Precalculus

$$\{(1,2),(2,4),(3,6),(4,8),(5,10)\}$$

$\{1,2,3,4,5\}.\{2,4,6,8,10\}$. x. y. f $\{1,2,3,4,5\},\{2,4,6,8,10\}$.

$$\{\ (\text{odd},1),(\text{even},2),(\text{odd},3),(\text{even},4),(\text{odd},5)\ \}$$

$\{\text{even},\text{odd}\}\{1,2,3,4,5\}.\{1,3,5\}\{2,4\}$. q r n. q ha f, g, h x, y,zA, B,C

h is f of a We name the function f; height is a function of age. h=f(a)

We use parentheses to indicate the function input. f(a)

We name the function f; the expression is read as “f of a.”

h(a) h a. a h f(a+b) y=f(x) f. “y x.” x y, f(x), f,days=f(month)d=f(m). f(March)=31, d=f(m) d m N=f(y) N, y. f(2005)=300 f(2005)=300, (N), N=f(y). f(2005)=300 d.w=f(d) y=f(x), y=y(x), f, y f x. y=f(x),P=W(d), f D=f(m) m D Q=g(n). g n Q .nQ a h

$$f(2)=1, f(5)=3, \text{and } f(8)=6$$

$$g(-3)=5, g(0)=1, \text{and } g(4)=5$$

f(x)=5−3 x 2 f(x)= x 2 +3x−4 2aa+hf(a+h)−f(a) h x

$$f(2)=2^2+3(2)-4 \quad =4+6-4 \quad =6$$

$$f(a)=a^2+3a-4$$

a+h,

$$f(a+h)=(a+h)^2+3(a+h)-4 \quad =a^2+2ah+h^2+3a+3h-4$$

$$f(a+h)=a^2+2ah+h^2+3a+3h-4$$

$$f(a)=a^2+3a-4$$

$$\frac{f(a+h)-f(a)}{h}=\frac{(a^2+2ah+h^2+3a+3h-4)-(a^2+3a-4)}{h} \quad =2ah+h^2+3h$$

$$=h(2a+h+3) \text{ Factor out } h. \quad =2a+h+3 \text{ Simplify.}$$

h(p)= p 2 +2p, h(4). h(4), p

$$h(p)=p^2+2p \quad h(4)=(4)^2+2(4) \quad =16+8 \quad =24$$

g(m)= m−4 , g(5). g(5)=1 h(p)= p 2 +2p, h(p)=3.

$$h(p)=3 \quad p^2+2p=3 \text{ Substitute the original function } h(p)=p^2+2p. \quad p^2+2p-3=0$$

Subtract 3 from each side. (p+3)(p−1)=0 Factor.

(p+3)(p−1)=0, (p+3)=0 (p−1)=0 p

$$(p+3)=0,\ p=-3 \quad (p-1)=0,\ p=1$$

h(p)=3 p=1 p=−3. h(1)=h(−3)=3 h(4)=24. g(m)= m−4 , g(m)=2. m=8 2n+6p=12 n p. p n. 2n+6p=12 p=f(n), p n, p=[expression involving n].

$$2n+6p=12 \quad 6p=12-2n \text{ Subtract } 2n \text{ from both sides.} \quad p=\frac{12-2n}{6}$$

$$\text{Divide both sides by 6 and simplify.} \quad p=\frac{12}{6}-\frac{2n}{6} \quad p=2-\frac{1}{3}n$$

p n

$$p=f(n)=2-\frac{1}{3}n$$

x 2 + y 2 =1 x y y=f(x). x 2

$$y^2=1-x^2$$

y

$$y=\pm\sqrt{1-x^2} \quad =+\sqrt{1-x^2} \text{ and } -\sqrt{1-x^2}$$

y=f(x). x−8 y 3 =0, y x.y=f(x)= x 3 2 x=y+ 2 y , y x, x y. x y, y y x, P. P P(goldfish)=2160. P g(3). g(n)=6.ng(n)g(3)gn=3.n=3g(3)=7.g(n)=6n,24.ng(n) g, g, g(1).g(1)=8 f(2). f(x)=4. f(2), x=2, (2,1), f(2)=1. f(x)=4, 4 y=4, 4:(−1,4) (3,4). f(x)=4:−1 3. f(−1)=4 f(3)=4, −1 3, 4. f(x)=1. x=0 x=2 q r n. r A=π r 2 , r, A. r. A A=π r 2 . A π . x y, y x, y=f(x) f. (x,y) y=f(x). f(0)=2 f(6)=1. (x,y) y=f(x) (0,2) (6,1) y=f(x)? x y=f(x) f(x)=c, cf(x)=xf(x)=| x |f(x)= x 2f(x)= x 3 f(x)= 1 xf(x)= 1 x 2f(x)= xf(x)= x 3f(x)=c, c f(x)=xf(x)=| x |f(x)= x 2f(x)= x 3f(x)= 1 x f(x)= 1 x 2f(x)= xf(x)= x 3 y=f(x). { (a,b), (c,d), (a,c) }{(a,b),(b,c),(c,c)} y x. 5x+2y=10y= x 2 x= y 23 x 2 +y=142x+ y 2 =6y=−2 x 2 +40xy= 1 xx= 3y+5 7y−1x= 1− y 2y= 3x+5 7x−1x 2 + y 2 =9 2 = 1− x 2x=± 1−yy=± 1−xy 2 = x 2y 3 = x 2 f f(−3),f(2),f(−a),−f(a),f(a+h).f(x)=2x−5

f(−3)=−11; f(2)=−1; f(−a)=−2a−5; −f(a)=−2a+5; f(a+h)=2a+2h−5f(x)=−5 x 2 +2x−1f(x)= 2−x +5
f(−3)= 5 +5; f(2)=5; f(−a)= 2+a +5; −f(a)=− 2−a −5; f(a+h)=2−a−h +5f(x)= 6x−1 5x+2
f(x)=| x−1 |−| x+1 |
f(−3)=2; f(2)=1−3=−2; f(−a)=| −a−1 |−| −a+1 |; −f(a)=−| a−1 | +| a+1 |; f(a+h)= | a+h−1 |−| a+h+1 |
g(x)=5− x 2 , g(x+h)−g(x) h , h≠0. g(x)= x 2 +2x, g(x)−g(a) x−a , x≠a.g(x)−g(a) x−a =x+a+2, x≠a
k(t)=2t−1: k(2). k(t)=7. f(x)=8−3x: f(−2). f(x)=−1. f(−2)=14; x=3 p(c)= c 2 +c: p(−3). p(c)=2.
f(x)= x 2 −3x: f(5). f(x)=4. f(5)=10; x=−1 x=4 f(x)= x+2 : f(7). f(x)=4. 3r+2t=18. r=f(t). f(−3). f(t)=2.
f(t)=6− 2 3 t; f(−3)=8; t=6 f(−1). f(x)=3. f(0). f(x)=−3. f(0)=1; f(x)=−3, x=−2 x=2 f(4). f(x)=1.
{ (−1,−1),(−2,−2),(−3,−3) }{ (3,4),(4,5),(5,6) }{ (2,5),(7,11),(15,8),(7,9) } y x.xyxyxy f xf(x)
f(3). f(x)=1.f(x)=1, x=2 f f(−2), f(−1), f(0), f(1), f(2).f(x)=4−2xf(x)=8−3x
f(−2)=14; f(−1)=11; f(0)=8; f(1)=5; f(2)=2f(x)=8 x 2 −7x+3f(x)=3+ x+3
f(−2)=4; f(−1)=4.414; f(0)=4.732; f(1)=4.5; f(2)=5.236f(x)= x−2 x+3f(x)= 3 x
f(−2)= 1 9 ; f(−1)= 1 3 ; f(0)=1; f(1)=3; f(2)=9f, g, h:f(x)=3x−2g(x)=5− x 2h(x)=−2 x 2 +3x−1
3f(1)−4g(−2)f(7 3)−h(−2) y= x 2 [−0.1, 0.1][−10, 10][0, 100][−100,100] y= x 3 [−0.1, 0.1]
[−0.001, 0.001][−10, 10][−100, 100][−1,000,000, 1,000,000] y= x [0, 0.01][0, 100][0, 10][0, 10,000]
y= x 3[−0.001,0.001][−0.1,0.1][−1000,1000][−1,000,000,1,000,000][−100, 100] G, p G=f(p). G p f.
f(5)=2. D, a D=g(a). g. g(100)=1. g(5000)=50; f(t) t f(5)=30f(10)=40 h(t) t h(1)=200h(2)=350
f(x)=3 (x−5) 2 +7 (0, 100]. { (2, 10),(3, 10),(4, 20),(5, 30),(6, 40) }

{2,3,4,5,6}

{ (−5,4),(0,0),(5,−4),(10,−8),(15,−12) }{−5, 0, 5, 10, 15} f(x)= x 2 −1. x f (−∞,∞). f(x)=5−x+ x 3 .
(−∞,∞) x f(x)= x+1 2−x . x.

$$2-x=0 \quad -x=-2 \quad x=2$$

x<2 x>2. ∪,(−∞,2)∪(2,∞). f (−∞,2)∪(2,∞). f(x)= 1+4x 2x−1 .(−∞, 1 2)∪(1 2 ,∞) x. f(x)= 7−x . x.

$$7-x\geq 0 \quad -x\geq -7 \quad x\leq 7$$

7, (−∞,7]. f(x)= 5+2x .[− 5 2 ,∞) f(x)=− 1 x { x|10≤x<30 } x {} { x|10≤x<30 } x, x ∪,{2,3,5} {4,6}
{2,3,4,5,6}.

$$\{ x| \ |x|\geq 3 \}=(-\infty,-3]\cup[3,\infty)$$

{x| statement about x} x x

$$\{ x|4<x\leq 12 \}$$
$$(4,12]$$

∪ x, x 1≤x≤3 or x>5{ x|1≤x≤3 or x>5 }[1,3]∪(5,∞){ x|x≤−2 or −1≤x<3 }(−∞,−2]∪[−1,3) −5 [−5,∞). 5
(−∞,5]. f f (−3,1]. [−4 , 0). f t b 1973≤t≤2008 180≤b≤2010. f(x)=c, c, {c} [c,c], c. f(x)=x, x.
f(x)=| x |, x. f(x)= x 2 , f(x)= x 3 , f(x)= 1 x ,{x| x≠0}, f(x)= 1 x 2 ,0,0x f(x)= x , x − x x. f(x)= x 3 ,
f(x)=2 x 3 −x. (−∞,∞) (−∞,∞). f(x)= 2 x+1 . −1 (−∞,−1)∪(−1,∞). (−∞,0)∪(0,∞). f(x)=2 x+4 .

$$x+4\geq 0 \text{ when } x\geq -4$$

f(x) [−4,∞). f(−4)=0, x f [0, ∞) . f. f(x)=− 2−x . (−∞,2]; (−∞,0] f(x)=| x |.

$$f(x)=x \text{ if } x\geq 0$$
$$f(x)=-x \text{ if } x<0$$

S 0.1S S≤$10,000 $1000+0.2(S−$10,000) S>$10,000.

f(x)={ formula 1 if x is in domain 1 formula 2 if x is in domain 2 formula 3 if x is in domain 3

$$|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$$

n, C. C=5n. n C=50.

$$C(n)=\begin{cases} 5n & \text{if } 0<n<10 \\ 50 & \text{if } n\geq 10 \end{cases}$$

n=0 n=10 n=10, C, g

$$C(g)=\begin{cases} 25 & \text{if } 0<g<2 \\ 25+10(g-2) & \text{if } g\geq 2 \end{cases}$$

C(1.5),

$$C(1.5)=\$25$$

C(4),

C(4)=25+10(4−2)=$45

g=2.

f(x)={ x 2 if x≤1 3 if 1<x≤2 x if x>2

f(x)= x 2 if x≤1; f(x)=3 if 1< x≤2; f(x)=x if x>2 x=1 x=2 (1,3)(2,2)(1,1)(2, 3)

f(x)={ x 3 if x<−1 −2 if −1<x<4 x if x>4

f(x)= x 3 f(x)= x . x f(x)= x 3 (−∞,∞). x f(x)= x [0,∞). x y −∞ ∞. f(x)=−2x(x−1)(x−2)f(x)=5−2 x 2 (−∞,∞)f(x)=3 x−2f(x)=3− 6−2x(−∞,3]f(x)= 4−3xf(x)= x 2 +4(−∞,∞)f(x)= 1−2x 3f(x)= x−1 3(−∞,∞) f(x)= 9 x−6f(x)= 3x+1 4x+2(−∞,− 1 2)∪(− 1 2 ,∞)f(x)= x+4 x−4f(x)= x−3 x 2 +9x−22 (−∞,−11)∪(−11,2)∪(2,∞)f(x)= 1 x 2 −x−6f(x)= 2 x 3 −250 x 2 −2x−15(−∞,−3)∪(−3,5)∪(5,∞)5 x−3 2x+1 5−x(−∞,5)f(x)= x−4 x−6f(x)= x−6 x−4[6,∞)f(x)= x xf(x)= x 2 −9x x 2 −81 (−∞,−9)∪(−9,9)∪(9,∞) f(x)= 2 x 3 −50x (2,8], [6,8) [−4, 4], [0, 2] [−5, 3), [0,2] (−∞,1], [0,∞) [−6,− 1 6]∪[1 6 ,6]; [−6,− 1 6]∪[1 6 ,6] [−3, ∞); [0,∞) f(x)={ x+1 if x<−2 −2x−3 if x≥−2 f(x)={ 2x−1 if x<1 1+x if x≥1 (−∞,∞)f(x)={ x+1 if x<0 x−1 if x>0f(x)={ 3 if x<0 x if x≥0 (−∞,∞)

f(x)={ x 2 if x<0 1−x if x>0

f(x)={ x 2 x+2 if x<0 if x≥0 (−∞,∞)f(x)={ x+1 if x<1 x 3 if x≥1f(x)={ |x| 1 if x<2 if x≥2 (−∞,∞)f,f(−3), f(−2), f(−1),f(0).f(x)={ x+1 if x<−2 −2x−3 if x≥−2f(x)={ 1 if x≤−3 0 if x>−3 f(−3)=1; f(−2)=0; f(−1)=0; f(0)=0f(x)={ −2 x 2 +3 if x≤−1 5x−7 if x>−1 f, f(−1), f(0), f(2), f(4). f(x)={ 7x+3 if x<0 7x+6 if x≥0f(−1)=−4; f(0)=6; f(2)=20; f(4)=34f(x)={ x 2 −2 if x<2 4+| x−5 | if x≥2 f(x)={ 5x if x<0 3 if 0≤x≤3 x 2 if x>3f(−1)=−5; f(0)=3; f(2)=3; f(4)=16

f(x)={ x+1 if x<−2 −2x−3 if x≥−2f(x)={ x 2 −2 if x<1 − x 2 +2 if x>1 (−∞,1)∪(1,∞)

f(x)={ 2x−3 −3 x 2 if x<0 if x≥2 y= 1 x 2 [−0.5,−0.1] [0.1,0.5]. [−0.5,−0.1]; [4, 100] [0.1, 0.5]; [4, 100] y= 1 x [−0.5,−0.1] [0.1, 0.5]. f [−5, 8]. |f(x)|?[0, 8] x>2. f(x)= 1 x−2 . h t t h(t)=−16 t 2 +96t. [0, 6]; x C(x)=10x+500. C(x)? {x| statement about x}yC(y)

Average rate of change= Change in output Change in input = Δy Δx

= y 2 − y 1 x 2 − x 1 = f(x 2)−f(x 1) x 2 − x 1

Δ y x. Δf Δy,

Δy Δx = $1.37 7 years ≈0.196 dollars per year

Δy Δx = f(x 2)−f(x 1) x 2 − x 1

x 1 x 2 .y 2 − y 1 =Δy.x 2 − x 1 =Δx. Δy Δx .

Δy Δx = y 2 − y 1 x 2 − x 1 = $2.41−$2.84 2009−2007 = −$0.43 2 years =−$0.22 per year $2.84−$2.31 5 years = $0.53 5 years =$0.106 g(t) [−1,2].t=−1,g(−1)=4. t=2,g(2)=1. Δt=3 Δg(t)=−3

1−4 2−(−1) = −3 3 =−1

y 2 − y 1 x 1 − x 2 , (x 1 , y 1) (x 2 , y 2).

292−10 6−0 = 282 6 =47

f(x)= x 2 − 1 x[2, 4].

f(2)= 2 2 − 1 2 f(4)= 4 2 − 1 4 =4− 1 2 =16− 1 4 = 7 2 = 63 4

Average rate of change= f(4)−f(2) 4−2 = 63 4 − 7 2 4−2 = 49 4 2 = 49 8

f(x)=x−2 x[1, 9].1 2 F, d, F(d)= 2 d 2 . F(d)= 2 d 2 [2,6].

Average rate of change = F(6)−F(2) 6−2 = 2 6 2 − 2 2 2 6−2 Simplify.

= 2 36 − 2 4 4 = − 16 36 4

Combine numerator terms. =− 1 9 Simplify

− 1 9 g(t)= t 2 +3t+1[0, a].a.

Average rate of change= g(a)−g(0) a−0 Evaluate. = (a 2 +3a+1)−(0 2 +3(0)+1) a −0 Simplify. = a 2 +3a+1−1 a Simplify and factor. = a(a+3) a Divide by the common factor a. =a+3

a t=0 t=a. [0,5], 5+3=8.f(x)= x 2 +2x−8[5, a]. a+7 f(x)= x 3 −12x (−∞, −2) ∪ (2, ∞) (−2, 2). x=−2. −16 x=2. f f(b)>f(a) ab b>a. f f(b)<f(a) abb>a.fb(a,c)f(b)f(x)xxbfb(a,c)f(b)f(x)xxb p(t) t=1 t=3 t=4 (4,∞). t=1 t=3 t=4 f(x)= 2 x + x 3 . x=2 x=3, x=−3 x=−2. (−∞,−2.449) (2.449,∞). ± 6 ,

f(x)= x 3 −6 x 2 −15x+20 (−1,28), (5,−80). (−∞,−1)∪(5,∞) (−1,5). f f. x=1 x=1. y x=1, 2. x=−1 x=−1. x=−1, −2. y- f(x)= x 3 f x=c f(c) f(c)≥f(x) x f. f x=d f(d) f(d)≤f(x) x f. f f. x=−2 x=2, x=−2 x=2, 16. x=3, x=3,−10.Δy Δx = f(x 2)−f(x 1) x 2 − x 1 f (a,b) (b,c), f (a,c)? y= x 2 , b h. f(x)=4 x 2 −7 [1, b]4(b+1)g(x)=2 x 2 −9 [4, b]p(x)=3x+4 [2, 2+h]k(x)=4x−2 [3, 3+h] f(x)=2 x 2 +1 [x,x+h]4x+2hg(x)=3 x 2 −2 [x,x+h]a(t)= 1 t+4 [9,9+h]−1 13(13+h)b(x)= 1 x+3 [1,1+h]j(x)=3 x 3 [1,1+h]3 h 2 +9h+9r(t)=4 t 3 [2,2+h]f(x+h)−f(x) h f(x)=2 x 2 −3x [x,x+h] 4x+2h−3 f x=1 x=4. x=2 x=5.4 3 (−∞,−2.5)∪(1,∞), (−2.5, 1) (−∞,1)∪(3,4), (1,3)∪(4,∞) f (−3, 60), (3, −60) (7, 150), (−7.5, −220)f(x)= x 2 [1, 5]h(x)=5−2 x 2 [−2,4]q(x)= x 3 [−4,2] g(x)=3 x 3 −1 [−3,3]y= 1 x [1, 3]p(t)= (t 2 −4)(t+1) t 2 +3 [−3,1]k(t)=6 t 2 + 4 t 3 [−1,3] f(x)= x 4 −4 x 3 +5 (3,−22), (−∞, 3), (3, ∞) h(x)= x 5 +5 x 4 +10 x 3 +10 x 2 −1g(t)=t t+3 (−2,−2), (−3,−2), (−2, ∞)k(t)=3 t 2 3 −tm(x)= x 4 +2 x 3 −12 x 2 −10x+4 (−0.5, 6), (−3.25,−47) (2.1,−32), (−∞,−3.25) (−0.5, 2.1), (−3.25, −0.5) (2.1, ∞) n(x)= x 4 −8 x 3 +18 x 2 −6x+2 f (1.333, 5.185) f(x)= 1 x . c f (1,c) − 1 4 . f(x)= 1 x b f (2,b) − 1 10 .b=5 d(t)=2.6667 t 2 , t d(t) t=1 t=2. t t=5 t=15. f(b)<f(a) a b b>a f(b)>f(a) a b b>a C(T) C T T(d) d Cost=C(T(d)) T(d). T(5) C(T(5)). w(y) h(y) y, T

$$T(y)=h(y)+w(y)$$

$$T=h+w$$

f(x) g(x) f+g, f−g, fg, f g

(f+g)(x)=f(x)+g(x) (f−g)(x)=f(x)−g(x) (fg)(x)=f(x)g(x) (f g)(x)= f(x) g(x)

(g−f)(x) (g f)(x), f(x)=x−1 g(x)= x 2 −1.

(g−f)(x)=g(x)−f(x) (g−f)(x)= x 2 −1−(x−1) = x 2 −x =x(x−1) (g f)(x)= g(x) f(x)

(g f)(x)= x 2 −1 x−1 = (x+1)(x−1) x−1 where x≠1 =x+1

(g f)(x), x≠1 x=1, (fg)(x) (f−g)(x).

$$f(x)=x-1 \quad \text{and} \quad g(x)= x^2-1$$

(fg)(x)=f(x)g(x)=(x−1)(x 2 −1)= x 3 − x 2 −x+1 (f−g)(x)=f(x)−g(x)=(x−1)−(x 2 −1)=x− x 2

$$(f\circ g)(x)=f(g(x))$$

“f g x,”“f g x.” ∘ f(g(x))≠f(x)g(x). g x g(x). f g(x) f(g(x)). f∘g g∘f f(g(x))≠g(f(x)) x.

f(x)= x 2 g(x)=x+2,

f(g(x))=f(x+2) = (x+2) 2 = x 2 +4x+4

g(f(x))=g(x 2) = x 2 +2

x, x=− 1 2 . x f g, f∘g

$$(f\circ g)(x)=f(g(x))$$

f∘g x x g g(x) f. fg f(g(x)), f(x)g(x)≠f(g(x)). f(g(x)) g(f(x)).

f(x)=2x+1 g(x)=3−x

g(x) f(x).

f(g(x))=2(3−x)+1 =6−2x+1 =7−2x

f(x) g(x).

g(f(x))=3−(2x+1) =3−2x−1 =−2x+2

g(f(x))≠f(g(x)), c(s) s s(t) t c(s(3)). s(3). t=3 s(3) s(3) c(s) f(x) x g(y) y f(g(y)) g(f(x))? y=f(x)

number of miles =f (number of hours)

g(y)

number of gallons =g (number of miles)

g(y) f(x) f(g(y)) f(x) g(y) f(x) g(y), g(f(x)) g, f(x), x f(g(y)) g(f(x)) G(r).Fa(F).a(G(r))G(a(F)) f(g(3)) g(f(3)).xf(x)g(x) f(g(3)), g(3) g: g(3)=2. f, g(3) f(2). f, f(2)=8.

$$g(3)=2 \quad f(g(3))=f(2)=8$$

g(f(3)), f(3) f(3)=3. g,

$$g(f(3))=g(3)=2$$

f∘g g∘f xg(x)f(g(x))f(x)g(f(x)) f(g(1)) g(f(4)).f(g(1))=f(3)=3 g(f(4))=g(1)=3 x- y- x- y- x- y- f(g(1)). f(g(1)), g(1) g(x), x- g(1)=3. f.

$$f(g(1))=f(3)$$

f(x), x- f(3)=6, f(g(1))=6. g(f(2)).g(f(2))=g(5)=3 f(g(x)). f(t)= t 2 –t, f(t)= t 2 –t h(x)=3x+2, f(h(1)). h(1), h(x)

$$h(1)=3(1)+2 \quad h(1)=5$$

f(h(1))=f(5), f(t)

$$f(h(1))=f(5) \quad f(h(1))= 5\ 2 -5 \quad f(h(1))=20$$

t x f(t)= t 2 –t h(x)=3x+2, h(f(2))h(f(–2)) f∘g g f. f∘g. f g x f(g(x)), x g g(x) f, f(g(x)) f∘g g g f. f g x x g g(x) f. f(g(x)) x g g(x) f. f(g(x)), g. f. x g g(x) f. x g g(x) f. f∘g.

$$(f\circ g)(x) \quad \text{where} \quad f(x)= 5\ x-1 \quad \text{and} \quad g(x)= 4\ 3x-2$$

g(x) x= 2 3 , f g(x) x g(x)=1.

$$4\ 3x-2 =1 \qquad 4=3x-2 \qquad 6=3x \qquad x=2$$

f∘g 2 3 2.

$$x\neq 2\ 3 \quad \text{or} \quad x\neq 2$$

$$(-\infty, 2\ 3)\cup(2\ 3 ,2)\cup(2,\infty)$$

$$(f\circ g)(x) \quad \text{where} \quad f(x)= x+2 \quad \text{and} \quad g(x)= 3-x$$

g (–∞,3].

$$(f\circ g)(x)= 3-x+2 \quad \text{or} \quad (f\circ g)(x)= 5-x$$

(–∞,5]. f∘g, (–∞,3] f∘g. f∘g g, (–∞,3]. f∘g f, g.

$$(f\circ g)(x) \quad \text{where} \quad f(x)= 1\ x-2 \quad \text{and} \quad g(x)= x+4$$

[–4,0)∪(0,∞) f(x)= 5– x 2 g h, f(x)=g(h(x)). f(x). 5– x 2

$$h(x)=5- x\ 2 \text{ and } g(x)= x$$

$$g(h(x))=g(5- x\ 2)= 5- x\ 2$$

f(x)= 4 3– 4+ x 2 g(x)= 4+ x 2h(x)= 4 3–xf=h∘g(f∘g)(x)=f(g(x)) f g ? g f g, f∘g? f(x)=x+1 and g(x)=x–1. f(g(x))=f(x–1)=(x–1)+1=x g(f(x))=g(x+1)=(x+1)–1=x. f∘g=g∘f. f∘g? f(x)= x 2 +2x g(x)=6– x 2 , f+g, f–g, fg, f g .(f+g)(x)=2x+6, (–∞,∞)(f–g)(x)=2 x 2 +2x–6, (–∞,∞) (fg)(x)=– x 4 –2 x 3 +6 x 2 +12x, (–∞,∞)(f g)(x)= x 2 +2x 6– x 2 , (–∞,– 6)∪(– 6 , 6)∪(6 ,∞) f(x)=–3 x 2 +x g(x)=5, f+g, f–g, fg, f g . f(x)=2 x 2 +4x g(x)= 1 2x , f+g, f–g, fg, f g . (f+g)(x)= 4 x 3 +8 x 2 +1 2x , (–∞,0)∪(0,∞)(f–g)(x)= 4 x 3 +8 x 2 –1 2x , (–∞,0)∪(0,∞) (fg)(x)=x+2, (–∞,0)∪(0,∞)(f g)(x)=4 x 3 +8 x 2 , (–∞,0)∪(0,∞) f(x)= 1 x–4 g(x)= 1 6–x , f+g, f–g, fg, f g . f(x)=3 x 2 g(x)= x–5 , f+g, f–g, fg, f g . (f+g)(x)=3 x 2 + x–5 , [5,∞) (f–g)(x)=3 x 2 – x–5 , [5,∞)(fg)(x)=3 x 2 x–5 , [5,∞)(f g)(x)= 3 x 2 x–5 , (5,∞) f(x)= x g(x)=|x–3|, g f . f(x)=2 x 2 +1 g(x)=3x–5, f(g(2))f(g(x))g(f(x))(g∘g)(x)(f∘f)(–2) f(g(x))=2 (3x–5) 2 +1; f(g(x))=6 x 2 –2; (g∘g)(x)=3(3x–5)–5=9x–20; (f∘f)(–2)=163 f(g(x)) g(f(x)). f(x)= x 2 +1, g(x)= x+2f(x)= x +2, g(x)= x 2 +3f(g(x))= x 2 +3 +2, g(f(x))=x+4 x +7f(x)=| x |, g(x)=5x+1 f(x)= x 3 , g(x)= x+1 x 3f(g(x))= x+1 x 3 3 = x+1 3 x , g(f(x))= x 3 +1 xf(x)= 1 x–6 , g(x)= 7 x +6 f(x)= 1 x–4 , g(x)= 2 x +4(f∘g)(x)= 1 2 x +4–4 = x 2 , (g∘f)(x)=2x–4 f(g(h(x))). f(x)= x 4 +6, g(x)=x–6, h(x)= xf(x)= x 2 +1, g(x)= 1 x , h(x)=x+3f(g(h(x)))= (1 x+3) 2 +1 f(x)= 1 x g(x)=x–3, (f∘g)(x) (f∘g)(x) (g∘f)(x) (g∘f)(x) (f g)x f(x)= 2–4x g(x)=– 3 x , (g∘f)(x) (g∘f)(x) (g∘f)(x)=– 3 2–4x ; (–∞, 1 2) f(x)= 1–x x and g(x)= 1 1+ x 2 , (g∘f)(x)(g∘f)(2) p(x)= 1 x m(x)= x 2 –4, p(x) m(x)p(m(x)) m(p(x)) (0,2)∪(2,∞); (–∞,–2)∪(2,∞); (0,∞) q(x)= 1 x h(x)= x 2 –9, q(x) h(x)q(h(x))h(q(x)) f(x)= 1 x g(x)= x–1 , (f∘g)(x) (1,∞) f(x) g(x) h(x)=f(g(x)).h(x)= (x+2) 2h(x)= (x–5) 3f(x)= x 3 g(x)=x–5 h(x)= 3 x–5h(x)= 4 (x+2) 2f(x)= 4 x g(x)= (x+2) 2h(x)=4+ x 3h(x)= 1 2x–3 3f(x)= x 3 g(x)= 1 2x–3 h(x)= 1 (3 x 2 –4) –3h(x)= 3x–2 x+5 4f(x)= x 4 g(x)= 3x–2 x+5h(x)= (8+ x 3 8– x 3) 4h(x)= 2x+6 f(x)= xg(x)=2x+6h(x)= (5x–1) 3h(x)= x–1 3f(x)= x 3g(x)=(x–1)h(x)=| x 2 +7 |h(x)= 1 (x–2) 3f(x)= x 3 g(x)= 1 x–2h(x)= (1 2x–3) 2h(x)= 2x–1 3x+4f(x)= xg(x)= 2x–1 3x+4 f, g,f(g(3))f(g(1))g(f(1))

g(f(0))f(f(5))f(f(4))g(g(2))g(g(0)) f(x), g(x), h(x),g(f(1))g(f(2))f(g(4))f(g(1))f(h(2)) h(f(2))f(g(h(4)))f(g(f(−2))) f and g xf(x)g(x)f(g(8))f(g(5))g(f(5))g(f(3))f(f(4)) f(f(1))g(g(2))g(g(6)) f and g xf(x)g(x)(f∘g)(1)(f∘g)(2)(g∘f)(2)(g∘f)(3)(g∘g)(1)(f∘f)(3) f(g(0)) g(f(0)).f(x)=4x+8, g(x)=7− x 2f(x)=5x+7, g(x)=4−2 x 2f(g(0))=27, g(f(0))=−94 f(x)= x+4 , g(x)=12− x 3f(x)= 1 x+2 , g(x)=4x+3f(g(0))= 1 5 , g(f(0))=5 f(x)=2 x 2 +1 g(x)=3x+5 f(g(2)) f(g(x))18 x 2 +60x+51g(f(−3))(g∘g)(x)g∘g(x)=9x+20 f(x)= x 3 +1 g(x)= x−1 3 .(f∘g)(x) (g∘f)(x). (f∘g)(2) (g∘f)(2). (g∘f)(x)? (f∘g)(x)?(−∞,∞) f(x)= 1 x .(f∘f)(x). (f∘f)(x) f F(x)= (x+1) 5 , f(x)= x 5 , g(x)=x+1. (g∘f)(x)=F(x). (f∘g)(x)=F(x). f(x)= x 2 +2 x≥0 g(x)= x−2 .(f∘g)(6); (g∘f)(6)(f∘g)(6)=6 (g∘f)(6)=6(g∘f)(a); (f∘g)(a)(f∘g)(11); (g∘f)(11)(f∘g)(11)=11 , (g∘f)(11)=11 D(p) p. C(x) x D(C(6)). C(D(6)). D(C(x))=6. C(D(p))=6. A(d) d t m(t). A(m(4)). m(A(4)). A(m(t))=4. m(A(d))=4. x P(x) x. r(t)=25 t+2 , t=2.A(t)=π (25 t+2) 2 A(2)=π (25 4) 2 =2500π r(t)=2t+1, t A(5)=π (2(5)+1) 2 =121π r, V, r(V)= 3V 4π 3 . t V(t)=10+20t. r(V(t)).N(T)=23 T 2 −56T+1, 3<T<33, TT(t)=5t+1.5,t N(T(t)). N(T(t))=23 (5t+1.5) 2 −56(5t+1.5)+1; x y g(x)=f(x)+k, f(x) k k=1 f(x)= x 3 . y=f(x). f(x)+k y+k. y y+k, y- k. f(x),g(x)=f(x)+k, kf(x).kkk V t. S(t),

$$S(t)=V(t)+20$$

t,S(t) V S V, tV(t)S(t) f(x) g(x)=f(x)−3.xf(x) g(x)=f(x)−3 g f.

f(2)=1 Given g(x)=f(x)−3 Given transformation g(2)=f(2)−3 =1−3 =−2

f(x) g(x) xf(x)g(x) h(t)=−4.9 t 2 +30t h t b(t) h(t), b(t).

$$b(t)=h(t)+10=-4.9t^2+30t+10$$

f(x)= x 3 . h=+1 x. f(x)= x 2 , g(x)= (x−2) 2 f. f, g(x)=f(x−h), h f. h h V(t) F(t)

V(t)= the original venting plan F(t)=starting 2 hrs sooner

V V, F, V(8)=F(6). 220 ft 2 220 ft 2 V(10)=F(8). F(t) h=−2. F(t)=V(t−(−2))=V(t+2). V(t+2) F(t) V(t), V(t). V F:F(t)=V(t+2). f(x) g(x)=f(x−3).xf(x) g(x)=f(x−3) g f f(2)=1. g, g(x) f.

$$g(5)=f(5-3)=f(2)=1$$

xx−3f(x)g(x) g(x) g(x) f(x), x, f(x)= x 2 . g(x) f(x), g(x). f(x)= x 2

$$g(x)=f(x-2)$$

x=2 y=0; f(x) g(x) f(x−2).

$$f(x)=x^2\quad g(x)=f(x-2)\quad g(x)=f(x-2)=(x-2)^2$$

+2 −2 f(0)=0. g(2)=0. f, g(2)=f(x−2)=f(0)=0. G(m) m G(m)+10 G(m+10).G(m)+10 m G(m+10) m f(x)= x , f(x) g(x)=f(x+2) f(x) g(x) y-x- f(x)=| x |, h(x)=f(x+1)−3. f h f f(x+1) f(x+1)−3 f(x)=| x |.(0,0) (0,0)→(−1,0)(−1,0)(−1,0)→(−1,−3) h. f(x)=| x |, h(x)=f(x−2)+4.

$$h(x)=f(x-1)+2$$

$$h(x)=|x-1|+2$$

[1,∞) [2,∞). f(x)= 1 x g(x)= 1 x-1 +1 f(x), g(x)=−f(x) f(x), f(x), g(x)=f(−x) f(x), s(t)= t

$$V(t)=-s(t) \text{ or } V(t)=-t$$

s(t)

$$H(t)=s(-t) \text{ or } H(t)=-t$$

[0,∞) [0,∞), V(t) (−∞, 0] H(t) (−∞, 0]. f(x)=|x−1| f(x) g(x)=−f(x) h(x)=f(−x)xf(x) g(x), x g(x) h(x), h(x) f(x) xh(x) f(x) g(x)=−f(x)h(x)=f(−x)xf(x)g(x)=−f(x)xg(x)−5−10−15−20h(x)=f(−x)xh(x) k(t)=− 2 −t +1, ktf(t)= 2 tk(t).f(−t)= 2 −t −f(−t)=− 2 −t −f(−t)+1=− 2 −t +1 t≥0, [0,1). f(x)= x 2 , g(x)=−f(x) h(x)=f(−x). g(x)=f(−x) f(x)f(x)= x 2f(x)=| x | f(x)= x 3 f(x)= 1 x f(x)= 2 x f(x)=0. x

$$f(x)=f(-x)$$

y- x

$$f(x)=-f(-x)$$

f(x)=f(−x). f(x)=−f(−x). f(x)= x 3 +2x

$$f(-x)=(-x)^3+2(-x)=-x^3-2x$$

$$-f(-x)=-(-x^3-2x)=x^3+2x$$

−f(−x)=f(x), f (x,y) (−x,−y) f, (−1,−3) f(s)= s 4 +3 s 2 +7 f(x), g(x)=af(x), a f(x). a>1, 0<a<1, a<0,

a. a. a>1, a. 0<a<1, a. a<0, P(t) Q,

(0, 1)→(0, 2) (3, 3)→(3, 6) (6, 2)→(6, 4) (7, 0)→(7, 0)

Q(t)=2P(t)

t, Q P. t, a. a. f g(x)= 1 2 f(x).xf(x) g(x)= 1 2 f(x) g f f(4)=3.

g(4)= 1 2 f(4)= 1 2 (3)= 3 2

x2468g(x)1 23 27 211 2 g(x) 1 2 . f g(x)= 3 4 f(x).xf(x)xg(x) f(x)= x 3 . g(x) f(x), g(x). g(2)=2. f(2)= 2 3 =8. g 1 4 f g(2)= 1 4 f(2). g(x)= 1 4 f(x). g f.

g(x)= 1 4 f(x)= 1 4 x 3

g(x)=3x-2 y=f(x), y=f(bx) y= x 2 . y= (0.5x) 2 y= x 2 y= (2x) 2 y= x 2 f(x), g(x)=f(bx), b f(x). b>1, 1 b . 0<b<1, 1 b . b<0, g(x)=f(bx) b>1 0<b<1 R,

R(1)=P(2), R(2)=P(4), and in general, R(t)=P(2t).

f(x) g(x)=f(1 2 x).xf(x) g(x)=f(1 2 x) g f g(2) g(2)=f(1 2 ·2)=f(1), f(1) g f g(4).

g(4)=f(1 2 ·4)=f(2)=1

xg(x) g(x) g(x) f(x) g(x) f(x) f(x) (6,4) g(x) (2,4), x- 1 3 , 6(1 3)=2. g(2)=f(6) g(1)=f(3). g(x)=f(3x). 1 3 . 1 4 f(1 4 x). g(x)=f(1 3 x) g(x)= 1 3 x 2f(x)+3, f(x), g(x)=f(2x+3), g f. f(7)=12. g x g(x)=f(2x+3)=12? 2x+3=7. x,

f(bx+p)=f(b(x+ p b))

f(x)= (2x+4) 2

f(x)= (2(x+2)) 2

af(x)+k, a k. f(bx+h), h 1 b . f(b(x+h)), 1 b h. f(x), g(x)=2f(3x)+1.xf(x) f(3x) 1 3 , x- 1 3 .xf(3x)x 2f(3x)xg(x)=2f(3x)+1 f(x) k(x)=f(1 2 x+1)−3.

f(1 2 x+1)−3=f(1 2 (x+2))−3

1 2 x+2. −3 g(x)=f(x)+k k>0g(x)=f(x−h) h>0g(x)=−f(x)g(x)=f(−x)g(x)=af(x) a>0g(x)=af(x) (0<a<1) g(x)=f(bx)(0<b<1)g(x)=f(bx) b>1 x-y-y- f(x)=f(−x). f(x)=−f(−x). f, (−x) (x) f(x). f(−x)=f(x), f(−x)=−f(x), f(x)= x f(x)=| x | g(x)=|x-1|−3 f(x)= 1 x f(x)= 1 x 2 g(x)= 1 (x+4) 2 +2 f.y=f(x−49) y=f(x+43) f(x+43) f.y=f(x+3)y=f(x−4) f(x-4) f.y=f(x)+5y=f(x)+8 f(x)+8 f.y=f(x)−2y=f(x)−7 f(x)−7 f. y=f(x−2)+3y=f(x+4)−1f(x+4)−1f.f(x)=4 (x+1) 2 −5g(x)=5 (x+3) 2 −2 (−∞,−3) (−3,∞)a(x)= −x+4 k(x)=−3 x −1(0, ∞) f(x)= 2 x f(x).g(x)= 2 x +1h(x)= 2 x −3w(x)= 2 x−1f(t)= (t+1) 2 −3h(x)=|x−1|+4 k(x)= (x−2) 3 −1m(t)=3+ t+2 f, g, h g(x) h(x) f(x).xf(x)xg(x)xh(x)g(x)=f(x-1), h(x)=f(x)+1 f, g, h g(x) h(x) f(x).xf(x)xg(x)xh(x)f(x)=|x-3|−2f(x)= x+3 −1f(x)= (x-2) 2f(x)=|x+3|−2f(x)=− xf(x)=− (x+1) 2 +2 f(x)= −x +1f(x)=3 x 4g(x)= xh(x)= 1 x +3xf(x)= (x−2) 2g(x)=2 x 4h(x)=2x− x 3 f.g(x)=−f(x) g x f. g(x)=f(−x)g(x)=4f(x) g f.g(x)=6f(x)g(x)=f(5x) g 1 5 f.g(x)=f(2x)g(x)=f(1 3 x) g f.g(x)=f(1 5 x) g(x)=3f(−x) g y f.g(x)=−f(3x) g f(x)=|x| y 1 4g(x)=|−4x| f(x)= x x f(x)= 1 x 2 1 3 , g(x)= 1 3 (x+2) 2 −3 f(x)= 1 x f(x)= x 2 1 2 , g(x)= 1 2 (x-5) 2 +1 f(x)= x 2 g(x)=4 (x+1) 2 −5 f(x)= x 2 g(x)=5 (x+3) 2 −2h(x)=−2|x−4|+3 f(x)=|x| k(x)=−3 x −1m(x)= 1 2 x 3 f(x)= x 3 1 2 . n(x)= 1 3 |x−2|p(x)= (1 3 x) 3 −3q(x)= (1 4 x) 3 +1a(x)= −x+4 f(x)= x x=4.g(x)=f(x)−2g(x)=−f(x) g(x)=f(x+1)g(x)=f(x−2) f(x)=f(−x), y- b>1 −1 0<b<1 f(x)=−f(−x), 0<a<1 −1 a>1 f(x)=| x |,

f(x)=| x |={ x if x≥0 −x if x<0

x x x | x−5 |. x | x−5 |≤4.

−4≤x−5 x−5≤4 1≤x x≤9

| x−5 |≤4 1≤x≤9. x | x−2 |≤3 ±1%, ±5%, ±10%. ±5%. R

| R−680 |≤34

p | p−80 |≤20 y=2| x−3 |+4. y=| x | (3,4)

f(x)=2|x−3|−2, treating the stretch as a vertical stretch, or f(x)=|2(x−3)|−2,

treating the stretch as a horizontal compression.

x f(x).

f(x)=a|x−3|−2

2=a| 1−3 |−2 4=2a a=2

f(x)=−| x+2 |+3 8=| 2x−6 |,

$$2x-6=8 \text{ or } 2x-6=-8 \qquad 2x=14 \qquad 2x=-2 \qquad x=7 \qquad x=-1$$

$$|x|=4, |2x-1|=3 \; 5x+2|-4=9$$

A B, | A |=B, B≥0, A=B A=−B. B<0, | A |=B | A |=B A=B −A=B, B>0. x. f(x)=| 4x+1 |−7x f(x)=0 0=|4x+1|−7 Substitute 0 for f(x). 7=|4x+1| Isolate the absolute value on one side of the equation. 7=4x+1 or −7=4x+1 Break into two separate equations and solve. 6=4x −8=4x x= 6 4 =1.5 x= −8 4 =−2 x=1.5 x=−2. f(x)=| 2x−1 |−3, x f(x)=0.x=−1 x=2 | A |=B? 2+| 3x−5 |=1. | A |=B A=B A=−B. x. 1=4| x−2 |+2.

$$1=4|x-2|+2 \quad -1=4|x-2| \; -1 \; 4 =|x-2|$$

f(x)=1 g(x)=4| x−2 |+2 f g 1=4| x−2 |+2 f(x)=−| x+2 |+3 f(0)=1, (0,1). f(x)=0 x=−5 x=1 (−5,0) (1,0).

$$|A|<B, |A|\le B, |A|>B, \text{ or } |A|\ge B,$$

A B x. x

$$|x|<200 \text{ or } -200<x<200$$

x x x | x−600 |.

$$|x-600|<200 \quad \text{or} \quad -200<x-600<200 \quad -200+600<x-600+600<200+600 \qquad 400<x<800$$

| x−A |≤B a b b | x−A |=B. | x−A |≤B. |x −5|≤4. | x−5 |=4. | x−5 |=4.

$$x-5=4 \qquad x=9 \quad \text{or} \quad x-5=-4 \quad x=1$$

x=1 x=9,

$$x<1, \; 1<x<9, \text{ and } \; x>9.$$

xf(x)<4 >4?x<1| 0−5 |=51<x<9| 6−5 |=1x>9| 11−5 |=6 1≤x≤9 | x−5 |≤4 1≤x≤9, [1,9]. f(x)=| x−5 |. g(x)=4 x=1 x=9. f g 1<x<9. f(x) g(x). 1≤x≤9. [1,9].

$$|x-A|<C, |x-A|>C, \quad -C<x-A<C, x-A<-C \text{ or } x-A>C.$$

< > ≤ or ≥.

| x−5|≤4 −4≤x−5≤4 Rewrite by removing the absolute value bars. −4+5≤x−5+5≤4+5 Isolate the x.

$$1\le x\le 9$$

| x+2 |≤6.4≤x≤8f(x)=− 1 2 | 4x−5 |+3, x- f(x)<0, − 1 2 |4x−5|+3<0.

− 1 2 |4x−5|<−3 Multiply both sides by −2, and reverse the inequality. |4x−5|>6

| 4x−5 |=6.

$$4x-5=6 \; 4x-5=-6 \; 4x-5=6 \quad \text{or} \qquad 4x=-1 \qquad x=\tfrac{11}{4} \qquad x=-\tfrac{1}{4}$$

f x=− 1 4 x= 11 4 x=− 1 4 x= 11 4 . x=− 1 4 , x= 11 4 .

$$x<-\tfrac{1}{4} \quad \text{or} \quad x>\tfrac{11}{4}$$

(−∞,−0.25)∪(2.75,∞). −2| k−4 |≤−6.k≤1 k≥7; (−∞,1]∪[7,∞) | A |<B, | A |≤B, | A |>B, or | A |≥B. |A|=B. A, B. A −B. x x x x 1 2 | x+4 |= 1 2 x f(x) f(x) |f(x)−8|<0.03|x+3|=9|6−x|=5{ 1,11 } |5x−2|=11|4x−2|=11{ 9 4 , 13 4 }2|4−x|=73|5−x|=5{ 10 3 , 20 3 }3|x+1|−4=55| x−4 |−7=2 { 11 5 , 29 5 }0=−| x−3 |+22| x−3 |+1=2{ 5 2 , 7 2 }| 3x−2 |=7| 3x−2 |=−7| 1 2 x−5 |=11| 1 3 x+5 |=14 { −57,27 }−| 1 3 x+5 |+14=0f(x)=2| x+1 |−10(0,−8); (−6,0), (4,0)f(x)=4| x−3 |+4f(x)=−3| x−2 |−1 (0,−7); xf(x)=−2| x+1 |+6| x−2 |>10(−∞,−8)∪(12,∞)2| v−7 |−4≥42| 3x−4 |≤8−4 3 ≤x≤4| x−4 |≥8 | 3x−5 |≥13(−∞,− 8 3]∪[6,∞)| 3x−5 |≥−13| 3 4 x−5 |≥7(−∞,− 8 3]∪[16,∞)| 3 4 x−5 |+1≤16y=|x−1| y=|x+1|y=|x|+1y=| x |−2y=−| x |y=−| x |−2y=−| x−3 |−2f(x)=−|x−1|−2f(x)=−|x+3|+4f(x)=2|x+3|+1 f(x)=3| x−2 |+3f(x)=| 2x−4 |−3f(x)=| 3x+9 |+2f(x)=−| x−1 |−3f(x)=−| x+4 |−3f(x)= 1 2 | x+4 |−3 f(x)=10|x−2|[0,4]. [0,20] f(x)=−100|x|+100 [−5,5]. f(x)=−0.1| 0.1(0.2−x) |+0.3x- f(x)=4× 10 9 | x−(5× 10 9) |+2× 10 9|−2x− 2 3 (x+1)|+3>−1(−∞,∞)ax-f(x)=2| x+1 |+a. a y f(x)=2| x+1 |+a. a y y x=0. x p | p−0.08 |≤0.015 x x | x−5.0 |≤0.01 x | A |=B, B≥0; A=B A=−B | A |<B, | A |≤B, | A |>B, or | A |≥B

$$C=\tfrac{5}{9}(F-32)$$

F

$$\tfrac{5}{9}(75-32)\approx 24°C.$$

F C.

$$26=\tfrac{5}{9}(F-32) \qquad 26\cdot\tfrac{9}{5}=F-32 \qquad F=26\cdot\tfrac{9}{5}+32\approx 79$$

f(x), f −1 (x), “f x.” −1 −1 f −1 (x) 1 f(x) 1 f(x) f a −1 a=1 a, f −1 ∘f

(f −1 ∘f)(x)= f −1 (f(x))= f −1 (y)=x

x f. f(x), g(x) f(x) g(f(x))=x f(g(x))=x y=4x y= 1 4 x

(f −1 ∘f)(x)= f −1 (4x)= 1 4 (4x)=x

(f ∘ f −1)(x)=f(1 4 x)=4(1 4 x)=x

y=4x y= 1 4 x f(x)=y, f −1 (x) f f −1 (y)=x. f −1 (f(x))=x x f. f(f −1 (x))=x x f −1 f −1 f. f −1“ f f −1 , f −1 (x),“fx.”

f −1 (x)≠ 1 f(x)

f(2)=4 f(5)=12,

f(2)=4, then f −1 (4)=2; f(5)=12, then f −1 (12)=5.

g, g(4)=2 g(12)=5.(x,f(x))(x,g(x))(2,4)(4,2)(5,12)(12,5) h −1 (6)=2, h? h(2)=6 f(x) g(x), f(g(x))=x g(f(x))=x.g= f −1f= g −1 . g≠ f −1 f≠ g −1 . f(x)= 1 x+2 g(x)= 1 x −2, g= f −1 ?

g(f(x))= 1 (1 x+2) −2 =x+2−2 =x

g= f −1 and f= g −1

f(g(x))= 1 1 x −2+2 = 1 1 x =x

f(x)= x 3 −4 g(x)= x+4 3 , g= f −1 ? f(x)= x 3 g(x)= 1 3 x, g= f −1 ?

f(g(x))= x 3 27 ≠x

x 3 = x 1 3 , f(x)= (x−1) 3 and g(x)= x 3 +1, g= f −1 ? f f −1 , f f −1 . f f −1 , f f −1 . f(x)= x f −1 (x)= x 2 , [0,∞), f(x)= x . f(x)= x 2 . f(x)= x 2 [0,∞), f(x)= (x−1) 2 [1,∞), f −1 (x)= x +1. f f −1 [1,∞). f −1 f [0,∞). g h, f, g=h. f(x) f −1 (x). f(x) f −1 (x).f(x)=cf(x)=xf(x)= x 2f(x)= x 3 f(x)= 1 xf(x)= 1 x 2f(x)= x 3f(x)= xf(x)=| x | [0,∞), (0,∞). f (1,∞) f (−∞,−2). f −1 (−∞,−2) f −1 (1,∞). f(t) t f −1 (70).t (minutes)f(t) (miles) f f. f −1 (70), f, f −1 (70)=90. f(a)=b, f −1 (b)=a. f −1 (70)=a, a f(a)=70. t f(t)=70, t=90. f(60), f −1 (60).t (minutes)f(t) (miles)f(60)=50. f −1 (60)=70. g(x) g(3) g −1 (3).g(3), (3,1) g(3)=1. g −1 (3), g −1 (3) g(x)=3. (5,3) g(5)=3, g −1 (3)=5. g −1 (1), g −1 (4). y x— x y. f x. x y.

