin(α−β)=sin α cos β−cos α sin βtan(α+β)= tan α+tan β 1−tan α tan β
tan(α−β)= tan α−tan β 1+tan α tan β
sin θ=cos(π 2 −θ) cos θ=sin(π 2 −θ) tan θ=cot(π 2 −θ) cot θ=tan(π 2 −θ) sec θ=csc(π 2 −θ) csc θ=sec(π 2 −θ)

x, π 2 −x. sinx=cos(π 2 −x). cos(5π 4)? f(x)=sin(x) g(x)=cos(x). 0−x=−xsin(−x)=−sinx, sinx cos(−x)=cos(0−x)=cosx, cosx cos(7π 12)cos(π 12)2 + 6 4sin(5π 12)sin(11π 12)6 − 2 4
tan(− π 12)tan(19π 12)−2− 3 sin x cos x.sin(x+ 11π 6)sin(x− 3π 4)− 2 2 sinx− 2 2 cosxcos(x− 5π 6)
cos(x+ 2π 3)− 1 2 cosx− 3 2 sinxcsc(π 2 −t)sec(π 2 −θ)cscθcot(π 2 −x)tan(π 2 −x)cotx
sin(2x)cos(5x)−sin(5x)cos(2x)tan(3 2 x)−tan(7 5 x) 1+tan(3 2 x)tan(7 5 x)tan(x 10) sin a= 2 3 cos b=− 1 4 , a b [π 2 ,π), sin(a+b) cos(a−b). sin a= 4 5 , cos b= 1 3 , a b [0, π 2), sin(a−b) cos(a+b).
sin(a−b)=(4 5)(1 3)−(3 5)(2 2 3)= 4−6 2 15cos(a+b)=(3 5)(1 3)−(4 5)(2 2 3)= 3−8 2 15
sin(cos −1 (0)− cos −1 (1 2))cos(cos −1 (2 2)+ sin −1 (3 2))2 − 6 4

tan(sin −1 (1 2)− cos −1 (1 2))cos(π 2 −x)sinxsin(π−x)tan(π 3 +x)cot(π 6 −x)sin(π 3 +x)
tan(π 4 −x)cot(π 4 +x)cos(7π 6 +x)sin(π 4 +x)sinx 2 + cosx 2cos(5π 4 +x) 2x=x+x.
f(x)=sin(4x)−sin(3x)cos x,g(x)=sin x cos(3x)f(x)=cos(4x)+sin x sin(3x),g(x)=−cos x cos(3x)
f(x)=sin(3x)cos(6x),g(x)=−sin(3x)cos(6x) g(x)=sin(9x)−cos(3x)sin(6x).
f(x)=sin(4x),g(x)=sin(5x)cos x−cos(5x)sin xf(x)=sin(2x),g(x)=2 sin x cos x
f(θ)=cos(2θ),g(θ)= cos 2 θ− sin 2 θf(θ)=tan(2θ),g(θ)= tan θ 1+ tan 2 θ g(θ)= 2 tanθ 1− tan 2 θ .
f(x)=sin(3x)sin x,g(x)= sin 2 (2x) cos 2 x− cos 2 (2x) sin 2 x
f(x)=tan(−x),g(x)= tan x−tan(2x) 1−tan x tan(2x) g(x)= tanx−tan(2x) 1+tanxtan(2x) .sin(75 ∘)
sin(195 ∘)− 3 −1 2 2 , or −0.2588cos(165 ∘)cos(345 ∘)1+ 3 2 2 ,tan(− 15 ∘)
tan(x+ π 4)= tan x+1 1−tan x
tan(x+ π 4)= tanx+tan(π 4) 1−tanxtan(π 4) = tanx+1 1−tanx(1) = tanx+1 1−tanx
tan(a+b) tan(a−b) = sin a cos a+sin b cos b sin a cos a−sin b cos bcos(a+b) cos a cos b =1−tan a tan b
cos(a+b) cosacosb = cosacosb cosacosb − sinasinb cosacosb =1−tanatanb
cos(x+y)cos(x−y)= cos 2 x− sin 2 ycos(x+h)−cos x h =cos x cos h−1 h −sin x sin h h
cos(x+h)−cosx h = cosxcosh−sinxsinh−cosx h = cosx(cosh−1)−sinxsinh h =cosx cosh−1 h −sinx sinh h
tan(u+v)= tan u+tan v 1−tan u tan vtan(u−v)= tan u−tan v 1+tan u tan v
tan(x+y) 1+tan x tan x = tan x+tan y 1− tan 2 x tan 2 y α, β, γ sin(α+β)=sin γ. sin(α+β)=sin(π−γ)
α,β, y tan α+tan β+tan γ=tan α tan β tan γ θ tan θ= 5 3 . α=β.

sin(α+β)=sin α cos β+cos α sin β

α=β=θ,

sin(θ+θ)=sin θ cos θ+cos θ sin θ sin(2θ)=2sin θ cos θ

cos(α+β)=cos α cos β−sin α sin β, α=β=θ,

cos(θ+θ)=cos θ cos θ−sin θsin θ cos(2θ)= cos 2 θ− sin 2 θ

cos(2θ)= cos 2 θ− sin 2 θ =(1− sin 2 θ)− sin 2 θ =1−2 sin 2 θ

cos(2θ)= cos 2 θ− sin 2 θ = cos 2 θ−(1− cos 2 θ) =2 cos 2 θ−1

α=β=θ

tan(α+β)= tan α+tan β 1−tan α tan β tan(θ+θ)= tan θ+tan θ 1−tan θ tan θ tan(2θ)= 2tan θ 1− tan 2 θ

sin(2θ)=2 sin θ cos θ

cos(2θ)= cos 2 θ− sin 2 θ =1−2 sin 2 θ =2 cos 2 θ−1

tan(2θ)= 2 tan θ 1− tan 2 θ

tan θ=− 3 4 θ sin(2θ)cos(2θ)tan(2θ) tan θ=− 3 4 , θ θ

(−4) 2 + (3) 2 = c 2 16+9= c 2 25= c 2 c=5

sin(2θ)=2 sin θ cos θ

sin θ cos θ. sin θ= 3 5 , cos θ=− 4 5 .

sin(2θ)=2(3 5)(− 4 5) =− 24 25

cos(2θ)= cos 2 θ− sin 2 θ

cos(2θ)= (− 4 5) 2 − (3 5) 2 = 16 25 − 9 25 = 7 25

tan(2θ)= 2 tan θ 1− tan 2 θ

tan θ=− 3 4 .

tan(2θ)= 2(− 3 4) 1− (− 3 4) 2 = − 3 2 1− 9 16 =− 3 2 (16 7) =− 24 7

sin α= 5 8 , θ cos(2α).cos(2α)= 7 32 cos(6x) cos(3x).

cos(6x)=cos(3x+3x) =cos 3x cos 3x−sin 3x sin 3x = cos 2 3x− sin 2 3x

1+sin(2θ)= (sin θ+cos θ) 2

(sin θ+cos θ) 2 = sin 2 θ+2 sin θ cos θ+ cos 2 θ =(sin 2 θ+ cos 2 θ)+2 sin θ cos θ

=1+2 sin θ cos θ =1+sin(2θ)

(a±b) 2 = a 2 ±2ab+ b 2

a=sin θ b=cos θ. cos 4 θ− sin 4 θ=cos(2θ).

cos 4 θ− sin 4 θ=(cos 2 θ+ sin 2 θ)(cos 2 θ− sin 2 θ)=cos(2θ) tan(2θ)= 2 cot θ−tan θ

tan(2θ)= 2 tan θ 1− tan 2 θ Double-angle formula = 2 tan θ(1 tan θ) (1− tan 2 θ)(1 tan θ) Multiply by a term that results in desired numerator. = 2 1 tan θ − tan 2 θ tan θ = 2 cot θ −tan θ Use reciprocal identity for 1 tan θ .

2tan θ 1− tan 2 θ = 2 cot θ−tan θ

2 cot θ−tan θ = 2 1 tan θ −tan θ (tan θ tan θ) = 2 tan θ 1 tan θ (tan θ)−tan θ(tan θ) = 2 tan θ 1− tan 2 θ

cos(2θ)cos θ= cos 3 θ−cos θ sin 2 θ.cos(2θ)cos θ=(cos 2 θ− sin 2 θ)cos θ= cos 3 θ−cos θ sin 2 θ

cos(2θ)=1−2 sin 2 θ. sin 2 θ:

cos(2θ)=1−2 sin 2 θ 2 sin 2 θ=1−cos(2θ) sin 2 θ= 1−cos(2θ) 2

cos(2θ)=2 cos 2 θ−1. cos 2 θ:

cos(2θ)=2 cos 2 θ−1 1+cos(2θ)=2 cos 2 θ 1+cos(2θ) 2 = cos 2 θ

tan 2 θ= sin 2 θ cos 2 θ = 1−cos(2θ) 2 1+cos(2θ) 2 Substitute the reduction formulas. =(1−cos(2θ) 2)(2 1+cos(2θ)) = 1−cos(2θ) 1+cos(2θ)

sin 2 θ= 1−cos(2θ) 2

cos 2 θ= 1+cos(2θ) 2

tan 2 θ= 1−cos(2θ) 1+cos(2θ)

cos 4 x

cos 4 x= (cos 2 x) 2 = (1+cos(2x) 2) 2 Substitute reduction formula for cos 2 x. = 1 4 (1+2cos(2x)+ cos 2 (2x)) = 1 4 + 1 2 cos(2x)+ 1 4 (1+cos2(2x) 2) Substitute reduction formula for cos 2 x. = 1 4 + 1 2 cos(2x)+ 1 8 + 1 8 cos(4x) = 3 8 + 1 2 cos(2x)+ 1 8 cos(4x)

sin 3 (2x)=[1 2 sin(2x)] [1−cos(4x)]

sin 3 (2x)=[sin(2x)][sin 2 (2x)] =sin(2x)[1−cos(4x) 2] Substitute the power-reduction formula. =sin(2x)(1 2)[1−cos(4x)] = 1 2 [sin(2x)][1−cos(4x)]

1−cos(4x) 2

sin 2 (2x).

sin 2 θ= 1−cos(2θ) 2

θ=2x, 2θ=4x. 10 cos 4 x= 15 4 +5 cos(2x)+ 5 4 cos(4x).

10 cos 4 x=10 cos 4 x=10 (cos 2 x) 2 =10 [1+cos(2x) 2] 2 Substitute reduction formula for cos 2 x. = 10 4 [1+2cos(2x)+ cos 2 (2x)] = 10 4 + 10 2 cos(2x)+ 10 4 (1+cos2(2x) 2) Substitute reduction formula for cos 2 x. = 10 4 + 10 2 cos(2x)+ 10 8 + 10 8 cos(4x) = 30 8 +5cos(2x)+ 10 8 cos(4x) = 15 4 +5cos(2x)+ 5 4 cos(4x)

θ α 2 , sin(α 2). ± α 2

sin 2 θ= 1−cos(2θ) 2 sin 2 (α 2)= 1−(cos2· α 2) 2 = 1−cos α 2 sin(α 2)=± 1−cos α 2

cos 2 θ= 1+cos(2θ) 2 cos 2 (α 2)= 1+cos(2· α 2) 2 = 1+cos α 2 cos(α 2)=± 1+cos α 2

tan 2 θ= 1−cos(2θ) 1+cos(2θ) tan 2 (α 2)= 1−cos(2· α 2) 1+cos(2· α 2) = 1−cos α 1+cos α

tan(α 2)=± 1−cos α 1+cos α

sin(α 2)=± 1−cos α 2

cos(α 2)=± 1+cos α 2

tan(α 2)=± 1−cos α 1+cos α = sin α 1+cos α = 1−cos α sin α

sin(15 ∘) 15 ∘ = 30 ∘ 2 ,

sin 30 ∘ 2 = 1−cos 30 ∘ 2 = 1− 3 2 2 = 2− 3 2 2 = 2− 3 4 = 2− 3 2

sin(15 o) tan α= 8 15 α sin(α 2)cos(α 2)tan(α 2) sin α=− 8 17 cos α=− 15 17 . α 180°<α<270°, 180° 2 < α 2 < 270° 2 . α 2 90°< α 2 <135°. sin α 2 ,

sin α 2 =± 1−cos α 2 =± 1−(− 15 17) 2 =± 32 17 2 =± 32 17 · 1 2 =± 16 17 =± 4 17 = 4 17 17

sin α 2 cos α 2 ,

cos α 2 =± 1+cos α 2 =± 1+(− 15 17) 2 =± 2 17 2 =± 2 17 · 1 2 =± 1 17 =− 17 17

cos α 2 tan α 2 ,

tan α 2 =± 1−cos α 1+cos α =± 1−(− 15 17) 1+(− 15 17) =± 32 17 2 17 =± 32 2 =− 16 =−4

tan α 2 α 2 sin α=− 4 5 α cos (α 2).− 2 5 θ tan θ= 5 3 tan θ= 5 3 cos θ 3 2 + 5 2 =34 c= 34

cos θ= 3 34 = 3 34 34 . tan θ 2 = 1−cos θ 1+cos θ . tan θ tan θ 2 .

tan θ 2 = 1− 3 34 34 1+ 3 34 34 = 34−3 34 34 34+3 34 34 = 34−3 34 34+3 34 ≈0.57

tan −1 (0.57)≈ 29.7 ◦ . ≈ 29.7 ◦ .

sin(2θ)=2sin θ cos θ cos(2θ)= cos 2 θ− sin 2 θ =1−2 sin 2 θ =2 cos 2 θ−1 tan(2θ)= 2tan θ 1− tan 2 θ

sin 2 θ= 1−cos(2θ) 2 cos 2 θ= 1+cos(2θ) 2 tan 2 θ= 1−cos(2θ) 1+cos(2θ)

sin α 2 =± 1−cos α 2 cos α 2 =± 1+cos α 2 tan α 2 =± 1−cos α 1+cos α = sin α 1+cos α = 1−cos α sin α

cos(2x)= cos 2 x− sin 2 x. tan(2x) cos(2x) sin(2x). tan(x 2)= 1−cos x 1+cos x sin(x 2) cos(x 2).

tan(x 2) 1−cos x sin x , sin x 1+cos x , 1−cos x 1+cos x , tan(x 2), cos x sin(2x), cos(2x), tan(2x)

x. sin x= 1 8 , x 3 7 32 31 32 3 7 31 cos x= 2 3 , x cos x=− 1 2 , x 3 2 − 1 2 − 3 tan x=−8, x

cos(2θ)= 3 5 90 ◦ ≤θ≤ 180 ◦cos θ=− 2 5 5 ,sin θ= 5 5 ,tan θ=− 1 2 ,csc θ= 5 ,sec θ=− 5 2 ,cot θ=−2

cos(2θ)= 1 2 180 ◦ ≤θ≤ 270 ◦2 sin(π 4) 2 cos(π 4)2 sin(π 2)4 sin(π 8) cos(π 8) sin(π 8) 2− 2 2

cos(− 11π 12)sin(11π 12)2− 3 2cos(7π 8)tan(5π 12)2+ 3tan(− 3π 12)tan(− 3π 8)−1− 2 sin(x 2),

cos(x 2), tan(x 2) x. tan x=− 4 3 , x sin x=− 12 13 , x 3 13 13 − 2 13 13 − 3 2 csc x=7, x sec x=−4,

x 10 4 6 4 15 3 sin(2θ),cos(2θ), tan(2θ). sin(2α),cos(2α), tan(2α).120 169 ,– 119 169 ,– 120 119

sin(θ 2),cos(θ 2), tan(θ 2). sin(α 2),cos(α 2), tan(α 2).2 13 13 , 3 13 13 , 2 3

cos 2 (28 ◦)− sin 2 (28 ◦)2 cos 2 (37 ◦)−1cos(74 ◦)1−2 sin 2 (17 ◦)cos 2 (9x)− sin 2 (9x)cos(18x)

4 sin(8x) cos(8x)6 sin(5x) cos(5x)3sin(10x)(sin t−cos t) 2 =1−sin(2t)sin(2x)=−2 sin(−x) cos(−x)

−2 sin(−x)cos(−x)=−2(−sin(x)cos(x))=sin(2x)cot x−tan x=2 cot(2x)

sin(2θ) 1+cos(2θ) tan 2 θ=tan θ

sin(2θ) 1+cos(2θ) tan 2 θ= 2sin(θ)cos(θ) 1+ cos 2 θ− sin 2 θ tan 2 θ= 2sin(θ)cos(θ) 2 cos 2 θ tan 2 θ= sin(θ) cos θ tan 2 θ= cot(θ) tan 2 θ=tan θ

cos 2 (5x)cos 2 (6x)1+cos(12x) 2sin 4 (8x)sin 4 (3x)3+cos(12x)−4cos(6x) 8cos 2 x sin 4 xcos 4 x sin 2 x 2+cos(2x)−2cos(4x)−cos(6x) 32tan 2 x sin 2 xtan 4 x3+cos(4x)−4cos(2x) 3+cos(4x)+4cos(2x)sin 2 (2x) sin 2 x cos 2 x1−cos(4x) 8tan 2 x sin xtan 4 x cos 2 x3+cos(4x)−4cos(2x) 4(cos(2x)+1)cos 2 x sin(2x) cos 2 (2x)sin x(1+cos(4x))sin x 2tan 2 (x 2) sin x sin x cos x,sin(4x)4sin xcos x(cos 2 x− sin 2 x) cos(4x)sin(2x)= 2 tan x 1+ tan 2 x

2tan x 1+ tan 2 x = 2sin x cos x 1+ sin 2 x cos 2 x = 2sin x cos x cos 2 x+ sin 2 x cos 2 x = 2sinx cosx . cos 2 x 1 =2sinxcosx=sin(2x)cos(2α)= 1− tan 2 α 1+ tan 2 αtan(2x)= 2 sin xcos x 2 cos 2 x−1 2sin xcos x 2 cos 2 x−1 = sin(2x) cos(2x) =tan(2x)(sin 2 x−1) 2 =cos(2x)+ sin 4 x

sin(3x)=3 sin x cos 2 x− sin 3 x

sin(x+2x)=sin xcos(2x)+sin(2x)cos x =sin x(cos 2 x− sin 2 x)+2sin xcos xcos x

=sin x cos 2 x− sin 3 x+2sin x cos 2 x =3sin x cos 2 x− sin 3 x

cos(3x)= cos 3 x−3 sin 2 xcos x1+cos(2t) sin(2t)−cos t = 2 cos t 2 sin t−1

1+cos(2t) sin(2t)−cost = 1+2 cos 2 t−1 2sintcost−cost = 2 cos 2 t cost(2sint−1)

= 2cost 2sint−1

sin(16x)=16 sin x cos x cos(2x)cos(4x)cos(8x)

cos(16x)=(cos 2 (4x)− sin 2 (4x)−sin(8x))(cos 2 (4x)− sin 2 (4x)+sin(8x))

(cos 2 (4x)− sin 2 (4x)−sin(8x))(cos 2 (4x)− sin 2 (4x)+sin(8x))=
=(cos(8x)−sin(8x))(cos(8x)+sin(8x))
= cos 2 (8x)− sin 2 (8x)
=cos(16x)

cos α cos β+sin α sin β=cos(α−β) + cos α cos β−sin α sin β=cos(α+β)
2 cos α cos β=cos(α−β)+cos(α+β)
2

cos α cos β= 1 2 [cos(α−β)+cos(α+β)]

2 cos(7x 2) cos 3x 2 .

cos α cos β= 1 2 [cos(α−β)+cos(α+β)]

2 cos(7x 2)cos(3x 2)=(2)(1 2)[cos(7x 2 − 3x 2)+cos(7x 2 + 3x 2)] =[cos(4x 2)+cos(10x 2)] =cos 2x+cos 5x

cos(2θ)cos(4θ).1 2 (cos6θ+cos2θ)

sin(α+β)=sin α cos β+cos α sin β + sin(α−β)=sin α cos β−cos α sin β
sin(α+β)+sin(α−β)=2 sin α cos β

sin α cos β= 1 2 [sin(α+β)+sin(α−β)]

sin(4θ)cos(2θ).

sin α cos β= 1 2 [sin(α+β)+sin(α−β)] sin(4θ)cos(2θ)= 1 2 [sin(4θ+2θ)+sin(4θ−2θ)]
= 1 2 [sin(6θ)+sin(2θ)]

sin(x+y)cos(x−y).1 2 (sin2x+sin2y)

cos(α−β)=cos α cos β+sin α sin β − cos(α+β)=−(cos α cos β−sin α sin β)
cos(α−β)−cos(α+β)=2 sin α sin β

sin α sin β= 1 2 [cos(α−β)−cos(α+β)]
cos α cos β= 1 2 [cos(α−β)+cos(α+β)]
sin α cos β= 1 2 [sin(α+β)+sin(α−β)]
sin α sin β= 1 2 [cos(α−β)−cos(α+β)]
cos α sin β= 1 2 [sin(α+β)−sin(α−β)]

cos(3θ) cos(5θ)

cos α cos β= 1 2 [cos(α−β)+cos(α+β)] cos(3θ)cos(5θ)= 1 2 [cos(3θ−5θ)+cos(3θ+5θ)]
= 1 2 [cos(2θ)+cos(8θ)] Use even-odd identity.

cos 11π 12 cos π 12 .−2− 3 4 u+v 2 =α u−v 2 =β.

α+β= u+v 2 + u−v 2 = 2u 2 =u α−β= u+v 2 − u−v 2 = 2v 2 =v

α β

sin α cos β= 1 2 [sin(α+β)+sin(α−β)] sin(u+v 2)cos(u−v 2)= 1 2 [sin u+sin v]
Substitute for(α+β) and (α−β) 2 sin(u+v 2)cos(u−v 2)=sin u+sin v

sin α+sin β=2sin(α+β 2)cos(α−β 2)
sin α−sin β=2sin(α−β 2)cos(α+β 2)
cos α−cos β=−2sin(α+β 2)sin(α−β 2)
cos α+cos β=2cos(α+β 2)cos(α−β 2)

sin(4θ)−sin(2θ).

sin α−sin β=2sin(α−β 2)cos(α+β 2)

sin(4θ)−sin(2θ)=2sin(4θ−2θ 2) cos(4θ+2θ 2) =2sin(2θ 2) cos(6θ 2)
=2 sin θ cos(3θ)

sin(3θ)+sin(θ).2sin(2θ)cos(θ) cos(15 ∘)−cos(75 ∘).

cos α−cos β=−2 sin(α+β 2) sin(α−β 2)

cos(15 ∘)−cos(75 ∘)=−2sin(15 ∘ + 75 ∘ 2) sin(15 ∘ − 75 ∘ 2) =−2sin(45 ∘) sin(− 30 ∘) =−2(2 2)(− 1 2) = 2 2

cos(4t)−cos(2t) sin(4t)+sin(2t) =−tan t

cos(4t)−cos(2t) sin(4t)+sin(2t) = −2 sin(4t+2t 2) sin(4t−2t 2) 2 sin(4t+2t 2) cos(4t−2t 2)
= −2 sin(3t)sin t 2 sin(3t)cos t = − 2 sin(3t) sin t 2 sin(3t) cos t
=− sin t cos t =−tan t
csc 2 θ−2= cos(2θ) sin 2 θ .
cos(2θ) sin 2 θ = 1−2 sin 2 θ sin 2 θ = 1 sin 2 θ − 2 sin 2 θ sin 2 θ = csc 2 θ−2
tan θ cot θ− cos 2 θ= sin 2 θ.
tan θ cot θ− cos 2 θ=(sin θ cos θ)(cos θ sin θ)− cos 2 θ =1− cos 2 θ
= sin 2 θ
cos α cos β= 1 2 [cos(α−β)+cos(α+β)] sin α cos β= 1 2 [sin(α+β)+sin(α−β)] sin α sin β= 1 2 [cos(α−β) −cos(α+β)] cos α sin β= 1 2 [sin(α+β)−sin(α−β)]
sin α+sin β=2 sin(α+β 2)cos(α−β 2) sin α−sin β=2 sin(α−β 2)cos(α+β 2) cos α−cos β=−2 sin(α+β 2)sin(α−β 2) cos α+cos β=2 cos(α+β 2)cos(α−β 2)
sin α cos β= 1 2 [sin(α+β)+sin(α−β)], cos α sin β. α β cos(195°)cos(105°), sin(3x)+sin x cos x =1.
2 sin(2x)cos x cos x =116 sin(16x)sin(11x)8(cos(5x)−cos(27x))20 cos(36t)cos(6t)
2 sin(5x)cos(3x)sin(2x)+sin(8x)10 cos(5x)sin(10x)sin(−x)sin(5x)1 2 (cos(6x)−cos(4x))
sin(3x)cos(5x)cos(6t)+cos(4t)2 cos(5t)cos tsin(3x)+sin(7x)cos(7x)+cos(−7x)2 cos(7x)
sin(3x)−sin(−3x)cos(3x)+cos(9x)2 cos(6x)cos(3x)sin h−sin(3h)cos(45°)cos(15°)1 4 (1+ 3)
cos(45°)sin(15°)sin(−345°)sin(−15°)1 4 (3 −2)sin(195°)cos(15°)sin(−45°)sin(−15°)1 4 (3 −1)
cos(23°)sin(17°)2 sin(100°)sin(20°)cos(80°)−cos(120°)2 sin(−100°)sin(−20°)sin(213°)cos(8°)
1 2 (sin(221°)+sin(205°))2 cos(56°)cos(47°)sin(76°)+sin(14°)2 cos(31°)cos(58°)−cos(12°)
sin(101°)−sin(32°)2 cos(66.5°)sin(34.5°)cos(100°)+cos(200°)sin(−1°)+sin(−2°)2 sin(−1.5°)cos(0.5°)
cos(a+b) cos(a−b) = 1−tan a tan b 1+tan a tan b4 sin(3x)cos(4x)=2 sin(7x)−2 sinx
2 sin(7x)−2 sinx=2 sin(4x+3x)−2 sin(4x−3x)= 2(sin(4x)cos(3x)+sin(3x)cos(4x))−2(sin(4x)cos(3x) −sin(3x)cos(4x))= 2 sin(4x)cos(3x)+2 sin(3x)cos(4x))−2 sin(4x)cos(3x)+2 sin(3x)cos(4x))=
4 sin(3x)cos(4x)
6 cos(8x)sin(2x) sin(−6x) =−3 sin(10x)csc(6x)+3sin x+sin(3x)=4 sin x cos 2 x
sinx+sin(3x)=2sin(4x 2)cos(−2x 2)=2sin(2x)cos x=2(2 sin x cos x)cos x=4 sinx cos 2 x
2(cos 3 x−cos x sin 2 x)=cos(3x)+cos x2 tan x cos(3x)=sec x(sin(4x)−sin(2x))
2tanxcos(3x)= 2sinxcos(3x) cosx = 2(.5(sin(4x)−sin(2x))) cosx
= 1 cosx (sin(4x)−sin(2x))=sec x(sin(4x)−sin(2x))cos(a+b)+cos(a−b)=2 cos a cos b
cos(58 ∘)+cos(12 ∘)2 cos(35 ∘)cos(23 ∘), 1.5081sin(2 ∘)−sin(3 ∘)cos(44 ∘)−cos(22 ∘)
−2 sin(33 ∘)sin(11 ∘), −0.2078cos(176 ∘)sin(9 ∘)sin(− 14 ∘)sin(85 ∘)
1 2 (cos(99 ∘)−cos(71 ∘)), −0.24102 sin(2x)sin(3x)=cos x−cos(5x)
cos(10θ)+cos(6θ) cos(6θ)−cos(10θ) =cot(2θ)cot(8θ)sin(3x)−sin(5x) cos(3x)+cos(5x) =tan x
2 cos(2x)cos x+sin(2x)sin x=2 sin x 2 cos 3 x sin(2x)+sin(4x) sin(2x)−sin(4x) =−tan(3x)cot x
sin(9t)−sin(3t) cos(9t)+cos(3t)tan(3t)2 sin(8x)cos(6x)−sin(2x)sin(3x)−sin x sin x2 cos(2x)
cos(5x)+cos(3x) sin(5x)+sin(3x)sin x cos(15x)−cos x sin(15x)−sin(14x)
sin x−sin y=2 sin(x−y 2)cos(x+y 2)cos x+cos y=2 cos(x+y 2)cos(x−y 2) cos x+cos y. x=α+β y=α−β,
cosx+cosycos(α+β)+cos(α−β)=cosαcosβ−sinαsinβ+cosαcosβ+sinαsinβ=2cosαcos β x=α+β y=α−β, α β 2cosαcosβ2cos(x+y 2)cos(x−y 2).sin(6x)+sin(4x) sin(6x)−sin(4x) =tan (5x)cot x
cos(3x)+cos x cos(3x)−cos x =−cot (2x)cot x
cos(3x)+cos x cos(3x)−cos x = 2 cos(2x)cos x −2 sin(2x)sin x =−cot(2x)cot x
cos(6y)+cos(8y) sin(6y)−sin(4y) =cot y cos (7y)sec (5y)cos(2y)−cos(4y) sin(2y)+sin(4y) =tan y
cos(2y)−cos(4y) sin(2y)+sin(4y) = −2 sin(3y)sin(−y) 2 sin(3y)cos y = 2 sin(3y)sin(y) 2 sin(3y)cos y =tan y
sin(10x)−sin(2x) cos(10x)+cos(2x) =tan(4x)cos x−cos(3x)=4 sin 2 xcos x
cos x−cos(3x)=−2 sin(2x)sin(−x)= 2(2 sin x cos x)sin x=4 sin 2 x cos x
(cos(2x)−cos(4x)) 2 + (sin(4x)+sin(2x)) 2 =4 sin 2 (3x)tan(π 4 −t)= 1−tan t 1+tan t

tan(π 4 −t)= tan(π 4)−tant 1+tan(π 4)tan(t) = 1−tant 1+tant 2π. 2π 2πk, k 2π:

sin θ=sin(θ±2kπ)

cos θ= 1 2 .

cos θ= 1 2 θ= π 3 , 5π 3

[0,2π].

π 3 ±2kπ and 5π 3 ±2kπ

k sin t= 1 2 . 2π. π 6 5π 6 .

π 6 ±2πk and 5π 6 ±2πk

k x u. 2 cos θ−3=−5,0≤θ<2π.

2 cos θ−3=−5 2 cos θ=−2 cos θ=−1 θ=π

[0,2π): 2 sin x+1=0.x= 7π 6 , 11π 6π,2π. π 2 , 2 sin 2 θ−1=0,0≤θ<2π. sinθ.

2 sin 2 θ−1=0 2 sin 2 θ=1 sin 2 θ= 1 2 sin 2 θ =± 1 2 sin θ=± 1 2 =± 2 2 θ= π 4 , 3π 4 , 5π 4 , 7π 4

csc θ=−2,0≤θ<4π. θ csc θ=−2 0≤θ<4π.

csc θ=−2 1 sin θ =−2 sin θ=− 1 2 θ= 7π 6 , 11π 6 , 19π 6 , 23π 6

sin θ=− 1 2 , tan(θ− π 2)=1,0≤θ<2π. π. [0,π), π 4 , (θ− π 2). tan(π 4)=1,

θ− π 2 = π 4 θ= 3π 4 ±kπ

[0,2π),

3π 4 and 3π 4 +π= 7π 4

tan x= 3 .π 3 ±πk 2(tan x+3)=5+tan x,0≤x<2π. tan x

2(tanx)+2(3) =5+tanx 2tan x+6 =5+tan x 2tanx−tanx =5−6 tanx =−1

−1:θ= 3π 4 θ= 7π 4 . sin θ=0.8, θ θ, sin −1 sin −1 (. sin −1 (0.8),

sin −1 (0.8)≈0.9273

0.9273±2πk

θ≈ 53.1 ∘ θ≈ 180 ∘ − 53.1 ∘ ≈ 126.9 ∘

π−θ. sec θ=−4,

sec θ=−4 1 cos θ =−4 cos θ=− 1 4

cos −1 (− 1 4)≈1.8235 θ≈1.8235+2πk

π 2 ≈1.57 π≈3.14, θ≈1.8235 θ '≈π−1.8235≈1.3181. π+1.3181≈4.4597. 1.8235±2πk 4.4597±2πk.

cos θ=−0.2.θ≈1.7722±2πk θ≈4.5110±2πk x u. cos 2 θ+3 cos θ−1=0,0≤θ<2π.θ x. cos θ=x.

x 2 +3x−1=0

x= −b± b 2 −4ac 2a .

x= −3± (−3) 2 −4(1)(−1) 2 = −3± 13 2

x cos θ,

cos θ= −3± 13 2 θ= cos −1 (−3+ 13 2)

θ= cos −1 (−3− 13 2) [−1,1].

cos −1 (−3+ 13 2) ≈1.26

2π− cos −1 (−3+ 13 2) ≈5.02

2 sin 2 θ−5 sin θ+3=0,0≤θ≤2π. sin θ=u,

2 sin 2 θ−5 sin θ+3=0 (2 sin θ−3)(sin θ−1)=0

2 sin θ−3=0 2 sin θ=3 sin θ= 3 2 sin θ−1=0 sin θ=1

θ:sin θ≠ 3 2 , [−1,1]. sin θ=1, π 2 . sin 2 θ=2 cos θ+2,0≤θ≤2π. cos θ=−1, θ=π

2 sin 2 θ+sin θ=0;0≤θ<2π

sin θ=x. 2 x 2 +x=0.

2 x 2 +x=0 x(2x+1)=0

x=0 (2x+1)=0 x=− 1 2

sinθ x.

sin θ=0 θ=0,π sin θ=− 1 2 θ= 7π 6 , 11π 6

0≤θ<2π 0,π, 7π 6 , 11π 6 .

2 sin 2 θ+sin θ=0 sin θ(2sin θ+1)=0 sin θ=0 θ=0,π 2 sin θ+1=0
2sin θ=−1 sin θ=− 1 2 θ= 7π 6 , 11π 6
0≤θ<2π, 2 sin 2 θ−3 sin θ+1=0,0≤θ<2π. θ
(2 sin θ−1)(sin θ−1)=0 2 sin θ−1=0 sin θ= 1 2 θ= π 6 ,
5π 6 sin θ=1 θ= π 2
2 cos 2 θ+cos θ=0.π 2 , 2π 3 , 4π 3 , 3π 2 0≤x<2π.
cos x cos(2x)+sin x sin(2x)= 3 2
cos x cos(2x)+sin x sin(2x)= 3 2 cos(x−2x)= 3 2 Difference formula for cosine
cos(−x)= 3 2 Use the negative angle identity. cos x= 3 2
cos x= 3 2 x= π 6 , 11π 6 . cos(2 θ)=cos θ.
cos(2θ)=cos θ 2 cos 2 θ−1=cos θ 2 cos 2 θ−cos θ−1=0 (2 cos θ+1)(cos θ
−1)=0 2 cos θ+1=0 cos θ=− 1 2 cos θ−1=0
cos θ=1
cos θ=− 1 2 , θ= 2π 3 ±2πk θ= 4π 3 ±2πk; cos θ=1, θ=0±2πk. 3 cos θ+3=2 sin 2 θ,0≤θ<2π.
3 cos θ+3 = 2 sin 2θ 3 cos θ+3 =2(1−cos2 θ) 3 cos θ+3 =2−2cos2 θ 2 cos2θ+3 cos θ+1 =0 (2 cos θ+1)
(cos θ+1) =0 2 cos θ+1 =0 cos θ =− 1 2 θ = 2π 3 , 4π 3 cos θ+1 =0 cos θ =−1 θ =π
2π 3 , 4π 3 ,π. sin(2x) cos(3x). y=sin(2x) y=sin x. 2π, y=sin(2x), y=sin x. sin(2x)=0 sin x=0.
cos(2x)= 1 2 [0,2π). cos(α)= 1 2 , α θ= cos −1 1 2 cos θ= 1 2 θ= π 3 θ= 5π 3 . 2x= π 3
2x= 5π 3 , x= π 6 x= 5π 6 . cos(2(π 6))=cos(π 3)= 1 2 . 2x= π 3 , x= π 6
2x= π 3 +2π = π 3 + 6π 3 = 7π 3
x= 7π 6 .
2x= π 3 +4π = π 3 + 12π 3 = 13π 3
x= 13π 6 >2π, x 2π, [0,2π). 2x= 5π 3 , x= 5π 6
2x= 5π 3 +2π = 5π 3 + 6π 3 = 11π 3
x= 11π 6 .
2x= 5π 3 +4π = 5π 3 + 12π 3 = 17π 3
x= 17π 6 >2π, x 2π, [0,2π). π 6 , 5π 6 , 7π 6 ,and 11π 6 . sin(nx)=c, n a 2 + b 2 = c 2 ,
a 2 + b 2 = c 2 (23) 2 + (69.5) 2 ≈5359 5359 ≈73.2 m
θ,
tan θ= 69.5 23 tan −1 (69.5 23)≈1.2522 ≈ 71.69 ∘
71.7 ∘ , θ 4a.
cos θ= a 4a = 1 4 cos −1 (1 4)≈ 75.5 ∘
75.5 ∘
a 2 + b 2 = (4a) 2 b 2 = (4a) 2 − a 2 b 2 =16 a 2 − a 2 b 2 =15 a 2 b= 15 a
15 a cos(x)=−5. 0≤θ<2π.2 sin θ=− 22 sin θ= 3π 3 , 2π 32 cos θ=12 cos θ=− 23π 4 , 5π 4tan θ=−1tan x=1
π 4 , 5π 4cot x+1=04 sin 2 x−2=0π 4 , 3π 4 , 5π 4 , 7π 4csc 2 x−4=0 [0,2π).2 cos θ= 2π 4 , 7π 4
2 cos θ=−12 sin θ=−17π 6 , 11π 62 sin θ=− 32 sin(3θ)=1π 18 , 5π 18 , 13π 18 , 17π 18 , 25π 18 , 29π 18
2 sin(2θ)= 32 cos(3θ)=− 23π 12 , 5π 12 , 11π 12 , 13π 12 , 19π 12 , 21π 12cos(2θ)=− 3 22 sin(πθ)=1
1 6 , 5 6 , 13 6 , 17 6 , 25 6 , 29 6 , 37 62 cos(π 5 θ)= 3 [0,2π).sec(x)sin(x)−2 sin(x)=00, π 3 ,π, 5π 3
tan(x)−2 sin(x)tan(x)=02 cos 2 t+cos(t)=1π 3 ,π, 5π 32 tan 2 (t)=3 sec(t)
2 sin(x)cos(x)−sin(x)+2 cos(x)−1=0π 3 , 3π 2 , 5π 3cos 2 θ= 1 2sec 2 x=10,πtan 2 (x)=−1+2 tan(−x)
8 sin 2 (x)+6 sin(x)+1=0π− sin −1 (− 1 4), 7π 6 , 11π 6 ,2π+ sin −1 (− 1 4)tan 5 (x)=tan(x) [0,2π).
sin(3x)cos(6x)−cos(3x)sin(6x)=−0.9
1 3 (sin −1 (9 10)), π 3 − 1 3 (sin −1 (9 10)), 2π 3 + 1 3 (sin −1 (9 10)),π− 1 3 (sin −1 (9 10)),
4π 3 + 1 3 (sin −1 (9 10)), 5π 3 − 1 3 (sin −1 (9 10))
sin(6x)cos(11x)−cos(6x)sin(11x)=−0.1cos(2x)cos x+sin(2x)sin x=106 sin(2t)+9 sin t=0
9 cos(2θ)=9 cos 2 θ−4π 6 , 5π 6 , 7π 6 , 11π 6sin(2t)=cos tcos(2t)=sin t3π 2 , π 6 , 5π 6
cos(6x)−cos(3x)=0 [0,2π). tan 2 x− 3 tan x=00, π 3 ,π, 4π 3sin 2 x+sin x−2=0sin 2 x−2 sin x−4=0

5 cos 2 x+3 cos x−1=03 cos 2 x−2 cos x−2=0cos −1 (1 3 (1− 7)),2π− cos −1 (1 3 (1− 7))
5 sin 2 x+2 sin x−1=0tan 2 x+5tan x−1=0
tan −1 (1 2 (29 −5)),π+ tan −1 (1 2 (− 29 −5)),π+ tan −1 (1 2 (29 −5)),2π+ tan −1 (1 2 (− 29 −5))
cot 2 x=−cot x− tan 2 x−tan x−2=0 [0,2π). sin 2 x− cos 2 x−sin x=0sin 2 x+ cos 2 x=0sin(2x)−sin x=0
cos(2x)−cos x=00, 2π 3 , 4π 32 tan x 2− sec 2 x − sin 2 x= cos 2 x1−cos(2x)=1+cos(2x)
π 4 , 3π 4 , 5π 4 , 7π 4sec 2 x=710 sin x cos x=6 cos xsin −1 (3 5), π 2 ,π− sin −1 (3 5), 3π 2
−3 sin t=15 cos t sin t4 cos 2 x−4=15 cos xcos −1 (− 1 4),2π− cos −1 (− 1 4)8 sin 2 x+6 sin x+1=0
8 cos 2 θ=3−2 cos θπ 3 , cos −1 (− 3 4),2π− cos −1 (− 3 4), 5π 36 cos 2 x+7 sin x−8=0
12 sin 2 t+cos t−6=0cos −1 (3 4), cos −1 (− 2 3),2π− cos −1 (− 2 3),2π− cos −1 (3 4)tan x=3 sin x
cos 3 t=cos t0, π 2 ,π, 3π 26 sin 2 x−5 sin x+1=08 cos 2 x−2 cos x−1=0
π 3 , cos −1 (− 1 4),2π− cos −1 (− 1 4), 5π 3100 tan 2 x+20 tan x−3=02 cos 2 x−cos x+15=0
20 sin 2 x−27 sin x+7=02 tan 2 x+7 tan x+6=0
π+ tan −1 (−2),π+ tan −1 (− 3 2),2π+ tan −1 (−2),2π+ tan −1 (− 3 2)130 tan 2 x+69 tan x−130=0
sin x=0.272πk+0.2734,2πk+2.8682sin x=−0.55tan x=−0.34πk−0.3277cos x=0.71 [0,2π).
tan 2 x+3 tan x−3=00.6694,1.8287,3.8110,4.97036 tan 2 x+13 tan x=−6tan 2 x−sec x=1
1.0472,3.1416,5.2360sin 2 x−2 cos 2 x=02 tan 2 x+9 tan x−6=00.5326,1.7648,3.6742,4.9064
4 sin 2 x+sin(2x)sec x−3=0 [0,2π).csc 2 x−3 csc x−4=0sin −1 (1 4),π− sin −1 (1 4), 3π 2
sin 2 x− cos 2 x−1=0sin 2 x(1− sin 2 x)+ cos 2 x(1− sin 2 x)=0π 2 , 3π 2
3 sec 2 x+2+ sin 2 x− tan 2 x+ cos 2 x=0sin 2 x−1+2 cos(2x)− cos 2 x=1tan 2 x−1− sec 3 x cos x=0
sin(2x) sec 2 x =00, π 2 ,π, 3π 2sin(2x) 2 csc 2 x =02 cos 2 x− sin 2 x−cos x−5=0
1 sec 2 x +2+ sin 2 x+4 cos 2 x=47.2 ∘5.7 ∘82.4 ∘31.0 ∘88.7 ∘59.0 ∘36.9 ∘ 2π. x,

sin(x±2πk)=sin x and cos(x±2πk)=cos x where k is an integer

y=A sin(Bt−C)+D or y=A cos(Bt−C)+D

amplitude=|A|,B period= 2π B ,C C B D y=a sin(ω t±C)+D y=a cos(ω t±C)+D, 2π ω . y=sin x
y=2 sin(4x− π 2)+2. y=sinx y=2sinx. B, period= 2π B . B=4, π 2 . π 2 C B , π 2 4 = π 8 . D.
y=2 sin(1 4 x)y=−3 sin(2x+ π 2)y=cos x+3y=2 sin(1 4 x)

y=A sin(Bt+C)+D

| A | 2π B ,

2π B = 2π 1 4 =8π

y=−3 sin(2x+ π 2)

y=A sin(Bt−C)+D

| A |, | −3 |=3. A 2π B ,

2π B = 2π 2 =π

C B = π 2 2 = π 4 y=cos x+3

y=A cos(Bt±C)+D

| A |, 2π. y=3 cos(3πx)? 3, 2 3 . 1 4 y=cos θ. 2π,

2π 4 = π 2

θ=0, π 2 π 2 θ0π 2π3π 22πy=cos θ10−101 y=−4 cos(πx) |−4|=4. 2π ω = 2π π =2. B ω.) [0,2].
x=0 1 2 x y. x01 213 22y=−4 cos(πx)−4040−4 y=3 sin(3x) 3sin(3x)π 6π 3π 2−32π 3
y=A sin(Bt−C)+D o F

A= largest value −smallest value 2
|A|= 69−42.5 2 =13.25

2π B =12 B= 2π 12 = π 6 .

D= highest value+lowest value 2
D= 69+42.5 2 =55.8

y=13.3 sin(π 6 x−C)+55.8. x y C.