C= 5 9 (F−32)

C= 5 9 (F−32) C· 9 5 =F−32 F= 9 5 C+32

C=h(F)= 5 9 (F−32),

F= h −1 (C)= 9 5 C+32.

h C −1 x y y= 1 3 (x−5)x=3y+5 f(x)= 2 x−3 +4.

y= 2 x−3 +4 Set up an equation. y−4= 2 x−3 Subtract 4 from both sides. x−3= 2 y−4 Multiply both sides by x−3 and divide by y−4. x= 2 y−4 +3 Add 3 to both sides.

f −1 (y)= 2 y−4 +3 f −1 (x)= 2 x−4 +3. f f f −1 xf −1 (y)f(x)y f(x)=2+ x−4 .

y=2+ x−4 (y−2) 2 =x−4 x= (y−2) 2 +4

f −1 (x)= (x−2) 2 +4. f [4,∞). f [2,∞), f −1 [2,∞). f −1 (x) x. f −1 f f −1 [2,∞) f −1 f −1 f. f(x)=2− x ?f −1 (x)= (2−x) 2 ; domain of f: [0,∞); domain of f −1 : (−∞,2] f(x)= x 2 [0,∞), [0,∞) f −1 (x)= x .f f −1x-f and f −1 ? f −1 (x) f(x) y=x, f(x) f −1 (x). (0,∞) (−∞,∞), (−∞,∞) (0,∞). y=x, (1,0) (0,1) (4,2) (2,4). f f −1 f= f −1 , f(f(x))=x,

1 1 x =x

f(x)=c−x, c g(x) f(x), g(f(x))=f(g(x))=x. y=f(x) x y. x y. y=x. y y f(x)= x 2 f(x)= 1 x y=f(x), x y. x y. y. y y= f −1 (x). f(x)=a−x a. f −1 (x) f(x)=x+3f −1 (x)=x−3f(x)=x+5f(x)=2−xf −1 (x)=2−x f(x)=3−xf(x)= x x+2f −1 (x)= −2x x−1f(x)= 2x+3 5x+4 f f f(x)= (x+7) 2f(x): [−7,∞); f −1 (x)= x −7 f(x)= (x−6) 2f(x)= x 2 −5 f(x): [0,∞); f −1 (x)= x+5 f(x)= x 2 +x g(x)= 2x 1−x : f(g(x)) g(f(x)). f(x) g(x)? f(g(x))=x g(f(x))=x. f g f(x) g(x) f(x)= x−1 3 g(x)= x 3 +1 f(g(x))=x, g(f(x))=xf(x)=−3x+5 g(x)= x−5 −3f(x)= xf(x)= 3x+1 3f(x)=−5x+1f(x)= x 3 −27 f f(0).3 f(x)=0. f −1 (0).2 f −1 (x)=0. f −1 . f(6) and f −1 (2). f f. [2,10] f f. f f(6)=7, f −1 (7).6 f(3)=2, f −1 (2). f −1 (−4)=−8, f(−8).−4 f −1 (−2)=−1, f(−1).xf(x) f(1).0 f(x)=3. f −1 (0). 1 f −1 (x)=7. f f −1 (x).xf(x)xf −1 (x) f(x)= 3 x−2f(x)= x 3 −1f −1 (x)= (1+x) 1/3 f(x)= 1 x−1 . x y f(x)= 9 5 x+32. f −1 (x)= 5 9 (x−32). x, C

C(r)=2πr. r(C). r(36π) t, d(t)=50t. t(d). t(180) t(d)= d 50 , t(180)= 180 50 . { (a,b),(c,d),(e,d) } { (5,2),(6,1),(6,2),(4,8) }y 2 +4=x, x y f(−3); f(2); f(−a); −f(a); f(a+h).f(x)=−2 x 2 +3xf(−3)=−27; f(2)=−2;f(−a)=−2 a 2 −3a;−f(a)=2 a 2 −3a;f(a+h)=−2 a 2 +3a−4ah+3h−2 h 2f(x)=2| 3x−1 |f(x)=−3x+5 f(x)=| x−3 |f(x)=| x+1 |f(x)= x 2 −2f(2)f(−2)2 f(x)=−2, x. f(x)=1, x.x=−1.8 or x=1.8 h(t)=−16 t 2 +80t h(2)−h(1) 2−1h(a)−h(1) a−1−64+80a−16 a 2 −1+a =−16a+64f(x)= 2 3x+2f(x)= x−3 x 2 −4x−12 (−∞,−2)∪(−2,6)∪(6,∞)f(x)= x−6 x−4f(x)={ x+1 x<−2 −2x−3 x≥−2 x=1 to x=2.f(x)=4x−3 f(x)=10 x 2 +x31f(x)=− 2 x 2 (2,∞); (−∞,2)(−3,1); (−∞,−3)∪(1,∞) (−2,−3); (1,3) [−3,3]. [−10,10]. (−1.8,10) (f∘g)(x) (g∘f)(x) f(x)=4−x, g(x)=−4xf(x)=3x+2, g(x)=5−6x (f∘g)(x)=17−18x; (g∘f)(x)=−7−18xf(x)= x 2 +2x, g(x)=5x+1f(x)= x+2 , g(x)= 1 x (f∘g)(x)= 1 x +2 ; (g∘f)(x)= 1 x+2 f(x)= x+3 2 , g(x)= 1−x (f∘g) (f∘g)(x) f(x)= x+1 x+4 , g(x)= 1 x (f∘g)(x)= 1+x 1+4x , x≠0, x≠− 1 4f(x)= 1 x+3 , g(x)= 1 x−9f(x)= 1 x , g(x)= x(f∘g)(x)= 1 x , x>0 f(x)= 1 x 2 −1 , g(x)= x+1 H f g H(x)=(f∘g)(x).H(x)= 2x−1 3x+4 g(x)= 2x−1 3x+4 ; f(x)= x H(x)= 1 (3 x 2 −4) −3f(x)= (x−3) 2f(x)= (x+4) 3f(x)= x +5f(x)=− x 3f(x)= −x 3f(x)=5 −x −4 f(x)=4[| x−2 |−6]f(x)=− (x+2) 2 −1 g f g(x)=f(x−1)g(x)=3f(x)f(x)=| x−3 |f(x)=3 x 4g(x)= xh(x)= 1 x +3x f(x)=| x |.f(x)= 1 2 | x+2 |+1f(x)=−3| x−3 |+3f(x)=| x−5 |f(x)=−| x−3 |f(x)=| 2x−4 || x+4 |=18x=−22, x=14 | 1 3 x+5 |=| 3 4 x−2 || 3x−2 |<7(− 5 3 ,3)| 1 3 x−2 |≤7 f −1 (x) f(x)=9+10xf(x)= x x+2f −1 (x)= −2x x−1 f f f(x)= x 2 +1f(x)= x 3 −5g(x)= x+5 3 : f(g(x))g(f(x)).f(x)g(x)? f(g(x))=xg(f(x))=x.fgf(x)= 1 x f(x)=−3 x 2 +xf(5)=2,f −1 (2).5f(1)=4,f −1 (4).y=2x+8{ (2,1),(3,2),(−1,1),(0,−2) } f(x)=−3 x 2 +2x f(−2) f(a) f(x)=−2 (x−1) 2 +3 f(x)= 3−x f(x)=2 x 2 −5x, f(a+1)−f(1).2 a 2 −a f(x)={ x+1 if −2<x<3 −x if x≥3 f(x)=3−2 x 2 +x f(b)−f(a) b−a .−2(a+b)+1 f(x)=3−2 x 2 +x and g(x)= x (g∘f)(x)(g∘f)(1)2 H(x)= 5 x 2 −3x 3 f g, (f∘g)(x)=H(x).f(x)= x+6 −1 f(x)= 1 x+2 −1f(x)=− 5 x 2 +9 x 6evenf(x)=− 5 x 3 +9 x 5f(x)= 1 xodd f(x)=−2| x−1 |+3.| 2x−3 |=17.x=−7 x=10 −| 1 3 x−3 |≥17. f(x)=3x−5f −1 (x)= x+5 3f(x)= 4 x+7 g (−∞,−1.1) and (1.1,∞)(1.1,−0.9) f(2). f(2)=2 f(−2).f(x)={ | x | if x≤2 3 if x>2xF(x) F(6). F(x)=5.x=2 F −1 (15). f(x)=−2x+11, f −1 (x). f −1 (x)=− x−11 2 f(x), f −1 (x) f −1 (f(x))=x x f; f(f −1 (x))=x x f −1xmb

Equation form y=mx+b Equation notation f(x)=mx+b

D(t)Dt.m,b

D(t)=83t+250

D(t)=83t+250,(0,250)D(t)=83t+250.f(x)=2 x +1.

f(x)=mx+b

bx=0m(0,b).P,d,P(d)=0.434d+14.696.f(x)=mx+b is an increasing function if m>0. f(x)=mx+b is an decreasing function if m<0.f(x)=mx+b is a constant function if m=0.f(x)=60xx f(x)=500−60xxxf(x)=50x 1x 2 ,y 1y 2(x 1 , y 1)(x 2 , y 2)m,

m= change in output (rise) change in input (run) = Δy Δx = y 2 − y 1 x 2 − x 1

ΔyΔxy1y2f,y1=f(x1)y2=f(x2),

m= f(x 2)−f(x 1) x 2 − x 1

(x 1, y 1)(x 2, y 2),yx.(x 2, y 2)(x 1 , y 1),units for the output units for the inputm

m= change in output (rise) change in input (run) = Δy Δx = y 2 − y 1 x 2 − x 1

x 1x 2y 1y 2f(x)(3,−2)(8,1)(3,−2)(8,1).

m= change in output change in input = 1−(−2) 8−3 = 3 5

m=0.6.m>0.f(x)(2, 3)(0, 4)m= 4−3 0−2 = 1 −2 =− 1 2m<0.27,800−23,400=4400

4,400 people 4 years =1,100 people year

m= 1,868−1,442 2,012−2,009 = 426 3 =142 people per year

y− y 1 =m(x− x 1)

m= y− y 1 x− x 1 assuming x≠ x 1 m(x− x 1)= y− y 1 x− x 1 (x− x 1) Multiply both sides by (x− x 1).

m(x− x 1)=y− y 1 Simplify. y− y 1 =m(x− x 1) Rearrange.

y−4=− 1 2 (x−6)

y−4=− 1 2 (x−6) y−4=− 1 2 x+3 Distribute the − 1 2 . y=− 1 2 x+7 Add 4 to each side.

y=− 1 2 x+7.

$$y-y_1=m(x-x_1)$$

mx 1 and y 1x and y(4,1).m=2x 1 =4y 1 =1.

$$y-y_1=m(x-x_1)\ y-1=2(x-4)$$

$y-1=2(x-4)$ $y-1=2x-8$ Distribute the 2. $y=2x-7$ Add 1 to each side.

y−1=2(x−4)y=2x−7,(6,−1).m=3.x1=6y1 =−1.

$$y-y_1=m(x-x_1)\ y-(-1)=3(x-6)$$ Substitute known values. $y+1=3(x-6)$ Distribute −1 to find point-slope form.

$y+1=3(x-6)$ $y+1=3x-18$ Distribute 3. $y=3x-19$ Simplify to slope-intercept form.

−2(−2, 2).y−2=−2(x+2)y=−2x−2(0, 1)(3, 2).

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-1}{3-0}=\frac{1}{3}$$

$$y-y_1=m(x-x_1)\ y-1=\frac{1}{3}(x-0)$$

$y-1=\frac{1}{3}(x-0)$ $y-1=\frac{1}{3}x$ Distribute the $\frac{1}{3}$. $y=\frac{1}{3}x+1$ Add 1 to each side.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{7-1}{8-5}=\frac{6}{3}=2$$

m=2.(5,1).

$$y-y_1=m(x-x_1)\ y-1=2(x-5)$$

y 2 −1=2(x 2 −5).

$y-1=2(x-5)$ $y-1=2x-10$ $y=2x-9$

y=2x−9.(−1,3) (0,0).y−0=−3(x−0)y=−3xf(0, 7)(4, 4).

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-7}{4-0}=-\frac{3}{4}$$

$$y-y_1=m(x-x_1)\ y-4=-\frac{3}{4}(x-4)$$

$y-4=-\frac{3}{4}(x-4)$ $y-4=-\frac{3}{4}x+3$ $y=-\frac{3}{4}x+7$

b=7.bmmbf(x)=− 3 4 x+7,y=− 3 4 x+7.f(0, 2)(−2,−4).

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-2}{-2-0}=\frac{-6}{-2}=3$$

$$y-y_1=m(x-x_1)\ y-(-4)=3(x-(-2))\ y+4=3(x+2)$$

$y+4=3(x+2)$ $y+4=3x+6$ $y=3x+2$

(0, 2)b=2.CC(x)x37.5.C(x)=1250+37.5x.

$$C(100)=1250+37.5(100)=5000$$

ff(3)=−2f(8)=1

$$f(3)=-2\rightarrow(3,-2)\ f(8)=1\rightarrow(8,1)$$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-(-2)}{8-3}=\frac{3}{5}$$

$$y-y_1=m(x-x_1)\ y-(-2)=\frac{3}{5}(x-3)$$

$y+2=\frac{3}{5}(x-3)$ $y+2=\frac{3}{5}x-\frac{9}{5}$ $y=\frac{3}{5}x-\frac{19}{5}$

f(x)f(2)=−11,f(4)=−25,y=−7x+3ff(c).f(x)=mx+b.x=c.N,t,N(0)=200,b=200.m=15.N(t)=15t+200.t=12.

$$N(12)=15(12)+200=180+200=380$$

I,n,I(n),(3,760)(5,920).

$$m=\frac{920-760}{5-3}=\$160\text{ 2 policies}=\$80\text{ per policy}$$

$I(n)=80n+b$ $760=80(3)+b$ when $n=3$, $I(3)=760$ $760-80(3)=b$ $520=b$

bn=0,

$$I(n)=80n+520$$

b=1000.m,

$$P(w)=40w+1000$$

(2,1080)(6,1240)

$$m=\frac{1240-1080}{6-2}=\frac{160}{4}=40$$

f(x)=mx+b,b.xH(x),xxH(x)H(x)=0.5x+12.5f(x)=mx+b

m= change in output (rise) change in input (run) $=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$ $y-y_1=m(x-x_1)$mbE(t),

tE(t)=3000−70t.E(t),tE(t)=1200+40t.d(t)=100−10tny= 1 4 x+6y=3x−5y=3 x 2 −23x+5y=15

3 x 2 +5y=153x+5 y 2 =15−2 x 2 +3 y 2 =6− x−3 5 =2yf(x)=4x+3g(x)=5x+6a(x)=5−2xb(x)=8−3x

h(x)=−2x+4k(x)=−4x+1j(x)=12x−3p(x)=14x−5n(x)=−13x−2m(x)=−38x+3(2, 4)(4, 10)(1, 5)(4, 11)(−1,4)

(5,2)−13(8,−2)(4,6)(6, 11)(−4, 3)45f(−5)=−4,f(5)=2f(−1)=4f(5)=1f(x)=−12x+72(2,4)(4,10)(1,5)(4,11)

y=2x+3(−1, 4)(5, 2)(−2, 8)(4, 6)y=−13x+223(−2, 0)(0,−3)(−5, 0)(0, 4)y=45x+4−54y= 2 3 x+1y=−2x+3

y=3

x
g(x)
g(x)=−3x+5xh(x)xf(x)f(x)=5x−5
x
k(x)
x
g(x)
g(x)=−252x+6
x
f(x)
x
f(x)
f(x)=10x−24
x
k(x)
ff(0.1)=11.5, and f(0.4)=−5.9,f(x)=−58x+17.3f[−10, 10]: f(x)=0.02x−0.01.x−10x10.f
[−10, 10]:fx)=2,500x+4,000w,k,k.k,wkp,q,q.k.pqa=11,900b=1001.1q(p)=1000p−100f[−10,10]183116
−1010.f[−0.1,0.1]−22.5−0.10.1.ff(x)=ax+b[−4,4]ab.a=2; b=3a=2; b=4a=2; b=−4a=2; b=−5x
(x,2),(−4,6), m=3x=−163(10,y),(25,100), m=−5(a, b)(a, b+1)x=a(2a, b)(a, b+1)(a, 0)(c, d)
y=dc−ax−adc−ann,p,$30p(n)=mn+bpnC(n)=24+0.1n,nC(n)n,y,y=mn+bn−400.P(t),tI(x)=1054x+23,286,x
C,F(C).F(28).F(−40).f(x)=mx+b, then m<0.f(x)=mx+b, then m>0.y−y1=m(x−x1)f(x)=mx+bf(x)=x.
f(x)=2x,(1,2).(2,4).f(x)=−23x+5

x=0 f(0)=− 2 3 (0)+5=5⇒(0,5) x=3 f(3)=− 2 3 (3)+5=3⇒(3,3) x=6 f(6)=− 2 3 (6)+5=1⇒(6,1)
f(x)=−23x+5.f(x)=−23x+5.f(x)=−34x+6x=0m,

f(x)=12x+1

12.x=0.(0,1).(0,1)m=riserun.m=12,(0,1),f(x)=mx+bb(0,b)m

m= change in output (rise) change in input (run) = Δy Δx = y 2 − y 1 x 2 − x 1
riserunf(x)=−23x+5x=0x=0(0,5).−23.−2(0,5)(−3,7),(−6,9),(−9,11).f(x)=xf(x)=mx,mmf(x)=xmfmm>1fm
0<m<1.m,f(x)=x.f(x)=mx+b,bbf(x)=xfbb|b|bf(x)=x.f(x)=mx+b.f(x)=x.m.bf(x)=12x−3m=1212.b=−3y=x,
12.y=12x,f(x)=4+2x,

f(2)= 1 2 (2)−3 =1−3 =−2

(0, 4)(−2,0).

m= rise run = 4 2 =2

y=2x+4
f(x)=2x+3g(x)=2x−3h(x)=−2x+3j(x)=12x+3g(0, 3)f−3.(0,−3)12(0, 3),jfjf(x)x.

f(x)=3x−6

x.

0=3x−6 6=3x 2=x x=2

(2, 0).y=c,cy=5xf(x)=0.0=mx+b.f(x)=12x−3.x.

0= 1 2 x−3 3= 1 2 x 6=x x=6

(6, 0).(6, 0)f(x)=12x−3.f(x)=14x−4.(16, 0)m=0f(x)=mx+b,f(x)=b.f(x)=2.f(x)=2.x=0.x=2.x=2,f(x)=b.x=a.
−4,y=−4.7,x=7.1.m1 and m2−1.

m1m2=−1

18,18

f(x)= 1 4 x+2 negative reciprocal of 1 4 is −4 f(x)=−4x+3 negative reciprocal of−4 is 1 4
−4(14)=−1

f(x)=m1x + b1 and g(x)=m2x + b2 are parallel if m1 = m2.

b1=b2m1=m2,

f(x)= m 1 x+ b 1 and g(x)= m 2 x+ b 2 are perpendicular if m 1 m 2 =−1, and so m 2 =− 1 m 1 .

f(x)=2x+3 h(x)=−2x+2 g(x)= 1 2 x−4 j(x)=2x−6

f(x)=2x+3j(x)=2x−612g(x)=12x−4h(x)=−2x+2f(x)=2x+3j(x)=2x−6g(x)=12x−4h(x)=−2x+2

f(x)=3x+1

(0,1).f(x).f(x).

g(x)=3x+6 h(x)=3x+1 p(x)=3x+ 2 3

f(1, 7).b

y− y 1 =m(x− x 1) y−7=3(x−1) y−7=3x−3 y=3x+4

g(x)=3x+4f(x)=3x+1(1, 7).f(x)=3x+6(3, 0).m=3,x=3,f(x)=0

g(x)=3x+b 0=3(3)+b b=−9

f(x)(3, 0)g(x)=3x−9.

f(x)=2x+4

−12.−12f(x).f(x).

g(x)=− 1 2 x+4 h(x)=− 1 2 x+2 p(x)=− 1 2 x− 1 2

f(x)(4, 0).−12.b.

g(x)=mx+b 0=− 1 2 (4)+b 0=−2+b 2=b b=2

−12

g(x)=−12x+2.

g(x)=−12x+2f(x)=2x+4(4, 0).xyg(x)=mx+b.b.f(x)=3x+3(3, 0).m=3,−13.

g(x)=− 1 3 x+b 0=− 1 3 (3)+b 1=b b=1

f(x)(3, 0)g(x)=−13x+1.h(x)=2x−4,(0,0)h(x)h(x)f(x)=2xg(x)=− 1 2 x(−2, 6)(4,5).(4,5).

m 1 = 5−6 4−(−2) = −1 6 =− 1 6

m 2 = −1 − 1 6 =−1(− 6 1) =6

(4,5).

g(x)=6x+b 5=6(4)+b 5=24+b −19=b b=−19

(4,5)

y=6x−19

(−2,−15)(2,−3).(6,4).y=−13x+6f(x)=g(x).h(t)=3t−4j(t)=5−t.h(t)=j(t).

3t−4=5−t 4t=9 t= 9 4

94.

j(9 4)=5− 9 4 = 11 4

(94,114).j(t):j(t)h(t)j(t)j(t)f(x)=x.(0,5)(5, 0)C,xR,xC and R.

C(x)=120x+250,000

xR(x)=140x.x.

C(x)=R(x) 250,000+120x=140x 250,000=20x 12,500=x

x=12,500

y,

R(20)=140(12,500) =$1,750,000

(12,500,1,750,000). f(x)=b.x=b.f(x)=mx+b,bx.f(x)=ax=a,(a,a).yaxa.y=mx+bb.y=mx+bmb.
4x−7y=10 7x+4y=13y+x=12 −y=8x+13y+4x=12 −6y=8x+16x−9y=10 3x+2y=1y= 2 3 x+1 3x+2y=1
y= 3 4 x+1 −3x+4y=1f(x)=−x+2g(x)=2x+4(−2, 0)(0, 4)h(x)=3x−5k(x)=−5x+1(15, 0)(0, 1)−2x+5y=20
7x+2y=56(8, 0)(0, 28)(0,6)(3,−24)(−1,19)(8,−71)(−8,−55)(10, 89)(9,−44)(4,−14)
Line 1: m=8 Line 2: m=−6 Neither(2,3)(4,−1)(6,3)(8,5)(1,7)(5,5)(−1,−3)(1,1)
Line 1: m=− 1 2 Line 2: m=2 Perpendicular(0,5)(3,3)(1,−5)(3,−2)(2,5)(5,−1)(−3,7)(3,−5)
Line 1: m=−2 Line 2: m=−2 Parallelf(x)=−5x−3(2, −12).g(x)=3x−1(4,9).g(x)=3x−3h(t)=−2t+4(-4, −1).
p(t)=3t+4(3,1).p(t)=−13t+2f(x)=−2x−1g(x)=−x.f(x)=2x+5g(x)=−3x−5.(−2,1)f(x)= −45x +27425
h(x)=94x + 7310.f(x)=74x + 45760g(x)=43x + 315.(−175,53)f(x)=−x−1f(x)=−2x−1f(x)=−12x−1
f(x)=2f(x)=2+xf(x)=3x+2(−4, 0)(0, −2)(−2, 0)(0, 4)(0, 7)−32(0, 3)25(−6, −2)(6, −6)(−3, −4)(3, 0)
f(x)=−2x−1g(x)=−3x+2h(x)=13x+2k(x)=23x−3f(t)=3+2tp(t)=−2 + 3tx=3x=−2r(x)=4q(x)=34x=−9y+36
x3−y4=13x−5y=153x=153y=12g(x)f(x)=x3 4 ,g(x).g(x)=0.75x−5.5(0,−5.5)g(x)f(x)=x13,g(x).y=3x=−3
y= 3 4 x+1 −3x+4y=122x−3y=12 5y+x=302x=y−3y+4x=15(2, 7)x−2y+2=3 x−y=35x+3y=−65 x−y=−5
(−10, −5)g(x)=−0.01x + 2.01(1, 2).g(x)=−0.01x+2.01(1, 2).y=100x−98f(x)=−0.1x+200 and g(x)=20x+0.1.

fg.f(x)g(x)?g(x)f(x)?x< 1999 201 x> 1999 201f(x)=b,bx=a,a M, t, M(t) M. M(t)=mt+b. x-

0=–400t+3500 t= 3500 400 =8.75

x- x- 0≤t≤8.75.y-y-x-y-

f(x)=mx+b =–250x+1000

xx-

0=–250x+1000 1000=250x 4=x x=4

x- x-x-x-y- y-t,P(t),(t=9),

m= change in output change in input

t=0,(0,6200).t=5,(5,8100). (0,6200) (5,8100).

m= 8100–6200 5–0 = 1900 5 =380 people per year

P(t)=380t+6200

t=9.

P(9)=380(9)+6,200 =9,620

P(t)=15000 t.

15000=380t+6200 8800=380t t≈23.158

C x, C(x)=0.25x+25,000(0,25,000). t, A(t), E(t), t=0,AE

A(t)=4tE(t)=3t

A,E,D, D, t, A(t),E(t), D(t)

D (t) 2 =A (t) 2 +E (t) 2 = (4t) 2 + (3t) 2 =16 t 2 +9 t 2 =25 t 2 D(t)=± 25 t 2

Solve for D(t) using the square root =±5|t|

t, D(t) D(t)=5t. DD(t)=2 t.

D(t)=2 5t=2 t= 2 5 =0.4

(30, 10), (20,0).

m= 10–0 30–0 = 1 3

W(x)= 1 3 x

m=–3.

E(x)=–3x+b 0=–3(20)+b Substitute in (20, 0) b=60 E(x)=–3x+60

1 3 x=–3x+60 10 3 x=60 10x=180 x=18 Substituting this back into W(x) y=W(18) = 1 3 (18)

=6

distance= (x 2 – x 1) 2 + (y 2 – y 1) 2 = (18–0) 2 + (6–0) 2 ≈18.974 miles

x,d,K(d):M(d)K(0)=20M(0)=16 K(d)=$0.59 P(d)=$0.63 f(x)=mx+b.

K(d)=0.59d+20 M(d)=0.63d+16

K(d)<M(d). K(d) K(d)

K(d)=M(d) 0.59d+20=0.63d+16 4=0.04d 100=d d=100

K(d)d>100. y=0, mx+b x=3, f(x)=1+2x, f(x) (2, 7). f(x)=12– 1 3 x, f(x) f(x)=9– 6 7 x, f(x) g(x)=2,

f(x)=3x, f(x) (6,1). P t, t P(t)=75,000+2500t P. P P W t.W(t)=7.5t+0.5WW(–15,0) (0, 7.5) C t.

C(t)=12,025–205tC. C (58.7, 0)(0,12,025) C y, t, t y, y t, y=–2t+180 y, t, t y, y t, y=30t–300P,

P(t)=305+174tPtC(x)=0.15x+10xP(t)=190t+4360P.R,t,R(t)=16–2.1tR,t,riserun.

m=6050=1.2

T(c)=1.2c+30

cT(c)x=50,

T(30)=30+1.2(30) =66 degrees

40=30+1.2c 10=1.2c c≈8.33

54°F

T(c)=30.281+1.143c

T(30)=30.281+1.143(30) =64.571 ≈64.6 degrees

r.rr,r.r=0.9509.t,

C(t)=113.318+2.209t

(t=14),

$$C(14)=113.318+2.209(14) \quad =144.244$$

r,(x)(y).

$$y=ax+b \; a=-1.341 \; b=32.234 \; r=-0.896$$

x,y,

$$y=ax+b \; a=6.301 \; b=-1.044 \; r=-0.970$$

60° F.r=0.95r=–0.89r=0.26r=–0.39r=0.985xyy=1.640x+13.800r=0.987xyxyxy

$$y=-0.962x+26.86, \quad r=-0.965$$

xyxyy=–1.981x+60.197r=–0.998xyxyy=0.121x–38.841, r=0.998f(x)=0.5x+10x=–2, 1, 5, 6, 9
f(x)=–2x–10x=–2, 1, 5, 6, 9(–2,–6),(1,–12),(5,–20),(6,–22),(9,–28)y=–2x–10
(46, 1,600), (48, 1,550), (50, 1,505), (52, 1,540), (54, 1,495)P(189.8,0)
(2500, 2000), (2650, 2001), (3000, 2003), (3500, 2006), (4200, 2010)y,y=0.00587x+1985.41
(46, 250), (48, 305), (50, 350), (52, 390), (54, 410)y=20.25x–671.5
(46, 250), (48, 225), (50, 205), (52, 180), (54, 165).y=–10.75x+742.502x+3y=76x2–y=5f(x)=7x–2
g(x)=–x+2(7,5)(3,17)y=–3x+26(6,0)(0,10)y=2x–2xg(x)xg(x)2x–6y=12 –x+3y=1y= 1 3 x–2 3x+y=–9
7x+9y=–63(–9,0);(0,–7)f(x)=2x–1(5,11)(10,1)(–1,3)(–5,11)m=–2;m=–2;(8,–10)(0,–26)(2,5)(4,4)
f(x)=5x–1y=–0.2x+21(0, 2)–12f(t)=2t–5x=y+62x–y=13f(x)=10–2xft.y,x,xx,y=–300x + 11,500
P(t)=100t + 1700P.xyxyy=–1.294x+49.412; r=–0.974
(3,600, 2000); (4,000, 2001); (4,700, 2003); (6,000, 2006)y,2x + 3y=7f(x)=–2x + 5f(x)=7x + 9y =–1.5x –6
y=–2x – 1xg(x)xg(x)ny= 3 4 x–9 –4x–3y=8–2x+y=3 3x+ 3 2 y=52x + 7y=–14.(–7,0)(0,–2)(–2,–6)(3,14)
(2,6)(4,14)f(x)=4x+3(8,10).y=–0.25x+12(0,5)–52f(x)=–x+6x=y+22x–3y=–1f(x)=12–4xfCt.yxxyyx
y=875x + 10,675y=–46.875t+1250xy(4,500, 2000); (4,700, 2001); (5,200, 2003); (5,800, 2006)
y=0.00455x + 1979.5r=0.999r,

$$x^2+4=0$$

i

$$\sqrt{-1}=i$$

$$i^2=(\sqrt{-1})^2=-1$$

i.

$$\sqrt{-25}=\sqrt{25}\cdot\sqrt{(-1)} \quad =25\sqrt{-1} \quad =5i$$

5i –5i 25 a+bi a bi 5+2i 3+4 3 i . a+bi a bi b=0, a+bi a=0 b –a a –1 . –1 i. a ·i –9

$$\sqrt{-9}=\sqrt{9}\sqrt{-1}=3i$$

0+3i. –24 –24 =0+2i 6 (a,b), a b –2+3i. –2 3i. (–2,3) –2+3i 3–4i 3, –4i. (3,–4) –4–i

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

3–4i 2+5i.

$$(a+bi)+(c+di)=(a+c)+(b+d)i \quad (3-4i)+(2+5i)=(3+2)+(-4+5)i \quad =5+i$$

2+5i 3–4i. (3–4i)–(2+5i)=1–9i 4(2+5i).

$$4(2+5i)=(4\cdot 2)+(4\cdot 5i) \quad =8+20i$$

–4(2+6i).–8–24i

$$(a+bi)(c+di)=ac+adi+bci+bd\,i^2$$

$i^2=-1$,

$$(a+bi)(c+di)=ac+adi+bci-bd$$

$$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$$

(4+3i)(2–5i). (a+bi)(c+di)=(ac–bd)+(ad+bc)i

$$(4+3i)(2-5i)=(4\cdot 2-3\cdot(-5))+(4\cdot(-5)+3\cdot 2)i \quad =(8+15)+(-20+6)i \quad =23-14i$$

(3–4i)(2+3i).18+i a+bi a–bi. a+bi a–bi, a–bi a+bi. c+di a+bi, a b
c+di a+bi where a≠0 and b≠0

$$(c+di)(a+bi)\cdot(a-bi)(a-bi)=(c+di)(a-bi)(a+bi)(a-bi)$$

$$=ca-cbi+adi-bd\,i^2\,a^2-abi+abi-b^2i^2$$

$i^2=-1$.

= ca−cbi+adi−bd(−1) a 2 −abi+abi− b 2 (−1) = (ca+bd)+(ad−cb)i a 2 + b 2
a+bi a−bi. 2+i 5− 1 2 i a+bi. a−bi, 2−i 5 . a+bi 0− 1 2 i. a−bi, 0+ 1 2 i. 1 2 i. i. (2+5i) (4−i).
(2+5i) (4−i)
(2+5i) (4−i) · (4+i) (4+i)
(2+5i) (4−i) · (4+i) (4+i) = 8+2i+20i+5 i 2 16+4i−4i− i 2 = 8+2i+20i+5(−1) 16+4i−4i
−(−1) Because i 2 =−1 = 3+22i 17 = 3 17 + 22 17 i
Separate real and imaginary parts.
f(x)= x 2 −5x+2. f(3+i). x=3+i f(x)= x 2 −5x+2 f(3+i)=−5+i. 3+i −5+i. f(x)=2 x 2 −3x. f(8−i).
102−29i f(x)= 2+x x+3 . f(10i). x=10i
2+10i 10i+3 Substitute 10i for x. 2+10i 3+10i Rewrite the denominator in standard form. 2+10i 3+10i ·
3−10i 3−10i Prepare to multiply the numerator and denominator by the complex conjugate
of the denominator. 6−20i+30i−100 i 2 9−30i+30i−100 i 2
Multiply using the distributive property or the FOIL method. 6−20i+30i−100(−1) 9−30i+30i−100(−1)
Substitute −1 for i 2 . 106+10i 109 Simplify. 106 109 + 10 109 i Separate the real and imaginary parts.
f(x)= x+1 x−4 . f(−i).− 3 17 + 5i 17 i i
i 1 =i i 2 =−1 i 3 = i 2 ·i=−1·i=−i i 4 = i 3 ·i=−i·i=− i 2 =−(−1)=1 i 5 = i 4 ·i=1·i=i
i, i i.
i 6 = i 5 ·i=i·i= i 2 =−1 i 7 = i 6 ·i= i 2 ·i= i 3 =−i i 8 = i 7 ·i= i 3 ·i= i 4 =1 i 9 = i 8 ·i= i 4 ·i= i 5 =i
i i 35 . i 4 =1, i 4 35=4·8+3.
i 35 = i 4·8+3 = i 4·8 · i 3 = (i 4) 8 · i 3 = 1 8 · i 3 = i 3 =−i
i 35 i 35 i 3 i 35 i 35i 34 ·ii 33 · i 2i 31 · i 4i 19 · i 16(i 2) 17 ·ii 33 ·(−1)i 31 ·1i 19 · (i 4) 4
(−1) 17 ·i− i 33i 31i 19 i. i i i If f(x)= x 2 +x−4, f(2i).−8+2iIf f(x)= x 3 −2, f(i).If f(x)= x 2 +3x+5,
f(2+i).14+7iIf f(x)=2 x 2 +x−3, f(2−3i).If f(x)= x+1 2−x , f(5i).− 23 29 + 15 29 iIf f(x)= 1+2x x+3 ,
f(4i).1−2i−2+3ii−3−4i(3+2i)+(5−3i)8−i(−2−4i)+(1+6i)(−5+3i)−(6−i)−11+4i(2−3i)−(3+2i)
(−4+4i)−(−6+9i)2−5i(2+3i)(4i)(5−2i)(3i)6+15i(6−2i)(5)(−2+4i)(8)−16+32i(2+3i)(4−i)
(−1+2i)(−2+3i)−4−7i(4−2i)(4+2i)(3+4i)(3−4i)3+4i 26−2i 32− 2 3 i−5+3i 2i6+4i i4−6i2−3i 4+3i
3+4i 2−i2 5 + 11 5 i2+3i 2−3i−9 +3 −1615i− −4 −4 −252+ −12 21+i 34+ −20 2i 81i 15i 22−1 (1+i) k
k=4, 8, and 12. k=16. (1−i) k k=2, 6, and 10. k=14. (1+i)k − (1−i) kk=4, 8, and 12 k=16. x 6 +1=0
3 2 + 1 2 i.(3 2 + 1 2 i) 6 =−1 x 8 −1=0 2 2 + 2 2 i.1 i + 4 i 33i1 i 11 − 1 i 21i 7 (1+ i 2)i −3 +5 i 7
(2+i)(4−2i) (1+i)(1+3i)(2−4i) (1+2i)(3+i) 2 (1+2i) 2−2i3+2i 2+i +(4+3i)4+i i + 3−4i 1−i
9 2 − 9 2 i3+2i 1+2i − 2−3i 3+i
a+bi,
a bi bi i= −1 x y=0. y- (3,1). x=3. x- y- (0,7)
f(x)=a x 2 +bx+c
a,b, c a≠0. a>0, a<0, x=− b 2a . x= −b± b 2 −4ac 2a , a x 2 +bx+c=0 x- x x=− b 2a , y= x 2 +4x+3.
a=1,b=4, c=3. a>0, x=− 4 2(1) =−2. x=−2 (−2,−1). x- x- (−3,0) (−1,0).
f(x)=a (x−h) 2 +k
(h, k) a>0, a<0, y=−3 (x+2) 2 +4. x−h=x+2 h=−2. a=−3,h=−2, k=4. a<0, (−2, 4). y= x 2 . k>0,
k<0, k>0, h>0, h<0, h<0, a | a |>1, x- | a |<1, x- | a |>1,
a (x−h) 2 +k=a x 2 +bx+c a x 2 −2ahx+(a h 2 +k)=a x 2 +bx+c
−2ah=b, so h=− b 2a .
a h 2 +k=c k=c−a h 2 =c−a− (b 2a) 2 =c− b 2 4a
h, f(h)=k. f(x)=a x 2 +bx+c a,b, c a≠0. f(x)=a (x−h) 2 +k. (h,k)
h=− b 2a , k=f(h)=f(−b 2a).
h. k. h k. f(x)=a (x−h) 2 +k. x f(x). | a |. a>0. a<0 x- g f(x)= x 2 , f(x)= x 2 g(x)=a (x+2) 2 −3. (0,−1),
−1=a (0+2) 2 −3 2=4a a= 1 2
(g)x= 1 2 (x+2) 2 −3.
g(x)= 1 2 (x+2) 2 −3 = 1 2 (x+2)(x+2)−3 = 1 2 (x 2 +4x+4)−3 = 1 2 x 2 +2x+2−3 = 1 2
x 2 +2x−1

Y1= 1 2 (x+2) 2 −3. TBLSET, TblStart=−6 ΔTbl = 2, TABLE. xy (−4, 7), (h)x=− 7 16 (x+4) 2 +7. h(−7.5) h(−7.5)≈1.64; a, b, and c. h, a b h=− b 2a . k, k=f(h)=f(− b 2a). f(x)=2 x 2 −6x+7.

$$h=-\frac{b}{2a}=-\frac{-6}{2(2)}=\frac{6}{4}=\frac{3}{2}$$

$$k=f(h)=f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^2-6\left(\frac{3}{2}\right)+7=\frac{5}{2}$$

a

$$f(x)=a x^2+bx+c \quad f(x)=2x^2-6x+7$$

$$f(x)=2\left(x-\frac{3}{2}\right)^2+\frac{5}{2}$$

(k), (x). g(x)=13+ x 2 −6x,g(x)= x 2 −6x+13 g(x)= (x−3) 2 +4 f(x)=a x 2 +bx+c a f(x)≥f(− b 2a), [f(− b 2a),∞); a f(x)≤f(− b 2a), (−∞,f(− b 2a)]. f(x)=a (x−h) 2 +k a f(x)≥k; a f(x)≤k. a a a k. f(x)≥k, [k,∞). f(x)≤k, (−∞,k]. f(x)=−5 x 2 +9x−1. a x-

$$h=-\frac{b}{2a}=-\frac{9}{2(-5)}=\frac{9}{10}$$

f(h).

$$f\left(\frac{9}{10}\right)=5\left(\frac{9}{10}\right)^2+9\left(\frac{9}{10}\right)-1=\frac{61}{20}$$

$f(x)\le\frac{61}{20}$, $(-\infty,\frac{61}{20}]$. $f(x)=2\left(x-\frac{4}{7}\right)^2+\frac{8}{11}$. $f(x)\ge\frac{8}{11}$, $[\frac{8}{11},\infty)$. L. W, L+W+L=80, 2L+W=80. W, L.

$$W=80-2L$$

$$A=LW=L(80-2L) \quad A(L)=80L-2L^2$$

L.

$$A(L)=-2L^2+80L.$$

a a=−2,b=80, c=0.

$$h=-\frac{80}{2(-2)} \quad k=A(20)=20 \text{ and } =80(20)-2(20)^2=800$$

L=20 p Q Revenue=pQ. p=30 Q=84,000. p=32 Q=79,000.

$$m=\frac{79{,}000-84{,}000}{32-30}=\frac{-5{,}000}{2}=-2{,}500$$

Q=−2500p+b Substitute in the point Q=84,000 and p=30 84,000=−2500(30)+b Solve for b

$$b=159{,}000$$

Q=−2,500p+159,000

Revenue=pQ Revenue=p(−2,500p+159,000) Revenue=−2,500 p 2 +159,000p

$$h=-\frac{159{,}000}{2(-2{,}500)}=31.8$$

maximum revenue=−2,500 (31.8) 2 +159,000(31.8) =2,528,100

y- x- x- f(x), y- f(0) y- f(x)=0 f(x)=3 x 2 +5x−2. f(0).

$$f(0)=3(0)^2+5(0)-2=-2$$

(0,−2). f(x)=0.

$$0=3x^2+5x-2$$

$$0=(3x-1)(x+2)$$

0=3x−1 0=x+2 x= 1 3 or x=−2

(1 3 ,0) (−2,0). (0,−2). (1 3 ,0) (−2,0). x- a b h=− b 2a . x=h k. h k. x- x- x- f(x)=2 x 2 +4x−4.

$$0=2x^2+4x-4$$

$$f(x)=a(x-h)^2+k$$

a=2. h k.

$$h=-\frac{b}{2a} \quad k=f(-1)=-\frac{4}{2(2)}=2(-1)^2+4(-1)-4=-1=-6$$

$$f(x)=2(x+1)^2-6$$

$$0=2(x+1)^2-6 \quad 6=2(x+1)^2 \quad 3=(x+1)^2 \quad x+1=\pm\sqrt{3} \quad x=-1\pm\sqrt{3}$$

x- (−1− 3 ,0) (−1+ 3 ,0). x- g(x)=13+ x 2 −6x. x- x- x 2 +x+2=0. x= −b± b 2 −4ac 2a . a, b and c. x 2 +x+2=0, a=1, b=1, and c=2.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-4\cdot1\cdot(2)}}{2\cdot1}=\frac{-1\pm\sqrt{1-8}}{2}=\frac{-1\pm\sqrt{-7}}{2}=\frac{-1\pm i\sqrt{7}}{2}$$

$x=\frac{-1+i\sqrt{7}}{2}$ $x=\frac{-1-i\sqrt{7}}{2}$ $x=\frac{-1}{2}+\frac{i\sqrt{7}}{2}$ $x=\frac{-1}{2}-\frac{i\sqrt{7}}{2}$. H(t)=−16 t 2 +80t+40.

$$h=-\frac{80}{2(-16)}=\frac{80}{32}=\frac{5}{2}=2.5$$

y-

$$k=H\left(-\frac{b}{2a}\right)=H(2.5)=-16(2.5)^2+80(2.5)+40=140$$

H(t)=0.

$$t= -80\pm \sqrt{80^2-4(-16)(40)}\,2(-16) = -80\pm \sqrt{8960}\,-32$$

$$t= -80-\sqrt{8960}\,-32 \approx 5.458 \text{ or } t= -80+\sqrt{8960}\,-32 \approx -0.458$$

H(t)=−16 t 2 +96t+112.

$$f(x)=a\,x^2+bx+c$$

$$x= -b\pm \sqrt{b^2-4ac}\,2a$$

$$f(x)=a\,(x-h)^2+k$$

x- x- y- y- y- a≠0 a=0 f(x)= x 2 −12x+32g(x)= x 2 +2x−3f(x)= (x+1) 2 −2, (−1,−4)f(x)= x 2 −x
f(x)= x 2 +5x−2f(x)= (x+ 5 2) 2 − 33 4 , (− 5 2 ,− 33 4)h(x)=2 x 2 +8x−10k(x)=3 x 2 −6x−9
f(x)=3 (x−1) 2 −12, (1,−12)f(x)=2 x 2 −6xf(x)=3 x 2 −5x−1f(x)=3 (x− 5 6) 2 − 37 12 , (5 6 ,− 37 12)
y(x)=2 x 2 +10x+12f(x)=2 x 2 −10x+4 − 17 2 5 2 . x= 5 2 .f(x)=− x 2 +4x+3f(x)=4 x 2 +x−1
− 17 16 − 1 8 . x=− 1 8 .h(t)=−4 t 2 +6t−1f(x)= 1 2 x 2 +3x+1 − 7 2 −3. x=−3.f(x)=− 1 3 x 2 −2x+3
f(x)= (x−3) 2 +2 (−∞,∞). [2,∞).f(x)=−2 (x+3) 2 −6f(x)= x 2 +6x+4 (−∞,∞). [−5,∞).f(x)=2 x 2 −4x+2
k(x)=3 x 2 −6x−9 (−∞,∞). [−12,∞).x 2 =−25x 2 =−8{ 2i 2 ,−2i 2 }x 2 +36=0x 2 +27=0{ 3i 3 ,−3i 3 }
x 2 +2x+5=0x 2 −4x+5=0{2+i,2−i}x 2 +8x+25=0x 2 −4x+13=0{2+3i,2−3i}x 2 +6x+25=0x 2 −10x+26=0
{5+i,5−i}x 2 −6x+10=0x(x−4)=20{2+2 6 , 2−2 6 }x(x−2)=102 x 2 +2x+5=0{ − 1 2 + 3 2 i, − 1 2 − 3 2 i }
5 x 2 −8x+5=05 x 2 +6x+2=0{ − 3 5 + 1 5 i, − 3 5 − 1 5 i }2 x 2 −6x+5=0x 2 +x+2=0
{ − 1 2 + 1 2 i 7 , − 1 2 − 1 2 i 7 }x 2 −2x+4=0 (h,k) (x,y) (h,k)=(2,0),(x,y)=(4,4)f(x)= x 2 −4x+4
(h,k)=(−2,−1),(x,y)=(−4,3)(h,k)=(0,1),(x,y)=(2,5)f(x)= x 2 +1(h,k)=(2,3),(x,y)=(5,12)
(h,k)=(−5,3),(x,y)=(2,9)f(x)= 6 49 x 2 + 60 49 x+ 297 49(h,k)=(3,2),(x,y)=(10,1)(h,k)=(0,1),(x,y)=(1,0)
f(x)=− x 2 +1(h,k)=(1,0),(x,y)=(0,1)f(x)= x 2 −2x(1, −1), x=1. (0,0), (2,0).f(x)= x 2 −6x−1
f(x)= x 2 −5x−6 (5 2 , −49 4), (0,−6),(−1,0),(6,0).f(x)= x 2 −7x+3f(x)=−2 x 2 +5x−8 (5 4 , − 39 8),
x= 5 4 . (0, −8).f(x)=4 x 2 −12x−3f(x)= x 2 −4x+1f(x)=−2 x 2 +8x−1f(x)= 1 2 x 2 −3x+ 7 2xy
f(x)= x 2 +1xyxyf(x)=2− x 2xyxyf(x)=2 x 2 f(x)= x 2 ,f(x)=2 x 2 , and f(x)= 1 3 x 2 .
f(x)= x 2 ,f(x)= x 2 +2 f(x)= x 2 ,f(x)= x 2 +5 f(x)= x 2 −3.
f(x)= x 2 ,f(x)= (x−2) 2 ,f (x−3) 2 , and f(x)= (x+4) 2 . h(x)= −32 (80) 2 x 2 +x x h(x) h(x)=.0001 x 2
−2000≤x≤2000 | x | h(x) (1,−2), (−∞,∞). [−2,∞). (−1,2) (−5,11), (−∞,∞) (−∞,11]. (−100,100), (1,1)
f(x)=2 x 2 . y-f(x)=2 x 2 −1 (−1,4) f(x)=2 x 2 . y- (2,3) f(x)=3 x 2 . y-f(x)=3 x 2 −9 (1,−3) f(x)=− x 2 .
y- (4,3) f(x)=5 x 2 . y-f(x)=5 x 2 −77 (1,−6) f(x)=3 x 2 . −1. f(x)=− x 2 +100x. f(x)=−2 x 2 +250x.6 −6;
f(x)= x 2 +12x. p=$45−0.0125x, x R=x·p. h(t)=−4.9 t 2 +229t+234. h(t)=−4.9 t 2 +24t+8. x=− b 2a .
f(x)=a x 2 +bx+c, a,b, c a≠0. f(x)=a (x−h) 2 +k, (h, k) x y=0,

2009
2010
2011
2012
2013
800
897
992
1,083
1,169

P(t)=−0.3 t 3 +97t+800, P(t) t r

$$A(r)=\pi\, r^2$$

r

$$V(r)= 4\,3\,\pi\, r^3$$

π 4 3 π, r

$$f(x)=k\,x^p$$

k p k f(x)= 2 x

f(x)=1 Constant function f(x)=x Identify function f(x)= x 2 Quadratic function f(x)= x 3 Cubic function f(x)= 1 x Reciprocal function f(x)= 1 x 2 Reciprocal squared function f(x)= x Square root function f(x)= x 3 Cube root function

f(x)= x 0 f(x)= x 1 f(x)= x 2 f(x)= x 3 . f(x)= x −1 f(x)= x −2 . f(x)= x 1/2 f(x)= x 1/3 . f(x)=2 x 2 ·4 x 3 g(x)=− x 5 +5 x 3 −4x h(x)= 2 x 5 −1 3 x 2 +4f(x) f(x)=8 x 5 . f(x)= x 2 , g(x)= x 4 and h(x)= x 6 , ∞ −∞ x x→∞, x x f(x)

as x→±∞, f(x)→∞

f(x)= x 3 , g(x)= x 5 , and h(x)= x 7 , f(x)= x n f(x)= x n , n even, y- f(x)= x n , n odd, x f(x) x f(x)

as x→−∞, f(x)→−∞ as x→∞, f(x)→∞

x→−∞ x→∞ f(x)=k x n n f(x)=k x n n f(x)= x 8 . x f(x) x→∞, f(x)→∞. x x→−∞, f(x)→∞. f(x)=− x 9 . −1 x- f(x)= x 9 . x x

as x→−∞, f(x)→∞ as x→∞, f(x)→−∞

xf(x) x, x, f(x)=−5 x 4 . x f(x) x→±∞, f(x)→−∞ r w

r(w)=24+8w

A

$$A(r)=\pi r^2$$

$$A(w)=A(r(w)) \quad =A(24+8w) \quad =\pi (24+8w)^2$$

$$A(w)=576\pi+384\pi w+64\pi w^2$$

n

$$f(x)= a_n x^n +...+ a_2 x^2 + a_1 x+ a_0$$

a i a i x i

f(x)=2 x 3 ·3x+4 g(x)=−x(x 2 −4) h(x)=5 x +2

f(x)= a n x n +...+ a 2 x 2 + a 1 x+ a 0 , f(x) f(x)=6 x 4 +4.g(x) g(x)=− x 3 +4x.h(x) x x

f(x)=3+2 x 2 −4 x 3 g(t)=5 t 5 −2 t 3 +7t h(p)=6p− p 3 −2

f(x), x −4 x 3 . −4. g(t), t 5, 5. 5 t 5 . 5. h(p), p 3, 3. −p 3 ; −1. f(x)=4 x 2 − x 6 +2x−6. − x 6 . −1. x f(x)=5 x 4 +2 x 3 −x−45 x 4f(x)=−2 x 6 − x 5 +3 x 4 + x 3−2 x 6f(x)=3 x 5 −4 x 4 +2 x 2 +13 x 5 f(x)=−6 x 3 +7 x 2 +3x+1−6 x 3 x f(x) x f(x)

as x→−∞, f(x)→−∞ as x→∞, f(x)→∞

x x x→∞, f(x)→−∞; as x→−∞, f(x)→−∞. f(x)=−3 x 2 (x−1)(x+4), f(x).

f(x)=−3 x 2 (x−1)(x+4) =−3 x 2 (x 2 +3x−4) =−3 x 4 −9 x 3 +12 x 2

f(x)=−3 x 4 −9 x 3 +12 x 2 . −3 x 4 ;

as x→−∞, f(x)→−∞ as x→∞, f(x)→−∞

f(x)=0.2(x−2)(x+1)(x−5), 0.2 x 3 , x f(x) x f(x) (0, a 0). x- x=0 x- f(x)=(x−2)(x+1)(x−4), y- x- x.

f(0)=(0−2)(0+1)(0−4) =(−2)(1)(−4) =8

0=(x−2)(x+1)(x−4) x−2=0 or x+1=0 or x−4=0 x=2 or x=−1 or x=4

x- (2,0),(−1,0), (4,0). f(x)= x 4 −4 x 2 −45, y- x-

f(0)= (0) 4 −4 (0) 2 −45 =−45

(0,−45).

f(x)= x 4 −4 x 2 −45 =(x 2 −9)(x 2 +5) =(x−3)(x+3)(x 2 +5)

0=(x−3)(x+3)(x 2 +5)

x−3=0 or x+3=0 or x 2 +5=0 x=3 or x=−3 or (no real solution)

(3,0) (−3,0). f(x)=f(−x). f(x)=2 x 3 −6 x 2 −20x, y- x- (0,0); (0,0),(−2,0), (5,0) x- nth n n x- n n−1 n n n−1 x- f(x)=−3 x 10 +4 x 7 − x 4 +2 x 3 . 10, n n−1 x- f(x)=108−13 x 9 −8 x 4 +14 x 12 +2 x 3 x- x- x- f(x)=−4x(x+3)(x−4), y- f(0).

f(0)=−4(0)(0+3)(0−4) =0

y- (0,0). x-

0=−4x(x+3)(x−4) x=0 or x+3=0 or x−4=0 x=0 or x=−3 or x=4

x- (0,0),(–3,0), (4,0). f(x)=0.2(x–2)(x+1)(x–5), x- (2,0),(–1,0), (5,0), (0,2), x-

$$f(x)= a_n x^n +...+ a_2 x^2 + a_1 x+ a_0$$

n n $n–1$ x $f(x)$. x $f(x)$. $x\to–\infty, f(x)\to–\infty$ $x\to\infty, f(x)\to–\infty$. $f(x)= x^5 f(x)= (x^2) 3 f(x)=x– x^4$
$f(x)= x^2 x^2 –1 f(x)=2x(x+2) (x–1) 2 f(x)= 3 x+1–3x^4 7–2 x^2–2 x^2 – 3 x^5 + x–6 x(4– x^2)(2x+1)$
$x^2 (2x–3) 2 f(x)= x^4$ As $x\to\infty$, $f(x)\to\infty$, as $x\to–\infty, f(x)\to\infty f(x)= x^3 f(x)=– x^4$
As $x\to–\infty$, $f(x)\to–\infty$, as $x\to\infty, f(x)\to–\infty f(x)=– x^9 f(x)=–2 x^4 – 3 x^2 + x–1$
As $x\to–\infty$, $f(x)\to–\infty$, as $x\to\infty, f(x)\to–\infty f(x)=3 x^2 + x–2 f(x)= x^2 (2 x^3 –x+1)$
As $x\to\infty$, $f(x)\to\infty$, as $x\to–\infty, f(x)\to–\infty f(x)= (2–x)^7 f(t)=2(t–1)(t+2)(t–3)^3$ (0,12),
(1,0);(–2,0);and (3,0). $g(n)=–2(3n–1)(2n+1) f(x)= x^4 –16$ (0,–16). (2,0) (–2,0). $f(x)= x^3 +27$
$f(x)=x(x^2 –2x–8)$ (0,0). (0,0),(4,0), (–2, 0). $f(x)=(x+3)(4 x^2 –1) f(x)=– x^3 f(x)= x^4 –5 x^2 x f(x)$
as $x\to–\infty$, $f(x)\to\infty$, as $x\to\infty, f(x)\to\infty f(x)= x^2 (1–x)^2 f(x)=(x–1)(x–2)(3–x)x f(x)$
as $x\to–\infty$, $f(x)\to\infty$, as $x\to\infty, f(x)\to–\infty f(x)= x^5 10 – x^4 f(x)= x^3 (x–2)$ y- (0, 0). x- (0, 0), (2, 0).
As $x\to–\infty$, $f(x)\to\infty$, as $x\to\infty, f(x)\to\infty f(x)=x(x–3)(x+3) f(x)=x(14–2x)(10–2x)$ y- (0,0) x-
(0, 0), (5, 0), (7, 0). As $x\to–\infty$, $f(x)\to–\infty$, as $x\to\infty, f(x)\to\infty f(x)=x(14–2x)$ (10–2x) $2 f(x)= x^3 –16x$ y-
(0, 0). x- (–4, 0), (0, 0), (4, 0). As $x\to–\infty$, $f(x)\to–\infty$, as $x\to\infty, f(x)\to\infty f(x)= x^3 –27 f(x)= x^4 –81$ y-
(0, –81). x- (3, 0), (–3, 0). As $x\to–\infty$, $f(x)\to\infty$, as $x\to\infty, f(x)\to\infty f(x)=– x^3 + x^2 +2x$
$f(x)= x^3 –2 x^2 –15x$ y- (0, 0). x- (–3, 0), (0, 0), (5, 0). As $x\to–\infty$, $f(x)\to–\infty$, as $x\to\infty, f(x)\to\infty$
$f(x)= x^3 –0.01x$ y- (0,–4). x- (–2,0), (2,0). as $x\to–\infty$, $f(x)\to\infty$, as $x\to\infty, f(x)\to\infty$. $f(x)= x^2 –4$ y- (0,9). x-
(–3,0), (3,0). as $x\to–\infty$, $f(x)\to–\infty$, as $x\to\infty, f(x)\to–\infty$. y- (0,0). x- (0,0), (2,0).
as $x\to–\infty$, $f(x)\to–\infty$, as $x\to\infty, f(x)\to\infty$. $f(x)= x^3 –4 x^2 +4x$ y- (0,1). x- (1,0).
as $x\to–\infty$, $f(x)\to\infty$, as $x\to\infty, f(x)\to–\infty$. y- (0,1). x- as $x\to–\infty$, $f(x)\to\infty$, as $x\to\infty, f(x)\to\infty$. $f(x)= x^4 +1$ d,
m, $V(m)=8 m^3 +36 m^2 +54m+27$ x x. x- x. $V(x)=4 x^3 –32 x^2 +64x$ x $f(x)=k x^p$ k p, $a_i x_i$
$f(x)= a_n x^n +...+ a_2 x^2 + a_1 x+ a_0$

$$R(t)=–0.037 t^4 +1.414 t^3 –19.777 t^2 +118.696t–205.332$$

R t t=6 f h g k g k f x $f(x)=0$ f. x- x- f, $f(x)=0$. x- $f(x)= x^6 –3 x^4 +2 x^2$. $f(x)=0$.