42.5=13.3 sin(π 6 (1)−C)+55.8 −13.3=13.3 sin(π 6 −C) −1=sin(π 6 −C) sinθ=−1→θ=− π 2 π 6
−C=− π 2 π 6 + π 2 =C = 2π 3

$y=13.3\sin(\pi 6 x-2\pi 3)+55.8$. y x xy(0,30)(3,54)(6,78)(9,54)(12,30)

$$|A|=|78-30\ 2| \quad =24$$
$$B=2\pi\ 12 \quad =\pi\ 6$$
$$D=78+30\ 2 \quad =54$$

C=0. y x=0 A. $y=A\cos(Bx\pm C)+D$,

$$y=-24\cos(\pi\ 6\ x)+54$$
$$|A|=|(15-7)\ 2| \quad =4$$

B

$$2\pi\ 12=\pi\ 6$$

(15+8) 2 =11.5. t=0, A.

$$y=4\cos(\pi\ 6)t+11$$

24 °F 40 °F. 32 °F. t=0 $y=8\sin(\pi\ 12\ t)+32$ $f(t)=20\sin(160\pi t)+100$, f(t) t,

$$2\pi\ \omega=2\pi\ 160\pi \quad =1\ 80$$
$$\omega\ 2\pi=160\pi\ 2\pi=80$$

120 80 (maximum minimum). 120 80 t=0,d=0.

$$d=a\cos(\omega t) \text{ or } d=a\sin(\omega t)$$

| a | 2π ω ω 2π $y=5\sin(3t)$ $y=6\cos(\pi t)$ $y=5\cos(\pi\ 2\ t)$ $y=5\sin(3t)$ | a |, 2π ω = 2π 3 . ω 2π = 3 2π . $y=6\cos(\pi t)$ 6. 2π ω = 2π π =2. ω 2π = π 2π = 1 2 .$y=5\cos(\pi\ 2)t$ 5. 2π ω = 2π π 2 =4. 1 4 . t

$$f(t)=a\ e\ -ct\ \sin(\omega t) \text{ or } f(t)=a\ e\ -ct\ \cos(\omega t)$$

c | a | 2π ω 0.5 0.5 0.1. t=0, f= ω 2π

$$\omega\ 2\pi=0.5 \quad \omega=(0.5)2\pi \quad =\pi$$

c=0.5.

$$f(t)=10\ e\ -0.5t\ \cos(\pi t)$$

c=0.1

$$f(t)=10\ e\ -0.1t\ \cos(\pi t)$$

c=0.5 c=0.1. $y=a\ e\ -ct\ \cos(\omega t)$ a=20,c=0.05,p=4a=2,c=1.5,f=3 2π ω ω 2π .$y=20\ e\ -0.05t\ \cos(\pi\ 2\ t)$. $y=2\ e\ -1.5t\ \cos(6\pi t)$. $f(t)=5\ e\ -6t\ \cos(4t)$ 2 π $y=a\ e\ -ct\ \sin(\omega t)$ a=7,c=10,p= π 6a=0.3,c=0.2,f=20 ω 2π ω ,

$$\pi\ 6=2\pi\ \omega \quad \omega\pi=6(2\pi) \quad \omega=12$$

$y=7\ e\ -10t\ \sin(12t)$. ω 2π ,

$$20=\omega\ 2\pi\ 40\pi=\omega$$

0.2 0.3. $y=0.3\ e\ -0.2t\ \sin(40\pi t)$. t=0, t=0. $y=20\ e\ -0.05t\ \cos(\pi\ 2\ t)$ t=0, y=20, t=0

$$y=20\ e\ -0.05(0)\ \cos(\pi\ 2)(0) \quad =20(1)(1) \quad =20$$
$$y=20\ e\ -0.05(0)\ \sin(\pi\ 2)(0) \quad =20(1)(0) \quad =0$$

a=10,c=0.5, p=2.$y=10\ e\ -0.5t\ \cos(\pi t)$ t

$$A(t)=5\ (1-0.30)\ t$$

(1−0.30) t e ct

$$0.7=e\ c \quad c=\ln.7 \quad c=-0.357$$
$$3=2\pi\ \omega\ \omega=2\pi\ 3$$
$$y=\cos(2\pi\ 3\ t)+10$$

t

$$y=-5\ e\ -0.357t\ \cos(2\pi\ 3\ t)+10$$

t=0 1 3 $y=5\cos(6\pi t)$ a e −ct e −ct .

$$a\ e\ -c(t+3)=1\ 2\ a\ e\ -ct$$

c.

a e −c(t+3) = 1 2 a e −ct e −ct · e −3c = 1 2 e −ct Divide out a. e −3c = 1 2 Divide out e −ct . e 3c =2 Take reciprocals.

$$e\ 3c=2 \quad 3c=\ln 2 \quad c=\ln 2\ 3$$

ln 2 3 . $f(x)=\cos(2\pi x)\cos(16\pi x)$. f(x) f(x) $y=\cos(2\pi x)$ $y=-\cos(2\pi x)$

y=A sin(Bt−C)+D or y=A cos(Bt−C)+Dd=a cos(ωt) or d=a sin(ωt)
f(t)=a e −c t sin(ωt) or f(t)=a e −ct cos(ωt)
xy0−43−1629−112−415−1182y=−3cos(π 6 x)−1xy05214−3618510112−3xy02π 47π 223π 4−3π25π 47
3π 225sin(2x)+2xy02π 47π 223π 4−3π25π 473π 22xy011−32−73−3415−36−74cos(xπ 2)−3xy0−2142
10344−254610xy051−325313455−3655−8sin(xπ 2)xy−3−1− 2−2−1−11− 20012 −12132 +1xy−13 −200
12− 323 3314352+ 3tan(xπ 12)f(x)=−30 cos(xπ 6)−20 cos 2 (xπ 6)+80 [0,12]
f(x)=−18 cos(xπ 12)−5 sin(xπ 12)+100 [0,24]f(x)=10−sin(xπ 6)+24 tan(xπ 240) [0,80] 105°F 85°F.
84°F 70°F. 56 °F 47°F 63°F 51°F? 64°F 86°F 70°F? 1.8024 h(t) h(t)=8sin(6πt), t h(t)
h(t)=11sin(12πt), t 1 6 , h(t) h(t)=4cos(π 2 t), t h(t), h(t)=−5 cos(60πt), t 1 30 , P, t. P, t.
P(t)=−15cos(π 6 t)+650+ 55 6 t P, t. P, t.P(t)=−40cos(π 6 t)+800 (1.04) t D, t, D, t,
D(t)=7 (0.89) t cos(40πt) D, t, D, t,D(t)=19 (0.9265) t cos(26πt)20.1 −0.1 0.1 cm, −0.1 0.1 cm,
0.1 cm. 0.1 cm. 22 ∘ 112 ∘ 60 ∘ , 150 ∘ . 90 ∘ y=a b x +csin(π 2 x) xyxyy=6 (5) x +4sin(π 2 x)xy
y=a b x cos(π 2 x)+c xyy=8 (1 2) x cos(π 2 x)+3xy [0,2π).csc 2 t=3
sin −1 (3 3),π− sin −1 (3 3),π+ sin −1 (3 3),2π− sin −1 (3 3)cos 2 x= 1 42 sin θ=−17π 6 , 11π 6
tan x sin x+sin(−x)=09 sin ω−2=4 sin 2 ωsin −1 (1 4),π− sin −1 (1 4)1−2 tan(ω)= tan 2 (ω)
sec x cos x+cos x− 1 sec x1sin 3 x+ cos 2 x sin xsin 2 x+ sec 2 x−1= (1− cos 2 x)(1+ cos 2 x) cos 2 x
tan 3 x csc 2 x cot 2 x cos x sin x=1tan(7π 12)−2− 3cos(25π 12)
sin(70 ∘)cos(25 ∘)−cos(70 ∘)sin(25 ∘)2 2cos(83 ∘)cos(23 ∘)+sin(83 ∘)sin(23 ∘)
cos(4x)−cos(3x)cosx= sin 2 x−4 cos 2 x sin 2 x
cos(4x)−cos(3x)cosx=cos(2x+2x)−cos(x+2x)cos x =cos(2x)cos(2x)−sin(2x
)sin(2x)−cos x cos(2x)cos x+sin x sin(2x)cos x = (cos 2 x− sin 2 x) 2 −4 cos 2
x sin 2 x− cos 2 x(cos 2 x− sin 2 x)+sin x(2)sin x cos x cos x = (cos 2 x− sin 2 x)
2 −4 cos 2 x sin 2 x− cos 2 x(cos 2 x− sin 2 x)+2 sin 2 x cos 2 x = cos 4 x−2 cos
2 x sin 2 x+ sin 4 x−4 cos 2 x sin 2 x− cos 4 x+ cos 2 x sin 2 x+2 sin 2 x cos 2 x =
sin 4 x−4 cos 2 x sin 2 x+ cos 2 x sin 2 x = sin 2 x(sin 2 x+ cos 2 x)−4 cos 2 x
sin 2 x = sin 2 x−4 cos 2 x sin 2 x
cos(3x)− cos 3 x=−cos x sin 2 x−sin x sin(2x)tan(1 2 x)+tan(1 8 x) 1−tan(1 8 x)tan(1 2 x)tan(5 8 x)
cos(sin −1 (0)− cos −1 (1 2))tan(sin −1 (0)+ sin −1 (1 2))3 3 sin(2θ), cos(2θ), tan(2θ)
cos θ=− 1 3 θ [π 2 ,π]. sin(2θ), cos(2θ), tan(2θ) sec θ=− 5 3 θ [π 2 ,π].− 24 25 ,− 7 25 , 24 7
sin(7π 8)sec(3π 8)2(2+ 2)sin(2β),cos(2β),tan(2β),sin(2α),cos(2α), and tan(2α)
sin(β 2),cos(β 2),tan(β 2),sin(α 2),cos(α 2), and tan(α 2)2 10 , 7 2 10 , 1 7 , 3 5 , 4 5 , 3 4
2cos(2x) sin(2x) =cot x−tan xcot x cos(2x)=−sin(2x)+cot x
cotxcos(2x)=cotx(1−2 sin 2 x) =cotx− cosx sinx (2) sin 2 x =−2sinxcosx+cotx
=−sin(2x)+cotx
cos 2 x sin 4 (2x)tan 2 x sin 3 x10sinx−5sin(3x)+sin(5x) 8(cos(2x)+1)cos(π 3) sin(π 4)
2 sin(2π 3) sin(5π 6)3 22 cos(π 5) cos(π 3)sin(π 12)−sin(7π 12)− 2 2cos(5π 12)+cos(7π 12)
sin(9x)cos(3x)1 2 (sin(6x)+sin(12x))cos(7x)cos(12x)sin(11x)+sin(2x)2sin(13 2 x)cos(9 2 x)
cos(6x)+cos(5x) [0,2π).tan x+1=03π 4 , 7π 42 sin(2x)+ 2 =0 [0,2π).2 sin 2 x−sin x=00, π 6 , 5π 6 ,π
cos 2 x−cos x−1=02 sin 2 x+5 sin x+3=03π 2cos x−5 sin(2x)=01 sec 2 x +2+ sin 2 x+4 cos 2 x=0
[0,2π). 3 cot 2 x+cot x=1csc 2 x−3 csc x−4=00.2527,2.8889,4.7124 [0,2π).20 cos 2 x+21 cos x+1=0
sec 2 x−2 sec x=151.3694, 1.9106, 4.3726, 4.9137x012345y1611616xy0−2112−23−54−251
3sin(xπ 2)−2xy−33+2 2−23−1 2 2 −1 0113−2 22−13−1−2 271.6 ∘ t,P(t)=950−450sin(π 6 t) 90°F 30°F
y=3 cos(xπ)1 2y=−2 sin(16xπ) t,C(t)=20sin(2πt)+100 (1.4427) t t,cos(−x)sin x cot x+ sin 2 x
sin(−x)cos(−2x)−sin(−x)cos(−2x)cos(7π 12)2 − 6 4tan(3π 8)tan(sin −1 (2 2)+ tan −1 3)− 2 − 3
2sin(π 4)sin(π 6) [0,2π).cos 2 x− sin 2 x−1=00,πcos 2 x=cos x 4 sin 2 x+2 sin x−3=0
sin −1 (1 4 (13 −1)),π− sin −1 (1 4 (13 −1))cos(2x)+ sin 2 x=02 sin 2 x−sin x=00, π 6 , 5π 6 ,π
cos(2x)+cos(−8x). tan(x)− 3 =0.π 3 +kπ sec 2 x−2 sec x=15 [0,2π) sin(2θ),cos(2θ), tan(2θ)
cot θ=− 3 4 θ [π 2 ,π]. − 24 25 ,− 7 25 , 24 7 sin(θ 2),cos(θ 2), tan(θ 2) cos θ= 7 25 θ sin 4 x

1 8 (3+cos(4x)−4cos(2x))tan 3 x−tan x sec 2 x=tan(−x)sin(3x)−cos x sin(2x)= cos 2 x sin x− sin 3 x sin(3x)−cosxsin(2x)= sin(x+2x)−cosx(2sinxcosx)= sinxcos(2x)+sin(2x)cosx−2sinx cos 2 x= sinx(cos 2 x− sin 2 x)+2sinxcosxcosx−2sinx cos 2 x= sinx cos 2 x− sin 3 +0= cos 2 xsinx− sin 3 x= cos 2 xsinx− sin 3 x

sin(2x) sin x − cos(2x) cos x =sec x y=Acos(Bx+C)+D x012345y−22−22−22y=2cos(πx+π) h(t) h(t)= 1 4 sin(120πt), t 81.5 ◦ , 78.7 ◦ n(t)=8 cos(20πt)cos(1000πt). 6+5 cos(π 6 (1−x)) t t D(t)=2 cos(π 6 t)+108+ 1 4 t, sin α= h b sin β= h a . h h.

h=bsin α and h=asin β

bsin α=asin β (1 ab)(bsin α)=(asin β)(1 ab) Multiply both sides by 1 ab . sin α a = sin β b

sin α a = sin γ c and sin β b = sin γ c

sin α a = sin β b = sin λ c

α a; β b; γ c.

sin α a = sin β b = sin γ c

a sin α = b sin β = c sin γ

β=180°−50°−30° =100°

α=50°a=10. c.

sin(50°) 10 = sin(30°) c c sin(50°) 10 =sin(30°) Multiply both sides by c. c=sin(30°) 10 sin(50°) Multiply by the reciprocal to isolate c. c≈6.5

b,

sin(50°) 10 = sin(100°) b bsin(50°)=10sin(100°) Multiply both sides by b. b= 10sin(100°) sin(50°) Multiply by the reciprocal to isolate b. b≈12.9

α=50° a=10 β=100° b≈12.9 γ=30° c≈6.5

α= 98 ◦ a=34.6 β= 39 ◦ b=22 γ= 43 ◦ c=23.8 a b α. β γ, c. β,

sin α a = sin β b sin(35°) 6 = sin β 8 8sin(35°) 6 =sin β 0.7648≈sin β sin −1 (0.7648)≈49.9° β≈49.9°

β β? γ β β=180°−49.9°=130.1°. γ,

γ=180°−35°−130.1°≈14.9°

γ ′ γ,

γ ′ =180°−35°−49.9°≈95.1°

c c ′ .

c sin(14.9°) = 6 sin(35°) c= 6sin(14.9°) sin(35°) ≈2.7

c ′ sin(95.1°) = 6 sin(35°) c ′ = 6sin(95.1°) sin(35°) ≈10.4

β. α=80°,a=120, b=121,

α=80° a=120 β≈83.2° b=121 γ≈16.8° c≈35.2

α ′ =80° a ′ =120 β ′ ≈96.8° b ′ =121 γ ′ ≈3.2° c ′ ≈6.8

γ=85°, c=12, b=9. β.

sin(85°) 12 = sin β 9 Isolate the unknown. 9sin(85°) 12 =sin β

β, β.

β= sin −1 (9sin(85°) 12) β≈ sin −1 (0.7471) β≈48.3°

β β=180°−48.3°≈131.7°.

α=180°−85°−131.7°≈−36.7°,

β≈48.3°. α=180°−85°−48.3°≈46.7°. a a

sin(85°) 12 = sin(46.7°) a a sin(85°) 12 =sin(46.7°) a= 12sin(46.7°) sin(85°) ≈8.8

α≈46.7° a≈8.8 β≈48.3° b=9 γ=85° c=12

α=80°,a=100, b=10, β≈5.7°,γ≈94.3°,c≈101.3

sin α 10 = sin(50°) 4 sin α= 10sin(50°) 4 sin α≈1.915

α. [−1,1], sin −1 (1.915) a=31, b=26, β=48°. Area= 1 2 bh, b h h sin α= opposite hypotenuse

$\sin\alpha = h\ c\ \ c\sin\alpha = h$. b α', $180-\alpha$.

$$\text{Area}= 1\ 2\ (\text{ base })(\text{ height })= 1\ 2\ b(\ c\sin\alpha\ )$$
$$\text{Area}= 1\ 2\ a(\ b\sin\gamma\ )= 1\ 2\ a(\ c\sin\beta\ )$$
$$\text{Area}= 1\ 2\ bc\sin\alpha \qquad = 1\ 2\ ac\sin\beta \qquad = 1\ 2\ ab\sin\gamma$$

$a=90, b=52,\ \gamma=102°$.

$$\text{Area}= 1\ 2\ ab\sin\gamma\ \text{Area}= 1\ 2\ (90)(52)\sin(102°)\ \text{Area}\approx 2289\ \text{ square units}$$

$\beta=42°,\ a=7.2$ ft, $c=3.4$ ft. 8.2 square feet a, h.

$$\sin(130°)\ 20 = \sin(35°)\ a\ \ a\sin(130°)=20\sin(35°) \qquad a= 20\sin(35°)\ \sin(130°) \qquad a\approx 14.98$$

a, h.

$$\sin(15°)= \text{opposite hypotenuse}\ \sin(15°)= h\ a\ \sin(15°)= h\ 14.98 \qquad h=14.98\sin(15°) \qquad h\approx 3.88$$

B, $\sin\alpha\ a = \sin\beta\ b = \sin\gamma\ c \quad a\sin\alpha = b\sin\beta = c\sin\gamma$

Area$= 1\ 2\ bc\sin\alpha \quad = 1\ 2\ ac\sin\beta \quad = 1\ 2\ ab\sin\gamma$ α a,β b, γ c. $\alpha=43°,\gamma=69°,a=20\alpha=35°,\gamma=73°,c=20$ $\beta=72°,a\approx 12.0,b\approx 19.9\alpha=60°,\ \beta=60°,\gamma=60°a=4,\ \alpha=\ 60°\ ,\beta=100°\ \gamma=20°,b\approx 4.5,c\approx 1.6b=10,\beta=95°,\gamma=\ 30°$ A a, B b, C c. b $A=37°,\ B=49°, c=5$. $b\approx 3.78$ a $A=132°,C=23°,b=10$. c $B=37°,C=21, b=23$. $c\approx 13.70$ α a,β b, γ c. $\alpha=119°,a=14,b=26\gamma=113°,b=10,c=32\ \alpha\approx 50.3°,\beta\approx 16.7°,a\approx 26.7b=3.5,\ c=5.3,\ \gamma=\ 80°$ $a=12,\ c=17,\ \alpha=\ 35°\ \ \gamma\approx 54.3°,\beta\approx 90.7°,b\approx 20.9\ \ \gamma'\approx 125.7°, \beta'\approx 19.3°, b'\approx 6.9a=20.5,\ b=35.0,\ \beta= 25°$ $a=7, c=9,\ \alpha= 43°\ \beta\approx 75.7°,\ \gamma\approx 61.3°,b\approx 9.9\ \ \beta'\approx 18.3°, \gamma'\approx 118.7°, b'\approx 3.2a=7,b=3,\beta=24°b=13,c=5,\gamma= 10°$ $\alpha\approx 143.2°,\beta\approx 26.8°,a\approx 17.3\ \ \ \alpha'\approx 16.8°, \beta'\approx 153.2°, a'\approx 8.3a=2.3,c=1.8,\gamma=28°\beta=119°,b=8.2,a=11.3$A $a=24,b=5,B=22°$.A $a=13,b=6,B=20°$.$A\approx 47.8°\ \ \ A'\approx 132.2°$ B $A=12°,a=2,b=9$.$a=5,c=6,\beta=\ 35°8.6$ $b=11,c=8,\alpha= 28°a=32,b=24,\gamma= 75°370.9a=7.2,b=4.5,\gamma= 43°$ x. 12.312.2 16.0 x, 29.7°$x=76.9°$or $x=103.1°$ x 110.6°$A\approx 39.4$, $C\approx 47.6$, $BC\approx 20.7$ 57.142.0 430.2 10.1 m$\angle$ADC AD AD$\approx$ 13.8 AB AB$\approx$2.8 H JK). N LM). $L\approx 49.7$, $N\approx 56.3$, $LN\approx 5.8$ ABCD $\angle$m 7° 55°, A A B, A B 86.2° 83.9°, A A 67°. 16°. 20° 38° 37° 44°, A A A B C A. BAC ACB A B, 3 1 2 A,B, C, A B. C B A B, A B? ABC A c C (x,y) C

$$\cos\theta= x(\text{adjacent})\ b(\text{hypotenuse})\ \text{ and } \sin\theta= y(\text{opposite})\ b(\text{hypotenuse})$$

θ, $x=b\cos\theta$ $y=b\sin\theta$. (x,y) C ($b\cos\theta$, $b\sin\theta$). (x−c) y a

$a\ 2 = (x-c)\ 2 + y\ 2 \quad = (b\cos\theta-c)\ 2 + (b\sin\theta)\ 2$ Substitute ($b\cos\theta$) for x and ($b\sin\theta$) for y. $=(\ b\ 2\cos 2\ \theta-2bc\cos\theta+ c\ 2\ )+ b\ 2\sin 2\ \theta$ Expand the perfect square. $= b\ 2\cos 2\ \theta+ b\ 2\sin 2\ \theta+ c\ 2\ -2bc\cos\theta$ Group terms noting that $\cos 2\ \theta+ \sin 2\ \theta=1$. $= b\ 2\ (\ \cos 2\ \theta+ \sin 2\ \theta\ )+ c\ 2\ -2bc\cos\theta$ Factor out b 2 . $a\ 2 = b\ 2 + c\ 2\ -2bc\cos\theta$

α,β,γ, a,b, c,

$$a\ 2 = b\ 2 + c\ 2\ -2bc\ \cos\alpha\ b\ 2 = a\ 2 + c\ 2\ -2ac\ \cos\beta\ c\ 2 = a\ 2 + b\ 2\ -2ab\ \cos\gamma$$
$$\cos\alpha= b\ 2 + c\ 2 - a\ 2\ 2bc\ \cos\beta= a\ 2 + c\ 2 - b\ 2\ 2ac\ \cos\gamma= a\ 2 + b\ 2 - c\ 2\ 2ab$$

b, β.

$$b\ 2 = a\ 2 + c\ 2\ -2ac\cos\beta\ b\ 2 = 10\ 2 + 12\ 2\ -2(10)(12)\cos(\ 30\ \circ\ )$$

Substitute the measurements for the known quantities. $b\ 2 =100+144-240(\ 3\ 2\ )$ Evaluate the cosine and begin to simplify. $b\ 2 =244-120\ 3 \quad b= 244-120\ 3$ Use the square root property. $b\approx 6.013$

b, α,

$$\sin\alpha\ a = \sin\beta\ b\ \sin\alpha\ 10 = \sin(30°)\ 6.013\ \ \sin\alpha= 10\sin(30°)\ 6.013$$

Multiply both sides of the equation by 10. $\alpha= \sin -1\ (\ 10\sin(30°)\ 6.013\ )$ Find the inverse sine of $10\sin(30°)\ 6.013$. $\alpha\approx 56.3°$

α $\alpha=180°-56.3°\approx 123.7°$. α α 123.7° 0° 180°. $\alpha\approx 56.3°$,

$$\gamma=180°-30°-56.3°\approx 93.7°$$
$$\alpha\approx 56.3°\ a=10\ \beta=30°\ b\approx 6.013\ \ \gamma\approx 93.7°\ c=12$$

$\alpha=30°,\ b=12,\ c=24$. $a\approx 14.9,\ \beta\approx 23.8°,\ \gamma\approx 126.2°$. α $a=20,\ b=25,\ c=18$. α,

a 2 = b 2 + c 2 −2bccos α 20 2 = 25 2 + 18 2 −2(25)(18)cos α
Substitute the appropriate measurements. 400=625+324−900cos α Simplify in each step.
400=949−900cos α −549=−900cos α Isolate cos α. −549 −900 =cos α
0.61≈cos α cos −1 (0.61)≈α Find the inverse cosine. α≈52.4°
a=5,b=7, c=10, α≈27.7°, β≈40.5°, γ≈111.8° θ. a=2420,b=5050, c=6000. θ a=2420.
a 2 = b 2 + c 2 −2bccos θ (2420) 2 = (5050) 2 +
(6000) 2 −2(5050)(6000)cos θ (2420) 2 − (5050) 2 − (6000) 2 =−2(5050)(6000)cos θ (2420) 2 −
(5050) 2 − (6000) 2 −2(5050)(6000) =cos θ cos θ≈0.9183
θ≈ cos −1 (0.9183) θ≈23.3°
θ= 23.3°
cos(23.3°)= x 5050 x=5050cos(23.3°) x≈4638.15 feet sin(23.3°)= y 5050
y=5050sin(23.3°) y≈1997.5 feet
180°−20°=160°.
x 2 = 8 2 + 10 2 −2(8)(10)cos(160°) x 2 =314.35 x= 314.35 x≈17.7 miles
a,b, c
Area= s(s−a)(s−b)(s−c)
s= (a+b+c) 2 s.
s= (a+b+c) 2 s= (10+15+7) 2 =16
Area= s(s−a)(s−b)(s−c) Area= 16(16−10)(16−15)(16−7) Area≈29.4
a=29.7 ft,b=42.3 ft, c=38.4 ft. s,
s= (62.4+43.5+34.1) 2 s=70 m
Area= 70(70−62.4)(70−43.5)(70−34.1) Area= 506,118.2 Area≈711.4
a=4.38 ft ,b=3.79 ft, c=5.22 ft.
a 2 = b 2 + c 2 −2bccos α b 2 = a 2 + c 2 −2accos β c 2 = a 2 + b 2 −2abcos γ
Area= s(s−a)(s−b)(s−c) where s= (a+b+c) 2 s s α a,β b, γ c. γ=41.2°,a=2.49,b=3.13α=120°,b=6,c=7
β=58.7°,a=10.6,c=15.7γ=115°,a=18,b=23α=119°,a=26,b=14γ=113°,b=10,c=32β=67°,a=49,b=38
α=43.1°,a=184.2,b=242.8α=36.6°,a=186.2,b=242.2β= 50° ,a=105,b=45 a=42,b=19,c=30; A.
a=14, b=13, c=20; C. a=16,b=31,c=20; B. a=13, b=22, c=28; A.a=108, b=132, c=160; C.
A=35°,b=8,c=11B≈45.9°,C≈99.1°,a≈6.4B=88°,a=4.4,c=5.2C=121°,a=21,b=37A≈20.6°,B≈38.4°,c≈51.1
a=13,b=11,c=15a=3.1,b=3.5,c=5A≈37.8°,B≈43.8,C≈98.4°a=51,b=25,c=29a= 1 2 m,b= 1 3 m,c= 1 4 m
a=12.4 ft, b=13.7 ft, c=20.2 fta=1.6 yd, b=2.6 yd, c=4.1 ydx. A. x 2 =25+36−60cos(52) (x,y) (r,θ) r
θ, r. θ, r θ. θ r, (2, π 4), π 4 (3, π 2) π 2 π 2 (2, π 3) (−2, π 6) π 6 r=−2. r (2, π 6) π 6
π 6 (2, π 6) (3,− π 6) (2, 9π 4) x, y, r, θ.
cos θ= x r →x=rcos θ sin θ= y r →y=rsin θ
cos θ sin θ (r, θ) (x, y),
cos θ= x r →x=rcos θ
sin θ= y r →y=rsin θ
(r,θ), x=rcos θ y=rsin θ. cos θ sin θ. cos θ r sin θ r (3, π 2)
x=rcos θ x=3cos π 2 =0 y=rsin θ y=3sin π 2 =3
(0,3). (−2,0)
x=rcos θ x=−2cos(0)=−2 y=rsin θ y=−2sin(0)=0
(−2,0). (−1, 2π 3) (x,y)=(1 2 ,− 3 2)
cos θ= x r or x=rcos θ sin θ= y r or y=rsin θ r 2 = x 2 + y 2 tan θ= y x
(3,3) (3,3) θ, tan θ= y x .
tan θ= 3 3 tan θ=1 tan −1 (1)= π 4
r, x y r= x 2 + y 2 . r π 4
r= 3 2 + 3 2 r= 9+9 r= 18 =3 2
r=3 2 θ= π 4 , (3 2 , π 4). (−3 2 , 5π 4) (3 2 ,− 7π 4) (3 2 , π 4). (−3 2 , 5π 4) π, π 4 . −3 2 .

5π 4 r (3 2 , π 4). (3 2 ,− 7π 4) − 7π 4 , π 4 . 3 2 , r 1 , r 2 , . . . , r 6 .r. x 2 + y 2 =9 x y r θ. r θ. (x,y) (r,θ). x=rcos θ y=rsin θ, r.

(rcos θ) 2 + (rsin θ) 2 =9 r 2 cos 2 θ+ r 2 sin 2 θ=9 r 2 (cos 2 θ+ sin 2 θ)=9 r 2 (1)=9 Substitute cos 2 θ+ sin 2 θ=1. r=±3 Use the square root property.

x 2 + y 2 =9,r=3, r=−3 x 2 + y 2 =9 r=3 y.

x 2 + y 2 =9 y 2 =9− x 2 y=± 9− x 2

Y 1 = 9− x 2 Y 2 =− 9− x 2 . x 2 + y 2 =6y

r 2 =6y Use x 2 + y 2 = r 2 . r 2 =6rsin θ Substitute y=rsin θ. r 2 −6rsin θ=0 Set equal to 0. r(r−6sin θ)=0 Factor and solve. r=0 We reject r=0, as it only represents one point, (0,0). or r=6sin θ

x 2 + y 2 =6y r=6sin θ x 2 + y 2 =6y r=6sin θ y=3x+2 x=rcos θ y=rsin θ.

y=3x+2 rsin θ=3rcos θ+2 rsin θ−3rcos θ=2 r(sin θ−3cos θ)=2 Isolate r. r= 2 sin θ−3cos θ Solve for r.

y 2 =3− x 2 r= 3 r=2sec θ

r=2sec θ r= 2 cos θ rcos θ=2 x=2

r=2sec θ x=2 x=c r=csec θ r=2csc θ y=2. r=ccsc θ y=c. r= 3 1−2cos θ θ r, x y. r x y, x 2 + y 2 = r 2 .

r= 3 1−2cos θ r(1−2cos θ)=3 r(1−2(x r))=3 Use cos θ= x r to eliminate θ. r−2x=3 r=3+2x Isolate r. r 2 = (3+2x) 2 Square both sides. x 2 + y 2 = (3+2x) 2 Use x 2 + y 2 = r 2 .

x 2 + y 2 = (3+2x) 2 . y.

x 2 + y 2 = (3+2x) 2 y 2 = (3+2x) 2 − x 2 y=± (3+2x) 2 − x 2

r θ x y, y

x 2 + y 2 = (3+2x) 2 x 2 + y 2 − (3+2x) 2 =0 x 2 + y 2 −(9+12x+4 x 2)=0 x 2 + y 2 −9−12x−4 x 2 =0 −3 x 2 −12x+ y 2 =9 Multiply through by −1. 3 x 2 +12x− y 2 =−9 3(x 2 +4x+)− y 2 =−9 Organize terms to complete the square for x. 3(x 2 +4x+4)− y 2 =−9+12 3 (x+2) 2 − y 2 =3 (x+2) 2 − y 2 3 =1

r=2sin θ x 2 + y 2 =2y x 2 + (y−1) 2 =1 r=sin(2θ)

r=sin(2θ) Use the double angle identity for sine. r=2sin θcos θ Use cos θ= x r and sin θ= y r . r=2(x r)(y r) Simplify. r= 2xy r 2 Multiply both sides by r 2 . r 3 =2xy (x 2 + y 2) 3 =2xy As x 2 + y 2 = r 2 ,r= x 2 + y 2 . (x 2 + y 2) 3 2 =2xy or x 2 + y 2 = (2xy) 2 3

cos θ= x r →x=rcos θ sin θ= y r →y=rsin θ r 2 = x 2 + y 2 tan θ= y x (r,θ), θ>0, θ, r θ. θ r θ. r θ. x=rcos θ y=rsin θ. cos θ= x r ,sin θ= y r ,tan θ= y x , r= x 2 + y 2 . θ r r r r θ (3, π 2) (−3, π 2) (−3, π 2) (3,− π 2) (−3, π 2) π 2 (3,− π 2) − π 2 r>0 0≤θ≤2π. θ (7, 7π 6)(5,π)(−5,0) (6,− π 4)(−3, π 6)(− 3 3 2 ,− 3 2)(4, 7π 4) r>0, 0≤θ<2π. (4,2)(2 5 , 0.464)(−4,6)(3,−5) (34 ,5.253)(−10,−13)(8,8)(8 2 , π 4)x=3y=4r=4cscθy=4 x 2y=2 x 4r= sinθ 2co s 4 θ 3x 2 + y 2 =4y x 2 + y 2 =3xr=3cosθx 2 − y 2 =xx 2 − y 2 =3yr= 3sinθ cos(2θ)x 2 + y 2 =9x 2 =9yr= 9sinθ cos 2 θ y 2 =9x9xy=1r= 1 9cosθsinθr=3sin θr=4cos θx 2 + y 2 =4x (x−2) 2 4 + y 2 4 =1;r= 4 sin θ+7cos θ r= 6 cos θ+3sin θ3y+x=6; r=2sec θr=3csc θy=3; r= rcos θ+2r 2 =4sec θ csc θxy=4; r=4r 2 =4x 2 + y 2 =4; r= 1 4cos θ−3sin θr= 3 cos θ−5sin θx−5y=3; (3, 3π 4)(5,π)(−2, π 3)(−1,− π 2)(3.5, 7π 4)(−4, π 3) (5, π 2)(4, −5π 4)(3, 5π 6)(−1.5, 7π 6)(−2, π 4)(1, 3π 2)5x−y=6r= 6 5cosθ−sinθ2x+7y=−3 x 2 + (y−1) 2 =1r=2sinθ(x+2) 2 + (y+3) 2 =13x=2r= 2 cosθx 2 + y 2 =5yx 2 + y 2 =3xr=3cosθr=6 r=−4x 2 + y 2 =16θ=− 2π 3θ= π 4y=xr=sec θr=−10sin θx 2 + (y+5) 2 =25r=3cos θ (2,− π 5). (1.618,−1.176) (−3, 3π 7). (−7,8) (10.630,131.186°) (3,−4) (−2,0) (2,3.14)or(2,π) r=asec θ;a>0. r=asec θ;a<0. a r=acsc θ;a>0. r=acsc θ;a<0. a r<40≤θ≤ π 4θ= π 4 , r ≥ 2θ= π 4 , r ≥−3 0≤θ≤ π 3 , r < 2−π 6 <θ≤ π 3 ,−3<r < 2 (r,θ), θ r θ (r,θ). r θ y= x 2 x y r θ (r,θ) θ, r θ. r) θ= π 2 (r,θ) (−r,−θ) r=2sin θ;

r=2sin θ −r=2sin(−θ) Replace (r,θ) with (−r,−θ). −r=−2sin θ Identity: sin(−θ)=−sin θ. r=2sin θ Multiply both sides by−1.

θ= π 2 . x (r,θ) (r,−θ) (−r,π−θ) r=1−2cos θ.

r=1−2cos θ r=1−2cos(−θ) Replace (r,θ) with (r,−θ). r=1−2cos θ Even/Odd identity

(r,θ) (−r,θ) r=2sin(3θ).

r=2sin(3θ) −r=2sin(3θ)

θ= π 2 , θ= π 2 (r,θ) (−r,−θ) (r,θ) (r,−θ) (−r,π−θ) (r,θ) (−r,θ) (r,θ): (−r,−θ) θ= π 2 (r,−θ) (−r,θ) r=2sin θ (r,θ) (−r,−θ) θ= π 2 .

−r=2sin(−θ) −r=−2sin θ Even-odd identity r=2sin θ Multiply by −1 Passed θ −θ

r=2sin(−θ) r=−2sin θ Even-odd identity r=−2sin θ≠2sin θ Failed r −r −r=2sin θ r=−2sin θ≠2sin θ Failed

r=2sin θ (0,1) r=1 θ= π 2 . r=−2cos θ. θ= π 2 x y θ r θ r. θ r θ r=0. x. r=0, θ. θ r=5cos θ; θ=0, 5cos θ, θ=0 | r |. θ= π 2 , r=5sin θ, θ= π 2 | r |. r θ=0. | r | r=2sin θ. r θ.

2sin θ=0 sin θ=0 θ= sin −1 0 θ=nπ where n is an integer

θ 0.

r=2sin(0) r=0

(0,0) (0,±nπ) sin θ, θ= π 2 ±2kπ sin(π 2)=1. π 2 θ.

r=2sin(π 2) r=2(1) r=2

(2, π 2) θr=2sin θrr=2sin(0)=00π 6r=2sin(π 6)=11π 3r=2sin(π 3)≈1.731.73π 2r=2sin(π 2)=222π 3 r=2sin(2π 3)≈1.731.735π 6r=2sin(5π 6)=11πr=2sin(π)=00 | r |: r=3cos θ. (0, π 2), (3,0). r=acos θ r=asin θ, a | a | 2 , r=acos θ, (a 2 ,0). r=asin θ, (a 2 ,π). r=4cos θ. | r | r=4cos θ. r=0, θ θ= π 2 ±kπ. (0, π 2) . r, θ=0±2kπ. θ=0

r=4cos θ r=4cos(0) r=4(1)=4

(4, 0). r=4cos θ θ [0, π]. θπ 6π 4π 3π 22π 33π 45π 6πr r=a±bcos θ r=a±bsin θ a>0, b>0, a b =1. r=0. r θ. r=2+2cos θ. r=0, θ=π+2kπ. (0,π). r=2+2cos θ cos θ cos θ=1 θ=0. θ=0 r.

r=2+2cos(0) r=2+2(1)=4

(4,0) [0, π]. θ0π 4π 22π 3πr 1< a b <2 a b ≥2. r=a±bcos θ r=a±bsin θ a>0, b>0, and 1< a b <2. r=4−3sin θ. θ= π 2 , r=0 θ θ θ sin θ>1. θ θ sin θ>1. r θ=0.

r(0)=4−3sin(0) r=4−3⋅0=4

(4,0). θ= π 2 , θ= π 2 r. r=1. θ0π 6π 3π 22π 35π 6π7π 64π 33π 25π 311π 62πr sin θ θ= π 2 , r=3−2cos θ. r=a±bcos θ r=a±bsin θ a>0, b>0, a<b. r=2+5cos θ. r=0, θ=1.98. | r | cos θ=1 θ=0. θ0π 6 π 3π 22π 35π 6π7π 64π 33π 25π 311π 62πr θ=π. ∞ r 2 = a 2 cos 2θ r 2 = a 2 sin 2θ a≠0. r 2 = a 2 sin 2θ r 2 = a 2 cos 2θ θ= π 2 , r 2 =4cos 2θ. θ= π 2 , u=2θ.

0=4cos 2θ 0=4cos u 0=cos u cos −1 0= π 2 u= π 2

Substitute 2θ back in for u. 2θ= π 2 θ= π 4

(0, π 4) cos u=1 u=0, cos 2θ=1 2θ=0.

r 2 =4cos(0) r 2 =4(1)=4 r=± 4 =2

θ= π 2 ,θπ 6π 4π 3π 2r22 u=2θ r. θ. 4cos(2θ) r=acos nθ r=asin nθ a≠0. n 2n n n r=2cos 4θ. θ= π 2 u=4θ.

0=2cos 4θ 0=cos 4θ 0=cos u cos −1 0=u u= π 2

4θ= π 2 θ= π 8

θ= π 8 . (0, π 8) | r |. cos u=1 θ=0.

r=2cos(4⋅0) r=2cos(0) r=2(1)=2

(2,0) θπ 8π 43π 8π 25π 83π 4r r=0 θ= π 8 , π 8 r=0, r=2, 2n n r=4sin(2θ). n r=2sin(5θ). θ= π 2 . u=5θ.

0=2sin(5θ) 0=sin u sin −1 0=0 u=0 5θ=0 θ=0

sin θ

r=2sin(5⋅ π 2) r=2(1)=2

n n, θπ 6π 3π 22π 35π 6πr n r=3cos(3θ). n r=θ θ≥0. θ r [0,2π], r θ r=θ [0,2π]. r θ, θπ 4π 2π3π 27π 4

2πr [0,2π]. (−∞,∞). r=−θ [0,4π]. θ= π 2 , θ r. θ r=0 θ. r=acos θ r=asin θ. r=a±bcos θ r=a±bsin θ, a>0, b>0, a b =1. r=a±bcos θ r=a±bsin θ 1< a b <2. r=a±bcos θ r=a±bsin θ a>0, b>0, a<b. r 2 = a 2 cos 2θ r 2 = a 2 sin 2θ, a≠0. r=acos nθ r=asin nθ, a≠0; n 2n n n r=0, θ≥0. x θ= π 2 y θ=0, π 2 , π and 3π 2 , r=5cos 3θr=3−3cos θr=3+2sin θr=3sin 2θθ= π 2 ,r=4r=2θr=4cos θ 2r= 2 θ r=3 1− cos 2 θr= 5sin 2θr=3cos θr=4sin θr=2+2cos θr=2−2cos θr=5−5sin θr=3+3sin θr=3+2sin θ r=7+4sin θr=4+3cos θr=5+4cos θr=10+9cos θr=1+3sin θr=2+5sin θr=5+7sin θr=2+4cos θr=5+6cos θ r 2 =36cos(2θ)r 2 =10cos(2θ)r 2 =4sin(2θ)r 2 =10sin(2θ)r=3sin(2θ)r=3cos(2θ)r=5sin(3θ)r=4sin(4θ) r=4sin(5θ)r=−θr=2θr=−3θr= 1 θr= 1 θr=2sin θtan θ,r=2 1− sin 2 θr=5+cos(4θ)r=2−sin(2θ)r= θ 2 r=θ+1r=θsin θr=θcos θ [0,4π] r=θ,r=−θr=θ,r=θ+sin θr=sin θ+θ,r=sin θ−θr=2sin(θ 2),r=θsin(θ 2) θ 4π r=sin(cos(3θ)) r=sin(3θ) r=sin(16 5 θ) [0,4π],[0,8π],[0,12π], [0,16π]. r=sin θ+ (sin(5 2 θ)) 3 [0,4π].

r 1 =3sin(3θ) r 2 =2sin(3θ) r 3 =sin(3θ)

r 1 =3+3cos θ r 2 =2+2cos θ r 3 =1+cos θ

r 1 =3θ r 2 =2θ r 3 =θ

r 1 =3+2sin θ, r 2 =2r 1 =6−4cos θ, r 2 =4(4, π 3),(4, 5π 3)r 1 =1+sin θ, r 2 =3sin θ r 1 =1+cos θ, r 2 =3cos θ(3 2 , π 3),(3 2 , 5π 3)r 1 =cos(2θ), r 2 =sin(2θ) r 1 = sin 2 (2θ), r 2 =1−cos(4θ)(0, π 2), (0,π), (0, 3π 2), (0,2π)r 1 = 3 , r 2 =2sin(θ) r 1 2 =sin θ, r 2 2 =cos θ(8 4 2 , π 4), (8 4 2 , 5π 4) θ= 3π 4 , 7π 4 r r 1 =1+cos θ, r 2 =1−sin θ r=θ. r=aθ. r=θ, r=a±bcos θ r=a±bsin θ, a b =1 r=a±bcos θ r=a±bsin θ a b ≥2 r=a±bcos θ r=a±bsin θ 1< a b <2 r=a±bcos θ r=a±b sin θ a<b r 2 = a 2 cos 2θ r 2 = a 2 sin 2θ, a≠0 r=a±bcos θ r=a±bsin θ a>0,b>0, a b >1; r=acos nθ r=asin nθ; n 2n n n a+bi a, bi. a+bi, a b 2−3i 1+5i | z |. z=2+4i, | z |. z=x+yi, z

| z |= x 2 + y 2

(x,y). (0, 0). z= 5 −i.

| z |= x 2 + y 2 | z |= (5) 2 + (−1) 2 | z |= 5+1 | z |= 6

z=12−5i. z=3−4i, | z |.

| z |= x 2 + y 2 | z |= (3) 2 + (−4) 2 | z |= 9+16 | z |= 25 | z |=5

z z=1−7i, | z |.| z |= 50 =5 2 θ r. z=x+yi,

x=rcos θ y=rsin θ r= x 2 + y 2

(x,y). r, θ

z=x+yi z=rcos θ+(rsin θ)i z=r(cos θ+isin θ)

x=rcos θ y=rsin θ r= x 2 + y 2

z=x+yi z=(rcos θ)+i(rsin θ) z=r(cos θ+isin θ)

r θ rcis θ r(cos θ+isin θ). 4i z=4i z=0+4i. r

r= x 2 + y 2 r= 0 2 + 4 2 r= 16 r=4

x. x=rcos θ, x=0, θ= π 2 . z=0+4i z=4(cos(π 2)+isin(π 2)) 4cis(π 2). z=3i r cis θ z=3(cos(π 2)+isin(π 2)) −4+4i. r.

r= x 2 + y 2 r= (−4) 2 +(4 2) r= 32 r=4 2

θ

cos θ= x r cos θ= −4 4 2 cos θ=− 1 2 θ= cos −1 (− 1 2)= 3π 4

4 2 cis(3π 4). z= 3 +i z=2(cos(π 6)+isin(π 6)) z=r(cos θ+isin θ), cos θ sin θ. r.

z=12(cos(π 6)+isin(π 6))

cos(π 6)= 3 2 and sin(π 6)= 1 2

z=12(3 2 + 1 2 i)

z=12(3 2 + 1 2 i) =(12) 3 2 +(12) 1 2 i =6 3 +6i

6 3 +6i. r=13 tan θ= 5 12 . tan θ= 5 12 , tan θ= y x , r= x 2 + y 2 = 12 2 + 5 2 =13. cos θ= x r sin θ= y r .

z=13(cos θ+isin θ) =13(12 13 + 5 13 i) =12+5i

12+5i.

z=4(cos 11π 6 +isin 11π 6)

z=2 3 −2i z 1 = r 1 (cos θ 1 +isin θ 1) z 2 = r 2 (cos θ 2 +isin θ 2),

z 1 z 2 = r 1 r 2 [cos(θ 1 + θ 2)+isin(θ 1 + θ 2)] z 1 z 2 = r 1 r 2 cis(θ 1 + θ 2)

z 1 z 2 , z 1 =4(cos(80°)+isin(80°)) z 2 =2(cos(145°)+isin(145°)).

z 1 z 2 =4·2[cos(80°+145°)+isin(80°+145°)] z 1 z 2 =8[cos(225°)+isin(225°)] z 1 z 2 =8[cos(5π 4)+isin(5π 4)] z 1 z 2 =8[− 2 2 +i(− 2 2)] z 1 z 2 =−4 2 −4i 2

z 1 = r 1 (cos θ 1 +isin θ 1) z 2 = r 2 (cos θ 2 +isin θ 2),

z 1 z 2 = r 1 r 2 [cos(θ 1 − θ 2)+isin(θ 1 − θ 2)], z 2 ≠0 z 1 z 2 = r 1 r 2 cis(θ 1 − θ 2), z 2 ≠0

r 1 r 2 . θ 1 − θ 2 . z=r(cos θ+isin θ). r r 1 r 2 , θ θ 1 − θ 2 . r. z 1 =2(cos(213°)+isin(213°))

z 2 =4(cos(33°)+isin(33°)).

z 1 z 2 = 2 4 [cos(213°−33°)+isin(213°−33°)] z 1 z 2 = 1 2 [cos(180°)+isin(180°)] z 1 z 2 = 1 2 [−1+0i] z 1 z 2 =− 1 2 +0i z 1 z 2 =− 1 2

z 1 =2 3 (cos(150°)+isin(150°)) z 2 =2(cos(30°)+isin(30°)). z 1 z 2 =−4 3 ; z 1 z 2 =− 3 2 + 3 2 i

n, z n nth n. z=r(cos θ+isin θ)

z n = r n [cos(nθ)+isin(nθ)] z n = r n cis(nθ)

n (1+i) 5 (1+i) r.

r= x 2 + y 2 r= (1) 2 + (1) 2 r= 2

θ. tan θ= y x

tan θ= 1 1 tan θ=1 θ= π 4

(a+bi) n = r n [cos(nθ)+isin(nθ)] (1+i) 5 = (2) 5 [cos(5· π 4)+isin(5· π 4)] (1+i) 5 =4 2 [cos(5π 4)+isin(5π 4)] (1+i) 5 =4 2 [− 2 2 +i(− 2 2)] (1+i) 5 =−4−4i

nth nth nth

z 1 n = r 1 n [cos(θ n + 2kπ n)+isin(θ n + 2kπ n)]

k=0, 1, 2, 3, . . . , n−1. 2kπ n θ n z=8(cos(2π 3)+isin(2π 3)).

z 1 3 = 8 1 3 [cos(2π 3 3 + 2kπ 3)+isin(2π 3 3 + 2kπ 3)] z 1 3 =2[cos(2π 9 + 2kπ 3)+isin(2π 9 + 2kπ 3)]

k=0, 1, 2. k=0,

z 1 3 =2(cos(2π 9)+isin(2π 9))

k=1,

z 1 3 =2[cos(2π 9 + 6π 9)+isin(2π 9 + 6π 9)] Add 2(1)π 3 to each angle. z 1 3 =2(cos(8π 9)+isin(8π 9))

k=2,

z 1 3 =2[cos(2π 9 + 12π 9)+isin(2π 9 + 12π 9)] Add 2(2)π 3 to each angle. z 1 3 =2(cos(14π 9)+isin(14π 9))

k=1,

2π 3 3 + 2(1)π 3 = 2π 3 (1 3)+ 2(1)π 3 (3 3) = 2π 9 + 6π 9 = 8π 9

16(cos(120°)+isin(120°)).z 0 =2(cos(30°)+isin(30°))z 1 =2(cos(120°)+isin(120°))

z 2 =2(cos(210°)+isin(210°))z 3 =2(cos(300°)+isin(300°)) a+bi | z |= a 2 + b 2 . x=rcos θ,y=rsin θ,

r= x 2 + y 2 . z=r(cos θ+isin θ). r. z n , r n, θ n. a+bi. i= −1 x=rcosθ y=rsinθ.

z n = r n (cos(nθ)+isin(nθ)) 5+3i−7+i5 2−3−3i2 −6i382i2.2−3.1i14.452+2i8−4i4 5 cis(333.4°)

− 1 2 − 1 2 i3 +i2cis(π 6)3iz=7cis(π 6)7 3 2 +i 7 2z=2cis(π 3)z=4cis(7π 6)−2 3 −2iz=7cis(25°)

z=3cis(240°)−1.5−i 3 3 2z= 2 cis(100°) z 1 z 2 z 1 =2 3 cis(116°); z 2 =2cis(82°)4 3 cis(198°)

z 1 = 2 cis(205°); z 2 =2 2 cis(118°)z 1 =3cis(120°); z 2 = 1 4 cis(60°)3 4 cis(180°)

z 1 =3cis(π 4); z 2 =5cis(π 6)z 1 = 5 cis(5π 8); z 2 = 15 cis(π 12)5 3 cis(17π 24)

z 1 =4cis(π 2); z 2 =2cis(π 4) z 1 z 2 z 1 =21cis(135°); z 2 =3cis(65°)7cis(70°)

z 1 = 2 cis(90°); z 2 =2cis(60°)z 1 =15cis(120°); z 2 =3cis(40°)5cis(80°)

z 1 =6cis(π 3); z 2 =2cis(π 4)z 1 =5 2 cis(π); z 2 = 2 cis(2π 3)5cis(π 3)

z 1 =2cis(3π 5); z 2 =3cis(π 4) z 3 z=5cis(45°).125cis(135°) z 4 z=2cis(70°). z 2

z=3cis(120°).9cis(240°) z 2 z=4cis(π 4). z 4 z=cis(3π 16).cis(3π 4) z 3 z=3cis(5π 3). z

z=27cis(240°). 3cis(80°),3cis(200°),3cis(320°) z z=16cis(100°). z z=32cis(2π 3).

2 4 3 cis(2π 9),2 4 3 cis(8π 9),2 4 3 cis(14π 9) z z=32cis(π). z z=8cis(7π 4).
2 2 cis(7π 8),2 2 cis(15π 8)2+4i−3−3i5−4i−1−5i3+2i2i−46−2i−2+i1−4i 5+5i 3−2i 3.61 e −0.59i
−3−8i 4cis(120°) −2+3.46i 2cis(45°) 5cis(210°) −4.33−2.50i nth n, z n nth n (x,y); θ r;
x=rcos θ, y=rsin θ, r= x 2 + y 2 x(t) y(t) t t, x y x y t. x t, x(t), y t, y(t). x(t) y(t) (x(t), y(t)). x y ,
x,y, t. x y. y=f(x), r 2 = x 2 + y 2 . y y=± r 2 − x 2 , y 1 = r 2 − x 2 y 2 =− r 2 − x 2 . y 1 y 2 t I.
(x(t), y(t)), x=f(t) y=g(t), t. x=f(t) y=g(t) y= x 2 −1 x(t)=t. x(t)=t, y(t) x x(t). y(t)= t 2 −1.tx(t)
y(t)−4−4y(−4)= (−4) 2 −1=15−3−3y(−3)= (−3) 2 −1=8−2−2y(−2)= (−2) 2 −1=3−1−1
y(−1)= (−1) 2 −1=000y(0)= (0) 2 −1=−111y(1)= (1) 2 −1=022y(2)= (2) 2 −1=333
y(3)= (3) 2 −1=844y(4)= (4) 2 −1=15 t y(t)= t 2 −1 y= x 2 −1 y= x 2 −1.
x(t)=t−3, y(t)=2t+4; −1≤t≤2.tx(t)y(t)−1−420−341−262−18 y=1− x 2 , x(t)=t. x(t)=t t x y
y(t)=1− t 2 .

x(t)=t y(t)=1− t 2

t=0, t=−3 t=3. x(t) t x(t)=t. y(t). tx(t)=ty(t)=1− t 2−3−3y(−3)=1− (−3) 2 =−8−2−2
y(−2)=1− (−2) 2 =−3−1−1y(−1)=1− (−1) 2 =000y(0)=1−0=111y(1)=1− (1) 2 =022y(2)=1− (2) 2 =−3
33y(3)=1− (3) 2 =−8 y=1− t 2 [−3, 3], t. t. x=0, x x= y 3 −2y.x(t)= t 3 −2t y(t)=t (−5, 3) (3, −1) −5
8 m 4 s , 2 m/s. x(t)=2t−5. y=mx+b, 2t=mx −5=b. −1, −4 m 4 s , −1m/s. y(t)=−t+3. x y t

x(t)=2t−5 y(t)=−t+3

t,x, y t t tx(t)=2t−5y(t)=−t+30x=2(0)−5=−5y=−(0)+3=31x=2(1)−5=−3y=−(1)+3=22x=2(2)−5=−1
y=−(2)+3=13x=2(3)−5=1y=−(3)+3=04x=2(4)−5=3y=−(4)+3=−1 x t, y t, y x, t. x y. t t. t x y.
x(t)= t 2 +1 y(t)=2+t, y t.

y=2+t y−2=t

y−2 t x(t).

x= t 2 +1 x= (y−2) 2 +1 Substitute the expression for t into x. x= y 2 −4y+4+1 x= y 2 −4y+5 x= y 2 −4y+5

x= y 2 −4y+5. x y. (1,2), x(t) y(t), t x y y x. y x.

x(t)=2 t 2 +6 y(t)=5−t

y=5− 1 2 x−3 x(t)= e −t y(t)=3 e t , t>0. e t .

x= e −t e t = 1 x

y(t).

y=3 e t y=3(1 x) y= 3 x

y= 3 x . t>0. y= 3 x x≠0. x(t)= t +2 y(t)=log(t). t.

x= t +2 x−2= t (x−2) 2 =t Square both sides.

ty

y=log(t) y=log (x−2) 2

y=log (x−2) 2 . x= t +2 t>0; x x>2. y=log(t) t>0; y=log (x−2) 2 x>2.

x(t)= t 2 y(t)=ln t t>0

y=ln x

x(t)=acos t y(t)=bsin t

cos t sin t,

x a =cos t y b =sin t
cos 2 t+ sin 2 t=1
cos 2 t+ sin 2 t= (x a) 2 + (y b) 2 =1

0≤t≤2π

x(t)=4cos t y(t)=3sin t

cos t sin t,

x=4cos t x 4 =cos t y=3sin t y 3 =sin t
cos 2 t+ sin 2 t=1 (x 4) 2 + (y 3) 2 =1 x 2 16 + y 2 9 =1

x 2 16 + y 2 9 =1 (0,0). t=0 (4,0), t= π 2 (0,3). t. x(t)=2cos t y(t)=3sin t.x 2 4 + y 2 9 =1 x(t)=t.
y(t)

x(t)=t y(t)= t 2 −3

x t. y= x 2 −3.

x(t)=3t−2 y(t)=t+1

x t. y

x=3t−2 x+2=3t x+2 3 =t

t y

y=t+1 y=(x+2 3)+1 y= x 3 + 2 3 +1 y= 1 3 x+ 5 3

y t x

y=t+1 y−1=t

y.

x=3(y−1)−2 x=3y−3−2 x=3y−5 x+5=3y x+5 3 =y y= 1 3 x+ 5 3

x(t)= t 3 y(t)= t 6 .y= x 2 x, y x y= (x+3) 2 +1. x(t)=t. y(t)= (t+3) 2 +1. x=t+3?

y= (x+3) 2 +1 y= ((t+3)+3) 2 +1 y= (t+6) 2 +1

x(t)=t+3 y(t)= (t+6) 2 +1

x y, t, x y t, t, t x x=f(t) y=f(t). t, t { x(t)=5−t y(t)=8−2ty=−2+2x{ x(t)=6−3t y(t)=10−t
{ x(t)=2t+1 y(t)=3 ty=3 x−1 2{ x(t)=3t−1 y(t)=2 t 2{ x(t)=2 e t y(t)=1−5tx=2 e 1−y 5
y=1−5ln(x 2){ x(t)= e −2t y(t)=2 e −t{ x(t)=4log(t) y(t)=3+2tx=4log(y−3 2){ x(t)=log(2t) y(t)= t−1
{ x(t)= t 3 −t y(t)=2tx= (y 2) 3 − y 2{ x(t)=t− t 4 y(t)=t+2{ x(t)= e 2t y(t)= e 6ty= x 3
{ x(t)= t 5 y(t)= t 10{ x(t)=4cos t y(t)=5sin t (x 4) 2 + (y 5) 2 =1{ x(t)=3sin t y(t)=6cos t
{ x(t)=2 cos 2 t y(t)=−sin t y 2 =1− 1 2 x{ x(t)=cos t+4 y(t)=2 sin 2 t{ x(t)=t−1 y(t)= t 2y= x 2 +2x+1
{ x(t)=−t y(t)= t 3 +1{ x(t)=2t−1 y(t)= t 3 −2y= (x+1 2) 3 −2x-y{ x(t)=2t−1 y(t)=t+4{ x(t)=4−t y(t)=3t+2
y=−3x+14{ x(t)=2t−1 y(t)=5t{ x(t)=4t−1 y(t)=4t+2y=x+3x(t)=t y(t)=t.y(x)=3 x 2 +3y(x)=2sin x+1
{ x(t)=t y(t)=2sint+1x(y)=3log(y)+yx(y)= y +2y{ x(t)= t +2t y(t)=tx(t)=acos t y(t)=bsin t.
x 2 4 + y 2 9 =1x 2 16 + y 2 36 =1{ x(t)=4cos t y(t)=6sin t ; x 2 + y 2 =16x 2 + y 2 =10
{ x(t)= 10 cost y(t)= 10 sint ; (3,0) (−2,−5) (3,0) t=0, (−2,−5) t=1. (−1,0) (3,−2) (−1,0) t=0, (3,−2)
t=1.{ x(t)=−1+4t y(t)=−2t (−1,5) (2,3) (−1,5) t=0, (2,3) t=1. (4,1) (6,−2) (4,1) t=0, (6,−2) t=1.
{ x(t)=4+2t y(t)=1−3t{ x 1 (t)=3t y 1 (t)=2t−1 and { x 2 (t)=t+3 y 2 (t)=4t−4
{ x 1 (t)= t 2 y 1 (t)=2t−1 and { x 2 (t)=−t+6 y 2 (t)=t+1t=2{ x 1 (t)=3 t 2 −3t+7 y 1 (t)=2t+3txy
{ x 1 (t)= t 2 −4 y 1 (t)=2 t 2 −1txytxy{ x 1 (t)= t 4 y 1 (t)= t 3 +4txy y= (x+1) 2 .
{ x(t)=t−1 y(t)= t 2 and { x(t)=t+1 y(t)= (t+2) 2 y=3x−2. y= x 2 −4x+4.
{ x(t)=t y(t)= t 2 −4t+4 and { x(t)=t+2 y(t)= t 2 x y 45° t,x(t),and y(t).xyt (x,y).
x(t)= t 2 +1, y(t)=2+t. t,x(t), y(t), tx(t)= t 2 +1y(t)=2+t−526−3−417−2−310−1−250−121012123254310
541765267 (1,2), t t t. t,t. x= t , y=2t+3, 0≤t≤3.

x=2cos t y=4sin t

t, x y. t tx=2cos ty=4sin tx=2cos(0)=2y=4sin(0)=0π 6x=2cos(π 6)= 3y=4sin(π 6)=2π 3
x=2cos(π 3)=1y=4sin(π 3)=2 3π 2x=2cos(π 2)=0y=4sin(π 2)=42π 3x=2cos(2π 3)=−1
y=4sin(2π 3)=2 35π 6x=2cos(5π 6)=− 3y=4sin(5π 6)=2πx=2cos(π)=−2y=4sin(π)=07π 6
x=2cos(7π 6)=− 3y=4sin(7π 6)=−24π 3x=2cos(4π 3)=−1y=4sin(4π 3)=−2 33π 2x=2cos(3π 2)=0
y=4sin(3π 2)=−45π 3x=2cos(5π 3)=1y=4sin(5π 3)=−2 311π 6x=2cos(11π 6)= 3y=4sin(11π 6)=−2
2πx=2cos(2π)=2y=4sin(2π)=0x y, t Y=

X 1T = Y 1T =

Y 1 =. x=5cos t, y=3sin t. x=5cos t y=2sin t. tx=5cos ty=2sin t0x=5cos(0)=5y=2sin(0)=01x=5cos(1)≈2.7
y=2sin(1)≈1.72x=5cos(2)≈−2.1y=2sin(2)≈1.83x=5cos(3)≈−4.95y=2sin(3)≈0.284x=5cos(4)≈−3.3
y=2sin(4)≈−1.55x=5cos(5)≈1.4y=2sin(5)≈−1.9−1x=5cos(−1)≈2.7y=2sin(−1)≈−1.7−2x=5cos(−2)≈−2.1
y=2sin(−2)≈−1.8−3x=5cos(−3)≈−4.95y=2sin(−3)≈−0.28−4x=5cos(−4)≈−3.3y=2sin(−4)≈1.5−5
x=5cos(−5)≈1.4y=2sin(−5)≈1.9 (x,y) t x(t) y(t), t y(x) t x, x(y) t y.

x=5cos t x 5 =cos t Solve for cos t. y=2sin t Solve for sin t. y 2 =sin t

cos 2 t+ sin 2 t=1 (x 5) 2 + (y 2) 2 =1 x 2 25 + y 2 4 =1

x=t+1 y= t , t≥0, y= x−1 y= t , t≥0 t t. x=2cos θ and y=4sin θ, θv 0 , hθv 0 , h

$x=(v 0 \cos\theta)t \quad y=- 1 2 g t 2 +(v 0 \sin\theta)t+h$

g h g=32 ft / s 2 g=9.8 m / s 2 . x y x=(v 0 cos θ)t. v 0 . cos θ cos θ. y=− 1 2 g t 2 +(v 0 sin θ)t+h. − 1 2 g t 2 g=32 ft/s 2 g=9.8 m/s 2 . v 0 , h. t. 45° x.

$x=(v 0 \cos \theta)t \quad x=(140\cos(45°))t$

y.

$y=-16 t 2 +(v 0 \sin \theta)t+h \quad y=-16 t 2 +(140\sin(45°))t+3$

$x=(140\cos(45°))(2) \quad x=198 \text{ feet} \quad y=-16 (2) 2 +(140\sin(45°))(2)+3 \quad y=137 \text{ feet}$

y=0.

$y=-16 t 2 +(140\sin(45 \circ))t+3 \quad y=0$ Set y(t)=0 and solve the quadratic. $t=6.2173$

t=6.2173 t, t y.

$x=(140\cos(45°))t \quad 400=(140\cos(45°))t \quad t=4.04 \quad y=-16 (4.04) 2 +(140\sin(45°))(4.04)+3 \quad y=141.8$

x y t,x(t), y(t). t x y. x=(v 0 cos θ)t y=−16 t 2 +(v 0 sin θ)t+h. v 0 . θ h t.{ x(t)=t y(t)= t 2 −1txy −3−2−10123

$\{ x(t)=t-1 \quad y(t)= t 2$

t−3−2−1012xy{ x(t)=2+t y(t)=3−2tt−2−10123xy{ x(t)=−2−2t y(t)=3+tt−3−2−101xy { x(t)= t 3 y(t)=t+2t−2−1012xy{ x(t)= t 2 y(t)=t+3t−2−1012xy{ x(t)=t y(t)= t{ x(t)=− t y(t)=t { x(t)=5−| t | y(t)=t+2{ x(t)=−t+2 y(t)=5−| t |{ x(t)=4sin t y(t)=2cos t{ x(t)=2sin t y(t)=4cos t { x(t)=3 cos 2 t y(t)=−3sin t{ x(t)=3 cos 2 t y(t)=−3 sin 2 t{ x(t)=sec t y(t)=tan t{ x(t)=sec t y(t)= tan 2 t { x(t)= 1 e 2t y(t)= e − t{ x(t)=t−1 y(t)=− t 2{ x(t)= t 3 y(t)=t+3{ x(t)=2cos t y(t)=−sin t { x(t)=7cos t y(t)=7sin t{ x(t)= e 2t y(t)=− e tx= t 2 , y = 3t, 0≤t≤5x=2t, y = t 2 , −5≤t≤5 x=t, y= 25− t 2 , 0<t≤5x(t)=−t,y(t)= t , t≥0x=−2cos t, y=6 sin t, 0≤t≤πx=−sec t, y=tan t, − π 2 <t< π 2

$x(t)=a\cos((a+b)t) \quad y(t)=a\cos((a-b)t)$

[−π,0], a=2 b=1, [−π,0], a=3 b=2 [−π,0], a=4 b=3 [−π,0], a=5 b=4 a b, a b a=100 b=99. b a? x(t)= t 2 y(t)=6−3t x(t) x(t)=− t 2 y(t) y(t)= t 2 x(t) y(t)=− t 2 x(t) (0,0), { x(t)=5cost y(t)=5sint (0,0), [−3,3] [−3,3] [0,2π) a b

$\{ x(t)=\sin(at) \quad y(t)=\sin(bt)$

a=1,b=2a=2,b=1a=3,b=3a=5,b=5a=2,b=5a=5,b=2 { x(t)=acos(bt) y(t)=csin(dt) . a,b,c,da=4, b=3, c=6, d=1 a=4, b=2, c=3, d=3{ x(t)=cost−1 y(t)=sint+t{ x(t)=cost+t y(t)=sint−1{ x(t)=t−sint y(t)=cost−1 [0, 2π]. [0,4π]. [−4π,6π]. sin t cos t y y(t)=−16 t 2 +20ty(x)=−16 (x 15) 2 +20(x 15) y(t)=−16 t 2 +10t+5. t. { x(t)=64tcos(52°) y(t)=−16 t 2 +64tsin(52°) x { x(t)=14cos t−cos(14t) y(t)=14sin t+sin(14t) [0,2π] { x(t)=6sin t+2sin(6t) y(t)=6cos t−2cos(6t) [0,2π]{ x(t)=2sin t+5cos(6t) y(t)=5cos t−2sin(6t) [0,2π] { x(t)=5sin(2t)sint y(t)=5sin(2t)cost [0,2π]vijv, u, w, v → , u → , w → . P Q, PQ → . (0,0) (a,b), ⟨ a,b ⟩.⟨ a,b ⟩(0,0) ⟨ a,b ⟩. CD → C(x 1 , y 1) D(x 2 , y 2),

$AB \rightarrow =\langle x 2 - x 1 , y 2 - y 1 \rangle \quad =\langle a,b \rangle$

CD → AB → . (0,0) ⟨ a,b ⟩. P(2,3) Q(6,4).

$v=\langle 6-2,4-3 \rangle \quad =\langle 4,1 \rangle$

(0,0) (4,1). ⟨ 4,1 ⟩. v (−3,2) (4,5),

$v=\langle 4-(-3),5-2 \rangle \quad =\langle 7,3 \rangle$

(0,0) (7,3). v (3,5). v=⟨ a,b ⟩,| v |= a 2 + b 2 . tan θ=(b a)⇒θ= tan −1 (b a), P(−8,1) Q(−2,−5).

$u=\langle -2,-(-8),-5-1 \rangle \quad =\langle 6,-6 \rangle$

$|u|= (6) 2 + (-6) 2 \quad = 72 \quad =6 2$

$\tan \theta= -6 6 =-1\Rightarrow\theta= \tan -1 (-1) \quad =-45°$

−45°+360°=315°. (5,−3) (−1,2) (−1,−3) (−7,2). v (5,−3) (−1,2). u (−1,−3) (−7,2).

$v=\langle -1-5,2-(-3)\rangle \quad =\langle -6,5\rangle \quad u=\langle -7-(-1),2-(-3)\rangle \quad =\langle -6,5\rangle$

$|v|= (-1-5) 2 + (2-(-3)) 2 \quad = (-6) 2 + (5) 2 \quad = 36+25 \quad = 61 \quad |u|= (-7-(-1)) 2 + (2-(-3)) 2 \quad = (-6) 2 + (5) 2 \quad = 36+25 \quad = 61$

$$\tan \theta = -5\ 6 \Rightarrow \theta = \tan^{-1}(-5\ 6) \qquad = -39.8°$$

180°. −39.8°+180°=140.2°.u=⟨ x,y ⟩ (x,y), u=⟨ 3,−2 ⟩v=⟨ −1,4 ⟩,

$$u+v = \langle 3,-2 \rangle + \langle -1,4 \rangle \qquad = \langle 3+(-1),-2+4 \rangle \qquad = \langle 2,2 \rangle$$

v u.

$$u+(-v) = \langle 3,-2 \rangle + \langle 1,-4 \rangle \qquad = \langle 3+1,-2+(-4) \rangle \qquad = \langle 4,-6 \rangle$$

v=⟨ a,b ⟩k

$$kv = \langle ka,kb \rangle$$

k v=⟨ 3,1 ⟩, 1 2v, v=⟨ 3,1 ⟩,

$$3v = \langle 3\cdot 3, 3\cdot 1 \rangle \qquad = \langle 9,3 \rangle \quad 1\ 2\ v = \langle 1\ 2\cdot 3, 1\ 2\cdot 1 \rangle \qquad = \langle 3\ 2, 1\ 2 \rangle \ -v = \langle -3,-1 \rangle$$

1 2v uu=⟨ 5,4 ⟩.3u=⟨ 15,12 ⟩u=⟨ 3,−2 ⟩v=⟨ −1,4 ⟩,

$$3u = 3\langle 3,-2 \rangle \qquad = \langle 9,-6 \rangle \ 2v = 2\langle -1,4 \rangle \qquad = \langle -2,8 \rangle$$

$$w = 3u+2v \qquad = \langle 9,-6 \rangle + \langle -2,8 \rangle \qquad = \langle 9-2,-6+8 \rangle \qquad = \langle 7,2 \rangle$$

w=⟨ 7,2 ⟩. x y ⟨ 2,3 ⟩ (0,0) (2,0).

$$v_1 = \langle 2-0,0-0 \rangle \qquad = \langle 2,0 \rangle$$

(0,0) (0, 3).

$$v_2 = \langle 0-0,3-0 \rangle \qquad = \langle 0,3 \rangle$$

$$v = \langle 2+0,3+0 \rangle \quad = \langle 2,3 \rangle$$

$$|v| = |v_1|^2 + |v_2|^2 \qquad = 2^2 + 3^2 \qquad = 13$$

13 . tan θ= y x .

$$\tan \theta = v_2\ v_1 \ \tan \theta = 3\ 2 \qquad \theta = \tan^{-1}(3\ 2) = 56.3°$$

v 13 56.3 ∘ v (3,2) (7,4).

$$v = \langle 7-3,4-2 \rangle \quad = \langle 4,2 \rangle$$

v 1=⟨ 4,0 ⟩ v 2=⟨ 0,2 ⟩.i=⟨ 1,0 ⟩j=⟨ 0,1 ⟩ v v | v | v. v=⟨ −5,12 ⟩.

$$|v| = (-5)^2 + (12)^2 \qquad = 25+144 \qquad = 169 \qquad = 13$$

| v |,

$$v\ |v| = -5\ 13\ i + 12\ 13\ j$$

$$v\ |v| = \langle -5\ 13, 12\ 13 \rangle$$

− 5 13 i+ 12 13 j

$$(-5\ 13)^2 + (12\ 13)^2 = 25\ 169 + 144\ 169 \qquad = 169\ 169 = 1$$

= 5 13+ 12 13=⟨ −5,12 ⟩. v P=(x 1 , y 1) Q=(x 2 , y 2),

$$v = (x_2 - x_1)i + (y_1 - y_2)j$$

(0,0) (a,b), (x 2 − x 1)=a (y 2 − y 1)=b, | v |= a 2 + b 2 . v P=(2,−6) Q=(−6,6), i j.

$$v = (x_2 - x_1)i + (y_2 - y_1)j \quad = (-6-2)i + (6-(-6))j \quad = -8i+12j$$

P 1 =(−1,3) P 2 =(2,7), v i j.

$$v = (x_2 - x_1)i + (y_2 - y_1)j \ v = (2-(-1))i + (7-3)j \quad = 3i+4j$$

u P=(−1,6) Q=(7,−5) i j.u=8i−11j i j,

$$v+u = (a+c)i + (b+d)j \ v-u = (a-c)i + (b-d)j$$

v 1 =2i−3j v 2 =4i+5j.

$$v_1 + v_2 = (2+4)i + (-3+5)j \qquad = 6i+2j$$

i and j. | v | r. v=⟨ x,y ⟩ θ,

$$\cos \theta = x\ |v| \text{ and } \sin \theta = y\ |v| \qquad x = |v|\cos \theta \qquad y = |v|\sin \theta$$

v=xi+yj=| v |cos θi+| v |sin θj, | v |= x 2 + y 2 . x=| v |cos θi y=| v |sin θj,

$$x = 7\cos(135°)i \quad = -7\ 2\ 2 \ y = 7\sin(135°)j \quad = 7\ 2\ 2$$

v=7cos(135°)i+7sin(135°)j

$$v = -7\ 2\ 2\ i + 7\ 2\ 2\ j$$

(3,5). v= 34 cos(59°)i+ 34 sin(59°)j 34θ= tan −1 (5 3)=59.04° v=⟨ a,b ⟩ u=⟨ c,d ⟩

v·u=ac+bd
cos θ= v | v | · u | u |

v=⟨ 5,12 ⟩ u=⟨ −3,4 ⟩.
v·u=⟨ 5,12 ⟩·⟨ −3,4 ⟩ =5·(−3)+12·4 =−15+48 =33
v 1 · v 2 =⟨ 5,2 ⟩·⟨ 3,7 ⟩ =5·3+2·7 =15+14 =29
cos θ= v |v| · u |u| .
v |v| · u |u| =⟨ 5 29 + 2 29 ⟩·⟨ 3 58 + 7 58 ⟩ = 5 29 · 3 58 + 2 29 · 7 58
= 15 1682 + 14 1682 = 29 1682 =0.707107 cos −1
(0.707107)=45°
u=⟨ −3,4 ⟩ v=⟨ 5,12 ⟩.
θ= cos −1 (u |u| · v |v|) (u |u| · v |v|)= −3i+4j 5 · 5i+12j 13 =(− 3 5 · 5 13)+(4 5 · 12 13) =− 15 65 + 48 65 = 33 65 θ= cos −1 (33 65) = 59.5 ∘
x α 140°+α . BCO AOC BCO x
x 2 = (16.2) 2 + (200) 2 −2(16.2)(200)cos(140°) x 2 =45,226.41 x= 45,226.41 x=212.7
sin α 16.2 = sin(140°) 212.7 sin α= 16.2sin(140°) 212.7
=0.04896 sin −1 (0.04896)=2.8°
| v |= a 2 + b 2 . ij i j v a and b. v=| v |cos θi+| v |sin θj. i j u,v,w i j,⟨ a,b ⟩, i j? i, j. (5,2) (−1,−3), (0,0). ⟨ a,b ⟩. (−4,2) (3,−3), (0,0). ⟨ a,b ⟩.⟨ 7,−5 ⟩ (7,−1) (−1,−7), (0,0). ⟨ a,b ⟩. u v u P 1 P 2 v P 3 P 4P 1 =(5,1), P 2 =(3,−2), P 3 =(−1,3), P 4 =(9,−4)P 1 =(2,−3), P 2 =(5,1), P 3 =(6,−1), P 4 =(9,3)P 1 =(−1,−1), P 2 =(−4,5), P 3 =(−10,6), P 4 =(−13,12)
P 1 =(3,7), P 2 =(2,1), P 3 =(1,2), P 4 =(−1,−4)P 1 =(8,3), P 2 =(6,5), P 3 =(11,8), P 4 =(9,10) P 1 =(−3,1) P 2 =(5,2), v i j. P 1 =(6,0) P 2 =(−1,−3), v i j. 7i−3j−6i−2ju=⟨ 2,−3 ⟩,v=⟨ 1,5 ⟩ u=⟨ −3,4 ⟩,v=⟨ −2,1 ⟩u+v=⟨ −5,5 ⟩,u−v=⟨ −1,3 ⟩,2u−3v=⟨ 0,5 ⟩ v. v.−10i−4j− 2 29 29 i+ 5 29 29 j d=− 1 3 i+ 5 2 j− 2 229 229 i+ 15 229 229 j− 7 2 10 i+ 2 10 j 0≤θ<2π.⟨ 0,4 ⟩⟨ 6,5 ⟩| v |=7.810,θ=39.806° ⟨ 2,−5 ⟩⟨ −4,−6 ⟩| v |=7.211,θ=236.310° u·v. u·v.−6 u=⟨ −2,4 ⟩ v=⟨ −3,1 ⟩, u·v.=⟨ −1,6 ⟩=⟨ 6,−1 ⟩, u·v.−12 v, v, 1 2 v.⟨ 2,−1 ⟩⟨ −1,4 ⟩⟨ −3,−2 ⟩⟨ 4,1 ⟩ P 1 =(2,1) P 2 =(−1,2), v i j, P 1 =(4,−1) P 2 =(−3,2), v i j. v=−7i+3j P 1 =(3,3) P 2 =(−3,3), v i j. | v |=6,θ=45°3 2 i+3 2 j| v |=8,θ=220°| v |=2,θ=300° i− 3 j| v |=5,θ=135°x=7.13 y=3.63 x=2.87 y=4.10 (5,7) (0.081,8.602) (7,3) α a,β b, γ c. β=50°,a=105,b=45α=43.1°,a=184.2,b=242.8C=120°,a=23.1,c=34.1 A A: α a,β b, γ c: a=4, b=6,c=8. B=71.0°,C=55.0°,a=12.8 (3, π 6). (5,− 2π 3) (6,− 3π 4) (−2, 3π 2) (0,2) (7,−2)(−9,−4) (9.8489,203.96°)x=−2x 2 + y 2 =64r=8x 2 + y 2 =−2yr=7cos θx 2 + y 2 =7xr= −2 4cos θ+sin θθ= 3π 4 y=−xr=5sec θr=4+4sin θθ= π 2r=7 r=1−5sin θ. r=5sin(7θ). r=3−3cos θ−2+6i4−3i5+9i1 2 − 3 2 i cis(− π 3)z=5cis(5π 6)z=3cis(40°)2.3+1.9i z 1 z 2 z 1 =2cis(89°)z 2 =5cis(23°)z 1 =10cis(π 6) z 2 =6cis(π 3)60cis(π 2) z 1 z 2 z 1 =12cis(55°)z 2 =3cis(18°)z 1 =27cis(5π 3)z 2 =9cis(π 3) 3cis(4π 3) z 4 z=2cis(70°) z 2 z=5cis(3π 4)25cis(3π 2) z z=64cis(210°). z z=25cis(3π 2). 5cis(3π 4),5cis(7π 4)6−2i−1+3i t { x(t)=3t−1 y(t)= t{ x(t)=−cos t y(t)=2 sin 2 t x 2 + 1 2 y=1 x(t)=acos t y(t)=bsin t x 2 25 + y 2 16 =1. (−2,3) (4,7) (−2,3) t=0 (4,7) t=1.{ x(t)=−2+6t y(t)=3+4t { x(t)=3 t 2 y(t)=2t−1{ x(t)= e t y(t)=−2 e 5 ty=−2 x 5{ x(t)=3cos t y(t)=2sin t { x(t)=(80cos(40°))t y(t)=−16 t 2 +(80sin(40°))t+4 u v, u P 1 P 2 , v P 3 P 4 . P 1 =(−1,4), P 2 =(3,1), P 3 =(5,5) P 4 =(9,2)P 1 =(6,11), P 2 =(−2,8), P 3 =(0,−1) P 4 =(−8,2) u=2i−j,v=4i−3j, w=−2i+5j − 3 10 10− 10 10⟨ 6,−2 ⟩⟨ −3,−3 ⟩ 3 2 , 225° u·v.16=⟨ −3,4 ⟩ 1 2 P 1 =(3,2) P 2 =(−5,−1), v i j. α a,β b, γ c. β=68°,b=21,c=16.α=67.1°,γ=44.9°,a=20.91712 miles (2,2) (2, π 3) (1, 3) x 2 + y 2 =5y.r=−3csc θ.y=−3 r=−4sin(2θ). r=3+3cos θ. r=3−5sin θ.5−9i.106 4+i. z=5cis(2π 3).−5 2 +i 5 3 2 z 1 =8cis(36°) z 2 =2cis(15°),z 1 z 2z 1 z 24cis(21°)(z 2) 3z 1 2 2 cis(18°),2 2 cis(198°) −5−i t { x(t)=t+1 y(t)=2 t 2 .y=2 (x−1) 2 x(t)=acos t y(t)=bsin t: x 2 36 + y 2 100 =1. { x(t)=−2sin t y(t)=5cos t . u·v. v.2 13 13 i+ 3 13 13 j v P 1 =(2,2) P 2 =(−1,0), v

i j. v, −v. (0,0) (a,b), v 1 =⟨ 1,0 ⟩ v 2 =⟨ 0,1 ⟩.

$2x+y=15$ $3x-y=5$

$2(4)+(7)=15$ True $3(4)-(7)=5$ True

(x,y). (5,1)

$x+3y=8$ $2x-9=y$

(5,1)

$(5)+3(1)=8$ $8=8$ True $2(5)-9=(1)$ $1=1$ True

(5,1) (8,5)

$5x-4y=20$ $2x+1=3y$

$2x+y=-8$ $x-y=-1$

y.

$2x+y=-8$ $y=-2x-8$

y.

$x-y=-1$ $y=x+1$

(−3,−2).

$2(-3)+(-2)=-8$ $-8=-8$ True $(-3)-(-2)=-1$ $-1=-1$ True

(−3,−2),

$2x-5y=-25$ $-4x+5y=35$

(−5,3).

$-x+y=-5$ $2x-5y=1$

y.

$-x+y=-5$ $y=x-5$

x−5 y

$2x-5y=1$ $2x-5(x-5)=1$ $2x-5x+25=1$ $-3x=-24$ $x=8$

x=8 y.

$-(8)+y=-5$ $y=3$

(8,3). (8,3)

$-x+y=-5$ $-(8)+(3)=-5$ True $2x-5y=1$ $2(8)-5(3)=1$ True

$x=y+3$ $4=3x-2y$

(−2,−5)

$x+2y=-1$ $-x+y=3$

x x x

$x+2y=-1$ $-x+y=3$ $3y=2$

x, y.

$3y=2$ $y=\frac{2}{3}$

y x.

$-x+y=3$ $-x+\frac{2}{3}=3$ $-x=3-\frac{2}{3}$ $-x=\frac{7}{3}$ $x=-\frac{7}{3}$

$\left(-\frac{7}{3},\frac{2}{3}\right)$.

$x+2y=-1$ $\left(-\frac{7}{3}\right)+2\left(\frac{2}{3}\right)=$ $-\frac{7}{3}+\frac{4}{3}=$ $-\frac{3}{3}=$ $-1=-1$ True

$3x+5y=-11$ $x-2y=11$

3x x. −3,

$x-2y=11$ $-3(x-2y)=-3(11)$ Multiply both sides by −3. $-3x+6y=-33$ Use the distributive property.

$3x+5y=-11$ $-3x+6y=-33$ ________________ $11y=-44$ $y=-4$

y=−4 x.

$3x+5y=-11$ $3x+5(-4)=-11$ $3x-20=-11$ $3x=9$ $x=3$

(3,−4).

$x-2y=11$ $(3)-2(-4)=3+8$ $=11$ True

$2x-7y=2$ $3x+y=-20$

(−6,−2)

$2x+3y=-16$ $5x-10y=30$

$2x$ $5x$. $10x$ x -5 2.

$-5(2x+3y)=-5(-16)$ $-10x-15y=80$ $2(5x-10y)=2(30)$ $10x-20y=60$

$-10x-15y=80$ $10x-20y=60$ ________________ $-35y=140$ $y=-4$

$y=-4$

$2x+3(-4)=-16$ $2x-12=-16$ $2x=-4$ $x=-2$

$(-2,-4)$.

$5x-10y=30$ $5(-2)-10(-4)=30$ $-10+40=30$ $30=30$

x 3 + y 6 =3 x 2 − y 4 = 1

6(x 3 + y 6)=6(3) $2x+y=18$ 4(x 2 − y 4)=4(1) $2x-y=4$

-1

$-1(2x-y)=-1(4)$ $-2x+y=-4$

$2x+y=18$ $-2x+y=-4$ ______________ $2y=14$ $y=7$

$y=7$

$2x+(7)=18$ $2x=11$ x= 11 2 =7.5

(11 2 ,7).

x 2 − y 4 =1 11 2 2 − 7 4 =1 11 4 − 7 4 =1 4 4 =1

$2x+3y=8$ $3x+5y=10$

$(10,-4)$

y 12=0.

$x=9-2y$ $x+2y=13$

x,

$x+2y=13$ $(9-2y)+2y=13$ $9+0y=13$ $9=13$

$9\neq13$.

$x=9-2y$ $2y=-x+9$ y=− 1 2 x+ 9 2

$x+2y=13$ $2y=-x+13$ y=− 1 2 x+ 13 2

y=− 1 2 x+ 9 2 y=− 1 2 x+ 13 2

$2y-2x=2$ $2y-2x=6$

$0=0$.

$x+3y=2$ $3x+9y=6$

x. -3, x

$x+3y=2$ $(-3)(x+3y)=(-3)(2)$ $-3x-9y=-6$

$-3x-9y=-6$ + $3x+9y=6$ _______________ $0=0$

$x+3y=2$ $3y=-x+2$ y=− 1 3 x+ 2 3 $3x+9y=6$ $9y=-3x+6$ y=− 3 9 x+ 6 9 y=− 1 3 x+ 2 3

(x, − 1 3 x+ 2 3).

$y-2x=5$ $-3y+6x=-15$

$(x,2x+5)$. $R=xp$, x= p= x $P(x)=R(x)-C(x)$. $C(x)=0.85x+35{,}000$ $R(x)=1.55x$, y

$y=0.85x+35{,}000$ $y=1.55x$

$0.85x+35{,}000$ x.

$0.85x+35{,}000=1.55x$ $35{,}000=0.7x$ $50{,}000=x$

$x=50{,}000$

$1.55(50{,}000)=77{,}500$

$(50{,}000,77{,}500)$. $P(x)=R(x)-C(x)$.

$P(x)=1.55x-(0.85x+35{,}000)$ $=0.7x-35{,}000$

$P(x)=0.7x-35{,}000$. $x=50{,}000$, \$25.00 \$50.00 2,000 \$70,000. 2,000.

$c+a=2{,}000$

\$25.00 25c. \$50.00 50a. \$70,000.

$25c+50a=70{,}000$

$$c+a=2{,}000 \quad 25c+50a=70{,}000$$

c a. a.

$$c+a=2{,}000 \qquad a=2{,}000-c$$

2,000−c a c.

$$25c+50(2{,}000-c)=70{,}000 \quad 25c+100{,}000-50c=70{,}000 \qquad -25c=-30{,}000 \qquad c=1{,}200$$

c=1,200 a.

$$1{,}200+a=2{,}000 \qquad a=800$$

1,200 800 \$4.00 \$12.00 1,650 \$14,200, x y 5x−y=4 x+6y=2 (4,0)−3x−5y=13 −x+4y=10(−6,1)
3x+7y=1 2x+4y=0 (2,3)−2x+5y=7 2x+9y=7(−1,1)x+8y=43 3x−2y=−1 (3,5) x+3y=5 2x+3y=4(−1,2)
3x−2y=18 5x+10y=−104x+2y=−10 3x+9y=0(−3,1)2x+4y=−3.8 9x−5y=1.3−2x+3y=1.2 −3x−6y=1.8
(− 3 5 ,0) x−0.2y=1 −10x+2y=5 3x+5y=9 30x+50y=−90 −3x+y=2 12x−4y=−8
1 2 x+ 1 3 y=16 1 6 x+ 1 4 y=9(72 5 , 132 5)− 1 4 x+ 3 2 y=11 − 1 8 x+ 1 3 y=3
−2x+5y=−42 7x+2y=30(6,−6)6x−5y=−34 2x+6y=4 5x−y=−2.6 −4x−6y=1.4(− 1 2 , 1 10)
7x−2y=3 4x+5y=3.25 −x+2y=−1 5x−10y=6 7x+6y=2 −28x−24y=−8
5 6 x+ 1 4 y=0 1 8 x− 1 2 y=− 43 120(− 1 5 , 2 3) 1 3 x+ 1 9 y= 2 9 − 1 2 x+ 4 5 y=− 1 3
−0.2x+0.4y=0.6 x−2y=−3(x, x+3 2)−0.1x+0.2y=0.6 5x−10y=15x+9y=16 x+2y=4(−4,4)
6x−8y=−0.6 3x+2y=0.95x−2y=2.25 7x−4y=3(1 2 , 1 8) x− 5 12 y=− 55 12 −6x+ 5 2 y= 55 2
7x−4y= 7 6 2x+4y= 1 3(1 6 ,0)3x+6y=11 2x+4y=9 7 3 x− 1 6 y=2 − 21 6 x+ 3 12 y=−3(x,2(7x−6))
1 2 x+ 1 3 y= 1 3 3 2 x+ 1 4 y=− 1 82.2x+1.3y=−0.1 4.2x+4.2y=2.1(− 5 6 , 4 3)
0.1x+0.2y=2 0.35x−0.3y=03x−y=0.6 x−2y=1.3−x+2y=4 2x−4y=1 x+2y=7 2x+6y=12
3x−5y=7 x−2y=3 3x−2y=5 −9x+6y=−15 0.1x+0.2y=0.3 −0.3x+0.5y=1
−0.01x+0.12y=0.62 0.15x+0.20y=0.52(−3.08,4.91) 0.5x+0.3y=4 0.25x−0.9y=0.46
0.15x+0.27y=0.39 −0.34x+0.56y=1.8(−1.52,2.29)−0.71x+0.92y=0.13 0.83x+0.05y=2.1 A,B,C,D,E,
F A−F A≠B AE≠BD.x+y=A x−y=B(A+B 2 , A−B 2)x+Ay=1 x+By=1Ax+y=0 Bx+y=1
(−1 A−B , A A−B)Ax+By=C x+y=1Ax+By=C Dx+Ey=F(CE−BF BD−AE , AF−CD BD−AE)
C=12x+30 R=20x. C(x)=11x+120 R(x)=5x. C(x)=150x+10,000 R(x)=200x. C(x)=64x+20,000, x
(1,250,100,000) C(x)=75x+50,000. 2 7 8 % (x,y) P(x)=R(x)−C(x), R=xp, x= p= (x,y,z),

$$Ax+By+Cz=D \qquad Ey+Fz=G \qquad Hz=K$$

z, y x. { (x,y,z) }. { (x,y,z) }. 0=0. 3=0. (3,−2,1)

$$x+y+z=2 \quad 6x-4y+5z=31 \quad 5x+2y+2z=13$$

x,y, z.

x+y+z=2 (3)+(−2)+(1)=2 True 6x−4y+5z=31 6(3)−4(−2)+5(1)=31 18+8+5=31 True 5x+2y+2z=13
5(3)+2(−2)+2(1)=13 15−4+2=13 True

(3,−2,1)

$$x-2y+3z=9\ (1) \quad -x+3y-z=-6\ (2) \quad 2x-5y+5z=17\ (3)$$

x

$$x-2y+3z=9\ (1) \quad -x+3y-z=-6\ (2) \qquad y+2z=3\ (3)$$

−2 x.

$$-2x+4y-6z=-18\ (1) \text{ multiplied by } -2 \quad 2x-5y+5z=17\ (3) \quad \underline{\hspace{10em}} \quad -y-z=-1\ (5)$$

z

$$y+2z=3\ (4) \quad -y-z=-1\ (5) \qquad z=2\ (6)$$

$$x-2y+3z=9\ (1) \quad y+2z=3\ (4) \qquad z=2\ (6)$$

z=2 y.

$$y+2(2)=3 \quad y+4=3 \qquad y=-1$$

z=2 y=−1 x.

$$x-2(-1)+3(2)=9 \qquad x+2+6=9 \qquad x=1$$

(1,−1,2).

x=amount invested in money-market fund y=amount invested in municipal bonds z=amount invested in mutual funds

$x+y+z=12{,}000$

$z=y+4{,}000$

$0.03x+0.04y+0.07z=670$

$x+y+z=12{,}000$ $-y+z=4{,}000$ $0.03x+0.04y+0.07z=670$

$x+ y+z =12{,}000$ (1) $-y+z =4{,}000$ (2) $3x+4y+7z=67{,}000$ (3)

$x+ y + z=12{,}000$ $3x+4y +7z=67{,}000$ $-y + z=4{,}000$

-3

$x+y+z =12{,}000$ $y+4z=31{,}000$ $-y+z =4{,}000$

$x+y+ z=12{,}000$ $y+4z=31{,}000$ $5z =35{,}000$

z y. z y x.

$5z=35{,}000$ $z=7{,}000$ $y+4(7{,}000)=31{,}000$ $y=3{,}000$

$x+3{,}000+7{,}000=12{,}000$ $x=2{,}000$

$2x+y-2z=-1$ $3x-3y-z=5$ $x-2y+3z=6$

$(1,-1,1)$ $3=7$

$x-3y+z=4$ (1) $-x+2y-5z=3$ (2) $5x-13y+13z=8$ (3)

x, x

$x-3y+z=4$ (1) $-x+2y-5z=3$ (2) $-y-4z=7$ (4)

-5

$-5x+15y-5z=-20$ (1) multiplied by -5 $5x-13y+13z=8$ (3)

$2y+8z=-12$ (5)

$-2y-8z=14$ (4) multiplied by 2 $2y+8z=-12$ (5)

$0=2$

$0=2$

$x+y+z=2$ $y-3z=1$ $2x+y+5z=0$

$2x+y-3z=0$ (1) $4x+2y-6z=0$ (2) $x-y+z=0$ (3)

-2

$-4x-2y+6z=0$ equation (1) multiplied by -2 $4x+2y-6z=0$ (2)

$0=0$

$0=0, -2, 0=0.$

$2x+y-3z=0$ $x-y+z=0$ $3x-2z=0$

z.

$3x-2z=0$ $z= 3\ 2\ x$

z y.

$2x+y-3(3\ 2\ x)=0$ $2x+y- 9\ 2\ x=0$ $y= 9\ 2\ x-2x$ $y= 5\ 2\ x$

$(x, 5\ 2\ x, 3\ 2\ x)$. x y z x. x? x y.

$x+y+z=7$ $3x-2y-z=4$ $x+6y+5z=24$

$(x,4x-11,-5x+18)$. $\{ (x,y,z) \}$ $(0,0,0)$ $2x+3y-6z=1$ $-4x-6y+12z=-2$ $x+2y+5z=10$

$2x-6y+6z=-12$ $x+4y+5z=-1$ $-x+2y+3z=-1$ $(0,1,-1)$ $6x-y+3z=6$ $3x+5y+2z=0$ $x+y=0(3,-3,-5)$

$6x-7y+z=2$ $-x-y+3z=4$ $2x+y-z=1$ $(4,2,-6)$ $x-y=0$ $x-z=5$ $x-y+z=-1$ $(4,4,-1)$

$-x-y+2z=3$ $5x+8y-3z=4$ $-x+3y-5z=-5$ $(4,1,-7)$ $3x-4y+2z=-15$ $2x+4y+z=16$ $2x+3y+5z=20(-1,4,2)$

$5x-2y+3z=20$ $2x-4y-3z=-9$ $x+6y-8z=21$ $5x+2y+4z=9$ $-3x+2y+z=10$ $4x-3y+5z=-3$

$(- 85\ 107 , 312\ 107 , 191\ 107)$ $4x-3y+5z=31$ $-x+2y+4z=20$ $x+5y-2z=-29$

$5x-2y+3z=4$ $-4x+6y-7z=-1$ $3x+2y-z=4(1, 1\ 2 ,0)$ $4x+6y+9z=0$ $-5x+2y-6z=3$ $7x-4y+3z=-3$

$2x-y+3z=17$ $-5x+4y-2z=-46$ $2y+5z=-7(4,-6,1)$ $5x-6y+3z=50$ $-x+4y=10$ $2x-z=10$

$2x+3y-6z=1$ $-4x-6y+12z=-2$ $x+2y+5z=10(x, 1\ 27\ (65-16x), x+28\ 27)$

$4x+6y-2z=8$ $6x+9y-3z=12$ $-2x-3y+z=-4$ $2x+3y-4z=5$ $-3x+2y+z=11$ $-x+5y+3z=4$

$(- 45\ 13 , 17\ 13 ,-2)$ $10x+2y-14z=8$ $-x-2y-4z=-1$ $-12x-6y+6z=-12$

x+y+z=14 2y+3z=−14 −16y−24z=−112 5x−3y+4z=−1 −4x+2y−3z=0 −x+5y+7z=−11
x+y+z=0 2x−y+3z=0 x−z=0(0,0,0)3x+2y−5z=6 5x−4y+3z=−12 4x+5y−2z=15
x+y+z=0 2x−y+3z=0 x−z=1(4 7 ,− 1 7 ,− 3 7)
3x− 1 2 y−z=− 1 2 4x+z=3 −x+ 3 2 y= 5 2
6x−5y+6z=38 1 5 x− 1 2 y+ 3 5 z=1 −4x− 3 2 y−z=−74(7,20,16)
1 2 x− 1 5 y+ 2 5 z=− 13 10 1 4 x− 2 5 y− 1 5 z=− 7 20 − 1 2 x− 3 4 y− 1 2 z=− 5 4
− 1 3 x− 1 2 y− 1 4 z= 3 4 − 1 2 x− 1 4 y− 1 2 z=2 − 1 4 x− 3 4 y− 1 2 z=− 1 2(−6,2,1)
1 2 x− 1 4 y+ 3 4 z=0 1 4 x− 1 10 y+ 2 5 z=−2 1 8 x+ 1 5 y− 1 8 z=2
4 5 x− 7 8 y+ 1 2 z=1 − 4 5 x− 3 4 y+ 1 3 z=−8 − 2 5 x− 7 8 y+ 1 2 z=−5(5,12,15)
− 1 3 x− 1 8 y+ 1 6 z=− 4 3 − 2 3 x− 7 8 y+ 1 3 z=− 23 3 − 1 3 x− 5 8 y+ 5 6 z=0
− 1 4 x− 5 4 y+ 5 2 z=−5 − 1 2 x− 5 3 y+ 5 4 z= 55 12 − 1 3 x− 1 3 y+ 1 3 z= 5 3(−5,−5,−5)
1 40 x+ 1 60 y+ 1 80 z= 1 100 − 1 2 x− 1 3 y− 1 4 z=− 1 5 3 8 x+ 3 12 y+ 3 16 z= 3 20
0.1x−0.2y+0.3z=2 0.5x−0.1y+0.4z=8 0.7x−0.2y+0.3z=8(10,10,10)
0.2x+0.1y−0.3z=0.2 0.8x+0.4y−1.2z=0.1 1.6x+0.8y−2.4z=0.2
1.1x+0.7y−3.1z=−1.79 2.1x+0.5y−1.6z=−0.13 0.5x+0.4y−0.5z=−0.07(1 2 , 1 5 , 4 5)
0.5x−0.5y+0.5z=10 0.2x−0.2y+0.2z=4 0.1x−0.1y+0.1z=2
0.1x+0.2y+0.3z=0.37 0.1x−0.2y−0.3z=−0.27 0.5x−0.1y−0.3z=−0.03(1 2 , 2 5 , 4 5)
0.5x−0.5y−0.3z=0.13 0.4x−0.1y−0.3z=0.11 0.2x−0.8y−0.9z=−0.32
0.5x+0.2y−0.3z=1 0.4x−0.6y+0.7z=0.8 0.3x−0.1y−0.9z=0.6(2,0,0)
0.3x+0.3y+0.5z=0.6 0.4x+0.4y+0.4z=1.8 0.4x+0.2y+0.1z=1.6
0.8x+0.8y+0.8z=2.4 0.3x−0.5y+0.2z=0 0.1x+0.2y+0.3z=0.6(1,1,1) x,y, z.
x+y+z=3 x−1 2 + y−3 2 + z+1 2 =0 x−2 3 + y+4 3 + z−3 3 = 2 3
5x−3y− z+1 2 = 1 2 6x+ y−9 2 +2z=−3 x+8 2 −4y+z=4(128 557 , 23 557 , 28 557)
x+4 7 − y−1 6 + z+2 3 =1 x−2 4 + y+1 8 − z+8 12 =0 x+6 3 − y+2 3 + z+4 2 =3
x−3 6 + y+2 2 − z−3 3 =2 x+2 4 + y−5 2 + z+4 2 =1 x+6 2 − y−3 2 +z+1=9(6,−1,0)
x−1 3 + y+3 4 + z+2 6 =1 4x+3y−2z=11 0.02x+0.015y−0.01z=0.065 3 4 3 1 8 % 2 1 2 %
Ax+By+C=0.

x−y=−1 y= x 2 +1

x

x−y=−1 x=y−1 Solve for x. y= x 2 +1 y= (y−1) 2 +1 Substitute expression for x.

y= (y−1) 2 =(y 2 −2y+1)+1 = y 2 −2y+2 0= y 2 −3y+2 =(y−2)(y−1)

y y=2 y=1. y x.

x−y=−1 x−(2)=−1 x=1 x−(1)=−1 x=0

(1,2) (0,1), (x,y) y x x x. y=1

y= x 2 +1 y= x 2 +1 x 2 =0 x=± 0 =0

y=2

y= x 2 +1 2= x 2 +1 x 2 =1 x=± 1 =±1

−1

3x−y=−2 2 x 2 −y=0

(− 1 2 , 1 2) (2,8)

x 2 + y 2 =5 y=3x−5

y. y=3x−5

x 2 + (3x−5) 2 =5 x 2 +9 x 2 −30x+25=5 10 x 2 −30x+20=0

x.