$x^6 –3 x^4 +2 x^2 =0$ Factor out the greatest common factor. $x^2 (x^4 –3 x^2 +2)=0$

Factor the trinomial. $x^2 (x^2 –1)(x^2 –2)=0$ Set each factor equal to zero.

$(x^2 –1)=0$ $(x^2 –2)=0$ $x^2 =0$ or $x^2 =1$ or $x^2 =2$ $x=0$ $x=\pm1$ $x=\pm\sqrt{2}$

x- (0,0),(1,0),(–1,0),($\sqrt{2}$,0), (– $\sqrt{2}$,0). x- $f(x)= x^3 –5 x^2 –x+5$. $f(x)=0$

$x^3 –5 x^2 –x+5=0$ Factor by grouping. $x^2 (x–5)–(x–5)=0$ Factor out the common factor. $(x^2 –1)(x–5)=0$ Factor the difference of squares. $(x+1)(x–1)(x–5)=0$ Set each factor equal to zero.

$x+1=0$ or $x–1=0$ or $x–5=0$ $x=–1$ $x=1$ $x=5$

x- (–1,0),(1,0), (5,0). y- $g(x)= (x–2)^2 (2x+3)$. $g(0)$.

$$g(0)= (0–2)^2 (2(0)+3) =12$$

(0,12). $g(x)=0$.

$$(x–2)^2 (2x+3)=0$$

$(x–2)^2 =0$ $(2x+3)=0$ $x–2=0$ or $x=–\frac{3}{2}$ $x=2$

x- (2,0) ($–\frac{3}{2}$,0). x- $h(x)= x^3 +4 x^2 +x–6$. x=–3,–2, 1. x

$$h(–3)=h(–2)=h(1)=0.$$

$h(x)= x^3 +4 x^2 +x–6$,

$h(–3)= (–3)^3 +4 (–3)^2 +(–3)–6=–27+36–3–6=0$ $h(–2)= (–2)^3 +4 (–2)^2 +(–2)–6=–8+16–2–6=0$

$$h(1)= (1)^3 +4 (1)^2 +(1)–6=1+4+1–6=0$$

x-

$$h(x)= x^3 +4 x^2 +x–6 \quad =(x+3)(x+2)(x–1)$$

y- $f(x)= x^4 –19 x^2 +30x$. (0,0); (0,0),(–5,0),(2,0), (3,0) x-

$$f(x)=(x+3) (x–2)^2 (x+1)^3 .$$

x- x- –3 $(x+3)=0$. x- $x=–3$. x- 2 $(x–2)^2 =0$.

$$(x–2)^2 =(x–2)(x–2)$$

(x−2) x=2, (x−2) x- −1 (x+1) 3 =0. f(x)= x 3 . x- x- x- x- x- (x−h) p , x- h p. x=h p. x- n, n. n. n. x=−3. −3 2. x=−1. x=4.

$$f(x)= a_n x^n + a_{n-1} x^{n-1} +...+ a_1 x+ a_0$$

x x a n x n , x f(x) x f(x) x f(x) f(x)= x 4 − x 3 −4 x 2 +4x f n n−1 f(x)=− x 3 +4 x 5 −3 x 2 ++1 f(x)=− (x−1) 2 (1+2 x 2) f(x)=−x + 3 4 x 5 −3 x 2 ++1 f(x)=4 x 5 − x 3 −3 x 2 ++1 5−1=4. f(x)=− (x−1) 2 (1+2 x 2) 4−1=3. y- f(−x)=f(x). f(−x)=−f(x). x- f(x)=−2 (x+3) 2 (x−5). x- x=−3, x- x=5, f(0).

$$f(0)=-2\,(0+3)^2\,(0-5) \quad =-2\cdot 9\cdot(-5) \quad =90$$

y- (0,90). −2 x 3 , x→−∞ f(x)→∞, x- f(−x)=−2 (−x+3) 2 (−x−5) f(x), (−3,0), x- (0,90), y- y- (5,0). x→∞ f(x)→−∞, f(x)=−2 (x+3) 2 (x−5) f(x)= 1 4 x (x−1) 4 (x+3) 3 . f a b f, a<b f(a)≠f(b), f f(a) f(b). f x=a x- x=b x- x=a x=b x- (c, f(c)). c f(c)=0. x- a b. f f(a) f(b) c a b f(c)=0. f(x)= x 3 −5 x 2 +3x+6 x=1 x=4. f(x) x=1,2,3,and4. xf(x) x=2. f(3) f(4) x=1 x=4. x=1 x=4. f(x)=7 x 5 −9 x 4 − x 2 x=1 x=2. f f(1) f(2) x=1 x=2. x- x- p x= x 1 , x 2 ,…, x n , f(x)=a (x− x 1) p 1 (x− x 2) p 2 ⋯ (x− x n) p n p i a x- x=−3,2, 5. y- (0,2). x=−3 x=5, x=2,

$$f(x)=a(x+3)\,(x-2)^2\,(x-5)$$

y- (0,−2), a.

$$f(0)=a(0+3)\,(0-2)^2\,(0-5) \;\; -2=a(0+3)\,(0-2)^2\,(0-5) \;\; -2=-60a \quad a=\frac{1}{30}$$

f(x)= 1 30 (x+3) (x−2) 2 (x−5).f(x)=− 1 8 (x−2) 3 (x+1) 2 (x−4) x=a x=a. a, f(a)≥f(x) x x=a. a, f(a)≤f(x) x x=a. a, f(a)≥f(x) x. a, f(a)≤f(x) x. f(x)=x w. (14−2w) (20−2w) w

$$V(w)=(20-2w)(14-2w)w \quad =280w-68\,w^2+4\,w^3$$

w, 20−2w 14−2w, w 0<w<7. V(w) w [0, 7]. [−1,4] f(x)=−0.2 (x−2) 3 (x+1) 2 (x−4). (0,−6.5), (3.5,7). x- x- x- n n−1 n−1 f(a) and f(b) c a b f(c)=0. x- f? x- x- f(x)=0. n n a b a b. x- x- C(t)=2(t−4)(t+1)(t−6) C(t)=3(t+2)(t−3)(t+5)(−2,0),(3,0),(−5,0) C(t)=4t (t−2) 2 (t+1) C(t)=2t(t−3) (t+1) 2 (3,0),(−1,0),(0,0) C(t)=2 t 4 −8 t 3 +6 t 2 C(t)=4 t 4 +12 t 3 −40 t 2 (0,0), (−5,0), (2,0) f(x)= x 4 − x 2 f(x)= x 3 + x 2 −20x(0,0), (−5,0), (4,0)f(x)= x 3 +6 x 2 −7x f(x)= x 3 + x 2 −4x−4(2,0), (−2,0), (−1,0)f(x)= x 3 +2 x 2 −9x−18f(x)=2 x 3 − x 2 −8x+4 (−2,0), (2,0), (1 2 ,0)f(x)= x 6 −7 x 3 −8f(x)=2 x 4 +6 x 2 −8(1,0), (−1,0)f(x)= x 3 −3 x 2 −x+3 f(x)= x 6 −2 x 4 −3 x 2(0,0), (3 ,0), (− 3 ,0)f(x)= x 6 −3 x 4 −4 x 2f(x)= x 5 −5 x 3 +4x (0,0), (1,0), (−1,0), (2,0), (−2,0)f(x)= x 3 −9x, x=−4 x=−2.f(x)= x 3 −9x, x=2 x=4.f(2)=−10 f(4)=28.f(x)= x 5 −2x, x=1 x=2.f(x)=− x 4 +4, x=1 x=3f(1)=3 f(3)=−77. f(x)=−2 x 3 −x, x=−1 x=1. f(x)= x 3 −100x+2, x=0.01 x=0.1f(0.01)=1.000001 f(0.1)=−7.999. f(x)= (x+2) 3 (x−3) 2 f(x)= x 2 (2x+3) 5 (x−4) 2 − 3 2 f(x)= x 3 (x−1) 3 (x+2)f(x)= x 2 (x 2 +4x+4) f(x)= (2x+1) 3 (9 x 2 −6x+1)f(x)= (3x+2) 5 (x 2 −10x+25) − 2 3 with multiplicity 5, 5 with multiplicity 2f(x)=x(4 x 2 −12x+9)(x 2 +8x+16)f(x)= x 6 − x 5 −2 x 4 0 with multiplicity 4, 2 with multiplicity 1, −1 with multiplicity 1f(x)=3 x 4 +6 x 3 +3 x 2 f(x)=4 x 5 −12 x 4 +9 x 33 2 f(x)=2 x 4 (x 3 −4 x 2 +4x)f(x)=4 x 4 (9 x 4 −12 x 3 +4 x 2) 0 with multiplicity 6, 2 3 with multiplicity 2 x- y-f(x)= (x+3) 2 (x−2)g(x)=(x+4) (x−1) 2(1, 0) (−4, 0)y-(0, 4)x→−∞f(x)→−∞x→∞f(x)→∞h(x)= (x−1) 3 (x+3) 2k(x)= (x−3) 3 (x−2) 2(3,0) (2,0)y-(0,−108)x→−∞f(x)→−∞x→∞f(x)→∞.m(x)=−2x(x−1)(x+3)n(x)=−3x(x+2)(x−4) (0, 0), (−2, 0), (4, 0)y(0, 0).x→−∞f(x)→∞x→∞f(x)→−∞.f(x)=− 2 9 (x−3)(x+1)(x+3) f(x)= 1 4 (x+2) 2 (x−3) x=−2, x=1, x=3. (0,−4).f(x)=− 2 3 (x+2)(x−1)(x−3) x=−5, x=−2, x=1. (0,6) x=3 x=1 x=−3. (0,9)f(x)= 1 3 (x−3) 2 (x−1) 2 (x+3) x=4, x=1 x=−2. (0,−3). x=1, x=3. (2,15). f(x)=−15 (x−1) 2 (x−3) 3 x=4, x=3, x=2. (0,−24). x=−3, x=−2 x=1. (0,12).f(x)=−2(x+3)(x+2)(x−1) x=−3 x=2 x=−2. (0, 4). x= 1 2 x=6 x=−2. (0,18).f(x)=− 3 2 (2x−1) 2 (x−6)(x+2) x=−3 x=0. (1,32).f(x)= x 3 −x−1 (−.58, −.62), (.58, −1.38) f(x)=2 x 3 −3x−1f(x)= x 4 +x (−.63, −.47) f(x)=− x 4 +3x−2f(x)= x 4 − x 3 +1 (.75, .89)f(x)= (x−500) 2 (x+200) x x x.f(x)=4 x 3 −36 x 2 +80x 2x 2x x. x +1 x +1 x.f(x)=4 x 3 −36 x 2 +60x+100 x+2 3x+6 V= 1 3 π r 2 h r h. f(x)=π(9 x 3 +45 x 2 +72x+36) f(a) f(a)≥f(x) x. f(a) f(a)≤f(x) x. a b f, a<b f(a)≠f(b), f f(a) f(b); x-

(x−h) p , x=h p.

$$V=l\cdot w\cdot h \quad =61.5\cdot 40\cdot 30 \quad =73{,}800$$

(m^3).

$$h=\text{ V l}\cdot w \quad = 73{,}800\ 61.5\cdot 40 \quad =30$$

3 x 4 −3 x 3 −33 x 2 +54x. 3x; x−2.

dividend = (divisor · quotient) + remainder 178=(3·59)+1 =177+1 =178

2 x 3 −3 x 2 +4x+5 x+2

$$2x^3-3x^2+4x+5 \text{ x+2} = 2x^2-7x+18-\frac{31}{x+2}$$

$$2x^3-3x^2+4x+5=(x+2)(2x^2-7x+18)-31$$

f(x) d(x) d(x) f(x), q(x) r(x)

$$f(x)=d(x)q(x)+r(x)$$

q(x) r(x) d(x). r(x)=0, d(x) f(x). d(x) q(x) f(x). 5 x 2 +3x−2 x+1. 5x−2.

$$5x^2+3x-2 \text{ x+1} = 5x-2$$

$$5x^2+3x-2=(x+1)(5x-2)$$

6 x 3 +11 x 2 −31x+15 3x−2.

$$6x^3+11x^2-31x+15 \text{ 3x−2} = 2x^2+5x-7+\frac{1}{3x-2}$$

$$(3x-2)(2x^2+5x-7)+1=6x^3+11x^2-31x+15$$

6 x 3 +11 x 2 −31x+15 3x−2 2 x 2 +5x−7 1 16 x 3 −12 x 2 +20x−3 4x+5.

$$4x^2-8x+15-\frac{78}{4x+5}$$

2 x 3 −3 x 2 +4x+5 x+2 $2x^2$−7x+18 −31. x−k. k k. k. 5 x 2 −3x−36 x−3. k k. k. 5x+12. x−3

$$(x-3)(5x+12)+0=5x^2-3x-36$$

4 x 3 +10 x 2 −6x−20 x+2. x+2 k=−2. 4 x 2 +2x−10. x+2 4 x 3 +10 x 2 −6x−20.

f(x)=4 x 3 +10 x 2 −6x−20 x=k=−2. x+2 4 x 3 +10 x 2 −6x−20. −9 x 4 +10 x 3 +7 x 2 −6 x−1.

−9 x 3 + x 2 +8x+8+ 2 x−1 . 3 x 4 +18 x 3 −3x+40 x+7.3 x 3 −3 x 2 +21x−150+ 1,090 x+7

3 x 4 −3 x 3 −33 x 2 +54x. 3x x−2.

$$V=l\cdot w\cdot h \ 3x^4-3x^3-33x^2+54x=3x\cdot(x-2)\cdot h$$

h, 3x.

$$3x\cdot(x-2)\cdot h \ 3x = 3x^4-3x^3-33x^2+54x \ 3x \ (x-2)h= x^3-x^2-11x+18$$

h

$$h= x^3-x^2-11x+18 \text{ x−2}$$

2 1 −1 −11 18 2 2 −18 1 1 −9 0

x^2+x-9 x^2+x-9. 3 x 3 +14 x 2 −23x+6. x+6. 3 x 2 −4x+1f(x)=d(x)q(x)+r(x)q(x)≠0 x−k. n

(x^2+5x-1)÷(x−1)x+6+ 5 x-1 , quotient: x+6, remainder: 5(2 x 2 −9x−5)÷(x−5)

(3 x 2 +23x+14)÷(x+7)3x+2, quotient: 3x+2, remainder: 0(4 x 2 −10x+6)÷(4x+2)

(6 x 2 −25x−25)÷(6x+5)x−5, quotient: x−5, remainder: 0(− x 2 −1)÷(x+1)(2 x 2 −3x+2)÷(x+2)

2x−7+ 16 x+2 , quotient: 2x−7, remainder: 16(x 3 −126)÷(x−5)(3 x 2 −5x+4)÷(3x+1)

x−2+ 6 3x+1 , quotient: x−2, remainder: 6(x 3 −3 x 2 +5x−6)÷(x−2)(2 x 3 +3 x 2 −4x+15)÷(x+3)

2 x 2 −3x+5, quotient: 2 x 2 −3x+5, remainder: 0(3 x 3 −2 x 2 +x−4)÷(x+3)

(2 x 3 −6 x 2 −7x+6)÷(x−4)2 x 2 +2x+1+ 10 x−4(6 x 3 −10 x 2 −7x−15)÷(x+1)

(4 x 3 −12 x 2 −5x−1)÷(2x+1)2 x 2 −7x+1− 2 2x+1(9 x 3 −9 x 2 +18x+5)÷(3x−1)

(3 x 3 −2 x 2 +x−4)÷(x+3)3 x 2 −11x+34− 106 x+3(−6 x 3 + x 2 −4)÷(2x−3)

(2 x 3 +7 x 2 −13x−3)÷(2x−3)x 2 +5x+1(3 x 3 −5 x 2 +2x+3)÷(x+2)(4 x 3 −5 x 2 +13)÷(x+4)

4 x 2 −21x+84− 323 x+4(x 3 −3x+2)÷(x+2)(x 3 −21 x 2 +147x−343)÷(x−7)x 2 −14x+49

(x 3 −15 x 2 +75x−125)÷(x−5)(9 x 3 −x+2)÷(3x−1)3 x 2 +x+ 2 3x−1(6 x 3 − x 2 +5x+2)÷(3x+1)

(x 4 + x 3 −3 x 2 −2x+1)÷(x+1)x 3 −3x+1(x 4 −3 x 2 +1)÷(x−1)(x 4 +2 x 3 −3 x 2 +2x+6)÷(x+3)

x 3 − x 2 +2(x 4 −10 x 3 +37 x 2 −60x+36)÷(x−2)(x 4 −8 x 3 +24 x 2 −32x+16)÷(x−2)

x 3 −6 x 2 +12x−8(x 4 +5 x 3 −3 x 2 −13x+10)÷(x+5)(x 4 −12 x 3 +54 x 2 −108x+81)÷(x−3)

x 3 −9 x 2 +27x−27(4 x 4 −2 x 3 −4x+2)÷(2x−1)(4 x 4 +2 x 3 −4 x 2 +2x+2)÷(2x+1)2 x 3 −2x+2

x−2, 4 x 3 −3 x 2 −8x+4x−2, 3 x 4 −6 x 3 −5x+10 (x−2)(3 x 3 −5)x+3, −4 x 3 +5 x 2 +8

x−2, 4 x 4 −15 x 2 −4 (x−2)(4 x 3 +8 x 2 +x+2)x− 1 2 , 2 x 4 − x 3 +2x−1x+ 1 3 , 3 x 4 + x 3 −3x+1 x 2 −x+3 (x 2 +2x+4)(x−1)(x 2 +2x+4) x 2 +2x+5 x 2 +x+1(x−5)(x 2 +x+1)x 2 +2x+24 x 3 −33 x−2 Quotient: 4 x 2 +8x+16, remainder: −12 x 3 +25 x+33 x 3 +2x−5 x−1Quotient: 3 x 2 +3x+5, remainder: 0 −4 x 3 − x 2 −12 x+4x 4 −22 x+2Quotient: x 3 −2 x 2 +4x−8, remainder: −6 x k −1 x−1 k=1, 2, 3. k=4? x k +1 x+1 k=1, 3, 5. k=7?x 6 − x 5 + x 4 − x 3 + x 2 −x+1 x 4 − k 4 x−k k=1, 2, 3. k=4? x k x+1 k=1, 2, 3. k=4?x 3 − x 2 +x−1+ 1 x+1 x k x−1 k=1, 2, 3. k=4?x+1 x−i1+ 1+i x−ix 2 +1 x−ix+1 x+i 1+ 1−i x+ix 2 +1 x+ix 3 +1 x−ix 2 −ix−1+ 1−i x−i x+5, 2 x 2 +9x−5. 2x + 5, 4 x 3 +10 x 2 +6x+15 2 x 2 +3 3x−4, 6 x 4 −8 x 3 +9 x 2 −9x−4 12 x 3 +20 x 2 −21x−36, 2x+3, 3x−4.2x+3 18 x 3 −21 x 2 −40x+48, 3x−4, 3x−4. 10 x 3 +27 x 2 +2x−24, 5x−4, 2x+3.x+2 10 x 3 +30 x 2 −8x−24, 2, x+3. π(25 x 3 −65 x 2 −29x−3), 5x+1.x−3 π(4 x 3 +12 x 2 −15x−50), 2x+5. π(3 x 4 +24 x 3 +46 x 2 −16x−32), x+4.3 x 2 −2 f(x) d(x) d(x) f(x), q(x) r(x) f(x)=d(x)q(x)+r(x) q(x) r(x) d(x). x−k x−k, k, f(k) f(x) d(x) d(x) f(x), q(x) r(x)

$$f(x)=d(x)q(x)+r(x)$$

d(x), x−k,

$$f(x)=(x-k)q(x)+r$$

x−k r. x=k,

$$f(k)=(k-k)q(k)+r \quad =0\cdot q(k)+r \quad =r$$

f(k) f(x) x−k. f(x) x−k, f(k). f, f(x) x=k x−k. f(k). f(x)=6 x 4 − x 3 −15 x 2 +2x−7 x=2. x−2.

2 6 −1 −15 2 −7 12 22 14 32 6 11 7 16 25

f(2)=25. f(2).

$$f(x)=6x^4-x^3-15x^2+2x-7 \quad f(2)=6(2)^4-(2)^3-15(2)^2+2(2)-7 \quad =25$$

f(x)=2 x 5 −3 x 4 −9 x 3 +8 x 2 +2 x=−3. f(−3)=−412

$$f(x)=(x-k)q(x)+r.$$

k r f(k)=0 f(x)=(x−k)q(x)+0 f(x)=(x−k)q(x). x−k f(x). k f(x), x−k f(x). x−k f(x), f(x)=(x−k)q(x)+r k n n n k f(x) (x−k) f(x). (x−k). (x−k) (x+2) x 3 −6 x 2 −x+30. (x+2)

−2 1 −6 −1 30 −2 16 −30 1 −8 15 0

(x+2)

$$(x+2)(x^2-8x+15)$$

$$(x+2)(x-3)(x-5)$$

x 3 −6 x 2 −x+30 f(x)= x 3 +4 x 2 −4x−16 (x−2) x= 2 5 x= 3 4 .

f(x)= a n x n + a n−1 x n−1 +...+ a 1 x+ a 0 f(x) p q p a 0 q a n . f(x), p q , p q f(p q).

f(x)=2 x 4 −5 x 3 + x 2 −4. f(x) p=±1,±2,±4. q=±1,±2.

p q =± 1 1 , ± 1 2 p q =± 2 1 ,± 2 2 p q = ± 4 1 ,± 4 2

2 2 =1 4 2 =2,

p q = Factors of the last Factors of the first =±1,±2,±4,± 1 2

f(x)=2 x 3 + x 2 −4x+1. p q f(x), p q

p q = factor of constant term factor of leading coefficient = factor of 1 factor of 2 ±1 ±1 ±2. p q ±1 ± 1 2 . x f(x).

f(−1)=2 (−1) 3 + (−1) 2 −4(−1)+1=4 f(1)=2 (1) 3 + (1) 2 −4(1)+1=0 f(− 1 2)=2 (− 1 2) 3 + (− 1 2) 2 −4(− 1 2)+1=3 f(1 2)=2 (1 2) 3 + (1 2) 2 −4(1 2)+1=− 1 2 −1,− 1 2 , and 1 2 f(x). f(x). f(x)= x 3 −5 x 2 +2x+1. f, f(x)=4 x 3 −3x−1. p q f(x), p q

p q = factor of constant term factor of leading coefficient = factor of −1 factor of 4 −1 ±1 4 ±1,±2, ±4. p q ±1, ± 1 2 , ± 1 4 .

1 4 0 −3 −1 4 4 1 4 4 1 0

(x−1)

$$(x-1)(4x^2+4x+1).$$

f(x)

$$(x-1)(2x+1)^2.$$

$$2x+1=0 \qquad x=-\frac{1}{2}$$

– 1 2 f x=–0.5, –0.5. x=1, x=1. f f(x)=0. c 1 . f(x) x– c 1 x– c 1 c 2 . x– c 2 f(x). f(x) n>0, a f(x) n

$$f(x)=a(x-c_1)(x-c_2)...(x-c_n)$$

c 1 , c 2 ,..., c n f(x) n f(x)=3 x 3 +9 x 2 +x+3. p q f(x), p q

p q = factor of constant term factor of leading coefficient = factor of 3 factor of 3

±1 ±3. p q , ±3,±1, and ± 1 3 .

–3 3 9 1 3 –9 0 –3 3 0 1 0

(x+3)

$$(x+3)(3x^2+1)$$

$$3x^2+1=0 \qquad x^2=-\frac{1}{3} \qquad x=\pm\sqrt{-\frac{1}{3}}=\pm\frac{i\sqrt{3}}{3}$$

f(x) ± i 3 3 .f x=–3, x=–3. x=–3 x=–3 f(x)=2 x 3 +5 x 2 –11x+4. –4, 1 2 , and 1. n n n (x–c), c f a+bi, b≠0, f(x). x–(a+bi) f(x). f x–(a–bi) f(x). x–(a–bi), x–(a+bi), f a+bi, a–bi f(x). (x–c), c f a+bi, a–bi, f f, (c,f(c)) i, f(–2)=100. x=i x=–i (x+3), (x–2), (x–i), (x+i).

f(x)=a(x+3)(x–2)(x–i)(x+i) f(x)=a(x 2 +x–6)(x 2 +1) f(x)=a(x 4 + x 3 –5 x 2 +x–6) f(–2)=100. x=–2 f(2)=100 f(x).

$$100=a((-2)^4+(-2)^3-5(-2)^2+(-2)-6) \quad 100=a(-20) \quad -5=a$$

$$f(x)=-5(x^4+x^3-5x^2+x-6)$$

$$f(x)=-5x^4-5x^3+25x^2-5x+30$$

i –i i –i –i i. 2+3i 2–3i –2i f(1)=10. f(x)=– 1 2 x 3 + 5 2 x 2 –2x+10 f(x) f(–x) f(–x) f(x) f(x)= a n x n + a n–1 x n–1 +...+ a 1 x+ a 0 f(x) f(–x) f(x)=– x 4 –3 x 3 +6 x 2 –4x–12. f(–x)

f(–x)=– (–x) 4 –3 (–x) 3 +6 (–x) 2 –4(–x)–12 f(–x)=– x 4 +3 x 3 +6 x 2 +4x–12

f(x)=2 x 4 –10 x 3 +11 x 2 –15x+12. V=lwh. l=w+4. h= 1 3 w.

$$V=(w+4)(w)(\tfrac{1}{3}w) \quad V=\tfrac{1}{3}w^3+\tfrac{4}{3}w^2$$

351= 1 3 w 3 + 4 3 w 2 Substitute 351 for V. 1053= w 3 +4 w 2 Multiply both sides by 3. 0= w 3 +4 w 2 –1053 Subtract 1053 from both sides.

±3, ±9, ±13, ±27, ±39, ±81, ±117, ±351, ±1053. x=1.

1 1 4 0 –1053 1 5 5 1 5 5 –1048

x=3. x=9.

$$l=w+4=9+4=13 \text{ and } h=\tfrac{1}{3}w=\tfrac{1}{3}(9)=3$$

f(k), f(x) x–k. k f(x) (x–k) f(x). (x–c), c f(–x) (x 4 –9 x 2 +14)÷(x–2)(3 x 3 –2 x 2 +x–4)÷(x+3) –106(x 4 +5 x 3 –4x–17)÷(x+1)(–3 x 2 +6x+24)÷(x–4) 0

(5 x 5 –4 x 4 +3 x 3 –2 x 2 +x–1)÷(x+6)(x 4 –1)÷(x–4)255(3 x 3 +4 x 2 –8x+2)÷(x–3) (4 x 3 +5 x 2 –2x+7)÷(x+2)–1f(x)=2 x 3 –9 x 2 +13x–6; x–1f(x)=2 x 3 + x 2 –5x+2; x+2–2, 1, 1 2 f(x)=3 x 3 + x 2 –20x+12; x+3f(x)=2 x 3 +3 x 2 +x+6; x+2–2f(x)=–5 x 3 +16 x 2 –9; x–3 x 3 +3 x 2 +4x+12; x+3–34 x 3 –7x+3; x–12 x 3 +5 x 2 –12x–30, 2x+5– 5 2 , 6 , – 6 x 3 –3 x 2 –10x+24=02 x 3 +7 x 2 –10x–24=02, –4, – 3 2x 3 +2 x 2 –9x–18=0x 3 +5 x 2 –16x–80=0 4, –4, –5x 3 –3 x 2 –25x+75=02 x 3 –3 x 2 –32x–15=05, –3, – 1 22 x 3 + x 2 –7x–6=0 2 x 3 –3 x 2 –x+1=01 2 , 1+ 5 2 , 1– 5 23 x 3 – x 2 –11x–6=02 x 3 –5 x 2 +9x–9=03 2 2 x 3 –3 x 2 +4x+3=0x 4 –2 x 3 –7 x 2 +8x+12=02, 3, –1, –2x 4 +2 x 3 –9 x 2 –2x+8=0 4 x 4 +4 x 3 –25 x 2 –x+6=01 2 , – 1 2 , 2, –32 x 4 –3 x 3 –15 x 2 +32x–12=0 x 4 +2 x 3 –4 x 2 –10x–5=0–1, –1, 5 , – 54 x 3 –3x+1=08 x 4 +26 x 3 +39 x 2 +26x+6– 3 4 , – 1 2 x 3 + x 2 +x+1=0x 3 –8 x 2 +25x–26=02, 3+2i, 3–2ix 3 +13 x 2 +57x+85=03 x 3 –4 x 2 +11x+10=0 – 2 3 , 1+2i, 1–2ix 4 +2 x 3 +22 x 2 +50x–75=02 x 3 –3 x 2 +32x+17=0– 1 2 , 1+4i, 1–4if(x)= x 3 –1 f(x)= x 4 – x 2 –1f(x)= x 3 –2 x 2 –5x+6f(x)= x 3 –2 x 2 +x–1f(x)= x 4 +2 x 3 –12 x 2 +14x–5 f(x)=2 x 3 +37 x 2 +200x+300f(x)= x 3 –2 x 2 –16x+32f(x)=2 x 4 –5 x 3 –5 x 2 +5x+3 f(x)=2 x 4 –5 x 3 –14 x 2 +20x+8f(x)=10 x 4 –21 x 2 +11f(x)= x 4 +3 x 3 –4x+4f(x)=2 x 3 +3 x 2 –8x+5 ±5, ±1, ± 5 2f(x)=3 x 3 +5 x 2 –5x+4f(x)=6 x 4 –10 x 2 +13x+1±1, ± 1 2 , ± 1 3 , ± 1 6 f(x)=4 x 5 –10 x 4 +8 x 3 + x 2 –8f(x)=6 x 3 –7 x 2 +11, 1 2 , – 1 3f(x)=4 x 3 –4 x 2 –13x–5 f(x)=8 x 3 –6 x 2 –23x+62, 1 4 , – 3 2f(x)=12 x 4 +55 x 3 +12 x 2 –117x+54 f(x)=16 x 4 –24 x 3 + x 2 –15x+255 4 (2,f(2))=(2,4) (2,f(2))=(2,4)f(x)= 4 9 (x 3 + x 2 –x–1)

1 2 (−3,f(−3))=(−3,5) − 1 2 1 2 (−2,f(−2))=(−2,6)f(x)=− 1 5 (4 x 3 −x) (−2,f(−2))=(−2,10) 16π 72π 48π 28.125π 1 3 98 9 π f(x) f(−x) k f(x) (x−k) f(x) (x−c), c p q p q f(x) x−k, f(k) x, C(x)=15,000x−0.1 x 2 +1000. x x. x

$$f(x)= 15,000x-0.1 x^2 +1000 x$$

f(x)= 1 x . (y=0) as x→−∞. x=0 (y=0) as x→∞. x f(x) x→ a −x a x<a ax→ a +x a x>a ax→∞x x x→−∞ x x f(x)→∞f(x)→−∞f(x)→a a f(x)= 1 x f(x)= 1 x . x=0; xf(x)= 1 x

$$\text{as } x\to 0^- ,f(x)\to-\infty$$

xf(x)= 1 x

$$\text{As } x\to 0^+ , f(x)\to\infty.$$

x=0 x=a a.

$$\text{As } x\to a, f(x)\to\infty, \text{ or as } x\to a, f(x)\to-\infty.$$

f(x)= 1 x x x

$$\text{As } x\to\infty, f(x)\to 0, \text{and as } x\to-\infty, f(x)\to 0.$$

y=0. y=b

$$\text{As } x\to\infty \text{ or } x\to-\infty, f(x)\to b.$$

x=2, x=2.

$$\text{As } x\to 2^- , f(x)\to-\infty, \text{ and as } x\to 2^+ , f(x)\to\infty.$$

y=4.

$$\text{As } x\to\infty, f(x)\to 4 \text{ and as } x\to-\infty, f(x)\to 4.$$

x→±∞, f(x)→0; x→0, f(x)→∞

$$f(x)= \frac{1}{x+2} +3$$

$$f(x)= \frac{3x+7}{x+2}$$

x=−2, x=−2.

$$\text{As } x\to -2^- , f(x)\to-\infty, \text{and as } x\to -2^+ , f(x)\to\infty.$$

y=3.

$$\text{As } x\to\pm\infty, f(x)\to 3.$$

x→3,f(x)→∞, x→±∞,f(x)→−4. f(x)= 1 (x−3) 2 −4. f(x)= 3x+7 x+2 . P(x) and Q(x).

f(x)= P(x) Q(x) = a p x p + a p−1 x p−1 +...+ a 1 x+ a 0 b q x q + b q−1 x q−1 +...+ b 1 x+ b 0 ,Q(x)≠0 t

water: W(t)=100+10t in gallons sugar: S(t)=5+1t in pounds

C,

$$C(t)= \frac{5+t}{100+10t}$$

C(t) t= 12.

$$C(12)= \frac{5+12}{100+10(12)} = 17\ 220$$

$$C(0)= \frac{5+0}{100+10(0)} = 1\ 20$$

17 220 ≈0.08> 1 20 =0.05,

$$1\ 10 =0.1$$

y= 0.1. C, 12 11 f(x)= x+3 x 2 −9 .

$$x^2 -9=0 \qquad x^2 =9 \qquad x=\pm 3$$

x=±3. x=±3. x=±3. x=3 x=−3. f(x)= 4x 5(x−1)(x−5) . x=1 x=5. k(x)= 5+2 x 2 2−x− x 2 .

$$k(x)= \frac{5+2 x^2}{2-x- x^2} = \frac{5+2 x^2}{(2+x)(1-x)}$$

$$(2+x)(1-x)=0 \qquad x=-2,1$$

x=−2 x=1 f(x)= x 2 −1 x 2 −2x−3

$$f(x)= \frac{(x+1)(x-1)}{(x+1)(x-3)}$$

x+1 x=−1, x−3 x=3, x=a a k(x)= x−2 x 2 −4 .

$$k(x)= \frac{x-2}{(x-2)(x+2)}$$

x−2. x=2. x+2. x=−2. x=−2. x=−2, x=2 f(x)= x 2 −25 x 3 −6 x 2 +5x . x=5. x=0, x=1. y=0.

Example: f(x)= 4x+2 x 2 +4x−5

f(x)≈ 4x x 2 = 4 x . g(x)= 4 x , y=0. y=0 f(x)= p(x) q(x) , q(x)≠0 where degree of p<degree of q.

Example: f(x)= 3 x 2 −2x+1 x−1

f(x)≈ 3 x 2 x =3x. g(x)=3x. g(x)=3x f g, y=3x. 3 x 2 −2x+1 x−1 . 3x+1, g(x)=3x+1.

f(x)= p(x) q(x) , q(x)≠0 p>degree of q by 1. y= a n b n , a n b n p(x) q(x) f(x)= p(x) q(x) ,q(x)≠0.

Example: f(x)= 3 x 2 +2 x 2 +4x−5

f(x)≈ 3 x 2 x 2 =3. g(x)=3, x→±∞,f(x)→3, y=3. f(x)= p(x) q(x) , q(x)≠0 where degree of p=degree of q.

f(x)= 3 x 5 − x 2 x+3

f(x)≈ 3 x 5 x =3 x 4 ,

x→±∞, f(x)→∞

y=0.g(x)= 6 x 3 −10x 2 x 3 +5 x 2h(x)= x 2 −4x+1 x+2k(x)= x 2 +4x x 3 −8 f(x)= p(x) q(x) , q(x)≠0.

g(x)= 6 x 3 −10x 2 x 3 +5 x 2 : p=degree of q=3, y= 6 2 y=3.h(x)= x 2 −4x+1 x+2 : p=2 q=1. p>q

x 2 −4x+1 x+2 .

2 1 −4 1 −2 12 1 −6 13

x−2 y=−x−2.k(x)= x 2 +4x x 3 −8 : p=2 < q=3, y=0. C(t)= 5+t 100+10t . t, 10t,

t→∞, C(t)→ 1 10

y= 1 10 . C 1 10 . 1 10

f(x)= (x−2)(x+3) (x−1)(x+2)(x−5)

x=1,−2,and 5, x→±∞, f(x)→0. y=0. f(x)= (2x−1)(2x+1) (x−2)(x+3) x=2 x=−3; y=4.

f(x)= (x−2)(x+3) (x−1)(x+2)(x−5) .

f(0)= (0−2)(0+3) (0−1)(0+2)(0−5) = −6 10 =− 3 5 =−0.6

0= (x−2)(x+3) (x−1)(x+2)(x−5) This is zero when the numerator is zero. 0=(x−2)(x+3) x=2, −3

(0,−0.6), (2,0) (−3,0).

f(x)= 1 (x−3) 2 −4= 1−4 (x−3) 2 (x−3) 2 = 1−4(x 2 −6x+9) (x−3)(x−3) = −4 x 2 +24x−35 x 2 −6x+9

x→±∞, f(x)→−4; so y=−4 x=3, x→3,f(x)→∞. (2.5,0) (3.5,0). (0, −35 9).

f(x)= (x+1) 2 (x−3) (x+3) 2 (x−2) x=−1 (x+1) 2 x=3 (x−3) x=−3 (x+3) 2 f(x)= 1 x 2 . x=2, (x−2)

f(x)= 1 x . f(x)= (x+2)(x−3) (x+1) 2 (x−2) .

f(0)= (0+2)(0−3) (0+1) 2 (0−2) =3

x=−2 x=3. (0,3) (−2,0) (3,0). x+1=0 x−2=0, x=−1 x=2. y=0. x=−1 x=2,

f(x)= (x+2) 2 (x−2) 2 (x−1) 2 (x−3) , y= 1 2 . x=1 and x=3. (0, 4 3 .) (2,0) and (−2,0). (−2,0) (2,0)

x= x 1 , x 2 , ..., x n , x= v 1 , v 2 ,…, v m , x i =any v j ,

f(x)=a (x− x 1) p 1 (x− x 2) p 2 ⋯ (x− x n) p n (x− v 1) q 1 (x− v 2) q 2 ⋯ (x− v m) q n

p i q i a x=−2 x=3. x=−1 1 x , x=2 1 x 2 ,

f(x)=a (x+2)(x−3) (x+1) (x−2) 2 .

(0,−2).

−2=a (0+2)(0−3) (0+1) (0−2) 2 −2=a −6 4 a= −8 −6 = 4 3

f(x)= 4(x+2)(x−3) 3(x+1) (x−2) 2 .

f(x)= P(x) Q(x) = a p x p + a p−1 x p−1 +...+ a 1 x+ a 0 b q x q + b q−1 x q−1 +...+ b 1 x+ b 0 , Q(x)≠0

f(x)= 1 x f(x)= 1 x 2 . x= x 1 , x 2 ,…, x n , x= v 1 , v 2 ,…, v m , x i =any v j ,

f(x)=a (x− x 1) p 1 (x− x 2) p 2 ⋯ (x− x n) p n (x− v 1) q 1 (x− v 2) q 2 ⋯ (x− v m) q n

f(x)= x−1 x+2f(x)= x+1 x 2 −1All reals x≠−1, 1f(x)= x 2 +4 x 2 −2x−8f(x)= x 2 +4x−3 x 4 −5 x 2 +4

All reals x≠−1, −2, 1, 2f(x)= 4 x−1f(x)= 2 5x+2 x=− 2 5 ; y=0; x≠− 2 5f(x)= x x 2 −9f(x)= x x 2 +5x−36

x=4, −9; y=0; x≠4, −9f(x)= 3+x x 3 −27f(x)= 3x−4 x 3 −16x x=0, 4, −4; y=0; x≠0,4, −4

f(x)= x 2 −1 x 3 +9 x 2 +14xf(x)= x+5 x 2 −25 x=−5; y=0; x≠5,−5f(x)= x−4 x−6f(x)= 4−2x 3x−1

x= 1 3 ; y=− 2 3 ; x≠ 1 3 .f(x)= x+5 x 2 +4f(x)= x x 2 −xf(x)= x 2 +8x+7 x 2 +11x+30

f(x)= x 2 +x+6 x 2 −10x+24x-intercepts none, y-intercept (0, 1 4)f(x)= 94−2 x 2 3 x 2 −12f(x)= x 2x+1

x→− 1 2 + ,f(x)→−∞,x→− 1 2 − ,f(x)→∞ x→±∞,f(x)→ 1 2f(x)= 2x x−6f(x)= −2x x−6

x→ 6 + ,f(x)→−∞,x→ 6 − ,f(x)→∞, x→±∞,f(x)→−2f(x)= x 2 −4x+3 x 2 −4x−5

f(x)= 2 x 2 −32 6 x 2 +13x−5 x→− 1 3 + ,f(x)→∞,x→− 1 3 − , f(x)→−∞,x→ 5 2 − ,f(x)→∞,x→ 5 2 +

f(x)→−∞x→±∞,f(x)→ 1 3f(x)= 24 x 2 +6x 2x+1f(x)= 4 x 2 −10 2x−4y=2x+4f(x)= 81 x 2 −18 3x−2

f(x)= 6 x 3 −5x 3 x 2 +4y=2xf(x)= x 2 +5x+4 x−1V.A. x=0,H.A. y=2V.A. x=2, H.A. y=0
p(x)= 2x−3 x+4V.A. x=−4, H.A. y=2;(3 2 ,0);(0,− 3 4)q(x)= x−5 3x−1s(x)= 4 (x−2) 2
V.A. x=2, H.A. y=0, (0,1)r(x)= 5 (x+1) 2f(x)= 3 x 2 −14x−5 3 x 2 +8x−16
V.A. x=−4, x= 4 3 , H.A. y=1;(5,0);(− 1 3 ,0);(0, 5 16)g(x)= 2 x 2 +7x−15 3 x 2 −14+15
a(x)= x 2 +2x−3 x 2 −1V.A. x=−1, H.A. y=1;(−3,0);(0,3)b(x)= x 2 −x−6 x 2 −4
h(x)= 2 x 2 + x−1 x−4V.A. x=4, S.A. y=2x+9;(−1,0);(1 2 ,0);(0, 1 4)k(x)= 2 x 2 −3x−20 x−5
w(x)= (x−1)(x+3)(x−5) (x+2) 2 (x−4)V.A. x=−2, x=4, H.A. y=1,(1,0);(5,0);(−3,0);(0,− 15 16)
z(x)= (x+2) 2 (x−5) (x−3)(x+1)(x+4) x=5 x=−5, (2,0) (−1,0), (0,4)y=50 x 2 −x−2 x 2 −25 x=−4
x=−1, (1,0) (5,0), (0,7) x=−4 x=−5, (4,0) (−6,0), y=7y=7 x 2 +2x−24 x 2 +9x+20 x=−3 x=6,
(−2,0) (1,0), y=−2 x=−1, x=2, (0,2)y= 1 2 x 2 −4x+4 x+1 x=3, x=1, (0,4)y=4 x−3 x 2 −x−12
y=−9 x−2 x 2 −9y= 1 3 x 2 +x−6 x−1y=−6 (x−1) 2 (x+3) (x−2) 2f(x)= 1 x−2xyxy x=2, y=0f(x)= x x−3
f(x)= 2x x+4xyxy x=−4, y=2f(x)= 2x (x−3) 2f(x)= x 2 x 2 +2x+1xyxy x=−1, y=1 f(x). f(x)>0.
f(x)= 2 x+1f(x)= 4 2x−3(3 2 ,∞)f(x)= 2 (x−1)(x+2)f(x)= x+2 (x−1)(x−4)(−2,1)∪(4,∞)
f(x)= (x+3) 2 (x−1) 2 (x+1)f(x)= x 2 −4 x−2(2,4)f(x)= x 3 +1 x+1f(x)= x 2 +x−6 x−2(2,5)
f(x)= 2 x 2 +5x−3 x+3f(x)= x 3 + x 2 x+1(−1,1) t t C(t)= 8+2t 300+20t C t C(t)= 2t 3+ t 2 . t C t
C(t)= 100t 2 t 2 +75 . x x A(x)=50 x 2 + 800 x . x x A(x)=π x 2 + 100 x . x y=b x=a a

$$V= 1 3 \pi r 2 h = 1 3 \pi r 2 (2r) = 2 3 \pi r 3$$

V r.

$$r= 3V 2\pi 3$$

V r. f g f, (a, b), g,(b, a). x y y(x)=a x 2 . a.

$$18=a 6 2 \quad a= 18 36 = 1 2$$

$$y(x)= 1 2 x 2$$

y 2x, x x

$$y= 1 2 x 2 \quad 2y= x 2 \quad x=\pm 2y$$

x

$$y= x 2 2 , x>0$$

x 2x.

$$\text{Area}=l\cdot w =36\cdot 2x =72x =72 2y$$

f −1 (x). f −1 (x) f(x). f(x), (f(x)) −1 = 1 f(x) . f −1 f, f f −1 . f x, f −1

$$f^{-1}(f(x))=x, \text{ for all } x \text{ in the domain of } f$$

$$f(f^{-1}(x))=x, \text{ for all } x \text{ in the domain of } f^{-1}$$

f g, x f g.

$$g(f(x))=f(g(x))=x$$

f(x) y. x y. y, f −1 (x). f(x)= 1 x+1 f −1 (x)= 1 x −1 x≠0,−1 f −1 (f(x))=x f(f −1 (x))=x.
f −1 (f(x))= f −1 (1 x+1) = 1 1 x+1 −1 =(x+1)−1 =x f(f −1 (x))=f(1 x −1)
= 1 (1 x −1)+1 = 1 1 x =x
f(x)= 1 x+1 f −1 (x)= 1 x −1 f(x)= x+5 3 f −1 (x)=3x−5
f −1 (f(x))= f −1 (x+5 3)=3(x+5 3)−5=(x−5)+5=x f(f −1 (x))=f(3x−5)= (3x−5)+5 3 = 3x 3 =x
f(x)=5 x 3 +1. x.

$$y=5 x 3 +1 \quad x=5 y 3 +1 \quad x−1=5 y 3 \quad x−1 5 = y 3 \quad f^{-1}(x)= x−1 5 3$$

f f −1 . y=x. x y, (a,b) f, (b,a) f −1 . (0,1) f, (1,0) f −1 . (1,6) f, (6,1) f −1 . f(x)= x+4 3 .
f −1 (x)= x 3 −4 f(x) with y. x and y. y, f −1 (x). f −1 (x) f:f(x)= (x−4) 2 , x≥4f(x)= (x−4) 2 , x≤4
f(x)= (x−4) 2 x≥4 x≤4 f(x) y.

$$y= (x−4) 2 \text{ Interchange x and y.} \quad x= (y−4) 2 \text{ Take the square root.} \quad \pm x =y−4$$

Add 4 to both sides. 4± x =y
x y f(x), x x y, y x≥4, y≥4, x≥4, f(x)≥4,

$$f^{-1}(x)=4+ x$$

x≤4, f(x)≤4,

$$f^{-1}(x)=4− x$$

y=x. (4, 0) f (0, 4) f −1 . (a, b) f, (b, a) f −1 . f f −1 y=x. f f −1 y=x.

f(x)= (x−2) 2 −3.

(2, −3) x≥2. f(x) y, x.

y= (x−2) 2 −3 Interchange x and y. x= (y−2) 2 −3 Add 3 to both sides.

x+3= (y−2) 2 Take the square root. ± x+3 =y−2 Add 2 to both sides. 2± x+3 =y

Rename the function. f −1 (x)=2± x+3

x≥2,

f −1 (x)=2+ x+3

x≥2. x≤2, f −1 (x)=2− x+3 . y=x. (2, −3) f (−3, 2) f −1 .

domain of f=range of f −1 =[2,∞)

domain of f −1 =range of f=[−3,∞)

f f −1 y=x. f(x)= x 2 +1, x≥0.f −1 (x)= x−1 f(x) y, x. f(x)= x−4 . f(x)≥0. f(x) y, x.

y = x−4 Replace f(x) with y. x = y−4 Interchange x and y. x = y−4 Square each side. x 2 =y−4 Add 4. x 2 +4 =y Rename the function f −1 (x). f −1 (x) = x 2 +4

f −1 (x)= x 2 +4,x≥0

y=x. f(x)= 2x+3 .f −1 (x)= x 2 −3 2 ,x≥0

V= 2 3 π r 3

V= 2 3 π r 3 V r. π=3.14. V. r≥0 V≥0. r V,

V= 2 3 π r 3 r 3 = 3V 2π Solve for r 3 . r= 3V 2π 3 Solve for r.

V=100 π=3.14.

r= 3V 2π 3 = 3·100 2·3.14 3 ≈ 47.7707 3 ≈3.63

f(x)= (x+2)(x−3) (x−1) . (x+2)(x−3) (x−1) ≥0. x (0, 6). x=−2, f(x) f(x) −2≤x<1 or x≥3, [−2,1)∪[3,∞).

C= 20+0.4n 100+n C n n C. n C.

C= 20+0.4n 100+n C(100+n)=20+0.4n 100C+Cn=20+0.4n 100C−20=0.4n−Cn

100C−20=(0.4−C)n n= 100C−20 0.4−C

C=0.35 (35%).

n= 100(0.35)−20 0.4−0.35 = 15 0.05 =300

f(x)= x+3 x−2 .f −1 (x)= 2x+3 x−1 f −1 f, f f −1 . x y.f(x)= (x−4) 2 , [4,∞) f −1 (x)= x +4

f(x)= (x+2) 2 , [−2,∞)f(x)= (x+1) 2 −3, [−1,∞) f −1 (x)= x+3 −1f(x)=2− 3+x

f(x)=3 x 2 +5, (− ∞,0],[0,∞)f −1 (x)=− x−5 3f(x)=12− x 2 , [0,∞)f(x)=9− x 2 , [0,∞)f(x)= 9−x

f(x)=2 x 2 +4, [0,∞)f(x)= x 3 +5 f −1 (x)= x−5 3f(x)=3 x 3 +1f(x)=4− x 3 f −1 (x)= 4−x 3

f(x)=4−2 x 3f(x)= 2x+1f −1 (x)= x 2 −1 2 , [0,∞)f(x)= 3−4xf(x)=9+ 4x−4

f −1 (x)= (x−9) 2 +4 4 , [9,∞)f(x)= 6x−8 +5f(x)=9+2 x 3f −1 (x)= (x−9 2) 3f(x)=3− x 3

f(x)= 2 x+8f −1 (x)= 2−8x xf(x)= 3 x−4f(x)= x+3 x+7 f −1 (x)= 7x−3 1−xf(x)= x−2 x+7

f(x)= 3x+4 5−4x f −1 (x)= 5x−4 4x+3f(x)= 5x+1 2−5xf(x)= x 2 +2x, [−1,∞)f −1 (x)= x+1 −1

f(x)= x 2 +4x+1, [−2,∞)f(x)= x 2 −6x+3, [3,∞)f −1 (x)= x+6 +3f(x)= x 2 +2, x≥0f(x)=4− x 2 , x≥0

f −1 (x)= 4−xf(x)= (x+3) 2 , x≥−3f(x)= (x−4) 2 , x≥4f −1 (x)= x +4f(x)= x 3 +3f(x)=1− x 3

f −1 (x)= 1−x 3f(x)= x 2 +4x, x≥−2f(x)= x 2 −6x+1, x≥3f −1 (x)= x+8 +3f(x)= 2 xf(x)= 1 x 2 , x≥0

f −1 (x)= 1 xf(x)= (x+1)(x−1) xf(x)= (x+2)(x−3) x−1[−2,1)∪[3,∞)f(x)= x(x+3) x−4f(x)= x 2 −x−20 x−2

[−4,2)∪[5,∞)f(x)= 9− x 2 x+4f(x)= x 3 −x−2, y=1, 2, 3(−2, 0); (4, 2); (22, 3)f(x)= x 3 +x−2, y=0, 1, 2

f(x)= x 3 +3x−4, y=0, 1, 2(−4, 0); (0, 1); (10, 2)f(x)= x 3 +8x−4, y=−1, 0, 1f(x)= x 4 +5x+1, y=−1, 0, 1

(−3, −1); (1, 0); (7, 1) a,b,c f(x)=a x 3 +bf(x)= x 2 +bxf −1 (x)= x+ b 2 4 − b 2f(x)= a x 2 +bf(x)= ax+b 3

f −1 (x)= x 3 −b af(x)= ax+b x+c h(t), t h(t)=200−4.9 t 2 . t h, t(h)= 200−h 4.9 , h(t), t

h(t)=600−16 t 2 . t h, V, r, V(r)= 4 3 π r 3 . r V, r(V)= 3V 4π 3 , A, r, A(r)=4π r 2 . r V, n

C(n)= 25+.6n 100+n C, n. n C n(C)= 100C−25 .6−C , T, l, T(l)=2π l 32.2 l T V, r, h, V=π r 2 h. V

r(V)= V 6π , A, r, h, A=2π r 2 +2πrh. V V, r, h, V= 1 3 π r 2 h. r h r(V)= V 4π , r, V, e=0.16s e, s

e=0.16se=0.16(4,600)=736e=0.16(9,200)=1,472e=0.16(18,400)=2,944 y=k x n k k=0.16 n=1.

x and y

y=k x n

y nth x. k= y x n , k x, y. y x y x. y=25 x=2, y x y=k x 3 . y x.

k= y x 3 = 25 2 3 = 25 8

y= 25 8 x 3

x=6 y.

y= 25 8 (6) 3 =675

(0,0). y x. y=24 x=3, y x 128 3 T= 14,000 d d, T= 14,000 d14,000 500 =2814,000 350 =40 14,000 250 =56 y= k x k=14,000. x y

y= k x n

k y nth x. k= x n y. t v vt=distance. vt=100.

t(v)= 100 v =100 v −1

x, y. y x y x. y=25 x=2, y x y= k x 3 . y x.

k= x 3 y = 2 3 ·25 =200

y= k x 3 , k=200 y= 200 x 3

x=6 y.

y= 200 6 3 = 25 27

y x. y=8 x=3, y x 9 2 c, n, d. x y z, x=kyz. x y z, x= ky z . x y z. x=6 y=2 z=8, x y=1 z=27.

x= k y 2 z 3

x=6, y=2, z=8 k.