10(x 2 −3x+2)=0 10(x−2)(x−1)=0 x=2 x=1

y.

y=3(2)−5 =1 y=3(1)−5 =−2

(2,1) (1,−2), (x,y)

x 2 + y 2 =10 x−3y=−10

(−1,3)

x 2 + y 2 =26 (1) 3 x 2 +25 y 2 =100 (2)

−3,

(−3)(x 2 + y 2)=(−3)(26) −3 x 2 −3 y 2 =−78 3 x 2 +25 y 2 =100 22 y 2 =22

y.

y 2 =1 y=± 1 =±1

y=±1 x.

x 2 + (1) 2 =26 x 2 +1=26 x 2 =25 x=± 25 =±5 x 2 + (−1) 2 =26 x 2 +1=26 x 2 =25=±5

(5,1),(−5,1),(5,−1),and (−5,−1).

4 x 2 + y 2 =13 x 2 + y 2 =10

{ (1,3),(1,−3),(−1,3),(−1,−3) } y>a, y<a, y≥a, y≤a, y>a; y≥a; y<a; y≤a ≤ ≥ y> x 2 +1. y= x 2 +1.

y> x 2 +1 (0,2) (2,0).

y> x 2 +1 2> (0) 2 +1 2>1 True 0> (2) 2 +1 0>5 False

x 2 −y≤0 2 x 2 +y≤12

y x.

x 2 −y=0 2 x 2 +y=12 ____________ 3 x 2 =12 x 2 =4 x=±2

y.

x 2 −y=0 (2) 2 −y=0 4−y=0 y=4 (−2) 2 −y=0 4−y=0 y=4

(2,4) (−2,4).

x 2 −y≤0 x 2 ≤y y≥ x 2 2 x 2 +y≤12 y≤−2 x 2 +12

2 x 2 +y≤12 x 2 −y≤0

y≥ x 2 −1 x−y≥−1

C(x) R(x). C(x)<R(x), x+y=4 x 2 + y 2 =9 y=x−3 x 2 + y 2 =9(0,−3),(3,0)

y=x x 2 + y 2 =9 y=−x x 2 + y 2 =9(− 3 2 2 , 3 2 2),(3 2 2 ,− 3 2 2) x=2 x 2 − y 2 =9

4 x 2 −9 y 2 =36 4 x 2 +9 y 2 =36(−3,0),(3,0)x 2 + y 2 =25 x 2 − y 2 =1

2 x 2 +4 y 2 =4 2 x 2 −4 y 2 =25x−10(1 4 ,− 62 8),(1 4 , 62 8)y 2 − x 2 =9 3 x 2 +2 y 2 =8

x 2 + y 2 + 1 16 =2500 y=2 x 2(− 398 4 , 199 4),(398 4 , 199 4)−2 x 2 +y=−5 6x−y=9

− x 2 +y=2 −x+y=2(0,2),(1,3)x 2 + y 2 =1 y=20 x 2 −1x 2 + y 2 =1 y=− x 2

(− 1 2 (5 −1) , 1 2 (1− 5)),(1 2 (5 −1) , 1 2 (1− 5))2 x 3 − x 2 =y y= 1 2 −x

9 x 2 +25 y 2 =225 (x−6) 2 + y 2 =1(5,0)x 4 − x 2 =y x 2 +y=02 x 3 − x 2 =y x 2 +y=0(0,0)

x 2 + y 2 =9 y=3− x 2x 2 − y 2 =9 x=3(3,0)x 2 − y 2 =9 y=3x 2 − y 2 =9 x−y=0

− x 2 +y=2 −4x+y=−1− x 2 +y=2 2y=−xx 2 + y 2 =25 x 2 − y 2 =36x 2 + y 2 =1 y 2 = x 2

(− 2 2 ,− 2 2),(− 2 2 , 2 2),(2 2 ,− 2 2),(2 2 , 2 2)16 x 2 −9 y 2 +144=0 y 2 + x 2 =16

3 x 2 − y 2 =12 (x−1) 2 + y 2 =1(2,0) 3 x 2 − y 2 =12 (x−1) 2 + y 2 =4

3 x 2 − y 2 =12 x 2 + y 2 =16(− 7 ,−3),(− 7 ,3),(7 ,−3),(7 ,3)

x 2 − y 2 −6x−4y−11=0 − x 2 + y 2 =5x 2 + y 2 −6y=7 x 2 +y=1

(− 1 2 (73 −5) , 1 2 (7− 73)),(1 2 (73 −5) , 1 2 (7− 73))x 2 + y 2 =6 xy=1x 2 +y<9

x 2 + y 2 <4x 2 +y<1 y>2xx 2 +y<−5 y>5x+10x 2 + y 2 <25 3 x 2 − y 2 >12x 2 − y 2 >−4 x 2 + y 2 <12

x 2 +3 y 2 >16 3 x 2 − y 2 <1y≥ e x y≤ln(x)+5y≤−log(x) y≤ e x4 x 2 + 1 y 2 =24 5 x 2 − 2 y 2 +4=0

6 x 2 − 1 y 2 =8 1 x 2 − 6 y 2 = 1 8

(−2 70 383 ,−2 35 29),(−2 70 383 ,2 35 29),(2 70 383 ,−2 35 29),(2 70 383 ,2 35 29)

x 2 −xy+ y 2 −2=0 x+3y=4x 2 −xy−2 y 2 −6=0 x 2 + y 2 =1

x 2 +4xy−2 y 2 −6=0 x=y+2xy<1 y> xx=0,y>0 0<x<1, x <y< 1 xx 2 +y<3 y>2x

C(x)=3 x 2 −10x+200 R(x)=−2 x 2 +100x+50. C(x)=8 x 2 −600x+21,500 R(x)=−3 x 2 +480x.

P(x) Q(x)

2 x−3 + −1 x+2

(x+2)(x−3).

2 x−3 (x+2 x+2)+ −1 x+2 (x−3 x−3)= 2x+4−x+3 (x+2)(x−3) = x+7 x 2 −x−6

x+7 x 2 −x−6 Simplified sum = 2 x−3 + −1 x+2 Partial fraction decomposition

x 2 −x−6 (x−3)(x+2), P(x) Q(x) :Q(x) P(x) Q(x) Q(x) P(x) Q(x)

P(x) Q(x) = A 1 (a 1 x+ b 1) + A 2 (a 2 x+ b 2) + A 3 (a 3 x+ b 3) +⋯+ A n (a n x+ b n) .

A,B, C, A n

P(x) Q(x) = A 1 (a 1 x+ b 1) + A 2 (a 2 x+ b 2) +⋯+ A n (a n x+ b n)

3x (x+2)(x−1)

A,B ,C.

3x (x+2)(x−1) = A (x+2) + B (x−1)

(x+2)(x−1)[3x (x+2)(x−1)]= (x+2) (x−1)[A (x+2)]+(x+2) (x−1) [B (x−1)]

3x=A(x−1)+B(x+2)

3x=Ax−A+Bx+2B 3x=(A+B)x−A+2B

3= A+B 0=−A+2B

B.

3= A+B 0=−A+2B ¯ 3= 0 +3B 1=B

B=1

3=A+1 2=A

3x (x+2)(x−1) = 2 (x+2) + 1 (x−1)

A B x x=1, A- B.

3x=A(x−1)+B(x+2) 3(1)=A[(1)−1]+B[(1)+2] 3=0+3B 1=B

B=1 A, x=−2

3x=A(x−1)+B(x+2) 3(−2)=A[(−2)−1]+B[(−2)+2] −6=−3A+0 −6 −3 =A 2=A

A B

3x (x+2)(x−1) = 2 (x+2) + 1 (x−1)

x (x−3)(x−2)

3 x−3 − 2 x−2 P(x) Q(x) P(x) Q(x) :Q(x) P(x) Q(x) , Q(x) n P(x) Q(x),

P(x) Q(x) = A 1 (ax+b) + A 2 (ax+b) 2 + A 3 (ax+b) 3 +⋯+ A n (ax+b) n

A,B, C

P(x) Q(x) = A 1 (ax+b) + A 2 (ax+b) 2 + . . . + A n (ax+b) n

− x 2 +2x+4 x 3 −4 x 2 +4x

x (x−2) 2 . (x−2), x,(x−2), (x−2) 2 .

− x 2 +2x+4 x 3 −4 x 2 +4x = A x + B (x−2) + C (x−2) 2

x (x−2) 2 [− x 2 +2x+4 x (x−2) 2]=[A x + B (x−2) + C (x−2) 2]x (x−2) 2 − x 2 +2x+4=A (x −2) 2 +Bx(x−2)+Cx

− x 2 +2x+4=A(x 2 −4x+4)+B(x 2 −2x)+Cx =A x 2 −4Ax+4A+B x 2 −2Bx+Cx

=(A+B) x 2 +(−4A−2B+C)x+4A

− x 2 +2x+4=(A+B) x 2 +(−4A−2B+C)x+4A

A+B=−1 (1) −4A−2B+C=2 (2) 4A=4 (3)

A

4A=4 A=1

A=1

A+B=−1 (1)+B=−1 B=−2

C, A B

−4A−2B+C=2 −4(1)−2(−2)+C=2 −4+4+C=2 C=2

− x 2 +2x+4 x 3 −4 x 2 +4x = 1 x − 2 (x−2) + 2 (x−2) 2

6x−11 (x−1) 2

6 x−1 − 5 (x−1) 2 P(x) Q(x) , A,B, C Ax+B,Bx+C, P(x) Q(x) :Q(x) P(x) Q(x) Q(x) P(x) Q(x)

P(x) Q(x) = A 1 x+ B 1 (a 1 x 2 + b 1 x+ c 1) + A 2 x+ B 2 (a 2 x 2 + b 2 x+ c 2) +⋯+ A n x+ B n (a n x 2 + b n x+ c n)

A,B,C, A,B, C A 1 x+ B 1 , A 2 x+ B 2 ,

P(x) Q(x) = A ax+b + A 1 x+ B 1 (a 1 x 2 + b 1 x+ c 1) + A 2 x+ B 2 (a 2 x 2 + b 2 x+ c 2) +⋯+ A n x+ B n (a n x 2 + b n x+ c n)

P(x) Q(x)

8 x 2 +12x−20 (x+3)(x 2 +x+2)

8 x 2 +12x−20 (x+3)(x 2 +x+2) = A (x+3) + Bx+C (x 2 +x+2)

(x+3)(x 2 +x+2)[8 x 2 +12x−20 (x+3)(x 2 +x+2)]=[A (x+3) + Bx+C (x 2 +x+2)](x+3)(x 2 +x+2)

8 x 2 +12x−20=A(x 2 +x+2)+(Bx+C)(x+3)

A x Bx+C x=−3

8 x 2 +12x−20=A(x 2 +x+2)+(Bx+C)(x+3) 8 (−3) 2 +12(−3)−20=A((−3) 2 +(−3)+2)+ (B(−3)+C)((−3)+3) 16=8A A=2

A,

8 x 2 +12x−20=2(x 2 +x+2)+(Bx+C)(x+3) 8 x 2 +12x−20=2 x 2 +2x+4+B x 2 +3B+Cx+3C 8 x 2 +12x −20=(2+B) x 2 +(2+3B+C)x+(4+3C)

2+B=8 (1) 2+3B+C=12 (2) 4+3C=−20 (3)

B C

2+B=8 (1) B=6 4+3C=−20 (3) 3C=−24 C=−8

8 x 2 +12x−20 (x+3)(x 2 +x+2) = 2 (x+3) + 6x−8 (x 2 +x+2)

A

8 x 2 +12x−20=A x 2 +Ax+2A+B x 2 +3B+Cx+3C 8 x 2 +12x−20=(A+B) x 2 +(A+3B+C)x+(2A+3C)

A+B=8 A+3B+C=12 2A+3C=−20

5 x 2 −6x+7 (x−1)(x 2 +1)

3 x−1 + 2x−4 x 2 +1 P(x) Q(x) P(x) Q(x) P(x) Q(x) , Q(x) P(x) Q(x),

P(x) (a x 2 +bx+c) n = A 1 x+ B 1 (a x 2 +bx+c) + A 2 x+ B 2 (a x 2 +bx+c) 2 + A 3 x+ B 3 (a x 2 +bx+c) 3 +⋯+ A n x+ B n (a x 2 +bx+c) n

A,B, C A 1 x+ B 1 , A 2 x+ B 2 ,

P(x) Q(x) = A ax+b + A 1 x+ B 1 (a x 2 +bx+c) + A 2 x+ B 2 (a x 2 +bx+c) 2 +⋯+ A n + B n (a x 2 +bx+c) n

x 4 + x 3 + x 2 −x+1 x (x 2 +1) 2

x,(x 2 +1), (x 2 +1) 2 . Ax+B.

x 4 + x 3 + x 2 −x+1 x (x 2 +1) 2 = A x + Bx+C (x 2 +1) + Dx+E (x 2 +1) 2

x (x 2 +1) 2 .

x 4 + x 3 + x 2 −x+1=A (x 2 +1) 2 +(Bx+C)(x)(x 2 +1)+(Dx+E)(x)

x 4 + x 3 + x 2 −x+1=A(x 4 +2 x 2 +1)+B x 4 +B x 2 +C x 3 +Cx+D x 2 +Ex

=A x 4 +2A x 2 +A+B x 4 +B x 2 +C x 3 +Cx+D x 2 +Ex

x 4 + x 3 + x 2 −x+1=(A+B) x 4 +(C) x 3 +(2A+B+D) x 2 +(C+E)x+A

A+B=1 C=1 2A+B+D=1 C+E=−1 A=1

A=1

1+B=1 B=0

A=1 B=0

2(1)+0+D=1 D=−1

C=1

1+E=−1 E=−2

A=1, B=0, C=1, D=−1, E=−2.

x 4 + x 3 + x 2 −x+1 x (x 2 +1) 2 = 1 x + 1 (x 2 +1) − x+2 (x 2 +1) 2

x 3 −4 x 2 +9x−5 (x 2 −2x+3) 2

x−2 x 2 −2x+3 + 2x+1 (x 2 −2x+3) 2 P(x) Q(x) A a 1 x+ b 1 + B a 2 x+ b 2 . P(x) Q(x)

P(x) Q(x) A x + Bx+C (a x 2 +bx+c) . P(x) Q(x) , Q(x)

Ax+B (a x 2 +bx+c) + A 2 x+ B 2 (a x 2 +bx+c) 2 +⋯+ A n x+ B n (a x 2 +bx+c) n .

1 x 2 +1 7x+13 3 x 2 +8x+15 = A x+1 + B 3x+5 , 7x+13=A(3x+5)+B(x+1). x A B A B. x=−1, A=3.
x=− 5 3 , −2.5x+16 x 2 +10x+243x−79 x 2 −5x−248 x+3 − 5 x−8−x−24 x 2 −2x−2410x+47 x 2 +7x+10
1 x+5 + 9 x+2x 6 x 2 +25x+2532x−11 20 x 2 −13x+23 5x−2 + 4 4x−1x+1 x 2 +7x+105x x 2 −9
5 2(x+3) + 5 2(x−3)10x x 2 −256x x 2 −43 x+2 + 3 x−22x−3 x 2 −6x+54x−1 x 2 −x−6
9 5(x+2) + 11 5(x−3)4x+3 x 2 +8x+153x−1 x 2 −5x+68 x−3 − 5 x−2−5x−19 (x+4) 2x (x−2) 2
1 x−2 + 2 (x−2) 27x+14 (x+3) 2−24x−27 (4x+5) 2− 6 4x+5 + 3 (4x+5) 2−24x−27 (6x−7) 2
5−x (x−7) 2− 1 x−7 − 2 (x−7) 25x+14 2 x 2 +12x+185 x 2 +20x+8 2x (x+1) 2
4 x − 3 2(x+1) + 7 2 (x+1) 24 x 2 +55x+25 5x (3x+5) 254 x 3 +127 x 2 +80x+16 2 x 2 (3x+2) 2
4 x + 2 x 2 − 3 3x+2 + 7 2 (3x+2) 2x 3 −5 x 2 +12x+144 x 2 (x 2 +12x+36)
4 x 2 +6x+11 (x+2)(x 2 +x+3)x+1 x 2 +x+3 + 3 x+24 x 2 +9x+23 (x−1)(x 2 +6x+11)
−2 x 2 +10x+4 (x−1)(x 2 +3x+8)4−3x x 2 +3x+8 + 1 x−1x 2 +3x+1 (x+1)(x 2 +5x−2)
4 x 2 +17x−1 (x+3)(x 2 +6x+1)2x−1 x 2 +6x+1 + 2 x+34 x 2 (x+5)(x 2 +7x−5)4 x 2 +5x+3 x 3 −1
1 x 2 +x+1 + 4 x−1−5 x 2 +18x−4 x 3 +83 x 2 −7x+33 x 3 +272 x 2 −3x+9 + 3 x+3x 2 +2x+40 x 3 −125
4 x 2 +4x+12 8 x 3 −27− 1 4 x 2 +6x+9 + 1 2x−3−50 x 2 +5x−3 125 x 3 −1
−2 x 3 −30 x 2 +36x+216 x 4 +216x1 x + 1 x+6 − 4x x 2 −6x+363 x 3 +2 x 2 +14x+15 (x 2 +4) 2
x 3 +6 x 2 +5x+9 (x 2 +1) 2x+6 x 2 +1 + 4x+3 (x 2 +1) 2x 3 − x 2 +x−1 (x 2 −3) 2x 2 +5x+5 (x+2) 2
x+1 x+2 + 2x+3 (x+2) 2x 3 +2 x 2 +4x (x 2 +2x+9) 2x 2 +25 (x 2 +3x+25) 2
1 x 2 +3x+25 − 3x (x 2 +3x+25) 22 x 3 +11x+7x+70 (2 x 2 +x+14) 25x+2 x (x 2 +4) 2
1 8x − x 8(x 2 +4) + 10−x 2 (x 2 +4) 2x 4 + x 3 +8 x 2 +6x+36 x (x 2 +6) 22x−9 (x 2 −x) 2
− 16 x − 9 x 2 + 16 x−1 − 7 (x−1) 25 x 3 −2x+1 (x 2 +2x) 2x 2 +4 (x+1) 3
1 x+1 − 2 (x+1) 2 + 5 (x+1) 3x 3 −4 x 2 +5x+4 (x−2) 37 x+8 + 5 x−2 − x−1 x 2 −6x−16
5 x−2 − 3 10(x+2) + 7 x+8 − 7 10(x−8)1 x−4 − 3 x+6 − 2x+7 x 2 +2x−24
2x x 2 −16 − 1−2x x 2 +6x+8 − x−5 x 2 −4x− 5 4x − 5 2(x+2) + 11 2(x+4) + 5 4(x+4) A,B, C

A=[1 2 3 4],B=[1 2 7 0 −5 6 7 8 2],C=[−1 0 3 3 2 1]

m × n m n A a ij , i, j. A, a 23 .

A=[a 11 a 12 a 13 a 21 a 22 a 23 a 31 a 32 a 33]

n × n, 3×3 1 × n.

[a 11 a 12 a 13]

m × 1.

[a 11 a 21 a 31]

A,B,C, a ij , i j m × n m n A: A? a 31 a 22 ?

A=[2 1 0 2 4 7 3 1 −2]

3 × 3 a 31 a 22 3 × 3 3 × 3 2 × 3 3 × 3 A B A B C D

A+B=C such that a ij + b ij = c ij
A−B=D such that a ij − b ij = d ij
A+B=B+A
(A+B)+C=A+(B+C)

A B,

A=[a b c d] and B=[e f g h]
A+B=[a b c d]+[e f g h] =[a+e b+f c+g d+h]

A B.

A=[4 1 3 2] and B=[5 9 0 7]

a 11 , A b 11 , B.

A+B=[4 1 3 2]+[5 9 0 7] =[4+5 1+9 3+0 2+7] =[9 10 3 9]

A B.

A=[−2 3 0 1] and B=[8 1 5 4]
A−B=[−2 3 0 1]−[8 1 5 4] =[−2−8 3−1 0−5 1−4] =[−10 2 −5 −3]

A B:

A=[2 −10 −2 14 12 10 4 −2 2] and B=[6 10 −2 0 −12 −4 −5 2 −2]

$A+B=[\ 2\ \ -10\ \ -2\ 14\ \ 12\ \ 10\ 4\ \ -2\ \ 2\ ]+[\ 6\ \ 10\ \ -2\ 0\ \ -12\ \ -4\ -5\ \ 2\ \ -2\ ]\quad =[\ 2+6\ \ -10+10\ \ -2-2\ 14+0\ \ 12-12\ \ 10-4\ 4-5\ \ -2+2\ \ 2-2\ ]\quad =[\ 8\ \ 0\ \ -4\ 14\ \ 0\ \ 6\ -1\ \ 0\ \ 0\ ]$

$A-B=[\ 2\ -10\ -2\ 14\ 12\ 10\ 4\ -2\ 2\ ]-[\ 6\ 10\ -2\ 0\ -12\ -4\ -5\ 2\ -2\ ]\quad =[\ 2-6\ \ -10-10\ \ -2+2\ 14-0\ \ 12+12\ \ 10+4\ 4+5\ \ -2-2\ \ 2+2\ ]\quad =[\ -4\ \ -20\ \ 0\ 14\ \ 24\ \ 14\ 9\ \ -4\ \ 4\ ]$

A B.

$$A=[\ 2\ 6\ 1\ 0\ 1\ -3\ ]\ \text{ and }\ B=[\ 3\ -2\ 1\ 5\ -4\ 3\ ]$$

$$A+B=[\ 2\ 1\ 1\ \ 6\ \ 0\ \ -3\ ]+[\ \ 3\ 1\ -4\ \ -2\ \ 5\ \ 3\ ]=[\ 2\ +3\ 1\ +\ 1\ 1+(-4)\ \ 6+(-2)\ 0\ +\ 5\ -3\ +\ 3\ ]=[\ 5\ \ 2\ -3\ \ 4\ 5\ 0\ ]$$

$$C\ 2013\ =[\ 15\ 16\ 16\ \ 27\ 34\ 34\ ]$$

C

$$(0.15)\ C\ 2013\ =[\ (0.15)15\ (0.15)16\ (0.15)16\ \ (0.15)27\ (0.15)34\ (0.15)34\ ]=[\ 2.25\ 2.4\ 2.4\ \ 4.05\ 5.1\ 5.1\ ]$$

$$[\ 3\ 3\ 3\ \ 5\ 6\ 6\ ]$$

$$[\ 15\ 16\ 16\ \ 27\ 34\ 34\ ]+[\ 3\ 3\ 3\ \ 5\ 6\ 6\ ]=[\ 18\ 19\ 19\ \ 32\ 40\ 40\ ]$$

$$C\ 2014\ =[\ 18\ 19\ 19\ \ 32\ 40\ 40\ ]$$

$$A=[\ a\ 11\ a\ 12\ a\ 21\ a\ 22\ ]$$

cA

$$cA=c[\ a\ 11\ a\ 12\ a\ 21\ a\ 22\ ]\quad =[\ c\ a\ 11\ c\ a\ 12\ c\ a\ 21\ c\ a\ 22\ ]$$

A,B, C a b,

$$a(A+B)=aA+aB\ (a+b)A=aA+bA$$

A

$$A=[\ 8\ 1\ 5\ 4\ ]$$

A

$$3A=3[\ 8\ \ 1\ 5\ \ 4\ ]\quad =[\ 3\cdot 8\ \ 3\cdot 1\ 3\cdot 5\ \ 3\cdot 4\ ]\quad =[\ 24\ 3\ 15\ 12\ ]$$

B, $-2B$

$$B=[\ 4\ 1\ 3\ 2\ ]$$

$-2B=[\ -8\ -2\ -6\ -4\ ]$ 3A+2B.

$$A=[\ 1\ -2\ 0\ 0\ -1\ 2\ 4\ 3\ -6\ ]\text{ and }B=[\ -1\ 2\ 1\ 0\ -3\ 2\ 0\ 1\ -4\ ]$$

3A, 2B.

$$3A=[\ 3\cdot 1\ \ 3(-2)\ \ 3\cdot 0\ 3\cdot 0\ \ 3(-1)\ \ 3\cdot 2\ 3\cdot 4\ \ 3\cdot 3\ \ 3(-6)\ ]\quad =[\ 3\ \ -6\ \ 0\ 0\ \ -3\ \ 6\ 12\ \ 9\ \ -18\ ]$$

$$2B=[\ 2(-1)\ \ 2\cdot 2\ \ 2\cdot 1\ 2\cdot 0\ \ 2(-3)\ \ 2\cdot 2\ 2\cdot 0\ \ 2\cdot 1\ \ 2(-4)\ ]\quad =[\ -2\ 4\ 2\ 0\ -6\ 4\ 0\ 2\ -8\ ]$$

3A+2B.

$$3A+2B=[\ 3\ -6\ 0\ 0\ -3\ 6\ 12\ 9\ -18\ ]+[\ -2\ 4\ 2\ 0\ -6\ 4\ 0\ 2\ -8\ ]\quad =[\ 3-2\ \ -6+4\ 0+2\ 0+0\ \ -3-6\ 6+4\ \ 12+0\ \ 9+2\ -18-8\ ]\quad =[\ 1\ \ -2\ 2\ 0\ \ -9\ 10\ 12\ \ 11\ -26\ ]$$

A $m\times r$ B $r\times n$ AB $m\times n$ AB A B. A B i AB, i A j B A B, A 2×3 B 3×3, AB 2×3

$$A=[\ a\ 11\ a\ 12\ a\ 13\ a\ 21\ a\ 22\ a\ 23\ ]\text{ and }B=[\ b\ 11\ b\ 12\ b\ 13\ b\ 21\ b\ 22\ b\ 23\ b\ 31\ b\ 32\ b\ 33\ ]$$

AB. AB, A B,

$$[\ a\ 11\ a\ 12\ a\ 13\ ]\cdot[\ b\ 11\ b\ 21\ b\ 31\ ]=a\ 11\cdot b\ 11+a\ 12\cdot b\ 21+a\ 13\cdot b\ 31$$

AB, A B,

$$[\ a\ 11\ a\ 12\ a\ 13\ ]\cdot[\ b\ 12\ b\ 22\ b\ 32\ ]=a\ 11\cdot b\ 12+a\ 12\cdot b\ 22+a\ 13\cdot b\ 32$$

AB, A B,

$$[\ a\ 11\ a\ 12\ a\ 13\ ]\cdot[\ b\ 13\ b\ 23\ b\ 33\ ]=a\ 11\cdot b\ 13+a\ 12\cdot b\ 23+a\ 13\cdot b\ 33$$

AB. A B; A B; A B.

$$AB=[\ a\ 11\cdot b\ 11+a\ 12\cdot b\ 21+a\ 13\cdot b\ 31\ a\ 21\cdot b\ 11+a\ 22\cdot b\ 21+a\ 23\cdot b\ 31\ \ a\ 11\cdot b\ 12+a\ 12\cdot b\ 22+a\ 13\cdot b\ 32\ a\ 21\cdot b\ 12+a\ 22\cdot b\ 22+a\ 23\cdot b\ 32\ \ a\ 11\cdot b\ 13+a\ 12\cdot b\ 23+a\ 13\cdot b\ 33\ a\ 21\cdot b\ 13+a\ 22\cdot b\ 23+a\ 23\cdot b\ 33\ ]$$

A,B, C $(\ AB\ )C=A(\ BC\ )$. $C(A+B)=CA+CB$, $(A+B)C=AC+BC$. A B.

$$A=[\ 1\ 2\ 3\ 4\ ]\ \text{ and }\ B=[\ 5\ 6\ 7\ 8\ ]$$

A 2 × 2 B 2 × 2. 2 × 2. A B: AB. BA.

A=[−1 2 3 4 0 5] and B=[5 −4 2 −1 0 3]

A 2 × 3 B 3 × 2, A B. 2 × 2 A B.

AB=[−1 2 3 4 0 5] [5 −1 −4 0 2 3] =[−1(5)+2(−4)+3(2) −1(−1)+2(0)+3(3) 4(5)+0(−4)+5(2) 4(−1)+0(0)+5(3)] =[−7 10 30 11]

B 3 × 2 A 2 × 3. 3 × 3

BA=[5 −1 −4 0 2 3] [−1 2 3 4 0 5] =[5(−1)+−1(4) 5(2)+−1(0) 5(3)+−1(5) −4(−1)+0(4) −4(2)+0(0) −4(3)+0(5) 2(−1)+3(4) 2(2)+3(0) 2(3)+3(5)] =[−9 10 10 4 −8 −12 10 4 21]

AB BA

AB=[−7 10 30 11]≠[−9 10 10 4 −8 −12 10 4 21]=BA

3 × 4 4 × 2.

E=[6 30 14 10 24 20]

C=[300 10 30]

CE=[300 10 30]·[6 10 30 24 14 20] =[300(6)+10(30)+30(14) 300(10)+10(24)+30(20)] =[2,520 3,840]

[A],[B],[C],... AB−C

A=[−15 25 32 41 −7 −28 10 34 −2],B=[45 21 −37 −24 52 19 6 −48 −31],and C=[−100 −89 −98 25 −56 74 −67 42 −75].

A [A], B [B], C [C].

[A]×[B]−[C]

[−983 −462 136 1,820 1,897 −856 −311 2,032 413]

3×2 A B, A B; A B, 2×2 2×3 [1 2 3 4]+[6 5 4 3 2 1] AB BA A m×n B n×m, AB=BA? AB, A B AB. BA, B A BA.

A=[1 3 0 7],B=[2 14 22 6],C=[1 5 8 92 12 6],D=[10 14 7 2 5 61],E=[6 12 14 5],F=[0 9 78 17 15 4]

A+BC+D[11 19 15 94 17 67]A+CB−E[−4 2 8 1]C+FD−B

A=[4 6 13 12],B=[3 9 21 12 0 64],C=[16 3 7 18 90 5 3 29],D=[18 12 13 8 14 6 7 4 21]

5A3B[9 27 63 36 0 192]−2B−4C[−64 −12 −28 −72 −360 −20 −12 −116]1 2 C100D

[1,800 1,200 1,300 800 1,400 600 700 400 2,100]

A=[−1 5 3 2],B=[3 6 4 −8 0 12],C=[4 10 −2 6 5 9],D=[2 −3 12 9 3 1 0 8 −10]

ABBC[20 102 28 28]CABD[60 41 2 −16 120 −216]DCCB[−68 24 136 −54 −12 64 −57 30 128]

A=[2 −5 6 7],B=[−9 6 −4 2],C=[0 9 7 1],D=[−8 7 −5 4 3 2 0 9 2],E=[4 5 3 7 −6 −5 1 0 9]

A+B−C4A+5D2C+B3D+4E[−8 41 −3 40 −15 −14 4 27 42]C−0.5D100D−10E

[−840 650 −530 330 360 250 −10 900 110] A 2 =A·A

A=[−10 20 5 25],B=[40 10 −20 30],C=[−1 0 0 −1 1 0]

ABBA[−350 1,050 350 350]CABCA 2B 2[1,400 700 −1,400 700]C 2B 2 A 2

[332,500 927,500 −227,500 87,500]A 2 B 2(AB) 2[490,000 0 0 490,000](BA) 2 A 2 =A·A

A=[1 0 2 3],B=[−2 3 4 −1 1 −5],C=[0.5 0.1 1 0.2 −0.5 0.3],D=[1 0 −1 −6 7 5 4 2 1]

AB[−2 3 4 −7 9 −7]BABD[−4 29 21 −27 −3 1]DCD 2[−3 −2 −2 −28 59 46 −4 16 7]A 2D 3

[1 −18 −9 −198 505 369 −72 126 91](AB)CA(BC)

[0 1.6 9 −1]

A=[−2 0 9 1 8 −3 0.5 4 5],B=[0.5 3 0 −4 1 6 8 7 2],C=[1 0 1 0 1 0 1 0 1]

ABBA[2 24 −4.5 12 32 −9 −8 64 61]CABC[0.5 3 0.5 2 1 2 10 7 10]ABC

B=[1 0 0 0 0 1 0 1 0]

B 2[1 0 0 0 1 0 0 0 1]B 3B 4[1 0 0 0 1 0 0 0 1]B 5 B n . B 201 B 202 ,

B n ={ [1 0 0 0 1 0 0 0 1], n even, [1 0 0 0 0 1 0 1 0], n odd. 2 × 2

3x+4y=7 4x−2y=5

[3 4 4 −2 | 7 5]

[3 4 4 −2]

3x−y−z=0 x+y=5 2x−3z=2

[3 −1 −1 1 1 0 2 0 −3]

[3 −1 −1 1 1 0 2 0 −3 | 0 5 2]

ax+by+cz=d

x+2y−z=3 2x−y+2z=6 x−3y+3z=4

[1 2 −1 2 −1 2 1 −3 3 | 3 6 4]

4x−3y=11 3x+2y=4

[4 −3 3 2 | 11 4]

[1 −3 −5 2 −5 −4 −3 5 4 | −2 5 6]

x, y, z,

[1 −3 −5 2 −5 −4 −3 5 4 | −2 5 6]→ x−3y−5z=−2 2x−5y−4z=5 −3x+5y+4z=6

[1 −1 1 2 −1 3 0 1 1 | 5 1 −9]

x − y + z=5 2x − y + 3z=1 y + z=−9

Row-echelon form [1 a b 0 1 d 0 0 1]

R i ⇔ R j c R i R i +c R j) A

A=[a 11 a 12 a 13 a 21 a 22 a 23 a 31 a 32 a 33] → After Gaussian elimination A=[1 b 12 b 13 0 1 b 23 0 0 1]

2×2

2x+3y=6 x−y= 1 2

[2 3 1 −1 | 6 1 2]

R 1 ⇔ R 2 →[1 −1 2 3 | 1 2 6]

−2,

−2 R 1 + R 2 = R 2 →[1 −1 0 5 | 1 2 5]

1 5 .

1 5 R 2 = R 2 →[1 −1 0 1 | 1 2 1]

y=1. y=1

x−(1)= 1 2 x= 3 2

(3 2 ,1).

4x+3y=11 x−3y=−1

(2, 1) 2 × 2

2x+y=1 4x+2y=6

[2 1 4 2 | 1 6]

1 2 .

1 2 R 1 = R 1 →[1 1 2 4 2 | 1 2 6]

−4

−4 R 1 + R 2 = R 2 →[1 1 2 0 0 | 1 2 4]

0=4.

3x+4y=12 6x+8y=24

A=[3 4 6 8 | 12 24]

− 1 2 R 2 + R 1 = R 1 →[0 0 6 8 | 0 24] R 1 ⇔ R 2 →[6 8 0 0 | 24 0]

0y=0. y.

3x+4y=12 4y=12−3x y=3− 3 4 x

(x,3− 3 4 x).

[1 −3 4 2 −5 6 −3 3 4 | 3 6 6]

−2

−2 R 1 + R 2 = R 2 →[1 −3 4 0 1 −2 −3 3 4 | 3 0 6]

3 R 1 + R 3 = R 3 →[1 −3 4 0 1 −2 0 −6 16 | 3 0 15]

6 R 2 + R 3 = R 3 →[1 −3 4 0 1 −2 0 0 4 | 3 0 15]

1 2 R 3 = R 3 →[1 −3 4 0 1 −2 0 0 1 | 3 −6 21 2]

x−2y+3z=9 −x+3y=−4 2x−5y+5z=17

[1 − 5 2 5 2 0 1 5 0 0 1 | 17 2 9 2]

x − y + z= 8 2x + 3y − z=−2 3x − 2y −9z= 9

[1 −1 1 2 3 −1 3 −2 −9 | 8 −2 9]

−2 R 1 + R 2 = R 2 →[1 −1 1 0 5 −3 3 −2 −9 | 8 −18 9] −3 R 1 + R 3 = R 3 →[1 −1 1 0 5 −3 0 1 −12 | 8 −18 −15]

R 2 R 3 .

Interchange R 2 and R 3 →[1 −1 1 8 0 1 −12 −15 0 5 −3 −18]

−5 R 2 + R 3 = R 3 →[1 −1 1 0 1 −12 0 0 57 | 8 −15 57] − 1 57 R 3 = R 3 →[1 −1 1 0 1 −12 0 0 1 | 8 −15 1]

x−y+z=8 y−12z=−15 z=1

(4,−3,1).

−x−2y+z=−1 2x+3y=2 y−2z=0

[−1 −2 1 2 3 0 0 1 −2 | −1 2 0]

−1

− R 1 →[1 2 −1 1 2 3 0 2 0 1 −2 0]

R 2 ↔ R 3 →[1 2 −1 0 1 −2 2 3 0 | 1 0 2]

−2 R 1 + R 3 = R 3 →[1 2 −1 0 1 −2 0 −1 2 | 1 0 0]

R 2 + R 3 = R 3 →[1 2 −1 0 1 −2 0 0 0 | 2 1 0]

x+2y−z=1 y−2z=0 0=0

0=0 y z x.

x+2y−z=1 y=2z x+2(2z)−z=1 x+3z=1 z= 1−x 3

z y x.

y−2z=0 z= 1−x 3 y−2(1−x 3)=0 y= 2−2x 3

(x, 2−2x 3 , 1−x 3).

x+4y−z=4 2x+5y+8z=15 x+3y−3z=1

(1, 1, 1) [A], [B], [C], ….

5x+3y+9z=−1 −2x+3y−z=−2 −x−4y+5z=1

[5 3 9 −2 3 −1 −1 −4 5 | 5 −2 −1]

[A].

[A]=[5 3 9 −1 −2 3 −1 −2 −1 −4 5 1]

[A].

ref([A])

[1 3 5 9 5 1 5 0 1 13 21 − 4 7 0 0 1 − 24 187]→ x+ 3 5 y+ 9 5 z=− 1 5 y+ 13 21 z=− 4 7 z=− 24 187

(61 187 ,− 92 187 ,− 24 187). x= y=

x+y=12,000 0.105x+0.12y=1,335

[1 1 0.105 0.12 | 12,000 1,335]

−0.105

[1 1 0 0.015 | 12,000 75]

0.015y=75 y=5,000

12,000−5,000=7,000. x y z

x+y+z=10,000 0.05x+0.08y+0.09z=770 2x−z=0

[1 1 1 0.05 0.08 0.09 2 0 −1 | 10,000 770 0]

−0.05 R 1 + R 2 = R 2 →[1 1 1 0 0.03 0.04 2 0 −1 | 10,000 270 0] −2 R 1 + R 3 = R 3 →[1 1 1 0 0.03 0.04 0 −2 −3 | 10,000 270 −20,000] 1 0.03 R 2 = R 2 →[0 1 1 0 1 4 3 0 −2 −3 | 10,000 9,000 −20,000] 2 R 2 + R 3 = R 3 →[1 1 1 0 1 4 3 0 0 − 1 3 | 10,000 9,000 −2,000]

− 1 3 z=−2,000; z=6,000. y+ 4 3 z=9,000. z=6,000,

y+ 4 3 (6,000)=9,000 y+8,000=9,000 y=1,000 x+y+z=10,000. y=1,000 z=6,000,

x+1,000+6,000=10,000 x=3,000

[9 3 1 −2 | 0 6]. R 2 = R 2 −9 R 1 . R 2 = R 1 −9 R 2 . 8x−37y=8 2x+12y=3 16y=4 9x−y=2

[0 16 9 −1 | 4 2] 3x+2y+10z=3 −6x+2y+5z=13 4x+z=18

x+5y+8z=19 12x+3y=4 3x+4y+9z=−7[1 5 8 12 3 0 3 4 9 | 16 4 −7]

6x+12y+16z=4 19x−5y+3z=−9 x+2y=−8[−2 5 6 −18 | 5 26]−2x+5y=5 6x−18y=26

[3 4 10 17 | 10 439][3 2 0 −1 −9 4 8 5 7 | 3 −1 8]3x+2y=13 −x−9y+4z=53 8x+5y+7z=80

[8 29 1 −1 7 5 0 0 3 | 43 38 10][4 5 −2 0 1 58 8 7 −3 | 12 2 −5]

4x+5y−2z=12 y+58z=2 8x+7y−3z=−5[1 0 0 0 | 3 0][1 0 1 0 | 1 2][1 2 4 5 | 3 6]

[−1 2 4 −5 | −3 6](−1,−2)[−2 0 0 2 | 1 −1] 2x−3y=−9 5x+4y=58(6,7)6x+2y=−4 3x+4y=−17

2x+3y=12 4x+y=14(3,2)−4x−3y=−2 3x−5y=−13−5x+8y=3 10x+6y=5(1 5 , 1 2)

3x+4y=12 −6x−8y=−24−60x+45y=12 20x−15y=−4(x, 4 15 (5x+1))11x+10y=43 15x+20y=65

2x−y=2 3x+2y=17(3,4)−1.06x−2.25y=5.51 −5.03x−1.08y=5.403 4 x− 3 5 y=4 1 4 x+ 2 3 y=1

(196 39 ,− 5 13)1 4 x− 2 3 y=−1 1 2 x+ 1 3 y=3[1 0 0 0 1 1 0 0 1 | 31 45 87](31,−42,87)

[1 0 1 1 1 0 0 1 1 | 50 20 −90][1 2 3 0 5 6 0 0 8 | 4 7 9](21 40 , 1 20 , 9 8)

[−0.1 0.3 −0.1 −0.4 0.2 0.1 0.6 0.1 0.7 | 0.2 0.8 −0.8] −2x+3y−2z=3 4x+2y−z=9 4x−8y+2z=−6

(18 13 , 15 13 ,− 15 13) x+y−4z=−4 5x−3y−2z=0 2x+6y+7z=30

2x+3y+2z=1 −4x−6y−4z=−2 10x+15y+10z=5(x,y, 1 2 (1−2x−3y))

x+2y−z=1 −x−2y+2z=−2 3x+6y−3z=5 x+2y−z=1 −x−2y+2z=−2 3x+6y−3z=3(x,− x 2 ,−1)

x+y=2 x+z=1 −y−z=−3x+y+z=100 x+2z=125 −y+2z=25(125,−25,0)

1 4 x− 2 3 z=− 1 2 1 5 x+ 1 3 y= 4 7 1 5 y− 1 3 z= 2 9

− 1 2 x+ 1 2 y+ 1 7 z=− 53 14 1 2 x− 1 2 y+ 1 4 z=3 1 4 x+ 1 5 y+ 1 3 z= 23 15(8,1,−2)

− 1 2 x− 1 3 y+ 1 4 z=− 29 6 1 5 x+ 1 6 y− 1 7 z= 431 210 − 1 8 x+ 1 9 y+ 1 10 z=− 49 45

x−1 7 + y−2 8 + z−3 4 =0 x+y+z=6 x+2 3 +2y+ z−3 3 =5(1,2,3)

x−1 4 − y+1 4 +3z=−1 x+5 2 + y+7 4 −z=4 x+y− z−2 2 =1

x−3 4 − y−1 3 +2z=−1 x+5 2 + y+5 2 + z+5 2 =8 x+y+z=1

(x, 31 28 − 3x 4 , 1 28 (−7x−3))x−3 10 + y+3 2 −2z=3 x+5 4 − y−1 8 +z= 3 2 x−1 4 + y+4 2 +3z= 3 2

x−3 4 − y−1 3 +2z=−1 x+5 2 + y+5 2 + z+5 2 =7 x+y+z=1 A B a a −1 ,

a a −1 = a −1 a=(1 a)a=1. 2 −1 = 1 2 (1 2)2=1. A A −1 I n n 2×2 3×3

I 2 =[1 0 0 1]

I 3 =[1 0 0 0 1 0 0 0 1]

AI=IA=A.

A A −1 =I A −1 A=I

A A −1 = A −1 A=I, A A −1 2 × 2 2 × 2 3 × 3 I n ,

I 2 =[1 0 0 1] I 3 =[1 0 0 0 1 0 0 0 1] 2 × 2 3 × 3

A n × n B n × n AB=BA= I n , B= A −1 , A. AI=IA=A.

A=[3 4 −2 5]

A

AI=[3 4 −2 5] [1 0 0 1]=[3·1+4·0 3·0+4·1 −2·1+5·0 −2·0+5·1]=[3 4 −2 5]

AI=[1 0 0 1] [3 4 −2 5]=[1·3+0·(−2) 1·4+0·5 0·3+1·(−2) 0·4+1·5]=[3 4 −2 5]

A n × n B n × n AB. AB=I, BA. BA=I, B= A −1 A= B −1 .

A=[1 5 −2 −9],B=[−9 −5 2 1]

AB BA.

AB=[1 5 −2 −9]·[−9 −5 2 1] =[1(−9)+5(2) 1(−5)+5(1) −2(−9)−9(2) −2(−5)−9(1)] =[1 0 0 1]

BA=[−9 −5 2 1]·[1 5 −2 −9] =[−9(1)−5(−2) −9(5)−5(−9) 2(1)+1(−2) 2(−5)+1(−9)] =[1 0 0 1]

A B

A=[1 4 −1 −3],B=[−3 −4 1 1]

AB=[1 4 −1 −3] [−3 −4 1 1]=[1(−3)+4(1) 1(−4)+4(1) −1(−3)+−3(1) −1(−4)+−3(1)]=[1 0 0 1] BA=[−3 −4 1 1] [1 4 −1 −3]=[−3(1)+−4(−1) −3(4)+−4(−3) 1(1)+1(−1) 1(4)+1(−3)]=[1 0 0 1]

A=[1 −2 2 −3]

A

[1 −2 2 −3] [a b c d]=[1 0 0 1]

[1 −2 2 −3] [a b c d]=[1a−2c 1b−2d 2a−3c 2b−3d]

1a−2c=1 R 1 2a−3c=0 R 2

(−2) R 1 + R 2 → R 2 . c.

1a−2c=1 0+1c=−2 c=−2

a.

a−2(−2)=1 a+4=1 a=−3

1b−2d=0 R 1 2b−3d=1 R 2

(−2) R 1 + R 2 = R 2 . d.

1b−2d=0 0+1d=1 d=1

b.

b−2(1)=0 b−2=0 b=2

A −1 =[−3 2 −2 1]

A I, I A −1 .

A=[2 1 5 3]

A

[2 1 5 3 | 1 0 0 1]

A

[5 3 2 1 | 0 1 1 0]

−2

[1 1 2 1 | −2 1 1 0]

−2

[1 1 0 −1 | −2 1 5 −2]

[1 0 0 −1 | 3 −1 5 −2]

−1.

[1 0 0 1 | 3 −1 −5 2]

A −1 .

A −1 =[3 −1 −5 2]

2 × 2 A 2×2

A=[a b c d]

A

A −1 = 1 ad−bc [d −b −c a]

ad−bc≠0. ad−bc=0, A

A=[1 −2 2 −3]

A −1 = 1 (1)(−3)−(−2)(2) [−3 2 −2 1] = 1 −3+4 [−3 2 −2 1] =[−3 2 −2 1]

A

[1 −2 2 −3 | 1 0 0 1]

A −2

[1 −2 0 1 | 1 0 −2 1]

[1 0 0 1 | −3 2 −2 1]

A −1 =[−3 2 −2 1]

A.

A=[1 −1 2 3]

A −1 =[3 5 1 5 − 2 5 1 5]

A=[3 6 1 2]

[3 6 1 3 | 1 0 0 1]

[1 3 3 6 | 0 1 1 0]

[1 2 0 0 | 1 0 −3 1]

2×2 3×3 3 × 3

A=[2 3 1 3 3 1 2 4 1]

A

A|I=[2 3 1 3 3 1 2 4 1 | 1 0 0 0 1 0 0 0 1]

A 3 × 3 A A −1 =I A −1 A=I. 3 × 3 A,

A=[2 3 1 3 3 1 2 4 1]

A A. A.

[2 3 1 3 3 1 2 4 1 | 1 0 0 0 1 0 0 0 1] → Interchange R 2 and R 1 [3 3 1 2 3 1 2 4 1 | 0 1 0 1 0 0 0 0 1]

− R 2 + R 1 = R 1 →[1 0 0 2 3 1 2 4 1 | −1 1 0 1 0 0 0 0 1]

− R 2 + R 3 = R 3 →[1 0 0 2 3 1 0 1 0 | −1 1 0 1 0 0 −1 0 1]

R 3 ↔ R 2 →[1 0 0 0 1 0 2 3 1 | −1 1 0 −1 0 1 1 0 0]

−2 R 1 + R 3 = R 3 →[1 0 0 0 1 0 0 3 1 | −1 1 0 −1 0 1 3 −2 0]

−3 R 2 + R 3 = R 3 →[1 0 0 0 1 0 0 0 1 | −1 1 0 −1 0 1 6 −2 −3]

A −1 =B=[−1 1 0 −1 0 1 6 −2 −3]

B= A −1 , A A −1 =I A −1 A=I.

A A −1 =[2 3 1 3 3 1 2 4 1] [−1 1 0 −1 0 1 6 −2 −3] =[2(−1)+3(−1)+1(6) 2(1)+3(0)+1(−2) 2(0)+3(1)+1(−3) 3(−1)+3(−1)+1(6) 3(1)+3(0)+1(−2) 3(0)+3(1)+1(−3) 2(−1)+4(−1)+1(6) 2(1)+4(0)+1(−2) 2(0)+4(1)+1(−3)] =[1 0 0 0 1 0 0 0 1]

A −1 A=[−1 1 0 −1 0 1 6 −2 −3] [2 3 1 3 3 1 2 4 1] =[−1(2)+1(3)+0(2) −1(3)+1(3)+0(4) −1(1)+1(1)+0(1) −1(2)+0(3)+1(2) −1(3)+0(3)+1(4) −1(1)+0(1)+1(1) 6(2)+−2(3)+−3(2) 6(3)+−2(3)+−3(4) 6(1)+−2(1)+−3(1)] =[1 0 0 0 1 0 0 0 1]

3×3

A=[2 −17 11 −1 11 −7 0 3 −2]

A −1 =[1 1 2 2 4 −3 3 6 −5] X B

AX=B

A X B AX=B.

a 1 x+ b 1 y= c 1 a 2 x+ b 2 y= c 2

A=[a 1 b 1 a 2 b 2]

X=[x y]

B=[c 1 c 2]

AX=B

[a 1 b 1 a 2 b 2] [x y]=[c 1 c 2]

(2 −1) 2=(1 2) 2=1. ax=b x, a.

ax=b (1 a)ax=(1 a)b (a −1)ax=(a −1)b [(a −1)a]x=(a −1)b 1x=(a −1)b x=(a −1)b

2 × 2 3 × 3 A, X, B.

AX=B

A

(A −1)AX=(A −1)B [(A −1)A]X=(A −1)B IX=(A −1)B X=(A −1)B

3x+8y=5 4x+11y=7

A=[3 8 4 11],X=[x y],B=[5 7]

[3 8 4 11] [x y]=[5 7]

A −1 .

A −1 = 1 ad−bc [d −b −c a] = 1 3(11)−8(4) [11 −8 −4 3] = 1 1 [11 −8 −4 3]

A −1 =[11 −8 −4 3]

A −1 .

(A −1)AX=(A −1)B [11 −8 −4 3] [3 8 4 11] [x y]=[11 −8 −4 3] [5 7]
[1 0 0 1] [x y]=[11(5)+(−8)7 −4(5)+3(7)] [x y]=[−1 1]
(−1,1). X B A −1 ? A −1 B≠B A −1 .
(A −1)AX=(A −1)B [(A −1)A]X=(A −1)B IX=(A −1)B X=(A −1)B
A −1 , A −1 A B
5x+15y+56z=35 −4x−11y−41z=−26 −x−3y−11z=−7
AX=B.
[5 15 56 −4 −11 −41 −1 −3 −11] [x y z]=[35 −26 −7]
A
[5 15 56 −4 −11 −41 −1 −3 −11 | 1 0 0 0 1 0 0 0 1]
1 5 .
[1 3 56 5 −4 −11 −41 −1 −3 −11 | 1 5 0 0 0 1 0 0 0 1]
[1 3 56 5 0 1 19 5 −1 −3 −11 | 1 5 0 0 4 5 1 0 0 0 1]
[1 3 56 5 0 1 19 5 0 0 1 5 | 1 5 0 0 4 5 1 0 1 5 0 1]
[1 0 − 1 5 0 1 19 5 0 0 1 5 | − 11 5 −3 0 4 5 1 0 1 5 0 1]
[1 0 − 1 5 0 1 19 5 0 0 1 | − 11 5 −3 0 4 5 1 0 1 0 5]
1 5
[1 0 0 0 1 19 5 0 0 1 | −2 −3 1 4 5 1 0 1 0 5]
− 19 5
[1 0 0 0 1 0 0 0 1 | −2 −3 1 −3 1 −19 1 0 5]
A −1 =[−2 −3 1 −3 1 −19 1 0 5]
A −1 . A −1 AX= A −1 B:
[−2 −3 1 −3 1 −19 1 0 5] [5 15 56 −4 −11 −41 −1 −3 −11] [x y z]=[−2 −3 1 −3 1 −19 1 0 5] [35 −26 −7]
A −1 B=[−70+78−7 −105−26+133 35+0−35]=[1 2 0]
(1,2,0).
2x−17y+11z=0 −x+11y−7z=8 3y−2z=−2
X=[4 38 58]
[A] [B].
2x+3y+z=32 3x+3y+z=−27 2x+4y+z=−2
[A], [B].
[A]=[2 3 1 3 3 1 2 4 1], [B]=[32 −27 −2]
X,
[A] −1 ×[B]
[−59 −34 252]
2 × 2I 2 =[1 0 0 1]3 × 3I 3 =[1 0 0 0 1 0 0 0 1]2 × 2A −1 = 1 ad−bc [d −b −c a], where ad−bc≠0
AI=IA=A. A A −1 = A −1 A=I. 2×2 3×3 AX=B, A: A −1 AX= A −1 B. AB≠BA A −1 A=A A −1 ?
A −1 A, A A −1 =I, A A −1 , A −1 A=I. 2×2 2×2 2×2 ad bc ad−bc=0, 2×2 [0 1 1 0].
A −1 = 1 0(0)−1(1) [0 −1 −1 0]=[0 1 1 0]. A B.A=[1 0 −1 1],B=[1 0 1 1]
A=[1 2 3 4],B=[−2 1 3 2 − 1 2]AB=BA=[1 0 0 1]=IA=[4 5 7 0],B=[0 1 7 1 5 − 4 35]
A=[−2 1 2 3 −1],B=[−2 −1 −6 −4]AB=BA=[1 0 0 1]=I
A=[1 0 1 0 1 −1 0 1 1],B= 1 2 [2 1 −1 0 1 1 0 −1 1]
A=[1 2 3 4 0 2 1 6 9],B= 1 4 [6 0 −2 17 −3 −5 −12 2 4]AB=BA=[1 0 0 0 1 0 0 0 1]=I
A=[3 8 2 1 1 1 5 6 12],B= 1 36 [−6 84 −6 7 −26 1 −1 −22 5][3 −2 1 9]1 29 [9 2 −1 3][−2 2 3 1]
[−3 7 9 2]1 69 [−2 7 9 3][−4 −3 −5 8][1 1 2 2][0 1 1 0][0.5 1.5 1 −0.5]4 7 [0.5 1.5 1 −0.5]
[1 0 6 −2 1 7 3 0 2][0 1 −3 4 1 0 1 0 5]1 17 [−5 5 −3 20 −3 12 1 −1 4][1 2 −1 −3 4 1 −2 −4 −5]
[1 9 −3 2 5 6 4 −2 7]1 209 [47 −57 69 10 19 −12 −24 38 −13][1 −2 3 −4 8 −12 1 4 2]
[1 2 1 2 1 2 1 3 1 4 1 5 1 6 1 7 1 8][18 60 −168 −56 −140 448 40 80 −280][1 2 3 4 5 6 7 8 9] 2 × 2
5x−6y=−61 4x+3y=−2(−5,6)8x+4y=−100 3x−4y=1 3x−2y=6 −x+5y=−2(2,0)5x−4y=−5 4x+y=2.3

−3x−4y=9 12x+4y=−6(1 3 ,− 5 2)−2x+3y= 3 10 −x+5y= 1 2 8 5 x− 4 5 y= 2 5 − 8 5 x+ 1 5 y= 7 10 (− 2 3 ,− 11 6)1 2 x+ 1 5 y=− 1 4 1 2 x− 3 5 y=− 9 4 3×3 3x−2y+5z=21 5x+4y=37 x−2y−5z=5 (7, 1 2 , 1 5) 4x+4y+4z=40 2x−3y+4z=−12 −x+3y+4z=9 6x−5y−z=31 −x+2y+z=−6 3x+3y+2z=13 (5,0,−1)6x−5y+2z=−4 2x+5y−z=12 2x+5y+z=124x−2y+3z=−12 2x+2y−9z=33 6y−4z=1 1 34 (−35,−97,−154)1 10 x− 1 5 y+4z= −41 2 1 5 x−20y+ 2 5 z=−101 3 10 x+4y− 3 10 z=23 1 2 x− 1 5 y+ 1 5 z= 31 100 − 3 4 x− 1 4 y+ 1 2 z= 7 40 − 4 5 x− 1 2 y+ 3 2 z= 1 4 1 690 (65,−1136,−229)0.1x+0.2y+0.3z=−1.4 0.1x−0.2y+0.3z=0.6 0.4y+0.9z=−2 2x−y=−3 −x+2y=2.3(− 37 30 , 8 15)− 1 2 x− 3 2 y=− 43 20 5 2 x+ 11 5 y= 31 4 12.3x−2y−2.5z=2 36.9x+7y−7.5z=−7 8y−5z=−10(10 123 ,−1, 2 5) 0.5x−3y+6z=−0.8 0.7x−2y=−0.06 0.5x+4y+5z=0[1 0 1 0 0 1 0 1 0 1 1 0 0 0 1 1] 1 2 [2 1 −1 −1 0 1 1 −1 0 −1 1 1 0 1 −1 1][−1 0 2 5 0 0 0 2 0 2 −1 0 1 −3 0 1] [1 −2 3 0 0 1 0 2 1 4 −2 3 −5 0 1 1]1 39 [3 2 1 −7 18 −53 32 10 24 −36 21 9 −9 46 −16 −5] [1 2 0 2 3 0 2 1 0 0 0 0 3 0 1 0 2 0 0 1 0 0 1 2 0] [1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 1 1 1 1 1 1] [1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 −1 −1 −1 −1 −1 1] 14 ft 2 . 3 ft 2 2 × 2

A=[a b c d]

det(A) | A |.

A=[5 2 −6 3]

det(A)=| 5 2 −6 3 | =5(3)−(−6)(2) =27

a 1 x+ b 1 y= c 1 (1) a 2 x+ b 2 y= c 2 (2)

x. y y y

b 2 a 1 x+ b 2 b 1 y= b 2 c 1 Multiply R 1 by b 2 − b 1 a 2 x− b 1 b 2 y=− b 1 c 2 Multiply R 2 by− b 1 ______________________________ b 2 a 1 x− b 1 a 2 x= b 2 c 1 − b 1 c 2

x.

b 2 a 1 x− b 1 a 2 x= b 2 c 1 − b 1 c 2 x(b 2 a 1 − b 1 a 2)= b 2 c 1 − b 1 c 2 x= b 2 c 1 − b 1 c 2 b 2 a 1 − b 1 a 2 = [c 1 b 1 c 2 b 2] [a 1 b 1 a 2 b 2]

y, x.

a 2 a 1 x+ a 2 b 1 y= a 2 c 1 Multiply R 1 by a 2 − a 1 a 2 x− a 1 b 2 y=− a 1 c 2 Multiply R 2 by− a 1 ______________________________ a 2 b 1 y− a 1 b 2 y= a 2 c 1 − a 1 c 2

y

a 2 b 1 y− a 1 b 2 y= a 2 c 1 − a 1 c 2 y(a 2 b 1 − a 1 b 2)= a 2 c 1 − a 1 c 2 y= a 2 c 1 − a 1 c 2 a 2 b 1 − a 1 b 2 = a 1 c 2 − a 2 c 1 a 1 b 2 − a 2 b 1 = | a 1 c 1 a 2 c 2 | | a 1 b 1 a 2 b 2 |

x y x y, D:D x :x

x= D x D

D y : y

y= D y D

x y

a 1 x+ b 1 y= c 1 a 2 x+ b 2 y= c 2

x= D x D = | c 1 b 1 c 2 b 2 | | a 1 b 1 a 2 b 2 | , D≠0; y= D y D = | a 1 c 1 a 2 c 2 | | a 1 b 1 a 2 b 2 | , D≠0.

x, x y, y 2 × 2

12x+3y=15 2x−3y=13

x.

x= D x D = | 15 3 13 −3 | | 12 3 2 −3 | = −45−39 −36−6 = −84 −42 =2

y.

y= D y D = | 12 15 2 13 | | 12 3 2 −3 | = 156−30 −36−6 =− 126 42 =−3

(2,−3).

$x+2y=-11\ -2x+y=-13$

$(3,-7)$

$A=[a\ 1\ b\ 1\ c\ 1\ a\ 2\ b\ 2\ c\ 2\ a\ 3\ b\ 3\ c\ 3]$

A

$\det(A)=|a\ 1\ b\ 1\ c\ 1\ a\ 2\ b\ 2\ c\ 2\ a\ 3\ b\ 3\ c\ 3\ \ |\ \ a\ 1\ a\ 2\ a\ 3\ \ b\ 1\ b\ 2\ b\ 3|$

$|A|=a\ 1\ b\ 2\ c\ 3+b\ 1\ c\ 2\ a\ 3+c\ 1\ a\ 2\ b\ 3-a\ 3\ b\ 2\ c\ 1-b\ 3\ c\ 2\ a\ 1-c\ 3\ a\ 2\ b\ 1$

$A=[0\ 2\ 1\ 3\ -1\ 1\ 4\ 0\ 1]$

$|A|=|0\ 2\ 1\ 3\ -1\ 1\ 4\ 0\ 1\ \ |0\ 3\ \ 4\ \ 2\ -1\ 0|\ \ =0(-1)(1)+2(1)(4)+1(3)(0)-4(-1)(1)-0(1)(0)-1(3)(2)\ \ =0+8+0+4-0-6\ \ =6$

$\det(A)=|1\ -3\ 7\ 1\ 1\ 1\ 1\ -2\ 3|$

$-10\ 2\times 2\ \ 3\times 3$

$x=D\ x\ D\ ,y=D\ y\ D\ ,z=D\ z\ D\ ,D\neq 0$

$D\ x\ ,x\ D\ y\ ,y\ \ D\ z\ ,z$

$x+y-z=6\ 3x-2y+z=-5\ x+3y-2z=14$

$D=|1\ \ 1\ -1\ 3\ -2\ \ 1\ 1\ \ 3\ -2\ |,D\ x=|6\ 1\ -1\ -5\ -2\ \ 1\ 14\ \ 3\ -2\ |,D\ y=|1\ \ 6\ -1\ 3\ -5\ \ 1\ 1\ 14\ -2\ |,D\ z=|1\ 1\ 6\ 3\ -2\ -5\ 1\ \ 3\ 14|$

$x=D\ x\ D=-3\ -3=1\ y=D\ y\ D=-9\ -3=3\ z=D\ z\ D=6\ -3=-2$

$(1,3,-2)$.

$x-3y+7z=13\ x+y+z=1\ \ x-2y+3z=4$

$(-2,3\ 5\ ,12\ 5)$

$3x-2y=4\ \ (1)\ 6x-4y=0\ \ (2)$

$D, D\ x$,and $D\ y$.

$D=|3\ -2\ 6\ -4|=3(-4)-6(-2)=0$

$-2.\ (2)$.

$-6x+4y\ =-8\ \ 6x-4y\ =\ 0$ _______________ $0\ =-8$

$0=-8$,

$x-2y+3z=0\ (1)\ 3x+y-2z=0\ (2)\ 2x-4y+6z=0\ (3)$

$|1\ -2\ 3\ 3\ 1\ -2\ 2\ -4\ 6\ \ |\ \ 1\ -2\ 3\ 1\ 2\ -4|$

$1(1)(6)+(-2)(-2)(2)+3(3)(-4)-2(1)(3)-(-4)(-2)(1)-6(3)(-2)=0$

-2

$-2x+4y-6x=0\ 2x-4y+6z=0\ \ \ \ 0=0$

$0=0$, $A\ -1$ A.

$A=[1\ \ 2\ 3\ 0\ \ 2\ 1\ 0\ \ 0\ -1]$

A

$A=[1\ 2\ 3\ 0\ 2\ 1\ 0\ 0\ -1\ |\ \ 1\ 0\ 0\ \ 2\ 2\ 0]$

$\det(A)=1(2)(-1)+2(1)(0)+3(0)(0)-0(2)(3)-0(1)(1)+1(0)(2)\ \ =-2$

$A=[-1\ 5\ 4\ -3]$, $\det(A)=(-1)(-3)-(4)(5)=3-20=-17$ $B=[4\ -3\ -1\ 5]$, $\det(B)=(4)(5)-(-1)(-3)=20-3=17$

$A=[1\ 2\ 2\ 2\ 2\ 2\ -1\ 2\ 2\ \ |\ \ 1\ 2\ -1\ \ 2\ 2\ 2]$ $\det(A)=1(2)(2)+2(2)(-1)+2(2)(2)+1(2)(2)-2(2)(1)-2(2)(2)\ \ =4-4+8+4-4-8=0$

$A=[1\ 2\ 0\ 0]$, $\det(A)=1(0)-2(0)=0$

$A\ -1$ A.

$A=[1\ 2\ 3\ 4]$,$\det(A)=1(4)-3(2)=-2$ $A\ -1=[-2\ 1\ 3\ 2\ -1\ 2]$,$\det(A\ -1)=-2(-1\ 2)-(3\ 2)(1)=-1\ 2$

$A=[1\ 2\ 3\ 4]$,$\det(A)=1(4)-2(3)=-2$ $B=[2(1)\ 2(2)\ 3\ 4]$,$\det(B)=2(4)-3(4)=-4$

$2x+4y+4z=2\ (1)\ 3x+7y+7z=-5\ (2)\ \ x+2y+2z=4\ (3)$

$D=|2\ 4\ 4\ 3\ 7\ 7\ 1\ 2\ 2|$

$-2x-4y-4x=-8\ \ 2x+4y+4z=2\ \ \ \ 0=-6$

$[a\ b\ c\ d]$ $ad-bc$. $x=D\ x\ D\ ,y=D\ y\ D$. $x=D\ x\ D\ ,y=D\ y\ D\ ,z=D\ z\ D$. $A\ -1$ A. $2\times 2\ \ 2\times 2$ A

$|1\ 2\ 3\ 4|-2|\ -1\ 2\ 3\ -4|$

| 2 −5 −1 6 |
7| −8 4 −1 5 || 1 0 3 −4 |−4| 10 20 0 −10 || 10 0.2 5 0.1 |0| 6 −3 8 4 || −2 −3 3.1 4,000 |−7,990.7
| −1.1 0.6 7.2 −0.5 || −1 0 0 0 1 0 0 0 −3 |3| −1 4 0 0 2 3 0 0 −3 || 1 0 1 0 1 0 1 0 0 |−1
| 2 −3 1 3 −4 1 −5 6 1 || −2 1 4 −4 2 −8 2 −8 −3 |224| 6 −1 2 −4 −3 5 1 9 −1 || 5 1 −1 2 3 1 3 −6 −3 |15
| 1.1 2 −1 −4 0 0 4.1 −0.4 2.5 || 2 −1.6 3.1 1.1 3 −8 −9.3 0 2 |−17.03| − 1 2 1 3 1 4 1 5 − 1 6 1 7 0 0 1 8 |
2x−3y=−1 4x+5y=9(1,1)5x−4y=2 −4x+7y=6 6x−3y=2 −8x+9y=−1(1 2 , 1 3)2x+6y=12 5x−2y=13
4x+3y=23 2x−y=−1(2,5)10x−6y=2 −5x+8y=−14x−3y=−3 2x+6y=−4(−1,− 1 3)
4x−5y=7 −3x+9y=04x+10y=180 −3x−5y=−105(15,12) 8x−2y=−3 −4x+6y=4
x+2y−4z=−1 7x+3y+5z=26 −2x−6y+7z=−6(1,3,2)
−5x+2y−4z=−47 4x−3y−z=−94 3x−3y+2z=94 4x+5y−z=−7 −2x−9y+2z=8 5y+7z=21
(−1,0,3)4x−3y+4z=10 5x−2z=−2 3x+2y−5z=−94x−2y+3z=6 −6x+y=−2 2x+7y+8z=24(1 2 ,1,2)
5x+2y−z=1 −7x−8y+3z=1.5 6x−12y+z=7
13x−17y+16z=73 −11x+15y+17z=61 46x+10y−30z=−18(2,1,4)
−4x−3y−8z=−7 2x−9y+5z=0.5 5x−6y−5z=−2 4x−6y+8z=10 −2x+3y−4z=−5 x+y+z=1
4x−6y+8z=10 −2x+3y−4z=−5 12x+18y−24z=−30| 1 0 8 9 0 2 1 0 1 0 3 0 0 2 4 3 |24
| 1 0 2 1 0 −9 1 3 3 0 −2 −1 0 1 1 −2 || 1 2 1 7 4 0 1 2 100 5 0 0 2 2,000 0 0 0 2 |1
| 1 0 0 0 2 3 0 0 4 5 6 0 7 8 9 0 | 3 4 3x−y=4 x+4y=−3 (−1,1)6x−2y=24 −3x+3y=18 (9,15)
10x+5y=−5 3x−2y=−12(−2,3)4 7 x+ 1 5 y= 43 70 5 6 x− 1 3 y=− 2 35x+6y=14 4x+8y=8(4,−1)
3x+2y=−7 2x+4y=63x+4y=2 9x+12y=38x+4y=2 6x−5y=0.7 C(x)=150x+15,000 R(x)=200x.
(300,60,000) C(x)=50x+10,000, x (400,30,000) 0.5x−0.5y=10 −0.2y+0.2x=4 0.1x+0.1z=2
(10,−10,10)5x+3y−z=5 3x−2y+4z=13 4x+3y+5z=22x+y+z=1 2x+2y+2z=1 3x+3y=2
2x−3y+z=−1 x+y+z=−4 4x+2y−3z=33 3x+2y−z=−10 x−y+2z=7 −x+3y+z=−2(−1,−2,3)
3x+4z=−11 x−2y=5 4y−z=−102x−3y+z=0 2x+4y−3z=0 6x−2y−z=0(x, 8x 5 , 14x 5)
6x−4y−2z=2 3x+2y−5z=4 6y−7z=5y= x 2 −7 y=5x−13(2,−3),(3,2)y= x 2 −4 y=5x+10
x 2 + y 2 =16 y=x−8x 2 + y 2 =25 y= x 2 +5x 2 + y 2 =4 y− x 2 =3y> x 2 −1
1 4 x 2 + y 2 <4x 2 + y 2 +2x<3 y>− x 2 −3
x 2 −2x+ y 2 −4x<4 y<−x+4x 2 + y 2 <1 y 2 <x−2x+6 x 2 +3x+2
2 x+2 , −4 x+110x+2 4 x 2 +4x+17x+20 x 2 +10x+257 x+5 , −15 (x+5) 2x−18 x 2 −12x+36
− x 2 +36x+70 x 3 −1253 x−5 , −4x+1 x 2 +5x+25−5 x 2 +6x−2 x 3 +27x 3 −4 x 2 +3x+11 (x 2 −2) 2
x−4 (x 2 −2) , 5x+3 (x 2 −2) 24 x 4 −2 x 3 +22 x 2 −6x+48 x (x 2 +4) 2
A=[4 −2 1 3],B=[6 7 −3 11 −2 4],C=[6 7 11 −2 14 0],D=[1 −4 9 10 5 −7 2 8 5],E=[7 −14 3 2 −1 3 0 1 9]
−4A[−16 8 −4 −12]10D−6EB+CABBABCCB[113 28 10 44 81 −41 84 98 −42]DEED
[−127 −74 176 −2 11 40 28 77 38]ECCEA 3[1 0 −3 0 1 2 0 0 0 | 7 −5 0]x−3z=7 y+2z=−5
[1 0 5 0 1 −2 0 0 0 | −9 4 3]−2x+2y+z=7 2x−8y+5z=0 19x−10y+22z=3
[−2 2 1 2 −8 5 19 −10 22 | 7 0 3] 4x+2y−3z=14 −12x+3y+z=100 9x−6y+2z=31
x+3z=12 −x+4y=0 y+2z=−7[1 0 3 −1 4 0 0 1 2 | 12 0 −7]3x−4y=−7 −6x+8y=14
3x−4y=1 −6x+8y=6−1.1x−2.3y=6.2 −5.2x−4.1y=4.32x+3y+2z=1 −4x−6y−4z=−2 10x+15y+10z=0
−x+2y−4z=8 3y+8z=−4 −7x+y+2z=1 [−0.2 1.4 1.2 −0.4]1 8 [2 7 6 1][1 2 − 1 2 − 1 4 3 4]
[12 9 −6 −1 3 2 −4 −3 2][2 1 3 1 2 3 3 2 1] 0.3x−0.1y=−10 −0.1x+0.3y=14(−20,40)
0.4x−0.2y=−0.6 −0.1x+0.05y=0.34x+3y−3z=−4.3 5x−4y−z=−6.1 x+z=−0.7(−1,0.2,0.3)
−2x−3y+2z=3 −x+2y+4z=−5 −2y+5z=−3| 100 0 0 0 || 0.2 −0.6 0.7 −1.1 || −1 4 3 0 2 3 0 0 −3 |
| 2 0 0 0 2 0 0 0 2 |4x−2y=23 −5x−10y=−35(6, 1 2)0.2x−0.1y=0 −0.3x+0.3y=2.5
−0.5x+0.1y=0.3 −0.25x+0.05y=0.15x+6y+3z=4 2x+y+2z=3 3x−2y+z=0
4x−3y+5z=− 5 2 7x−9y−3z= 3 2 x−5y−5z= 5 2 (0,0,− 1 2)
3 10 x− 1 5 y− 3 10 z=− 1 50 1 10 x− 1 10 y− 1 2 z=− 9 50 2 5 x− 1 2 y− 3 5 z=− 1 5−5x−y=12 x+4y=9
(−3,3)1 2 x− 1 3 y=4 3 2 x−y=0− 1 2 x−4y=4 2x+16y=25x−y=1 −10x+2y=−2
4x−6y−2z= 1 10 x−7y+5z=− 1 4 3x+6y−9z= 6 51 20 (10,5,4)x+z=20 x+y+z=20 x+2y+z=10
5x−4y−3z=0 2x+y+2z=0 x−6y−7z=0(x, 16x 5 − 13x 5)y= x 2 +2x−3 y=x−1y 2 + x 2 =25 y 2 −2 x 2 =1

(−2 2 ,− 17),(−2 2 , 17),(2 2 ,− 17),(2 2 , 17)y< x 2 +9x 2 + y 2 >4 y< x 2 +1−8x−30 x 2 +10x+25
13x+2 (3x+1) 25 3x+1 − 2x+3 (3x+1) 2x 4 − x 3 +2x−1 x (x 2 +1) 25[4 9 −2 3]+ 1 2 [−6 12 4 −8]
[17 51 −8 11][1 4 −7 −2 9 5 12 0 −4] [3 −4 1 3 5 10][1 2 1 3 1 4 1 5] −1[12 −20 −15 30]
det| 0 0 400 4,000 |det| 1 2 − 1 2 0 − 1 2 0 1 2 0 1 2 0 |− 1 8 det(A)=−6,
14x−2y+13z=140 −2x+3y−6z=−1 x−5y+12z=11
[14 −2 13 −2 3 −6 1 −5 12 | 140 −1 11]
[1 0 3 −2 4 9 −6 1 2 | 12 −5 8]
x−6y=4 2x−12y=0
2x+y+z=−3 x−2y+3z=6 x−y−z=6
4x−5y=−50 −x+2y=80 (100,90)
1 100 x− 3 100 y+ 1 20 z=−49 3 100 x− 7 100 y− 1 100 z=13 9 100 x− 9 100 y− 9 100 z=99
200x−300y=2 400x+715y=4(1 100 ,0)
0.1x+0.1y−0.1z=−1.2 0.1x−0.2y+0.4z=−1.2 0.5x−0.3y+0.8z=−5.9 C(x)= x 2 +75x+2,688
R(x)= x 2 +160x. (x,y) (−c,0) (c,0). (x,y) (x,y) (a,0) (−c,0) (a,0) a−(−c)=a+c. (c,0) (a,0) a−c
(a+c)+(a−c)=2a
(x,y)
d 1 =the distance from (−c,0) to (x,y) d 2 =the distance from (c,0) to (x,y)
d 1 + d 2 (x,y) 2a (a,0). d 1 + d 2 =2a
d 1 + d 2 = (x−(−c)) 2 + (y−0) 2 + (x−c) 2 + (y−0) 2 =2a Distance formula (x+c) 2 + y 2 + (x−c) 2 + y 2 =2a Simplify expressions. (x+c) 2 + y 2 =2a− (x−c) 2 + y 2 Move radical to opposite side. (x+c) 2 + y 2 = [2a− (x−c) 2 + y 2] 2 Square both sides. x 2 +2cx+ c 2 + y 2 =4 a 2 −4a (x−c) 2 + y 2 + (x−c) 2 + y 2 Expand the squares. x 2 +2cx+ c 2 + y 2 =4 a 2 −4a (x−c) 2 + y 2 + x 2 −2cx+ c 2 + y 2 Expand remaining squares. 2cx=4 a 2 −4a (x−c) 2 + y 2 −2cx Combine like terms. 4cx−4 a 2 =−4a (x−c) 2 + y 2 Isolate the radical. cx− a 2 =−a (x−c) 2 + y 2 Divide by 4. [cx− a 2] 2 = a 2 [(x−c) 2 + y 2] 2 Square both sides. c 2 x 2 −2 a 2 cx+ a 4 = a 2 (x 2 −2cx+ c 2 + y 2) Expand the squares. c 2 x 2 −2 a 2 cx+ a 4 = a 2 x 2 −2 a 2 cx+ a 2 c 2 + a 2 y 2 Distribute a 2 . a 2 x 2 − c 2 x 2 + a 2 y 2 = a 4 − a 2 c 2 Rewrite. x 2 (a 2 − c 2)+ a 2 y 2 = a 2 (a 2 − c 2) Factor common terms. x 2 b 2 + a 2 y 2 = a 2 b 2 Set b 2 = a 2 − c 2 . x 2 b 2 a 2 b 2 + a 2 y 2 a 2 b 2 = a 2 b 2 a 2 b 2 Divide both sides by a 2 b 2 . x 2 a 2 + y 2 b 2 =1 Simplify.
x 2 a 2 + y 2 b 2 =1. a>b, b>a, (0,0)
x 2 a 2 + y 2 b 2 =1
a>b 2a (±a,0) 2b (0,±b) (±c,0) c 2 = a 2 − b 2 . (0,0)
x 2 b 2 + y 2 a 2 =1
a>b 2a (0,±a) 2b (±b,0) (0,±c) c 2 = a 2 − b 2 . c 2 = a 2 − b 2 . (0,0) (0,0) (±a,0) (±c,0) x 2 a 2 + y 2 b 2 =1. (0,±a) (±c,0), x 2 b 2 + y 2 a 2 =1. c 2 = a 2 − b 2 , b 2 . a 2 b 2 (±8,0) (±5,0)?
x 2 a 2 + y 2 b 2 =1
(±8,0), a=8 a 2 =64. (±5,0), c=5 c 2 =25. c 2 = a 2 − b 2 . b 2 ,
c 2 = a 2 − b 2 25=64− b 2 Substitute for c 2 and a 2 . b 2 =39 Solve for b 2 .
a 2 =64 b 2 =39 x 2 64 + y 2 39 =1. (0,±4) (0,± 15)?x 2 + y 2 16 =1 (±a,0) (0, ±a). (±c,0) (0, ±c). a c c 2 = a 2 − b 2 , b 2 . h k (h,k). x (x−h) (y−k). (h, k)
(x−h) 2 a 2 + (y−k) 2 b 2 =1
a>b 2a (h±a,k) 2b (h,k±b) (h±c,k), c 2 = a 2 − b 2 . (h,k)
(x−h) 2 b 2 + (y−k) 2 a 2 =1
a>b 2a (h,k±a) 2b (h±b,k) (h,k±c), c 2 = a 2 − b 2 . (h,k) c 2 = a 2 − b 2 . (h,k) (h,k) (x−h) 2 a 2 + (y−k) 2 b 2 =1. (x−h) 2 b 2 + (y−k) 2 a 2 =1. (h,k) a 2 2a, c 2 h k, b 2

c 2 = a 2 − b 2 . h, k, a 2 , b 2 (−2,−8) (−2,2) (−2,−7) (−2,1)?
(x−h) 2 b 2 + (y−k) 2 a 2 =1
(h,k). (−2,−8) (−2,2).
(h,k)=(−2+(−2) 2 , −8+2 2) =(−2,−3)
a 2 . 2a, a
2a=2−(−8) 2a=10 a=5
a 2 =25. c 2 . (h,k±c). (h,k−c)=(−2,−7) (h,k+c)=(−2,1). k=−3 c.
k+c=1 −3+c=1 c=4
c 2 =16. b 2 c 2 = a 2 − b 2 .
c 2 = a 2 − b 2 16=25− b 2 b 2 =9
h,k, a 2 , b 2
(x+2) 2 9 + (y+3) 2 25 =1
(−3,3) (5,3) (1−2 3 ,3) (1+2 3 ,3)?(x−1) 2 16 + (y−3) 2 4 =1 x 2 a 2 + y 2 b 2 =1, a>b
x 2 b 2 + y 2 a 2 =1, a>b (0, 0), x 2 a 2 + y 2 b 2 =1, a>b, (±a,0) (0,±b) (±c,0)
x 2 b 2 + y 2 a 2 =1, a>b, (0,±a) (±b,0) (0,±c) c c 2 = a 2 − b 2 . x 2 9 + y 2 25 =1. 25>9,
x 2 b 2 + y 2 a 2 =1, b 2 =9 a 2 =25. (0,0) (0,±a)=(0,± 25)=(0,±5) (±b,0)=(± 9 ,0)=(±3,0)
(0,±c), c 2 = a 2 − b 2 c,
c=± a 2 − b 2 =± 25−9 =± 16 =±4
(0,±4). x 2 36 + y 2 4 =1. (0,0); (±6,0); (0,±2); (±4 2 ,0) 4 x 2 +25 y 2 =100.
4 x 2 +25 y 2 =100 4 x 2 100 + 25 y 2 100 = 100 100 x 2 25 + y 2 4 =1
25>4, x 2 a 2 + y 2 b 2 =1, a 2 =25 b 2 =4. (0,0) (±a,0)=(± 25 ,0)=(±5,0)
(0,±b)=(0,± 4)=(0,±2) (±c,0), c 2 = a 2 − b 2 . c,
c=± a 2 − b 2 =± 25−4 =± 21
(± 21 ,0). 49 x 2 +16 y 2 =784. x 2 16 + y 2 49 =1; (0,0); (0,±7); (±4,0); (0,± 33) (h,k),
(x−h) 2 a 2 + (y−k) 2 b 2 =1, a>b (x−h) 2 b 2 + (y−k) 2 a 2 =1, a>b (h, k),
(x−h) 2 a 2 + (y−k) 2 b 2 =1, a>b, (h,k) (h±a,k) (h,k±b) (h±c,k) (x−h) 2 b 2 + (y−k) 2 a 2 =1,
a>b, (h,k) (h,k±a) (h±b,k) (h,k±c) c c 2 = a 2 − b 2 . (x+2) 2 4 + (y−5) 2 9 =1. 9>4,
(x−h) 2 b 2 + (y−k) 2 a 2 =1, b 2 =4 a 2 =9. (h,k)=(−2,5) (h,k±a)=(−2,5± 9)=(−2,5±3), (−2,2)
(−2,8) (h±b,k)=(−2± 4 ,5)=(−2±2,5), (−4,5) (0,5) (h,k±c), c 2 = a 2 − b 2 . c,
c=± a 2 − b 2 =± 9−4 =± 5
(−2,5− 5) (−2,5+ 5). (x−4) 2 36 + (y−2) 2 20 =1. (4,2); (−2,2) (10,2); (4,2−2 5) (4,2+2 5);
(0,2) (8,2) a x 2 +b y 2 +cx+dy+e=0 x 2 y 2 m 1 (x−h) 2 + m 2 (y−k) 2 = m 3 , m 1 , m 2 ,
m 3 4 x 2 +9 y 2 −40x+36y+100=0.
4 x 2 +9 y 2 −40x+36y+100=0
(4 x 2 −40x)+(9 y 2 +36y)=−100
4(x 2 −10x)+9(y 2 +4y)=−100
4(x 2 −10x+25)+9(y 2 +4y+4)=−100+100+36
4 (x−5) 2 +9 (y+2) 2 =36
(x−5) 2 9 + (y+2) 2 4 =1
9>4, (x−h) 2 a 2 + (y−k) 2 b 2 =1, a 2 =9 b 2 =4. (h,k)=(5,−2) (h±a,k)=(5± 9 ,−2)=(5±3,−2),
(2,−2) (8,−2) (h,k±b)=(5,−2± 4)=(5,−2±2), (5,−4) (5,0) (h±c,k), c 2 = a 2 − b 2 . c,
c=± a 2 − b 2 =± 9−4 =± 5
(5− 5 ,−2) (5+ 5 ,−2).
4 x 2 + y 2 −24x+2y+21=0
(x−3) 2 4 + (y+1) 2 16 =1; (3,−1); (3,−5) (3,3); (1,−1) (5,−1); (3,−1−2 3) (3,−1+2 3) (0,0).
(0,0), x 2 a 2 + y 2 b 2 =1, a>b. 2a, 2b. a, 2a=96, a=48, a 2 =2304. b, 2b=46, b=23, b 2 =529.
x 2 2304 + y 2 529 =1. (±c,0), c 2 = a 2 − b 2 . c,

c 2 = a 2 − b 2 c 2 =2304−529 Substitute using the values found in part (a). c=± 2304−529 Take the square root of both sides. c=± 1775 Subtract. c≈±42 Round to the nearest foot.

(±42,0) 2(42)=84 (0,0).x 2 57,600 + y 2 25,600 =1x 2 a 2 + y 2 b 2 =1, a>bx 2 b 2 + y 2 a 2 =1, a>b
(h,k)(x−h) 2 a 2 + (y−k) 2 b 2 =1, a>b (h,k)(x−h) 2 b 2 + (y−k) 2 a 2 =1, a>b (x,y) 2 x 2 +y=4
4 x 2 +9 y 2 =36 x 2 3 2 + y 2 2 2 =14 x 2 − y 2 =44 x 2 +9 y 2 =1x 2 (1 2) 2 + y 2 (1 3) 2 =1
4 x 2 −8x+9 y 2 −72y+112=0x 2 4 + y 2 49 =1x 2 2 2 + y 2 7 2 =1; (0,7) (0,−7). (2,0) (−2,0).
(0,3 5),(0,−3 5).x 2 100 + y 2 64 =1x 2 +9 y 2 =1x 2 (1) 2 + y 2 (1 3) 2 =1; (1,0) (−1,0).
(0, 1 3),(0,− 1 3). (2 2 3 ,0),(− 2 2 3 ,0).4 x 2 +16 y 2 =1(x−2) 2 49 + (y−4) 2 25 =1
(x−2) 2 7 2 + (y−4) 2 5 2 =1; (9,4),(−5,4). (2,9),(2,−1). (2+2 6 ,4),(2−2 6 ,4).
(x−2) 2 81 + (y+1) 2 16 =1(x+5) 2 4 + (y−7) 2 9 =1(x+5) 2 2 2 + (y−7) 2 3 2 =1;
(−5,10),(−5,4). (−3,7),(−7,7). (−5,7+ 5),(−5,7− 5).(x−7) 2 49 + (y−7) 2 49 =1
4 x 2 −8x+9 y 2 −72y+112=0(x−1) 2 3 2 + (y−4) 2 2 2 =1; (4,4),(−2,4). (1,6),(1,2).
(1+ 5 ,4),(1− 5 ,4).9 x 2 −54x+9 y 2 −54y+81=04 x 2 −24x+36 y 2 −360y+864=0
(x−3) 2 (3 2) 2 + (y−5) 2 (2) 2 =1; (3+3 2 ,5),(3−3 2 ,5). (3,5+ 2),(3,5− 2). (7,5),(−1,5).
4 x 2 +24x+16 y 2 −128y+228=04 x 2 +40x+25 y 2 −100y+100=0(x+5) 2 (5) 2 + (y−2) 2 (2) 2 =1;
(0,2),(−10,2). (−5,4),(−5,0). (−5+ 21 ,2),(−5− 21 ,2).x 2 +2x+100 y 2 −1000y+2401=0
4 x 2 +24x+25 y 2 +200y+336=0(x+3) 2 (5) 2 + (y+4) 2 (2) 2 =1; (2,−4),(−8,−4).
(−3,−2),(−3,−6). (−3+ 21 ,−4),(−3− 21 ,−4).9 x 2 +72x+16 y 2 +16y+4=0
(x+3) 2 25 + (y+1) 2 36 =1 (−3,−1+ 11),(−3,−1− 11)(x+1) 2 100 + (y−2) 2 4 =1x 2 + y 2 =1 (0,0)
x 2 +4 y 2 +4x+8y=110 x 2 + y 2 +200x=0 (−10,30),(−10,−30)x 2 25 + y 2 36 =1x 2 16 + y 2 9 =1
(0,0), (4,0),(−4,0),(0,3),(0,−3), (7 ,0),(− 7 ,0)4 x 2 +9 y 2 =181 x 2 +49 y 2 =1 (0,0),
(1 9 ,0),(− 1 9 ,0),(0, 1 7),(0,− 1 7), (0, 4 2 63),(0,− 4 2 63)(x−2) 2 64 + (y−4) 2 16 =1
(x+3) 2 9 + (y−3) 2 9 =1 (−3,3), (0,3),(−6,3),(−3,0),(−3,6), (−3,3) x 2 2 + (y+1) 2 5 =1
4 x 2 −8x+16 y 2 −32y−44=0 (1,1), (5,1),(−3,1),(1,3),(1,−1), (1,1+4 3),(1,1−4 3)
x 2 −8x+25 y 2 −100y+91=0x 2 +8x+4 y 2 −40y+112=0 (−4,5), (−2,5),(−6,4),(−4,6),(−4,4),
(−4+ 3 ,5),(−4− 3 ,5)64 x 2 +128x+9 y 2 −72y−368=016 x 2 +64x+4 y 2 −8y+4=0 (−2,1),
(0,1),(−4,1),(−2,5),(−2,−3), (−2,1+2 3),(−2,1−2 3)100 x 2 +1000x+ y 2 −10y+2425=0
4 x 2 +16x+4 y 2 +16y+16=0 (−2,−2), (0,−2),(−4,−2),(−2,0),(−2,−4), (−2,−2) (4,0), (0,3). (0,−2),
(5,0).x 2 25 + y 2 29 =1 (3,0), (4,2) (9,2) (4+2 6 ,2)(x−4) 2 25 + (y−2) 2 1 =1 (3,5) (3,11)
(3, 5+4 2) (−3,4) (1,4) (−3+2 3 ,4)(x+3) 2 16 + (y−4) 2 4 =1x 2 81 + y 2 9 =1
(x+2) 2 4 + (y−2) 2 9 =1 Area=a·b·π.(x−3) 2 9 + (y−3) 2 16 =1Area = 12π square units
(x+6) 2 16 + (y−6) 2 36 =1(x+1) 2 4 + (y−2) 2 5 =1Area = 2 5 π square units
4 x 2 −8x+9 y 2 −72y+112=09 x 2 −54x+9 y 2 −54y+81=0Area = 9π square units h,
x 2 4 h 2 + y 2 1 4 h 2 =1x 2 400 + y 2 144 =1 (x,y) (x,y) (x,y) (x,y) (−c,0) (c,0) (x,y) (x,y)
(a,0) (−c,0) (a,0) a−(−c)=a+c. (c,0) (a,0) c−a.

(a+c)−(c−a)=2a

(x,y)

d 2 =the distance from (−c,0) to (x,y) d 1 =the distance from (c,0) to (x,y)

d 2 − d 1 (x,y) 2a (a,0). d 2 − d 1 =2a

d 2 − d 1 = (x−(−c)) 2 + (y−0) 2 − (x−c) 2 + (y−0) 2 =2a Distance Formula (x+c) 2 + y 2 − (x−c) 2 + y 2 =2a Simplify expressions. (x+c) 2 + y 2 =2a+ (x−c) 2 + y 2 Move radical to opposite side. (x+c) 2 + y 2 = (2a+ (x−c) 2 + y 2) 2 Square both sides. x 2 +2cx+ c 2 + y 2 =4 a 2 +4a (x−c) 2 + y 2 + (x−c) 2 + y 2 Expand the squares. x 2 +2cx+ c 2 + y 2 =4 a 2 +4a (x−c) 2 + y 2 + x 2 −2cx+ c 2 + y 2 Expand remaining square. 2cx=4 a 2 +4a (x−c) 2 + y 2 −2cx Combine like terms. 4cx−4 a 2 =4a (x−c) 2 + y 2 Isolate the radical. cx− a 2 =a (x−c) 2 + y 2 Divide by 4. (cx− a 2) 2 = a 2 [(x −c) 2 + y 2] 2 Square both sides. c 2 x 2 −2 a 2 cx+ a 4 = a 2 (x 2 −2cx+ c 2 + y 2) Expand the squares. c 2 x 2 −2 a 2 cx+ a 4 = a 2 x 2 −2 a 2 cx+ a 2 c 2 + a 2 y 2 Distribute a 2 . a 4 + c 2 x 2 = a 2 x 2 + a 2 c 2 + a 2 y 2 Combine like terms. c 2 x 2 − a 2 x 2 − a 2 y 2 = a 2 c 2 − a 4 Rearrange terms. x 2 (c 2 − a 2)− a 2 y 2 = a 2 (c 2 − a 2) Factor common terms. x 2 b 2 − a 2 y 2 = a 2 b 2 Set b 2 = c 2 − a 2 . x 2 b 2 a 2 b 2 − a 2 y 2 a 2 b 2 = a 2 b 2 a 2 b 2 Divide both sides by a 2 b 2
x 2 a 2 − y 2 b 2 =1

(±a,0) (0±b). (0,0)

x 2 a 2 − y 2 b 2 =1

2a (±a,0) 2b (0,±b) 2c, c 2 = a 2 + b 2 (±c,0) y=± b a x (0,0)

y 2 a 2 − x 2 b 2 =1

2a (0,±a) 2b (±b,0) 2c, c 2 = a 2 + b 2 (0,±c) y=± a b x c 2 = a 2 + b 2 . (0,0) (0,0) a 2
x 2 a 2 − y 2 b 2 =1, (±a,0), (±c,0). y 2 a 2 − x 2 b 2 =1, (0,±a), (0,±c). a a= a 2 . c c= a 2 + b 2 .
y 2 49 − x 2 32 =1. y 2 a 2 − x 2 b 2 =1, x=0, y.