6= k 2 2 8 3 6= 4k 2 3=k

x= 3 y 2 z 3

x y=1 z=27, y z

x= 3 (1) 2 27 3 =1

x y z. x=40 y=4 z=2, x y=10 z=25. x=20

y=k x n ,k is a nonzero constant.

y= k x n ,k is a nonzero constant.

y x x=6, y=12. y x x=4, y=80. y=5 x 2 y x x=36, y=24. y x x=36, y=24.y=10 x 3 y x x=27, y=15. y x x=1, y=6.y=6 x 4 y x x=4, y=2. y x x=3, y=2.y= 18 x 2 y x x=2, y=5. y x x=3, y=1. y= 81 x 4 y x x=25, y=3. y x x=64, y=5.y= 20 x 3 y x z x=2 and z=3, y=36. y x, z, and w x=1, z=2, w=5, then y=100.y=10xzw y x z x=3 and z=4, then y=72. y x z x=2 and z=25, then y=100. y=10x z y x z W. x=1,z=2, and w=36, then y=48. y x and z w. x=3, z=5, and w=6, then y=10.y=4 xz w y x z w. x=3,z=4, and w=3, then y=6. y x z w t . x=3,z=1,w=25, and t=2, then y=6.y=40 xz w t 2 y x. x=3, then y=12. Find y when x=20. y x. x=2, then y=16. Find y when x=8.y=256 y x. x=3, then y=5. Find y when x=4. y x. x=16, then y=4. Find y when x=36.y=6 y x. x=125, then y=15. Find y when x=1,000. y x. x=3, then y=2. Find y when x=1.y=6 y x. x=4, then y=3. Find y when x=2. y x. x=3, then y=1. Find y when x=1.y=27 y x. x=64, y=12. y x=36. y x. x=27, y=5. y x=125.y=3 y x and z. x=4 z=2, y=16. y x=3 z=3. y x, z, and w. x=2, z=1, w=12, y=72. y x=1, z=2, w=3.y=18 y x z. x=2 z=4, y=144. y x=4 z=5. y x z. x=2 z=9, y=24. y x=3 z=25. y=90 y x z w. x=5, z=2, w=20, y=4. y x=3 z=8, w=48. y x z w. x=2, z=2, w=64, y=12. y x=1, z=3, w=4.y= 81 2 y x z w t . x=2, z=3, w=16, t=3, y=1. y x=3, z=2, w=36, t=5. y x x=2, y=3. y= 3 4 x 2 y x x=2, y=4. y x x=36, y=2.y= 1 3 x y x x=6, y=2. y x x=1, y=4.y= 4 x 2 T, a, T a. T a. s t, v t, K m v. (4+3i)+(−2−5i)2−2i(6−5i)−(10+3i)(2−3i)(3+6i)24+3i2−i 2+ix 2 −4x+5=0 {2+i, 2−i}x 2 +2x+10=0f(x)= x 2 −4x−5f(x)= (x−2) 2 −9 vertex (2,−9), intercepts (5,0); (−1,0); (0,−5) f(x)=−2 x 2 −4x(−2,3) (3,6).f(x)= 3 25 (x+2) 2 +3 (−3,6.5) (2,6). h, x, h(x)= −32 (120) 2 x 2 +x. f(x)=4 x 5 −3 x 3 +2x−1f(x)= 5 x+1 − x 2f(x)= x 2 (3−6x+ x 2)f(x)=2 x 4 +3 x 3 −5 x 2 +7 f(x)=4 x 3 −6 x 2 +2As x→−∞, f(x)→−∞, as x→∞, f(x)→∞f(x)=2 x 2 (1+3x− x 2) f(x)= (x+3) 2 (2x−1) (x+1) 3− 1 2f(x)= x 5 +4 x 4 +4 x 3f(x)= x 3 −4 x 2 +x−41 2 f(x)= x 3 −5x+1 x 3 −2 x 2 +4x+4 x−2 x 2 +4 3 x 4 −4 x 2 +4x+8 x+1x 3 −2 x 2 +5x−1 x+3x 2 −5x+20− 61 x+3 x 3 +4x+10 x−32 x 3 +6 x 2 −11x−12 x+42 x 2 −2x−3(x+4)(2 x 2 −2x−3)3 x 4 +3 x 3 +2x+2 x+1

2 x 3 −3 x 2 −18x−8=0{ −2, 4, − 1 2 }3 x 3 +11 x 2 +8x−4=02 x 4 −17 x 3 +46 x 2 −43x+12=0 { 1, 3, 4, 1 2 }4 x 4 +8 x 3 +19 x 2 +32x+12=0x 3 −3 x 2 −2x+4=02 x 4 − x 3 +4 x 2 −5x+1=0 f(x)= x+2 x−5(−2,0)and(0,− 2 5) x=5 y=1.f(x)= x 2 +1 x 2 −4f(x)= 3 x 2 −27 x 2 +x−2 (0, 27 2) x=1, x=−2, y=3.f(x)= x+2 x 2 −9f(x)= x 2 −1 x+2y= x−2f(x)= 2 x 3 − x 2 +4 x 2 +1f(x)= (x−2) 2 , x≥2 f −1 (x)= x +2f(x)= (x+4) 2 −3, x≥−4f(x)= x 2 +6x−2, x≥−3f −1 (x)= x+11 −3f(x)=2 x 3 −3f(x)= 4x+5 −3 f −1 (x)= (x+3) 2 −5 4 , x≥−3f(x)= x−3 2x+1 y x. x=3, y=36, y x=4.y=64 y x x=25, y=2, y x=4. y x z. x=1 z=2, y=6, y x=2 z=3.y = 72 y x z w. x=3, z=4, w=2, y=48, y x=4, z=5, w=3. V T (3−4i)(4+2i) 20−10i1−4i 3+4ix 2 −4x+13=0{2+3i, 2−3i}f(x)= x 3 (3−6 x 2 −2 x 2)f(x)=8 x 3 −3 x 2 +2x−4 As x→−∞, f(x)→−∞, as x→∞, f(x)→∞f(x)=−2 x 2 (4−3x−5 x 2)f(x)= x 2 +2x−8f(x)= (x+1) 2 −9 (−1,−9) (2,0);(−4,0); (0,−8) (2,0) (4,12).f(x)= (x−3) 3 (3x−1) (x−1) 2f(x)=2 x 6 −6 x 5 +18 x 4 2 x 3 +3x−4 x+22 x 2 −4x+11− 26 x+2x 4 +3 x 2 −4 x−22 x 3 +5 x 2 −7x−12 x+32 x 2 −x−4 (x+3)(2 x 2 −x−4)f(x)=2 x 3 +5 x 2 −6x−9f(x)=4 x 4 +8 x 3 +21 x 2 +17x+4− 1 2 −1±i 15 2 f(x)=4 x 4 +16 x 3 +13 x 2 −15x−18f(x)= x 5 +6 x 4 +13 x 3 +14 x 2 +12x+8 −2 ±i x=3 x=1 x=−2 (0,12). x= 1 2 x=−3 (1,8).f(x)=2 (2x−1) 3 (x+3)8 x 3 −21 x 2 +6=0f(x)= x+4 x 2 −2x−3 (−4,0), (0,− 4 3) x=3, x =−1, y=0f(x)= x 2 +2x−3 x 2 −4f(x)= x 2 +3x−3 x−1y=x+4f(x)= x−2 +4 f(x)=3 x 3 −4f −1 (x)= x+4 3 3f(x)= 2x+3 3x−1 y x x=3, y=2. y x=1.y=18 y x z. x=2 z=27, y=12, y x=5 z=8. k 1.25 1.2% 2031. f(x)=3x+4,xf(x)= 2 xg(x)=2x f(x)=a b x , a b b>1, 0<b<1, f(x)= 2 x −3 3. x−3−2−10123f(x)= 2 x2 −3 = 1 82 −2 = 1 42 −1 = 1 22 0 =12 1 =22 2 =42 3 =8 f f(x)= 2 x (−∞,∞), (0,∞), x→∞,f(x)→∞, x→−∞,f(x)→0, f(x) f(x) y=0 x,

$$f(x)=a\ b\ x$$

a b b≠1. f f a>0. f a<0. (0,a), y=0.f(x)= 4 3(x−2)g(x)= x 3h(x)= (1 3) xj(x)= (−2) x g(x)= x 3 g(x)= x 3 b≠1. j(x)= (−2) x −2, 0.f(x)=2 x 2 −3x+1g(x)= 0.875 xh(x)=1.75x+2j(x)= 1095.6 −2x g(x)= 0.875 x j(x)= 1095.6 −2x 1.b b=−9 x= 1 2 . f(x)=f(1 2)= (−9) 1 2 = −9 ,1?1 1: b=1. f(x)= 1 x =1 x. f(x)= b x ,x f(x)= 2 x . f(3)?

$$f(\ x\ )=2\ x\ f(\ 3\ )=2\ 3$$ Substitute x=3. =8 Evaluate the power.

f(x)=30 (2) x . f(3)?

f(x) =30 (2) x f(3) =30 (2) 3 Substitute x=3. =30(8) Simplify the power first. =240 Multiply.

$$f(3)=30\ (\ 2\ )\ 3\neq 60\ 3\ =216{,}000$$

f(x)=5 (3) x+1 . f(2)

f(x) =5 (3) x+1 f(2) =5 (3) 2+1 Substitute x=2. =5 (3) 3 Add the exponents. =5(27) Simplify the power. =135 Multiply.

f(x)=8 (1.2) x−5 . f(3) 5.5556 x a b b≠1,

$$f(x)=a\ b\ x$$

a b x A(x)=100+50x. B(x)=100 (1+0.5) x .x0100+50(0)=100100 (1+0.5) 0 =1001100+50(1)=150 100 (1+0.5) 1 =1502100+50(2)=200100 (1+0.5) 2 =2253100+50(3)=250100 (1+0.5) 3 =337.5x A(x)=100+50xB(x)=100 (1+0.5) x [0,∞), [100,∞). B(x)=100 (1+0.5) x . 1+0.5=1.5 B(x)=100 (1.5) x , 1.5 x 1.25 1.2%. P(t)=1.25 (1.012) t , t 2013. 2031? t=18, 18

$$P(18)=1.25\ (\ 1.012\ )\ 18\approx 1.549$$

0.6%. P(t)=1.39 (1.006) t , t 2013. 1.548 a b, (0,a), a a, f(x)=a (b) x , b. (0,a), f(x)=a (b) x . a b. a b f(x)=a (b) x . N(t) (N) t. t a=80. N(t)=80 b t b:

N(t) =80 b t 180 =80 b 6 Substitute using point (6, 180). 9 4 = b 6 Divide and write in lowest terms. b = (9 4) 1 6 Isolate b using properties of exponents. b ≈1.1447 Round to 4 decimal places. N(t)=80 (1.1447) t . (0, 80) (6, 180). [0,∞), [80,∞). N(t)=80 (1.1447) t ,t 129 2013, N t.(0,129) (2,236); N(t)=129 (1.3526) t (−2,6) (2,1). f(x)=a b x , a b. (−2,6) 6=a b −2 (2,1) 1=a b 2 a b: a b: b a: f(x)=2.4492 (0.6389) x . (−2, 6) (2, 1). f(x)=2.4492 (0.6389) x (1,3) (2,4.5),f(x)=2 (1.5) x x, (0,a) a a, f(x)=a (b) x , b. (0,a), f(x)=a (b) x . a b. f(x)=a (b) x . (0,3), a=3. (0,3) (2,12).

y=a b x Write the general form of an exponential equation. y=3 b x Substitute the initial value 3 for a. 12=3 b 2 Substitute in 12 for y and 2 for x. 4= b 2 Divide by 3. b=±2 Take the square root. b, b=2. a b f(x)=3 (2) x .f(x)= 2 (2) x . 1.4142 (1.4142) x . y=a· b x (2,24.8) (5,198.4). a=6.2 b=2

y=6.2· 2 x .y≈12· 1.85 x t, P, r, n:

$$A(t)=P\left(1+\frac{r}{n}\right)^{nt}$$

$$A(t)=P\left(1+\frac{r}{n}\right)^{nt}$$

A(t) t P r n P=3000. r = 0.03. n=4. A(10), t = 10.

$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ Use the compound interest formula. $A(10) = 3000\left(1+\frac{0.03}{4}\right)^{4\cdot 10}$

Substitute using given values. ≈$4045.05 Round to two decimal places.

r=0.06. k=2. P, 18 P.

$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ Use the compound interest formula. $40{,}000 = P\left(1+\frac{0.06}{2}\right)^{2(18)}$

Substitute using given values A, r, n, and t. $40{,}000 = P(1.03)^{36}$ Simplify. $40{,}000 (1.03)^{36} = P$ Isolate P.

P ≈$13,801 Divide and round to the nearest dollar.

$A(t)=\left(1+\frac{1}{n}\right)^{n}\left(1+\frac{1}{1}\right)^{1}\left(1+\frac{1}{2}\right)^{2}\left(1+\frac{1}{4}\right)^{4}\left(1+\frac{1}{12}\right)^{12}\left(1+\frac{1}{365}\right)^{365}\left(1+\frac{1}{8766}\right)^{8766}$
$\left(1+\frac{1}{525960}\right)^{525960}\left(1+\frac{1}{31557600}\right)^{31557600}$ n n $\left(1+\frac{1}{n}\right)^{n}$ e.

$\left(1+\frac{1}{n}\right)^{n}$,as n increases without bound

e≈2.718282. e 3.14 . [e x]. [e^( ]. 3.14 [ )]. 5 e 3.14 ≈23.10387. e. e –0.5 . e –0.5 ≈0.60653 e t, a r,

$$A(t)=a\,e^{rt}$$

a r t r>0 r<0

$$A(t)=P\,e^{rt}$$

P r t t, a r. r>0. r<0. t. A(t). r=0.10. P=1000. t=1

$A(t) = P\,e^{rt}$ Use the continuous compounding formula. =1000 (e) 0.1

Substitute known values for P, r, and t. ≈1105.17 Use a calculator to approximate.

17.3% r = –0.173. 100 a=100. t=3

$A(t) = a\,e^{rt}$ Use the continuous growth formula. $= 100\,e^{-0.173(3)}$ Substitute known values for a, r, and t.

≈59.5115 Use a calculator to approximate.

3.77× 10 –26 $f(x)=b^{x}$, where $b>0, b\neq 1$ $f(x)=a\,b^{x}$, where $a>0, b>0, b\neq 1$

$A(t)=P\left(1+\frac{r}{n}\right)^{nt}$, where A(t) is the account value at time t t is the number of years

P is the initial investment, often called the principal r is the annual percentage rate (APR), or nominal rate

n is the number of compounding periods in one year

$A(t)=a\,e^{rt}$, wheretae e≈2.718282 1 t e e≈2.718282. [e x] [exp(x)] e. e 1 8 24 3.25% 20 $5 t

h(t)=–4.9 t 2 +18t+40. t, A(t)=115 (1.025) t . B(t)=82 (1.029) t . B(t)=82 (1.029) t . 20 t=20 43 100

y=300 (1–t) 5y=220 (1.06) x 1.06, 1.y=16.5 (1.025) 1 xy=11,701 (0.97) t 0.97, 0 1.(0,6) (3,750)

(0,2000) (2,20)f(x)=2000 (0.1) x(–1, 3 2) (3,24)(–2,6) (3,1)

f(x)= (1 6) – 3 5 (1 6) x 5 ≈2.93 (0.699) x(3,1) (5,4)xf(x)xh(x)xm(x)xf(x)xg(x) $A(t)=P\left(1+\frac{r}{n}\right)^{nt}$.

10,250 (1+ 0.04 12) 120 . $10,250 3.6% 20 $13,268.58 $PP=A(t)\cdot\left(1+\frac{r}{n}\right)^{-nt}$ $14,472.74 5.5% 5 5

$4,572.56 r.4%y=3742 (e) 0.75t 0.y=150 (e) 3.25 ty=2.25 (e) –2t 0. $12,000 7.2% 30 30 $669.42

f(x)=2 (5) x , f(–3)f(x)=– 4 2x+3 , f(–1)f(–1)=–4f(x)= e x , f(3)f(x)=–2 e x–1 , f(–1)f(–1)≈–0.2707

f(x)=2.7 (4) –x+1 +1.5,f(–2)f(x)=1.2 e 2x –0.3, f(3)f(3)≈483.8146f(x)=– 3 2 (3) –x + 3 2 , f(2)

(0,3) (3,375)y=3· 5 x(3,222.62) (10,77.456)(20,29.495) (150,730.89)y≈18· 1.025 x(5,2.909) (13,0.005)

(11,310.035) (25,356.3652)y≈0.2· 1.95 x APY= $\left(1+\frac{r}{12}\right)^{12}-1$. I(n) n

APY= A(t)–a a = a $\left(1+\frac{r}{365}\right)^{365}(1)-a$ a = a$\left[\left(1+\frac{r}{365}\right)^{365}-1\right]$ a = $\left(1+\frac{r}{365}\right)^{365}-1$;

$I(n)=\left(1+\frac{r}{n}\right)^{n}-1$ $f(x)=a\cdot b^{x}$ a b $b\neq 1$. b b= e n n e $b>1$, $f(x)=a\cdot\left(\frac{1}{b}\right)^{x}$. b= e n , $f(x)=a\left(e\right)^{-nx}$

n f $f(x)=a\cdot\left(\frac{1}{b}\right)^{x}$ $b>1$. $n>0$,$f(x)=a\cdot\left(\frac{1}{b}\right)^{x}=a\left(b^{-1}\right)^{x}=a\left(\left(e^{n}\right)^{-1}\right)^{x}=a\left(e^{-n}\right)^{x}=a\left(e\right)^{-nx}$.

A r t $A(t)=a\left(e\right)^{rt}$, a t I(t)= e rt –1.47,622 1.39%; $155,368.09$35,838.76$82,247.78; $449.75

$f(x)=b^{x}$ $f(x)=2^{x}$. 1.x–3–2–10123f(x)= 2 x1 81 41 21248 2. 2 $f(x)=a\,b^{x}$, b a.x; x x $f(x)=2^{x}$.

$f(x)=2^{x}$ $(0,\infty)$, y=0. $f(x)=b^{x}$ $g(x)=\left(\frac{1}{2}\right)^{x}$. 1.x-3-2-10123g (x) =(1 2) x84211 21 41 8 1 2 . x; x

x $g(x)=\left(\frac{1}{2}\right)^{x}$. $g(x)=\left(\frac{1}{2}\right)^{x}$ $(0,\infty)$, y=0. $f(x)=b^{x}$, $b>0, b\neq 1$, y=0 $(-\infty,\infty)$ $(0,\infty)$ (0,1) $b>1$ $b<1$

$f(x)=b^{x}$, 3 (0,1). $(-\infty,\infty)$, $(0,\infty)$, y=0. $f(x)=0.25^{x}$. b=0.25 y=0.x–3–2–10123f(x)= 0.25 x641641

0.250.06250.015625 (0,1), (–1,4) (1,0.25). $(-\infty,\infty)$; $(0,\infty)$; y=0. $f(x)=4^{x}$. $(-\infty,\infty)$; $(0,\infty)$; y=0.

f(x)= b x d f(x)= b x , d f(x)= 2 x , d=3: g(x)= 2 x +3 h(x)= 2 x −3. f(x)= 2 x (−∞,∞) 3 g(x)= 2 x +3: 3 (0,4). 3 y=3. (3,∞). 3 h(x)= 2 x −3: 3 (0,−2). 3 y=−3. (−3,∞). c f(x)= b x , c f(x)= 2 x , c=3: g(x)= 2 x+3 , h(x)= 2 x−3 . f(x)= 2 x (−∞,∞), y=0, 3 g(x)= 2 x+3 , (0,8). 2 x+3 =(8) 2 x , 8. 3 h(x)= 2 x−3 , (0, 1 8). 2 x−3 =(1 8) 2 x , 1 8 . c d, f(x)= b x+c +d f(x)= b x d d. c c. (0, b c +d). y=d. (d,∞). (−∞,∞), f(x)= b x+c +d, y=d. (−c,d). f(x)= b x c c c c f(x)= b x d d d d (−∞,∞), (d,∞), y=d. f(x)= 2 x+1 −3. f(x)= b x+c +d, b=2, c=1, d=−3. y=d y=−3. (−c,d), (−1,−3). f(x)= b x (−∞,∞); (−3,∞); y=−3. f(x)= 2 x−1 +3. (−∞,∞); (3,∞); y=3. f(x)= b x+c +d x, f(x) f(x). x, 42=1.2 (5) x +2.8 1.2 (5) x +2.8 x y. x=2. x≈2.166. 4=7.85 (1.15) x −2.27 x≈−1.608 f(x)= b x |a|>0. f(x)= 2 x , a=3, g(x)=3 (2) x a= 1 3 , h(x)= 1 3 (2) x g(x)=3 (2) x f(x)= 2 x 3. h(x)= 1 3 (2) x f(x)= 2 x 1 3 . a>0, f(x)=a (b) x a |a|>1. a |a|<1. (0,a). y=0, (0,∞), (−∞,∞), f(x)=4 (1 2) x . b= 1 2 x x a=4, f(x)= (1 2) x 4.x−3−2−10123

f(x)=4(1 2) x

321684210.5 (0,4), (−1,8) (1,2). (−∞,∞); (0,∞); y=0. f(x)= 1 2 (4) x . (−∞,∞); (0,∞); y=0. f(x)= b x −1, −1, f(x)= 2 x , g(x)= −2 x , h(x)= 2 −x , g(x)=− 2 x f(x)= 2 x g(x)= 2 −x f(x)= 2 x f(x)=− b x f(x)= b x (0,−1). (−∞,0) y=0 (−∞,∞), f(x)= b −x f(x)= b x (0,1), y=0, (0,∞), (−∞,∞), g(x), f(x)= (1 4) x f(x)= (1 4) x f(x) −1 g(x)=− (1 4) x . x−3−2−10123

g(x)=−(1 4) x

−64−16−4−1−0.25−0.0625−0.0156 (0,−1), (−1,−4) (1,−0.25). (−∞,∞); (−∞,0); y=0. g(x), f(x)= 1.25 x (−∞,∞); (0,∞); y=0. f(x)= b x c d

f(x)= b x+c +d

| a |>1 0<| a |<1

f(x)=a b x

f(x)=− b x

f(x)= b −x = (1 b) x

f(x)=a b x+c +d

f(x)=a b x+c +d

y= b x , b>1, c | a | | a |>0. | a | 0<| a |<1. d a<0.f(x)= e x 2 4 f(x)=a b x+c +d. a,b,c, d. f(x)= e x , b=e. 2 a=2. x −x e −x . d=4.

f(x) =a b x+c +d =2 e −x+0 +4 =2 e −x +4

(−∞,∞); (4,∞); y=4.f(x)= e x 1 3 , 2f(x)=− 1 3 e x −2; (−∞,∞); (−∞,2); y=2. f(x)= b x f(x)=a b x+c +d f(x)= b x (0, 1), (−∞, ∞), (0, ∞), y=0. b>1, y=0, 0<b<1, y=0. f(x)= b x +d f(x)= b x . f(x)= b x+c f(x)= b x . f(x)= b x+c +d f(x)=a b x , a>0, | a |>1 0<| a |<1 f(x)= b x . f(x)= b x −1, f(x)=− b x , −1, f(x)= b −x , f(x)=a b x+c +d. f(x)=a b x+c +d, x f(x)= 3 x 4. g(x)? g(x)=4 (3) −x ; (0,4); 0. f(x)= (1 2) −x 1 5 . g(x)? f(x)= 10 x 7 g(x)? g(x)=− 10 x +7; (0,6); 7. f(x)= (1.68) x 3 2, 3 g(x)? f(x)=− 1 2 (1 4) x−2 +4 2 4, 4 g(x)? g(x)=2 (1 4) x ; (0, 2); 0.f(x)=3 (1 2) x g(x)=−2 (0.25) x (0,−2)h(x)=6 (1.75) −xf(x)=3 (1 4) x ,g(x)=3 (2) x , h(x)=3 (4) xf(x)= 1 4 (3) x , g(x)=2 (3) x , h(x)=4 (3) xf(x)=2 (0.69) xf(x)=2 (1.28) xf(x)=2 (0.81) xf(x)=4 (1.28) x f(x)=2 (1.59) xf(x)=4 (0.69) x f(x)=a b x . b? b? a? a?f(x)= 1 2 (4) xf(x)=3 (0.75) x −1 f(x)=−4 (2) x +2 f(x)= 2 x . f(x)= 2 −xh(x)= 2 x +3 h(x)=3; 3.f(x)= 2 x−2f(x)=−5 (4) x −1x→∞ f(x)→−∞x→−∞f(x)→−1f(x)=3 (1 2) x −2f(x)=3 (4) −x +2x→∞f(x)→2x→−∞f(x)→∞ f(x)= 4 x . f(x) f(x) f(x)= 4 x −3 f(x) f(x) f(x)= 4 x−5 f(x) f(x) f(x)= 4 −x y= 2 x . y=− 2 x +3 y=−2 (3) x +7 x.g(x)= 1 3 (7) x−2 g(6).g(6)=800+ 1 3 ≈800.3333f(x)=4 (2) x−1 −2 f(5). h(x)=− 1 2 (1 2) x +6 h(−7).h(−7)=−58 f(x)=a b x +d.−50=− (1 2) −x116= 1 4 (1 8) xx≈−2.953 12=2 (3) x +15=3 (1 2) x−1 −2x≈−0.222−30=−4 (2) x+2 +2 F(x)= (b) x G(x)= (1 b) x . b x (1 b) x b>0. G(x)= (1 b) x F(x)= b x ; b>0 f(x)= b x , (1 b) x F(−x). f(x)= 4 x , g(x)= 4 x−2 , h(x)=(1 16) 4 x . b x (1 b n) b x b>0. g(x) h(x) f(x); b>0, f(x)= b x , (1 b n) b x f(x−n). 10 8−4 = 10 4 =10,000 10 x =500, x x? 10 x =500. 10 2 =100 10 3 =1000, x y= 10 x y= b x x= b y x y y y x, y= log b (x). b b b x y, b x y.b y x, 2 5 =32, log 2 32=5.

$$\log b(x)=y \Leftrightarrow b^y=x,\ b>0,b\neq 1$$

b log b (x), f(x). log b x. x. y= log b (x) y= b x b x x>0,b>0,b≠1,

y= log b (x) is equivalent to b y =x

log b (x) b x b x." y b x. x y b is (0,∞). b is (−∞,∞). log b (x)=y, y= log b x b,y,andx. log b x=y b y =x.log 6 (6)= 1 2log 3 (9)=2 b,y,and x. b y =x.log 6 (6)= 1 2 b=6,y= 1 2 ,and x= 6. log 6 (6)= 1 2

$$6^{\frac{1}{2}}=6.$$

log 3 (9)=2 b=3,y=2,and x=9. log 3 (9)=2 3 2 =9. log 10 (1,000,000)=6log 5 (25)=2 log 10 (1,000,000)=6 10 6 =1,000,000log 5 (25)=2 5 2 =25 b, x, y. x= log b (y).2 3 =85 2 =25 10 −4 = 1 10,000 b,y,andx. x= log b (y).2 3 =8 b=2, x=3, y=8. 2 3 =8 log 2 (8)=3.5 2 =25 b=5, x=2, y=25. 5 2 =25 log 5 (25)=2.10 −4 = 1 10,000 b=10, x=−4, y= 1 10,000 . 10 −4 = 1 10,000 log 10 (1 10,000)=−4.3 2 =95 3 =1252 −1 = 1 23 2 =9 log 3 (9)=25 3 =125 log 5 (125)=32 −1 = 1 2 log 2 (1 2)=−1 log 2 8. 2 2 3 =8, log 2 8=3. log 7 49 log 3 27 7 2 =49. log 7 49=2 3 3 =27. log 3 27=3 log 2 3 4 9 2 3 4 9 ? 2 2 =4 3 2 =9, (2 3) 2 = 4 9 . log 2 3 (4 9)=2. y= log b (x), x b: b y =x. b y b x? y= log 4 (64) 4 y =64.

$$4^3=64$$

$$\log(64)\,4=3$$

y= log 121 (11) log 121 (11)= 1 2 121 = (121) 1 2 =11 y= log 3 (1 27) 3 y = 1 27 . 1 27 ? 3 3 =27, 1 27 ? b −a = 1 b a .

$$3^{-3}=\frac{1}{3^3}=\frac{1}{27}$$

log 3 (1 27)=−3. y= log 2 (1 32) log 2 (1 32)=−5 log(x) log 10 (x). 10. log 10 (x) log(x). x x>0,

y=log(x) is equivalent to 10 y =x

log(x) 10 x x. y 10 x. y=log(x), x 10: 10 y =x. 10 y 10 x? y=log(1000) 10 y =1000. 10 10 3 =1000

log(1000)=3. y=log(1,000,000).log(1,000,000)=6 y=log(x), x, y=log(321) log(321)≈2.5065. 10 2 =100 10 3 =1000. log(321) log(100) log(1000).

$$100<321<1000\quad 2<2.5065<3$$

y=log(123) log(123)≈2.0899 10 x =500 x

10 x =500 log(500) =x Use the definition of the common log.

500, log(500)≈2.699. 2.699. 8,500 10 x =8500 x 3.929. e. e e log e (x), ln(x). ln(x) ln1=0. ln e e. log e (x)ln(x).x x>0,

y=ln(x) is equivalent to e y =x

ln(x) e x x. y e x. y=e x y=ln(x) ln(e x)=x x e = ln(x) x x>0. y=ln(x), x, y=ln(500) 500, ln(500)≈6.2146 ln(−500). x>0,b>0,b≠1,y= log b (x) b y =x. x>0,y=log(x) 10 y =x. x>0,y=ln(x) e y =x. b b. 10. 10 b b y =x log b x=y b>0,b≠1. b b y, b y log b x, x b y. f(x)= log b x g(x)= b x ? log b x=y x b y =x, x. b, b, b e. log e (x), ln(x).log 4 (q)=mlog a (b)=ca c =blog 16 (y)=x log x (64)=yx y =64log y (x)=−11log 15 (a)=b15 b =alog y (137)=xlog 13 (142)=a13 a =142 log(v)=tln(w)=ne n =w4 x =yc d =klog c (k)=dm −7 =n19 x =ylog 19 y=xx − 10 13 =yn 4 =103 log n (103)=4(7 5) m =ny x = 39 100log y (39 100)=x10 a =be k =hln(h)=k x log 3 (x)=2log 2 (x)=−3 x= 2 −3 = 1 8log 5 (x)=2log 3 (x)=3x= 3 3 =27log 2 (x)=6log 9 (x)= 1 2x= 9 1 2 =3log 18 (x)=2 log 6 (x)=−3x= 6 −3 = 1 216log(x)=3ln(x)=2x= e 2log(100 8)10 log(32)322log(.0001)e ln(1.06)1.06 ln(e −5.03)e ln(10.125) +414.125 b log 3 (1 27)log 6 (6)1 2log 2 (1 8)+46 log 8 (4)4log(10,000) log(0.001)−3log(1)+72log(100 −3)−12ln(e 1 3)ln(1)0ln(e −0.225)−325ln(e 2 5)10log(0.04)ln(15) 2.708ln(4 5)log(2)0.151ln(2) x=0 f(x)=log(x)? x=0? x=0. x=0 f(x)=log(x). n n=log(0). 10 n =0, n x=0 f(x)=log(x). f(x)=0 f(x)=log(x)? x? x lnx=2? x lnx=2. x= e 2 , x= e 2 . ln(x)=ln(e 2)=2. log 3 (27) log 4 (1 64) =−1? ln(e 1.725) ln(1) =1.725? ln(1)=0, ln(e 1.725) ln(1) EI EI= log 2 (f 2 t), f t 8 2 EI −2,2 log I 1 I 2 = M 1 − M 2 M x; log 10 (x) log(x). b x; y= log b (x) e x; log e (x) ln(x). $2500 5%, t A=2500 e 0.05t . y= b x x b>0,b≠1, y (−∞,∞). y (0,∞).

y= log b (x) y= b x . y= log b (x) y= b x : (0,∞). y= log b (x) y= b x : (−∞,∞). y= log b (x) y= b x . y= log b (x) f(x)= log 4 (2x−3). x 2x−3, x:

2x−3>0 Show the argument greater than zero. 2x>3 Add 3. x>1.5 Divide by 2.

f(x)= log 4 (2x−3) (1.5,∞). x. f(x)= log 2 (x+3)? x+3>0.

x+3>0 The input must be positive. x>−3 Subtract 3.

f(x)= log 2 (x+3) (−3,∞). f(x)= log 5 (x−2)+1?(2,∞) f(x)=log(5−2x)? 5−2x>0.

5−2x>0 The input must be positive. −2x>−5 Subtract 5. x< 5 2

Divide by −2 and switch the inequality.

f(x)=log(5−2x) (−∞, 5 2). f(x)=log(x−5)+2?(5,∞) y= log b (x) y= log b (x). y= b x , y=x. y= 2 x x= log 2 (y) x−3−2−101232 x =y1 81 41 21248log 2 (y)=x−3−2−10123 f(x)= 2 x g(x)= log 2 (x). f(x)= 2 x(−3, 1 8)(−2, 1 4)(−1, 1 2)(0,1)(1,2)(2,4)(3,8)g(x)= log 2 (x)(1 8 ,−3)(1 4 ,−2) (1 2 ,−1)(1,0)(2,1)(4,2)(8,3) f g. f(x)= 2 x g(x)= log 2 (x) y=x.f(x)= 2 x (0,1) g(x)= log 2 (x) (1,0). f(x)= 2 x ,(−∞,∞), g(x)= log 2 (x). f(x)= 2 x ,(0,∞), g(x)= log 2 (x). x b>0, b≠1, f(x)= log b (x): x=0 (0,∞) (−∞,∞) (1,0) (b,1) b>1 0<b<1 b f(x)= log b (x) ln(x) e≈2.718.) f(x)= log b (x), x=0. (1,0). (b,1). (0,∞), (−∞,∞), x=0. f(x)= log 5 (x). b=5 x=0, (1,0). (5,1) (0,∞), (−∞,∞), x=0. f(x)= log 1 5 (x). (0,∞), (−∞,∞), x=0. y= log b (x) c f(x)=lo g b (x), c c. f(x)= log b (x) c>0 g(x)= log b (x+c), h(x)= log b (x−c). c, f(x)= log b (x+c) y= log b (x) c c>0. y= log b (x) c c<0. x=−c. (−c,∞). (−∞,∞). f(x)= log b (x+c), c>0, f(x)= log b (x) c c<0, f(x)= log b (x) c x=−c. c x (−c,∞), (−∞,∞), x=−c. f(x)= log 3 (x−2) f(x)= log 3 (x−2), x+(−2)=x−2. c=−2, c<0. f(x)= log 3 (x) x=−(−2) x=2. (1 3 ,−1),(1,0), (3,1). x (7 3 ,−1),(3,0), (5,1). (2,∞), (−∞,∞), x=2. f(x)= log 3 (x+4) (−4,∞), (−∞,∞), x=−4. d f(x)= log b (x), d d. f(x)= log b (x) g(x)= log b (x)+d h(x)= log b (x)−d. d, f(x)= log b (x)+d y= log b (x) d d>0. y= log b (x) d d<0. x=0. (0,∞). (−∞,∞). f(x)= log b (x)+d, d>0, f(x)= log b (x) d d<0, f(x)= log b (x) d x=0. d y (0,∞), (−∞,∞), x=0. f(x)= log 3 (x)−2 f(x)= log 3 (x)−2, d=−2. d<0. f(x)= log 3 (x) x=0. (1 3 ,−1), (1,0), (3,1). (1 3 ,−3),(1,−2), (3,−1). (0,∞), (−∞,∞), x=0. (0,∞), (−∞,∞), x=0. f(x)= log 2 (x)+2 (0,∞), (−∞,∞), x=0. f(x)= log b (x) a>0, a>1 f(x)= log b (x) g(x)=a log b (x) h(x)= 1 a log b (x). a>1, f(x)=a log b (x) y= log b (x) a a>1. y= log b (x) a 0<a<1. x=0. (1,0). (0,∞). (−∞,∞). f(x)=a log b (x),a>0, |a|>1, f(x)= log b (x) a |a|<1, f(x)= log b (x) a x=0. y a. (0,∞), (−∞,∞), x=0. f(x)=2 log 4 (x) f(x)=2 log 4 (x), a=2. f(x)= log 4 (x) x=0. (1 4 ,−1),(1,0), (4,1). y (1 4 ,−2),(1,0) , (4,2). (0, ∞), (−∞,∞), x=0. (0,∞), (−∞,∞), x=0. f(x)= 1 2 log 4 (x) (0,∞), (−∞,∞), x=0. f(x)=5log(x+2). x=−2. (−1,0). (−2,∞). (−1,0) (8,5). x=8 x=8, x+2=10, (−2,∞), (−∞,∞), x=−2. f(x)=3log(x−2)+1. (2,∞), (−∞,∞), x=2. f(x)= log b (x) −1, −1, b>1, f(x)= log b (x) g(x)= −log b (x) h(x)= log b (−x). f(x)= −log b (x) y= log b (x) (0,∞), (−∞,∞), x=0, f(x)= log b (−x) y= log b (x) (−∞,0). (−∞,∞), x=0, f(x)= log b (x),If f(x)=− log b (x) If f(x)= log b (−x) x=0. x=0. (1,0). (1,0). f(x)= log b (x) f(x)= log b (x) (0,∞), (−∞,∞), x=0. (−∞,0), (−∞,∞), x=0. f(x)=log(−x) f(x)=log(−x), b=10 −1,f f(x)=log(−x) x x=0. (−1,0). (−∞,0), (−∞,∞), x=0. f(x)=−log(−x). (−∞,0), (−∞,∞), x=0. x, x, 4ln(x)+1=−2ln(x−1) 4ln(x)+1 −2ln(x−1) x y. x=1. x≈1.339. 5log(x+2)=4−log(x) x≈3.049 y= log b (x) c d y= log b (x+c)+d | a |>1 | a |<1y=a log b (x)y=− log b (x)y= log b (−x)y=a log b (x+c)+d y= log b (x),

f(x)=a log b (x+c)+d

y= log b (x),b>1, d c | a | | a |>0. | a | 0<| a |<1. a<0. f(x)=log(−x), f(x)=−2 log 3 (x+4)+5? x=−4. x=−4. f(x)=3+ln(x−1)?x=1 x=−2

f(x)=−alog(x+2)+k

(−1,1) (2,−1). (−1,1),

1=−alog(−1+2)+k Substitute (−1,1). 1=−alog(1)+k Arithmetic. 1=k log(1)=0.

(2,−1)

−1=−alog(2+2)+1 Plug in (2,−1). −2=−alog(4) Arithmetic. a= 2 log(4) Solve for a.

f(x)=− 2 log(4) log(x+2)+1.xf(x)xf(x)f(x)=2ln(x+3)−1 x=−3 x=−3 {x | x>−3}. x→− 3 + ,f(x)→−∞

x→∞,f(x)→∞. f(x)= log b (x) f(x)=a log b (x+c)+d x. f(x)= log b (x) (1,0), (0,∞), (−∞,∞), x=0, b>1, 0<b<1, f(x)= log b (x+c) y= log b (x) c c>0. c c<0. f(x)= log b (x)+d y= log b (x) d d>0. d d<0. a>0, f(x)=a log b (x) y= log b (x) a |a|>1. y= log b (x) a |a|<1. y= log b (x) −1, −1, f(x)=− log b (x) f(x)= log b (−x) f(x)=a log b (x+c)+d. f(x)=a log b (x+c)+d, x=−c f(x)=a log b (x+c)+d, y=x. (a,b) (b,a) f(x)= log b (x). x f(x)= log 3 (x+4)h(x)=ln(1 2 −x) (−∞, 1 2); (−∞,∞)g(x)= log 5 (2x+9)−2h(x)=ln(4x+17)−5 (− 17 4 ,∞); (−∞,∞) f(x)= log 2 (12−3x)−3 f(x)= log b (x−5) (5,∞); x=5 g(x)=ln(3−x) f(x)=log(3x+1) (− 1 3 ,∞); x=− 1 3 f(x)=3log(−x)+2 g(x)=−ln(3x+9)−7 (−3,∞); x=−3f(x)=ln(2−x)f(x)=log(x− 3 7)(3 7 , ∞)x= 3 7 x→ (3 7) + ,f(x)→−∞x→∞,f(x)→∞h(x)=−log(3x−4)+3g(x)=ln(2x+6)−5(−3,∞)x=−3x→− 3 + f(x)→−∞x→∞f(x)→∞f(x)= log 3 (15−5x)+6h(x)= log 4 (x−1)+1 (1,∞); (−∞,∞); x=1; (5 4 ,0); f(x)=log(5x+10)+3g(x)=ln(−x)−2 (−∞,0); (−∞,∞); x=0; (− e 2 ,0); f(x)= log 2 (x+2)−5 h(x)=3ln(x)−9 (0,∞); (−∞,∞); x=0; (e 3 ,0); d(x)=log(x)f(x)=ln(x)g(x)= log 2 (x)h(x)= log 5 (x) j(x)= log 25 (x)f(x)= log 1 3 (x)g(x)= log 2 (x)h(x)= log 3 4 (x)f(x)=log(x) g(x)= 10 xf(x)=log(x) g(x)= log 1 2 (x)f(x)= log 4 (x) g(x)=ln(x)f(x)= e x g(x)=ln(x)f(x)= log 4 (−x+2)g(x)=− log 4 (x+2) h(x)= log 4 (x+2) f(x)= log 2 (x+2) f(x)=2log(x) f(x)=ln(−x)g(x)=log(4x+16)+4g(x)=log(6−3x)+1 h(x)=− 1 2 ln(x+1)−3 y= log 2 (x) f(x)= log 2 (−(x−1)) f(x)= log 3 (x) f(x)= log 4 (x) f(x)=3 log 4 (x+2) f(x)= log 5 (x) log(x−1)+2=ln(x−1)+2x=2log(2x−3)+2=−log(2x−3)+5ln(x−2)=−ln(x+1)x≈2.303 2ln(5x+1)= 1 2 ln(−5x)+11 3 log(1−x)=log(x+1)+ 1 3x≈−0.472 b b≠1. log b 1 f(x)= log 1 2 (x) g(x)=− log 2 (x). f(x)= log 1 2 (x) g(x)=− log 2 (x) b≠1, log b (x)=− log 1 b (x). f(x)=ln(x+2 x−4)? x+2 x−4 >0 f(x)= x+2 x−4 , (−∞,−2) (4,∞). (−∞,−2)∪(4,∞). f(x)=log(x 2 +4x+4) a

pH=−log([H +]) =log(1 [H +])

−log([H +]) log(1 [H +])

log b 1=0 log b b=1

log 5 1=0 5 0 =1. log 5 5=1 5 1 =5.

log b (b x)=x b log b x =x,x>0

log(100), log 10 (10 2), log b (b x)=x log 10 (10 2)=2. e ln(7) , e log e 7 , b log b x =x e log e 7 =7.

log b M= log b N if and only if M=N

log 3 (3x)= log 3 (2x+5) x. x:

3x=2x+5 Set the arguments equal. x=5 Subtract 2x.

log 3 (3x)+ log 3 (2x+5)=2? x a x b = x a+b . x M,N, b, b≠1,

log b (MN)= log b (M)+ log b (N).

m= log b M n= log b N. b m =M b n =N.

log b (MN) = log b (b m b n) Substitute for M and N. = log b (b m+n)

Apply the product rule for exponents. =m+n Apply the inverse property of logs. = log b (M)+ log b (N)

Substitute for m and n.

log b (wxyz).

log b (wxyz)= log b w+ log b x+ log b y+ log b z

log b (MN)= log b (M)+ log b (N) for b>0

log 3 (30x(3x+4)). 30

log 3 (30x(3x+4))= log 3 (2·3·5·x·(3x+4))

log 3 (30x(3x+4))= log 3 (2)+ log 3 (3)+ log 3 (5)+ log 3 (x)+ log 3 (3x+4)

log b (8k).log b 2+ log b 2+ log b 2+ log b k=3 log b 2+ log b k x a b = x a−b . x M,N, b, b≠1,

log b (M N)= log b (M)− log b (N).

m= log b M n= log b N. b m =M b n =N.

log b (M N) = log b (b m b n) Substitute for M and N. = log b (b m−n) Apply the quotient rule for exponents. =m−n Apply the inverse property of logs. = log b (M)− log b (N) Substitute for m and n.

log(2 x 2 +6x 3x+9),

log(2 x 2 +6x 3x+9)=log(2x(x+3) 3(x+3)) Factor the numerator and denominator. =log(2x 3) Cancel the common factors.

log(2x 3)=log(2x)−log(3) =log(2)+log(x)−log(3)

log b (M N)= log b M− log b N

log 2 (15x(x−1) (3x+4)(2−x)).

log 2 (15x(x−1) (3x+4)(2−x))= log 2 (15x(x−1))− log 2 ((3x+4)(2−x))

log 2 (15x(x−1))− log 2 ((3x+4)(2−x))=[log 2 (3)+ log 2 (5)+ log 2 (x)+ log 2 (x−1)]−[log 2 (3x+4)+ log 2 (2−x)] = log 2 (3)+ log 2 (5)+ log 2 (x)+ log 2 (x−1)− log 2 (3x+4)− log 2 (2−x)

x=− 4 3 x=2. x>0,x>1,x>− 4 3 , x<2. log 3 (7 x 2 +21x 7x(x−1)(x−2)).

log 3 (x+3)− log 3 (x−1)− log 3 (x−2) x 2 ?

log b (x 2) = log b (x·x) = log b x+ log b x =2 log b x

100= 10 2 3 = 3 1 2 1 e = e −1

log b (M n)=n log b M

log 2 x 5 . x,

log 2 (x 5)=5 log 2 x

ln x 2 . 2lnx log 3 (25) log 3 (25)= log 3 (5 2).

log 3 (5 2)=2 log 3 (5)

ln(1 x 2).−2ln(x) 4ln(x) 4ln(x), x, 4ln(x)=ln(x 4). 2 log 3 4 log 3 16

log b (6x y) = log b (6x)− log b y = log b 6+ log b x− log b y

log b (A C) = log b (A C −1) = log b (A)+ log b (C −1) = log b A+(−1) log b C = log b A− log b C

ln(x 4 y 7)

ln(x 4 y 7)=ln(x 4 y)−ln(7)

ln(x 4 y)−ln(7)=ln(x 4)+ln(y)−ln(7)

ln(x 4)+ln(y)−ln(7)=4ln(x)+ln(y)−ln(7)

log(x 2 y 3 z 4).2logx+3logy−4logz log(x).

log(x) =log x (1 2) = 1 2 logx

ln(x 2 3).2 3 lnx ln(x 2 + y 2)? log 6 (64 x 3 (4x+1) (2x−1)).

log 6 (64 x 3 (4x+1) (2x−1)) = log 6 64+ log 6 x 3 + log 6 (4x+1)− log 6 (2x−1)

Apply the Quotient Rule. = log 6 2 6 + log 6 x 3 + log 6 (4x+1)− log 6 (2x−1)

Simplify by writing 64 as 2 6 . =6 log 6 2+3 log 6 x+ log 6 (4x+1)− log 6 (2x−1) Apply the Power Rule.

ln((x−1) (2x+1) 2 (x 2 −9)).1 2 ln(x−1)+ln(2x+1)−ln(x+3)−ln(x−3)

log 3 (5)+ log 3 (8)− log 3 (2)

log 3 (5)+ log 3 (8)= log 3 (5·8)= log 3 (40)

log 3 (40)− log 3 (2)

log 3 (40)− log 3 (2)= log 3 (40 2)= log 3 (20)

log3−log4+log5−log6.log(3·5 4·6); log(5 8) log 2 (x 2)+ 1 2 log 2 (x−1)−3 log 2 ((x+3) 2).

log 2 (x 2)+ 1 2 log 2 (x−1)−3 log 2 ((x+3) 2)= log 2 (x 2)+ log 2 (x−1)− log 2 ((x+3) 6)

log 2 (x 2)+ log 2 (x−1)− log 2 ((x+3) 6)= log 2 (x 2 x−1)− log 2 ((x+3) 6)

log 2 (x 2 x−1)− log 2 ((x+3) 6)= log 2 x 2 x−1 (x+3) 6

2logx−4log(x+5)+ 1 x log(3x+5)

2logx−4log(x+5)+ 1 x log(3x+5)=log(x 2)−log((x+5) 4)+log((3x+5) x −1)

log(x 2)−log((x+5) 4)+log((3x+5) x −1)=log(x 2)−log((x+5) 4 (3x+5) x −1)

log(x 2)−log((x+5) 4 (3x+5) x −1)=log(x 2 (x+5) 4 ((3x+5) x −1))

log(5)+0.5log(x)−log(7x−1)+3log(x−1) log(5 (x−1) 3 x (7x−1))

4(3log(x)+log(x+5)–log(2x+3)).log x 12 (x+5) 4 (2x+3) 4 ; log (x 3 (x+5) (2x+3)) 4 .
pH=–log[H +]. C P P=–log(C). 2C.

pH=–log(2C)

pH=–log(2C)=–(log(2)+log(C))=–log(2)–log(C)

P=–log(C),

pH=P–log(2)≈P–0.301

e, M,b, n, n≠1 b≠1,

log b M= log n M log n b

y= log b M. n b y =M.

log n (b y) = log n M Apply the one-to-one property. y log n b = log n M Apply the power rule for logarithms. y = log n M log n b Isolate y. log b M = log n M log n b Substitute for y.

log 5 36

log 5 36 = log(36) log(5) Apply the change of base formula using base 10. ≈2.2266 Use a calculator to evaluate to 4 decimal places.

M,b, n, n≠1 b≠1,

log b M= log n M log n b .

log b M= lnM lnb

log b M= logM logb

log b M, n, n≠1. n, log(x), ln(x), e. n M. n b. log 5 3 log 5 3 n=e.

log b M = lnM lnb log 5 3 = ln3 ln5

log 0.5 8 ln8 ln0.5 log9 log 10 9. log9= ln9 ln10 . log 2 (10) e.

log 2 10= ln10 ln2 Apply the change of base formula using base e. ≈3.3219 Use a calculator to evaluate to 4 decimal places.

log 5 (100) ln100 ln5 ≈ 4.6051 1.6094 =2.861log b (MN)= log b (M)+ log b (N)
log b (M N)= log b M– log b Nlog b (M n)=n log b Mlog b M= log n M log n b n>0,n≠1,b≠1 e
log b (x n)? log b (x 1 n)= 1 n log b (x).log b (7x·2y)log b (2)+ log b (7)+ log b (x)+ log b (y)
ln(3ab·5c)log b (13 17)log b (13)– log b (17)log 4 (x z w)ln(1 4 k)–kln(4)log 2 (y x)
ln(7)+ln(x)+ln(y)ln(7xy)log 3 (2)+ log 3 (a)+ log 3 (11)+ log 3 (b)log b (28)– log b (7)log b (4)
ln(a)–ln(d)–ln(c)– log b (1 7)log b (7)1 3 ln(8)log(x 15 y 13 z 19)15log(x)+13log(y)–19log(z)
ln(a –2 b –4 c 5)log(x 3 y –4)3 2 log(x)–2log(y)ln(y y 1–y)log(x 2 y 3 x 2 y 5 3)
8 3 log(x)+ 14 3 log(y)log(2 x 4)+log(3 x 5)ln(6 x 9)–ln(3 x 2)ln(2 x 7)2log(x)+3log(x+1)
log(x)– 1 2 log(y)+3log(z)log(x z 3 y)4 log 7 (c)+ log 7 (a) 3 + log 7 (b) 3log 7 (15) e
log 7 (15)= ln(15) ln(7)log 14 (55.875) 10 log 5 (6)=a log 5 (11)=b. a b. log 11 (5)
log 11 (5)= log 5 (5) log 5 (11) = 1 blog 6 (55)log 11 (6 11)
log 11 (6 11)= log 5 (6 11) log 5 (11) = log 5 (6)– log 5 (11) log 5 (11) = a–b b = a b –1
log 3 (1 9)–3 log 3 (3)6 log 8 (2)+ log 8 (64) 3 log 8 (4)32 log 9 (3)–4 log 9 (3)+ log 9 (1 729)
log 3 (22)2.81359log 8 (65)log 6 (5.38)0.93913log 4 (15 2)log 1 2 (4.7)–2.23266 x
log 12 (2x+6)+ log 12 (x+2)=2. x log 6 (x+2)– log 6 (x–3)=1. x=4;
log 6 (x+2)– log 6 (x–3)= log 6 (x+2 x–3)=1. x:
6 1 = x+2 x–3 0 = x+2 x–3 –6 0 = x+2 x–3 – 6(x–3) (x–3) 0 = x+2–6x+18 x–3 0 = x–4 x–3 x =4
log 6 (4+2)– log 6 (4–3)= log 6 (6)– log 6 (1) x=4. b x =m? log b (n)= 1 log n (b) b>1 n>1. b
n 1. log b (n)= log n (n) log n (b) = 1 log n (b) . log 81 (2401)= log 3 (7)? b,S, T, b>0, b≠1,
b S = b T S=T. 3 4x–7 = 3 2x 3 . x, 3. x:

3 4x–7 = 3 2x 3 3 4x–7 = 3 2x 3 1 Rewrite 3 as 3 1 . 3 4x–7 = 3 2x–1 Use the division property of exponents. 4x–7 =2x–1 Apply the one-to-one property of exponents. 2x =6 Subtract 2x and add 7 to both sides. x =3 Divide by 3.