1= y 2 49 − x 2 32 1= y 2 49 − 0 2 32 1= y 2 49 y 2 =49 y=± 49 =±7

(0,±c). c,

c= a 2 + b 2 = 49+32 = 81 =9

(0,±7), (0,9). x 2 9 − y 2 25 =1. (±3,0); (± 34 ,0) (0,0), c 2 = a 2 + b 2 . b 2 = c 2 − a 2 . (0,0), (±a,0) (±c,0), x 2 a 2 − y 2 b 2 =1. (0,±a) (0,±c), y 2 a 2 − x 2 b 2 =1. b 2 b 2 = c 2 − a 2 . a 2 b 2 (±6,0) (±2 10 ,0)? x 2 a 2 − y 2 b 2 =1. (±6,0), a=6 a 2 =36. (±2 10 ,0), c=2 10 c 2 =40. b 2 ,

b 2 = c 2 − a 2 b 2 =40−36 Substitute for c 2 and a 2 . b 2 =4 Subtract.

a 2 =36 b 2 =4 x 2 a 2 − y 2 b 2 =1. x 2 36 − y 2 4 =1, (0,±2) (0,±2 5)?y 2 4 − x 2 16 =1 h k (h,k). x (x−h) y (y−k). (h,k)

(x−h) 2 a 2 − (y−k) 2 b 2 =1

2a (h±a,k) 2b (h,k±b) 2c, c 2 = a 2 + b 2 (h±c,k) 2a 2b. ± b a , (h,k). y=± b a (x−h)+k. (h,k)

(y−k) 2 a 2 − (x−h) 2 b 2 =1

2a (h,k±a) 2b (h±b,k) 2c, c 2 = a 2 + b 2 (h,k±c) y=± a b (x−h)+k. (h,k) (h,k) (h,k)
c 2 = a 2 + b 2 . (h,k), (x−h) 2 a 2 − (y−k) 2 b 2 =1. (y−k) 2 a 2 − (x−h) 2 b 2 =1. (h,k), a 2 2a c 2 h k b 2 b 2 = c 2 − a 2 . h, k, a 2 , b 2 (0,−2)(6,−2)(−2,−2)(8,−2)?

(x−h) 2 a 2 − (y−k) 2 b 2 =1

(h,k). (0,−2) (6,−2).

(h,k)=(0+6 2 , −2+(−2) 2)=(3,−2)

a 2 . 2a, a 2

2a=| 0−6 | 2a=6 a=3 a 2 =9

c 2 . (h±c,k). (h−c,k)=(−2,−2) (h+c,k)=(8,−2). c. (8,−2), h=3,

h+c=8 3+c=8 c=5 c 2 =25

b 2 b 2 = c 2 − a 2 :

b 2 = c 2 − a 2 =25−9 =16

h,k, a 2 , b 2

(x−3) 2 9 − (y+2) 2 16 =1

(1,−2) (1,8) (1,−10) (1,16)?(y−3) 2 25 + (x−1) 2 144 =1 x 2 a 2 − y 2 b 2 =1
y 2 a 2 − x 2 b 2 =1 (0,0), x 2 a 2 − y 2 b 2 =1, (±a,0) (0,±b) (±c,0) y=± b a x y 2 a 2 − x 2 b 2 =1,
(0,±a) (±b,0) (0,±c) y=± a b x c=± a 2 + b 2 . y 2 64 − x 2 36 =1. y 2 a 2 − x 2 b 2 =1.
(0,±a)=(0,± 64)=(0,±8) (±b,0)=(± 36 , 0)=(±6,0) (0,±c), c=± a 2 + b 2 . c,
c=± a 2 + b 2 =± 64+36 =± 100 =±10
(0,±10) y=± a b x=± 8 6 x=± 4 3 x x 2 144 − y 2 81 =1. (±12,0); (0,±9); (±15,0); y=± 3 4 x; (h,k)
(x−h) 2 a 2 − (y−k) 2 b 2 =1 (y−k) 2 a 2 − (x−h) 2 b 2 =1 (h, k), (x−h) 2 a 2 − (y−k) 2 b 2 =1,
(h,k) (h±a,k) (h,k±b) (h±c,k) y=± b a (x−h)+k (y−k) 2 a 2 − (x−h) 2 b 2 =1, (h,k) (h,k±a)
(h±b,k) (h,k±c) y=± a b (x−h)+k c=± a 2 + b 2 . 9 x 2 −4 y 2 −36x−40y−388=0.
(9 x 2 −36x)−(4 y 2 +40y)=388
9(x 2 −4x)−4(y 2 +10y)=388
9(x 2 −4x+4)−4(y 2 +10y+25)=388+36−100
9 (x−2) 2 −4 (y+5) 2 =324
(x−2) 2 36 − (y+5) 2 81 =1
(x−h) 2 a 2 − (y−k) 2 b 2 =1, a 2 =36 b 2 =81, a=6 b=9. (h,k)=(2,−5) (h±a,k)=(2±6,−5),
(−4,−5) (8,−5) (h,k±b)=(2,−5±9), (2,−14) (2,4) (h±c,k), c=± a 2 + b 2 . c,
c=± 36+81 =± 117 =±3 13
(2−3 13 ,−5) (2+3 13 ,−5). y=± b a (x−h)+k=± 3 2 (x−2)−5. (y+4) 2 100 − (x−3) 2 64 =1.
(3,−4); (3,−14) (3,6); (−5,−4); (11,−4); (3,−4−2 41) (3,−4+2 41); y=± 5 4 (x−3)−4
x 2 a 2 − y 2 b 2 =1, a 2 b 2 a 2 . 2a. 65.3 2a=60. a=30 a 2 =900. b 2 , x y (x,y)
x 2 a 2 − y 2 b 2 =1 Standard form of horizontal hyperbola. b 2 = y 2 x 2 a 2 −1 Isolate b 2
= (79.6) 2 (36) 2 900 −1 Substitute for a 2 ,x, and y ≈14400.3636
Round to four decimal places
x 2 900 − y 2 14400.3636 =1,or x 2 30 2 − y 2 120.0015 2 =1
x 2 400 − y 2 3600 =1or x 2 20 2 − y 2 60 2 =1.x 2 a 2 − y 2 b 2 =1y 2 a 2 − x 2 b 2 =1 (h,k),
(x−h) 2 a 2 − (y−k) 2 b 2 =1 (h,k),(y−k) 2 a 2 − (x−h) 2 b 2 =1 (x,y) (x,y) 3 y 2 +2x=6
x 2 36 − y 2 9 =1 x 2 6 2 − y 2 3 2 =15 y 2 +4 x 2 =6x25 x 2 −16 y 2 =400 x 2 4 2 − y 2 5 2 =1
−9 x 2 +18x+ y 2 +4y−14=0x 2 25 − y 2 36 =1x 2 5 2 − y 2 6 2 =1; (5,0),(−5,0); (61 ,0),(− 61 ,0);
y= 6 5 x,y=− 6 5 x x 2 100 − y 2 9 =1y 2 4 − x 2 81 =1y 2 2 2 − x 2 9 2 =1; (0,2),(0,−2);
(0, 85),(0,− 85); y= 2 9 x,y=− 2 9 x9 y 2 −4 x 2 =1(x−1) 2 9 − (y−2) 2 16 =1
(x−1) 2 3 2 − (y−2) 2 4 2 =1; (4,2),(−2,2); (6,2),(−4,2); y= 4 3 (x−1)+2,y=− 4 3 (x−1)+2
(y−6) 2 36 − (x+1) 2 16 =1(x−2) 2 49 − (y+7) 2 49 =1(x−2) 2 7 2 − (y+7) 2 7 2 =1;
(9,−7),(−5,−7); (2+7 2 ,−7),(2−7 2 ,−7); y=x−9,y=−x−54 x 2 −8x−9 y 2 −72y+112=0
−9 x 2 −54x+9 y 2 −54y+81=0(x+3) 2 3 2 − (y−3) 2 3 2 =1; (0,3),(−6,3);
(−3+3 2 ,1),(−3−3 2 ,1); y=x+6,y=−x4 x 2 −24x−36 y 2 −360y+864=0
−4 x 2 +24x+16 y 2 −128y+156=0(y−4) 2 2 2 − (x−3) 2 4 2 =1; (3,6),(3,2); (3,4+2 5),(3,4−2 5);
y= 1 2 (x−3)+4,y=− 1 2 (x−3)+4−4 x 2 +40x+25 y 2 −100y+100=0x 2 +2x−100 y 2 −1000y+2401=0
(y+5) 2 7 2 − (x+1) 2 70 2 =1; (−1,2),(−1,−12); (−1,−5+7 101),(−1,−5−7 101);
y= 1 10 (x+1)−5,y=− 1 10 (x+1)−5−9 x 2 +72x+16 y 2 +16y+4=04 x 2 +24x−25 y 2 +200y−464=0
(x+3) 2 5 2 − (y−4) 2 2 2 =1; (2,4),(−8,4); (−3+ 29 ,4),(−3− 29 ,4);
y= 2 5 (x+3)+4,y=− 2 5 (x+3)+4y 2 3 2 − x 2 3 2 =1(x−3) 2 5 2 − (y+4) 2 2 2 =1
y= 2 5 (x−3)−4,y=− 2 5 (x−3)−4(y−3) 2 3 2 − (x+5) 2 6 2 =19 x 2 −18x−16 y 2 +32y−151=0
y= 3 4 (x−1)+1,y=− 3 4 (x−1)+116 y 2 +96y−4 x 2 +16x+112=0x 2 49 − y 2 16 =1x 2 64 − y 2 4 =1
y 2 9 − x 2 25 =181 x 2 −9 y 2 =1(y+5) 2 9 − (x−4) 2 25 =1(x−2) 2 8 − (y+3) 2 27 =1
(y−3) 2 9 − (x−3) 2 9 =1−4 x 2 −8x+16 y 2 −32y−52=0x 2 −8x−25 y 2 −100y−109=0
− x 2 +8x+4 y 2 −40y+88=064 x 2 +128x−9 y 2 −72y−656=016 x 2 +64x−4 y 2 −8y−4=0
−100 x 2 +1000x+ y 2 −10y−2575=04 x 2 +16x−4 y 2 +16y+16=0 (3,0) (−3,0) (5,0).
x 2 9 − y 2 16 =1 (0,6) (0,−6) (0,−8). (1,1) (11,1) (12,1).(x−6) 2 25 − (y−1) 2 11 =1 (0,0);
(0,−13); (0, 313). (4,2); (9,2); (4+ 26 ,2).(x−4) 2 25 − (y−2) 2 1 =1 (3,5); (3,11); (3,5+2 10).

y 2 16 − x 2 25 =1y 2 9 − (x+1) 2 9 =1(x+3) 2 25 − (y+3) 2 25 =1 y x. x 2 4 − y 2 9 =1
y 2 9 − x 2 1 =1y(x)=3 x 2 +1 ,y(x)=−3 x 2 +1(x−2) 2 16 − (y+3) 2 25 =1
−4 x 2 −16x+ y 2 −2y−19=0y(x)=1+2 x 2 +4x+5 ,y(x)=1−2 x 2 +4x+54 x 2 −24x− y 2 −4y+16=0
y=x and y=−x,x 2 25 − y 2 25 =1 y=2x and y=−2x, y= 1 2 x y=− 1 2 x,x 2 100 − y 2 25 =1 y= 2 3 x
y=− 2 3 x, y= 3 4 x and y=− 3 4 x,x 2 400 − y 2 225 =1 y=x−2 y=−x+2. y=2x−2 y=−2x+2.
(x−1) 2 0.25 − y 2 0.75 =1 y=0.5x+2 y=−0.5x−2. y= 1 3 x−1 y=− 1 3 x+1.(x−3) 2 4 − y 2 5 =1
y=3x−9 y=−3x+9. (x,y) (x,y) (x,y) d P P (x,y) (0,0), (0,p), y= −p d (x,y) (x,−p) d=y+p. (0,p) (x,y) d

d= (x−0) 2 + (y−p) 2 = x 2 + (y−p) 2

d y (x,y) (0,p) (x,y) (x, −p).

x 2 + (y−p) 2 =y+p

x 2 + (y−p) 2 = (y+p) 2 x 2 + y 2 −2py+ p 2 = y 2 +2py+ p 2 x 2 −2py=2py x 2 =4py

(0,0) y 2 =4px x 2 =4py y 2 =4px(p, 0)x=−p(p, ±2p)x 2 =4py(0, p)y=−p(±2p, p) p>0 p<0 p<0 p<0 y 2 =4px x 2 =4py. y 2 =4px, y=0 4p p. p>0, p<0, p (p,0) p x=−p p (p,±2p). x=p x 2 =4py, x=0 4p p. p>0, p<0, p (0,p) p y=−p p (±2p,p) y 2 =24x. y 2 =4px. 24=4p, p=6. p>0, (p,0)=(6,0) x=−p=−6 x=6 (6,±12) y 2 =−16x. (−4,0); x=4; (−4,±8) x 2 =−6y. x 2 =4py. −6=4p, p=− 3 2 . p<0, (0,p)=(0,− 3 2) y=−p= 3 2 y= 3 2 (±3,− 3 2) x 2 =8y. (0,2); y=−2; (±4,2). (p,0), y 2 =4px. (0,p), x 2 =4py. 4p. (− 1 2 ,0) x= 1 2 ? (p,0), y 2 =4px. 4p, 4p=4(− 1 2)=−2. 4p, y 2 =4px=−2x. y 2 =−2x. (0, 7 2) y=− 7 2 ?x 2 =14y. h k (h,k). x (x−h) y (y−k). (h,k) (y−k) 2 =4p(x−h) (x−h) 2 =4p(y−k) (h,k).y=k(y−k) 2 =4p(x−h)(h+p, k)x=h−p(h+p, k±2p)x=h(x−h) 2 =4p(y−k) (h, k+p)y=k−p(h±2p, k+p) p>0, p<0, p>0, p<0, (y−k) 2 =4p(x−h) (x−h) 2 =4p(y−k). (y−k) 2 =4p(x−h), h k (h,k) k y=k 4p (x−h) p. p>0, p<0, h,k, p (h+p, k) h p x=h−p h,k, p (h+p,k±2p) (x−h) 2 =4p(y−k), h k (h,k) h x=h 4p (y−k) p. p>0, p<0, h,k, p (h, k+p) k p y=k−p h, k, p (h±2p, k+p) (y−1) 2 =−16(x+3). (y−k) 2 =4p(x−h). (h,k)=(−3,1) y=k=1−16=4p, p=−4. p<0, (h+p,k)=(−3+(−4),1)=(−7,1) x=h−p=−3−(−4)=1 (h+p,k±2p)=(−3+(−4),1±2(−4)), (−7,−7) (−7,9) (y+1) 2 =4(x−8). (8,−1); y=−1; (9,−1); x=7; (9,−3) (9,1).
x 2 −8x−28y−208=0. (x−h) 2 =4p(y−k). x

x 2 −8x−28y−208=0 x 2 −8x=28y+208 x 2 −8x+16=28y+208+16 (x −4) 2 =28y+224 (x−4) 2 =28(y+8) (x−4) 2 =4⋅7⋅(y+8)

(h,k)=(4,−8) x=h=4 p=7,p>0 (h,k+p)=(4,−8+7)=(4,−1) y=k−p=−8−7=−15 (h±2p,k+p)=(4±2(7),−8+7), (−10,−1) (18,−1) (x+2) 2 =−20(y−3). (−2,3); x=−2; (−2,−2); y=8; (−12,−2) (8,−2). x 2 =4py, p>0. p=1.7.

x 2 =4py Standard form of upward-facing parabola with vertex (0,0) x 2 =4(1.7)y Substitute 1.7 for p. x 2 =6.8y Multiply.

4.5 2 =2.25 x

x 2 =6.8y Equation found in part (a). (2.25) 2 =6.8y Substitute 2.25 for x. y≈0.74 Solve for y.

y 2 =1280xy 2 =4pxx 2 =4py (h,k),(y−k) 2 =4p(x−h) (h,k),(x−h) 2 =4p(y−k) (x,y) (0,0) p>0, p<0, (0,0) p>0, p<0, (h,k) p>0, p<0, (h,k) p>0, p<0, p p p?y 2 =4− x 2y=4 x 2 y=4(1) x 23 x 2 −6 y 2 =12 (y−3) 2 =8(x−2) (y−3) 2 =4(2)(x−2)y 2 +12x−6y−51=0 (V), (F), (d) x=8 y 2
y 2 = 1 8 x,V:(0,0);F:(1 32 ,0);d:x=− 1 32y= 1 4 x 2y=−4 x 2
x 2 =− 1 4 y,V:(0,0);F:(0,− 1 16);d:y= 1 16x= 1 8 y 2x=36 y 2
y 2 = 1 36 x,V:(0,0);F:(1 144 ,0);d:x=− 1 144x= 1 36 y 2(x−1) 2 =4(y−1)
(x−1) 2 =4(y−1),V:(1,1);F:(1,2);d:y=0(y−2) 2 = 4 5 (x+4)(y−4) 2 =2(x+3)
(y−4) 2 =2(x+3),V:(−3,4);F:(− 5 2 ,4);d:x=− 7 2(x+1) 2 =2(y+4)(x+4) 2 =24(y+1)
(x+4) 2 =24(y+1),V:(−4,−1);F:(−4,5);d:y=−7(y+4) 2 =16(x+4)y 2 +12x−6y+21=0
(y−3) 2 =−12(x+1),V:(−1,3);F:(−4,3);d:x=2x 2 −4x−24y+28=05 x 2 −50x−4y+113=0
(x−5) 2 = 4 5 (y+3),V:(5,−3);F:(5,− 14 5);d:y=− 16 5y 2 −24x+4y−68=0x 2 −4x+2y−6=0
(x−2) 2 =−2(y−5),V:(2,5);F:(2, 9 2);d:y= 11 2y 2 −6y+12x−3=03 y 2 −4x−6y+23=0
(y−1) 2 = 4 3 (x−5),V:(5,1);F:(16 3 ,1);d:x= 14 3x 2 +4x+8y−4=0x= 1 8 y 2y=36 x 2y= 1 36 x 2

y=−9 x 2(y−2) 2 =− 4 3 (x+2)−5 (x+5) 2 =4(y+5)−6 (y+5) 2 =4(x−4)y 2 −6y−8x+1=0
x 2 +8x+4y+20=03 x 2 +30x−4y+95=0y 2 −8x+10y+9=0x 2 +4x+2y+2=0y 2 +2y−12x+61=0
−2 x 2 +8x−4y−24=0 (0,0); y=4, (0,−4).x 2 =−16y (0,0); x=4, (−4,0). (2,2); x=2− 2 , (2+ 2 ,2).
(y−2) 2 =4 2 (x−2) (−2,3); x=− 7 2 , (− 1 2 ,3). (2 ,− 3); x=2 2 , (0,− 3).(y+ 3) 2 =−4 2 (x− 2)
(1,2); y= 11 3 , (1, 1 3).x 2 =y(y−2) 2 = 1 4 (x+2)(y− 3) 2 =4 5 (x+ 2)
V(0,0), Endpoints (2,1),(−2,1)V(0,0), Endpoints (−2,4),(−2,−4)y 2 =−8x
V(1,2), Endpoints (−5,5),(7,5)V(−3,−1),Endpoints (0,5),(0,−7)(y+1) 2 =12(x+3)
V(4,−3), Endpoints (5,− 7 2),(3,− 7 2) x 2 =4y. (0,1) (0,0.25),x 2 =−125(y−20), y=− x 2 +96x, x
y y=−0.5 x 2 +80x. (x,y)

A x 2 +Bxy+C y 2 +Dx+Ey+F=0

A,B, C xy xy B 4 x 2 +9 y 2 =14 x 2 +4 y 2 =14 x 2 −9 y 2 =14 x 2 =9y or 4 y 2 =9x4x+9y=1
(x−4)(y+4)=0(x−4)(x−9)=04 x 2 +4 y 2 =04 x 2 +4 y 2 = − 1

A x 2 +Bxy+C y 2 +Dx+Ey+F=0

A,B, C B=0, A C A x 2 +C y 2 +Dx+Ey+F=0, A≠C and AC>0A x 2 +C y 2 +Dx+Ey+F=0, A=C
A x 2 −C y 2 +Dx+Ey+F=0 or −A x 2 +C y 2 +Dx+Ey+F=0, A C
A x 2 +Dx+Ey+F=0 or C y 2 +Dx+Ey+F=0A x 2 +Bxy+C y 2 +Dx+Ey+F=0. A C A C A C A C A C
A x 2 +By2=0,A x 2 +By2=0,A x 2 +By2+1=0,4 x 2 −9 y 2 +36x+36y−125=09 y 2 +16x+36y−10=0
3 x 2 +3 y 2 −2x−6y−4=0−25 x 2 −4 y 2 +100x+16y+20=0A=4 C=−9, A C A=0 C=9. A A=3 C=3.
A=C,A=−25 C=−4. AC>0 A≠C,16 y 2 − x 2 +x−4y−9=016 x 2 +4 y 2 +16x+49y−81=0 xy xy θ, (x,y)
(x ′ , y ′) x 2 + y 2 −xy−15=0 x y x ′ y ′ θ. i j . i ′ j ′ . θ

i ′ =cos θi+sin θj j ′ =−sin θi+cos θj

u

u= x ′ i ′ + y ′ j ′ u= x ′ (i cos θ+j sin θ)+ y ′ (−i sin θ+j cos θ) Substitute. u=ix' cos θ+jx' sin θ−iy' sin θ +jy' cos θ Distribute. u=ix' cos θ−iy' sin θ+jx' sin θ+jy' cos θ Apply commutative property. u=(x' cos θ −y' sin θ)i+(x' sin θ+y' cos θ)j Factor by grouping.

u= x ′ i ′ + y ′ j ′ , x y

x= x ′ cos θ− y ′ sin θ and y= x ′ sin θ+ y ′ cos θ

(x,y) θ (x ′ , y ′). (x,y) (x ′ , y ′):

x= x ′ cos θ− y ′ sin θ
y= x ′ sin θ+ y ′ cos θ

x y x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ. x y x ′ y ′ 2 x 2 −xy+2 y 2 −30=0 θ=45°. x y,
x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ. θ=45°,

x= x ′ cos(45°)− y ′ sin(45°) x= x ′ (1 2)− y ′ (1 2) x= x ′ − y ′ 2
y= x ′ sin(45°)+ y ′ cos(45°) y= x ′ (1 2)+ y ′ (1 2) y= x ′ + y ′ 2

x= x ′ cosθ− y ′ sinθ y= x ′ sin θ+ y ′ cos θ 2 x 2 −xy+2 y 2 −30=0.

2 (x ′ − y ′ 2) 2 −(x ′ − y ′ 2)(x ′ + y ′ 2)+2 (x ′ + y ′ 2) 2 −30=0
2 (x ′ − y ′)(x ′ − y ′) 2 − (x ′ − y ′)(x ′ + y ′) 2 + 2 (x ′ + y ′)(x ′ + y ′) 2 −30=0 FOIL method
x ′ 2 −2 x ′ y ′ + y ′ 2 − (x ′ 2 − y ′ 2) 2 + x ′ 2 +2 x ′ y ′ + y ′ 2 −30=0 Combine like terms.
2 x ′ 2 +2 y ′ 2 − (x ′ 2 − y ′ 2) 2 =30 Combine like terms.
2(2 x ′ 2 +2 y ′ 2 − (x ′ 2 − y ′ 2) 2)=2(30) Multiply both sides by 2.
4 x ′ 2 +4 y ′ 2 −(x ′ 2 − y ′ 2)=60 Simplify.
4 x ′ 2 +4 y ′ 2 − x ′ 2 + y ′ 2 =60 Distribute.
3 x ′ 2 60 + 5 y ′ 2 60 = 60 60 Set equal to 1.

x ′ y ′

x ′ 2 20 + y ′ 2 12 =1

A x 2 +Bxy+C y 2 +Dx+Ey+F=0 x ′ y ′ x ′ y ′ θ

cot(2θ)= A−C B
A x 2 +Bxy+C y 2 +Dx+Ey+F=0

A,B, C B≠0, xy θ cot(2θ)= A−C B . cot(2θ)>0, 2θ θ (0°,45°). cot(2θ)<0, 2θ θ (45°,90°). A=C, θ=45°. x ′ y ′ x ′ y ′ x ′ y ′ , x ′ y ′ θ cot(2θ). sin θ cos θ. sin θ cos θ x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ. x y x ′ y ′ 8 x 2 −12xy+17 y 2 =20 x ′ y ′ x ′ y ′ cot(2θ).

8 x 2 −12xy+17 y 2 =20⇒A=8, B=−12 and C=17 cot(2θ)= A−C B = 8−17 −12

cot(2θ)= −9 −12 = 3 4

cot(2θ)= 3 4 = adjacent opposite

3 2 + 4 2 = h 2 9+16= h 2 25= h 2 h=5

sin θcos θ.

sin θ= 1−cos(2θ) 2 = 1− 3 5 2 = 5 5 − 3 5 2 = 5−3 5 · 1 2 = 2 10 = 1 5 sin θ= 1 5 cos θ= 1+cos(2θ) 2 = 1+ 3 5 2 = 5 5 + 3 5 2 = 5+3 5 · 1 2 = 8 10 = 4 5 cos θ= 2 5

sin θ cos θ x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ.

x= x ′ cos θ− y ′ sin θ x= x ′ (2 5)− y ′ (1 5) x= 2 x ′ − y ′ 5

y= x ′ sin θ+ y ′ cos θ y= x ′ (1 5)+ y ′ (2 5) y= x ′ +2 y ′ 5

x y

8 (2 x ′ − y ′ 5) 2 −12(2 x ′ − y ′ 5)(x ′ +2 y ′ 5)+17 (x ′ +2 y ′ 5) 2 =20 8((2 x ′ − y ′)(2 x ′ − y ′) 5)−12((2 x ′ − y ′)(x ′ +2 y ′) 5)+17((x ′ +2 y ′)(x ′ +2 y ′) 5)=20 8(4 x ′ 2 −4 x ′ y ′ + y ′ 2)−12(2 x ′ 2 +3 x ′ y ′ −2 y ′ 2)+17(x ′ 2 +4 x ′ y ′ +4 y ′ 2)=100 32 x ′ 2 −32 x ′ y ′ +8 y ′ 2 −24 x ′ 2 −36 x ′ y ′ +24 y ′ 2 +17 x ′ 2 +68 x ′ y ′ +68 y ′ 2 =100

25 x ′ 2 +100 y ′ 2 =100

25 100 x ′ 2 + 100 100 y ′ 2 = 100 100

x ′ y ′

x ′ 2 4 + y ′ 2 1 =1

13 x 2 −6 3 xy+7 y 2 =16 x ′ y ′ x ′ y ′ x ′ 2 4 + y ′ 2 1 =1 x ′ y ′

x 2 +12xy−4 y 2 =30

cot(2θ).

x 2 +12xy−4 y 2 =20⇒A=1, B=12,and C=−4

cot(2θ)= A−C B cot(2θ)= 1−(−4) 12 cot(2θ)= 5 12

cot(2θ)= 5 12 ,

cot(2θ)= 5 12 = adjacent opposite

5 2 + 12 2 = h 2 25+144= h 2 169= h 2 h=13

sin θ cos θ.

sin θ= 1−cos(2θ) 2 = 1− 5 13 2 = 13 13 − 5 13 2 = 8 13 · 1 2 = 2 13 cos θ= 1+cos(2θ) 2 = 1+ 5 13 2 = 13 13 + 5 13 2 = 18 13 · 1 2 = 3 13

x y.

x= x ′ cos θ− y ′ sin θ x= x ′ (3 13)− y ′ (2 13) x= 3 x ′ −2 y ′ 13

y= x ′ sin θ+ y ′ cos θ y= x ′ (2 13)+ y ′ (3 13) y= 2 x ′ +3 y ′ 13

x= 3 x ′ −2 y ′ 13 y= 2 x ′ +3 y ′ 13 x 2 +12xy−4 y 2 =30.

(3 x ′ −2 y ′ 13) 2 +12(3 x ′ −2 y ′ 13)(2 x ′ +3 y ′ 13)−4 (2 x ′ +3 y ′ 13) 2 =30 (1 13)[(3 x ′ −2 y ′) 2 +12(3 x ′ −2 y ′)(2 x ′ +3 y ′)−4 (2 x ′ +3 y ′) 2]=30 Factor. (1 13)[9 x ′ 2 −12 x ′ y ′ +4 y ′ 2 +12(6 x ′ 2 +5 x ′ y ′ −6 y ′ 2)−4(4 x ′ 2 +12 x ′ y ′ +9 y ′ 2)]=30 Multiply. (1 13)[9 x ′ 2 −12 x ′ y ′ +4 y ′ 2 +72 x ′ 2 +60 x ′ y ′ −72 y ′ 2 −16 x ′ 2 −48 x ′ y ′ −36 y ′ 2]=30 Distribute. (1 13)[65 x ′ 2 −104 y ′ 2]=30 Combine like terms. 65 x ′ 2 −104 y ′ 2 =390 Multiply.

x ′ 2 6 − 4 y ′ 2 15 =1

Divide by 390.

x ′ 2 6 − 4 y ′ 2 15 =1.

A x 2 +Bxy+C y 2 +Dx+Ey+F=0

A ′ x ′ 2 + B ′ x ′ y ′ + C ′ y ′ 2 + D ′ x ′ + E ′ y ′ + F ′ =0

B 2 −4AC= B ′ 2 −4 A ′ C ′ . B 2 −4AC, A x 2 +Bxy+C y 2 +Dx+Ey+F=0

A ′ x ′ 2 + B ′ x ′ y ′ + C ′ y ′ 2 + D ′ x ′ + E ′ y ′ + F ′ =0, B 2 −4AC= B ′ 2 −4 A ′ C ′ .

A x 2 +Bxy+C y 2 +Dx+Ey+F=0 B 2 −4AC,<0,=0,>0,5 x 2 +2 3 xy+2 y 2 −5=0

5 x 2 +2 3 xy+12 y 2 −5=0 A,B, C.

5 A x 2 + 2 3 B xy+ 2 C y 2 −5=0

B 2 −4AC= (2 3) 2 −4(5)(2) =4(3)−40 =12−40 =−28<0

5 x 2 +2 3 xy+2 y 2 −5=0 A,B, C.

5 A x 2 + 2 3 B xy+ 12 C y 2 −5=0

B 2 −4AC= (2 3) 2 −4(5)(12) =4(3)−240 =12−240 =−228<0

5 x 2 +2 3 xy+12 y 2 −5=0 x 2 −9xy+3 y 2 −12=010 x 2 −9xy+4 y 2 −4=0

A x 2 +Bxy+C y 2 +Dx+Ey+F=0

x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ

θ,where cot(2θ)= A−C B A x 2 +Bxy+C y 2 +Dx+Ey+F=0 A,B C A,B, C xy x ′ y ′ x ′ y ′ xy xy

A x 2 +B y 2 +Cx+Dy+E=0 AB=0, A x 2 +Bxy+C y 2 +Dx+Ey+F=0, B 2 −4AC>0,

a x 2 +4x+3 y 2 −12=0, a>0? A x 2 +Bxy+C y 2 +Dx+Ey+F=0, θ cot(2θ)= A−C B xy

9 x 2 +4 y 2 +72x+36y−500=0x 2 −10x+4y−10=0AB=0,2 x 2 −2 y 2 +4x−6y−2=04 x 2 − y 2 +8x−1=0

AB=−4<0,4 y 2 −5x+9y+1=02 x 2 +3 y 2 −8x−12y+2=0AB=6>0,4 x 2 +9xy+4 y 2 −36y−125=0

3 x 2 +6xy+3 y 2 −36y−125=0B 2 −4AC=0,−3 x 2 +3 3 xy−4 y 2 +9=02 x 2 +4 3 xy+6 y 2 −6x−3=0

B 2 −4AC=0,− x 2 +4 2 xy+2 y 2 −2y+1=08 x 2 +4 2 xy+4 y 2 −10x+1=0B 2 −4AC=−96<0,

3 x 2 +xy+3 y 2 −5=0,θ=45°4 x 2 −xy+4 y 2 −2=0,θ=45°7 x ′ 2 +9 y ′ 2 −4=02 x 2 +8xy−1=0,θ=30°

−2 x 2 +8xy+1=0,θ=45°3 x ′ 2 +2 x ′ y ′ −5 y ′ 2 +1=04 x 2 + 2 xy+4 y 2 +y+2=0,θ=45° θ xy xy

x 2 +3 3 xy+4 y 2 +y−2=0θ= 60 ◦ ,11 x ′ 2 − y ′ 2 + 3 x ′ + y ′ −4=04 x 2 +2 3 xy+6 y 2 +y−2=0

9 x 2 −3 3 xy+6 y 2 +4y−3=0θ= 150 ◦ ,21 x ′ 2 +9 y ′ 2 +4 x ′ −4 3 y ′ −6=0−3 x 2 − 3 xy−2 y 2 −x=0

16 x 2 +24xy+9 y 2 +6x−6y+2=0θ≈ 36.9 ◦ ,125 x ′ 2 +6 x ′ −42 y ′ +10=0x 2 +4xy+4 y 2 +3x−2=0

x 2 +4xy+ y 2 −2x+1=0θ= 45 ◦ ,3 x ′ 2 − y ′ 2 − 2 x ′ + 2 y ′ +1=04 x 2 −2 3 xy+6 y 2 −1=0

y=− x 2 ,θ=− 45 ◦2 2 (x ′ + y ′)= 1 2 (x ′ − y ′) 2x= y 2 ,θ= 45 ◦x 2 4 + y 2 1 =1,θ= 45 ◦

(x ′ − y ′) 2 8 + (x ′ + y ′) 2 2 =1y 2 16 + x 2 9 =1,θ= 45 ◦y 2 − x 2 =1,θ= 45 ◦

(x ′ + y ′) 2 2 − (x ′ − y ′) 2 2 =1y= x 2 2 ,θ= 30 ◦x= (y−1) 2 ,θ= 30 ◦

3 2 x ′ − 1 2 y ′ = (1 2 x ′ + 3 2 y ′ −1) 2x 2 9 + y 2 4 =1,θ= 30 ◦ x ′ y ′ x ′ y ′ xy=9

x 2 +10xy+ y 2 −6=0x 2 −10xy+ y 2 −24=04 x 2 −3 3 xy+ y 2 −22=06 x 2 +2 3 xy+4 y 2 −21=0

11 x 2 +10 3 xy+ y 2 −64=021 x 2 +2 3 xy+19 y 2 −18=016 x 2 +24xy+9 y 2 −130x+90y=0

16 x 2 +24xy+9 y 2 −60x+80y=013 x 2 −6 3 xy+7 y 2 −16=04 x 2 −4xy+ y 2 −8 5 x−16 5 y=0 xy

6 x 2 −5 3 xy+ y 2 +10x−12y=06 x 2 −5xy+6 y 2 +20x−y=0θ= 45 ◦6 x 2 −8 3 xy+14 y 2 +10x−3y=0

4 x 2 +6 3 xy+10 y 2 +20x−40y=0θ= 60 ◦8 x 2 +3xy+4 y 2 +2x−4=016 x 2 +24xy+9 y 2 +20x−44y=0

θ≈ 36.9 ◦ k 4 x 2 +kxy+16 y 2 +8x+24y−48=0, k 2 x 2 +kxy+12 y 2 +10x−16y+28=0, k −4 6 <k<4 6

3 x 2 +kxy+4 y 2 −6x+20y+128=0, k k x 2 +8xy+8 y 2 −12x+16y+18=0, k k=2

6 x 2 +12xy+k y 2 +16x+10y+4=0, k cot(2θ)>0, θ (0°,45°); cot(2θ)<0, θ (45°,90°); cot(2θ)=0, θ=45°

x=2+ y 2 P(r,θ) F D e P e= PF PD P P F P D e. e, 0≤e<1, e=1, e>1, x=±p, e, θ. r θ. x=±p, p e,

r= ep 1±e cos θ

y=±p, p e,

r= ep 1±e sin θ

e e x=p y=p ep x y.r= 6 3+2 sin θr= 12 4+5 cos θr= 7 2−2 sin θ 1 c , c 1 3 .

r= 6 3+2sin θ ⋅ (1 3) (1 3) = 6(1 3) 3(1 3)+2(1 3)sin θ = 2 1+ 2 3 sin θ

sin θ y=p. e= 2 3 .

2=ep 2= 2 3 p (3 2)2=(3 2) 2 3 p 3=p

e<1, e= 2 3 y=3. 1 4 .

r= 12 4+5 cos θ ⋅ (1 4) (1 4) r= 12(1 4) 4(1 4)+5(1 4)cos θ r= 3 1+ 5 4 cos θ cos θ x=p. e= 5 4 .

3=ep 3= 5 4 p (4 5)3=(4 5) 5 4 p 12 5 =p

e>1, e= 5 4 x= 12 5 =2.4. 1 2 .

r= 7 2−2 sin θ · (1 2) (1 2) r= 7(1 2) 2(1 2)−2(1 2) sin θ r= 7 2 1−sin θ

y=−p. e=1.

7 2 =ep 7 2 =(1)p 7 2 =p

e=1, e=1 y=− 7 2 =−3.5. r= 2 3−cos θ . e= 1 3 ; x=−2 e θ r θ 0, π 2 ,π, 3π 2 r= 5 3+3 cos θ . 1 3 .

r= 5 3+3 cos θ = 5(1 3) 3(1 3)+3(1 3)cos θ r= 5 3 1+cos θ

e=1, cos θ, x=p.

5 3 =ep 5 3 =(1)p 5 3 =p

x= 5 3 .θ0π 2π3π 2r= 5 3+3 cos θ5 6 ≈0.835 3 ≈1.675 3 ≈1.67 r= 8 2−3 sin θ . 1 2 .

r= 8 2−3sin θ = 8(1 2) 2(1 2)−3(1 2)sin θ r= 4 1− 3 2 sin θ

e= 3 2 ,e>1, sin θ y=−p.

4=ep 4=(3 2)p 4(2 3)=p 8 3 =p

y=− 8 3 .θ0π 2π3π 2

r= 8 2−3sin θ

4−848 5 =1.6 r= 10 5−4 cos θ . 1 5 .

r= 10 5−4cos θ = 10(1 5) 5(1 5)−4(1 5)cos θ r= 2 1− 4 5 cos θ

e= 4 5 ,e<1, cos θ, x=−p.

2=ep 2=(4 5)p 2(5 4)=p 5 2 =p

x=− 5 2 .θ0π 2π3π 2r= 10 5−4 cos θ10210 9 ≈1.12r= 10 5−4 cos θ [−3,12,1] [−4,4,1],θ min = 0

θ max = 2π. r= 2 4−cos θ . y, x, p<0, p>0, p e=3 y=−2. y=−p, y=−2,−2<0,

r= ep 1−e sin θ

e=3 | −2 |=2=p.

r= (3)(2) 1−3 sin θ r= 6 1−3 sin θ

e= 3 5 , x=4. x=p, x=4,4>0,

r= ep 1+e cos θ

e= 3 5 | 4 |=4=p.

r= (3 5)(4) 1+ 3 5 cos θ r= 12 5 1+ 3 5 cos θ r= 12 5 1(5 5)+ 3 5 cos θ r= 12 5 5 5 + 3 5 cos θ r= 12 5 · 5 5+3 cos θ r= 12 5+3 cos θ

e=1, x=−1.r= 1 1−cosθ r= 1 5−5sin θ r= x 2 + y 2 ,x=r cos θ,and y=r sin θ.

r= 1 5−5 sin θ r·(5−5 sin θ)= 1 5−5 sin θ ·(5−5 sin θ) Eliminate the fraction. 5r −5r sin θ=1 Distribute. 5r=1+5r sin θ Isolate 5r. 25 r 2 = (1+5r sin θ) 2 Square both sides. 25(x 2 + y 2)= (1+5y) 2 Substitute r= x 2 + y 2 and y=r sin θ. 25 x 2 +25 y 2 =1+10y+25 y 2 Distribute and use FOIL. 25 x 2 −10y=1 Rearrange terms and set equal to 1.

r= 2 1+2 cos θ 4−8x+3 x 2 − y 2 =0 P(r,θ) e= PF PD , e r= x 2 + y 2 ,x=r cos θ, y=r sin θ sin θ,

r= 6 1−2 cos θr= 3 4−4 sin θ e=1 3 4 r= 8 4−3 cos θr= 5 1+2 sin θ e=2 5 2 r= 16 4+3 cos θ

r= 3 10+10 cos θ e=1 3 10 r= 2 1−cos θr= 4 7+2 cos θ e= 2 7 2 r(1−cos θ)=3r(3+5sin θ)=11 e= 5 3 11 5 r(4−5sin θ)=1r(7+8cos θ)=7 e= 8 7 7 8 r= 4 1+3 sin θr= 2 5−3 sin θ25 x 2 +16 y 2 −12y−4=0

r= 8 3−2 cos θr= 3 2+5 cos θ21 x 2 −4 y 2 −30x+9=0r= 4 2+2 sin θr= 3 8−8 cos θ64 y 2 =48x+9

r= 2 6+7 cos θr= 5 5−11 sin θ96 y 2 −25 x 2 +110y+25=0r(5+2 cos θ)=6r(2−cos θ)=1

3 x 2 +4 y 2 −2x−1=0r(2.5−2.5 sin θ)=5r= 6sec θ −2+3 sec θ5 x 2 +9 y 2 −24x−36=0r= 6csc θ 3+2 csc θ

r= 5 2+cos θr= 2 3+3 sin θr= 10 5−4 sin θr= 3 1+2 cos θr= 8 4−5 cos θr= 3 4−4 cos θr= 2 1−sin θ

r= 6 3+2 sin θr(1+cos θ)=5r(3−4sin θ)=9r(3−2sin θ)=6r(6−4cos θ)=5x=4; e= 1 5r= 4 5+cosθx=−4; e=5

y=2; e=2r= 4 1+2sinθy=−2; e= 1 2x=1; e=1r= 1 1+cosθx=−1; e=1x=− 1 4 ; e= 7 2r= 7 8−28cosθ

y= 2 5 ; e= 7 2y=4; e= 3 2r= 12 2+3sinθx=−2; e= 8 3x=−5; e= 3 4r= 15 4−3cosθy=2; e=2.5x=−3; e= 1 3

r= 3 3−3cosθ xy r θ.xy=2x 2 +xy+ y 2 =4r=± 2 1+sinθcosθ2 x 2 +4xy+2 y 2 =916 x 2 +24xy+9 y 2 =4

r=± 2 4cosθ+3sinθ2xy+y=1x 2 25 + y 2 64 =1x 2 5 2 + y 2 8 2 =1; (0,0); (5,0),(−5,0),(0,8),(0,−8);

(0, 39),(0,− 39)(x−2) 2 100 + (y+3) 2 36 =19 x 2 + y 2 +54x−4y+76=0

(x+3) 2 1 2 + (y−2) 2 3 2 =1 (−3,2); (−2,2),(−4,2),(−3,5),(−3,−1); (−3,2+2 2),(−3,2−2 2)
9 x 2 +36 y 2 −36x+72y+36=0x 2 36 + y 2 9 =1 (0,0); (6,0),(−6,0),(0,3),(0,−3); (3 3 ,0),(−3 3 ,0)
(x−4) 2 25 + (y+3) 2 49 =14 x 2 + y 2 +16x+4y−44=0 (−2,−2); (2,−2),(−6,−2),(−2,6),(−2,−10);
(−2,−2+4 3 ,),(−2,−2−4 3) 2 x 2 +3 y 2 −20x+12y+38=0 (0,0), (3,0), (−5,0)x 2 25 + y 2 16 =1
(2,−2), (7,−2), (4,−2)x 2 81 − y 2 9 =1(y+1) 2 16 − (x−4) 2 36 =1(y+1) 2 4 2 − (x−4) 2 6 2 =1;
(4,−1); (4,3),(4,−5); (4,−1+2 13),(4,−1−2 13)9 y 2 −4 x 2 +54y−16x+29=0
3 x 2 − y 2 −12x−6y−9=0(x−2) 2 2 2 − (y+3) 2 (2 3) 2 =1; (2,−3); (4,−3),(0,−3);
(6,−3),(−2,−3)x 2 9 − y 2 16 =1(y−1) 2 49 − (x+1) 2 4 =1x 2 −4 y 2 +6x+32y−91=0
2 y 2 − x 2 −12y−6=0 (0,0), (0,4), (0,−6) (3,7) (7,7), (6,7)(x−5) 2 1 − (y−7) 2 3 =1y 2 =12x
(x+2) 2 = 1 2 (y−1)(x+2) 2 = 1 2 (y−1); (−2,1); (−2, 9 8); y= 7 8y 2 −6y−6x−3=0
x 2 +10x−y+23=0(x+5) 2 =(y+2); (−5,−2); (−5,− 7 4); y=− 9 4x 2 +4y=0(y−1) 2 = 1 2 (x+3)
x 2 −8x−10y+46=02 y 2 +12y+6x+15=0 (−4,0); x=4 (2, 9 8); y= 7 8(x−2) 2 =(1 2)(y−1)
16 x 2 +24xy+9 y 2 +24x−60y−60=0B 2 −4AC=0,4 x 2 +14xy+5 y 2 +18x−6y+30=0
4 x 2 +xy+2 y 2 +8x−26y+9=0B 2 −4AC=−31<0, θ xy xy x 2 +4xy−2 y 2 −6=0x 2 −xy+ y 2 −6=0
θ= 45 ∘ , x ′ 2 +3 y ′ 2 −12=0 x ′ y ′ x ′ y ′ 9 x 2 −24xy+16 y 2 −80x−60y+100=0x 2 −xy+ y 2 −2=0
θ= 45 ∘6 x 2 +24xy− y 2 −12x+26y+11=0r= 10 1−5 cos θ e=5 2 r= 6 3+2 cos θr= 1 4+3 sin θ e= 3 4
1 3 r= 3 5−5 sin θr= 3 1−sin θr= 8 4+3 sin θr= 10 4+5 cos θr= 9 3−6 cos θ x=3 e=1r= 3 1+cos θ y=−2
e=4x 2 9 + y 2 4 =1x 2 3 2 + y 2 2 2 =1; (0,0); (3,0),(−3,0),(0,2),(0,−2); (5 ,0),(− 5 ,0)
9 y 2 +16 x 2 −36y+32x−92=0(x−3) 2 64 + (y−2) 2 36 =1 (3,2); (11,2),(−5,2),(3,8),(3,−4);
(3+2 7 ,2),(3−2 7 ,2)2 x 2 + y 2 +8x−6y−7=0 (1,2), (7,2), (4,2).(x−1) 2 36 + (y−2) 2 27 =1
x 2 49 − y 2 81 =1x 2 7 2 − y 2 9 2 =1; (0,0); (7,0),(−7,0); (130 ,0),(− 130 ,0); y=± 9 7 x
16 y 2 −9 x 2 +128y+112=0(x−3) 2 25 − (y+3) 2 1 =1 (3,−3); (8,−3),(−2,−3);
(3+ 26 ,−3),(3− 26 ,−3); y=± 1 5 (x−3)−3y 2 − x 2 +4y−4x−18=0 (1,0) (1,6), (1,2).
(y−3) 2 1 − (x−1) 2 8 =1y 2 +10x=03 x 2 −12x−y+11=0(x−2) 2 = 1 3 (y+1); (2,−1); (2,− 11 12);
y=− 13 12(x−1) 2 =−4(y+3)y 2 +8x−8y+40=0 (2,3) y=−1. 8.49 θ xy 3 x 2 −2xy+3 y 2 =4
x 2 +4xy+4 y 2 +6x−8y=0 θ≈ 63.4 ∘ x ′ y ′ x ′ y ′ 11 x 2 +10 3 xy+ y 2 =416 x 2 +24xy+9 y 2 −125x=0
x ′ 2 −4 x ′ +3 y ′ =0r= 3 2−sin θr= 5 4+6 cos θ e= 3 2 , 5 6 r= 12 4−8 sin θr= 2 4+4 sin θ e=2, x=3. P F
D e= PF PD , e r θ

{2,4,8,16,32,…}.

nthnnth 2 1 =2, 2 2 =4, 2 3 =8, nthntha,b,c...n.

a n = 2 n .

nthn

a 31 = 2 31 =2,147,483,648

2,147,483,648nthnnntha n2 n

{2,4,8,16,32,…, 2 n ,…}.

{2,4,8,16,32,…, 2 n ,…,1024}.

na

a 1 , a 2 , a 3 ,…, a n ,…

a 1a 2a 3a nnthnth a 1 ?a 0 a 1 . n nn=1a 1 .a 2 ,n=2.na n =−3n+8.n=1n.

n=1 a 1 =−3(1)+8=5 n=2 a 2 =−3(2)+8=2 n=3 a 3 =−3(3)+8=−1 n=4 a 4 =−3(4)+8=−4 n=5 a 5 = −3(5)+8=−7

{5, 2, −1, −4, −7}.na n t n =5n−4.{ 1,6, 11, 16, 21 }.n

{2,−4,6,−8}

nnn=1a 1 .(−1) n a 2 , n=2. n

a n = (−1) n n 2 n+1

n=1,n=2,

n=1 a 1 = (−1) 1 2 2 1+1 =− 1 2 n=2 a 2 = (−1) 2 2 2 2+1 = 4 3 n=3 a 3 = (−1) 3 3 2 3+1 =− 9 4 n=4 a 4 = (−1) 4 4 2 4+1 = 16 5 n=5 a 5 = (−1) 5 5 2 5+1 =− 25 6

$\{ -\frac{1}{2}, \frac{4}{3}, -\frac{9}{4}, \frac{16}{5}, -\frac{25}{6} \}$.nn,n+1,n−1,

$$a_n = \frac{4^n}{(-2)^n}$$

$\{ -2, 2, -\frac{3}{2}, 1, -\frac{5}{8} \}$. n n=1 a_1, n=1 n=2 a_2, n=2 n

$$a_n = \begin{cases} n^2 & \text{if } n \text{ is not divisible by 3} \\ \frac{n}{3} & \text{if } n \text{ is divisible by 3} \end{cases}$$

n=1,n=2, n 2nn 3n

$a_1 = 1^2 = 1$ 1 is not a multiple of 3. Use n^2. $a_2 = 2^2 = 4$ 2 is not a multiple of 3. Use n^2. $a_3 = \frac{3}{3} = 1$ 3 is a multiple of 3. Use $\frac{n}{3}$. $a_4 = 4^2 = 16$ 4 is not a multiple of 3. Use n^2. $a_5 = 5^2 = 25$ 5 is not a multiple of 3. Use n^2. $a_6 = \frac{6}{3} = 2$ 6 is a multiple of 3. Use $\frac{n}{3}$.

$\{ 1, 4, 1, 16, 25, 2 \}$.

$$a_n = \begin{cases} 2^n & \text{if } n \text{ is odd} \\ \frac{5n}{2} & \text{if } n \text{ is even} \end{cases}$$

$\{ 2, 5, 54, 10, 250, 15 \}$. nth a nn.n=1, n=2,n=3.nth$\{ -\frac{2}{11}, \frac{3}{13}, -\frac{4}{15}, \frac{5}{17}, -\frac{6}{19}, \ldots \}$

$\{ -\frac{2}{25}, -\frac{2}{125}, -\frac{2}{625}, -\frac{2}{3{,}125}, -\frac{2}{15{,}625}, \ldots \}$$\{ e^4, e^5, e^6, e^7, e^8, \ldots \}$ $(-1)^n$ n+1.2n+9.

$a_n = \frac{(-1)^n (n+1)}{2n+9}$ 5 n+1 .

$$a_n = -\frac{2}{5^{n+1}}$$

e.n=1,e 4n+3.

$$a_n = e^{n+3}$$

nth

$$\{9, -81, 729, -6{,}561, 59{,}049, \ldots\}$$

$a_n = (-1)^{n+1} 9^n$ nth

$$\{ -\frac{3}{4}, -\frac{9}{8}, -\frac{27}{12}, -\frac{81}{16}, -\frac{243}{20}, \ldots \}$$

$a_n = -\frac{3^n}{4n}$ nth

$$\{ \frac{1}{e^2}, \frac{1}{e}, 1, e, e^2, \ldots \}$$

$a_n = e^{n-3}$a n

$$a_1 = 3 \quad a_n = 2a_{n-1} - 1, \text{ for } n \ge 2$$

$$a_1 = 3 \quad a_2 = 2a_1 - 1 = 2(3) - 1 = 5 \quad a_3 = 2a_2 - 1 = 2(5) - 1 = 9 \quad a_4 = 2a_3 - 1 = 2(9) - 1 = 17$$

$\{ 3, 5, 9, 17 \}$

$$a_1 = 1 \quad a_2 = 1 \quad a_n = a_{n-1} + a_{n-2}, \text{ for } n \ge 3$$

$$a_{10} = a_9 + a_8 = 34 + 21 = 55$$

n a_1, a_2,a_{n-1}. a_3,nth

$$a_1 = 9 \quad a_n = 3a_{n-1} - 20, \text{ for } n \ge 2$$

a_{n-1}

$$n=1 \quad a_1 = 9 \quad n=2 \quad a_2 = 3a_1 - 20 = 3(9) - 20 = 27 - 20 = 7 \quad n=3 \quad a_3 = 3a_2 - 20 = 3(7) - 20 = 21 - 20 = 1 \quad n=4 \quad a_4 = 3a_3 - 20 = 3(1) - 20 = 3 - 20 = -17 \quad n=5 \quad a_5 = 3a_4 - 20 = 3(-17) - 20 = -51 - 20 = -71$$

$\{ 9, 7, 1, -17, -71 \}$.

$$a_1 = 2 \quad a_n = 2a_{n-1} + 1, \text{ for } n \ge 2$$

$\{ 2, 5, 11, 23, 47 \}$n a_1, a_2, nth

$$a_1 = 1 \quad a_2 = 2 \quad a_n = 3a_{n-1} + 4a_{n-2}, \text{ for } n \ge 3$$

$a_{n-1}$$a_{n-2}$

$$n=3 \quad a_3 = 3a_2 + 4a_1 = 3(2) + 4(1) = 10 \quad n=4 \quad a_4 = 3a_3 + 4a_2 = 3(10) + 4(2) = 38 \quad n=5 \quad a_5 = 3a_4 + 4a_3 = 3(38) + 4(10) = 154 \quad n=6 \quad a_6 = 3a_5 + 4a_4 = 3(154) + 4(38) = 614$$

$\{1, 2, 10, 38, 154, 614\}$.

$$a_1 = 0 \quad a_2 = 1 \quad a_3 = 1 \quad a_n = \frac{a_{n-1}}{a_{n-2}} + a_{n-3}, \text{ for } n \ge 4$$

$\{ 0, 1, 1, 1, 2, 3, \frac{5}{2}, \frac{17}{6} \}$.nn!,n.

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24 \quad 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$a_n = (n+1)!$.n.

$$a_6 = (6+1)! = 7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

nn(n−1)!5! 5·4!. n, n!, n

$$0! = 1 \quad 1! = 1 \quad n! = n(n-1)(n-2)\cdots(2)(1), \text{ for } n \ge 2$$

0!0!=1.$a_n = \frac{5n}{(n+2)!}$. n=1,n=2, n=1 a 1 = 5(1) (1+2)! = 5 3! = 5 3·2·1 = 5 6 n=2 a 2 = 5(2) (2+2)! = 10 4! = 10 4·3·2·1 = 5 12 n=3 a 3 = 5(3) (3+2)! = 15 5! = 15 5·4·3·2·1 = 1 8 n=4 a 4 = 5(4) (4+2)! = 20 6! = 20 6·5·4·3·2·1 = 1 36 n=5 a 5 = 5(5) (5+2)! = 25 7! = 25 7·6·5·4·3·2·1 = 5 1,008

{ 5 6 , 5 12 , 1 8 , 1 36 , 5 1,008 }.na n = (n+1)! 2n .{ 1, 3 2 , 4, 15, 72 }.

0!=1 1!=1 n!=n(n−1)(n−2)⋯(2)(1), for n≥2

nth n n a nn?13·12·11·10·9·8·7·6·5·4·3·2·1.a n = 2 n −2a n =− 16 n+1−8, − 16 3 , −4, − 16 5
a n =− (−5) n−1a n = 2 n n 32, 1 2 , 8 27 , 1 4a n = 2n+1 n 3a n =1.25· (−4) n−11.25, −5, 20, −80
a n =−4· (−6) n−1a n = n 2 2n+11 3 , 4 5 , 9 7 , 16 9a n = (−10) n +1a n =−(4· (−5) n−1 5)
− 4 5 , 4, −20, 100a n ={ (−2) n −2 if n is even (3) n−1 if n is odda n ={ n 2 2n+1 if n ≤5 n 2 −5 if n >5
1 3 , 4 5 , 9 7 , 16 9 , 25 11 , 31, 44, 59a n ={ (2n+1) 2 if n is divisible by 4 2 n if n is not divisible by 4
a n ={ −0.6· 5 n−1 if n is prime or 1 2.5· (−2) n−1 if n is composite
−0.6,−3,−15,−20,−375,−80,−9375,−320a n ={ 4(n 2 −2) if n ≤3 or n > 6 n 2 −2 4 if 3<n≤6
4, 7, 12, 19, 28,…a n = n 2 +3−4,2,−10,14,−34,…1,1, 4 3 ,2, 16 5 ,…a n = 2 n 2n or 2 n−1 n
0, 1− e 1 1+ e 2 , 1− e 2 1+ e 3 , 1− e 3 1+ e 4 , 1− e 4 1+ e 5 ,…1,− 1 2 , 1 4 ,− 1 8 , 1 16 ,…
a n = (− 1 2) n−1a 1 =9, a n = a n−1 +na 1 =3, a n =(−3) a n−13, −9, 27, −81, 243
a 1 =−4, a n = a n−1 +2n a n−1 −1a 1 =−1, a n = (−3) n−1 a n−1 −2−1, 1, −9, 27 11 , 891 5
a 1 =−30, a n =(2+ a n−1) (1 2) na 1 = 1 24 , a 2 =1, a n =(2 a n−2)(3 a n−1)
1 24 , 1, 1 4 , 3 2 , 9 4 , 81 4 , 2187 8 , 531,441 16a 1 =−1, a 2 =5, a n = a n−2 (3− a n−1)
a 1 =2, a 2 =10, a n = 2(a n−1 +2) a n−22, 10, 12, 14 5 , 4 5 , 2, 10, 12−2.5,−5,−10,−20,−40,…
−8,−6,−3,1,6,…a 1 =−8, a n = a n−1 +n2, 4, 12, 48, 240, …35, 38, 41, 44, 47, …a 1 =35, a n = a n−1 +3
15,3, 3 5 , 3 25 , 3 125 ,⋯6!720(12 6)!12! 6!665,280100! 99!a n = n! n 21, 1 2 , 2 3 , 3 2a n = 3·n! 4·n!
a n = n! n 2 −n−1−1,2, 6 5 , 24 11a n = 100·n n(n−1)!a n = (−1) n n +n
a n ={ 4+n 2n if n is even 3+n if n is odda 1 =2, a n = (− a n−1 +1) 2a n =1, a n = a n−1 +8
a n = (n+1)! (n−1)!a n = 2 n−2a 1 =6, a n =2 a n−1 −5 a 1 a n−1 .
a 1 = 87 111 , a n = 4 3 a n−1 + 12 37 .29 37 , 152 111 , 716 333 , 3188 999 , 13724 2997
a 1 =625, a n =0.8 a n−1 +18. a 1 =2, a n = 2 [(a n −1)−1] +1.2,3,5,17,65537
a 1 =8, a n = (a n−1 +1)! a n−1 ! .a 1 =2, a n =n a n−1a 10 =7,257,600 [X,T,θ,n] n n n
a n =− 28 9 n+ 5 3 . a n = n 3 −3.5 n 2 + 4.1n−1.5 2.4n .0.042,0.146,0.875,2.385,4.708
a n = 15n· (−2) n−1 47a n = 5.7 n +0.275(n−1)!5.975,32.765,185.743,1057.25,6023.521a n = n! n .
a n =−6−8n.a n =−421 a n =−421 −421=−6−8nn −421=−6−8n, n=51.875a n =−421
a n = n 2 +4n+4 2(n+2)41?1, 0, −1, −1, 0, 1, 1, 0, −1, −1, 0, 1, 1, ... a n
a 1 =1, a 2 =0, a n = a n−1 − a n−2a n = (n+2)! (n−1)!b n = n 3 +3 n 2 +2n,
(n+2)! (n−1)! = (n+2)·(n+1)·(n)·(n−1)·...·3·2·1 (n−1)·...·3·2·1 =n(n+1)(n+2)= n 3 +3 n 2 +2n {1,2,…n} n n
a 1d

{ a n }={ a 1 , a 1 +d, a 1 +2d, a 1 +3d,...}

{1,2,4,8,16,...}{−3,1,5,9,13,...}ab

{18, 16, 14, 12, 10,…}

−2.

{1, 3, 6, 10, 15,…}

3−1≠6−3.nd

a n = a 1 +(n−1)d

a 1 =17d=−3 −3 {17, 14, 11, 8, 5}a 1 =1d=5{ 1 , 6 , 11 , 16 , 21 }a 1 , a n ,n a n = a 1 +(n−1)d d.
a 1 ,n, d a n = a 1 +(n−1)d.a 1 =8a 4 =14a 5d

{ 8,8+d,8+2d,8+3d }

a 1 +3d=8+3dd

a n = a 1 +(n−1)d a 4 = a 1 +3d a 4 =8+3d Write the fourth term of the sequence in terms of a 1 and d.
14=8+3d Substitute 14 for a 4 . d=2 Solve for the common difference.

a 5 = a 4 +2=16

a n = a 1 +(n−1)d.a 3 =7a 5 =17a 2a 2 = 2

$$a_n = a_{n-1} + d \quad n \geq 2$$

d

$$a_n = a_{n-1} + d \quad n \geq 2$$

$$\{-18, -7, 4, 15, 26, \ldots\}$$

−18

$$d = -7-(-18) = 11$$

$$a_1 = -18 \quad a_n = a_{n-1} + 11, \text{ for } n \geq 2$$

$$\{25, 37, 49, 61, \ldots\}$$

$a_1 = 25 \quad a_n = a_{n-1} + 12, \text{ for } n \geq 2$

$$a_n = a_1 + d(n-1)$$

−50−50y−50200: 200−(−50)=200+50=250y y=mx+b. a nynx. mb −50 250

$$a_n = -50n + 250$$

$a_n = 200 - 50(n-1)$ $a_n = -50n + 250$.nth

$$a_n = a_1 + d(n-1)$$

$a_2 - a_1$.$a_n = a_1 + d(n-1)$.

$$\{2, 12, 22, 32, 42, \ldots\}$$

$$d = a_2 - a_1 = 12-2 = 10$$

$$a_n = 2+10(n-1) \quad a_n = 10n-8$$

−8

$$\{50,47,44,41,\ldots\}$$

$a_n = 53 - 3n$ d.$a_n = a_1 + d(n-1)$.a nn.

$$\{8, 1, -6, \ldots, -41\}$$

$$1-8=-7$$

−7nth

$$a_n = a_1 + d(n-1) \quad a_n = 8 + -7(n-1) \quad a_n = 15 - 7n$$

−41a nn

$$-41 = 15-7n \qquad 8=n$$

$$\{6, 11, 16, \ldots, 56\}$$

a 0a 1 .

$$a_n = a_0 + dn$$

An

$$A_n = 1 + 2n$$

$$16 - 5 = 11$$

$$A_{11} = 1 + 2(11) = 23$$

$T_n = 10+4n$, $a_n = a_{n-1} + d$ $n\geq 2$ $a_n = a_1 + d(n-1)$ d $a_n = a_{n-1} + d, n \geq 2$. d $a_n = a_1 + d(n-1)$. $a_n = a_0 + dn$.
$\{5, 11, 17, 23, 29, \ldots\}$ $\{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots\}$ $\frac{1}{2}$ $\{11.4, 9.3, 7.2, 5.1, 3, \ldots\}$
$\{4, 16, 64, 256, 1024, \ldots\}$ $16-4 \neq 64-16$. $a_1 = -25$ $d = -9$ $a_1 = 0$ $d = \frac{2}{3}$

$$0, \frac{2}{3}, \frac{4}{3}, 2, \frac{8}{3}$$

$a_1 = 17$, $a_7 = -31$ $a_{13} = -60$, $a_{33} = -160$ $0, -5, -10, -15, -20$ $a_4 = 19$ $a_6 = 41$ a_1 $a_6 = 12$ $a_{14} = 28$.
$a_1 = 2$ a_1 $a_7 = 21$ $a_{15} = 42$. a_1 $a_8 = 40$ $a_{23} = 115$. $a_1 = 5$ a_1 $a_9 = 54$ $a_{17} = 102$. a_1 $a_{11} = 11$ $a_{21} = 16$. $a_1 = 6$
$a_1 = 33$ $a_7 = -15$. a_4 . $a_3 = -17.1$ $a_{10} = -15.7$. a_{21} . $a_{21} = -13.5$ $a_1 = 39$; $a_n = a_{n-1} - 3$
$a_1 = -19$; $a_n = a_{n-1} - 1.4$ $-19, -20.4, -21.8, -23.2, -24.6$ $a_n = \{40,60,80,\ldots\}$ $a_n = \{17,26,35,\ldots\}$
$a_1 = 17$; $a_n = a_{n-1} + 9$ $n\geq 2$ $a_n = \{-1,2,5,\ldots\}$ $a_n = \{12,17,22,\ldots\}$ $a_1 = 12$; $a_n = a_{n-1} + 5$ $n\geq 2$
$a_n = \{-15,-7,1,\ldots\}$ $a_n = \{8.9,10.3,11.7,\ldots\}$ $a_1 = 8.9$; $a_n = a_{n-1} + 1.4$ $n\geq 2$ $a_n = \{-0.52,-1.02,-1.52,\ldots\}$
$a_n = \{\frac{1}{5}, \frac{9}{20}, \frac{7}{10}, \ldots\}$ $a_1 = \frac{1}{5}$; $a_n = a_{n-1} + \frac{1}{4}$ $n\geq 2$ $a_n = \{-\frac{1}{2}, -\frac{5}{4}, -2, \ldots\}$
$a_n = \{\frac{1}{6}, -\frac{11}{12}, -2, \ldots\}$ $1 = \frac{1}{6}$; $a_n = a_{n-1} - \frac{13}{12}$ $n\geq 2$ $a_n = \{7, 4, 1, \ldots\}$; $a_n = \{4, 11, 18, \ldots\}$;
$a_1 = 4$; $a_n = a_{n-1} + 7$; $a_{14} = 95$ $a_n = \{2, 6, 10, \ldots\}$; $a_n = 24-4n$ $20,16,12,8,4$. $a_n = \frac{1}{2}n - \frac{1}{2}$
$a_n = \{3,5,7,\ldots\}$ $a_n = 1+2n$ $a_n = \{32,24,16,\ldots\}$ $a_n = \{-5, 95, 195, \ldots\}$ $a_n = -105+100n$
$a_n = \{-17, -217, -417,\ldots\}$ $a_n = \{1.8, 3.6, 5.4, \ldots\}$ $a_n = 1.8n$ $a_n = \{-18.1,-16.2,-14.3,\ldots\}$
$a_n = \{15.8,18.5,21.2,\ldots\}$ $a_n = 13.1+2.7n$ $a_n = \{\frac{1}{3}, -\frac{4}{3}, -3, \ldots\}$ $a_n = \{0, \frac{1}{3}, \frac{2}{3}, \ldots\}$ $a_n = \frac{1}{3}n - \frac{1}{3}$

a n ={ −5,− 10 3 ,− 5 3 ,… }a n ={3,−4,−11, ...,−60}a n ={1.2,1.4,1.6,...,3.8}a n ={ 1 2 ,2, 7 2 ,...,8 }
a 1 =0,d=4a 1 =9; a n = a n−1 −10a n =−12+5na n =3n−2nMin nMin=1u(n) u(n)=3n−2u(nMin)
u(nMin)=1 TblStart=1 ΔTbl=1u(n)?1,4,7,10,13,16,19n=50.u(n)?
nMin=1,nMax=5,xMin=0,xMax=6,yMin=−1, yMax=14. a n = 1 2 n+5 u(n) 9.206.a n =20.6n
a n =2+20.4n.{9b,5b,b,…}.{3a−2b,a+2b,−a+6b…}.a 11 =−17a+38b{5.4,14.5,23.6,...}
{ 17 3 , 31 6 , 14 3 ,... }a 13 =− 1 3{ 5 2 , 19 8 , 9 4 ,..., 1 8 }a 1 =3, a n = a n−1 −3.3,0,−3,−6 a 31 =−87
a 1r

{ a 1 , a 1 r, a 1 r 2 , a 1 r 3 , ...}.

1, 2, 4, 8, 16, ...48, 12, 4, 2, ...2 1 =2 4 2 =2 8 4 =2 16 8 =212 48 = 1 4 4 12 = 1 3 2 4 = 1 2

5,10,15,20,...

10 5 ≠ 15 10

100,20,4, 4 5 ,...

1 5a 1 =−2r=4,−2·4−8−8·4−32

a 1 =−2 a 2 =(−2·4)=−8 a 3 =(−8·4)=−32 a 4 =(−32·4)−128

{−2, −8, −32, −128}.a 1 ,a 2 .a n = a 2a 3a 3a 4,a 1 =5r=−2.a 1−2a 2 .a 2a 3 ,

a 1 =5 a 2 =−2 a 1 =−10 a 3 =−2 a 2 =20 a 4 =−2 a 3 =−40

{ 5,−10,20,−40 }.a 1 =18r= 1 3 .

{ 18,6,2, 2 3 , 2 9 }

ra 1

a n =r a n−1 ,n≥2

{6, 9, 13.5, 20.25, ...}

r= 9 6 =1.5

a 1 .

a n =r a n−1 a n =1.5 a n−1 for n≥2 a 1 =6

{2, 4 3 , 8 9 , 16 27 , ...}

a 1 =2 a n = 2 3 a n−1 for n≥2

a n = a 1 r n−1

{18, 36, 72, 144, 288, ...}.

a n =18· 2 n−1

a n = a 1 r n−1

a 1 =3 a 4 =24, a 2 . r.

3,3r,3 r 2 ,3 r 3 ,...

a n = a 1 r n−1 a 4 =3 r 3 Write the fourth term of sequence in terms of α 1 and r 24=3 r 3
Substitute 24 for a 4 8= r 3 Divide r=2 Solve for the common ratio

a 2 =2 a 1 =2(3) =6

a 2 =4a 3 =32a 6 .a 6 =16,384nth

{2, 10, 50, 250, ...}

10 2 =5

a n = a 1 r (n−1) a n =2· 5 n−1

{−1, 3, −9, 27, ...}

a n =− (−3) n−1a 0a 1 .

a n = a 0 r n

Pn

P n =284· 1.04 n

2020−2013=7

n

P 7 =284· 1.04 7 ≈374

P n = 293·1.026 a nntha n =r a n−1 ,n≥ 2 nth

a n = a 1 r n−1

r a n =r a n–1 n≥2r a n = a 1 r n–1 . a n = a 0 r n . 1,3,9,27,81,...−0.125,0.25,−0.5,1,−2,...−2

−2,− 1 2 ,− 1 8 ,− 1 32 ,− 1 128 ,...−6,−12,−24,−48,−96,...5,5.2,5.4,5.6,5.8,...−1, 1 2 ,− 1 4 , 1 8 ,− 1 16 ,...
− 1 2 .6,8,11,15,20,...0.8,4,20,100,500,...5.a 1 =8, r=0.3a 1 =5, r= 1 55,1, 1 5 , 1 25 , 1 125
a 7 =64, a 10 =512a 6 =25, a 8 =6.25800,400,200,100,502,3.− 1 3 .a 4 =− 16 27a n ={ −1,2,−4,8,... }.
a 12 .a n ={ −2, 2 3 ,− 2 9 , 2 27 ,... }.a 7 .a 7 =− 2 729a 1 =−486, a n =− 1 3 a n−1a 1 =7, a n =0.2 a n−1
7,1.4,0.28,0.056,0.0112a n ={ −1,5,−25,125,... }a n ={ −32,−16,−8,−4,... }a = 1 −32, a n = 1 2 a n−1
a n ={ 14,56,224,896,... }a n ={ 10,−3,0.9,−0.27,... }a 1 =10, a n =−0.3 a n−1
a n ={ 0.61,1.83,5.49,16.47,... }a n ={ 3 5 , 1 10 , 1 60 , 1 360 ,... }a 1 = 3 5 , a n = 1 6 a n−1
a n ={ −2, 4 3 ,− 8 9 , 16 27 ,... }a n ={ 1 512 ,− 1 128 , 1 32 ,− 1 8 ,... }a 1 = 1 512 , a n =−4 a n−1
a n =−4· 5 n−1a n =12· (− 1 2) n−112,−6,3,− 3 2 , 3 4a n ={ −2,−4,−8,−16,... }a n ={ 1,3,9,27,... }
a n = 3 n−1a n ={ −4,−12,−36,−108,... }a n ={ 0.8,−4,20,−100,... }a n =0.8· (−5) n−1
a n ={−1.25,−5,−20,−80,...}a n ={ −1,− 4 5 ,− 16 25 ,− 64 125 ,... }a n =− (4 5) n−1
a n ={ 2, 1 3 , 1 18 , 1 108 ,... }a n ={ 3,−1, 1 3 ,− 1 9 ,... }a n =3· (− 1 3) n−1a 1 =4,a n =−3 a n−1 .a 8 .
a n =− (− 1 3) n−1 .a 12 .a 12 = 1 177,147a n ={ −1,3,−9,...,2187 }a n ={ 2,1, 1 2 ,..., 1 1024 }12
a 1 =1, r= 1 2a 1 =3, a n =2 a n−1a n =27· 0.3 n−1 200.a 1 =800, a n =0.5a n−1a 1 =12.5, a n =4a n−1
1024.{b,4b,16b,...}.a 5 =256b{64a(−b),32a(−3b),16a(−9b),...}.{10,12,14.4,17.28, ...}100?100a 14 ≈107.
{ 1 2187 , 1 729 , 1 243 , 1 81 ... }a n =−36 (2 3) n−1a 4 =− 32 3 a n =400· 0.5 n−1 ;
400,200,100,50; a 8 =3.125

3+7+11+15+19+...

nth S n

S 1 =3 S 2 =3+7=10 S 3 =3+7+11=21 S 4 =3+7+11+15=36

Σ, a k =2kk=1k=5. k

a 1 =2(1)=2 a 2 =2(2)=4 a 3 =2(3)=6 a 4 =2(4)=8 a 5 =2(5)=10
∑ k=1 5 2k =2+4+6+8+10=30

n

∑ k=1 n a k

a kk=1k=n.k nk∑ k=3 7 k 2 .k 2k=3k=7.k=3,4,5,6, 7k 2 .

∑ k=3 7 k 2 = 3 2 + 4 2 + 5 2 + 6 2 + 7 2 =9+16+25+36+49 =135

∑ k=2 5 (3k–1) .d.n

S n = a 1 +(a 1 +d)+(a 1 +2d)+...+(a n –d)+ a n .
S n = a n +(a n –d)+(a n –2d)+...+(a 1 +d)+ a 1 .

nn

S n = a 1 +(a 1 +d)+(a 1 +2d)+...+(a n –d)+ a n + S n = a n +(a n –d)+(a n –2d)+...+(a 1 +d)+ a 1
2 S n =(a 1 + a n)+(a 1 + a n)+...+(a 1 + a n)

n

2 S n =n(a 1 + a n).

n

S n = n(a 1 + a n) 2

n

S n = n(a 1 + a n) 2

na 1a n .n.a 1 , a n , n S n = n(a 1 + a n) 2 .S n .5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32
20 + 15 + 10 +…+ −50∑ k=1 12 3k−8a 1 =5 a n =32.n=10. a 1 , a n ,n

S n = n(a 1 + a n) 2 S 10 = 10(5+32) 2 =185

a 1 =20a n =−50.n.

a n = a 1 +(n−1)d −50=20+(n−1)(−5) −70=(n−1)(−5) 14=n−1 15=n

a 1 , a n , n

S n = n(a 1 + a n) 2 S 15 = 15(20−50) 2 =−225

a 1 , k=1

a k =3k−8 a 1 =3(1)−8=−5

n=12.a 12 , k=12

a k =3k−8 a 12 =3(12)−8=28

a 1 , a n ,n

S n = n(a 1 + a n) 2 S 12 = 12(−5+28) 2 =138

1.4 + 1.6 + 1.8 + 2.0 + 2.2 + 2.4 + 2.6 + 2.8 + 3.0 + 3.2 + 3.426.413 + 21 + 29 + …+ 69328

∑ k=1 10 5 −6k−280 a 1 = 1 2 d= 1 4 . n=8, S 8 . a 8 ,

a n = a 1 +d(n−1) a 8 = 1 2 + 1 4 (8−1)= 9 4

S n = n(a 1 + a n) 2 S 8 = 8(1 2 + 9 4) 2 =11

r. n

S n = a 1 +r a 1 + r 2 a 1 +...+ r n–1 a 1 .

n r.

r S n =r a 1 + r 2 a 1 + r 3 a 1 +...+ r n a 1

S n = a 1 +r a 1 + r 2 a 1 +...+ r n–1 a 1 −r S n =−(r a 1 + r 2 a 1 + r 3 a 1 +...+ r n a 1) (1−r) S n = a 1 − r n a 1

S n ,(1−r).

S n = a 1 (1− r n) 1−r r≠1

n

S n = a 1 (1− r n) 1−r r≠1

a 1 , r, and n. a 1 , r,nS n = a 1 (1− r n) 1−r .S n .S 11 8 + -4 + 2 + …∑ 6 k=1 3⋅ 2 ka 1 =8,n=11.r

r= −4 8 =− 1 2

a 1 , r, and n

S n = a 1 (1− r n) 1−r S 11 = 8(1− (− 1 2) 11) 1−(− 1 2) ≈5.336

a 1k=1

a 1 =3⋅ 2 1 =6

r=2.n=6.a 1 , r,n

S n = a 1 (1− r n) 1−r S 6 = 6(1− 2 6) 1−2 =378

S 20 1,000 + 500 + 250 + …≈2,000.00∑ k=1 8 3 ka 1 =26,750; n=5; r=1.016. a 1 , r,n

S n = a 1 (1− r n) 1−r S 5 = 26,750(1− 1.016 5) 1−1.016 ≈138,099.03

n2+4+6+8+...∑ k=1 ∞ 2k,

1+0.2+0.04+0.008+0.0016+...

r = 0.2. nr n−1<r<1−1<r<1.−1<r<1.r,−1<r<112 + 8 + 4 + …3 4 + 1 2 + 1 3 +...∑ k=1 ∞ 27⋅ (1 3) k

∑ k=1 ∞ 5k2 3 ,1 2 .2 3 .1 3 ;1 3 .1 3 + 1 2 + 3 4 + 9 8 +...24+(−12)+6+(−3)+...∑ k=1 ∞ 15⋅ (–0.3) kn

S n = a 1 (1− r n) 1−r

r= 1 2 .r nn

(1 2) 2 = 1 4 (1 2) 3 = 1 8 (1 2) 4 = 1 16

r n n?

(1 2) 10 = 1 1,024 (1 2) 20 = 1 1,048,576 (1 2) 30 = 1 1,073,741,824

nr nnr nr n1− r n a 1 .−1<r<1

S= a 1 1−r

a 1r.−1<r<1.a 1rS= a 1 1−r . S.10+9+8+7+…248.6+99.44+39.776+ …∑ k=1 ∞ 4,374⋅ (– 1 3) k–1

∑ k=1 ∞ 1 9 ⋅ (4 3) ka 1 =248.6r= 99.44 248.6 =0.4,a 1 =248.6r=0.4

S= a 1 1−r S= 248.6 1−0.4 =414. 3 ¯

r=– 1 3 .a 1k=1

a 1 =4,374⋅ (– 1 3) 1–1 =4,374

a 1 =4,374r=– 1 3

S= a 1 1−r S= 4,374 1−(– 1 3) =3,280.5

r>1. 0.3¯0.3¯=0.333…

0.3¯=0.3+0.03+0.003+...

S n = a 1 1−r = 0.3 1−0.1 = 0.3 0.9 = 1 3 .

2+ 2 3 + 2 9 +...∑ k=1 ∞ 0.76k+1∑ k=1 ∞ (− 3 8) k− 3 11a 1 =50r=100.5%=1.005.nnn=72.

a 1 =50, r=1.005, and n=72

S 72 = 50(1− 1.005 72) 1−1.005 ≈4,320.44

72(50) = \$3,600. a 1 , n, r.r. a 1 , r, and n nS n = a 1 (1− r n) 1−r .S n ,n a 1 =100. n=120.r,

r=1+ 0.09 12 =1.0075

a 1 =100, r=1.0075, and n=120 n

S 120 = 100(1− 1.0075 120) 1−1.0075 ≈19,351.43

n S n = n(a 1 + a n) 2 n

S n = a 1 (1− r n) 1−r ·r≠1

−1<r< 1

S n = a 1 1−r ·r≠1

nn−1<r<1.nthnthnnthm 2 +3mm=1m=5n=0n=45n∑ n=0 4 5n6k−5k=−2k=1∑ k=1 5 4
5+10+15+20+25+30+35+40+45+5010+18+26+…+162∑ k=1 20 8k+21 2 +1+ 3 2 +2+…+4n
3 2 +2+ 5 2 +3+ 7 2S 5 = 5(3 2 + 7 2) 219+25+31+…+733.2+3.4+3.6+…+5.6S 13 = 13(3.2+5.6) 2
1+3+9+27+81+243+729+21878+4+2+…+0.125∑ k=1 7 8· 0.5 k−1− 1 6 + 1 12 − 1 24 +…+ 1 768n
9+3+1+ 1 3 + 1 9S 5 = 9(1− (1 3) 5) 1− 1 3 = 121 9 ≈13.44∑ n=1 9 5· 2 n−1∑ a=1 11 64· 0.2 a−1
S 11 = 64(1− 0.2 11) 1−0.2 = 781,249,984 9,765,625 ≈8012+18+24+30+...2+1.6+1.28+1.024+...
S= 2 1−0.8∑ m=1 ∞ 4 m−1∑ ∞ k=1 − (− 1 2) k−1S= −1 1−(− 1 2)∑ k=1 ∞ (1 2) k .S nS n
∑ k=1 ∞ (1 2) kS= 1 2 1−(1 2) =1.∑ a=1 14 a∑ n=1 6 n(n−2)∑ k=1 17 k 2∑ k=1 7 2 kn
−1.7+−0.4+0.9+2.2+3.5+4.86+ 15 2 +9+ 21 2 +12+ 27 2 +15S 7 = 147 2−1+3+7+...+31
∑ k=1 11 (k 2 − 1 2)S 11 = 55 2nS 6−2−10−50−250...S 70.4−2+10−50...S 7 =5208.4∑ k=1 9 2 k−1
∑ n=1 10 −2· (1 2) n−1S 10 =− 1023 2564+2+1+ 1 2 ...−1− 1 4 − 1 16 − 1 64 ...S=− 4 3
∑ ∞ k=1 3· (1 4) k−1∑ n=1 ∞ 4.6· 0.5 n−1S=9.2\$50;60;5%,\$150;24;3%,\$450;60;4.5%,\$100;120;10%,
50− k 2k=x7115.a k∑ k=0 6 a k =189.a k =30−k∑ k=1 n (3k−5)>100.−1−3−5−7.... −75?0. 65 ¯0. 65 ¯
r= 4 5\$10,000 1 30 3 4nmnm+n

\# of appetizer options × # of entree options × # of dessert options 2 ×

3 × 2 =12

mnm×n n n n!. n r P(n,r). n P r , P(n,r), n!, n (n−r)! (n−r) 6×5×4=120.

6! 3! = 6·5·4·3! 3! =6·5·4=120

P(n,r)= n! (n−r)!

n (n−n)! 0!, n n n! 1 n!. n r

P(n,r)= n! (n−r)!

n r n r n=12 r=9

P(n,r)= n! (n−r)! P(12,9)= 12! (12−9)! = 12! 3! =79,833,600

n P r n P r 15·14·13·12·11·10·9·8·7·6·5·4 P(7,7)=5,040 P(7,5)=2,520 r n C(n,r). C(n,r) n C r .

C(n,r)= n! r!(n−r)!

3!=3·2·1=624 6 , n r

C(n,r)= n! r!(n−r)!

n r n r

C(5,2)= 5! 2!(5−2)! =10

C(5,3)= 5! 3!(5−3)! =10

n C r , n C r , (n−r) C(n,r)=C(n,n−r). C(10,3)=120 rC(5,0)=1C(5,1)=5

C(5,0)+C(5,1)+C(5,2)+C(5,3)+C(5,4)+C(5,5)=32

2 5 . nn 2·2·2·…·22 nn=4

2 n = 2 4 =16

12!12!

n! r 1 ! r 2 !… r k !

4!3!

12! 4!3! =3,326,400

nr 1 r 2 r 3 r k ,

n! r 1 ! r 2 !… r k !

n=8, r 1 =2, r 2 =2

8! 2!2! =10,080

n r

P(n,r)= n! (n−r)!

n r

C(n,r)= n! r!(n−r)!

n

n! r 1 ! r 2 !… r k !

mnm+nmnm×nnnrP(n,r).P(n,r).nrC(n,r)n2 nnAmBA and BA or B m+n A B A and B nrnrnr
C(n,r)= n! (n−r)!r! A={−5,−3,−1,2,3,4,5,6}.A?B={−23,−16,−7,−2,20,36,48,72}.A? 4+2=6 5+4+7=16
2×6=12 A B— A={b,c,d} B={a,e,i,o,u}? 10 3 =1000 P(5,2) P(5,2)=20 P(8,4) P(3,3) P(3,3)=6 P(9,6)
P(11,5) P(11,5)=55,440 C(8,5) C(12,4) C(12,4)=495 C(26,3) C(7,6) C(7,6)=7 C(10,3)
{1,2,3,4,5,6,7,8,9,10} 2 10 =1024 {a,b,c,…,z} 2 12 =4096 2 9 =512 8! 3! =6720 12! 3!2!3!4! S
900,000,000 S? n n? C(n,r)P(n,r)?r=0r=1.r=0,C(n,r)=P(n,r)=1. r=1,r=1,C(n,r)=P(n,r)=n.AA?
6! 2! ×4!=8640 6−3+8−3=8 4×2×5=40 4×12×3=144 8 11 P(15,9)=1,816,214,400
C(10,3)×C(6,5)×C(5,2)=7,200 2 11 =2048 20! 6!6!8! =116,396,280 mnm+nmnm×nmnm×n (x+y) n n
(x+y) n , (n r) C(n,r),

(n r)=C(n,r)= n! r!(n−r)!

(n r) (5 2)=C(5,2)=10. nrn≥r,

(n r)=C(n,r)= n! r!(n−r)!

(5 3)(9 2)(9 7)n C r

(n r)=C(n,r)= n! r!(n−r)!

(5 3)= 5! 3!(5−3)! = 5·4·3! 3!2! =10(9 2)= 9! 2!(9−2)! = 9·8·7! 2!7! =36
(9 7)= 9! 7!(9−7)! = 9·8·7! 7!2! =36

(n r)=(n n−r)

(7 3) (11 4) (x+y) n(x+y) 52 ,(x+y)
(x+y) 2 = x 2 +2xy+ y 2 (x+y) 3 = x 3 +3 x 2 y+3x y 2 + y 3 (x+y) 4 = x 4 +4 x 3 y+6 x 2 y 2 +4x y 3 + y 4

xyn

(n 0),(n 1),(n 2),...,(n n).

(x+y) n = ∑ k=0 n (n k) x n−k y k = x n +(n 1) x n−1 y+(n 2) x n−2 y 2 +...+(n n−1)x y n−1 + y n
x+y,
(x+y) 1 =x+y (x+y) 2 = x 2 +2xy+ y 2 (x+y) 3 = x 3 +3 x 2 y+3x y 2 + y 3 (x+y) 4 = x 4 +4 x 3 y+6 x 2 y 2 +4x y 3 + y 4
(x+y) 5 ? n+1(x+y) n .n.xnyn.(x+y) 5 ,n=5,x,x 5 ,xx 0 =1y 0 =1,yy 5

x 5 , x 4 y, x 3 y 2 , x 2 y 3 , x y 4 , y 5

(x+y) 5 = x 5 +5 x 4 y+10 x 3 y 2 +10 x 2 y 3 +5x y 4 + y 5 .

1+1nth

(x+y) n = ∑ k=0 n (n k) x n−k y k = x n +(n 1) x n−1 y+(n 2) x n−2 y 2 +...+(n n−1)x y n−1 + y n
nk=0k=n (x+y) 5 (3x−y) 4 n=5k=0k=5

(x+y) 5 =(5 0) x 5 y 0 +(5 1) x 4 y 1 +(5 2) x 3 y 2 +(5 3) x 2 y 3 +(5 4) x 1 y 4 +(5 5) x 0 y 5
(x+y) 5 = x 5 +5 x 4 y+10 x 3 y 2 +10 x 2 y 3 +5x y 4 + y 5

n=4k=0k=43xx−yy.

(3x−y) 4 =(4 0) (3x) 4 (−y) 0 +(4 1) (3x) 3 (−y) 1 +(4 2) (3x) 2 (−y) 2 +(4 3) (3x) 1 (−y) 3 +(4 4) (3x) 0 (−y) 4 (3x−y) 4 =81 x 4 −108 x 3 y+54 x 2 y 2 −12x y 3 + y 4

(−y) (−y) (x−y) 5(2x+5y) 3x 5 −5 x 4 y+10 x 3 y 2 −10 x 2 y 3 +5x y 4 − y 5
8 x 3 +60 x 2 y+150x y 2 +125 y 3 (x+2y) 16 (x+y) 5 .

(x+y) 5 = x 5 +(5 1) x 4 y+(5 2) x 3 y 2 +(5 3) x 2 y 3 +(5 4)x y 4 + y 5

(5 1) x 4 y. (5 2) x 3 y 2 .

(n r) x n−r y r

(r+1)th (x+y) n

(n r) x n−r y r

n(r+1).r.r(r+1)th (x+2y) 16 r+1=10, r=9

(n r) x n−r y r

(16 9) x 16−9 (2y) 9 =5,857,280 x 7 y 9

(3x−y) 9 −10,206 x 4 y 5(x+y) n = ∑ k−0 n (n k) x n−k y k(r+1)th

(n r) x n−r y r

(n r) C(n,r). C(n,r). (n r)= C(n,r) = n! r!(n−r)! . (x+y) n = ∑ k=0 n (n k) x n−k y k (6 2)(5 3)(7 4) (9 7)(10 9)(25 11)(17 6)(200 199)(4a−b) 364 a 3 −48 a 2 b+12a b 2 − b 3(5a+2) 3(3a+2b) 3 27 a 3 +54 a 2 b+36a b 2 +8 b 3(2x+3y) 4(4x+2y) 5 1024 x 5 +2560 x 4 y+2560 x 3 y 2 +1280 x 2 y 3 +320x y 4 +32 y 5(3x−2y) 4(4x−3y) 5 1024 x 5 −3840 x 4 y+5760 x 3 y 2 −4320 x 2 y 3 +1620x y 4 −243 y 5(1 x +3y) 5(x −1 +2 y −1) 4 1 x 4 + 8 x 3 y + 24 x 2 y 2 + 32 x y 3 + 16 y 4(x − y) 5(a+b) 17a 17 +17 a 16 b+136 a 15 b 2(x−1) 18 (a−2b) 15a 15 −30 a 14 b+420 a 13 b 2(x−2y) 8(3a+b) 20 3,486,784,401 a 20 +23,245,229,340 a 19 b+73,609,892,910 a 18 b 2(2a+4b) 7(x 3 − y) 8 x 24 −8 x 21 y +28 x 18 y (2x−3y) 4 (3x−2y) 5−720 x 2 y 3 (6x−3y) 7 (7+5y) 14 220,812,466,875,000 y 7 (a+b) 11 (x−y) 735 x 3 y 4 (x−1) 12 (a−3 b 2) 111,082,565 a 3 b 16 (x 3 − 1 2) 10 (y 2 + 2 x) 91152 y 2 x 7f(x)= (x+3) 4 . f 1 (x), f 1 (x) f 2 (x), f 2 (x) f 2 (x)= x 4 +12 x 3 f 3 (x), f 3 (x) f 4 (x), f 4 (x) f 4 (x)= x 4 +12 x 3 +54 x 2 +108x f 5 (x), f 5 (x) (5x+3y) n , (n k) a n–k b k , where k 0,1,2, ..., n. (n k)=(7 2), 590,625 x 5 y 2 (a+b) n , a n−k b k (x+b) 40 . bkthk−1 (n k−1)+(n k) (n k). p, p≥1, p!=p(p−1)!.(n k−1)+(n k)=(n+1 k);

(n k−1)+(n k) = n! k!(n−k)! + n! (k−1)!(n−(k−1))! = n! k!(n−k)! + n! (k−1)!(n−k+1)! = (n −k+1)n! (n−k+1)k!(n−k)! + kn! k(k−1)!(n−k+1)! = (n−k+1)n!+kn! k!(n−k+1)! = (n+1)n! k!((n+1)−k)! = (n+1)! k!((n+1)−k)! =(n+1 k)

(x 2 −2x+1)(a +4 a −5) 8(x 3 +2 y 2 −z) 5(3 x 2 − 2 y 3) 12 (x 3 +2 y 2 −z) 5 C(n,r), (n r) (x+y) n {1,2,3,4,5,6 }. p0≤p≤1, 1 6 .1 61 61 61 61 61 6 S S. S. S, 4 6 = 2 3 . ES

P(E)= number of elements in E number of elements in S = n(E) n(S)

ES,0≤P(E)≤1.

P(E)= 3 6 = 1 2

2 3 E and F,written E∪F,

P(E∪F)=P(E)+P(F)−P(E∩F)

b. 3 6 = 1 2 . b. b2 6 = 1 3 .bb,b.