S and T, b≠1,

b S = b T if and only if S=T

b S = b T , S T b S = b T . S=T, 2 x−1 = 2 2x−4 .
2 x−1 = 2 2x−4 The common base is 2. x−1=2x−4 By the one-to-one property the exponents must be equal. x=3 Solve for x.
5 2x = 5 3x+2 .x=−2 256= 4 x−5 . 2. x:
256= 4 x−5 2 8 = (2 2) x−5 Rewrite each side as a power with base 2. 2 8 = 2 2x−10 Use the one-to-one property of exponents. 8=2x−10 Apply the one-to-one property of exponents. 18=2x Add 10 to both sides. x=9 Divide by 2.
b S = b T . S=T, 8 x+2 = 16 x+1 .
8 x+2 = 16 x+1 (2 3) x+2 = (2 4) x+1 Write 8 and 16 as powers of 2. 2 3x+6 = 2 4x+4 To take a power of a power, multiply exponents. 3x+6=4x+4 Use the one-to-one property to set the exponents equal. x=2 Solve for x.
5 2x = 25 3x+2 .x=−1 2 5x = 2 .
2 5x = 2 1 2 Write the square root of 2 as a power of 2. 5x= 1 2 Use the one-to-one property. x= 1 10 Solve for x.
5 x = 5 .x= 1 2 3 x+1 =−2. x 2 x =−100. log(a)=log(b) a=b, 5 x+2 = 4 x .
5 x+2 = 4 x There is no easy way to get the powers to have the same base. ln 5 x+2 =ln 4 x Take ln of both sides. (x+2)ln5=xln4 Use laws of logs. xln5+2ln5=xln4 Use the distributive law. xln5−xln4=−2ln5 Get terms containing x on one side, terms without x on the other. x(ln5−ln4)=−2ln5 On the left hand side, factor out an x. xln(5 4)=ln(1 25) Use the laws of logs. x= ln(1 25) ln(5 4) Divide by the coefficient of x.
2 x = 3 x+1 .x= ln3 ln(2 3) 2 x = 3 x ?0. e. e y=A e kt , t. A. k. 100=20 e 2t .
100 =20 e 2t 5 = e 2t Divide by the coefficient of the power. ln5 =2t Take ln of both sides. Use the fact that ln(x) and e x are inverse functions. t = ln5 2 Divide by the coefficient of t.
t=ln 5 . 3 e 0.5t =11.t=2ln(11 3) ln (11 3) 2 y=A e kt k≠0, y A 2=−3 e t . 4 e 2x +5=12.
4 e 2x +5=12 4 e 2x =7 Combine like terms. e 2x = 7 4 Divide by the coefficient of the power. 2x=ln(7 4) Take ln of both sides. x= 1 2 ln(7 4) Solve for x.
3+ e 2t =7 e 2t .t=ln(1 2)=− 1 2 ln(2) e 2x − e x =56.
e 2x − e x =56 e 2x − e x −56 =0 Get one side of the equation equal to zero. (e x +7)(e x −8) =0 Factor by the FOIL method. e x +7 =0 or e x −8=0 If a product is zero, then one factor must be zero. e x =−7 or e x =8 Isolate the exponentials. e x =8 Reject the equation in which the power equals a negative number. x =ln8 Solve the equation in which the power equals a positive number.
e x =−7 ln(−7) e 2x = e x +2.x=ln2 log b (x)=y b y =x. log 2 (2)+ log 2 (3x−5)=3. x:
log 2 (2)+ log 2 (3x−5)=3 log 2 (2(3x−5))=3 Apply the product rule of logarithms. log 2 (6x−10)=3 Distribute. 2 3 =6x−10 Apply the definition of a logarithm. 8=6x−10 Calculate 2 3 . 18=6x Add 10 to both sides. x=3 Divide by 6.

S b c, b>0, b≠1,
log b (S)=c if and only if b c =S

2lnx+3=7.
2lnx+3=7 2lnx=4 Subtract 3. lnx=2 Divide by 2. x= e 2 Rewrite in exponential form.
6+lnx=10.x= e 4 2ln(6x)=7.
2ln(6x)=7 ln(6x)= 7 2 Divide by 2. 6x= e (7 2) Use the definition of ln. x= 1 6 e (7 2) Divide by 6.
2ln(x+1)=10.x= e 5 −1 lnx=3.
lnx=3 x= e 3 Use the definition of the natural logarithm.

e 3 ≈20. e 3 ≈20.0855. y=lnx y=3 (e 3 ,3), 2 x =1000 x≈9.97 x>0,S>0,T>0 b, b≠1,

log b S= log b T if and only if S=T.

If log 2 (x−1)= log 2 (8),then x−1=8.

x−1=8, x, x=9. x=9 log 2 (9−1)= log 2 (8)=3. log(3x−2)−log(2)=log(x+4). x:

log(3x−2)−log(2)=log(x+4) log(3x−2 2)=log(x+4) Apply the quotient rule of logarithms.

3x−2 2 =x+4 Apply the one to one property of a logarithm. 3x−2=2x+8

Multiply both sides of the equation by 2. x=10 Subtract 2x and add 2.

x=10 log(3x−2)−log(2)=log(x+4).

log(3(10)−2)−log(2)=log((10)+4) log(28)−log(2)=log(14) log(28 2)=log(14)

The solution checks.

S T b, b≠1,

log b S= log b T if and only if S=T

log b S= log b T. S=T, ln(x 2)=ln(2x+3).

ln(x 2)=ln(2x+3) x 2 =2x+3 Use the one-to-one property of the logarithm. x

2 −2x−3=0 Get zero on one side before factoring. (x−3)(x+1)=0 Factor using FOIL. x

−3=0 or x+1=0 If a product is zero, one of the factors must be zero. x=3 or x=−1

Solve for x.

3 −1. −1 ln(x 2)=ln1.x=1 x=−1

A(t)= A 0 e ln(0.5) T t A(t)= A 0 e ln(0.5) t T A(t)= A 0 (e ln(0.5)) t T A(t)= A 0 (1 2) t T

A 0 T t y t

y=1000e ln(0.5) 703,800,000 t 900=1000 e ln(0.5) 703,800,000 t

After 10% decays, 900 grams are left. 0.9= e ln(0.5) 703,800,000 t Divide by 1000. ln(0.9)=ln(e

ln(0.5) 703,800,000 t) Take ln of both sides. ln(0.9)= ln(0.5) 703,800,000 t ln(e M)=M

t=703,800,000× ln(0.9) ln(0.5) years Solve for t. t≈106,979,777 years

t=703,800,000× ln(0.8) ln(0.5) years ≈ 226,572,993 years. S T b, b S = b T S=T. b c, b≠1,

log b (S)=c b c =S. b, b≠1,log b S= log b T S=T. e, log b (S)=c, S b c =S, log b (S)=c. y= log b (S)

y=c log b S= log b T, S T S=T 4 −3v−2 = 4 −v64· 4 3x =16x=− 1 33 2x+1 · 3 x =243

2 −3n · 1 4 = 2 n+2n=−1625· 5 3x+3 =12536 3b 36 2b = 216 2−bb= 6 5(1 64) 3n ·8= 2 69 x−10 =1x=10

2 e 6x =13e r+10 −10=−422· 10 9a =29−8· 10 p+7 −7=−24p=log(17 8)−77 e 3n−5 +5=−89e −3k +6=44

k=− ln(38) 3−5 e 9x−8 −8=−62−6 e 9x+8 +2=−74x= ln(38 3)−8 92 x+1 = 5 2x−1e 2x − e x −132=0

x=ln12 7 e 8x+8 −5=−9510 e 8x+3 +2=8x= ln(3 5)−3 84 e 3x+3 −7=538 e −5x−2 −4=−90

3 2x+1 = 7 x−2e 2x − e x −6=0x=ln(3)3 e 3−3x +6=−31log(1 100)=−210 −2 = 1 100log 324 (18)= 1 2

5 log 7 n=10n=49−8 log 9 x=164+ log 2 (9k)=2k= 1 362log(8n+4)+6=1010−4ln(9−8x)=6x= 9−e 8

ln(10−3x)=ln(−4x)log 13 (5n−2)= log 13 (8−5n)n=1log(x+3)−log(x)=log(74)

ln(−3x)=ln(x 2 −6x)log 4 (6−m)= log 4 3mln(x−2)−ln(x)=ln(54)

log 9 (2 n 2 −14n)= log 9 (−45+ n 2)ln(x 2 −10)+ln(9)=ln(10)x=± 10 3 x.log(x+12)=log(x)+log(12)

ln(x)+ln(x−3)=ln(7x)x=10log 2 (7x+6)=3ln(7)+ln(2−4 x 2)=ln(14)x=0

log 8 (x+6)− log 8 (x)= log 8 (58)ln(3)−ln(3−3x)=ln(4)x= 3 4log 3 (3x)− log 3 (6)= log 3 (77)

x,log 9 (x)−5=−4x=9log 3 (x)+3=2ln(3x)=2x= e 2 3 ≈2.5ln(x−5)=1log(4)+log(−5x)=2x=−5

−7+ log 3 (4−x)=−6ln(4x−10)−6=−5x= e+10 4 ≈3.2log(4−2x)=log(−4x)

log 11 (−2 x 2 −7x)= log 11 (x−2)ln(2x+9)=ln(−5x)log 9 (3−x)= log 9 (4x−8)x= 11 5 ≈2.2

log(x 2 +13)=log(7x+3)3 log 2 (10) −log(x−9)=log(44)x= 101 11 ≈9.2ln(x)−ln(x+3)=ln(6)

$6,500 7.25% $27,710.24 D D=10log(I I 0), I I 0 = 10 −12 8.3· 10 2 P=1650 e 0.5t t 20,000? x

1000 (1.03) t =5000 e 5x =17 ln(17) 5 ≈0.5673 (1.04) 3t =8 3 4x−5 =38

x= log(38)+5log(3) 4log(3) ≈2.07850 e −0.12t =10 7 e 3x−5 +7.9=47x≈2.2401

ln(3)+ln(4.4x+6.8)=2log(−0.7x−9)=1+5log(5)x≈−44655.7143 P P=14.7 e −0.21x ,x 8.369

M= 2 3 log(E E 0) E E 0 = 10 4.4 1.4· 10 13 5.83 b log b x =x. y=A e kt . t t t=ln((y A) 1 k)

A=a (1+ r k) kt . t. T T= T s +(T 0 − T s) e −kt , T s T 0 k t t t=ln((T− T s T 0 − T s) − 1 k) e

y=A0ekt

A0eky=A0ektA0eky=2e3x.y=3e−2x.4.01134972 × 1013.1013.y=A0ekty=0(−∞, ∞)(0,∞)(0,A0)k>0k<0 k>0k<0.A0A0A0=10.k,(t=1)1020.

20=10 e k·1 2= e k Divide by 10 ln2=k Take the natural logarithm

k=ln(2). y=10 e (ln2)t =10 (e ln2) t =10· 2 t . y=10e(ln2)t104.107,

12A0=Aoekt

kA0.

1 2 A 0 = A o e kt 1 2 = e kt Divide by A 0 . ln(1 2)=kt Take the natural log. –ln(2)=kt Apply laws of logarithms. − ln(2) k =t Divide by k.

t,k

t=−ln(2)k

A=Aoekt.A12A0tk.kk=−ln(2)t.t.

A= A 0 e kt The continuous growth formula. 0.5 A 0 = A 0 e k·5730 Substitute the half-life for t and 0.5 A 0 for f(t). 0.5= e 5730k Divide by A 0 . ln(0.5)=5730k Take the natural log of both sides. k= ln(0.5) 5730 Divide by the coefficient of k. A= A 0 e (ln(0.5) 5730)t Substitute for r in the continuous growth formula.

f(t)=A0e(ln(0.5)5730)t.t,ln(0.5)5730≈−1.2097f(t)=A0e−0.0000000087tt

A≈ A 0 e (ln(0.5) 5730)t

AA0

A= A 0 e kt The continuous growth formula. 0.5 A 0 = A 0 e k·5730 Substitute the half-life for t and 0.5 A 0 for f(t). 0.5= e 5730k Divide by A 0 . ln(0.5)=5730k Take the natural log of both sides. k= ln(0.5) 5730 Divide by the coefficient of k. A= A 0 e (ln(0.5) 5730)t Substitute for r in the continuous growth formula.

t:

t=ln(AA0)−0.000121

rA≈A0e−0.000121tr=AA0≈e−0.000121t.t,

t=ln(r)−0.000121

k. t= ln(r) −0.000121 t. 20%=0.20 k t:

t= ln(r) −0.000121 Use the general form of the equation. = ln(0.20) −0.000121 Substitute for r. ≈13301 Round to the nearest year.

13,301 years±1% or 13,301 years±133 years. A= A 0 e kt , 2 A 0 = A 0 e kt .

2 A 0 = A 0 e kt 2= e kt Divide by A 0 . ln2=kt Take the natural logarithm. t= ln2 k Divide by the coefficient of t.

t= ln2 k

t= ln2 k The doubling time formula. 2= ln2 k Use a doubling time of two years. k= ln2 2 Multiply by k and divide by 2. A = A 0 e ln2 2 t Substitute k into the continuous growth formula.

A= A 0 e ln2 2 t .f(t)= A 0 e ln2 3 t

T(t)=a e kt + T s

T(t)=A b ct + T s T(t)=A e ln(b ct) + T s Laws of logarithms. T(t)=A e ctlnb + T s Laws of logarithms. T(t)=A e kt + T s Rename the constant c ln b, calling it k.

T, T s

T(t)=A e kt + T s

t A k T s T(t)=A e k t + T s A k. 165°F, 35°F 150°F. 70°F

T(t)=A e kt +35

T(0)=165.

165=A e k0 +35 Substitute (0,165). A=130 Solve for A.

T(10)=150, k.

150=130 e k10 +35 Substitute (10, 150). 115=130 e k10 Subtract 35. 115 130 = e 10k Divide by 130. ln(115 130)=10k Take the natural log of both sides. k= ln(115 130) 10 =−0.0123 Divide by the coefficient of k.

T(t)=130 e –0.0123t +35.

70=130 e –0.0123t +35 Substitute in 70 for T(t). 35=130 e –0.0123t Subtract 35. 35 130 = e –0.0123t Divide by 130. ln(35 130)=–0.0123t Take the natural log of both sides t= ln(35 130) –0.0123 ≈106.68 Divide by the coefficient of t.

70°F.a, b,c, x

$$f(x)= c\ 1+a\ e\ –bx$$

$$f(x)= c\ 1+a\ e\ –bx$$

c 1+a c b t=0 b=0.6030.

$$f(x)= c\ 1+a\ e\ –bx$$

c=1000. a, t=0 c 1+a =1, a=999. f(x)= 1000 1+999 e –0.6030x ≈293.8. c=1000.

f(x)= 1000 1+999 e –0.6030xxy y=aln(bx). (1,0), 0=alnb. a=0 ln(b)=0. b=1 y=aln(x). (9,4.394) a:

y=aln(x) 4.394=aln(9) a= 4.394 ln(9)

a= 4.394 ln(9) ≈2, y=2ln(x). y=2lnx. y=ln(x 2) y=ln(x 2) x>0. y=ln(x 2)=2ln(x) x>0. x<0, y=ln(x 2) y=2ln(x) y=ln(x 2) xy y=2 e 0.5x . 10 e. e e. y=a b x , y= A 0 e kx . y=a b x y=a e ln(b x) . y=a e xln(b) =a e ln(b)x . a= A 0 k=ln(b) y= A 0 e kx . y=2.5 (3.1) x y= A 0 e kx . y=2.5 (3.1) x =2.5 e ln(3.1 x) Insert exponential and its inverse. =2.5 e xln3.1 Laws of logs. =2.5 e (ln3.1) x Commutative law of multiplication

y=3 (0.5) x e y=3 e (ln0.5)x A= A 0 e kt ,k<0, t=– ln(2) k .t= ln(A A 0) –0.000121 .A 0 A t A= A 0 e kt ,k>0, t= ln2 kT(t)=A e kt + T s , T s A=T(0)– T s , k f(x)=a b x . b>1, 0<b<1, A= A 0 e kx , A 0 A 0 k>0 k<0. t, k, t= ln(k) –0.000121 t. y=a b x y= A 0 e kx k=lnb. 10 2 T(t)=68 e –0.0174t +72. f(x)= 150 1+8 e –2x . f(0). f(0)≈16.7; f(4). xf(x) f(x)= 1.2 x f(x)=1.68 (0.65) x e xf(x)xf(x)xf(x)xf(x) t P(t)= 1000 1+9 e –0.6t . 1.4 2 900? 7.3 P.4 8.18 P(t)= P 0 e rt P 0 r>0. M= 2 3 log(S S 0). S.

M= 2 3 log(S S 0) log(S S 0)= 3 2 M S S 0 = 10 3M 2 S= S 0 10 3M 2

y= c 1+a e –rx ? b x = e xln(b) b≠1. y= b x b b≠1.

ln(y)=ln(b x) ln(y)=xln(b) e ln(y) = e xln(b) y= e xln(b) t A=125 e (–0.3567t) ;A≈43 f(10) 0.5 1.15% 60 t 250 250 32 f(t)=250 e (–0.00914t) ; 76 1590 10.4 r≈–0.0667, 6.67% 5730 1350 3 f(t)=1350 e (0.03466t) ; P(180)≈691,200 360 5 1000 20 f(t)=256 e (0.068110t) ; 10 100° 69° F 95° F. 80° F? 88 2 165°F 75°F 145°F.T(t)=90 e (–0.008377t) +75, t 110° F? 113 log(x)=1.5; x≈31.623 10 –10 W m 2 , 10 –4 W m 2 , 10 2 W m 2 M= 2 3 log(S S 0). 3.9 750 5.82 N(t)= 500 1+49 e –0.7t N(3)≈71 t N(t) 13 12 4.75 f(t)=13 (0.0805) tf(t)=13 e 0.9195t f(t)=13 e (–0.0839t)f(t)= 4.75 1+13 e –0.83925t f(x)= c 1+a e –bx c 1+a c b y=a b x y= A 0 e kx . y=a b x . y=a b x a>0):b y=a. b>1, x 0<b<1, x r, r 2 . r 2 r,

$$y=a\ b\ x$$

b b>1, 0<b<1, y=a b x . x y 0.16.

$$y=0.58304829\ (\ 2.20720213E10\ )\ x$$

$$y=0.58304829\ (\ 22{,}072{,}021{,}300\ )\ x$$

r 2 ≈0.97 0.16. 0.16 x y.

y =0.58304829 (22,072,021,300) x Use the regression model found in part (a). =0.58304829 (22,072,021,300) 0.16 Substitute 0.16 for x. ≈26.35 Round to the nearest hundredth.

y=522.88585984 (1.19645256) x . y=a+bln(x). x, (1,a) b>0, b<0,

$$y=a+b\ln(\ x\ )$$

x, b>0, b<0, y=a+bln(x). x x=1 x=2 y

$$y=42.52722583+13.85752327\ln(x)$$

x=14 y:

y =42.52722583+13.85752327ln(x) Use the regression model found in part (a). =42.52722583+13.85752327ln(14) Substitute 14 for x. ≈79.1 Round to the nearest tenth.

x x=1 y y=141.91242949+10.45366573ln(x)

$$y=\ c\ 1+a\ e\ –bx$$

c 1+a b>0, (ln(a) b , c 2). y=c.c

$$y= c\ 1+a\ e\ -bx$$

c 1+a . y=c a, b, c y= c 1+a e –bx . x x=0 y c.

$$y= 105.7379526\ 1+6.88328979\ e\ -0.2595440013x$$

x=18 y:

y = 105.7379526 1+6.88328979 e –0.2595440013x Use the regression model found in part (a). = 105.7379526 1+6.88328979 e –0.2595440013(18) Substitute 18 for x. ≈99.3 Round to the nearest tenth c=100 x x=0 y y= 25.65665979 1+6.113686306 e –0.3852149008x . 25,634 y=a b x y=a+bln(x) y= c 1+a e –bx y=10.209 e –0.294xy=5.598–1.912ln(x)y=2.104 (1.479) xy=4.607+2.733ln(x) y= 14.005 1+2.79 e –0.812x P(t)= 175 1+6.995 e –0.68t ? P(0)=22 A(t)=1550 (1.085) x e. h(p)=67.682–5.792ln(p). p h(p)=62?p≈2.67 P(t)= 90 1+5 e –0.42t . P(t)=45? (0,15) P x P(x)= 68 1+16 e –0.28x . 3 4 20 6.8 P P(x)= 558 1+54.8 e –0.462x , x 10 10 3 100 e. f(x)=776.682 e 0.3549x x f(x)=4000. f(x)=4000,x≈4.6.f(x)=731.92 (0.738) x e. x f(x)=250. y=a+bln(x) x=10.f(10)≈9.5 x f(x)=7. f(x)=7,x≈2.7. y=a+bln(x) f(x)=7.544–2.268ln(x) x=10. x f(x)=8. y= c 1+a e –bx x f(x)=12.5,x≈2.1.xf(x) y= c 1+a e –bx f(x)= 136.068 1+10.324 e –0.480x136 x P(t)= c 1+a e –bt , t=0 P(0)= P 0 . c–P(t) P(t) = c– P 0 P 0 e –bt . a e –bt : c–P(t) P(t) = c– c 1+a e –bt c 1+a e –bt = c(1+a e –bt)–c 1+a e –bt c 1+a e –bt = c(1+a e –bt –1) 1+a e –bt c 1+a e –bt =1+a e –bt –1=a e –bt a e –bt . t=0, P 0 = c 1+a e –b(0) = c 1+a . c– P 0 P 0 e –bt = c– c 1+a c 1+a e –bt = c(1+a)–c 1+a c 1+a e –bt = c(1+a–1) 1+a c 1+a e –bt =(1+a –1) e –bt =a e –bt c–P(t) P(t) = c– P 0 P 0 e –bt . f(x) g(x) (1.5,1.5) (8.5, 8.5). y=x f(x)=1.034341 e 0.247800x . f(g(x))=x, g(f(x)) =4.035510ln(1.034341 e 0.247800x)–0.136259 =4.03551(ln(1.034341)+ln(e 0.2478x)) –0.136259 =4.03551(ln(1.034341)+0.2478x)–0.136259 =0.136257+0.999999x–0.136259 = –0.000002+0.999999x ≈0+x =x f –1 (x) f(x)= c 1+a e –bx . P(t)= 20 1+4 e –0.5t P(t) P –1 (t) y=156 (0.825) t 0.825, 0 1. A(t)=205 (1.13) t , t 6 (2, 2.25) (5,60.75).y=0.25 (3) x 8.12% 20 $42,888.18 7.5% 3 y=2.294 e –0.654t $10,500 6.25% 25 f(x)=3.5 (2) x . f(x)=4 (1 8) x f(x)= 6.5 x 7. g(x)? g(x)=7 (6.5) –x ; (0, 7); 0. f(x)= 2 x . log 17 (4913)=x 17 x =4913 ln(s)=t a – 2 5 =b log a b=– 2 5 e –3.5 =h x log 64 (x)= 1 3 x= 64 1 3 =4 log 5 (1 125) log(0.000001) log(0.000001)=–6 log(4.005) ln(e –0.8648) ln(e –0.8648)=–0.8648 ln(18 3) g(x)=log(7x+21)–4. h(x)=2ln(9–3x)+1. g(x)=ln(4x+20)–17. x>–5; x=–5; x→– 5 + ,f(x)→–∞ x→∞,f(x)→∞. ln(7r·11st) log 8 (x)+ log 8 (5)+ log 8 (y)+ log 8 (13) log 8 (65xy) log m (67 83) ln(z)–ln(x)–ln(y) ln(z xy) ln(1 x 5) – log y (1 12) log y (12) log(r 2 s 11 t 14). ln(2b b+1 b–1). ln(2)+ln(b)+ ln(b+1)–ln(b–1) 2 5ln(b)+ln(c)+ ln(4–a) 2 3 log 7 v+6 log 7 w– log 7 u 3 log 7 (v 3 w 6 u 3) log 3 (12.75) e. 5 12x–17 =125 x x= log(125) log(5) +17 12 = 5 3 216 3x · 216 x = 36 3x+2 125 (1 625) –x–3 = 5 3 x=–3 7· 17 –9x –7=49. 3 e 6n–2 +1=–60. 5 e 3x –4=6 2 e 5x–2 –9=–56. 5 2x–3 = 7 x+1 . e 2x – e x –110=0. x=ln(11) –5 log 7 (10n)=5. 9+6ln(a+3)=33.a= e 4 –3 log 8 (7)+ log 8 (–4x)= log 8 (5). ln(5)+ln(5 x 2 –5)=ln(56). x=± 9 5 D D=10log(I I 0), I I 0 = 10 –12 6.3· 10 –3 P(t)=256,114 e 0.25t t 5.45 f –1 f(x)=2· e x+1 –5. f –1 f(x)=0.25· log 2 (x 3 +1).f –1 (x)= 2 4x –1 3 300 17% t 24 f(t)=300 (0.83) t ;f(24)≈3.43 g 350° 71°F 175°F. 85°F? 45 N(t)= 1200 1+199 e –0.625t t 8.5 (–2,100) (0,4). y=4 (0.2) x ; y=4 e -1.609438x P(t)= 250,000 1 + 499 e –0.45t ? P(t)= 14,250 1 + 29 e –0.62t , t 75% 7.2 y=16.68718–9.71860ln(x) A(t)=8 (1.17) t , t 3 13 (0, 4) (2, 9). 6.25% 4 $1,947 7.4% 15 f(x)=5 (0.5) –x (0, 5) f(x)= (1 2) x . log 8.5 (614.125)=a 8.5 a =614.125 e 1 2 =m x lo g 1 7 (x)=2 x= (1 7) 2 = 1 49 log(10,000,000) ln(0.716) ln(0.716)≈–0.334 g(x)=log(12–6x)+3. f(x)= log 5 (39–13x)+7. x<3; x=3; x→ 3 – ,f(x)→–∞ x→–∞,f(x)→∞ log(17a·2b)

log t (96)– log t (8) log t (12) log 8 (a 1 b) ln(y 3 z 2 · x–4 3).3 ln(y)+2ln(z)+ ln(x–4) 3 4ln(c)+ln(d)+ ln(a) 3 + ln(b+3) 3 16 3x–5 =1000 x x= ln(1000) ln(16) +5 3 ≈2.497 (1 81) x · 1 243 = (1 9) –3x–1 –9 e 10a–8 –5=–41a= ln(4)+8 10 10 e 4x+2 +5=56. –5 e –4x–1 –4=64. 2 x–3 = 6 2x–1 . e 2x – e x –72=0. x=ln(9) 4log(2n)–7=–11 log(4 x 2 –10)+log(3)=log(51) x=± 3 3 2 D D=10log(I I 0), I I 0 = 10 –12 4.7· 10 –1 112 17 80 f(t)=112 e –.019792t ; 35 e. 71° 35° F. 63° F. T(t)=36 e –0.025131t +35;T(60)≈ 43 o F P(t)= 360 1+6.2 e –0.35t , t P(t)= 16,120 1+25 e –0.75t , t 80% y=15.10062 (1.24621) x y= 18.41659 1+7.54644 e –0.68375xEF→. ED → EF → . ∠DEF.θϕorφαβγ ∠θ 1 360 90° 360° = 1 4 . 360° 360° =1.

30° 360° = 1 12

1 12 = 1 3 (1 4)

–135° 360° =– 3 8

– 3 8 =– 3 2 (1 4)

C=2πr. r, 2π 2π≈6.28 2π 2π

2π radians= 360 ◦ π radians= 360 ◦ 2 = 180 ◦ 1 radian= 180 ◦ π ≈ 57.3 ◦

t s s r.

s=rθ θ= s r

s=r, θ= r r = 1 radian.s r.s=2r.2πC=2πr,r2π(2)=4π2π(3)=6π.

Smaller circle: 1 2 π 2 = 1 4 π Larger circle: 3 4 π 3 = 1 4 π

1 4 π, 2π π s r s r 2π 1 360° 2π ≈57.3°. C=2πr, C=2π.

1 rotation =360° =2π radians 1 2 rotation=180° =π radians 1 4 rotation=90° = π 2 radians

1 rotation=2πr

s= 1 3 (2πr) = 2πr 3

radian measure= 2πr 3 r = 2πr 3r = 2π 3

3π 2

θ 180 = θ R π

θ θ π. π.

Degrees 180 = Radians π

θ 180 = θ R π

π 6

θ 180 = θ R π θ 180 = π 6 π θ= 180 6 θ= 30 ◦

θ 180 = θ R π θ 180 = 3 π θ= 3(180) π θ≈ 172 ◦

– 3π 4 15

θ 180 = θ R π 15 180 = θ R π 15π 180 = θ R π 12 = θ R

30 ◦ = π 6 . 15 ◦ = 1 2 (30 ◦), 1 2 (π 6) π 12 . 7π 10 2π. t t t ′ , t θ0°≤θ<360°. θ=80° α 0°≤α<360°.α=150°α

–45°+360°=315°

β 0°≤β<360°. β=60° 2π, 2π. 2π 2π, 2π 0 2π. β 19π 4 , 0≤β<2π. 2π

19π 4 –2π= 19π 4 – 8π 4 = 11π 4

11π 4 2π,

11π 4 –2π= 11π 4 – 8π 4 = 3π 4

3π 4 19π 4 , θ – 17π 6 0≤θ<2π. 7π 6 θ s r θ= s r . s θ

s=rθ

r,sθ. θ r θ:s=rθ.

C=2πr =2π(36 million miles) ≈226 million miles

(0.0114)226 million miles = 2.58 million miles

radian= arclength radius = 2.58 million miles 36 million miles =0.0717

215π 18 =37.525 units r A=π r 2 . θ, θ 2π θ 2π θ

Area of sector=(θ 2π)π r 2 = θπ r 2 2π = 1 2 θ r 2

r θ,

$$A= 1\ 2\ \theta\ r\ 2$$

r, θ. θ θ r: A= 1 2 θ r 2 .

30 degrees=30· π 180 = π 6 radians

Area = 1 2 (π 6) (20) 2 ≈104.72

104.72 ft 2 . 10π 10π v s t

$$v= s\ t$$

360 degrees 4 seconds = ω θ t

$$\omega= \theta\ t$$

s=rθ, θ, θ=ωt.

$$s=r\theta = r\omega t$$

$$v= s\ t = r\omega t\ t = r\omega$$

r, ω, θ t.

$$\omega= \theta\ t$$

v, s, t.

$$v= s\ t$$

$$v=r\omega$$

ω, r ω= θ t 2π ω= 2π 5 ≈1.257 −3π 2 ω= θ t . v=rω.

180 rotations minute · 2π radians rotation =360π radians minute

v=(14 inches)(360π radians minute) =5040π inches minute

5040π inches minute · 1 feet 12 inches · 1 mile 5280 feet · 60 minutes 1 hour

≈14.99 miles per hour (mph)

s=rθA= 1 2 θ r 2ω= θ tv= s tv=rω θ 180 = θ R π . 2π. 2π 37π 45π 6π 2− π 1022π 34π 3− π 6− 4π 32π 3 7π 2 ≈11.00 in 281π 20 ≈12.72 cm 23π 4π 9 − 5π 4π 3− 7π 3− 5π 1211π 6π 2 −3π π 5π 6 π 4 π 3 . 5.02π 3 ≈5.265π 6 .25π 9 ≈8.7321π 10 ≈6.60 π 2 π 2π − π 910π 34π 313π 644π 98π 9 v,v, 1,809,557.37 mm/min=30.16 m/s(1 minute= 1 60 degree)5.76 (1 minute= 1 60 degree). 3960 20 120° θ 2π 30° or (π 6), 45° or (π 4) 60° or (π 3). t s. s=rt,r=1, s=t. t, (x,y). x y f(t)=cos t f(t)=sin t, x=cos t y=sin t. t (0,0) 1 1. (x,y) s. (x,y) (x,y) t t t. y. t t. x. sin tsin(t)cos tcos(t). cos 2 t (cos(t)) 2 . t (x,y) t,

$$\cos t=x$$

$$\sin t=y$$

(x,y) t,tP:sin t=y.tP: cos t=x. P t, cos(t) sin(t). cos t sin t

$$x=\cos t= 1\ 2\ y=\sin t= 3\ 2$$

t (− 2 2 , 2 2) cos tsin t.cos(t)=− 2 2 ,sin(t)= 2 2 x y. cos(90°) sin(90°). 90° (x,y)

$$x=\cos t=\cos(90°)=0\ y=\sin t=\sin(90°)=1$$

π. cos(π)=−1,sin(π)=0 x 2 + y 2 =1. x=cos t y=sin t, x y cos 2 t+ sin 2 t=1. cos 2 t+ sin 2 t=1, t,

$$\cos 2\ t+ \sin 2\ t=1$$

t t. sin(t) cos(t). t sin(t)= 3 7 t cos(t). t,

cos 2 (t)+ sin 2 (t)=1 cos 2 (t)+ 9 49 =1 cos 2 (t)= 40 49 cos(t)=± 40 49 = ± 40 7 =± 2 10 7

cos(t)=− 2 10 7

cos(t)= 24 25tsin(t).sin(t)=− 7 25 45° π 4 , 45°–45°–90° t= π 4 y=x. y=x x y (y=x),

$$x\ 2 + y\ 2 =1$$

y=x,

$$x\ 2 + x\ 2 =1$$

$$2\ x\ 2 =1$$

x,

x 2 = 1 2 x=± 1 2

x= 1 2 . t= π 4

(x,y)=(x,x)=(1 2 , 1 2) x= 1 2 ,y= 1 2 cos t= 1 2 ,sin t= 1 2

cos t= 1 2 2 2 = 2 2 sin t= 1 2 2 2 = 2 2

(x,y) 1 45° (2 2 , 2 2). 30°, π 6 30°, –30°, 60°, 2y, r=2y y= 1 2 r. sin t=y

sin(π 6)= 1 2 r

r=1

sin(π 6)= 1 2 (1) = 1 2

cos 2 π 6 + sin 2 (π 6)=1 cos 2 (π 6)+ (1 2) 2 =1 cos 2 (π 6)= 3 4

Use the square root property. cos(π 6)= ± 3 ± 4 = 3 2

Since y is positive, choose the positive root.

(x,y) 1 30° (3 2 , 1 2). t= π 3 BAD, A 60°. B, ABC 60°. 180°, C 60°. ABC ABD ABC BD AC, AC AD 1 2 1 2 . AD B, BAD x 1 2 .

x 2 + y 2 =1

x= 1 2 ,

(1 2) 2 + y 2 =1

y,

1 4 + y 2 =1 y 2 =1– 1 4 y 2 = 3 4 y=± 3 2

t= π 3 y= 3 2 , t= π 3 (x,y) 1 60° (1 2 , 3 2),

(x,y)=(1 2 , 3 2) x= 1 2 ,y= 3 2 cos t= 1 2 ,sin t= 3 2

π 6 , π 4 , π 3 , π 2 ,3 22 2 1 2 1 2 2 2 3 2 cos(30) cos(5π 3) COS(5 × π ÷ 3) ENTER

cos(5π 3)=0.5

20°,

SIN(20 × π ÷ 180) ENTER

sin(π 3). 2π x, y, r, [–1,1]. [–1,1]. [–1,1]. α t; β t;

sin(t)= sin(α) and cos(t)=–cos(α) sin(t)=–sin(β) and cos(t)= cos(β)

t, t 0 90°, 0 π 2 0 2π, | π–t | | 180°–t |. 2π–t 360°–t. 0 2π, 2π 0 2π. 225° 225°

| (180°–225°) |=| –45° |=45°

5π 3 . π 3 (x,y) cos(150°) sin(150°). cos 5π 4 sin 5π 4 .

cos(30°)= 3 2 and sin(30°)= 1 2 .

cos(150°)=– 3 2 and sin(150°)= 1 2

5π 4 5π 4 –π= π 4 . π 4 2 2 . x y

cos 5π 4 =– 2 2 and sin 5π 4 =– 2 2

315° cos(315°) sin(315°). – π 6 cos(– π 6) sin(– π 6). cos(315°)= 2 2 , sin(315°)= – 2 2

cos(– π 6)= 3 2 , sin(– π 6)=– 1 2 (x,y) (x,y)

x=cos t y=sin t

(x,y) x y 7π 6 . 7π 6 π.

7π 6 –π= π 6

cos(π 6)= 3 2 sin(π 6)= 1 2

x y

cos(7π 6)=– 3 2 sin(7π 6)=– 1 2

(x,y) x=cos θy=sin θ. (– 3 2 ,– 1 2) 5π 3 . (1 2 ,– 3 2) cos t=xsin t=ycos 2 t+ sin 2 t=1 t t t [–1,1].

t, t t, t tsin(t)<0 cos(t)<0 sin(t)>0 cos(t)>0sin(t)>0 cos(t)<0sin(t)<0cos(t)>0sin π 2sin π 33 2cos π 2 cos π 31 2sin π 4cos π 42 2sin π 6sin πsin 3π 2cos πcos 0cos π 63 2sin 0240°60°–170°100°80°–315° 135°45°5π 42π 3π 35π 6–11π 3π 3– 7π 4–π 8π 8225°300°60°,sin(300°)=– 3 2 ,cos(300°)= 1 2 320°135° 45°, sin(135°)= 2 2 , cos(135°)=– 2 2210°120°60°, sin(120°)= 3 2 , cos(120°)=– 1 2250°150° 30°, sin(150°)= 1 2 , cos(150°)=– 3 25π 47π 6π 6 , sin(7π 6)=– 1 2 ,cos(7π 6)=– 3 25π 33π 4π 4 , sin(3π 4)= 2 2 , cos(4π 3)=– 2 24π 32π 3π 3 , sin(2π 3)= 3 2 , cos(2π 3)=– 1 25π 67π 4π 4 , sin(7π 4)=– 2 2 ,cos(7π 4)= 2 2 cos(t)= 1 7 t sin(t). cos(t)= 2 9 t sin(t). 77 9 sin(t)= 3 8 t cos(t).

sin(t)=– 1 4 t cos(t).– 15 4 220°. 120°. (–10,10 3) 7π 4 . 5π 9 . (–2.778,15.757) [–1,1] t .
sint= 1 2 ,cost=– 3 2sin t=– 2 2 ,cos t=– 2 2sin t= 3 2 ,cos t=– 1 2sin t=– 2 2 ,cos t= 2 2sin t=0,cos t=–1
sin t=–0.596,cos t=0.803sin t= 1 2 ,cos t= 3 2sin t=– 1 2 ,cos t= 3 2sin t=0.761,cos t=–0.649sin t=1,cos t=0
sin 5π 9cos 5π 9sin π 10cos π 10sin 3π 4cos 3π 4sin 98°cos 98°cos 310°sin 310°
sin(11π 3)cos(–5π 6)sin(3π 4)cos(5π 3)2 4sin(– 4π 3)cos(π 2)sin(–9π 4)cos(–π 6)– 6 4
sin(π 6)cos(–π 3)sin(7π 4)cos(–2π 3)2 4cos(5π 6)cos(2π 3)cos(–π 3)cos(π 4)2 4
sin(–5π 4)sin(11π 6)sin(π)sin(π 6) (0,1),(0,–1) (0.707,–0.707) (–0.866,–0.5) (0,0) π 3 , π 4 ,
π 6 . 1 12 (x,y) t, (x,y) t y x ,x≠0. t, t, t sin t cos t ,cos t≠0. tan. t 1 cos t = 1 x ,x≠0. sec. t
cos t sin t = x y , y≠0. cot. t 1 sin t = 1 y ,y≠0. csc. t (x,y) t

tan t= y x ,x≠0 sec t= 1 x ,x≠0 csc t= 1 y ,y≠0 cot t= x y ,y≠0

(– 3 2 , 1 2) sin t,cos t,tan t,sec t,csc t, cot t. (x,y) t,
sin t=y= 1 2 cos t=x=– 3 2 tan t= y x = 1 2 – 3 2 = 1 2 (– 2 3)=– 1 3 =– 3 3 sec t= 1 x = 1 – 3 2 =– 2 3 =
– 2 3 3 csc t= 1 y = 1 1 2 =2 cot t= x y = – 3 2 1 2 =– 3 2 (2 1)=– 3
(2 2 ,– 2 2) sin t,cos t,tan t,sec t,csc t, cot t.sin t=– 2 2 ,cos t= 2 2 ,tan t=–1,sec t= 2 ,csc t=– 2 ,cot t=–1
sin t,cos t,tan t,sec t,csc t, cot t t= π 6 . sin π 6 = 1 2 cos π 6 = 3 2 .

tan π 6 = sin π 6 cos π 6 = 1 2 3 2 = 1 3 = 3 3
sec π 6 = 1 cos π 6 = 1 3 2 = 2 3 = 2 3 3
csc π 6 = 1 sin π 6 = 1 1 2 =2
cot π 6 = cos π 6 sin π 6 = 3 2 1 2 = 3

sin t,cos t,tan t,sec t,csc t, cot t t= π 3 .
sin π 3 = 3 2 cos π 3 = 1 2 tan π 3 = 3 sec π 3 =2 csc π 3 = 2 3 3 cot π 3 = 3 3 x y 0 π 6 , or 30°
π 4 , or 45°π 3 , or 60°π 2 , or 90°3 22 21 21 22 23 23 332 3 3222 3 333 3 – 5π 6 . π 6 , – 5π 6 x y
cos(– 5π 6)=– 3 2 ,sin(– 5π 6)=– 1 2 ,tan(– 5π 6)= 3 3 sec(– 5π 6)=– 2 3 3 ,csc(– 5π 6)=–2,cot(–
5π 6)= 3
– 7π 4 . sin(–7π 4)= 2 2 ,cos(–7π 4)= 2 2 ,tan(–7π 4)=1,
sec(–7π 4)= 2 ,csc(–7π 4)= 2 ,cot(–7π 4)=1 f(x)= x 2 , (4) 2 = (–4) 2 ,(–5) 2 = (5) 2 , f(x)= x 2
f(–x)=f(x). f(x)= x 2 f(x)= x 3 , f(x)= x 3 f(–x)=–f(x). f(x)= x 3 y.

sin t=y sin(–t)=–y sin t≠sin(–t) cos t=x cos(–t)=x cos t=cos(–t)
tan(t)= y x tan(–t)=– y x tan t≠tan(–t) sec t= 1 x sec(–t)= 1 x sec t=sec(–t)
csc t= 1 y csc(–t)= 1 –y csc t≠csc(–t) cot t= x y cot(–t)= x –y cot t≠cot(–t) f(–x)=f(x).
f(–x)=–f(x).

cos(–t)=cos t sec(–t)=sec t
sin(–t)=–sin t tan(–t)=–tan t csc(–t)=–csc t cot(–t)=–cot t

t –t? –t t 3 , –t? – 3

tan t= sin tcos t
sec t= 1 cos t
csc t= 1 sin t
cot t= 1 tan t = cos t sin t

sin(45°)= 2 2 ,cos(45°)= 2 2 ,tan(45°). sin(5π 6)= 1 2 ,cos(5π 6)=– 3 2 ,evaluate sec(5π 6).

tan(45°)= sin(45°) cos(45°) = 2 2 2 2 =1
sec(5π 6)= 1 cos(5π 6) = 1 – 3 2 = –2 3 1 = –2 3 =– 2 3 3
csc(7π 6).–2sec t tan t .
sec t tan t = 1 cos t sin t cos t To divide the functions, we multiply by the reciprocal. = 1 cos t cos t
sin t Divide out the cosines. = 1 sin t Simplify and use the identity. =csc t
sec t tan t csc t,

sec t tan t =csc t

tan t(cos t).sin t cos 2 t+ sin 2 t=1. cos 2 t:

cos 2 t cos 2 t + sin 2 t cos 2 t = 1 cos 2 t 1+ tan 2 t= sec 2 t

sin 2 t:

cos 2 t sin 2 t + sin 2 t sin 2 t = 1 sin 2 t cot 2 t+1= csc 2 t

1+ tan 2 t= sec 2 t

cot 2 t+1= csc 2 t

cos(t)= 12 13 t cos 2 t+ sin 2 t=1,

(12 13) 2 + sin 2 t=1 sin 2 t=1– (12 13) 2 sin 2 t=1– 144 169 sin 2 t= 25 169

sin t=± 25 169 sin t=± 25 169 sin t=± 5 13

– 5 13 .

tan t= sin t cos t = – 5 13 12 13 =– 5 12 sec t= 1 cos t = 1 12 13 = 13 12 csc t= 1 sin t = 1 – 5 13 =– 13 5

cot t= 1 tan t = 1 – 5 12 =– 12 5

sec(t)=– 17 8 0<t<π,cost=– 8 17 ,sint= 15 17 ,tant=– 15 8csct= 17 15 ,cott=– 8 15 2π, x f(x)

f(x+4)=f(x). P f f(x+P)=f(x) x. 2π. π. t

sin t=y=– 3 2 cos t=x=– 1 2 tan t= sint cost = – 3 2 – 1 2 = 3 sec t= 1 cost = 1 – 1 2 =–2 csc t= 1 sint = 1 –

3 2 =– 2 3 3 cot t= 1 tant = 1 3 = 3 3

t sin t=–1,cos t=0,tan t=Undefined sec t= Undefined,csc t=–1,cot t=0 sin(t)=– 3 2 cos(t)= 1 2 ,

sec(t),csc(t),tan(t), cot(t).

sec t= 1 cos t = 1 1 2 =2 csc t= 1 sin t = 1 – 3 2 – 2 3 3 tan t= sin t cos t = – 3 2 1 2 =– 3 cot t= 1 tan t = 1 –

3 =– 3 3

sin(t)= 2 2 cos(t)= 2 2 , sec(t),csc(t),tan(t), and cot(t).sec t= 2 ,csc t= 2 ,tan t=1,cot t=1 π 180 30°,

(for a scientific calculator): 1 30× π 180 COS

(for a graphing calculator): 1 cos(30π 180)

1 / 1 / 5π 7 .

1 / (5 × π / 7) SIN =

csc(5π 7)≈1.279

– π 8 .≈–2.414tan t= sint costsec t= 1 costcsc t= 1 sintcot t= 1 tan t = cos t sin t f(–x)=f(x) f(–x)=–f(x).

P f f(x+P)=f(x) x. [0,2π), π 4 x= π 4 , 5π 4 , π y x 2 + y 2 =1. x π. π tan π 6sec π 62 3 3csc π 6

cot π 63tan π 4sec π 42csc π 4cot π 4tan π 3sec π 3csc π 3cot π 33 3tan 5π 6sec 7π 6– 2 3 3

csc 11π 6cot 13π 63tan 7π 4sec 3π 4– 2csc 5π 4cot 11π 4tan 8π 3sec 4π 3csc 2π 3cot 5π 3– 3 3

tan 225°sec 300°csc 150°cot 240°3 3tan 330°sec 120°csc 210°cot 315° sin t= 3 4 , t

cos t,sec t,csc t,tan t,cot t. cos t=– 1 3 , t sin t,sec t,csc t,tan t,cot t.

sin t=– 2 2 3 ,sec t=–3,csc t=– 3 2 4 ,tan t=2 2 ,cot t= 2 4 tan t= 12 5 , 0≤t< π 2 , sin t,cos t,sec t,csc t, cot t.

sin t= 3 2 cos t= 1 2 , sec t,csc t,tan t, cot t.sec t=2,csc t= 2 3 3 , tan t= 3 , cot t= 3 3

sin 40°≈0.643 cos 40°≈0.766 sec 40°,csc 40°,tan 40°,and cot 40°. sin t= 2 2 , sin(–t)? – 2 2 cos t= 1 2 ,

cos(–t)? sec t=3.1, sec(–t)? csc t=0.34, csc(–t)? tan t=–1.4, tan(–t)? cot t=9.23, cot(–t)?

sin t= 2 2 ,cos t= 2 2 ,tan t=1,cot t=1,sec t= 2 ,csc t= 2

sin t=– 3 2 ,cos t=– 1 2 ,tan t= 3 ,cot t= 3 3 ,sec t=–2,csc t=– 2 3 3csc 5π 9cot 4π 7sec π 10tan 5π 8

sec 3π 4csc π 4tan 98°cot 33°cot 140°sec 310° tan(t)≈2.7, sin(t)≈0.94, cos(t). tan(t)≈1.3,

cos(t)≈0.61, sin(t). sin(t)≈0.79 csc(t)≈3.2, cos(t)≈0.95, tan(t). cot(t)≈0.58, cos(t)≈0.5, csc(t).

csct≈1.16 f(x)=2sin x cos x f(x)=3 sin 2 x cos x + sec xf(x)=sin x –2 cos 2 x f(x)= csc 2 x+sec x csc t tan t

sec t csc tsin t cos t =tan t h=15cos(1 600 d), h d h=16cos(1 500 d), h d 1000π P=20sin(2πt)+100 P,

t h, y=2cos x+6, x 55°. h, y=2cos x+5, x 55°. csc t= 1 y ,y≠0 cot t= x y ,y≠0 P f f(x+P)=f(x)

sec t= 1 x ,x≠0 tan t= y x ,x≠0 30°(π 6), 45°(π 4), 60°(π 3).

cos t=x sin t=y

t t (x,y) y x.

cos t= x 1 =x

sin t= y 1 =y

(x,y) x, t. y,t. 1,t. t,

sin(t)= opposite hypotenuse cos(t)= adjacent hypotenuse tan(t)= opposite adjacent

cos α.

$$\cos(\alpha) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}$$

sin t.7 25sin α,cos α,tan α,sec α,csc α, cot α.

sin α= opposite α hypotenuse = 4 5 cos α= adjacent to α hypotenuse = 3 5 tan α= opposite α adjacent to α = 4 3 sec α= hypotenuse adjacent to α = 5 3 csc α= hypotenuse opposite α = 5 4 cot α= adjacent to α opposite α = 3 4

sin t,cos t,tan t,sec t,csc t, cot t.

sin t= 33 65 ,cos t= 56 65 ,tan t= 33 56 , sec t= 65 56 ,csc t= 65 33 ,cot t= 56 33 30°,60°, 45°, 0° and 90°.

30°,60°,90° π 6 , π 3 , π 2 s, 3 s,2s. 45°,45°,90° π 4 , π 4 , π 2 s,s, 2 s. π 3 ,

sin(π 3)= opp hyp = 3 s 2s = 3 2 cos(π 3)= adj hyp = s 2s = 1 2 tan(π 3)= opp adj = 3 s s = 3 sec(π 3)= hyp adj = 2s s =2 csc(π 3)= hyp opp = 2s 3 s = 2 3 = 2 3 3 cot(π 3)= adj opp = s 3 s = 1 3 = 3 3

π 4 ,sin(π 4)= 2 2 ,cos(π 4)= 2 2 ,tan(π 4)=1,sec(π 4)= 2 ,csc(π 4)= 2 ,cot(π 4)=1 π 6 π 3 , π 3 , 3 2 , π 6 , π 6 , 1 2 , π 3 .

sin π 3 =cos π 6 = 3 s 2s = 3 2 sin π 6 =cos π 3 = s 2s = 1 2

π 3 π 6 π 3 π 6 , sin(π 3) cos(π 6) 3 s 2s. cos(π 3) sin(π 6) s 2s. π 6 π 3 π, π 2 , π 2 .

π 2 π 12 5π 12 , 5π 12 π 12 . t,cos t= 5 13 , sin(π 2 –t)= 5 13

cos t=sin(π 2 –t)
sin t=cos(π 2 –t)
tan t=cot(π 2 –t)
cot t=tan(π 2 –t)
sec t=csc(π 2 –t)
csc t=sec(π 2 –t)

sin t= 5 12 , (cos π 2 –t).

sin t=cos(π 2 –t).
cos(π 2 –t)= 5 12 .

csc(π 6)=2, sec(π 3).

tan(30°)= 7 a

a.

a= 7 tan(30°) =12.1

sin(30°)= 7 c

c.

c= 7 sin(30°) =14

π 3 adjacent=10; opposite=10 3 π 657° 57° 57°, h

tan θ= opposite adjacent tan(57°)= h 30 Solve for h. h=30tan(57°) Multiply. h≈46.2 Use a calculator.

5π 12

cos t=sin(π 2 –t) sin t=cos(π 2 –t) tan t=cot(π 2 –t) cot t=tan(π 2 –t) sec t=csc(π 2 –t) csc t=sec(π 2 –t)

cos(34°)=sin(__°)cos(π 3)=sin(___)π 6csc(21°)=sec(___°)tan(π 4)=cot(__)π 4 a A, b B, c

cos B= 4 5 ,a=10sin B= 1 2 , a=20b= 20 3 3 ,c= 40 3 3tan A= 5 12 ,b=6tan A=100,b=100

a=10,000,c=10,000.5sin B= 1 3 , a=2a=5, ∡ A= 60 ◦b= 5 3 3 ,c= 10 3 3c=12, ∡ A= 45 ◦ A.sin A5 29 29

cos Atan A5 2csc Asec A29 2cot A A.sin A5 41 41cos Atan A5 4csc Asec A41 4cot Ac=14, b=7 3

a=15, b=15b=9.9970, c=12.2041a=2.0838, b=11.8177b=15, ∡B= 15 ◦a=55.9808,c=57.9555

c=200, ∡B= 5 ◦c=50, ∡B= 21 ◦a=46.6790,b=17.9184a=30, ∡A= 27 ◦b=3.5, ∡A= 78 ◦

a=16.4662,c=16.8341 x. x. x. x. 36°, 23°. 43°, 31°. 15°, 2°. 18°, 3°. 40°. 43°. 36°. 38°. 80°. 80°. 60°,

π 4 45°– 5π 3– 7π 6 3π 5 420°60°–80° 2π – 20π 112π 1114π 55π 4– π 3 sin π 3 .3 2 cos π 4 . cos π.

300°. 3π 4 .π 4 330°. 5π 4 .– 2 2[–1,1]cos π 6tan π 4csc π 3sec π 42sec 11π 3sec 315°2 sec(t)=–2.5

sec(–t)? tan(t)=–0.6, tan(–t)? tan(t)= 1 3 , tan(t–π). cos(t)= 2 2 , sin(t+2π).2 2 – 2 2cos π 4cot π 33 3

tan π 6cos(π 2)=sin(__°)csc(18°)=sec(__°)cos B= 3 5 ,a=6b=8,c=10tan A= 5 9 ,b=6sin A11 157 157 tan Ba=4, b=4 70°. 5π 6 150° −620° 30°. 5π 4 0° 360° 375°.15° 2π − 4π 7 . 315° − π 6 2π 75 sin π 6 . 240°.− 3 2[−1,1] cot π 4 . tan π 3 .3 csc 7π 4 . tan 210°.3 3 csc t=0.68, csc(−t)? cos t= 3 2 , cos(t−2π).3 2 cos(π 6)=sin(___)π 3 ABC:sin B= 3 4 ,c=12a= 9 2 ,b= 9 3 2x0π 6π 4π 3π 22π 33π 45π 6 πsin(x)01 2

2 2

3 2

1

3 2

2 2

1 20 π, π 2π, x0π 6π 4π 3π 22π 33π 45π 6πcos(x)1

3 2

2 2

1 2

0

− 1 2

− 2 2

− 3 2

−1 [−1,1]. 2π, 2π. f(x+P)=f(x) x f. P>0 sin(−x)=−sin x. cos(−x)=cos x. 2π. (−∞,∞) [−1,1]. y=sin x y=cos x y-

y=Asin(Bx−C)+D and y=Acos(Bx−C)+D

B P= 2π | B | . | B |>1, 2π | B |<1, 2π f(x)=sin(x), B=1, 2π, f(x)=sin(2x), B=2, π f(x)=sin(x 2), B= 1 2 , 4π | B |. C=0 D=0

y=Asin(Bx)

y=Acos(Bx)

2π | B | . f(x)=sin(π 6 x). y=Asin(Bx). B= π 6 ,

P= 2π |B| = 2π π 6 =2π· 6 π =12

g(x)=cos(x 3). 6π B A A | A | | A | x=D; D=0 | A |>1, f(x)=4 sin x

f(x)=2 sin x.

| A |<1, C=0 D=0

y=Asin(Bx) and y=Acos(Bx)

A, | A |.

| A | = amplitude = 1 2 | maximum − minimum |

f(x)=−4sin(x)? y=Asin(Bx). A=−4, | A |=| −4 |=4. A f(x)= 1 2 sin(x)? 1 2 A B C D.

y=Asin(Bx−C)+D and y=Acos(Bx−C)+D or y=Asin(B(x− C B))+D and y=Acos(B(x− C B))+D

C B C>0, C<0, | C |, f(x)=sin(x−π) π f(x)=sin(x− π 4), π 4 C D y=cos(x)+D y=D. D f(x)=sin x f(x)=sin x+2, f(x)=Asin(Bx−C)+D f(x)=Acos(Bx−C)+D, C B D f(x)=sin(x+ π 6)−2. y=Asin(Bx−C)+D. B=1 C=− π 6 .

C B =− π 6 1 =− π 6

π 6 C. f(x)=sin(x+ π 6)−2 f(x)=sin(x−(− π 6))−2. C f(x)=3cos(x− π 2).π 2 ; f(x)=cos(x)−3. y=Acos(Bx−C)+D. D=−3 f(x)=3sin(x)+2. f(x)=Asin(Bx−C)+D, | A |. P= 2π | B | . C B . y=D. y=3sin(2x)+1. y=Asin(Bx−C)+D.A=3, | A |=3. B=2, P= 2π | B | = 2π 2 =π. C=0 C B = 0 2 =0. D=1, y=1. π, y=1, y= 1 2 cos(x 3 − π 3). y=0; | A |= 1 2 ; P= 2π | B | =6π; C B =π

y=Asin(Bx−C)+D y=Acos(Bx−C)+D

x=0, (0,0). x=0, y=0.5. D D=0.5. | A |=0.5. | A |= 1 2 =0.5. A=−0.5. B=1; C=0.

g(x)=−0.5cos(x)+0.5

f(x)=sin(x)+2 −5, −2. D=−2. | A |=3. x=1 x=7,P= 2π | B | =6. B,

B= 2π P = 2π 6 = π 3

y=3sin(π 3 x−C)−2 y=3cos(π 3 x−C)−2.

y=3cos(π 3 x− π 3)−2 or y=−3cos(π 3 x+ 2π 3)−2
y=4sin(π 5 x− π 5)+4 y=−4sin(π 5 x+ 4π 5)+4
y=Asin(Bx−C)+D and y=Acos(Bx−C)+D,
C=0 D=0 y=Asin(Bx), | A |. P= 2π | B | . A A x= π 2| B | A>0 A<0, y=A. x= π | B | . A>0 A<0
x= 3π 2| B | y=−A. x= π 2| B | . f(x)=−2sin(πx 2). y=Asin(Bx). A=−2,
| A |=2
B= π 2 ,
P= 2π π 2 =2π· 2 π =4
A x=0, x=2 x=4. x=1 x=3. x=1, x=3. g(x)=−0.8cos(2x). y=0; | A |=0.8; P= 2π | B | =π; C B =0
y=Asin(Bx−C)+D or y=Acos(Bx−C)+D. | A |. P= 2π | B | . C B . f(x)=Asin(Bx) C B D.
f(x)=3sin(π 4 x− π 4). f(x)=3sin(π 4 x− π 4). | A |=| 3 |=3. | B |=| π 4 |= π 4 ,
P= 2π | B | = 2π π 4 =2π· 4 π =8
C= π 4 ,
C B = π 4 π 4 =1.
g(x)=−2cos(π 3 x+ π 6). y=0; | A |=2; P= 2π | B | =6; C B =− 1 2 y=−2cos(π 2 x+π)+3,
y=Acos(Bx−C)+D
A=−2, | A |=2. | B |= π 2 , P= 2π | B | = 2π π 2 =2π· 2 π =4. C=−π, C B = −π, π 2 =−π· 2 π =−2. −2.
D=3, y=3, A y=r sin(x), y(x)=3 sin(x). 2π; (3,0) x=2π,4π,6π,.... −3 3, 3. f(x)=7cos(x)? P
y=−3cos(x)+4
y −1 x=0) −7 x=π) y x. y x.y=3cos(x)−4 67.5+2=69.5 67.5, A=67.5 69.5, D=69.5 30,
B= 2π 30 = π 15 −cos(t)
y=−67.5cos(π 15 t)+69.5
t y f(x)=Asin(Bx−C)+D f(x)=Acos(Bx−C)+D 2π.sin x cos x P= 2π | B | . | A | | A |>1, | A |<1,
C B D f(x+P)=f(x) P. P y=sin x y=cos x? y=sin x y=cos x. A cos(Bx+C)+D, A D
y=A sin(Bx+C)+D? f(t)=sin t? t sin t x>0. f(x)=2sin xf(x)= 2 3 cos x 2 3 ; 2π; y=0; y= 2 3 x=0;
y=− 2 3 x=π; 2πf(x)=−3sin xf(x)=4sin x 2π; y=0; y=4 x= π 2 ; y=−4 x= 3π 2 ; x=0 x=2πf(x)=2cos x
f(x)=cos(2x) π; y=0; y=1 x=π; y=−1 x= π 2 ; x=0 x=πf(x)=2 sin(1 2 x)f(x)=4 cos(πx) y=0; y=4
x=0; y=−4 x=1f(x)=3 cos(6 5 x)y=3 sin(8(x+4))+5 π 4 ; y=5; y=8 x=0.12; y=2 x=0.516; −4; x=0
x= π 4y=2 sin(3x−21)+4y=5 sin(5x+20)−2 2π 5 ; y=−2; y=3 x=0.08; y=−7 x=0.71; −4; −2; x=0
x= 2π 5 x=0. x>0. f(t)=2sin(t− 5π 6)f(t)=−cos(t+ π 3)+1 2π; y=1; y=2 x=2.09; y=2 t=2.09; y=0
t=5.24; − π 3 ; t=0 t=2πf(t)=4cos(2(t+ π 4))−3f(t)=−sin(1 2 t+ 5π 3) 4π; y=0; y=1 t=11.52; y=−1
t=5.24; − 10π 3 ; f(x)=4sin(π 2 (x−3))+7 y=−3; f(x)=2sin(π 2 x)−3 y=3; f(x)=−2cos(2π 5 x)+3
y=0; f(x)=−4cos(π(x− π 2)) y=1; f(x)=2cos(πx)+1 f(x)=sin x. [0,2π), f(x)=0. [0,2π), f(x)= 1 2 .
π 6 , 5π 6 f(π 2). [0,2π),f(x)= 2 2 . x.π 4 , 3π 4 [0,2π), [0,2π),3π 2 f(−x)=−f(x). f(x)=sin x f(x)=cos x.
[0,2π), f(x)=cos x=0.π 2 , 3π 2 [0,2π), f(x)= 1 2 . [0,2π), f(x)=cos x.π 2 , 3π 2 [0,2π), [0,2π),
f(x)= 3 2 .π 6 , 11π 6 h(x)=x+sin x [0,2π]. h(x)=x+sin x [−100,100]. x. f(x)=x sin x [0,2π]
f(x)=sin x. f(x)=x sin x [−10,10] f(x)= sin x x [−5π,5π] h(t) h(t). h(t). y=13.5;
h(t)=12.5sin(π 5 (t−2.5))+13.5; A y=D, D f(x) f(x+P)=f(x) P x C B f(x)=Asin(Bx−C)+D
f(x)=Acos(Bx−C)+D
tan x= sin x cos x
π kπ k − π 2 π 2 ,
tan(−x)= sin(−x) cos(−x) Definition of tangent. = −sin x cos x
Sine is an odd function, cosine is even. =− sin x cos x
The quotient of an odd and an even function is odd. =−tan x Definition of tangent.
x− π 2− π 3− π 4− π 6π 6π 4π 3π 2tan(x)− 3− 3 33 33 π 3 <x< π 2 , π 3 ≈1.05 π 2 ≈1.57, x
1.05<x<1.57 xtan x x π 2 , y=tan x xtan x x − π 2 , x cos x=0. cos(π 2)=0 cos(3π 2)=0. y=tan x
x= π 2 and 3π 2 . y=tan x. π 2 π 3π 2 ,
y=Atan(Bx)

A B. A. | A |. P= π | B | . x, x≠ π 2| B | + π | B | k k (−∞,∞). x= π 2| B | + π | B | k, k y=Atan(Bx)
f(x)=Atan(Bx). (− P 2 , P 2) ± P 2 P= π B . (− π 2 , π 2), x=− π 2 , x= π 2 .

f(P 4)=Atan(B P 4)=Atan(B π 4B)=A

tan(π 4)=1. f(x)=Atan(Bx), | A |. B P= π | B | . x=− P 2 x= P 2 . A>0, A<0 (P 4 ,A), (0,0),
(− P 4 ,−A), y=0.5tan(π 2 x). A B. A=0.5 B= π 2 , π π 2 =2, x=±1.

f(0.5)=0.5tan(0.5π 2) =0.5tan(π 4) =0.5

(0.5,0.5),(0,0), (−0.5,−0.5). f(x)=3tan(π 6 x). C D

f(x)=Atan(Bx−C)+D

tan x | A |. π | B | . x≠ C B + π | B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π 2| B | k, k y=A tan(Bx)
y=Atan(Bx−C)+D, y=Atan(Bx−C)+D. | A |. B P= π | B | . C C B . y=Atan(Bx) C B D.
x= C B + π 2| B | k, k y=−2tan(πx+π) −1. y=Atan(Bx−C)+D. A=−2, | A |=2. B=π, P= π | B | = π π =1.
C=−π, C B = −π π =−1. x=− 3 2 x=− 1 2 (−1.25,1), (−1,−1), (−0.75,−3). A<0. A=2 −2? y=−1, P
f(x)=Atan(π P x). (x,f(x)) A. P=8. P= π | B | , B= π P = π 8 .f(x)=Atan(π 8 x). A, (2,2).

2=Atan(π 8 ·2)=Atan(π 4)

tan(π 4)=1, A=2. f(x)=2tan(π 8 x).g(x)=4tan(2x) sec x= 1 cos x . π 2 , 3π 2 , y=sec x x
sec(−x)=sec x. f(x)=secx= 1 cosx | A | | A |. 2π | B | . x≠ π 2| B | k, k (−∞,−| A |]∪[| A |,∞).
x= π 2| B | k, k y=Asec(Bx) csc x= 1 sin x . 0, π, y=csc x x csc(−x)=−cscx. f(x)=cscx= 1 sinx | A |.
2π | B | . x≠ π | B | k, k (−∞,−| A |]∪[| A |,∞). x= π | B | k, k y=Acsc(Bx)

y=Asec(Bx−C)+D

y=Acsc(Bx−C)+D

| A |. 2π | B | . x≠ C B + π 2| B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π 2| B | k, k y=Asec(Bx) | A |.
2π | B | . x≠ C B + π 2| B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π |B| k, k y=Acsc(Bx) y=Asec(Bx),
y=Asec(Bx). | A |. B P= 2π | B | . y=Acos(Bx). y=cos x y=sec x y=Asec(Bx). f(x)=2.5sec(0.4x).
y=Asec(Bx). A=2.5 2.5. B=0.4 P= 2π 0.4 =5π. 5π g(x)=2.5cos(0.4x). x=1.25π x=3.75π. (0,2.5)
(2.5π,−2.5). f(x)=−2.5sec(0.4x). A f(x)=Asec(Bx−C)+D (−∞,−| A |+D]∪[| A |+D,∞).
f(x)=Asec(Bx−C)+D, y=A sec(Bx−C)+D. | A |. B 2π | B | . C C B . y=A sec(Bx) . C B D.
x= C B + π 2| B | k, k y=4sec(π 3 x− π 2)+1. y=4sec(π 3 x− π 2)+1.| A |=4.

2π |B| = 2π π 3 = 2π 1 · 3 π =6

C B = π 2 π 3 = π 2 · 3 π =1.5

y=Asec(Bx), C B =1.5 D=6. x=0,x=3, x=6. (1.5,5) (4.5,−3). f(x)=−6sec(4x+2)−8. csc x x x≠kπ k.
y=Acsc(Bx−C)+D be x≠ C+kπ B ? y=Acsc(Bx), y=Acsc(Bx). | A |. B P= 2π | B | . y=Asin(Bx).
y=sin x y=csc x y=Acsc(Bx). f(x)=−3csc(4x). y=Acsc(Bx). | A |=| −3 |=3, B=4, P= 2π 4 = π 2 . π 2
g(x)=−3sin(4x). x=0, x= π 4 , x= π 2 . (π 8 ,−3) (3π 8 ,3). f(x)=0.5csc(2x). f(x)=Acsc(Bx−C)+D,
y=Acsc(Bx−C)+D. | A |. B 2π | B | . C C B . y=Acsc(Bx) D. x= C B + π | B | k, k y=2csc(π 2 x)+1.
y=2csc(π 2 x)+1. | A |=2. 2π | B | = 2π π 2 = 2π 1 · 2 π =4. 0 π 2 =0. y=Acsc(Bx) D=1. x=0,x=2,x=4.
f(x)=2sin(π 2 x)+1, f(x)=2cos(π 2 x)+1 g(x)=2sec(π 2 x)+1 cot x= 1 tan x . 0,π, y=cot x x tan x=0;
cot x x tan x=0, cot x=0 x tan x | A |. P= π | B | . x≠ π | B | k, k (−∞,∞). x= π | B | k, k y=Acot(Bx)

y=Acot(Bx−C)+D

| A |. π | B | . x≠ C B + π | B | k, k (−∞,−| A |]∪[| A |,∞). x= C B + π | B | k, k y=Acot(Bx)
f(x)=Acot(Bx), f(x)=Acot(Bx). | A |. P= π | B | . y=Atan(Bx). y=Acot(Bx). y=3cot(4x),
f(x)=Acot(Bx) f(x)=3cot(4x). | A |=3. P= π 4 . y=3tan(4x). (π 16 ,3) (3π 16 ,−3). y=3cot(4x).
x=0, x= π 4 . y=3tan(4x) y=3cot(4x). f(x)=Acot(Bx−C)+D, f(x)=Acot(Bx−C)+D. | A |.
P= π | B | . C B . y=Atan(Bx) C B D. x= C B + π | B | k, k f(x)=4cot(π 8 x− π 2)−2.
f(x)=Acot(Bx−C)+D. A=4, B= π 8 , P= π | B | = π π 8 =8. C= π 2 , C B = π 2 π 8 =4.
f(x)=4tan(π 8 x− π 2)−2. (6,2),(8,−2), (10,−6). f(x)=4cot(π 8 x− π 2)−2. x=4 x=12. y=5tan(π 4 t)
t y [0,5]. f(1) y=Atan(Bt) | A | π B π π 4 = π 1 · 4 π =4. t=2 f(1)=5tan(π 4 (1))=5(1)=5;
y=A tan(Bx−C)+Dy=A sec(Bx−C)+Dy=A csc(Bx−C)+Dy=A cot(Bx−C)+D π.
f(x)=Atan(Bx−C)+D 2π. f(x)=Asec(Bx−C)+D f(x)=Acsc(Bx−C)+D π 0,±π,±2π,.... (−∞,∞),

± π 2 ,± 3π 2 ,....f(x)=Acot(Bx−C)+D y=csc x. y=csc x y=sin x, y=sin x y=csc x. y=sin x y=csc x. y=cos x y=sec x? tan x π. tan(x+π)=tan x. y=csc x? y=csc x y=sin x? 2π.f(x)=tan xf(x)=sec x f(x)=csc xf(x)=cot xf(x)=2tan(4x−32)h(x)=2sec(π 4 (x+1))m(x)=6csc(π 3 x+π) tan x=−1.5, tan(−x). sec x=2, sec(−x). csc x=−5, csc(−x). xsin x=2, (−x)sin(−x). x cot(−x)cos(−x)+sin(−x) −cotxcosx−sinxcos(−x)+tan(−x)sin(−x)f(x)=2tan(4x−32) π 4 ;
x= 1 4 (π 2 +πk)+8, where k is an integer h(x)=2sec(π 4 (x+1)) m(x)=6csc(π 3 x+π)
x=3k, where k is an integerj(x)=tan(π 2 x)p(x)=tan(x− π 2) π; x=πk, where k is an integer
f(x)=4tan(x)f(x)=tan(x+ π 4) π; x= π 4 +πk, where k is an integerf(x)=πtan(πx−π)−πf(x)=2csc(x) 2π; x=πk, where k is an integerf(x)=− 1 4 csc(x)f(x)=4sec(3x) 2π 3 ;
x= π 6 k, where k is an odd integerf(x)=−3cot(2x)f(x)=7sec(5x) 2π 5 ;
x= π 10 k, where k is an odd integerf(x)= 9 10 csc(πx)f(x)=2csc(x+ π 4)−1 2π;
x=− π 4 +πk, where k is an integerf(x)=−sec(x− π 3)−2f(x)= 7 5 csc(x− π 4) 7 5 ; 2π;
x= π 4 +πk, where k is an integerf(x)=5(cot(x+ π 2)−3) | A |, A=1, π 3 ; (h, k)=(π 4 ,2)
y=tan(3(x− π 4))+2 A=−2, π 4 , (h, k)=(− π 4 ,−2)f(x)=csc(2x)f(x)=csc(4x)f(x)=2cscx
f(x)= 1 2 tan(100πx) csc x 1 sin x .f(x)=| csc(x) |f(x)=| cot(x) |f(x)= 2 csc(x)f(x)= csc(x) sec(x)
f(x)=1+ sec 2 (x)− tan 2 (x). f(x)=sec(0.001x)f(x)=cot(100πx)f(x)= sin 2 x+ cos 2 x
f(x)=20tan(π 10 x) x, f(x), [0, 5]. f(1) f(2.5) x, x d(x), d(x)=1.5sec(x). d(x)? d(x) d(x). d(− π 3). d(π 6). (− π 2 , π 2); x=− π 2 x= π 2 ; | x | π 2 x=− π 3 , x= π 6 , x=0 g(x), x x g(x)=250,000csc(π 30 x). g(x) [0, 35]. g(5) x π 120 x. h(x), x h(x) (0, 60). h(0) h(30). h(x) x h(x)=2tan(π 120 x);h(0)=0: h(30)=2: x h(x) f(x)=sin x, f −1 (x)= sin −1 x. sin −1 x 1 sinx . sin(π 6)= 1 2 , π 6 = sin −1 (1 2). cos(π)=−1, π= cos −1 (−1). tan(π 4)=1, π 4 = tan −1 (1). f(a)=b, f −1 (b)=a. [− π 2 , π 2] [0,π]. [− π 2 , π 2]; [0,π] (− π 2 , π 2). (− π 2 , π 2) y= sin −1 x x=sin y. arcsinx.

y= sin −1 x has domain [−1,1] and range [− π 2 , π 2]

y= cos −1 x x=cos y. arccos x.

y= cos −1 x has domain [−1,1] and range [0,π]

y= tan −1 x x=tan y. arctan x.

y= tan −1 x has domain (−∞,∞) and range (− π 2 , π 2)

sin −1 x [−1,1] [− π 2 , π 2],cos −1 x [−1,1] [0,π], tan −1 x (− π 2 , π 2). y=x. [− π 2 , π 2], sin y=x, sin −1 x=y. [0,π], cos y=x, cos −1 x=y. (− π 2 , π 2), tan y=x, tan −1 x=y.
sin(5π 12)≈0.96593, sin y=x, sin −1 x=y x=0.96593, y= 5π 12 .

sin −1 (0.96593)≈ 5π 12

cos(0.5)≈0.8776,arccos(0.8776)≈0.5 π 6 π 4 π 3 x x y x,sin −1 (1 2)sin −1 (− 2 2)cos −1 (− 3 2) tan −1 (1) sin −1 (1 2) 1 2 . x sin(x)= 1 2 ? π 6 5π 6 , [− π 2 , π 2], sin −1 (1 2)= π 6 . sin −1 (− 2 2), 5π 4 7π 4 − 2 2 , [− π 2 , π 2]. 7π 4 :sin −1 (− 2 2)=− π 4 . cos −1 (− 3 2), [0,π] − 3 2 . cos −1 (− 3 2)= 5π 6 . tan −1 (1), (− π 2 , π 2) tan −1 (1)= π 4 .sin −1 (−1) tan −1 (−1)cos −1 (−1)cos −1 (1 2) − π 2 ; − π 4 ; π; π 3 θ sin −1 (0.97) sin −1 (0.97)≈1.3252. sin −1 (0.97)≈75.93°. cos −1 (−0.4) h a θ= cos −1 (a h). h p θ= sin −1 (p h). θ= tan −1 (p a). θ.

cos θ= 9 12 θ= cos −1 (9 12) Apply definition of the inverse. θ≈0.7227 or about 41.4096°

Evaluate.

θ.sin −1 (0.6)=36.87°=0.6435 f(x) g(x) { sin(x),cos(x),tan(x) } f −1 (y) g −1 (y) f(f −1 (y))=y y f f −1 . f −1 (f(x)).

sin(sin −1 x)=x for −1≤x≤1 cos(cos −1 x)=x for −1≤x≤1 tan(tan −1 x)=x for −∞<x<∞

sin −1 (sin x)=x only for − π 2 ≤x≤ π 2 cos −1 (cos x)=x only for 0≤x≤π tan −1 (tan x)=x only for − π 2 <x< π 2

sin −1 (sin x)=x? x [− π 2 , π 2], x [− π 2 , π 2]. sin −1 (sin(3π 4))= π 4 .
f(θ)=sin θ, cos θ, or tan θ, θ f, then f −1 (f(θ))=θ. φ f f(φ)=f(θ). f −1 (f(θ))=φ.sin −1 (sin(π 3)) sin −1 (sin(2π 3))cos −1 (cos(2π 3))cos −1 (cos(− π 3))π 3 is in [− π 2 , π 2],

sin −1 (sin(π 3))= π 3 .2π 3 is not in [− π 2 , π 2], sin(2π 3)=sin(π 3), sin −1 (sin(2π 3))= π 3 . 2π 3 is in [0,π], cos −1 (cos(2π 3))= 2π 3 .− π 3 is not in [0,π], cos(− π 3)=cos(π 3) π 3 is in [0,π], cos −1 (cos(− π 3))= π 3 . tan −1 (tan(π 8)) and tan −1 (tan(11π 9)).π 8 ; 2π 9 f −1 (g(x)). x, θ, π 2 −θ. cos θ= b c =sin(π 2 −θ), sin −1 (cos θ)= π 2 −θ 0≤θ≤π. θ θ π 2 . sin θ= a c =cos(π 2 −θ), cos −1 (sin θ)= π 2 −θ − π 2 ≤θ≤ π 2 . sin −1 (cos x) cos −1 (sin x), x is in [0,π], sin −1 (cos x)= π 2 −x. x is not in [0,π], y in [0,π] cos y=cos x.

sin −1 (cos x)= π 2 −y

x is in [− π 2 , π 2], cos −1 (sin x)= π 2 −x. x is not in [− π 2 , π 2], y in [− π 2 , π 2] sin y=sin x.

cos −1 (sin x)= π 2 −y

sin −1 (cos(13π 6))

cos(13π 6)=cos(π 6 +2π) =cos(π 6) = 3 2

sin −1 (3 2)= π 3

x= 13π 6 , y= π 6 ,

sin −1 (cos(13π 6))= π 2 − π 6 = π 3

cos −1 (sin(− 11π 4)).3π 4 f(g −1 (x)), f g x g −1 , sin(cos −1 x)= 1− x 2 . sin 2 x+ cos 2 x=1, sin(cos −1 (4 5)). θ= cos −1 (4 5), cos θ= 4 5 , sin θ.

sin 2 θ+ cos 2 θ=1 Use our known value for cosine. sin 2 θ+ (4 5) 2 =1 Solve for sine. sin 2 θ=1− 16 25 sin θ=± 9 25 =± 3 5

θ= cos −1 (4 5) sin θ 3 5 . cos θ= 4 5 , sin θ= 3 5 [0,π], sin(cos −1 (4 5))=sin θ= 3 5 . cos(tan −1 (5 12)).12 13 sin(tan −1 (7 4)). tan θ= 7 4 .

4 2 + 7 2 = hypotenuse 2 hypotenuse= 65

sin θ= 7 65

sin(tan −1 (7 4))=sin θ = 7 65 = 7 65 65

cos(sin −1 (7 9)).4 2 9 cos(sin −1 (x 3)) −3≤x≤3. θ sin θ= x 3 .

sin 2 θ+ cos 2 θ=1 Use the Pythagorean Theorem. (x 3) 2 + cos 2 θ=1 Solve for cosine. cos 2 θ=1− x 2 9 cosθ=± 9− x 2 9 =± 9− x 2 3

[− π 2 , π 2],

cos(sin −1 (x 3))= 9− x 2 3

sin(tan −1 (4x)) − 1 4 ≤x≤ 1 4 .4x 16 x 2 +1 f(x), x= f −1 (y), f(x)=y. f(x)=y x= f −1 (y) x f. π 4 = tan −1 (1) and π 6 = sin −1 (1 2). sin(cos −1 (x))= 1− x 2 . sin −1 (cos x)= π 2 −x 0≤x≤π cos −1 (sin x)= π 2 −x − π 2 ≤x≤ π 2 . f(x)= sin −1 x g(x)= cos −1 x y=sinx [− π 2 , π 2]; y=sinx, f(x)= sin −1 x. y=cosx [0,π]; y=cosx,f(x)= cos −1 x. y=cos x y= cos −1 x cos −1 (cos(− π 6)) − π 6 ? π 6 =arcsin(0.5).π 6 − π 2 π 2 sec −1 (2). sin x, [− π 2 , π 2] [− π 2 , π 2] arccos(cos x)=x x.arccos(−x)=π−arccos x. θ 1 arccos(−x) x>0 θ 2 θ 2 arccosxx>0 θ 2 θ 1 θ 2 =π− θ 1 arccos(−x) π−arccosxsin −1 (2 2)sin −1 (− 1 2)− π 6cos −1 (1 2)cos −1 (− 2 2)3π 4 tan −1 (1)tan −1 (− 3)− π 3tan −1 (−1)tan −1 (3)π 3tan −1 (−1 3)cos −1 (−0.4)arcsin(0.23) arccos(3 5)cos −1 (0.8)tan −1 (6) θ sin −1 (cos(π))tan −1 (sin(π))cos −1 (sin(π 3)) tan −1 (sin(π 3))sin −1 (cos(−π 2))tan −1 (sin(4π 3))sin −1 (sin(5π 6))tan −1 (sin(−5π 2)) − π 4cos(sin −1 (4 5))sin(cos −1 (3 5))sin(tan −1 (4 3))cos(tan −1 (12 5))5 13 cos(sin −1 (1 2)) x tan(sin −1 (x−1))x−1 − x 2 +2xsin(cos −1 (1−x))cos(sin −1 (1 x))x 2 −1 x cos(tan −1 (3x−1))tan(sin −1 (x+ 1 2))x+0.5 − x 2 −x+ 3 4 sin −1 (1 2)− cos −1 (2 2)+ sin −1 (3 2)− cos −1 (1) cos −1 (3 2)− sin −1 (2 2)+ cos −1 (1 2)− sin −1 (0)

sin t= x x+1 .cos t2x+1 x+1sec tcot t2x+1 xcos(sin −1 (x x+1))tan −1 (x 2x+1)t y= sin −1 x y=arccos x [−1,1]; [0,π] y= tan −1 x x sin x= sin −1 x? x=0.00 x cos x= cos −1 x? arctan(10,000). y= 3 5 x y= −3 7 x f(x)=−3cos x+3 2π; y=3; f(x)= 1 4 sin xf(x)=3cos(x+ π 6) 2π; y=0; f(x)=−2sin(x− 2π 3)f(x)=3sin(x− π 4)−4 2π; y=−4; f(x)=2(cos(x− 4π 3)+1) f(x)=6sin(3x− π 6)−1 2π 3 ; y=−1; f(x)=−100sin(50x−20)f(x)=tan x−4 π; y=−4; x= π 2 +πk, k f(x)=2tan(x− π 6)f(x)=−3tan(4x)−2 π 4 ; y=−2; x= π 8 + π 4 k, k f(x)=0.2cos(0.1x)+0.3

f(x)= 1 3 sec x2π; x= π 2 k, k f(x)=3cot xf(x)=4csc(5x) 2π 5 ; x= π 5 k, k f(x)=8sec(1 4 x)
f(x)= 2 3 csc(1 2 x) 4π; x=2πk, k f(x)=−csc(2x+π) y=12,000+8,000sin(0.628x), [0,40]sin −1 (1)
cos −1 (3 2)π 6tan −1 (−1)cos −1 (1 2)π 4sin −1 (− 3 2)sin −1 (cos(π 6))π 3cos −1 (tan(3π 4))
sin(sec −1 (3 5))cot(sin −1 (3 5))tan(cos −1 (5 13))12 5sin(cos −1 (x x+1)) f(x)=cos x
f(x)=sec x [0,2π) y=x. y f(x)=sin x f(x)=csc x f (x)= x 1 − x 3 3! + x 5 5! − x 7 7! [−1,1]
f(x)=sin x f(x)=0.5sin x 2π; y=0 f(x)=5cos xf(x)=5sin x 2π; y=0f(x)=sin(3x)
f(x)=−cos(x+ π 3)+1 2π; y=1f(x)=5sin(3(x− π 6))+4f(x)=3cos(1 3 x− 5π 6) 6π; y=0
f(x)=tan(4x)f(x)=−2tan(x− 7π 6)+2 π; y=0, x= 2π 3 +πk, k f(x)=πcos(3x+π)f(x)=5csc(3x)
2π 3 ; y=0, x= π 3 k, k f(x)=πsec(π 2 x)f(x)=2csc(x+ π 4)−3 2π; y=−3 y=0;f(x)=2sin(π(x−1))
y=sin(π 6 x+π)−3 −6; y=−3y=8sin(7π 6 x+ 7π 2)+6 t D, t.D(t)=68−12sin(π 12 x)
g(x)=3tan(6x+42) π 6 ; −7n(x)=4csc(5π 3 x− 20π 3)f(x)=sec(πx); tan x=3, tan(−x). sec x=4,
sec(−x).4 m(x)=sin(2x)+cos(3x) [−10,10] [−3,3]. n(x)=0.02sin(50πx) x:[0,1] [0,3]. 1 25 .
f(x)= sin x x [−0.5,0.5] f(x)= 3 5 cos(6x). f(x)?3 5 f(x)? [0,2π]?
(0.5,1),(1.6,2.1),(2.6,3.1),(3.7,4.2),(4.7,5.2),(5.6,6.28) π 3 , (h,k)=(π 4 ,2) π 6 ,
(h,k)=(− π 4 ,3)f(x)=2cos(12(x+ π 4))+3f(x)=5cos(3x)+4sin(2x)f(x)= e sint 2π.sin −1 (3 2)
tan −1 (3)π 3cos −1 (− 3 2)cos −1 (sin(π))π 2cos −1 (tan(7π 4))cos(sin −1 (1−2x))1− (1−2x) 2
cos −1 (−0.4)cos(tan −1 (x 2))1 1+ x 4 sin t= x x+1 .tan tcsc tx+1 x θ arcsin(sin(5π 6))= 5π 6
arccos(cos(5π 6))= 5π 6 arccos x= cos −1 x arcsin x= sin −1 x arctan x= tan −1 x cos −1 x, sin −1 x,
tan −1 x, sin 2 θ+ cos 2 θ=11+ cot 2 θ= csc 2 θ1+ tan 2 θ= sec 2 θ 1+ cot 2 θ= csc 2 θ
1+ cot 2 θ= csc 2 θ

1+ cot 2 θ=(1+ cos 2 θ sin 2 θ) Rewrite the left side. =(sin 2 θ sin 2 θ)+(cos 2 θ sin 2 θ)
Write both terms with the common denominator. = sin 2 θ+ cos 2 θ sin 2 θ = 1 sin 2 θ = csc 2 θ

1+ tan 2 θ= sec 2 θ

1+ tan 2 θ=1+ (sin θ cos θ) 2 Rewrite left side. = (cos θ cos θ) 2 + (sin θ cos θ) 2
Write both terms with the common denominator. = cos 2 θ+ sin 2 θ cos 2 θ = 1 cos 2 θ = sec 2 θ

tan(−θ)=−tan θ cot(−θ)=−cot θsin(−θ)=−sin θ csc(−θ)=−csc θcos(−θ)=cos θ sec(−θ)=sec θ
f(− x)= −f(x) x f. sin(−θ)=−sin θ. π 2 − π 2 . sin(π 2) sin(− π 2).
sin(π 2)=1 and sin(− π 2)=−sin(π 2) =−1
y=sin θ

f(−x)=f(x) for all x in the domain of f

cos(−θ)=cos θ. π 4 − π 4 . cos(π 4) cos(− π 4).
cos(− π 4)=cos(π 4) ≈0.707
y=cos θ θ sin(−θ)=−sin θ, cos(− θ)=cos θ, tan(− θ)=−tan θ.
tan(− θ)= sin(−θ) cos(− θ) = −sin θ cos θ =−tan θ. tan(−θ)=−tan(θ) θ cot(−θ)=−cot θ,
cot(−θ)= cos(−θ) sin(−θ) = cos θ −sin θ =−cot θ. cot(−θ)=−cot(θ) θ
csc(−θ)= 1 sin(−θ) = 1 −sin θ =−csc θ. sec(−θ)= 1 cos(−θ) = 1 cos θ =sec θ. sin θ= 1 csc θ
csc θ= 1 sin θcos θ= 1 sec θsec θ= 1 cos θtan θ= 1 cot θcot θ= 1 tan θtan θ= sin θ cos θcot θ= cos θ sin θ

cos 2 θ+ sin 2 θ=1
1+ cot 2 θ= csc 2 θ
1+ tan 2 θ= sec 2 θ
tan(−θ)=−tan θ
cot(−θ)=−cot θ
sin(−θ)=−sin θ
csc(−θ)=−csc θ
cos(−θ)=cos θ
sec(−θ)=sec θ

$$\sin\theta = 1\csc\theta$$
$$\cos\theta = 1\sec\theta$$
$$\tan\theta = 1\cot\theta$$
$$\csc\theta = 1\sin\theta$$
$$\sec\theta = 1\cos\theta$$
$$\cot\theta = 1\tan\theta$$
$$\tan\theta = \sin\theta\cos\theta$$
$$\cot\theta = \cos\theta\sin\theta$$

$\cot\theta = 1\tan\theta$. $y = \cot\theta$ $y = 1\tan\theta$. $\tan\theta\cos\theta = \sin\theta$.

$$\tan\theta\cos\theta = (\sin\theta\cos\theta)\cos\theta = (\sin\theta\cos\theta)\cos\theta = \sin\theta$$

$\tan\theta$ $\sin\theta$ $\cos\theta$. $\csc\theta\cos\theta\tan\theta = 1$.

$$\csc\theta\cos\theta\tan\theta = (1\sin\theta)\cos\theta(\sin\theta\cos\theta) = \cos\theta\sin\theta(\sin\theta\cos\theta) = \sin\theta\cos\theta\sin\theta\cos\theta = 1$$

$$(1+\sin x)[1+\sin(-x)] = \cos 2x$$

$(1+\sin x)[1+\sin(-x)] = (1+\sin x)(1-\sin x)$ Since $\sin(-x) = -\sin x$ $= 1-\sin 2x$ Difference of squares $= \cos 2x\cos 2x = 1-\sin 2x$

$\sec 2\theta - 1\sec 2\theta = \sin 2\theta$

$\sec 2\theta - 1\sec 2\theta = (\tan 2\theta + 1) - 1\sec 2\theta\sec 2\theta = \tan 2\theta + 1$ $= \tan 2\theta\sec 2\theta$ $= \tan 2\theta(1\sec 2\theta)$ $= \tan 2\theta(\cos 2\theta)\cos 2\theta = 1\sec 2\theta$ $= (\sin 2\theta\cos 2\theta)(\cos 2\theta)\tan 2\theta = \sin 2\theta\cos 2\theta$ $= (\sin 2\theta\cos 2\theta)(\cos 2\theta)$ $= \sin 2\theta$

$\sec 2\theta - 1\sec 2\theta = \sec 2\theta\sec 2\theta - 1\sec 2\theta$ $= 1-\cos 2\theta$ $= \sin 2\theta$

$\sec 2\theta = \tan 2\theta + 1$ $\cot\theta\csc\theta = \cos\theta$.

$\cot\theta\csc\theta = \cos\theta\sin\theta\,1\sin\theta$ $= \cos\theta\sin\theta\cdot\sin\theta\,1$ $= \cos\theta$

$2\tan\theta\sec\theta$

$2\tan\theta\sec\theta = 2(\sin\theta\cos\theta)(1\cos\theta)$ $= 2\sin\theta\cos 2\theta$ $= 2\sin\theta\,1-\sin 2\theta$

Substitute $1-\sin 2\ \theta$ for $\cos 2\ \theta$

$$2\tan\theta\sec\theta = 2\sin\theta\,1-\sin 2\ \theta$$

$$\sin 2(-\theta) - \cos 2(-\theta)\sin(-\theta) - \cos(-\theta) = \cos\theta - \sin\theta$$

$\sin 2(-\theta) - \cos 2(-\theta)\sin(-\theta) - \cos(-\theta) = [\sin(-\theta)]2 - [\cos(-\theta)]2\sin(-\theta) - \cos(-\theta)$ $= (-\sin\theta)2 - (\cos\theta)2 - \sin\theta - \cos\theta$ $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$ $= (\sin\theta)2 - (\cos\theta)2 - \sin\theta - \cos\theta$ Difference of squares $= (\sin\theta - \cos\theta)(\sin\theta + \cos\theta) - (\sin\theta + \cos\theta)$ $= (\sin\theta - \cos\theta)(\sin\theta + \cos\theta) - (\sin\theta + \cos\theta)$ $= \cos\theta - \sin\theta$

$\sin 2\theta - 1\tan\theta\sin\theta - \tan\theta = \sin\theta + 1\tan\theta$.

$\sin 2\theta - 1\tan\theta\sin\theta - \tan\theta = (\sin\theta + 1)(\sin\theta - 1)\tan\theta(\sin\theta - 1) = \sin\theta + 1\tan\theta$

$(1-\cos 2x)(1+\cot 2x) = 1$.

$(1-\cos 2x)(1+\cot 2x) = (1-\cos 2x)(1+\cos 2x\sin 2x)$ $= (1-\cos 2x)(\sin 2x\sin 2x + \cos 2x\sin 2x)$ Find the common denominator. $= (1-\cos 2x)(\sin 2x + \cos 2x\sin 2x)$ $= (\sin 2x)(1\sin 2x)$ $= 1$

$(\sin x+1)(\sin x-1) = 0$ $(x+1)(x-1) = 0$, $a2 - b2 = (a-b)(a+b)$, $2\cos 2\theta + \cos\theta - 1$. $ax2 + bx + c$.

$\cos\theta = x$,

$$2x2 + x - 1$$

$(2x+1)(x-1)$. x. $x\ \cos\theta\ \theta$. $4\ \cos 2\theta - 1$.

$4\ \cos 2\theta - 1 = (2\cos\theta)2 - 1$ $= (2\cos\theta - 1)(2\cos\theta + 1)$

$\cos\theta = x, 4x2 - 1, (2x-1)(2x+1)$. $x\ \cos\theta\ 25-9\ \sin 2\ \theta$. $25-9\ \sin 2\ \theta = (5-3\sin\theta)(5+3\sin\theta)$.

$$\csc 2\theta - \cot 2\theta$$
$$1+\cot 2\theta = \csc 2\theta$$

$1+\cot 2\theta$ $\csc 2\theta$.

$\csc 2\theta - \cot 2\theta = 1+\cot 2\theta - \cot 2\theta$ $= 1$

cos θ 1+sin θ = 1−sin θ cos θ . 1−sin θ.)

cos θ 1+sin θ (1−sin θ 1−sin θ)= cos θ(1−sin θ) 1− sin 2 θ = cos θ(1−sin θ) cos 2 θ = 1−sin θ cos θ

sin 2 θ+ cos 2 θ=1 1+ cot 2 θ= csc 2 θ 1+ tan 2 θ= sec 2 θ

tan(−θ)=−tan θ cot(−θ)=−cot θ sin(−θ)=−sin θ csc(−θ)=−csc θ cos(−θ)=cos θ sec(−θ)=sec θ

sin θ= 1 csc θ cos θ= 1 sec θ tan θ= 1 cot θ csc θ= 1 sin θ sec θ= 1 cos θ cot θ= 1 tan θ

tan θ= sin θ cos θ cot θ= cos θ sin θ

g(x)=cos x f(x)=sin x h(x)=tan x G(x)= cos 2 x,F(x)= sin 2 x, H(x)= tan 2 x? F,G, H,

F(−x)=sin(−x)sin(−x)=(−sin x)(−sin x)= sin 2 x=F(x),G(−x)=cos(−x)cos(−x)=cos xcos x= cos 2 x=G(x)

H(−x)=tan(−x)tan(−x)=(−tan x)(−tan x)= tan 2 x=H(x). f(x)=sec x [−π,π]. f(x)=sec x? sec t,

cos t=0, sec t= 1 0 , sin 2 t+ cos 2 t=1 sin x cos x sec xsin xsin(−x)cos(−x)csc(−x)tan xsin x+sec x cos 2 x sec xcsc x+cos xcot(−x)cot t+tan t sec(−t)csc t3 sin 3 t csc t+ cos 2 t+2 cos(−t)cos t−tan(−x)cot(−x)−1 −sin(−x)cos x sec x csc x tan x cot x1+ tan 2 θ csc 2 θ + sin 2 θ+ 1 sec 2 θsec 2 x

(tan x csc 2 x + tan x sec 2 x)(1+tan x 1+cot x)− 1 cos 2 x1− cos 2 x tan 2 x +2 sin 2 xsin 2 x+1 tan x+cot x csc x ; cos xsec x+csc x 1+tan x ; sin x1 sin xcos x 1+sin x +tan x; cos x 1 sin xcos x −cot x; cot x1 cot x1 1−cos x − cos x 1+cos x ; csc x (sec x+csc x)(sin x+cos x)−2−cot x; tan xtan x1 csc x−sin x ; sec x and tan x 1−sin x 1+sin x − 1+sin x 1−sin x ; sec x and tan x−4sec xtan xtan x; sec xsec x; cot x± 1 cot 2 x +1 sec x; sin xcot x; sin x± 1− sin 2 x sin xcot x; csc xcos x− cos 3 x=cos x sin 2 x cos x− cos 3 x=cos x(1− cos 2 x)=cosx sin 2 xcos x(tan x−sec(−x))=sin x−1 1+ sin 2 x cos 2 x = 1 cos 2 x + sin 2 x cos 2 x =1+2 tan 2 x

1+ sin 2 x cos 2 x = 1 cos 2 x + sin 2 x cos 2 x = sec 2 x+ tan 2 x= tan 2 x+1+ tan 2 x=1+2 tan 2 x

(sin x+cos x) 2 =1+2 sin xcos xcos 2 x− tan 2 x=2− sin 2 x− sec 2 x

cos 2 x− tan 2 x=1− sin 2 x−(sec 2 x−1)=1− sin 2 x− sec 2 x+1=2− sin 2 x− sec 2 x

1 1+cos x − 1 1−cos(−x) =−2 cot x csc xcsc 2 x(1+ sin 2 x)= cot 2 x

(sec 2 (−x)− tan 2 x tan x)(2+2 tan x 2+2 cot x)−2 sin 2 x=cos 2xtan x sec x sin(−x)= cos 2 x

sec(−x) tan x+cot x =−sin(−x)1+sin x cos x = cos x 1+sin(−x)cos 2 θ− sin 2 θ 1− tan 2 θ = sin 2 θ

3 sin 2 θ+4 cos 2 θ=3+ cos 2 θ

3 sin 2 θ+4 cos 2 θ=3 sin 2 θ+3 cos 2 θ+ cos 2 θ=3(sin 2 θ+ cos 2 θ)+ cos 2 θ=3+ cos 2 θ

sec θ+tan θ cot θ+cos θ = sec 2 θ f(−x)=−f(x), f(−x)=f(x),cos(α+β)=cos α cos β−sin α sin β

cos(α−β)=cos α cos β+sin α sin β P α (cos α,sin α) Q β (cos β,sin β). POQ α−β. A (α−β) (cos(α−β),sin(α−β)); B (1,0). POQ AOB P Q A B.P Q

d PQ = (cos α−cos β) 2 + (sin α−sin β) 2 = cos 2 α−2 cos α cos β+ cos 2 β+ sin 2 α−2 sin α sin β+ sin 2 β

= (cos 2 α+ sin 2 α)+(cos 2 β+ sin 2 β)−2 cos α cos β−2 sin α sin β = 1+1−2 cos α cos β−2 sin α sin β = 2−2 cos α cos β−2 sin α sin β

A B.

d AB = (cos(α−β)−1) 2 + (sin(α−β)−0) 2 = cos 2 (α−β)−2 cos(α−β)+1+ sin 2 (α−β)

= (cos 2 (α−β)+ sin 2 (α−β))−2 cos(α−β)+1 = 1−2 cos(α−β)+1 = 2−2 cos(α−β)

2−2 cos α cos β−2 sin αsin β = 2−2 cos(α−β) 2−2 cos α cos β−2 sin αsin β=2−2 cos(α−β)

2 −2.

cos αcos β+sin αsin β=cos(α−β)

cos(α+β)=cos αcos β−sin αsin β

cos(α−β)=cos αcos β+sin αsin β

cos(5π 4 − π 6).

cos(α−β)=cos αcos β+sin αsin β cos(5π 4 − π 6)=cos(5π 4)cos(π 6)+sin(5π 4)sin(π 6)

=(− 2 2)(3 2)−(2 2)(1 2) =− 6 4 − 2 4 = − 6 − 2 4

cos(π 3 – π 4).2 + 6 4 cos(75 ∘). 75 ∘ = 45 ∘ + 30 ∘ , cos(75 ∘) cos(45 ∘ + 30 ∘).

cos(45 ∘ + 30 ∘)=cos(45 ∘)cos(30 ∘)–sin(45 ∘)sin(30 ∘) = 2 2 (3 2)– 2 2 (1 2)

= 6 4 – 2 4 = 6 – 2 4

cos(105 ∘).2 – 6 4

sin(α+β)=sin α cos β+cos α sin β

sin(α–β)=sin α cos β–cos α sin β

sin(45 ∘ – 30 ∘)sin(135 ∘ – 120 ∘)

sin(α–β)=sin α cos β–cos α sin β sin(45 ∘ – 30 ∘)=sin(45 ∘)cos(30 ∘)–cos(45 ∘)sin(30 ∘)

sin(45 ∘)= 2 2 , cos(30 ∘)= 3 2 , cos(45 ∘)= 2 2 , sin(30 ∘)= 1 2

sin(45 ∘ – 30 ∘)= 2 2 (3 2)– 2 2 (1 2) = 6 – 2 4

sin(α–β)=sin α cos β–cos α sin β sin(135 ∘ – 120 ∘)=sin(135 ∘)cos(120 ∘)–cos(135 ∘)sin(120 ∘)

sin(135 ∘)= 2 2 ,cos(120 ∘)=– 1 2 ,cos(135 ∘)= 2 2 ,sin(120 ∘)= 3 2

sin(135 ∘ – 120 ∘)= 2 2 (– 1 2)–(– 2 2)(3 2) = – 2 + 6 4 = 6 – 2 4

sin(135 ∘ – 120 ∘)= 2 2 (– 1 2)–(– 2 2)(3 2) = – 2 + 6 4 = 6 – 2 4

sin(cos –1 1 2 + sin –1 3 5).

sin(α+β). α= cos –1 1 2 β= sin –1 3 5 .

cos α= 1 2 ,0≤α≤π sin β= 3 5 ,– π 2 ≤β≤ π 2

sin α cos β.

sin α= 1– cos 2 α = 1– 1 4 = 3 4 = 3 2 cos β= 1– sin 2 β = 1– 9 25 = 16 25 = 4 5

sin(cos –1 1 2 + sin –1 3 5)=sin(α+β) =sin α cos β+cos α sin β

= 3 2 · 4 5 + 1 2 · 3 5 = 4 3 +3 10

tan x= sin x cos x ,cos x≠0.

tan(α+β)= sin(α+β) cos(α+β) = sin α cos β+cos α sin β cos α cos β–sin α sin β =

sin α cos β+cos α sin β cos α cos β cos α cos β–sin α sin β cos α cos β

Divide the numerator and denominator by cos α cos β = sin α cos β cos α cos β + cos α sin β cos α cos β cos α cos β – sin α sin β cos α cos β = sin α cos α + sin β cos β 1– sin α sin β cos α cos β = tan α+tan β 1–tan α tan β

tan(α+β)= tan α+tan β 1–tan α tan β

tan(α–β)= tan α–tan β 1+tan α tan β

tan(π 6 + π 4).

tan(α+β)= tan α+tan β 1–tan α tan β tan(π 6 + π 4)= tan(π 6)+tan(π 4) 1–(tan(π 6))(tan(π 4))

tan(π 6)= 1 3 ,tan(π 4)=1

tan(π 6 + π 4)= 1 3 +1 1–(1 3)(1) = 1+ 3 3 3 –1 3 = 1+ 3 3 (3 3 –1)

= 3 +1 3 –1

tan(2π 3 + π 4).1– 3 1+ 3 sin α= 3 5 ,0<α< π 2 ,cos β=– 5 13 ,π<β< 3π 2 ,sin(α+β)cos(α+β)

tan(α+β)tan(α–β) sin(α+β), sin α= 3 5 0<α< π 2 . α α a:

a 2 + 3 2 = 5 2 a 2 =16 a=4

cos β=– 5 13 π<β< 3π 2 , β −5, β

(–5) 2 + a 2 = 13 2 25+ a 2 =169 a 2 =144 a=±12

β a=–12. α β. α cos α= 4 5 . β sin β=– 12 13 . sin(α+β).

sin(α+β)=sin αcos β+cos αsin β =(3 5)(– 5 13)+(4 5)(– 12 13) =– 15 65 – 48 65

=– 63 65

cos(α+β)

cos(α+β)=cos α cos β–sin α sin β =(4 5)(– 5 13)–(3 5)(– 12 13) =– 20 65 + 36 65 = 16 65

tan(α+β), sin α= 3 5 cos α= 4 5 ,

$$\tan\alpha = 3\,5\;4\,5 = 3\,4$$

$\sin\beta = -\,12\,13$ $\cos\beta = -\,5\,13$,

$$\tan\beta = -12\,13\;-5\,13 = 12\,5$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{3\,4+12\,5}{1-3\,4\,(12\,5)} = \frac{63\,20-16\,20}{} = -\,63\,16$$

$\tan(\alpha-\beta)$,

$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} = \frac{3\,4-12\,5}{1+3\,4\,(12\,5)} = -\,33\,20\;56\,20 = -\,33\,56$$

α β

$$\tan(\alpha+\beta) = \sin(\alpha+\beta)\cos(\alpha+\beta)$$

$\pi\,2$, $\pi\,2$, θ, $(\pi\,2-\theta)$. $\sin\theta=\cos(\pi\,2-\theta)$: θ θ. $\sin\theta=\cos(\pi\,2-\theta)\cos\theta=\sin(\pi\,2-\theta)$
$\tan\theta=\cot(\pi\,2-\theta)\cot\theta=\tan(\pi\,2-\theta)\sec\theta=\csc(\pi\,2-\theta)\csc\theta=\sec(\pi\,2-\theta)$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta,$$

$$\cos(\pi\,2-\theta)=\cos\tfrac{\pi}{2}\cos\theta+\sin\tfrac{\pi}{2}\sin\theta = (0)\cos\theta+(1)\sin\theta = \sin\theta$$

$\tan\frac{\pi}{9}$ $\tan\theta=\cot(\pi\,2-\theta)$.

$$\tan(\pi\,9)=\cot(\pi\,2-\pi\,9) = \cot(9\pi\,18-2\pi\,18) = \cot(7\pi\,18)$$

$\sin\frac{\pi}{7}$ $\cos(5\pi\,14)$ $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$.

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \quad \sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta = 2\sin\alpha\cos\beta$$

$$\sin(\alpha-\beta)\cos\alpha\cos\beta = \tan\alpha-\tan\beta$$

$\sin(\alpha-\beta)\cos\alpha\cos\beta = \sin\alpha\cos\beta-\cos\alpha\sin\beta\cos\alpha\cos\beta = \sin\alpha\cos\beta\cos\alpha\cos\beta-\cos\alpha\sin\beta\cos\alpha\cos\beta$ Rewrite using a common denominator. $= \sin\alpha\cos\alpha-\sin\beta\cos\beta$ Cancel. $=\tan\alpha-\tan\beta$ Rewrite in terms of tangent.