1 2 + 1 3 − 1 6 = 2 3

b 2 3 .EF E∪FEFEF(EFE∩F

P(E∪F)=P(E)+P(F)−P(E∩F)

1 4 . 1 13 . 1 52 . P(H)= 1 4 , P(7)= 1 13 , and P(H∩7)= 1 52

P(E ∪ F)=P(E)+P(F)−P(E ∩ F) = 1 4 + 1 13 − 1 52 = 4 13

4 13 . 7 13 d. d,

P(E∪F)=P(E)+P(F)

E F 3 6 = 1 2 d 1 6 .d

P(E ∪ F)=P(E)+P(F) = 1 2 + 1 6 = 2 3

d 2 3 . E and F

P(E∪F)=P(E)+P(F)

1 4 , 1 4 ,

1 4 + 1 4 = 1 2

2 13 E, E ′ , E. W W

P(E ′)=1−P(E)

1 9 ,

1− 1 9 = 8 9

P(E ′)=1−P(E)

6×6, 36 1-11-21-31-41-51-62-12-22-32-42-52-63-13-23-33-43-53-64-14-24-34-44-54-65-15-25-35-45-5 5-66-16-26-36-46-56-6

3 36 = 1 12

P(E ′)=1−P(E) =1− 1 12 = 11 12

5 6 C(5,2) C(8,2)

ways to select 2 phones that are not defective ways to select 2 phones = C(5,2) C(8,2)

= 10 28 = 5 14

C(6,5) C(14,5)

C(6,5) C(14,5) = 6 2,002 = 3 1,001

C(6,2) C(5,3) C(6,2)·C(5,3)

C(6,2)C(5,3) C(14,5) = 15·10 2,002 = 75 1,001

C(9,5) C(14,5)

C(9,5) C(14,5) = 63 1,001

C(5,1)·C(9,4)

C(5,1)C(9,4) C(14,5) = 5·126 2,002 = 315 1,001

63 1,001 + 315 1,001 = 378 1,001

1− 378 1,001 = 623 1,001

a. 1 91 ; b. 5 91 ; c. 86 91

P(E)= n(E) n(S)

P(E∪F)=P(E)+P(F)−P(E∩F)

P(E∪F)=P(E)+P(F)

P(E')=1−P(E)

0 1, 0 1. A and B A and B, A or B 0 1. 1 2 . 5 8 . 1 2 . 3 8 . 1 4 . 3 4 . 3 8 . 1 8 . 15 16 . 5 8 . 52 1 13 . 1 26 . 12 13 . 3. 8. 5 12 .9. 15. 0. 15. 6 9. 4 9 . 6 9, 5 6. 1 4 . 5 6. 3 4 21 26 12 6 10 8 8 4 C(12,5) C(48,5) = 1 2162 4 3 C(12,3)C(36,2) C(48,5) = 175 2162 20 1 80. 20 1 80. 3,4, 5 20 C(20,3)C(60,17) C(80,20) ≈12.49% C(20,5)C(60,15) C(80,20) ≈23.33% 20.50+23.33−12.49=31.34% C(40000000,1)C(277000000,4) C(317000000,5) =36.78%
C(40000000,4)C(277000000,1) C(317000000,5) =0.11% a 1 =2, a n = a n−1 +n.2,4,7,11 6! (5−3)!3! . a n = 10 n +3.13,103,1003,10003 a n = n! n(n+1) . 4 7 , 47 21 , 82 21 , 39 7 , ... d= 5 3 . 2,4,8,16, ... a 1 =18 d=−8. 18,10,2,−6,−14a 3 =11.7a 8 =−14.6.−20,−10,0,10,…a 1 =−20, a n = a n−1 +10 0, − 1 2 , −1, − 3 2 ,…,7 8 , 29 24 , 37 24 , 15 8 ,…a n = 1 3 n+ 13 24 12,20,28,…,172?2.5, 5, 10, 20,… r=24, 16, 28, 40,… a 7 =16,384 a 9 =262,144 .4, 16, 64, 256, 1024 a 1 =−3 r= 1 2 . a 1 =3, a n =4· a n−1 ?3, 12, 48, 192, 7681, 1 3 , 1 9 , 1 27 ,…− 1 5 , − 1 15 , − 1 45 , − 1 135 ,… a n =− 1 5 · (1 3) n−1−5, − 5 3 , − 5 9 ,…, − 5 59,049 ?1 2 m+5m=0m=5.∑ m=0 5 (1 2 m+5).13n S 11 =11015S 912, 6, 3, 3 2 ,…S 9 ≈23.95∑ k=1 ∞ 45· (− 1 3) k−1 .S= 135 43 555 {−10,−6, 4, 10, 12, 18, 24, 32} 46?20127210 4 =10,000P(18,4).P(18,4)=73,44051032C(15,6). C(15,6)=5005{ 1, 3, 5, …, 99 }2 50 =1.13× 10 158! 3!2! =3360 (23 8).490,314(3x+ 1 2 y) 6 . (2a+b) 17 .131,072 a 17 +1,114,112 a 16 b+4,456,448 a 15 b 2(3 a 2 −2b) 112?1 65 94 9 1− C(350,8) C(500,8) ≈94.4%C(150,3)C(350,5) C(500,8) ≈25.6%a=−14, a n = 2+ a n−1 2 . −14,−6,−2,0a n = n 2 −n−1 n! .0.3, 1.2, 2.1, 3,…d=0.9.a 1 =−4d=− 4 3 .−2, − 7 2 , −5, − 13 2 ,… a 1 =−2, a n = a n−1 − 3 2 ; a 22 =− 67 215.6, 15, 14.4, 13.8,… −2,−1,− 1 2 ,− 1 4 ,…r= 1 2 . −1.5,−3,−6,−12,…? 1, − 1 2 , 1 4 , − 1 8 ,…a 1 =1, a n =− 1 2 · a n −14, − 4 3 , 4 9 , − 4 27 ,… 3 k 2 − 5 6 k k=−3 k=15.∑ k=−3 15 (3 k 2 − 5 6 k)n∑ k=1 7 −0.2· (−5) k−1 .S 7 =−2604.2 ∑ k=1 ∞ 1 3 · (− 1 5) k−1 .$140,355.75;$14,355.755×3×2×3×2=180C(15,3)=45510! 2!3!2! =151,200 (3 2 x− 1 2 y) 5 .(x 2 − 1 2) 13429 x 14 164 75 7C(14,3)C(26,4) C(40,7) ≈29.2% E

1, 1 2 , 1 4 , 1 8 ...

f(x)=L, x a, L. L. x a, f(x) L. L a.

$$\lim_{x\to a} f(x)=L.$$

x a x=a x=a, L.

$$f(x)= x 2 -6x-7 x-7 .$$

f(x)= (x−7) (x+1) x−7 Cancel like factors in numerator and denominator. f(x)=x+1,x≠7 Simplify.

x x≠7, x=7. x=7 x=7

$$\lim_{x\to 7} f(x)=8$$

x x=7? x=7,

f(7) does not exist.

$$f(x)=x+1,\ x\neq 7.$$

f(x) x=a. x a, x=a L f(x) x a x a a), f(x) L. f(x) a. a L

$$\lim_{x\to a} f(x)=L$$

a,f(x), L.

$$\lim_{x\to 2} (3x+5)=11$$

x a,

$$\lim_{x\to a} f(x)=L.$$

$$\lim_{x\to 2} (3x+5)=11.$$

a=2,f(x)=3x+5, and L=11. y=3x+5 lim x→2 (3x+5)=11 , x f(x)=3x+5 3(2)+5, L, x x=2. a,f(x), L.

$$\lim_{x\to 5} (2 x 2 -4)=46$$

a=5, f(x)=2 x 2 −4, L=46.

$$f(x)=x+1,x\neq 7$$

6.9, 6.99, 6.999. 7.9,7.99, 7.999. f(x) x f(x)=x+1,x≠7 x 7.1, 7.01, 7.001. 8.1, 8.01, 8.001.

f(x) x f(x)=x+1,x≠7 x x 6.9<x<7.1 f(x) 7.9<f(x)<8.1. f(x) f(x), L=8 x x

$$\lim_{x\to 7^-} f(x)=8.$$

$$\lim_{x\to 7^+} f(x)=8.$$

f(x) x a L,

$$\lim_{x\to a^-} f(x)=L.$$

f(x) L x a x<a x≠a. f(x), x a L,

$$\lim_{x\to a^+} f(x)=L.$$

f(x) L x a a. a L x a f(x) x a. f(x), x a, L,

$$\lim_{x\to a} f(x)=L$$

$$\lim_{x\to a^-} f(x)= \lim_{x\to a^+} f(x).$$

f(x) x a x a. x a, x x=a. a. x=a, x=a. x<a. L x=a, x=a. x>a. L, f(x) x a. (a,f(a)).

f(a) f(a) f(x), x a. x=a, f(a) f lim x→ 2 − f(x)lim x→ 2 + f(x)lim x→2 f(x)f(2) f

lim x→ 2 − f(x)lim x→ 2 + f(x)lim x→2 f(x)f(2)lim x→ 2 − f(x)=8; x<2, y=8.lim x→ 2 + f(x)=3;

x>2, y=3.lim x→ 2 f(x) lim x→ 2 − f(x)≠ lim x→ 2 + f(x); f(2)=3 f (2,f(2)) (2,3).

lim x→ 2 − f(x)=8; x<2 y=8.lim x→ 2 + f(x)=8; x>2 y=8.lim x→ 2 f(x)=8

lim x→ 2 − f(x)= lim x→ 2 + f(x)=8; f(2)=4 f (2,f(2)) (2,4). y=f(x) −2; x a L.

$$\lim_{x\to 5} (x 3 -125 x-5)$$

x=5. x>5 x>5

$$\lim_{x\to 5^-} f(x)=75= \lim_{x\to 5^+} f(x),$$

$$\lim_{x\to 5} f(x)=75.$$

f(5) f, x a f(a), a x a f(a).

$$\lim_{x\to 0} (5\sin(x)\ 3x)$$

x=0. x=0

$$f(x)= 5\sin(x)\ 3x$$

x=0, x=0 x=0. x<0 5 3 . x>0 5 3 .

$$\lim_{x\to 0^-} f(x)= 5\ 3 = \lim_{x\to 0^+} f(x),$$

lim x→0 f(x)= 5 3 .

f(x)= x 3 −125 x−5 x x x=5 75.

lim x→0 (20sin(x) 4x)

lim x→0 (20sin(x) 4x)=5 x x

f(x)=3sin(π x)

x=0. x=0, x=0 [−2,2] [−3,3]. [−0.1,0.1] [−3,3]. f(x) x

lim x→ 0 − (3sin(π x)) does not exist.

lim x→ 0 + (3sin(π x)) does not exist.

lim x→0 (3sin(π x)) does not exist.

lim x→0 (sin(2 x)). L a. lim x→ a f(x)=L, x a, x=a x=a, L. f(x) L x a x<a. f(x) L x a x>a. x a, y- x=a a x=a x a. x=a f(a). lim x→a f(x) x a a. x a x a x a, f lim x→− 2 − f(x)
lim x→− 2 + f(x)lim x→−2 f(x)f(−2)lim x→− 1 − f(x)lim x→ 1 + f(x)lim x→1 f(x)f(1)
lim x→ 4 − f(x)lim x→ 4 + f(x)lim x→4 f(x)f(4)
lim x→ 0 − f(x)=2, lim x→ 0 + f(x)=–3, lim x→2 f(x)=2, f(0)=4, f(2)=–1, f(–3) does not exist.
lim x→ 2 − f(x)=0, lim x→ 2 + =–2, lim x→0 f(x)=3, f(2)=5, f(0)
lim x→ 2 − f(x)=2, lim x→ 2 + f(x)=–3, lim x→0 f(x)=5, f(0)=1, f(1)=0
lim x→ 3 − f(x)=0, lim x→ 3 + f(x)=5, lim x→5 f(x)=0, f(5)=4, f(3) does not exist.
lim x→4 f(x)=6, lim x→ 6 + f(x)=–1, lim x→0 f(x)=5, f(4)=6, f(2)=6
lim x→–3 f(x)=2, lim x→ 1 + f(x)=–2, lim x→3 f(x)=–4, f(–3)=0, f(0)=0
lim x→π f(x)= π 2 , lim x→–π f(x)= π 2 , lim x→ 1 − f(x)=0, f(π)= 2 , f(0) does not exist. x
f(x)= (1+x) 1 xg(x)= (1+x) 2 xh(x)= (1+x) 3 xi(x)= (1+x) 4 xj(x)= (1+x) 5 x f(x)= (1+x) 6 x ,
g(x)= (1+x) 7 x , and h(x)= (1+x) n x .e 6 ≈403.428794, e 7 ≈1096.633158, e n x a. x a,
(x)={ | x |−1, if x≠1 x 3 , if x=1 a=1(x)={ 1 x+1 , if x=−2 (x+1) 2 , if x≠−2 a=−2lim x→−2 f(x)=1 x=a.
x=a. f(x)= x 2 −4x 16− x 2 ;a=4f(x)= x 2 −x−6 x 2 −9 ;a=3lim x→3 (x 2 −x−6 x 2 −9)= 5 6 ≈0.83
f(x)= x 2 −6x−7 x 2 − 7x ;a=7f(x)= x 2 −1 x 2 −3x+2 ;a=1lim x→1 (x 2 −1 x 2 −3x+2)=−2.00
f(x)= 1− x 2 x 2 −3x+2 ;a=1f(x)= 10−10 x 2 x 2 −3x+2 ;a=1lim x→1 (10−10 x 2 x 2 −3x+2)=20.00
f(x)= x 6 x 2 −5x−6 ;a= 3 2f(x)= x 4 x 2 +4x+1 ;a=− 1 2lim x→ −1 2 (x 4 x 2 +4x+1) x
f(x)= 2 x−4 ; a=4 x lim x→0 7tanx 3xlim x→0 7tanx 3x = 7 3lim x→4 x 2 x−4lim x→0 2sinx 4tanx
lim x→0 2sinx 4tanx = 1 2 x a. x a,lim x→0 e e 1 xlim x→0 e e − 1 x 2lim x→0 e e − 1 x 2 =1.0
lim x→0 | x | xlim x→−1 | x+1 | x+1lim x→− 1 − | x+1 | x+1 = −(x+1) (x+1) =−1
lim x→− 1 + | x+1 | x+1 = (x+1) (x+1) =1; lim x→−1 | x+1 | x+1 lim x→5 | x−5 | 5−x
lim x→−1 1 (x+1) 2lim x→−1 1 (x+1) 2 x −1 lim x→1 1 (x−1) 3lim x→0 5 1− e 2 x
lim x→0 5 1− e 2 x f(x)=| 1−x x | g(x)=| 1+x x | x f(x) g(x) x x m v

m= m o 1−(v 2 / c 2)

m o c m, v c − . v→c, m→∞.

lim v→ c − m= lim v→ c − m o 1−(v 2 / c 2) =∞

c, m, m v→c? v vm

f(x)

x a

lim x→ a − f(x)=L.

f(x)

L x a

x<a

x≠a.

a L L, f(x) x a a.

f(x),

L x x=a, x=a. a L L

lim x→a f(x)=L.

f(x)

x

a

lim x→ a + f(x)=L.

f(x)

L

x a

x>a,

x≠a.

a

L

f(x),

x a, L,

lim x→a f(x)=L

lim x→ a − f(x)= lim x→ a + f(x).

f(x)= x 2 −6x−7 x−7

f(x)= (x−7) (x+1) x−7 , which gives us f(x)=x+1,x≠7.

f g(x)=x+1? f x=7 g f(x) x=7, g(x), f x=7 x=7. g x x a, x=a. x a. x

lim x→7 f(x)= lim x→7 g(x).

x=a, x=a. a, k, A, B f g lim x→a f(x)=A lim x→a g(x)=B. lim x→a k=k

lim x→a [k·f(x)]=k lim x→a f(x)=kAlim x→a [f(x)+g(x)]= lim x→a f(x)+ lim x→a g(x)=A+B

lim x→a [f(x)−g(x)]= lim x→a f(x)− lim x→a g(x)=A−B

lim x→a [f(x)·g(x)]= lim x→a f(x)· lim x→a g(x)=A·B

lim x→a f(x) g(x) = lim x→a f(x) lim x→a g(x) = A B ,B≠0

lim x→a [f(x)] n = [lim x→∞ f(x)] n = A n , n lim x→a f(x) n = lim x→a [f(x)] n = A n

lim x→a p(x)=p(a) lim x→3 (2x+5).

lim x→3 (2x+5)= lim x→3 (2x)+ lim x→3 (5) Sum of functions property = 2lim x→3 (x)+ lim x→3 (5) Constant times a function property =2(3)+5 Evaluate =11

lim x→−12 (−2x+2). x a a a lim x→3 (5 x 2).

lim x→3 (5 x 2)=5 lim x→3 (x 2) Constant times a function property =5(3 2) Function raised to an exponent property =45

lim x→4 (x 3 −5). lim x→5 (2 x 3 −3x+1).

lim x→5 (2 x 3 −3x+1)= lim x→5 (2 x 3)− lim x→5 (3x)+ lim x→5 (1) Sum of functions = 2lim x→5 (x 3)− 3lim x→5 (x)+ lim x→5 (1) Constant times a function =2(5 3)−3(5)+1 Function raised to an exponent =236 Evaluate

lim x→−1 (x 4 −4 x 3 +5). lim x→2 (3x+1) 5 . x

lim x→2 (3x+1) 5 = (lim x→2 (3x+1)) 5 = (3(2)+1) 5 = 7 5 =16,807

lim x→−4 (10x+36) 3 .−64lim x→2 (x 2 +6x+8 x−2)xa lim x→2 (x 2 −6x+8 x−2).

lim x→2 (x 2 −6x+8 x−2)= lim x→2 ((x−2)(x−4) x−2) Factor the numerator. = lim x→2 ((x−2) (x−4) x−2) Cancel the common factors. = lim x→2 (x−4) Evaluate. =2−4=−2

f(x)= x 2 −6x+8 x−2

f(x)=x−4,x≠2.

x = 2. lim x→7 (x 2 −11x+28 7−x).−3 lim x→5 (1 x − 1 5 x−5). lim x→−5 (1 5 + 1 x 10+2x).

− 1 50 (0 0) a± b lim x→0 (25−x −5 x).

lim x→0 (25−x −5 x)= lim x→0 ((25−x −5) x · (25−x +5) (25−x +5))
Multiply numerator and denominator by the conjugate. = lim x→0 ((25−x)−25 x(25−x +5)) Multiply: (25−x −5)·(25−x +5)=(25−x)−25. = lim x→0 (−x x(25−x +5)) Combine like terms. = lim x→0 (− x x (25−x +5)) Simplify −x x =−1.
= −1 25−0 +5 Evaluate. = −1 5+5 =− 1 10
lim h→0 (16−h −4 h).− 1 8 lim x→4 (4−x x −2).
lim x→4 (4−x x −2) = lim x→4 ((2+ x)(2− x) x −2) Factor. = lim x→4 ((2+ x) (2− x) − (2− x)) Factor −1 out of the denominator. Simplify. = lim x→4 −(2+ x) Evaluate.
=−(2+ 4) =−4
a 2 − b 2
(a+b)(a−b).
lim x→3 (x−3 x − 3).2 3 lim x→7 | x−7 | x−7 . x=7 ,
| 6.9−7 | 6.9−7 = | 6.99−7 | 6.99−7 = | 6.999−7 | 6.999−7 =−1
| 7.1−7 | 7.1−7 = | 7.01−7 | 7.01−7 = | 7.001−7 | 7.001−7 =1 lim x→ 6 + 6−x | x−6 | .−1 f x a, f(a). f x a f(a). x a f(a)= 0 0 , f f(x) , x c, x c,lim x→0 (3)lim x→2 (−5x x 2 −1)−10 3
lim x→2 (x 2 −5x+6 x+2)lim x→3 (x 2 −9 x−3)lim x→−1 (x 2 −2x−3 x+1)
lim x→ 3 2 (6 x 2 −17x+12 2x−3)1 2lim x→− 7 2 (8 x 2 +18x−35 2x+7)lim x→3 (x 2 −9 x−5x+6)
lim x→−3 (−7 x 4 −21 x 3 −12 x 4 +108 x 2)lim x→3 (x 2 +2x−3 x−3)lim h→0 ((3+h) 3 −27 h)
lim h→0 ((2−h) 3 −8 h)−12lim h→0 ((h+3) 2 −9 h)lim h→0 (5−h − 5 h)− 5 10
lim x→0 (3−x − 3 x)lim x→9 (x 2 −81 3− x)−108lim x→1 (x − x 2 1− x)lim x→0 (x 1+2x −1)
lim x→ 1 2 (x 2 − 1 4 2x−1)lim x→4 (x 3 −64 x 2 −16)lim x→ 2 − (|x−2| x−2)
lim x→ 2 + (| x−2 | x−2)lim x→2 (| x−2 | x−2)lim x→ 4 − (| x−4 | 4−x)lim x→ 4 + (| x−4 | 4−x)
lim x→4 (| x−4 | 4−x)lim x→2 (−8+6x− x 2 x−2) lim x→c f(x)=3, lim x→c g(x)=5
lim x→c [2f(x)+ g(x)]6+ 5lim x→c [3f(x)+ g(x)]lim x→c f(x) g(x)3 5lim x→2 cos(πx)
lim x→2 sin(πx)lim x→2 sin(π x)f(x)={ 2 x 2 +2x+1, x≤0 x−3, x>0 ; lim x→ 0 + f(x)−3
f(x)={ 2 x 2 +2x+1, x≤0 x−3, x>0 ; lim x→ 0 − f(x)f(x)={ 2 x 2 +2x+1, x≤0 x−3, x>0 ; lim x→0 f(x)
lim x→4 x+5 −3 x−4lim x→ 2 + (2x− ⟦x⟧)lim x→2 x+7 −3 x 2 −x−2lim x→ 3 + x 2 x 2 −9
f(x+h)−f(x) h .f(x)=x+1f(x)=2 x 2 −14x+2hf(x)= x 2 +3x+4f(x)= x 2 +4x−1002x+h+4f(x)=3 x 2 +1
f(x)=cos(x)cos(x+h)−cos(x) hf(x)=2 x 3 −4xf(x)= 1 x−1 x(x+h)f(x)= 1 x 2f(x)= x−1 x+h + x
f(x)= x 2 +5x+6 x+3 x x s(t)=−16 t 2 +144t [1,2] s(t)=−64 t 2 +192t t=1 t=1.5 t
A= A 0 e 0.0425t , A 0 t=1 t=2 118 ∘ F 95 ∘ F. T, T(x) x 96 ∘ F 116 ∘ F, 118 ∘ F.
110.5 ∘ F. 96 ∘ F 118 ∘ F 96 ∘ F 118 ∘ F D, D(x) x y=f(x) f(a) x=a x=a x=a
lim x→a f(x) , x=a f x=a x=a y x=a f. lim x→a f(x)=f(a). x=a. x=a x=a f(x) x=a f(a)
lim x→a f(x) x=a lim x→a f(x)=f(a) f(x) x=a , x=a x=a y=f(x) x a f(x) x=a
lim x→ a − f(x)≠ lim x→ a + f(x) y=f(x) x=a x=a x=a f x=a f(x) x=a lim x→a f(x) ,f(a)
f(a), x=a f(a)≠ lim x→a f(x).f(x)= x 2 −2x−15 x−5g(x)={ x+1, x<2 −x, x≥2 x=5. f(5) x x=5.
g(2)=−2. x lim x→ 2 − (x+1)=2+1=3. lim x→ 2 + (−x)=−2.
lim x→ 2 − f(x)≠ lim x→ 2 + f(x).
lim x→2 f(x) x=2.f(x)= x 2 −6x x−6g(x)={ x , 0≤x<4 2x, x≥4 x=6; x=4 f(x) x a f(x) x=a.
lim x→a f(x)=f(a). f(x)= x 4 −9 x 2 f(x)= 4 x+2 −5 f(x)=sin(2x)−4 f(x)=−cos(x+ π 3) f(x)=2ln(x) x>0 f(x)=tan(x)+2, x≠ π 2 +kπ, k f(x)= x 2 −25 x−7 , x≠7 f(x), x=a. f(a) lim x→a f(x) x=a.
lim x→a f(x)=f(a). x=a. x=a. f(x)={ 4x, x≤3 8+x, x>3 x=3x= 8 3 f x=a, x=a f(a)
f(3)=4(3)=12 ⇒Condition 1 is satisfied.
lim x→3 f(x) x=3, f(x)=4x; x=3, f(x)=8+x. x lim x→ 3 − f(x)= lim x→ 3 − 4(3)=12
lim x→ 3 + f(x)= lim x→ 3 + (8+x)=8+3=11 lim x→ 1 − f(x)≠ lim x→ 1 + f(x), lim x→1 f(x)
⇒ Condition 2 fails.
x=3. x=3, f(x) x=3.x= 8 3 f(8 3)

f(8 3)=4(8 3)= 32 3 ⇒Condition 1 is satisfied.

lim x→ 8 3 f(x) x= 8 3 , f(x)=4x; x= 8 3 , f(x)=8+x. x 8 3 .

lim x→ 8 3 − f(x)= lim x→ 8 3 − 4(8 3)= 32 3 lim x→ 8 3 + f(x)= lim x→ 8 3 + (8+x)=8+ 8 3 = 32 3

lim x→ 8 3 f(x)

⇒Condition 2 is satisfied.

f(8 3)= lim x→ 8 3 f(x)?

f(32 3)= 32 3 = lim x→ 8 3 f(x) ⇒Condition 3 is satisfied.

x= 8 3 , f(x) x= 8 3 . f(x)={ 1 x , x≤2 9x−11.5, x>2 x=2. f(x)= x 2 −25 x−5 x=5. f x=5, x=5.

f(5) does not exist. ⇒Condition 1 fails.

x=5. x=5, f x=5.

lim x→5 x 2 −25 x−5 = lim x→3 (x−5) (x+5) x−5 = lim x→5 (x+5) =5+5=10

⇒Condition 2 is satisfied.

x=5, f(x)= 9− x 2 x 2 −3x x=3. x=3. x=3. a x a, a. f(a) lim x→a f(x) lim x→a f(x)=f(a). x=a.

x=a. f

fx={ x+1, x<2 3, 2≤x<4 x 2 −11, x≥4

f(x)=x+1 x<2, f(x)=3 2≤x<4, f(x)= x 2 −5 x≥4. x=2 x=4. x=2,

f(2)=3 ⇒Condition 1 is satisfied.

x=2, lim x→ 2 − f(x)= lim x→ 2 + f(x)? lim x→ 2 − f(x)= lim x→ 2 − (x+1)=2+1=3

lim x→ 2 + f(x)= lim x→ 2 + 3=3 3=3 lim x→ 2 − f(x)= lim x→ 2 + f(x)

⇒Condition 2 is satisfied.

lim x→2 f(x)=3=f(2) ⇒Condition 3 is satisfied.

x=2, f(x) x=2. x=4, x=4, lim x→ 4 − f(x)= lim x→ 4 + f(x)? lim x→ 4 − f(x)= lim x→ 4 − 3=3

lim x→ 4 + f(x)= lim x→ 4 + (x 2 −11)= 4 2 −11=5 3≠5 lim x→ 4 − f(x)≠ lim x→ 4 + f(x) ,

lim x→4 f(x)

⇒Condition 2 fails.

x=4, f(x) x=4. x=4, x=2. x=2 x=4. f(x)={ πx 4 , x<2 π x , 2≤x≤6 2πx, x>6 x=6 x=a

fx={ sin(x), x<0 x 3 , x>0

f(x)=sin(x) x<0 f(x)= x 3 x>0. x=0,

f(0) does not exist. ⇒Condition 1 fails.

x=0, f(x) x=0. x=0; lim x→0 f(x)=0, f(a) f(a) lim x→a f(x) x=a. lim x→a f(x)=f(a). f x=c.

x=c , f(c), f(c) (a,b). f a

f(x)=ln | x+3 |,a=−3

a=−3 f(−3)

f(x)=ln | 5x−2 |,a= 2 5

f(x)= x 2 −16 x+4 ,a=−4 a=−4 f(−4) f(x)= x 2 −16x x ,a=0f(x)={ x, x≠3 2x,x=3 a=3 a=3

lim x→3 f(x)=3 , f(3)=6 ,f(x)={ 5, x≠0 3, x=0 a=0f(x)={ 1 2−x , x≠2 3, x=2 a=2lim x→2 f(x)

f(x)={ 1 x+6 , x=−6 x 2 , x≠−6 a=−6f(x)={ 3+x, x<1 x, x=1 x 2 , x>1 a=1

lim x→ 1 − f(x)=4; lim x→ 1 + f(x)=1 lim x→1 f(x) f(x)={ 3−x, x<1 x, x=1 2 x 2 , x>1 a=1

f(x)={ 3+2x, x<1 x, x=1 − x 2 , x>1 a=1lim x→ 1 − f(x)=5≠ lim x→ 1 + f(x)=−1 lim x→1 f(x)

f(x)={ x 2 , x<−2 2x+1, x=−2 x 3 , x>−2 a=−2

f(x)={ x 2 −9 x+3 , x<−3 x−9, x=−3 1 x , x>−3 a=−3lim x→− 3 − f(x)=−6 lim x→− 3 + f(x)=− 1 3

lim x→−3 f(x) f(x)={ x 2 −9 x+3 , x<−3 x−9, x=−3 −6, x>−3 a=3f(x)= x 2 −4 x−2 , a=2f(2)

f(x)= 25− x 2 x 2 −10x+25 , a=5f(x)= x 3 −9x x 2 +11x+24 , a=−3f(−3)

f(x)= x 3 −27 x 2 −3x , a=3f(x)= x |x| , a=0f(0) f(x)= 2| x+2 | x+2 , a=−2 f f(x)= x 3 −2x−15

(−∞,∞)f(x)= x 2 −2x−15 x−5f(x)=2· 3 x+4 (−∞,∞)f(x)=−sin(3x)f(x)= | x−2 | x 2 −2x x=0 x=2

f(x)=tan(x)+2f(x)=2x+ 5 x x=0f(x)= log 2 (x)f(x)=ln x 2 (0,∞)f(x)= e 2xf(x)= x−4 [4,∞)

f(x)=sec(x)−3f(x)= x 2 +sin(x) (−∞,∞) b c f(x)= { x+1, 1<x<3 x 2 +bx+c, | x−2 |≥1 }

a, x=a x=−3x=2x=4 f(x)=sin(12π x) f(0).f(0) x f(x)=0. f(x)?(−∞,0)∪(0,∞) x=−1, x=1, f(1) x=2, x=−7 x=1.x 3 +6 x 2 −7x (x+7)(x−1) f(x)= x 3 −1 x−1 [−3,3], x lim x→1 f(x) x=1:

fx={ x 2 +4 x≠1 2 x=1

x=1 x f(1)=2. f(x)= sin(2x) x f(x) x=0? x=a f(x) x=a lim x→ a − f(x)≠ lim x→ a + f(x) f(x) f(x)= x 3 −4x. x=−2 x=−1 x=2 f x=a (a,f(a)). x=a, h. (a+h,f(a+h)) h. a x=a. (a,f(a)) (a+h,f(a+h)),

slope = change in y change in x

slope = change in y change in x

m sec

m sec = f(a+h)−f(a) (a+h)−(a) = f(a+h)−f(a) a +h− a

m sec (a,f(a)) (a+h,f(a+h)).

m sec = f(a+h)−f(a) h

(a,f(a)) (a+h,f(a+h)) f

AROC= f(a+h)−f(a) h

(2,−6) (−1,5).

AROC= f(a+h)−f(a) h .

(2,−6), (2,f(2)), f(2)=−6. h 2 −1, −1−2=−3. f(a+h) a+h , 2+(−3) −1, f(a+h)=f(−1)=5.

AROC= f(a+h)−f(a) h = 5−(−6) −3 = 11 −3 =− 11 3

(−5,1.5) (−2.5,9). h a+h a h (a+h,f(a+h)) (a,f(a)). x=a, x=a. x=a. x=a, f x=a. x=a f x=a f'(a),

f ′ (a)= lim h→0 f(a+h)−f(a) h

f x=a ,

f'(a)= lim h→0 f(a+h)−f(a) h

f(a+h)−f(a) h h f(x) x=a f(x) x=a. f(x) x=a f ′ (a). f ′ (a) (a,f(a)). f ′ (a) f(x) x=a. t=a. f(x) y ′ = f ′ (x), y=f(x). f ′ (x) f prime of x.

f ′ (x)= y ′ = dy dx = df dx = d dx f(x)=Df(x)

f ′ (x) x y=f(x) x. f(x) x=a f ′ (a). f, f(a+h). f(a). f(a+h)−f(a) h . f ′ (a)= lim h→0 f(a+h)−f(a) h . f(x)= x 2 −3x+5 x=a.

f ′ (a)= lim h→0 f(a+h)−f(a) h Definition of a derivative

f(a+h)= (a+h) 2 −3(a+h)+5 f(a)= a 2 −3a+5.

f ′ (a)= lim h→0 (a+h)(a+h)−3(a+h)+5−(a 2 −3a+5) h = lim h→0 a 2 +2ah+ h 2 −3a−3h+5− a 2 +3a −5 h Evaluate to remove parentheses. = lim h→0 a 2 +2ah+ h 2 −3a −3h +5 − a 2 +3a −5 h Simplify. = lim h→0 2ah+ h 2 −3h h = lim h→0 h (2a+h−3) h Factor out an h. =2a+0−3 Evaluate the limit. =2a−3

f(x)=3 x 2 +7x x=a.f ′ (a)=6a+7 f(x)= 3+x 2−x x=a.

f ′ (a)= lim h→0 f(a+h)−f(a) h = lim h→0 3+(a+h) 2−(a+h) −(3+a 2−a) h Substitute f(a+h) and f(a) = lim h→0 (2−(a+h))(2−a)[3+(a+h) 2−(a+h) −(3+a 2−a)] (2−(a+h)) (2−a)(h) Multiply numerator and denominator by (2−(a+h))(2−a) = lim h→0 (2−(a+h)) (2−a)(3+(a+h) (2−(a+h)))−(2−(a+h)) (2−a) (3+a 2−a) (2−(a+h))(2−a)(h) Distribute = lim h→0 6−3a+2a− a 2 +2h−ah−6+3a+3h−2a+ a 2 +ah (2−(a+h))(2−a)(h) Multiply = lim h→0 5 h (2−(a+h))(2−a)(h) Combine like terms = lim h→0 5 (2−(a+h))(2−a) Cancel like factors = 5 (2−(a+0))(2−a) = 5 (2−a)(2−a) = 5 (2−a) 2 Evaluate the limit

f(x)= 10x+11 5x+4 x=a.f ′ (a)= −15 (5a+4) 2 f(x)=4 x x=36.

f ′ (a)= lim h→0 f(a+h)−f(a) h = lim h→0 4 a+h −4 a h Substitute f(a+h) and f(a)

4 a+h +4 a 4 a+h +4 a .

f ′ (a)= lim h→0 (4 a+h −4 a h)·(4 a+h +4 a 4 a+h +4 a) = lim h→0 (16(a+h)−16a h4(a+h +4 a)) Multiply. = lim h→0 (16a +16h− 16a h4(a+h +4 a)) Distribute and combine like terms. = lim h→0 (16 h h (4 a+h +4 a)) Simplify. = lim h→0 (16 4 a+h +4 a) Evaluate the limit by letting h=0. = 16 8 a = 2 a f ′ (36)= 2 36 Evaluate the derivative at x=36. = 2 6 = 1 3

f(x)=9 x x=9.3 2 s(t)=−16 t 2 +64t+6, t s(t) s(t)=−16 t 2 +64t+6, t=1 t=3 t=1 t=3: f ′ (a)= lim h → 0 f(a+h)−f(a) h = lim h → 0 −16 (t+h) 2 +64(t+h)+6−(−16 t 2 +64t+6) h Substitute s(t+h) and s(t). = lim h → 0 −16 t 2 −32ht− h 2 +64t+64h+6+16 t 2 −64t−6 h Distribute. = lim h → 0 −32ht− h 2 +64h h Simplify. = lim h → 0 h (−32t−h+64) h Factor the numerator. = lim h → 0 −32t−h+64 Cancel out the common factor h. s ′ (t)=−32t+64 Evaluate the limit by letting h=0.

t s ′ (t) t. t=1 t=3.

s ′ (1)=−32(1)+64=32 s ′ (3)=−32(3)+64=−32

−32 s(t)=−16 t 2 +64t+6. x=a f(x) x=a. x=a f(x), x=a. x=a f(x). x=a a change in y change in x .

y=f(x) f(0)f(2)f'(0)f'(2) f(a), x=a. x=a, f ′ (a), x=a,f(0) x=0. (0,1), f(0)=1.f(2) x=2. (2,1), f(2)=1.f ′ (0) x=0. x=0 f ′ (0)=0.f ′ (2) x=2. x=2. x y f ′ (2)=4. f(x)= x 3 −3x f(1), f ′ (1), f(0), f ′ (0).−2 , −3 x=a f(x)

output units input unit

x C(x), x C ′ (x) x C ′ (11) C ′ (11)=2.50 x f(x)= x 2 −100x. f(x)= x 2 −100x x f ′ (x) f(a+b)= (x+h) 2 −100(x+h) f(a)= a 2 −100a f ′ (x)= f(a+h)−f(a) h Formula for a derivative = (x+h) 2 −100(x+h)−(x 2 −100x) h Substitute f(a+h) and f(a). = x 2 +2xh+ h 2 −100x−100h− x 2 +100x h Multiply polynomials, distribute. = 2xh+ h 2 −100h h Collect like terms. = h (2x+h−100) h Factor and cancel like terms. =2x+h−100 Simplify. =2x−100 Evaluate when h=0. f ′ (x)=2x−100 Formula for marginal cost f ′ (200)=2(200)−100=300 Evaluate for 200 units.

f(t), t f(0)=0f ′ (1)=60f(1)=70f(2.5)=150 f(t) f ′ (t), f(t), t f ′ (t), t f(0)=0 f ′ (1)=60 f(1)=70 f(2.5)=150 f(t) tf(0)=0f(10)=150f ′ (10)=15f ′ (20)=−10f(40)=−100 f(x) x=a x=a f(x) f(x)=| x |, x=0, x=0. f(x)=| x |, x=0. f(x)=| x |, f(x) x=a.

f(x)={ x 2 , x≤2 8−x, x>2 .

x x x=2. f(x) x=2. f(x)=| x |. x=0. f(x)=| x | x=0 f(x)= x 2 3 x=0. x f(x)= x 2 3 x=0. f(x)= x 1 3 x=0. f(x)= x 1 3 x=0. f(x) x=a x=a, f ′ (a) f(x) x=a. x=a. x=a. x=a. x=a.f(x) (−∞,−2)∪(−2, 1)∪(1,∞).f(x)x=−2x=1.(−∞,−2)∪(−2,−1)∪(−1,1)∪(1,2)∪(2,∞).f(x)x=−2x=−1 x=1x=2 y=f(x) f (−∞,1)∪(1,3)∪(3,∞). f x=1 x=3. f (−∞,1)∪(1,3)∪(3,∞). f x=1 x=3. f(x) x=a y=m(x− x 1)+ y 1 . x=a f ′ (a), f(x) x=a. x=a (a,f(a)).

y=f'(a)(x−a)+f(a)

f x=a

y=f'(a)(x−a)+f(a)

f, x=a. f(x) x=a f ′ (a)= lim h→0 f(a+h)−f(a) h . x=a. f(a). (a,f(a)) f ′ (a) y=f'(a)(x−a)+f(a). y=mx+b. f(x)= x 2 −4x x=3.

f'(a)= lim h→0 f(a+h)−f(a) h

f(a+h)= (a+h) 2 −4(a+h) f(a)= a 2 −4a.

f ′ (a)= lim h→0 (a+h)(a+h)−4(a+h)−(a 2 −4a) h = lim h→0 a 2 +2ah+ h 2 −4a−4h− a 2 +4a h Remove parentheses. = lim h→0 a 2 +2ah+ h 2 −4a −4h − a 2 +4a h Combine like terms. = lim h→0 2ah+ h 2 −4h h = lim h→0 h (2a+h−4) h Factor out h. =2a+0−4 f ′ (a)=2a−4 Evaluate the limit. f ′ (3)=2(3)−4=2

x=3:

y=f'(a)(x−a)+f(a) y=f'(3)(x−3)+f(3) y=2(x−3)+(−3) y=2x−9

x=3 x=3. f(x)=5 x 2 −x+4 x=2.y=19x−16 s(t) t. t=a
s ′ (a)= lim h→0 s(a+h)−s(a) h
t s(t)=−16 t 2 +36t+200, t=2. s ′ (t) t=2, s(a+h)=−16 (a+h) 2 +36(a+h)+200
s(a)=−16 a 2 +36a+200.
s ′ (a)= lim h→0 s(a+h)−s(a) h = lim h→0 −16 (a+h) 2 +36(a+h)+200−(−16 a 2 +36a+200) h = lim h→0 −16(a 2 +2ah+ h 2)+36(a+h)+200−(−16 a 2 +36a+200) h = lim h→0 −16 a 2 −32ah−16 h 2 +36a+36h+200+16 a 2 −36a−200 h = lim h→0 −16 a 2 −32ah−16 h 2 +36a +36h +200 +16 a 2 −36a −200 h = lim h→0 −32ah−16 h 2 +36h h = lim h→0 h (−32a−16h+36) h = lim h→0 (−32a−16h+36) =−32a−16·0+36 s ′ (a)=−32a+36 s ′ (2)=−32(2)+36 =−28
t=2 t s=−16 t 2 +60t−12. AROC= f(a+h)−f(a) hf ′ (a)= lim h→0 f(a+h)−f(a) hf(a+h)−f(a) h x. [x,x+h] x? x. f(x) t, f(x) t=a t=b f(a)+45(b−a). lim h→0 f(x+h)−f(x) h f(x)=3x−4 f(x)=−2x+1f ′ (x)=−2f(x)= x 2 −2x+1f(x)=2 x 2 +x−3f ′ (x)=4x+1f(x)=2 x 2 +5f(x)= −1 x−2 f ′ (x)= 1 (x−2) 2f(x)= 2+x 1−xf(x)= 5−2x 3+2x−16 (3+2x) 2f(x)= 1+3xf(x)=3 x 3 − x 2 +2x+5 f ′ (x)=9 x 2 −2x+2f(x)=5f(x)=5πf ′ (x)=0(−2,0) (−4,5)(4,−3) (−2,−1)− 1 3(0,5) (6,5)(7,−2) (7,10)f(x)= x 3 +1f(x)=−3 x 2 −7x=6f ′ (x)=−6x−7f(x)=7 x 2f(x)=3 x 3 +2 x 2 +x−26 f ′ (x)=9 x 2 +4x+1 x f(x)=2 x 2 −3x x=3f(x)= x 3 +1 x=2y=12x−15f(x)= x x=9 kf(x)= x 2 −kx, y=4x−9 k=−10 k=2 f x=−2 x=0. x=5. x x,f(−1)f(0)f(0)=−2f(1)f(2)f(2)=−6f(3)f ′ (−1)f ′ (−1)=9 f ′ (0)f ′ (1)f ′ (1)=−3f ′ (2)f ′ (3)f ′ (3)=9f ′ (x)=2x f(2)=4 f(x)= x 2 x=1 [0.9,1.1] [0.99,1.01] [0.999,1.001] , [0.9999, 1.0001] x=1, x=1. x=1 f(t) t f(0)=600f'(30)=−20f(30)=0 f'(200)=30f(240)=500 s, t s(t)=−16 t 2 +80t.s(2)=96s'(2)=16s(3)=96 t=3 s'(3)=−16s(0)=0,s(5)=0. t=0 t=5. V r V= 4 3 π r 3 . V r V r=3 cm.36π x R(x)=2 x 2 +10x. x x=10 x=20. R'(10) R'(15) R'(15) R'(10), x C(x)= x 2 −4x+1000. x x=10 to x=15. x=a, lim x→a f(x)−f(a) x−a ,f(x)= 1 x 2 f(x)=5 x 2 −x+4f'(x)=10a−1f(x)=− x 2 +4x+7f(x)= −4 3− x 24 (3−x) 2lim x→ −1 + f(x) lim x→ −1 − f(x)lim x→−1 f(x)lim x→3 f(x) x

Discontinuous at x=−1(lim x→ a f(x) does not exist),x=3 (jump discontinuity), and x=7 (lim x→ a f(x) does not exist).

lim x→0 f(x).xF(x) x a. x a, f(x)={ | x |−1, if x≠1 x 3 , if x=1 a=1 f(x)={ 1 x+1 , if x=−2 (x+1) 2 , if x≠−2 a=−2lim x→−2 f(x)=1f(x)={ x+3 , if x<1 − x 3 , if x>1 a=1 lim x→c f(x)=−3 lim x→c g(x)=5.lim x→c (f(x)+g(x))lim x→c f(x) g(x)lim x→c (f(x)·g(x))−15 lim x→ 0 + f(x),f(x)={ 3 x 2 +2x+1 5x+3 x>0 x<0lim x→ 0 − f(x),f(x)={ 3 x 2 +2x+1 5x+3 x>0 x<0 lim x→ 3 + (3x− ⟦x⟧)lim h→0 ((h+6) 2 −36 h)lim x→25 (x 2 −625 x −5)lim x→1 (− x 2 −9x x) −10lim x→4 7− 12x+1 x−4lim x→−3 (1 3 + 1 x 3+x)− 1 9 x=a. x=a.f(x)= −2 x−4 ; a=4 f(x)= −2 (x−4) 2 ; a=4 x=4,f(x)= −x x 2 −x−6 ; a=3f(x)= 6 x 2 +23x+20 4 x 2 −25 ; a=− 5 2 a=− 5 2 f(x)= x −3 9−x ; a=9 f(x) f(x)= x 2 −2x−15 (−∞,∞)f(x)= x 2 −2x−15 x−5f(x)= x 2 −2x x 2 −4x+4 x=2. f(2) f(x)= x 3 −125 2 x 2 −12x+10f(x)= x 2 − 1 x 2−x x=0 x=2. f(0) f(2) f(x)= x+2 x 2 −3x−10 f(x)= x+2 x 3 +8 x=−2. f(−2) f(x+h)−f(x) h .f(x)=3x+2f(x)=5f(x)= 1 x+1f(x)=ln(x)ln(x+h)−ln(x) h f(x)= e 2xf(x)=4x−6=4f(x)=5 x 2 −3x f(x) x f(x)=− x 3 +4x x=2.y=−8x+16f(x)= x | x | V= 1 3 π r 2 h π12π f f(1)lim x→ −1 + f(x)lim x→ −1 − f(x)lim x→−1 f(x)lim x→−2 f(x)−1 x f x a. x a, f(x)={ 1 x −3, if x≤2 x 3 +1,if x>2 a=2lim x→ 2 − f(x)=− 5 2 a lim x→ 2 + f(x)=9 x f(x)={ x 3 +1, if x<1 3 x 2 −1, if x=1 − x+3 +4, if x>1 a=1lim x→−5 (1 5 + 1 x 10+2x)− 1 50 lim h→0 (h 2 +25 −5 h 2)lim h→0 (1 h − 1 h 2 +h) f f(x)= x 2 −4 f(x)= x 3 −4 x 2 −9x+36 x 3 −3 x 2 +2x−6 x=3 x=a.f(x)= 3 5+2xf(x)= 3 xf'(x)=− 3 2 a 3 2f(x)=2 x 2 +9x f(x)=| x−2 |−| x+2 |f(x)= 2 1+ e 2 x x=0 s, t s(t).s(0)s(2) t=2 s'(2)s(2)−s(1) 2−1 t=1 to t=2s(t)=0 lim x→0 sin(x) 3x1 3lim x→0 tan 2 (x) 2xlim x→0 sin(x)(1−cos(x)) 2 x 2 lim x→1 f(x), where f(x)={ 4x−7 x≠1 x 2 −4 x=1 x f(x)={ 4x−7 x≠1 x 2 −4 x=1 x=1 to x=3. x f'(x)=0. x=1 x f'(x) f f(x)=3 x 2 −2x−6, x=−2y=−14x−18 f(x)=x (1−x) 2 5 f(x)=x (1−x) 2 5 f(x)=x ((1−x) 2) 1 5 f(x)=x ((1−x) 1 5) 2 f(x) x=1 x=1. x=1 lim h→0 f(x+h)−f(x) h f(x)=2x−8f(x)=4 x 2 −7f ' (x)=8xf(x)=x− 1 2 x 2f(x)= 1 x+2f ' (x)=− 1 (2+x) 2f(x)= 3 x−1f(x)=− x 3 +1 f ' (x)=−3 x 2f(x)= x 2 + x 3f(x)= x−1f'(x)= 1 2 x−1 (a,f(a)) (a+h,f(a+h)) f(x);

AROC= f(a+h)−f(a) h .

f ′ (a), x=a f ′ (a)= lim h→0 f(a+h)−f(a) h , f(x) x=a. f ′ (a) x=a

f ′ (a)= lim h→0 f(a+h)−f(a) h . s(t) t, t=a s ′ (a)= lim h→0 s(a+h)−s(a) h .

cos 2 t+ sin 2 t=1 1+ tan 2 t= sec 2 t 1+ cot 2 t= csc 2 t

cos(−t)=cos t sec(−t)=sec t sin(−t)=−sin t tan(−t)=−tan t csc(−t)=−csc t cot(−t)=−cot t

cos t=sin(π 2 −t) sin t=cos(π 2 −t) tan t=cot(π 2 −t) cot t=tan(π 2 −t) sec t=csc(π 2 −t) csc t=sec(π 2 −t)

tan t= sin t cos t sec t= 1 cos t csc t= 1 sin t cot t= 1 tan t = cos t sin t

cos(α+β)=cos α cos β−sin α sin β cos(α−β)=cos α cos β+sin α sin β sin(α+β)=sin α cos β+cos α sin β sin(α−β)=sin α cos β−cos α sin β tan(α+β)= tan α+tan β 1−tan α tan β tan(α−β)= tan α−tan β 1+tan α tan β

sin(2θ)=2 sin θ cos θ cos(2θ)= cos 2 θ− sin 2 θ cos(2θ)=1−2 sin 2 θ cos(2θ)=2 cos 2 θ−1 tan(2θ)= 2 tan θ 1− tan 2 θ

sin α 2 =± 1−cos α 2 cos α 2 =± 1+cos α 2 tan α 2 =± 1−cos α 1+cos α tan α 2 = sin α 1+cos α tan α 2 = 1−cos α sin α

sin 2 θ= 1−cos(2θ) 2 cos 2 θ= 1+cos(2θ) 2 tan 2 θ= 1−cos(2θ) 1+cos(2θ)

cos α cos β= 1 2 [cos(α−β)+cos(α+β)] sin α cos β= 1 2 [sin(α+β)+sin(α−β)] sin α sin β= 1 2 [cos(α −β)−cos(α+β)] cos α sin β= 1 2 [sin(α+β)−sin(α−β)]

sin α+sin β=2 sin(α+β 2) cos(α−β 2) sin α−sin β=2 sin(α−β 2) cos(α+β 2) cos α−cos β=−2 sin(α+β 2) sin(α−β 2) cos α+cos β=2 cos(α+β 2) cos(α−β 2)

sin α a = sin β b = sin γ c a sin α = b sin β = c sin γ

a 2 = b 2 + c 2 −2bc cos α b 2 = a 2 + c 2 −2ac cos β c 2 = a 2 + b 2 −2ab cos γ