$\tan(\pi-\theta)=-\tan\theta$.

$$\tan(\pi-\theta)=\frac{\tan(\pi)-\tan\theta}{1+\tan(\pi)\tan\theta} = \frac{0-\tan\theta}{1+0\cdot\tan\theta} = -\tan\theta$$

$L\,1$ $L\,2$ θ $L\,1$ $L\,2$.

$$\tan\theta = \frac{m\,2-m\,1}{1+m\,1\,m\,2}$$

$m\,1$ $m\,2$ $L\,1$ $L\,2$ $\tan\theta\,1 = m\,1$ $\tan\theta\,2 = m\,2$.

$$\tan\theta=\tan(\theta\,2-\theta\,1) = \frac{\tan\theta\,2-\tan\theta\,1}{1+\tan\theta\,1\tan\theta\,2} = \frac{m\,2-m\,1}{1+m\,1\,m\,2}$$

R S α $\tan\beta = 47\,50$, $\tan(\beta-\alpha) = 40\,50 = 4\,5$.

$$\tan(\beta-\alpha)=\frac{\tan\beta-\tan\alpha}{1+\tan\beta\tan\alpha}$$

$$4\,5 = \frac{47\,50-\tan\alpha}{1+47\,50\tan\alpha} \quad 4(1+47\,50\tan\alpha)=5(47\,50-\tan\alpha)$$

$$4(1)+4(47\,50)\tan\alpha=5(47\,50)-5\tan\alpha \quad 4+3.76\tan\alpha=4.7-5\tan\alpha \quad 5\tan\alpha+3.76\tan\alpha=0.7$$

$$8.76\tan\alpha=0.7 \quad \tan\alpha\approx 0.07991 \quad \tan^{-1}(0.07991)\approx .079741$$

$$\alpha\approx 0.079741\left(\frac{180}{\pi}\right)\approx 4.57^\circ$$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$
$\sin\theta=\cos(\pi\,2-\theta)\cos\theta=\sin(\pi\,2-\theta)\tan\theta=\cot(\pi\,2-\theta)\cot\theta=\tan(\pi\,2-\theta)\sec\theta=\csc(\pi\,2-\theta)\csc\theta=\sec(\pi\,2-\theta)$
x, $\pi\,2-x$. $\sin x=\cos(\pi\,2-x)$. $\cos(5\pi\,4)$? $f(x)=\sin(x)$ $g(x)=\cos(x)$. $0-x=-x\sin(-x)=-\sin x$, $\sin x$
$\cos(-x)=\cos(0-x)=\cos x$, $\cos x$ $\cos(7\pi\,12)\cos(\pi\,12)\,2+6\,4\sin(5\pi\,12)\sin(11\pi\,12)\,6-2\,4$
$\tan(-\pi\,12)\tan(19\pi\,12)-2-3\sin x\cos x.\sin(x+11\pi\,6)\sin(x-3\pi\,4)-2\,2\sin x-2\,2\cos x\cos(x-5\pi\,6)$
$\cos(x+2\pi\,3)-1\,2\cos x-3\,2\sin x\csc(\pi\,2-t)\sec(\pi\,2-\theta)\csc\theta\cot(\pi\,2-x)\tan(\pi\,2-x)\cot x$
$\sin(2x)\cos(5x)-\sin(5x)\cos(2x)\tan(3\,2\,x)-\tan(7\,5\,x)\,1+\tan(3\,2\,x)\tan(7\,5\,x)\tan(x\,10)$ $\sin a=2\,3$ $\cos b=-\,1\,4$, $a\;b\;[\pi\,2,\pi)$, $\sin(a+b)$ $\cos(a-b)$. $\sin a=4\,5$, $\cos b=1\,3$, $a\;b\;[0,\pi\,2)$, $\sin(a-b)$ $\cos(a+b)$.
$\sin(a-b)=(4\,5)(1\,3)-(3\,5)(2\,2\,3)=4-6\,2\,15\cos(a+b)=(3\,5)(1\,3)-(4\,5)(2\,2\,3)=3-8\,2\,15$
$\sin(\cos^{-1}(0)-\cos^{-1}(1\,2))\cos(\cos^{-1}(2\,2)+\sin^{-1}(3\,2))\,2-6\,4$

tan(sin −1 (1 2)− cos −1 (1 2))cos(π 2 −x)sinxsin(π−x)tan(π 3 +x)cot(π 6 −x)sin(π 3 +x)
tan(π 4 −x)cot(π 4 +x)cos(7π 6 +x)sin(π 4 +x)sinx 2 + cosx 2cos(5π 4 +x) 2x=x+x.
f(x)=sin(4x)−sin(3x)cos x,g(x)=sin x cos(3x)f(x)=cos(4x)+sin x sin(3x),g(x)=−cos x cos(3x)
f(x)=sin(3x)cos(6x),g(x)=−sin(3x)cos(6x) g(x)=sin(9x)−cos(3x)sin(6x).
f(x)=sin(4x),g(x)=sin(5x)cos x−cos(5x)sin xf(x)=sin(2x),g(x)=2 sin x cos x
f(θ)=cos(2θ),g(θ)= cos 2 θ− sin 2 θf(θ)=tan(2θ),g(θ)= tan θ 1+ tan 2 θ g(θ)= 2 tanθ 1− tan 2 θ .
f(x)=sin(3x)sin x,g(x)= sin 2 (2x) cos 2 x− cos 2 (2x) sin 2 x
f(x)=tan(−x),g(x)= tan x−tan(2x) 1−tan x tan(2x) g(x)= tanx−tan(2x) 1+tanxtan(2x) .sin(75 ◦)
sin(195 ◦)− 3 −1 2 2 , or −0.2588cos(165 ◦)cos(345 ◦)1+ 3 2 2 ,tan(− 15 ◦)
tan(x+ π 4)= tan x+1 1−tan x
tan(x+ π 4)= tanx+tan(π 4) 1−tanxtan(π 4) = tanx+1 1−tanx(1) = tanx+1 1−tanx
tan(a+b) tan(a−b) = sin a cos a+sin b cos b sin a cos a−sin b cos bcos(a+b) cos a cos b =1−tan a tan b
cos(a+b) cosacosb = cosacosb cosacosb − sinasinb cosacosb =1−tanatanb
cos(x+y)cos(x−y)= cos 2 x− sin 2 ycos(x+h)−cos x h =cos x cos h−1 h −sin x sin h h
cos(x+h)−cosx h = cosxcosh−sinxsinh−cosx h = cosx(cosh−1)−sinxsinh h =cosx cosh−1 h −sinx sinh h
tan(u+v)= tan u+tan v 1−tan u tan vtan(u−v)= tan u−tan v 1+tan u tan v
tan(x+y) 1+tan x tan x = tan x+tan y 1− tan 2 x tan 2 y α, β, γ sin(α+β)=sin γ. sin(α+β)=sin(π−γ)
α,β, y tan α+tan β+tan γ=tan α tan β tan γ θ tan θ= 5 3 . α=β.

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

α=β=θ,

$$\sin(\theta+\theta)=\sin\theta\cos\theta+\cos\theta\sin\theta \qquad \sin(2\theta)=2\sin\theta\cos\theta$$

cos(α+β)=cos α cos β−sin α sin β, α=β=θ,

$$\cos(\theta+\theta)=\cos\theta\cos\theta-\sin\theta\sin\theta \qquad \cos(2\theta)=\cos^2\theta-\sin^2\theta$$

$$\cos(2\theta)=\cos^2\theta-\sin^2\theta=(1-\sin^2\theta)-\sin^2\theta=1-2\sin^2\theta$$

$$\cos(2\theta)=\cos^2\theta-\sin^2\theta=\cos^2\theta-(1-\cos^2\theta)=2\cos^2\theta-1$$

α=β=θ

tan(α+β)= tan α+tan β 1−tan α tan β tan(θ+θ)= tan θ+tan θ 1−tan θ tan θ tan(2θ)= 2tan θ 1− tan 2 θ

$$\sin(2\theta)=2\sin\theta\cos\theta$$

$$\cos(2\theta)=\cos^2\theta-\sin^2\theta=1-2\sin^2\theta=2\cos^2\theta-1$$

$$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$$

tan θ=− 3 4 θ sin(2θ)cos(2θ)tan(2θ) tan θ=− 3 4 , θ θ

$$(-4)^2+(3)^2=c^2 \quad 16+9=c^2 \quad 25=c^2 \quad c=5$$

$$\sin(2\theta)=2\sin\theta\cos\theta$$

sin θ cos θ. sin θ= 3 5 ,cos θ=− 4 5 .

$$\sin(2\theta)=2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)=-\frac{24}{25}$$

$$\cos(2\theta)=\cos^2\theta-\sin^2\theta$$

$$\cos(2\theta)=\left(-\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2=\frac{16}{25}-\frac{9}{25}=\frac{7}{25}$$

$$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$$

tan θ=− 3 4 .

$$\tan(2\theta)=\frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}=\frac{-\frac{3}{2}}{1-\frac{9}{16}}=-\frac{3}{2}\left(\frac{16}{7}\right)=-\frac{24}{7}$$

sin α= 5 8 , θ cos(2α).cos(2α)= 7 32 cos(6x) cos(3x).

$$\cos(6x)=\cos(3x+3x)=\cos 3x\cos 3x-\sin 3x\sin 3x=\cos^2 3x-\sin^2 3x$$

$$1+\sin(2\theta)=(\sin\theta+\cos\theta)^2$$

$$(\sin\theta+\cos\theta)^2=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta=(\sin^2\theta+\cos^2\theta)+2\sin\theta\cos\theta$$

$$=1+2\sin\theta\cos\theta=1+\sin(2\theta)$$

$$(a\pm b)^2=a^2\pm 2ab+b^2$$

a=sin θ b=cos θ. cos 4 θ− sin 4 θ=cos(2θ).
cos 4 θ− sin 4 θ=(cos 2 θ+ sin 2 θ)(cos 2 θ− sin 2 θ)=cos(2θ)

$$\tan(2\theta)=\frac{2\cot\theta-\tan\theta}{}$$

$\tan(2\theta)=2\tan\theta\,1-\tan^2\theta$ Double-angle formula $=\frac{2\tan\theta}{(1\tan\theta)(1-\tan^2\theta)(1\tan\theta)}$ Multiply by a term that results in desired numerator. $=\frac{2\,1\tan\theta-\tan^2\theta\tan\theta}{}=2\cot\theta-\tan\theta$ Use reciprocal identity for $1\tan\theta$.

$$\frac{2\tan\theta\,1-\tan^2\theta}{}=2\cot\theta-\tan\theta$$

$2\cot\theta-\tan\theta=\frac{2\,1\tan\theta-\tan\theta(\tan\theta\tan\theta)}{}=\frac{2\tan\theta\,1\tan\theta(\tan\theta)-\tan\theta(\tan\theta)}{}$

$$=2\tan\theta\,1-\tan^2\theta$$

$\cos(2\theta)\cos\theta=\cos^3\theta-\cos\theta\sin^2\theta.\cos(2\theta)\cos\theta=(\cos^2\theta-\sin^2\theta)\cos\theta=\cos^3\theta-\cos\theta\sin^2\theta$

$\cos(2\theta)=1-2\sin^2\theta.\ \sin^2\theta$:

$$\cos(2\theta)=1-2\sin^2\theta\ \ 2\sin^2\theta=1-\cos(2\theta)\quad\sin^2\theta=\frac{1-\cos(2\theta)}{2}$$

$\cos(2\theta)=2\cos^2\theta-1.\ \cos^2\theta$:

$$\cos(2\theta)=2\cos^2\theta-1\ \ 1+\cos(2\theta)=2\cos^2\theta\ \frac{1+\cos(2\theta)}{2}=\cos^2\theta$$

$\tan^2\theta=\frac{\sin^2\theta}{\cos^2\theta}=\frac{1-\cos(2\theta)}{2}\frac{1+\cos(2\theta)}{2}$ Substitute the reduction formulas. $=\frac{(1-\cos(2\theta)}{2})(2\,1+\cos(2\theta))}{}=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

$$\sin^2\theta=\frac{1-\cos(2\theta)}{2}$$

$$\cos^2\theta=\frac{1+\cos(2\theta)}{2}$$

$$\tan^2\theta=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

$\cos 4x$

$\cos 4x=(\cos 2x)^2=\frac{(1+\cos(2x)}{2})^2$ Substitute reduction formula for $\cos 2x$. $=\frac{1}{4}(1+2\cos(2x)+\cos^2(2x))=\frac{1}{4}+\frac{1}{2}\cos(2x)+\frac{1}{4}(\frac{1+\cos 2(2x)}{2})$ Substitute reduction formula for $\cos 2x$. $=\frac{1}{4}+\frac{1}{2}\cos(2x)+\frac{1}{8}+\frac{1}{8}\cos(4x)=\frac{3}{8}+\frac{1}{2}\cos(2x)+\frac{1}{8}\cos(4x)$

$$\sin^3(2x)=[\frac{1}{2}\sin(2x)][1-\cos(4x)]$$

$\sin^3(2x)=[\sin(2x)][\sin^2(2x)]=\sin(2x)[\frac{1-\cos(4x)}{2}]$ Substitute the power-reduction formula. $=\sin(2x)(\frac{1}{2})[1-\cos(4x)]=\frac{1}{2}[\sin(2x)][1-\cos(4x)]$

$$\frac{1-\cos(4x)}{2}$$

$\sin^2(2x)$.

$$\sin^2\theta=\frac{1-\cos(2\theta)}{2}$$

$\theta=2x, 2\theta=4x$. 10 $\cos^4 x=\frac{15}{4}+5\cos(2x)+\frac{5}{4}\cos(4x)$.

$10\cos^4 x=10\cos^4 x=10(\cos^2 x)^2=10[\frac{1+\cos(2x)}{2}]^2$ Substitute reduction formula for $\cos^2 x$. $=\frac{10}{4}[1+2\cos(2x)+\cos^2(2x)]=\frac{10}{4}+\frac{10}{2}\cos(2x)+\frac{10}{4}(\frac{1+\cos 2(2x)}{2})$ Substitute reduction formula for $\cos 2x$. $=\frac{10}{4}+\frac{10}{2}\cos(2x)+\frac{10}{8}+\frac{10}{8}\cos(4x)=\frac{30}{8}+5\cos(2x)+\frac{10}{8}\cos(4x)=\frac{15}{4}+5\cos(2x)+\frac{5}{4}\cos(4x)$

$\theta\ \frac{\alpha}{2}$, $\sin(\frac{\alpha}{2})$. $\pm\ \frac{\alpha}{2}$

$\sin^2\theta=\frac{1-\cos(2\theta)}{2}\ \sin^2(\frac{\alpha}{2})=\frac{1-(\cos 2\cdot\frac{\alpha}{2})}{2}=\frac{1-\cos\alpha}{2}\quad\sin(\frac{\alpha}{2})=\pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\cos^2\theta=\frac{1+\cos(2\theta)}{2}\ \cos^2(\frac{\alpha}{2})=\frac{1+\cos(2\cdot\frac{\alpha}{2})}{2}=\frac{1+\cos\alpha}{2}\quad\cos(\frac{\alpha}{2})=\pm\sqrt{\frac{1+\cos\alpha}{2}}$

$\tan^2\theta=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}\ \tan^2(\frac{\alpha}{2})=\frac{1-\cos(2\cdot\frac{\alpha}{2})}{1+\cos(2\cdot\frac{\alpha}{2})}=\frac{1-\cos\alpha}{1+\cos\alpha}$

$$\tan(\frac{\alpha}{2})=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\sin(\frac{\alpha}{2})=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos(\frac{\alpha}{2})=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$\tan(\frac{\alpha}{2})=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}$

$\sin(15^\circ)\ 15^\circ=\frac{30^\circ}{2}$,

$\sin\ 30^\circ 2=\sqrt{\frac{1-\cos 30^\circ}{2}}=\sqrt{\frac{1-\frac{3}{2}}{2}}=\sqrt{\frac{2-\frac{3}{2}}{2}}=\sqrt{\frac{2-\frac{3}{4}}{}}=\frac{\sqrt{2-\frac{3}{2}}}{}$

$\sin(15^o)\ \tan\alpha=\frac{8}{15}\alpha\sin(\frac{\alpha}{2})\cos(\frac{\alpha}{2})\tan(\frac{\alpha}{2})\ \sin\alpha=-\frac{8}{17}\ \cos\alpha=-\frac{15}{17}$. $\alpha\ 180°<\alpha<270°$, $180°\ 2<\alpha\ 2<270°\ 2$. $\alpha\ 2\ 90°<\alpha\ 2<135°$. $\sin\ \alpha\ 2$,

$\sin\ \alpha\ 2=\pm\sqrt{\frac{1-\cos\alpha}{2}}=\pm\sqrt{\frac{1-(-\frac{15}{17})}{2}}=\pm\sqrt{\frac{32}{17}\ 2}=\pm\sqrt{\frac{32}{17}\cdot\frac{1}{2}}=\pm\sqrt{\frac{16}{17}}=\pm\frac{4}{\sqrt{17}}=\frac{4\sqrt{17}}{17}$

$\sin\ \alpha\ 2\ \cos\ \alpha\ 2$,

$\cos\ \alpha 2 = \pm\sqrt{\frac{1+\cos\alpha}{2}} = \pm\sqrt{\frac{1+(-15\,17)}{2}} = \pm\sqrt{\frac{2\,17}{2}} = \pm\sqrt{\frac{2\,17\cdot 1}{2}} = \pm\frac{1}{17} = -\frac{17}{17}$

$\cos\ \alpha 2\ \tan\ \alpha 2$,

$\tan\ \alpha 2 = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \pm\sqrt{\frac{1-(-15\,17)}{1+(-15\,17)}} = \pm\sqrt{\frac{32\,17}{2\,17}} = \pm\sqrt{\frac{32}{2}} = -16 = -4$

$\tan\ \alpha 2\quad \alpha 2\ \sin\alpha = -4\,5\ \ \alpha\ \cos(\alpha 2).-2\,5\ \theta\ \tan\theta = 5\,3\ \ \tan\theta = 5\,3\ \ \cos\theta$

$3\,2 + 5\,2 = 34 \qquad c = 34$

$\cos\theta = 3\,34 = 3\,34\,34$. $\tan\ \theta 2 = \frac{1-\cos\theta}{1+\cos\theta}$. $\tan\theta\ \tan\ \theta 2$.

$\tan\ \theta 2 = \frac{1-3\,34\,34}{1+3\,34\,34} = \frac{34-3\,34\,34}{34+3\,34\,34} = \frac{34-3\,34}{34+3\,34} \approx 0.57$

$\tan^{-1}(0.57) \approx 29.7^\circ$. $\approx 29.7^\circ$.

$\sin(2\theta) = 2\sin\theta\cos\theta \quad \cos(2\theta) = \cos^2\theta - \sin^2\theta = 1-2\sin^2\theta = 2\cos^2\theta - 1 \quad \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2\theta = \frac{1-\cos(2\theta)}{2} \quad \cos^2\theta = \frac{1+\cos(2\theta)}{2} \quad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

$\sin\ \alpha 2 = \pm\sqrt{\frac{1-\cos\alpha}{2}} \quad \cos\ \alpha 2 = \pm\sqrt{\frac{1+\cos\alpha}{2}} \quad \tan\ \alpha 2 = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$

$\cos(2x) = \cos^2 x - \sin^2 x$. $\tan(2x)\ \cos(2x)\ \sin(2x)$. $\tan(x\,2) = \frac{1-\cos x}{1+\cos x}\ \ \sin(x\,2)\ \ \cos(x\,2)$.

$\tan(x\,2)\ \ \frac{1-\cos x}{\sin x}$, $\frac{\sin x}{1+\cos x}$, $\frac{1-\cos x}{\quad 1+\cos x}$, $\tan(x\,2), \cos x\ \ \sin(2x), \cos(2x), \tan(2x)$

$x.\ \sin x = 1\,8$, $x\ \ 3\,7\,32\quad 31\,32\quad 3\,7\,31\ \cos x = 2\,3$, $x\ \ \cos x = -1\,2$, $x\quad 3\,2\ \ -1\,2\ \ -3\ \ \tan x = -8$, x

$\cos(2\theta) = 3\,5\quad 90^\circ \le\theta\le 180^\circ \cos\theta = -2\,5\,5$,$\sin\theta = 5\,5$,$\tan\theta = -1\,2$,$\csc\theta = 5$,$\sec\theta = -5\,2$,$\cot\theta = -2$

$\cos(2\theta) = 1\,2\quad 180^\circ \le\theta\le 270^\circ 2\sin(\pi 4)\ 2\cos(\pi 4)\ 2\sin(\pi 2)4\sin(\pi 8)\ \cos(\pi 8)\ \sin(\pi 8)\ 2-2\ 2$

$\cos(-11\pi 12)\sin(11\pi 12)2-3\ 2\cos(7\pi 8)\tan(5\pi 12)2+3\tan(-3\pi 12)\tan(-3\pi 8)-1-2\sin(x\,2)$,

$\cos(x\,2), \tan(x\,2)\ x.\ \tan x = -4\,3$, $x\ \ \sin x = -12\,13$, $x\ \ 3\,13\,13\ \ -2\,13\,13\ \ -3\,2\ \ \csc x = 7, x\ \ \sec x = -4$,

$x\ \ 10\,4\quad 6\,4\quad 15\,3\ \ \sin(2\theta), \cos(2\theta), \tan(2\theta).\ \sin(2\alpha), \cos(2\alpha), \tan(2\alpha).120\,169$,$-119\,169$,$-120\,119$

$\sin(\theta 2), \cos(\theta 2), \tan(\theta 2).\ \sin(\alpha 2), \cos(\alpha 2), \tan(\alpha 2).2\,13\,13$, $3\,13\,13$, $2\,3$

$\cos^2(28^\circ) - \sin^2(28^\circ)2\cos^2(37^\circ) - 1\cos(74^\circ)1 - 2\ \sin^2(17^\circ)\cos^2(9x) - \sin^2(9x)\cos(18x)$

$4\sin(8x)\cos(8x)6\sin(5x)\cos(5x)3\sin(10x)(\sin t - \cos t)^2 = 1-\sin(2t)\sin(2x) = -2\sin(-x)\cos(-x)$

$-2\sin(-x)\cos(-x) = -2(-\sin(x)\cos(x)) = \sin(2x)\cot x - \tan x = 2\cot(2x)$

$\sin(2\theta)\ 1+\cos(2\theta)\ \tan^2\theta = \tan\theta$

$\sin(2\theta)\ 1+\cos(2\theta)\ \tan^2\theta = 2\sin(\theta)\cos(\theta)\ 1+\ \cos^2\theta - \sin^2\theta\ \tan^2\theta = 2\sin(\theta)\cos(\theta)\ 2\cos^2\theta\ \tan^2\theta = \sin(\theta)\ \cos\theta\ \tan^2\theta = \cot(\theta)\ \tan^2\theta = \tan\theta$

$\cos^2(5x)\cos^2(6x)1+\cos(12x)\ 2\sin^4(8x)\sin^4(3x)3+\cos(12x)-4\cos(6x)\ 8\cos^2 x\ \sin^4 x\cos^4 x\ \sin^2 x$

$2+\cos(2x)-2\cos(4x)-\cos(6x)\ 32\tan^2 x\ \sin^2 x\tan^4 x3+\cos(4x)-4\cos(2x)\ 3+\cos(4x)+4\cos(2x)\sin^2(2x)$

$\sin^2 x\ \cos^2 x1-\cos(4x)\ 8\tan^2 x\ \sin x\tan^4 x\ \cos^2 x3+\cos(4x)-4\cos(2x)\ 4(\cos(2x)+1)\cos^2 x\ \sin(2x)$

$\cos^2(2x)\sin x(1+\cos(4x))\sin x\ 2\tan^2(x\,2)\ \sin x\ \sin x\ \cos x, \sin(4x)4\sin x\cos x(\cos^2 x - \sin^2 x)$

$\cos(4x)\sin(2x) = 2\tan x\ 1+\tan^2 x$

$2\tan x\ 1+\tan^2 x = 2\sin x\cos x\ 1+\sin^2 x\cos^2 x = 2\sin x\cos x\cos^2 x + \sin^2 x\cos^2 x =$

$2\sin x\ \cos x\ .\ \cos^2 x\ 1 = 2\sin x\cos x = \sin(2x)\cos(2\alpha) = 1-\tan^2\alpha\ 1+\tan^2\alpha\tan(2x) = 2\sin x\cos x\ 2\cos^2 x - 1$

$2\sin x\cos x\ 2\cos^2 x - 1 = \sin(2x)\ \cos(2x) = \tan(2x)(\sin^2 x - 1)\ 2 = \cos(2x) + \sin 4\ x$

$\sin(3x) = 3\sin x\ \ \cos^2 x - \sin^3 x$

$\sin(x+2x) = \sin x\cos(2x) + \sin(2x)\cos x = \sin x(\cos^2 x - \sin^2 x) + 2\sin x\cos x\cos x$

$= \sin x\cos^2 x - \sin^3 x + 2\sin x\cos^2 x \qquad = 3\sin x\cos^2 x - \sin^3 x$

$\cos(3x) = \cos^3 x - 3\sin^2 x\cos x1+\cos(2t)\ \sin(2t) - \cos t = 2\cos t\ 2\sin t - 1$

$1+\cos(2t)\ \sin(2t) - \cos t = 1+2\cos^2 t - 1\ 2\sin t\cos t - \cos t \qquad = 2\cos^2 t\ \cos t(2\sin t - 1)$

$= 2\cos t\ 2\sin t - 1$

$\sin(16x) = 16\sin x\cos x\cos(2x)\cos(4x)\cos(8x)$

$\cos(16x) = (\cos^2(4x) - \sin^2(4x) - \sin(8x))(\cos^2(4x) - \sin^2(4x) + \sin(8x))$

(cos 2 (4x)− sin 2 (4x)−sin(8x))(cos 2 (4x)− sin 2 (4x)+sin(8x))=

=(cos(8x)−sin(8x))(cos(8x)+sin(8x))

= cos 2 (8x)− sin 2 (8x)

=cos(16x)

cos α cos β+sin α sin β=cos(α−β) + cos α cos β−sin α sin β=cos(α+β)

_____ 2 cos α cos β=cos(α−β)+cos(α+β)

2

cos α cos β= 1 2 [cos(α−β)+cos(α+β)]

2 cos(7x 2) cos 3x 2 .

cos α cos β= 1 2 [cos(α−β)+cos(α+β)]

2 cos(7x 2)cos(3x 2)=(2)(1 2)[cos(7x 2 − 3x 2)+cos(7x 2 + 3x 2)] =[cos(4x 2)+cos(10x 2)] =cos 2x+cos 5x

cos(2θ)cos(4θ).1 2 (cos6θ+cos2θ)

sin(α+β)=sin α cos β+cos α sin β + sin(α−β)=sin α cos β−cos α sin β

_____ sin(α+β)+sin(α−β)=2 sin α cos β

sin α cos β= 1 2 [sin(α+β)+sin(α−β)]

sin(4θ)cos(2θ).

sin α cos β= 1 2 [sin(α+β)+sin(α−β)] sin(4θ)cos(2θ)= 1 2 [sin(4θ+2θ)+sin(4θ−2θ)]

= 1 2 [sin(6θ)+sin(2θ)]

sin(x+y)cos(x−y).1 2 (sin2x+sin2y)

cos(α−β)=cos α cos β+sin α sin β − cos(α+β)=−(cos α cos β−sin α sin β)

_____ cos(α−β)−cos(α+β)=2 sin α sin β

sin α sin β= 1 2 [cos(α−β)−cos(α+β)]

cos α cos β= 1 2 [cos(α−β)+cos(α+β)]

sin α cos β= 1 2 [sin(α+β)+sin(α−β)]

sin α sin β= 1 2 [cos(α−β)−cos(α+β)]

cos α sin β= 1 2 [sin(α+β)−sin(α−β)]

cos(3θ) cos(5θ)

cos α cos β= 1 2 [cos(α−β)+cos(α+β)] cos(3θ)cos(5θ)= 1 2 [cos(3θ−5θ)+cos(3θ+5θ)]

= 1 2 [cos(2θ)+cos(8θ)] Use even-odd identity.

cos 11π 12 cos π 12 .−2− 3 4 u+v 2 =α u−v 2 =β.

α+β= u+v 2 + u−v 2 = 2u 2 =u α−β= u+v 2 − u−v 2 = 2v 2 =v

α β

sin α cos β= 1 2 [sin(α+β)+sin(α−β)] sin(u+v 2)cos(u−v 2)= 1 2 [sin u+sin v]

Substitute for(α+β) and (α−β) 2 sin(u+v 2)cos(u−v 2)=sin u+sin v

sin α+sin β=2sin(α+β 2)cos(α−β 2)

sin α−sin β=2sin(α−β 2)cos(α+β 2)

cos α−cos β=−2sin(α+β 2)sin(α−β 2)

cos α+cos β=2cos(α+β 2)cos(α−β 2)

sin(4θ)−sin(2θ).

sin α−sin β=2sin(α−β 2)cos(α+β 2)

sin(4θ)−sin(2θ)=2sin(4θ−2θ 2) cos(4θ+2θ 2) =2sin(2θ 2) cos(6θ 2)

=2 sin θ cos(3θ)

sin(3θ)+sin(θ).2sin(2θ)cos(θ) cos(15 ∘)−cos(75 ∘).

cos α−cos β=−2 sin(α+β 2) sin(α−β 2)

cos(15 ∘)−cos(75 ∘)=−2sin(15 ∘ + 75 ∘ 2) sin(15 ∘ − 75 ∘ 2) =−2sin(45 ∘) sin(− 30 ∘) =−2(2 2)(− 1 2) = 2 2

cos(4t)−cos(2t) sin(4t)+sin(2t) =−tan t

$$\cos(4t)-\cos(2t)\ \sin(4t)+\sin(2t) = -2\sin(\tfrac{4t+2t}{2})\sin(\tfrac{4t-2t}{2})\ 2\sin(\tfrac{4t+2t}{2})\cos(\tfrac{4t-2t}{2}) = -2\sin(3t)\sin t\ 2\sin(3t)\cos t = -2\sin(3t)\sin t\ 2\sin(3t)\cos t = -\sin t\cos t = -\tan t$$

$\csc 2\theta - 2 = \cos(2\theta)\ \sin 2\theta$.

$$\cos(2\theta)\ \sin 2\theta = 1-2\ \sin 2\theta\ \sin 2\theta = 1\ \sin 2\theta - 2\ \sin 2\theta\ \sin 2\theta = \csc 2\theta - 2$$

$\tan\theta\cot\theta - \cos 2\theta = \sin 2\theta$.

$$\tan\theta\cot\theta - \cos 2\theta = (\sin\theta\cos\theta)(\cos\theta\sin\theta) - \cos 2\theta = 1-\cos 2\theta = \sin 2\theta$$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$ $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$ $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) -\cos(\alpha+\beta)]$ $\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$

$\sin\alpha+\sin\beta = 2\sin(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$ $\sin\alpha-\sin\beta = 2\sin(\frac{\alpha-\beta}{2})\cos(\frac{\alpha+\beta}{2})$ $\cos\alpha-\cos\beta = -2\sin(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$ $\cos\alpha+\cos\beta = 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$, $\cos\alpha\sin\beta$. $\alpha\ \ \beta\ \ \cos(195°)\cos(105°)$, $\sin(3x)+\sin x\cos x = 1$.

$2\sin(2x)\cos x\cos x = 116\sin(16x)\sin(11x)8(\cos(5x)-\cos(27x))20\cos(36t)\cos(6t)$

$2\sin(5x)\cos(3x)\sin(2x)+\sin(8x)10\cos(5x)\sin(10x)\sin(-x)\sin(5x)\frac{1}{2}(\cos(6x)-\cos(4x))$

$\sin(3x)\cos(5x)\cos(6t)+\cos(4t)2\cos(5t)\cos t\sin(3x)+\sin(7x)\cos(7x)+\cos(-7x)2\cos(7x)$

$\sin(3x)-\sin(-3x)\cos(3x)+\cos(9x)2\cos(6x)\cos(3x)\sin h-\sin(3h)\cos(45°)\cos(15°)\frac{1}{4}(1+\sqrt{3})$

$\cos(45°)\sin(15°)\sin(-345°)\sin(-15°)\frac{1}{4}(\sqrt{3}-2)\sin(195°)\cos(15°)\sin(-45°)\sin(-15°)\frac{1}{4}(\sqrt{3}-1)$

$\cos(23°)\sin(17°)2\sin(100°)\sin(20°)\cos(80°)-\cos(120°)2\sin(-100°)\sin(-20°)\sin(213°)\cos(8°)$

$\frac{1}{2}(\sin(221°)+\sin(205°))2\cos(56°)\cos(47°)\sin(76°)+\sin(14°)2\ \cos(31°)\cos(58°)-\cos(12°)$

$\sin(101°)-\sin(32°)2\cos(66.5°)\sin(34.5°)\cos(100°)+\cos(200°)\sin(-1°)+\sin(-2°)2\sin(-1.5°)\cos(0.5°)$

$\cos(a+b)\cos(a-b) = \frac{1-\tan a\tan b}{1+\tan a\tan b}4\sin(3x)\cos(4x) = 2\sin(7x)-2\sin x$

$$2\sin(7x)-2\sin x = 2\sin(4x+3x)-2\sin(4x-3x) = 2(\sin(4x)\cos(3x)+\sin(3x)\cos(4x))-2(\sin(4x)\cos(3x) -\sin(3x)\cos(4x)) = 2\sin(4x)\cos(3x)+2\sin(3x)\cos(4x))-2\sin(4x)\cos(3x)+2\sin(3x)\cos(4x)) = 4\sin(3x)\cos(4x)$$

$6\cos(8x)\sin(2x)\sin(-6x) = -3\sin(10x)\csc(6x)+3\sin x+\sin(3x) = 4\sin x\ \cos 2x$

$\sin x+\sin(3x) = 2\sin(\frac{4x}{2})\cos(\frac{-2x}{2}) = 2\sin(2x)\cos x = 2(2\sin x\cos x)\cos x = 4\sin x\cos 2x$

$2(\cos 3x-\cos x\ \sin 2x) = \cos(3x)+\cos x2\tan x\cos(3x) = \sec x(\sin(4x)-\sin(2x))$

$2\tan x\cos(3x) = 2\sin x\cos(3x)\ \cos x = 2(.5(\sin(4x)-\sin(2x)))\ \cos x = 1\ \cos x\ (\sin(4x)-\sin(2x)) = \sec x(\sin(4x)-\sin(2x))\cos(a+b)+\cos(a-b) = 2\cos a\cos b$

$\cos(58°)+\cos(12°)2\cos(35°)\cos(23°)$, $1.5081\sin(2°)-\sin(3°)\cos(44°)-\cos(22°)$

$-2\sin(33°)\sin(11°)$, $-0.2078\cos(176°)\sin(9°)\sin(-14°)\sin(85°)$

$\frac{1}{2}(\cos(99°)-\cos(71°))$, $-0.24102\ \sin(2x)\sin(3x) = \cos x-\cos(5x)$

$\cos(10\theta)+\cos(6\theta)\ \cos(6\theta)-\cos(10\theta) = \cot(2\theta)\cot(8\theta)\sin(3x)-\sin(5x)\ \cos(3x)+\cos(5x) = \tan x$

$2\cos(2x)\cos x+\sin(2x)\sin x = 2\sin x\ 2\ \cos 3x\ \sin(2x)+\sin(4x)\ \sin(2x)-\sin(4x) = -\tan(3x)\cot x$

$\sin(9t)-\sin(3t)\ \cos(9t)+\cos(3t)\tan(3t)2\sin(8x)\cos(6x)-\sin(2x)\sin(3x)-\sin x\ \sin x2\cos(2x)$

$\cos(5x)+\cos(3x)\ \sin(5x)+\sin(3x)\sin x\ \cos(15x)-\cos x\ \sin(15x)-\sin(14x)$

$\sin x-\sin y = 2\sin(\frac{x-y}{2})\cos(\frac{x+y}{2})\cos x+\cos y = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})\ \cos x+\cos y.\ x=\alpha+\beta\ \ y=\alpha-\beta,$

$\cos x+\cos y\cos(\alpha+\beta)+\cos(\alpha-\beta) = \cos\alpha\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\sin\beta = 2\cos\alpha\cos\beta\ x=\alpha+\beta\ \ y=\alpha-\beta,\ \alpha\ \beta\ 2\cos\alpha\cos\beta 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2}).\frac{\sin(6x)+\sin(4x)}{\sin(6x)-\sin(4x)} = \tan(5x)\cot x$

$\cos(3x)+\cos x\ \cos(3x)-\cos x = -\cot(2x)\cot x$

$\cos(3x)+\cos x\ \cos(3x)-\cos x = 2\cos(2x)\cos x - 2\sin(2x)\sin x = -\cot(2x)\cot x$

$\cos(6y)+\cos(8y)\ \sin(6y)-\sin(4y) = \cot y\ \cos(7y)\sec(5y)\cos(2y)-\cos(4y)\ \sin(2y)+\sin(4y) = \tan y$

$\cos(2y)-\cos(4y)\ \sin(2y)+\sin(4y) = -2\sin(3y)\sin(-y)\ 2\sin(3y)\cos y = 2\sin(3y)\sin(y)\ 2\sin(3y)\cos y = \tan y$

$\sin(10x)-\sin(2x)\ \cos(10x)+\cos(2x) = \tan(4x)\cos x-\cos(3x) = 4\ \sin 2x\cos x$

$\cos x-\cos(3x) = -2\sin(2x)\sin(-x) = 2(2\sin x\cos x)\sin x = 4\ \sin 2\ x\cos x$

$(\cos(2x)-\cos(4x))\ 2 + (\sin(4x)+\sin(2x))\ 2 = 4\ \sin 2(3x)\tan(\frac{\pi}{4}-t) = \frac{1-\tan t}{1+\tan t}$

tan(π 4 −t)= tan(π 4)−tant 1+tan(π 4)tan(t) = 1−tant 1+tant 2π. 2π 2πk, k 2π:

$$\sin\theta=\sin(\theta\pm 2k\pi)$$

cos θ= 1 2 .

$$\cos\theta=\frac{1}{2}\qquad \theta=\frac{\pi}{3},\frac{5\pi}{3}$$

[0,2π].

$$\frac{\pi}{3}\pm 2k\pi \text{ and } \frac{5\pi}{3}\pm 2k\pi$$

k sin t= 1 2 . 2π. π 6 5π 6 .

$$\frac{\pi}{6}\pm 2\pi k \text{ and } \frac{5\pi}{6}\pm 2\pi k$$

k x u. 2 cos θ−3=−5,0≤θ<2π.

$$2\cos\theta-3=-5 \qquad 2\cos\theta=-2 \qquad \cos\theta=-1 \qquad \theta=\pi$$

[0,2π): 2 sin x+1=0.x= 7π 6 , 11π 6π,2π. π 2 , 2 sin 2 θ−1=0,0≤θ<2π. sinθ.

2 sin 2 θ−1=0 2 sin 2 θ=1 sin 2 θ= 1 2 sin 2 θ =± 1 2 sin θ=± 1 2 =± 2 2 θ=

$$\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$$

csc θ=−2,0≤θ<4π. θ csc θ=−2 0≤θ<4π.

$$\csc\theta=-2 \quad 1\sin\theta=-2 \quad \sin\theta=-\frac{1}{2} \qquad \theta=\frac{7\pi}{6},\frac{11\pi}{6},\frac{19\pi}{6},\frac{23\pi}{6}$$

sin θ=− 1 2 , tan(θ− π 2)=1,0≤θ<2π. π. [0,π), π 4 , (θ− π 2). tan(π 4)=1,

$$\theta-\frac{\pi}{2}=\frac{\pi}{4} \qquad \theta=\frac{3\pi}{4}\pm k\pi$$

[0,2π),

$$\frac{3\pi}{4} \text{ and } \frac{3\pi}{4}+\pi=\frac{7\pi}{4}$$

tan x= 3 .π 3 ±πk 2(tan x+3)=5+tan x,0≤x<2π. tan x

$$2(\tan x)+2(3)=5+\tan x \quad 2\tan x+6=5+\tan x \quad 2\tan x-\tan x=5-6 \quad \tan x=-1$$

−1:θ= 3π 4 θ= 7π 4 . sin θ=0.8, θ θ, sin −1 sin −1 (. sin −1 (0.8),

$$\sin^{-1}(0.8)\approx 0.9273$$

$$0.9273\pm 2\pi k$$

$$\theta\approx 53.1^\circ \quad \theta\approx 180^\circ-53.1^\circ \approx 126.9^\circ$$

π−θ. sec θ=−4,

$$\sec\theta=-4 \quad 1\cos\theta=-4 \quad \cos\theta=-\frac{1}{4}$$

$$\cos^{-1}\left(-\frac{1}{4}\right)\approx 1.8235 \qquad \theta\approx 1.8235+2\pi k$$

π 2 ≈1.57 π≈3.14, θ≈1.8235 θ '≈π−1.8235≈1.3181. π+1.3181≈4.4597. 1.8235±2πk 4.4597±2πk.

cos θ=−0.2.θ≈1.7722±2πk θ≈4.5110±2πk x u. cos 2 θ+3 cos θ−1=0,0≤θ<2π.θ x. cos θ=x.

$$x^2+3x-1=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

$$x=\frac{-3\pm\sqrt{(-3)^2-4(1)(-1)}}{2}=\frac{-3\pm\sqrt{13}}{2}$$

x cos θ,

$$\cos\theta=\frac{-3\pm\sqrt{13}}{2} \qquad \theta=\cos^{-1}\left(\frac{-3+\sqrt{13}}{2}\right)$$

$$\theta=\cos^{-1}\left(\frac{-3-\sqrt{13}}{2}\right)\ [-1,1\ ].$$

$$\cos^{-1}\left(\frac{-3+\sqrt{13}}{2}\right)\approx 1.26$$

$$2\pi-\cos^{-1}\left(\frac{-3+\sqrt{13}}{2}\right)\approx 5.02$$

2 sin 2 θ−5 sin θ+3=0,0≤θ≤2π. sin θ=u,

$$2\sin^2\theta-5\sin\theta+3=0 \quad (2\sin\theta-3)(\sin\theta-1)=0$$

$$2\sin\theta-3=0 \qquad 2\sin\theta=3 \qquad \sin\theta=\frac{3}{2} \quad \sin\theta-1=0 \qquad \sin\theta=1$$

θ:sin θ≠ 3 2 , [−1,1]. sin θ=1, π 2 . sin 2 θ=2 cos θ+2,0≤θ≤2π. cos θ=−1, θ=π

$$2\sin^2\theta+\sin\theta=0;0\le\theta<2\pi$$

sin θ=x. 2 x 2 +x=0.

$$2x^2+x=0 \quad x(2x+1)=0$$

$$x=0 \quad (2x+1)=0 \qquad x=-\frac{1}{2}$$

sinθ x.

$$\sin\theta=0 \qquad \theta=0,\pi \quad \sin\theta=-\frac{1}{2} \qquad \theta=\frac{7\pi}{6},\frac{11\pi}{6}$$

0≤θ<2π 0,π, 7π 6 , 11π 6 .

2 sin 2 θ+sin θ=0 sin θ(2sin θ+1)=0 sin θ=0 θ=0,π 2 sin θ+1=0 2sin θ=−1 sin θ=− 1 2 θ= 7π 6 , 11π 6 0≤θ<2π, 2 sin 2 θ−3 sin θ+1=0,0≤θ<2π. θ (2 sin θ−1)(sin θ−1)=0 2 sin θ−1=0 sin θ= 1 2 θ= π 6 , 5π 6 sin θ=1 θ= π 2 2 cos 2 θ+cos θ=0.π 2 , 2π 3 , 4π 3 , 3π 2 0≤x<2π. cos x cos(2x)+sin x sin(2x)= 3 2 cos x cos(2x)+sin x sin(2x)= 3 2 cos(x−2x)= 3 2 Difference formula for cosine cos(−x)= 3 2 Use the negative angle identity. cos x= 3 2 cos x= 3 2 x= π 6 , 11π 6 . cos(2 θ)=cos θ. cos(2θ)=cos θ 2 cos 2 θ−1=cos θ 2 cos 2 θ−cos θ−1=0 (2 cos θ+1)(cos θ −1)=0 2 cos θ+1=0 cos θ=− 1 2 cos θ−1=0 cos θ=1 cos θ=− 1 2 , θ= 2π 3 ±2πk θ= 4π 3 ±2πk; cos θ=1, θ=0±2πk. 3 cos θ+3=2 sin 2 θ,0≤θ<2π. 3 cos θ+3 = 2 sin 2θ 3 cos θ+3 =2(1−cos2 θ) 3 cos θ+3 =2−2cos2 θ 2 cos2θ+3 cos θ+1 =0 (2 cos θ+1) (cos θ+1) =0 2 cos θ+1 =0 cos θ =− 1 2 θ = 2π 3 , 4π 3 cos θ+1 =0 cos θ =−1 θ =π 2π 3 , 4π 3 ,π. sin(2x) cos(3x). y=sin(2x) y=sin x. 2π, y=sin(2x), y=sin x. sin(2x)=0 sin x=0. cos(2x)= 1 2 [0,2π). cos(α)= 1 2 , α θ= cos −1 1 2 cos θ= 1 2 θ= π 3 θ= 5π 3 . 2x= π 3 2x= 5π 3 , x= π 6 x= 5π 6 . cos(2(π 6))=cos(π 3)= 1 2 . 2x= π 3 , x= π 6 2x= π 3 +2π = π 3 + 6π 3 = 7π 3 x= 7π 6 . 2x= π 3 +4π = π 3 + 12π 3 = 13π 3 x= 13π 6 >2π, x 2π, [0,2π). 2x= 5π 3 , x= 5π 6 2x= 5π 3 +2π = 5π 3 + 6π 3 = 11π 3 x= 11π 6 . 2x= 5π 3 +4π = 5π 3 + 12π 3 = 17π 3 x= 17π 6 >2π, x 2π, [0,2π). π 6 , 5π 6 , 7π 6 ,and 11π 6 . sin(nx)=c, n a 2 + b 2 = c 2 , a 2 + b 2 = c 2 (23) 2 + (69.5) 2 ≈5359 5359 ≈73.2 m θ, tan θ= 69.5 23 tan −1 (69.5 23)≈1.2522 ≈ 71.69 ∘ 71.7 ∘ , θ 4a. cos θ= a 4a = 1 4 cos −1 (1 4)≈ 75.5 ∘ 75.5 ∘ a 2 + b 2 = (4a) 2 b 2 = (4a) 2 − a 2 b 2 =16 a 2 − a 2 b 2 =15 a 2 b= 15 a 15 a cos(x)=−5. 0≤θ<2π.2 sin θ=− 22 sin θ= 3π 3 , 2π 32 cos θ=12 cos θ=− 23π 4 , 5π 4tan θ=−1tan x=1 π 4 , 5π 4cot x+1=04 sin 2 x−2=0π 4 , 3π 4 , 5π 4 , 7π 4csc 2 x−4=0 [0,2π).2 cos θ= 2π 4 , 7π 4 2 cos θ=−12 sin θ=−17π 6 , 11π 62 sin θ=− 32 sin(3θ)=1π 18 , 5π 18 , 13π 18 , 17π 18 , 25π 18 , 29π 18 2 sin(2θ)= 32 cos(3θ)=− 23π 12 , 5π 12 , 11π 12 , 13π 12 , 19π 12 , 21π 12cos(2θ)=− 3 22 sin(πθ)=1 1 6 , 5 6 , 13 6 , 17 6 , 25 6 , 29 6 , 37 62 cos(π 5 θ)= 3 [0,2π).sec(x)sin(x)−2 sin(x)=00, π 3 ,π, 5π 3 tan(x)−2 sin(x)tan(x)=02 cos 2 t+cos(t)=1π 3 ,π, 5π 32 tan 2 (t)=3 sec(t) 2 sin(x)cos(x)−sin(x)+2 cos(x)−1=0π 3 , 3π 2 , 5π 3cos 2 θ= 1 2sec 2 x=10,πtan 2 (x)=−1+2 tan(−x) 8 sin 2 (x)+6 sin(x)+1=0π− sin −1 (− 1 4), 7π 6 , 11π 6 ,2π+ sin −1 (− 1 4)tan 5 (x)=tan(x) [0,2π). sin(3x)cos(6x)−cos(3x)sin(6x)=−0.9 1 3 (sin −1 (9 10)), π 3 − 1 3 (sin −1 (9 10)), 2π 3 + 1 3 (sin −1 (9 10)),π− 1 3 (sin −1 (9 10)), 4π 3 + 1 3 (sin −1 (9 10)), 5π 3 − 1 3 (sin −1 (9 10)) sin(6x)cos(11x)−cos(6x)sin(11x)=−0.1cos(2x)cos x+sin(2x)sin x=106 sin(2t)+9 sin t=0 9 cos(2θ)=9 cos 2 θ−4π 6 , 5π 6 , 7π 6 , 11π 6sin(2t)=cos tcos(2t)=sin t3π 2 , π 6 , 5π 6 cos(6x)−cos(3x)=0 [0,2π). tan 2 x− 3 tan x=00, π 3 ,π, 4π 3sin 2 x+sin x−2=0sin 2 x−2 sin x−4=0

5 cos 2 x+3 cos x−1=03 cos 2 x−2 cos x−2=0cos −1 (1 3 (1− 7)),2π− cos −1 (1 3 (1− 7))
5 sin 2 x+2 sin x−1=0tan 2 x+5tan x−1=0
tan −1 (1 2 (29 −5)),π+ tan −1 (1 2 (− 29 −5)),π+ tan −1 (1 2 (29 −5)),2π+ tan −1 (1 2 (− 29 −5))
cot 2 x=−cot x− tan 2 x−tan x−2=0 [0,2π). sin 2 x− cos 2 x−sin x=0sin 2 x+ cos 2 x=0sin(2x)−sin x=0
cos(2x)−cos x=00, 2π 3 , 4π 32 tan x 2− sec 2 x − sin 2 x= cos 2 x1−cos(2x)=1+cos(2x)
π 4 , 3π 4 , 5π 4 , 7π 4sec 2 x=710 sin x cos x=6 cos xsin −1 (3 5), π 2 ,π− sin −1 (3 5), 3π 2
−3 sin t=15 cos t sin t4 cos 2 x−4=15 cos xcos −1 (− 1 4),2π− cos −1 (− 1 4)8 sin 2 x+6 sin x+1=0
8 cos 2 θ=3−2 cos θπ 3 , cos −1 (− 3 4),2π− cos −1 (− 3 4), 5π 36 cos 2 x+7 sin x−8=0
12 sin 2 t+cos t−6=0cos −1 (3 4), cos −1 (− 2 3),2π− cos −1 (− 2 3),2π− cos −1 (3 4)tan x=3 sin x
cos 3 t=cos t0, π 2 ,π, 3π 26 sin 2 x−5 sin x+1=08 cos 2 x−2 cos x−1=0
π 3 , cos −1 (− 1 4),2π− cos −1 (− 1 4), 5π 3100 tan 2 x+20 tan x−3=02 cos 2 x−cos x+15=0
20 sin 2 x−27 sin x+7=02 tan 2 x+7 tan x+6=0
π+ tan −1 (−2),π+ tan −1 (− 3 2),2π+ tan −1 (−2),2π+ tan −1 (− 3 2)130 tan 2 x+69 tan x−130=0
sin x=0.272πk+0.2734,2πk+2.8682sin x=−0.55tan x=−0.34πk−0.3277cos x=0.71 [0,2π).
tan 2 x+3 tan x−3=00.6694,1.8287,3.8110,4.97036 tan 2 x+13 tan x=−6tan 2 x−sec x=1
1.0472,3.1416,5.2360sin 2 x−2 cos 2 x=02 tan 2 x+9 tan x−6=00.5326,1.7648,3.6742,4.9064
4 sin 2 x+sin(2x)sec x−3=0 [0,2π).csc 2 x−3 csc x−4=0sin −1 (1 4),π− sin −1 (1 4), 3π 2
sin 2 x− cos 2 x−1=0sin 2 x(1− sin 2 x)+ cos 2 x(1− sin 2 x)=0π 2 , 3π 2
3 sec 2 x+2+ sin 2 x− tan 2 x+ cos 2 x=0sin 2 x−1+2 cos(2x)− cos 2 x=1tan 2 x−1− sec 3 x cos x=0
sin(2x) sec 2 x =00, π 2 ,π, 3π 2sin(2x) 2 csc 2 x =02 cos 2 x− sin 2 x−cos x−5=0
1 sec 2 x +2+ sin 2 x+4 cos 2 x=47.2 ◦5.7 ◦82.4 ◦31.0 ◦88.7 ◦59.0 ◦36.9 ◦ 2π. x,

$$\sin(x\pm 2\pi k)=\sin x \quad \text{and} \quad \cos(x\pm 2\pi k)=\cos x \quad \text{where k is an integer}$$

$$y=A\sin(Bt-C)+D \text{ or } y=A\cos(Bt-C)+D$$

amplitude=|A|,B period= 2π B ,C C B D y=a sin(ω t±C)+D y=a cos(ω t±C)+D, 2π ω . y=sin x
y=2 sin(4x− π 2)+2. y=sinx y=2sinx. B, period= 2π B . B=4, π 2 . π 2 C B , π 2 4 = π 8 . D.
y=2 sin(1 4 x)y=−3 sin(2x+ π 2)y=cos x+3y=2 sin(1 4 x)

$$y=A\sin(Bt+C)+D$$

| A | 2π B ,

$$2\pi B = 2\pi 1 4 = 8\pi$$

y=−3 sin(2x+ π 2)

$$y=A\sin(Bt-C)+D$$

| A |, | −3 |=3. A 2π B ,

$$2\pi B = 2\pi 2 = \pi$$

C B = π 2 2 = π 4 y=cos x+3

$$y=A\cos(Bt\pm C)+D$$

| A |, 2π. y=3 cos(3πx)? 3, 2 3 . 1 4 y=cos θ. 2π,

$$2\pi 4 = \pi 2$$

θ=0, π 2 π 2 θ0π 2π3π 22πy=cos θ10−101 y=−4 cos(πx) |−4|=4. 2π ω = 2π π =2. B ω.) [0,2].
x=0 1 2 x y. x01 213 22y=−4 cos(πx)−4040−4 y=3 sin(3x) 3sin(3x)π 6π 3π 2−32π 3
y=A sin(Bt−C)+D o F

$$A=\frac{\text{largest value}-\text{smallest value}}{2}$$

$$|A|=\frac{69-42.5}{2}=13.25$$

2π B =12 B= 2π 12 = π 6 .

$$D=\frac{\text{highest value}+\text{lowest value}}{2}$$

$$D=\frac{69+42.5}{2}=55.8$$

y=13.3 sin(π 6 x−C)+55.8. x y C.

$$42.5=13.3\sin\left(\frac{\pi}{6}(1)-C\right)+55.8 \quad -13.3=13.3\sin\left(\frac{\pi}{6}-C\right) \quad -1=\sin\left(\frac{\pi}{6}-C\right) \quad \sin\theta=-1\rightarrow\theta=-\frac{\pi}{2}$$

$$\frac{\pi}{6}-C=-\frac{\pi}{2} \quad -C=-\frac{\pi}{2}-\frac{\pi}{6}+\frac{\pi}{2}=C \quad =\frac{2\pi}{3}$$

$y=13.3\sin(\pi 6 x-2\pi 3)+55.8$. y x $xy(0,30)(3,54)(6,78)(9,54)(12,30)$

$$|A|=|78-30\ 2| \quad =24$$

$$B=2\pi 12 \quad =\pi 6$$

$$D=78+30\ 2 \quad =54$$

$C=0$. y $x=0$ A. $y=A\cos(Bx\pm C)+D$,

$$y=-24\cos(\pi 6 x)+54$$

$$|A|=|(15-7)\ 2| \quad =4$$

B

$$2\pi 12=\pi 6$$

$(15+8)\ 2=11.5$. $t=0$, A.

$$y=4\cos(\pi 6)t+11$$

24 °F 40 °F. 32 °F. $t=0$ $y=8\sin(\pi 12 t)+32$ $f(t)=20\sin(160\pi t)+100$, $f(t)$ t,

$$2\pi\omega=2\pi 160\pi \quad =1\ 80$$

$$\omega 2\pi=160\pi 2\pi=80$$

120 80 (maximum minimum). 120 80 $t=0$,$d=0$.

$$d=a\cos(\omega t) \text{ or } d=a\sin(\omega t)$$

$|a|$ $2\pi\omega$ $\omega 2\pi$ $y=5\sin(3t)$$y=6\cos(\pi t)$$y=5\cos(\pi 2 t)$$y=5\sin(3t)$ $|a|$, $2\pi\omega=2\pi 3$. $\omega 2\pi=3\ 2\pi$. $y=6\cos(\pi t)$ 6. $2\pi\omega=2\pi\pi=2$. $\omega 2\pi=\pi 2\pi=1\ 2$.$y=5\cos(\pi 2)t$ 5. $2\pi\omega=2\pi\pi 2=4$. $1\ 4$. t

$$f(t)=a e^{-ct}\sin(\omega t) \text{ or } f(t)=a e^{-ct}\cos(\omega t)$$

c $|a|$ $2\pi\omega$ 0.5 0.5 0.1. $t=0$, $f=\omega 2\pi$

$$\omega 2\pi=0.5 \quad \omega=(0.5)2\pi \quad =\pi$$

$c=0.5$.

$$f(t)=10 e^{-0.5t}\cos(\pi t)$$

$c=0.1$

$$f(t)=10 e^{-0.1t}\cos(\pi t)$$

$c=0.5$ $c=0.1$. $y=a e^{-ct}\cos(\omega t)$ $a=20$,$c=0.05$,$p=4a=2$,$c=1.5$,$f=3$ $2\pi\omega$ $\omega 2\pi$.$y=20 e^{-0.05t}\cos(\pi 2 t)$. $y=2 e^{-1.5t}\cos(6\pi t)$. $f(t)=5 e^{-6t}\cos(4t)\ 2\pi$ $y=a e^{-ct}\sin(\omega t)$ $a=7$,$c=10$,$p=\pi 6a=0.3$,$c=0.2$,$f=20$ ω $2\pi\omega$,

$$\pi 6=2\pi\omega \quad \omega\pi=6(2\pi) \quad \omega=12$$

$y=7 e^{-10t}\sin(12t)$. $\omega 2\pi$,

$$20=\omega 2\pi \quad 40\pi=\omega$$

0.2 0.3. $y=0.3 e^{-0.2t}\sin(40\pi t)$. $t=0$, $t=0$. $y=20 e^{-0.05t}\cos(\pi 2 t)$ $t=0$, $y=20$, $t=0$

$$y=20 e^{-0.05(0)}\cos(\pi 2)(0) \quad =20(1)(1) \quad =20$$

$$y=20 e^{-0.05(0)}\sin(\pi 2)(0) \quad =20(1)(0) \quad =0$$

$a=10$,$c=0.5$, $p=2$.$y=10 e^{-0.5t}\cos(\pi t)$ t

$$A(t)=5(1-0.30)t$$

$(1-0.30)t$ e ct

$$0.7=e c \quad c=\ln.7 \quad c=-0.357$$

$$3=2\pi\omega \quad \omega=2\pi 3$$

$$y=\cos(2\pi 3 t)+10$$

t

$$y=-5 e^{-0.357t}\cos(2\pi 3 t)+10$$

$t=0$ $1\ 3$ $y=5\cos(6\pi t)$ $a e^{-ct}$ e^{-ct} .

$$a e^{-c(t+3)}=1\ 2\ a e^{-ct}$$

c.

$a e^{-c(t+3)}=1\ 2\ a e^{-ct}$ $e^{-ct}\cdot e^{-3c}=1\ 2\ e^{-ct}$ Divide out a. $e^{-3c}=1\ 2$ Divide out e $-ct$. $e\ 3c=2$ Take reciprocals.

$$e\ 3c=2 \quad 3c=\ln 2 \quad c=\ln 2\ 3$$

$\ln 2\ 3$. $f(x)=\cos(2\pi x)\cos(16\pi x)$. $f(x)$ $f(x)$ $y=\cos(2\pi x)$ $y=-\cos(2\pi x)$

y=A sin(Bt−C)+D or y=A cos(Bt−C)+Dd=a cos(ωt) or d=a sin(ωt)
f(t)=a e −c t sin(ωt) or f(t)=a e −ct cos(ωt)
xy0−43−1629−112−415−1182y=−3cos(π 6 x)−1xy05214−3618510112−3xy02π 47π 223π 4−3π25π 47 3π 225sin(2x)+2xy02π 47π 223π 4−3π25π 473π 22xy011−32−73−3415−36−74cos(xπ 2)−3xy0−2142 10344−254610xy051−325313455−3655−8sin(xπ 2)xy−3−1− 2−2−1−11− 20012 −12132 +1xy−13 −200 12− 323 3314352+ 3tan(xπ 12)f(x)=−30 cos(xπ 6)−20 cos 2 (xπ 6)+80 [0,12] f(x)=−18 cos(xπ 12)−5 sin(xπ 12)+100 [0,24]f(x)=10−sin(xπ 6)+24 tan(xπ 240) [0,80] 105°F 85°F. 84°F 70°F. 56 °F 47°F 63°F 51°F? 64°F 86°F 70°F? 1.8024 h(t) h(t)=8sin(6πt), t h(t) h(t)=11sin(12πt), t 1 6 , h(t) h(t)=4cos(π 2 t), t h(t), h(t)=−5 cos(60πt), t 1 30 , P, t. P, t. P(t)=−15cos(π 6 t)+650+ 55 6 t P, t. P, t.P(t)=−40cos(π 6 t)+800 (1.04) t D, t, D, t, D(t)=7 (0.89) t cos(40πt) D, t, D, t,D(t)=19 (0.9265) t cos(26πt)20.1 −0.1 0.1 cm, −0.1 0.1 cm, 0.1 cm. 0.1 cm. 22 ◦ 112 ◦ 60 ◦ , 150 ◦ . 90 ◦ y=a b x +csin(π 2 x) xyxyy=6 (5) x +4sin(π 2 x)xy y=a b x cos(π 2 x)+c xyy=8 (1 2) x cos(π 2 x)+3xy [0,2π).csc 2 t=3
sin −1 (3 3),π− sin −1 (3 3),π+ sin −1 (3 3),2π− sin −1 (3 3)cos 2 x= 1 42 sin θ=−17π 6 , 11π 6
tan x sin x+sin(−x)=09 sin ω−2=4 sin 2 ωsin −1 (1 4),π− sin −1 (1 4)1−2 tan(ω)= tan 2 (ω)
sec x cos x+cos x− 1 sec x1sin 3 x+ cos 2 x sin xsin 2 x+ sec 2 x−1= (1− cos 2 x)(1+ cos 2 x) cos 2 x
tan 3 x csc 2 x cot 2 x cos x sin x=1tan(7π 12)−2− 3cos(25π 12)
sin(70 ◦)cos(25 ◦)−cos(70 ◦)sin(25 ◦)2 2cos(83 ◦)cos(23 ◦)+sin(83 ◦)sin(23 ◦)
cos(4x)−cos(3x)cosx= sin 2 x−4 cos 2 x sin 2 x
cos(4x)−cos(3x)cosx=cos(2x+2x)−cos(x+2x)cos x =cos(2x)cos(2x)−sin(2x)sin(2x)−cos x cos(2x)cos x+sin x sin(2x)cos x = (cos 2 x− sin 2 x) 2 −4 cos 2 x sin 2 x− cos 2 x(cos 2 x− sin 2 x)+sin x(2)sin x cos x cos x = (cos 2 x− sin 2 x) 2 −4 cos 2 x sin 2 x− cos 2 x(cos 2 x− sin 2 x)+2 sin 2 x cos 2 x = cos 4 x−2 cos 2 x sin 2 x+ sin 4 x−4 cos 2 x sin 2 x− cos 4 x+ cos 2 x sin 2 x+2 sin 2 x cos 2 x = sin 4 x−4 cos 2 x sin 2 x+ cos 2 x sin 2 x = sin 2 x(sin 2 x+ cos 2 x)−4 cos 2 x sin 2 x = sin 2 x−4 cos 2 x sin 2 x
cos(3x)− cos 3 x=−cos x sin 2 x−sin x sin(2x)tan(1 2 x)+tan(1 8 x) 1−tan(1 8 x)tan(1 2 x)tan(5 8 x)
cos(sin −1 (0)− cos −1 (1 2))tan(sin −1 (0)+ sin −1 (1 2))3 3 sin(2θ), cos(2θ), tan(2θ) cos θ=− 1 3 θ [π 2 ,π]. sin(2θ), cos(2θ), tan(2θ) sec θ=− 5 3 θ [π 2 ,π].− 24 25 ,− 7 25 , 24 7 sin(7π 8)sec(3π 8)2(2+ 2)sin(2β),cos(2β),tan(2β),sin(2α),cos(2α), and tan(2α)
sin(β 2),cos(β 2),tan(β 2),sin(α 2),cos(α 2), and tan(α 2)2 10 , 7 2 10 , 1 7 , 3 5 , 4 5 , 3 4
2cos(2x) sin(2x) =cot x−tan xcot x cos(2x)=−sin(2x)+cot x
cotxcos(2x)=cotx(1−2 sin 2 x) =cotx− cosx sinx (2) sin 2 x =−2sinxcosx+cotx =−sin(2x)+cotx
cos 2 x sin 4 (2x)tan 2 x sin 3 x10sinx−5sin(3x)+sin(5x) 8(cos(2x)+1)cos(π 3) sin(π 4) 2 sin(2π 3) sin(5π 6)3 22 cos(π 5) cos(π 3)sin(π 12)−sin(7π 12)− 2 2cos(5π 12)+cos(7π 12) sin(9x)cos(3x)1 2 (sin(6x)+sin(12x))cos(7x)cos(12x)sin(11x)+sin(2x)2sin(13 2 x)cos(9 2 x) cos(6x)+cos(5x) [0,2π).tan x+1=03π 4 , 7π 42 sin(2x)+ 2 =0 [0,2π).2 sin 2 x−sin x=00, π 6 , 5π 6 ,π cos 2 x−cos x−1=02 sin 2 x+5 sin x+3=03π 2cos x−5 sin(2x)=01 sec 2 x +2+ sin 2 x+4 cos 2 x=0 [0,2π). 3 cot 2 x+cot x=1csc 2 x−3 csc x−4=00.2527,2.8889,4.7124 [0,2π).20 cos 2 x+21 cos x+1=0 sec 2 x−2 sec x=151.3694, 1.9106, 4.3726, 4.9137x012345y1611616xy0−2112−23−54−251 3sin(xπ 2)−2xy−33+2 2−23−1 2 2 −1 0113−2 22−13−1−2 271.6 ◦ t,P(t)=950−450sin(π 6 t) 90°F 30°F y=3 cos(xπ)1 2y=−2 sin(16xπ) t,C(t)=20sin(2πt)+100 (1.4427) t t,cos(−x)sin x cot x+ sin 2 x sin(−x)cos(−2x)−sin(−x)cos(−2x)cos(7π 12)2 − 6 4tan(3π 8)tan(sin −1 (2 2)+ tan −1 3)− 2 − 3 2sin(π 4)sin(π 6) [0,2π).cos 2 x− sin 2 x−1=00,πcos 2 x=cos x 4 sin 2 x+2 sin x−3=0 sin −1 (1 4 (13 −1)),π− sin −1 (1 4 (13 −1))cos(2x)+ sin 2 x=02 sin 2 x−sin x=00, π 6 , 5π 6 ,π cos(2x)+cos(−8x). tan(x)− 3 =0.π 3 +kπ sec 2 x−2 sec x=15 [0,2π) sin(2θ),cos(2θ), tan(2θ) cot θ=− 3 4 θ [π 2 ,π].− 24 25 ,− 7 25 , 24 7 sin(θ 2),cos(θ 2), tan(θ 2) cos θ= 7 25 θ sin 4 x

1 8 (3+cos(4x)−4cos(2x))tan 3 x−tan x sec 2 x=tan(−x)sin(3x)−cos x sin(2x)= cos 2 x sin x− sin 3 x sin(3x)−cosxsin(2x)= sin(x+2x)−cosx(2sinxcosx)= sinxcos(2x)+sin(2x)cosx−2sinx cos 2 x= sinx(cos 2 x− sin 2 x)+2sinxcosxcosx−2sinx cos 2 x= sinx cos 2 x− sin 3 +0= cos 2 xsinx− sin 3 x= cos 2 xsinx− sin 3 x

sin(2x) sin x − cos(2x) cos x =sec x y=Acos(Bx+C)+D x012345y−22−22−22y=2cos(πx+π) h(t) h(t)= 1 4 sin(120πt), t 81.5 ◦ , 78.7 ◦ n(t)=8 cos(20πt)cos(1000πt). 6+5 cos(π 6 (1−x)) t t D(t)=2 cos(π 6 t)+108+ 1 4 t, sin α= h b sin β= h a . h h.

$$h=b\sin\alpha \text{ and } h=a\sin\beta$$

bsin α=asin β (1 ab)(bsin α)=(asin β)(1 ab) Multiply both sides by 1 ab . sin α a = sin β b

$$\sin\alpha\ a=\sin\gamma\ c \text{ and } \sin\beta\ b=\sin\gamma\ c$$

$$\sin\alpha\ a=\sin\beta\ b=\sin\lambda\ c$$

α a; β b; γ c.

$$\sin\alpha\ a=\sin\beta\ b=\sin\gamma\ c$$

$$a\sin\alpha=b\sin\beta=c\sin\gamma$$

$$\beta=180°-50°-30°=100°$$

α=50°a=10. c.

sin(50°) 10 = sin(30°) c c sin(50°) 10 =sin(30°) Multiply both sides by c. c=sin(30°) 10 sin(50°) Multiply by the reciprocal to isolate c. c≈6.5

b,

sin(50°) 10 = sin(100°) b bsin(50°)=10sin(100°) Multiply both sides by b. b= 10sin(100°) sin(50°) Multiply by the reciprocal to isolate b. b≈12.9

α=50° a=10 β=100° b≈12.9 γ=30° c≈6.5

α= 98 ◦ a=34.6 β= 39 ◦ b=22 γ= 43 ◦ c=23.8 a b α. β γ, c. β,

sin α a = sin β b sin(35°) 6 = sin β 8 8sin(35°) 6 =sin β 0.7648≈sin β sin −1 (0.7648)≈49.9° β≈49.9° β β? γ β β=180°−49.9°=130.1°. γ,

$$\gamma=180°-35°-130.1°\approx14.9°$$

γ ′ γ,

$$\gamma'=180°-35°-49.9°\approx95.1°$$

c c ′.

c sin(14.9°) = 6 sin(35°) c= 6sin(14.9°) sin(35°) ≈2.7

c ′ sin(95.1°) = 6 sin(35°) c ′ = 6sin(95.1°) sin(35°) ≈10.4

β. α=80°,a=120, b=121,

α=80° a=120 β≈83.2° b=121 γ≈16.8° c≈35.2

α ′ =80° a ′ =120 β ′ ≈96.8° b ′ =121 γ ′ ≈3.2° c ′ ≈6.8

γ=85°, c=12, b=9. β.

sin(85°) 12 = sin β 9 Isolate the unknown. 9sin(85°) 12 =sin β

β, β.

$$\beta=\sin^{-1}\left(\frac{9\sin(85°)}{12}\right)\beta\approx\sin^{-1}(0.7471)\ \beta\approx48.3°$$

β β=180°−48.3°≈131.7°.

α=180°−85°−131.7°≈−36.7°,

β≈48.3°. α=180°−85°−48.3°≈46.7°. a a

sin(85°) 12 = sin(46.7°) a a sin(85°) 12 =sin(46.7°) a= 12sin(46.7°) sin(85°) ≈8.8

α≈46.7° a≈8.8 β≈48.3° b=9 γ=85° c=12

α=80°,a=100, b=10, β≈5.7°,γ≈94.3°,c≈101.3

sin α 10 = sin(50°) 4 sin α= 10sin(50°) 4 sin α≈1.915

α. [−1,1], sin −1 (1.915) a=31, b=26, β=48°. Area= 1 2 bh, b h h sin α= opposite hypotenuse

sin α= h c csin α=h. b α ′ , 180−α.

Area= 1 2 (base)(height)= 1 2 b(csin α)

Area= 1 2 a(bsin γ)= 1 2 a(csin β)

Area= 1 2 bcsin α = 1 2 acsin β = 1 2 absin γ

a=90,b=52, γ=102°.

Area= 1 2 absin γ Area= 1 2 (90)(52)sin(102°) Area≈2289 square units

β=42°, a=7.2 ft, c=3.4 ft. 8.2 square feet a, h.

sin(130°) 20 = sin(35°) a asin(130°)=20sin(35°) a= 20sin(35°) sin(130°) a≈14.98

a, h.

sin(15°)= opposite hypotenuse sin(15°)= h a sin(15°)= h 14.98 h=14.98sin(15°) h≈3.88

B, sin α a = sin β b = sin γ c a sin α = b sin β = c sin γ

Area= 1 2 bcsin α = 1 2 acsin β = 1 2 absin γ α a,β b, γ c. α=43°,γ=69°,a=20α=35°,γ=73°,c=20

β=72°,a≈12.0,b≈19.9α=60°, β=60°, γ=60°a=4, α= 60° ,β=100° γ=20°,b≈4.5,c≈1.6b=10, β=95°,γ= 30°

A a, B b, C c. b A=37°, B=49°, c=5.b≈3.78 a A=132°,C=23°,b=10. c B=37°,C=21, b=23.c≈13.70 α

a,β b, γ c. α=119°,a=14,b=26γ=113°,b=10,c=32 α≈50.3°,β≈16.7°,a≈26.7b=3.5, c=5.3, γ= 80°

a=12, c=17, α= 35° γ≈54.3°,β≈90.7°,b≈20.9 γ ′ ≈125.7°, β ′ ≈19.3°, b ′ ≈6.9a=20.5, b=35.0, β= 25°

a=7, c=9, α= 43° β≈75.7°, γ≈61.3°,b≈9.9 β ′ ≈18.3°, γ ′ ≈118.7°, b ′ ≈3.2a=7,b=3,β=24°b=13,c=5,γ= 10°

α≈143.2°,β≈26.8°,a≈17.3 α ′ ≈16.8°, β ′ ≈153.2°, a ′ ≈8.3a=2.3,c=1.8,γ=28°β=119°,b=8.2,a=11.3A

a=24,b=5,B=22°.A a=13,b=6,B=20°.A≈47.8° A ′ ≈ 132.2° B A=12°,a=2,b=9.a=5,c=6,β= 35°8.6

b=11,c=8,α= 28°a=32,b=24,γ= 75°370.9a=7.2,b=4.5,γ= 43° x. 12.312.2 16.0 x, 29.7°x=76.9°or x=103.1°

x 110.6°A≈39.4, C≈47.6, BC≈20.7 57.142.0 430.2 10.1 m∠ADC AD AD≈ 13.8 AB AB≈2.8 H JK). N

LM). L≈49.7, N≈56.3, LN≈5.8 ABCD ∠m 7° 55°, A A B, A B 86.2° 83.9°, A A 67°. 16°. 20° 38°

37° 44°, A A A B C A. BAC ACB A B, 3 1 2 A,B,C, A B. C B A B, A B? ABC A c C (x,y) C

cos θ= x(adjacent) b(hypotenuse) and sin θ= y(opposite) b(hypotenuse)

θ, x=bcos θ y=bsin θ. (x,y) C (bcos θ, bsin θ). (x−c) y a

a 2 = (x−c) 2 + y 2 = (bcos θ−c) 2 + (bsin θ) 2 Substitute (bcos θ) for x and (bsin θ) for y. =(b

2 cos 2 θ−2bccos θ+ c 2)+ b 2 sin 2 θ Expand the perfect square. = b 2 cos 2 θ+ b 2 sin 2 θ+ c 2

−2bccos θ Group terms noting that cos 2 θ+ sin 2 θ=1. = b 2 (cos 2 θ+ sin 2 θ)+ c 2 −2bccos θ

Factor out b 2 . a 2 = b 2 + c 2 −2bccos θ

α,β, γ, a,b, c,

a 2 = b 2 + c 2 −2bc cos α b 2 = a 2 + c 2 −2ac cos β c 2 = a 2 + b 2 −2ab cos γ

cos α= b 2 + c 2 − a 2 2bc cos β= a 2 + c 2 − b 2 2ac cos γ= a 2 + b 2 − c 2 2ab

b, β.

b 2 = a 2 + c 2 −2accos β b 2 = 10 2 + 12 2 −2(10)(12)cos(30 ◦)

Substitute the measurements for the known quantities. b 2 =100+144−240(3 2)

Evaluate the cosine and begin to simplify. b 2 =244−120 3 b= 244−120 3 Use the square root property.

b≈6.013

b, α,

sin α a = sin β b sin α 10 = sin(30°) 6.013 sin α= 10sin(30°) 6.013

Multiply both sides of the equation by 10. α= sin −1 (10sin(30°) 6.013) Find the inverse sine of

10sin(30°) 6.013 . α≈56.3°

α α=180°−56.3°≈123.7°. α α 123.7° 0° 180°. α≈56.3°,

γ=180°−30°−56.3°≈93.7°

α≈56.3° a=10 β=30° b≈6.013 γ≈93.7° c=12

α=30°, b=12, c=24.a≈14.9, β≈23.8°, γ≈126.2°. α a=20, b=25, c=18. α,

$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $20^2 = 25^2 + 18^2 - 2(25)(18)\cos\alpha$ Substitute the appropriate measurements. $400 = 625 + 324 - 900\cos\alpha$ Simplify in each step. $400 = 949 - 900\cos\alpha$ $-549 = -900\cos\alpha$ Isolate $\cos\alpha$. $\frac{-549}{-900} = \cos\alpha$ $0.61 \approx \cos\alpha$ $\cos^{-1}(0.61) \approx \alpha$ Find the inverse cosine. $\alpha \approx 52.4°$

$a=5, b=7, c=10, \alpha \approx 27.7°, \beta \approx 40.5°, \gamma \approx 111.8°$ θ. $a=2420, b=5050, c=6000$. θ $a=2420$.

$a^2 = b^2 + c^2 - 2bc\cos\theta$ $(2420)^2 = (5050)^2 + (6000)^2 - 2(5050)(6000)\cos\theta$ $(2420)^2 - (5050)^2 - (6000)^2 = -2(5050)(6000)\cos\theta$ $\frac{(2420)^2 - (5050)^2 - (6000)^2}{-2(5050)(6000)} = \cos\theta$ $\cos\theta \approx 0.9183$ $\theta \approx \cos^{-1}(0.9183)$ $\theta \approx 23.3°$

$\theta = 23.3°$

$\cos(23.3°) = \frac{x}{5050}$ $x = 5050\cos(23.3°)$ $x \approx 4638.15$ feet $\sin(23.3°) = \frac{y}{5050}$ $y = 5050\sin(23.3°)$ $y \approx 1997.5$ feet

$180° - 20° = 160°$.

$x^2 = 8^2 + 10^2 - 2(8)(10)\cos(160°)$ $x^2 = 314.35$ $x = 314.35$ $x \approx 17.7$ miles

a, b, c

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$s = \frac{a+b+c}{2}$ s.

$s = \frac{(a+b+c)}{2}$ $s = \frac{(10+15+7)}{2} = 16$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ $\text{Area} = \sqrt{16(16-10)(16-15)(16-7)}$ $\text{Area} \approx 29.4$

$a=29.7$ ft, $b=42.3$ ft, $c=38.4$ ft. s,

$s = \frac{(62.4+43.5+34.1)}{2}$ $s = 70$ m

$\text{Area} = \sqrt{70(70-62.4)(70-43.5)(70-34.1)}$ $\text{Area} = \sqrt{506{,}118.2}$ $\text{Area} \approx 711.4$

$a=4.38$ ft, $b=3.79$ ft, $c=5.22$ ft.

$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = a^2 + c^2 - 2ac\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{(a+b+c)}{2}$ s s α a, β b, γ c. $\gamma = 41.2°, a=2.49, b=3.13$ $\alpha = 120°, b=6, c=7$ $\beta = 58.7°, a=10.6, c=15.7$ $\gamma = 115°, a=18, b=23$ $\alpha = 119°, a=26, b=14$ $\gamma = 113°, b=10, c=32$ $\beta = 67°, a=49, b=38$ $\alpha = 43.1°, a=184.2, b=242.8$ $\alpha = 36.6°, a=186.2, b=242.2$ $\beta = 50°$, $a=105, b=45$ $a=42, b=19, c=30$; A. $a=14, b=13, c=20$; C. $a=16, b=31, c=20$; B. $a=13, b=22, c=28$; A. $a=108, b=132, c=160$; C. $A = 35°, b=8, c=11$ $B \approx 45.9°, C \approx 99.1°, a \approx 6.4$ $B = 88°, a=4.4, c=5.2$ $C = 121°, a=21, b=37$ $A \approx 20.6°, B \approx 38.4°, c \approx 51.1$ $a=13, b=11, c=15$ $a=3.1, b=3.5, c=5$ $A \approx 37.8°, B \approx 43.8, C \approx 98.4°$ $a=51, b=25, c=29$ $a = 1\ 2$ m, $b = 1\ 3$ m, $c = 1\ 4$ m $a=12.4$ ft, $b=13.7$ ft, $c=20.2$ ft $a=1.6$ yd, $b=2.6$ yd, $c=4.1$ ydx. A. $x^2 = 25 + 36 - 60\cos(52)$ (x,y) (r,θ) r θ, r. θ, r θ. θ r, $(2, \pi 4)$, $\pi 4$ $(3, \pi 2)$ $\pi 2$ $\pi 2$ $(2, \pi 3)$ $(-2, \pi 6)$ $\pi 6$ $r = -2$. r $(2, \pi 6)$ $\pi 6$ $\pi 6$ $(2, \pi 6)$ $(3, -\pi 6)$ $(2, 9\pi 4)$ x, y, r, θ.

$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$ $\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$

$\cos\theta$ $\sin\theta$ (r,θ) (x,y),

$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$

(r,θ), $x = r\cos\theta$ $y = r\sin\theta$. $\cos\theta$ $\sin\theta$. $\cos\theta$ r $\sin\theta$ r $(3, \pi 2)$

$x = r\cos\theta$ $x = 3\cos\frac{\pi}{2} = 0$ $y = r\sin\theta$ $y = 3\sin\frac{\pi}{2} = 3$

$(0,3)$. $(-2,0)$

$x = r\cos\theta$ $x = -2\cos(0) = -2$ $y = r\sin\theta$ $y = -2\sin(0) = 0$

$(-2,0)$. $(-1, 2\pi 3)$ $(x,y) = (1\ 2, -3\ 2)$

$\cos\theta = \frac{x}{r}$ or $x = r\cos\theta$ $\sin\theta = \frac{y}{r}$ or $y = r\sin\theta$ $r^2 = x^2 + y^2$ $\tan\theta = \frac{y}{x}$

$(3,3)$ $(3,3)$ θ, $\tan\theta = \frac{y}{x}$.

$\tan\theta = \frac{3}{3}$ $\tan\theta = 1$ $\tan^{-1}(1) = \pi 4$

r, x y $r = \sqrt{x^2 + y^2}$. r $\pi 4$

$r = \sqrt{3^2 + 3^2}$ $r = \sqrt{9+9}$ $r = \sqrt{18} = 3\sqrt{2}$

$r = 3\sqrt{2}$ $\theta = \pi 4$, $(3\sqrt{2}, \pi 4)$. $(-3\sqrt{2}, 5\pi 4)$ $(3\sqrt{2}, -7\pi 4)$ $(3\sqrt{2}, \pi 4)$. $(-3\sqrt{2}, 5\pi 4)$ π, $\pi 4$. $-3\sqrt{2}$.

5π 4 r (3 2 , π 4). (3 2 ,– 7π 4) – 7π 4 , π 4 . 3 2 , r 1 , r 2 , . . . , r 6 .r. x 2 + y 2 =9 x y r θ. r θ. (x,y) (r,θ). x=rcos θ y=rsin θ, r.

(rcos θ) 2 + (rsin θ) 2 =9 r 2 cos 2 θ+ r 2 sin 2 θ=9 r 2 (cos 2 θ+ sin 2 θ)=9 r 2 (1)=9 Substitute cos 2 θ+ sin 2 θ=1. r=±3 Use the square root property.

x 2 + y 2 =9,r=3, r=–3 x 2 + y 2 =9 r=3 y.

x 2 + y 2 =9 y 2 =9– x 2 y=± 9– x 2

Y 1 = 9– x 2 Y 2 =– 9– x 2 . x 2 + y 2 =6y

r 2 =6y Use x 2 + y 2 = r 2 . r 2 =6rsin θ Substitute y=rsin θ. r 2 –6rsin θ=0 Set equal to 0. r(r–6sin θ)=0 Factor and solve. r=0

We reject r=0, as it only represents one point, (0,0). or r=6sin θ

x 2 + y 2 =6y r=6sin θ x 2 + y 2 =6y r=6sin θ y=3x+2 x=rcos θ y=rsin θ.

y=3x+2 rsin θ=3rcos θ+2 rsin θ–3rcos θ=2 r(sin θ–3cos θ)=2 Isolate r.

r= 2 sin θ–3cos θ Solve for r.

y 2 =3– x 2 r= 3 r=2sec θ

r=2sec θ r= 2 cos θ rcos θ=2 x=2

r=2sec θ x=2 x=c r=csec θ r=2csc θ y=2. r=ccsc θ y=c. r= 3 1–2cos θ θ r, x y. r x y,

x 2 + y 2 = r 2 .

r= 3 1–2cos θ r(1–2cos θ)=3 r(1–2(x r))=3 Use cos θ= x r to eliminate θ. r–2x=3

r=3+2x Isolate r. r 2 = (3+2x) 2 Square both sides. x 2 + y 2 = (3+2x) 2 Use x 2 + y 2 = r 2 .

x 2 + y 2 = (3+2x) 2 . y.

x 2 + y 2 = (3+2x) 2 y 2 = (3+2x) 2 – x 2 y=± (3+2x) 2 – x 2

r θ x y, y

x 2 + y 2 = (3+2x) 2 x 2 + y 2 – (3+2x) 2 =0 x 2 + y 2 –(9+12x+4 x 2)=0 x 2 + y 2 –9–12x–4 x 2 =0 –3 x 2 –12x+ y 2 =9 Multiply through by –1. 3 x 2 +12x– y 2 =–9 3(x 2 +4x+)– y 2 =–9 Organize terms to complete the square for x. 3(x 2 +4x+4)– y 2 =–9+12 3 (x+2) 2 – y 2 =3 (x+2) 2 – y 2 3 =1

r=2sin θ x 2 + y 2 =2y x 2 + (y–1) 2 =1 r=sin(2θ)

r=sin(2θ) Use the double angle identity for sine. r=2sin θcos θ Use cos θ= x r and sin θ= y r . r=2(x r)(y r) Simplify. r= 2xy r 2 Multiply both sides by r 2 .

r 3 =2xy (x 2 + y 2) 3 =2xy As x 2 + y 2 = r 2 ,r= x 2 + y 2 .

(x 2 + y 2) 3 2 =2xy or x 2 + y 2 = (2xy) 2 3

cos θ= x r →x=rcos θ sin θ= y r →y=rsin θ r 2 = x 2 + y 2 tan θ= y x (r,θ), θ>0, θ, r θ. θ r θ.r θ. x=rcos θ y=rsin θ. cos θ= x r ,sin θ= y r ,tan θ= y x , r= x 2 + y 2 . θ r r r r θ (3, π 2) (–3, π 2) (–3, π 2) (3,– π 2) (–3, π 2) π 2 (3,– π 2) – π 2 r>0 0≤θ≤2π. θ (7, 7π 6)(5,π)(–5,0) (6,– π 4)(–3, π 6)(– 3 3 2 ,– 3 2)(4, 7π 4) r>0, 0≤θ<2π. (4,2)(2 5 , 0.464)(–4,6)(3,–5) (34 ,5.253)(–10,–13)(8,8)(8 2 , π 4)x=3y=4r=4cscθy=4 x 2y=2 x 4r= sinθ 2co s 4 θ 3x 2 + y 2 =4y x 2 + y 2 =3xr=3cosθx 2 – y 2 =xx 2 – y 2 =3yr= 3sinθ cos(2θ)x 2 + y 2 =9x 2 =9yr= 9sinθ cos 2 θ y 2 =9x9xy=1r= 1 9cosθsinθr=3sin θr=4cos θx 2 + y 2 =4x (x–2) 2 4 + y 2 4 =1;r= 4 sin θ+7cos θ r= 6 cos θ+3sin θ3y+x=6; r=2sec θr=3csc θy=3; r= rcos θ+2r 2 =4sec θ csc θxy=4; r=4r 2 =4x 2 + y 2 =4; r= 1 4cos θ–3sin θr= 3 cos θ–5sin θx–5y=3; (3, 3π 4)(5,π)(–2, π 3)(–1,– π 2)(3.5, 7π 4)(–4, π 3) (5, π 2)(4, –5π 4)(3, 5π 6)(–1.5, 7π 6)(–2, π 4)(1, 3π 2)5x–y=6r= 6 5cosθ–sinθ2x+7y=–3 x 2 + (y–1) 2 =1r=2sinθ(x+2) 2 + (y+3) 2 =13x=2r= 2 cosθx 2 + y 2 =5yx 2 + y 2 =3xr=3cosθr=6 r=–4x 2 + y 2 =16θ=– 2π 3θ= π 4y=xr=sec θr=–10sin θx 2 + (y+5) 2 =25r=3cos θ (2,– π 5). (1.618,–1.176) (–3, 3π 7). (–7,8) (10.630,131.186°) (3,–4) (–2,0) (2,3.14)or(2,π) r=asec θ;a>0. r=asec θ;a<0. a r=acsc θ;a>0. r=acsc θ;a<0. a r<40≤θ≤ π 4θ= π 4 , r ≥ 2θ= π 4 , r ≥–3 0≤θ≤ π 3 , r < 2–π 6 <θ≤ π 3 ,–3<r < 2 (r,θ), θ r θ (r,θ). r θ y= x 2 x y r θ (r,θ) θ, r θ. r) θ= π 2 (r,θ) (–r,–θ) r=2sin θ;

r=2sin θ −r=2sin(−θ) Replace (r,θ) with (−r,−θ). −r=−2sin θ Identity: sin(−θ)=−sin θ. r=2sin θ Multiply both sides by−1.

θ= π 2 . x (r,θ) (r,−θ) (−r,π−θ) r=1−2cos θ.

r=1−2cos θ r=1−2cos(−θ) Replace (r,θ) with (r,−θ). r=1−2cos θ Even/Odd identity

(r,θ) (−r,θ) r=2sin(3θ).

r=2sin(3θ) −r=2sin(3θ)

θ= π 2 , θ= π 2 (r,θ) (−r,−θ) (r,θ) (r,−θ) (−r,π−θ) (r,θ) (−r,θ) (r,θ): (−r,−θ) θ= π 2 (r,−θ) (−r,θ) r=2sin θ (r,θ) (−r,−θ) θ= π 2 .

−r=2sin(−θ) −r=−2sin θ Even-odd identity r=2sin θ Multiply by −1 Passed θ −θ

r=2sin(−θ) r=−2sin θ Even-odd identity r=−2sin θ≠2sin θ Failed r −r −r=2sin θ r=−2sin θ≠2sin θ Failed

r=2sin θ (0,1) r=1 θ= π 2 . r=−2cos θ. θ= π 2 x y θ r θ r. θ r θ r=0. x. r=0, θ. θ r=5cos θ; θ=0, 5cos θ, θ=0 | r |. θ= π 2 , r=5sin θ, θ= π 2 | r |. r θ=0. | r | r=2sin θ. r θ.

2sin θ=0 sin θ=0 θ= sin −1 0 θ=nπ where n is an integer

θ 0.

r=2sin(0) r=0

(0,0) (0,±nπ) sin θ, θ= π 2 ±2kπ sin(π 2)=1. π 2 θ.

r=2sin(π 2) r=2(1) r=2

(2, π 2) θr=2sin θrr=2sin(0)=00π 6r=2sin(π 6)=11π 3r=2sin(π 3)≈1.731.73π 2r=2sin(π 2)=222π 3 r=2sin(2π 3)≈1.731.735π 6r=2sin(5π 6)=11πr=2sin(π)=00 | r |: r=3cos θ. (0, π 2), (3,0). r=acos θ r=asin θ, a | a | 2 , r=acos θ, (a 2 ,0). r=asin θ, (a 2 ,π). r=4cos θ. | r | r=4cos θ. r=0, θ θ= π 2 ±kπ. (0, π 2) . r, θ=0±2kπ. θ=0

r=4cos θ r=4cos(0) r=4(1)=4

(4,0). r=4cos θ θ [0, π]. θπ 6π 4π 3π 22π 33π 45π 6πr r=a±bcos θ r=a±bsin θ a>0, b>0, a b =1. r=0. r θ. r=2+2cos θ. r=0, θ=π+2kπ. (0,π). r=2+2cos θ cos θ cos θ=1 θ=0. θ=0 r.

r=2+2cos(0) r=2+2(1)=4

(4,0) [0, π]. θ0π 4π 22π 3πr 1< a b <2 a b ≥2. r=a±bcos θ r=a±bsin θ a>0, b>0, and 1< a b <2. r=4−3sin θ. θ= π 2 , r=0 θ θ θ sin θ>1. θ θ sin θ>1. r θ=0.

r(0)=4−3sin(0) r=4−3·0=4

(4,0). θ= π 2 , θ= π 2 r. r=1.θ0π 6π 3π 22π 35π 6π7π 64π 33π 25π 311π 62πr sin θ θ= π 2 , r=3−2cos θ. r=a±bcos θ r=a±bsin θ a>0, b>0, a<b. r=2+5cos θ. r=0, θ=1.98. | r | cos θ=1 θ=0. θ0π 6 π 3π 22π 35π 6π7π 64π 33π 25π 311π 62πr θ=π. ∞ r 2 = a 2 cos 2θ r 2 = a 2 sin 2θ a≠0. r 2 = a 2 sin 2θ r 2 = a 2 cos 2θ θ= π 2 , r 2 =4cos 2θ. θ= π 2 , u=2θ.

0=4cos 2θ 0=4cos u 0=cos u cos −1 0= π 2 u= π 2

Substitute 2θ back in for u. 2θ= π 2 θ= π 4

(0, π 4) cos u=1 u=0, cos 2θ=1 2θ=0.

r 2 =4cos(0) r 2 =4(1)=4 r=± 4 =2

θ= π 2 ,θπ 6π 4π 3π 2r22 u=2θ r. θ. 4cos(2θ) r=acos nθ r=asin nθ a≠0. n 2n n n r=2cos 4θ. θ= π 2 u=4θ.

0=2cos 4θ 0=cos 4θ 0=cos u cos −1 0=u u= π 2

4θ= π 2 θ= π 8

θ= π 8 . (0, π 8) | r |. cos u=1 θ=0.

r=2cos(4·0) r=2cos(0) r=2(1)=2

(2,0) θπ 8π 43π 8π 25π 83π 4r r=0 θ= π 8 , π 8 r=0, r=2, 2n n r=4sin(2θ). n r=2sin(5θ). θ= π 2 . u=5θ.

0=2sin(5θ) 0=sin u sin −1 0=0 u=0 5θ=0 θ=0

sin θ

r=2sin(5· π 2) r=2(1)=2

n n, θπ 6π 3π 22π 35π 6πr n r=3cos(3θ). n r=θ θ≥0. θ r [0,2π], r θ r=θ [0,2π]. r θ, θπ 4π 2π3π 27π 4

2πr [0,2π]. (−∞,∞). r=−θ [0,4π]. θ= π 2 , θ r. θ r=0 θ. r=acos θ r=asin θ. r=a±bcos θ r=a±bsin θ, a>0, b>0, a b =1. r=a±bcos θ r=a±bsin θ 1< a b <2. r=a±bcos θ r=a±bsin θ a>0, b>0, a<b. r 2 = a 2 cos 2θ r 2 = a 2 sin 2θ, a≠0. r=acos nθ r=asin nθ, a≠0; n 2n n n r=θ, θ≥0. x θ= π 2 y θ=0, π 2 , π and 3π 2 , r=5cos 3θr=3−3cos θr=3+2sin θr=3sin 2θθ= π 2 ,r=4r=2θr=4cos θ 2r= 2 θ r=3 1− cos 2 θr= 5sin 2θr=3cos θr=4sin θr=2+2cos θr=2−2cos θr=5−5sin θr=3+3sin θr=3+2sin θ r=7+4sin θr=4+3cos θr=5+4cos θr=10+9cos θr=1+3sin θr=2+5sin θr=5+7sin θr=2+4cos θr=5+6cos θ r 2 =36cos(2θ)r 2 =10cos(2θ)r 2 =4sin(2θ)r 2 =10sin(2θ)r=3sin(2θ)r=3cos(2θ)r=5sin(3θ)r=4sin(4θ) r=4sin(5θ)r=−θr=2θr=−3θr= 1 θr= 1 θr=2sin θtan θ,r=2 1− sin 2 θr=5+cos(4θ)r=2−sin(2θ)r= θ 2 r=θ+1r=θsin θr=θcos θ [0,4π] r=θ,r=−θr=θ,r=θ+sin θr=sin θ+θ,r=sin θ−θr=2sin(θ 2),r=θsin(θ 2) θ 4π r=sin(cos(3θ)) r=sin(3θ) r=sin(16 5 θ) [0,4π],[0,8π],[0,12π], [0,16π]. r=sin θ+ (sin(5 2 θ)) 3 [0,4π].

r 1 =3sin(3θ) r 2 =2sin(3θ) r 3 =sin(3θ)

r 1 =3+3cos θ r 2 =2+2cos θ r 3 =1+cos θ

r 1 =3θ r 2 =2θ r 3 =θ

r 1 =3+2sin θ, r 2 =2r 1 =6−4cos θ, r 2 =4(4, π 3),(4, 5π 3)r 1 =1+sin θ, r 2 =3sin θ r 1 =1+cos θ, r 2 =3cos θ(3 2 , π 3),(3 2 , 5π 3)r 1 =cos(2θ), r 2 =sin(2θ) r 1 = sin 2 (2θ), r 2 =1−cos(4θ)(0, π 2), (0,π), (0, 3π 2), (0,2π)r 1 = 3 , r 2 =2sin(θ) r 1 2 =sin θ, r 2 2 =cos θ(8 4 2 , π 4), (8 4 2 , 5π 4) θ= 3π 4 , 7π 4 r r 1 =1+cos θ, r 2 =1−sin θ r=θ. r=aθ. r=θ, r=a±bcos θ r=a±bsin θ, a b =1 r=a±bcos θ r=a±bsin θ a b ≥2 r=a±bcos θ r=a±bsin θ 1< a b <2 r=a±bcos θ r=a±b sin θ a<b r 2 = a 2 cos 2θ r 2 = a 2 sin 2θ, a≠0 r=a±bcos θ r=a±bsin θ a>0,b>0, a b >1; r=acos nθ r=asin nθ; n 2n n n a+bi a, bi. a+bi, a b 2−3i 1+5i | z |. z=2+4i, | z |. z=x+yi, z

| z |= x 2 + y 2

(x,y). (0, 0). z= 5 −i.

| z |= x 2 + y 2 | z |= (5) 2 + (−1) 2 | z |= 5+1 | z |= 6

z=12−5i. z=3−4i, | z |.

| z |= x 2 + y 2 | z |= (3) 2 + (−4) 2 | z |= 9+16 | z |= 25 | z |=5

z z=1−7i, | z |.| z |= 50 =5 2 θ r. z=x+yi,

x=rcos θ y=rsin θ r= x 2 + y 2

(x,y). r, θ

z=x+yi z=rcos θ+(rsin θ)i z=r(cos θ+isin θ)

x=rcos θ y=rsin θ r= x 2 + y 2

z=x+yi z=(rcos θ)+i(rsin θ) z=r(cos θ+isin θ)

r θ rcis θ r(cos θ+isin θ). 4i z=4i z=0+4i. r

r= x 2 + y 2 r= 0 2 + 4 2 r= 16 r=4

x. x=rcos θ, x=0, θ= π 2 . z=0+4i z=4(cos(π 2)+isin(π 2)) 4cis(π 2). z=3i r cis θ z=3(cos(π 2)+isin(π 2)) −4+4i. r.

r= x 2 + y 2 r= (−4) 2 +(4 2) r= 32 r=4 2

θ

cos θ= x r cos θ= −4 4 2 cos θ=− 1 2 θ= cos −1 (− 1 2)= 3π 4

4 2 cis(3π 4). z= 3 +i z=2(cos(π 6)+isin(π 6)) z=r(cos θ+isin θ), cos θ sin θ. r.

z=12(cos(π 6)+isin(π 6))

cos(π 6)= 3 2 and sin(π 6)= 1 2

z=12(3 2 + 1 2 i)

z=12(3 2 + 1 2 i) =(12) 3 2 +(12) 1 2 i =6 3 +6i

6 3 +6i. r=13 tan θ= 5 12 . tan θ= 5 12 , tan θ= y x , r= x 2 + y 2 = 12 2 + 5 2 =13. cos θ= x r sin θ= y r .

z=13(cos θ+isin θ) =13(12 13 + 5 13 i) =12+5i

12+5i.

z=4(cos 11π 6 +isin 11π 6)

z=2 3 –2i z 1 = r 1 (cos θ 1 +isin θ 1) z 2 = r 2 (cos θ 2 +isin θ 2),
z 1 z 2 = r 1 r 2 [cos(θ 1 + θ 2)+isin(θ 1 + θ 2)] z 1 z 2 = r 1 r 2 cis(θ 1 + θ 2)
z 1 z 2 , z 1 =4(cos(80°)+isin(80°)) z 2 =2(cos(145°)+isin(145°)).
z 1 z 2 =4·2[cos(80°+145°)+isin(80°+145°)] z 1 z 2 =8[cos(225°)+isin(225°)] z 1 z 2 =8[cos(5π 4)+isin(5π 4)] z 1 z 2 =8[– 2 2 +i(– 2 2)] z 1 z 2 =–4 2 –4i 2
z 1 = r 1 (cos θ 1 +isin θ 1) z 2 = r 2 (cos θ 2 +isin θ 2),
z 1 z 2 = r 1 r 2 [cos(θ 1 – θ 2)+isin(θ 1 – θ 2)], z 2 ≠0 z 1 z 2 = r 1 r 2 cis(θ 1 – θ 2), z 2 ≠0
r 1 r 2 . θ 1 – θ 2 . z=r(cos θ+isin θ). r r 1 r 2 , θ θ 1 – θ 2 . r. z 1 =2(cos(213°)+isin(213°))
z 2 =4(cos(33°)+isin(33°)).
z 1 z 2 = 2 4 [cos(213°–33°)+isin(213°–33°)] z 1 z 2 = 1 2 [cos(180°)+isin(180°)] z 1 z 2 = 1 2 [–1+0i] z 1 z 2 =– 1 2 +0i z 1 z 2 =– 1 2
z 1 =2 3 (cos(150°)+isin(150°)) z 2 =2(cos(30°)+isin(30°)). z 1 z 2 =–4 3 ; z 1 z 2 =– 3 2 + 3 2 i
n, z n nth n. z=r(cos θ+isin θ)
z n = r n [cos(nθ)+isin(nθ)] z n = r n cis(nθ)
n (1+i) 5 (1+i) r.
r= x 2 + y 2 r= (1) 2 + (1) 2 r= 2
θ. tan θ= y x
tan θ= 1 1 tan θ=1 θ= π 4
(a+bi) n = r n [cos(nθ)+isin(nθ)] (1+i) 5 = (2) 5 [cos(5· π 4)+isin(5· π 4)] (1+i) 5 =4 2 [cos(5π 4)+isin(5π 4)] (1+i) 5 =4 2 [– 2 2 +i(– 2 2)] (1+i) 5 =–4–4i
nth nth nth
z 1 n = r 1 n [cos(θ n + 2kπ n)+isin(θ n + 2kπ n)]
k=0, 1, 2, 3, . . . , n–1. 2kπ n θ n z=8(cos(2π 3)+isin(2π 3)).
z 1 3 = 8 1 3 [cos(2π 3 3 + 2kπ 3)+isin(2π 3 3 + 2kπ 3)] z 1 3 =2[cos(2π 9 + 2kπ 3)+isin(2π 9 + 2kπ 3)]
k=0, 1, 2. k=0,
z 1 3 =2(cos(2π 9)+isin(2π 9))
k=1,
z 1 3 =2[cos(2π 9 + 6π 9)+isin(2π 9 + 6π 9)] Add 2(1)π 3 to each angle. z 1 3 =2(cos(8π 9)+isin(8π 9))
k=2,
z 1 3 =2[cos(2π 9 + 12π 9)+isin(2π 9 + 12π 9)] Add 2(2)π 3 to each angle. z 1 3 =2(cos(14π 9)+isin(14π 9))
k=1,
2π 3 3 + 2(1)π 3 = 2π 3 (1 3)+ 2(1)π 3 (3 3) = 2π 9 + 6π 9 = 8π 9
16(cos(120°)+isin(120°)).z 0 =2(cos(30°)+isin(30°))z 1 =2(cos(120°)+isin(120°))
z 2 =2(cos(210°)+isin(210°))z 3 =2(cos(300°)+isin(300°)) a+bi | z |= a 2 + b 2 . x=rcos θ,y=rsin θ,
r= x 2 + y 2 . z=r(cos θ+isin θ). r. z n , r n, θ n. a+bi. i= –1 x=rcosθ y=rsinθ.
z n = r n (cos(nθ)+isin(nθ)) 5+3i–7+i5 2–3–3i2 –6i382i2.2–3.1i14.452+2i8–4i4 5 cis(333.4°)
– 1 2 – 1 2 i3 +i2cis(π 6)3iz=7cis(π 6)7 3 2 +i 7 2z=2cis(π 3)z=4cis(7π 6)–2 3 –2iz=7cis(25°)
z=3cis(240°)–1.5–i 3 3 2z= 2 cis(100°) z 1 z 2 z 1 =2 3 cis(116°); z 2 =2cis(82°)4 3 cis(198°)
z 1 = 2 cis(205°); z 2 =2 2 cis(118°)z 1 =3cis(120°); z 2 = 1 4 cis(60°)3 4 cis(180°)
z 1 =3cis(π 4); z 2 =5cis(π 6)z 1 = 5 cis(5π 8); z 2 = 15 cis(π 12)5 3 cis(17π 24)
z 1 =4cis(π 2); z 2 =2cis(π 4) z 1 z 2 z 1 =21cis(135°); z 2 =3cis(65°)7cis(70°)
z 1 = 2 cis(90°); z 2 =2cis(60°)z 1 =15cis(120°); z 2 =3cis(40°)5cis(80°)
z 1 =6cis(π 3); z 2 =2cis(π 4)z 1 =5 2 cis(π); z 2 = 2 cis(2π 3)5cis(π 3)
z 1 =2cis(3π 5); z 2 =3cis(π 4) z 3 z=5cis(45°).125cis(135°) z 4 z=2cis(70°). z 2
z=3cis(120°).9cis(240°) z 2 z=4cis(π 4). z 4 z=cis(3π 16).cis(3π 4) z 3 z=3cis(5π 3). z
z=27cis(240°). 3cis(80°),3cis(200°),3cis(320°) z z=16cis(100°). z z=32cis(2π 3).

2 4 3 cis(2π 9),2 4 3 cis(8π 9),2 4 3 cis(14π 9) z z=32cis(π). z z=8cis(7π 4).
2 2 cis(7π 8),2 2 cis(15π 8)2+4i−3−3i5−4i−1−5i3+2i2i−46−2i−2+i1−4i 5+5i 3−2i 3.61 e −0.59i
−3−8i 4cis(120°) −2+3.46i 2cis(45°) 5cis(210°) −4.33−2.50i nth n, z n nth n (x,y); θ r;
x=rcos θ, y=rsin θ, r= x 2 + y 2 x(t) y(t) t t, x y x y t. x t, x(t), y t, y(t). x(t) y(t) (x(t), y(t)). x y ,
x,y, t. x y. y=f(x), r 2 = x 2 + y 2 . y y=± r 2 − x 2 , y 1 = r 2 − x 2 y 2 =− r 2 − x 2 . y 1 y 2 t I.
(x(t), y(t)), x=f(t) y=g(t), t. x=f(t) y=g(t) y= x 2 −1 x(t)=t. x(t)=t, y(t) x x(t). y(t)= t 2 −1.tx(t)
y(t)−4−4y(−4)= (−4) 2 −1=15−3−3y(−3)= (−3) 2 −1=8−2−2y(−2)= (−2) 2 −1=3−1−1
y(−1)= (−1) 2 −1=000y(0)= (0) 2 −1=−111y(1)= (1) 2 −1=022y(2)= (2) 2 −1=333
y(3)= (3) 2 −1=844y(4)= (4) 2 −1=15 t y(t)= t 2 −1 y= x 2 −1 y= x 2 −1.
x(t)=t−3, y(t)=2t+4; −1≤t≤2.tx(t)y(t)−1−420−341−262−18 y=1− x 2 , x(t)=t. x(t)=t t x y
y(t)=1− t 2 .

x(t)=t y(t)=1− t 2

t=0, t=−3 t=3. x(t) t x(t)=t. y(t). tx(t)=ty(t)=1− t 2−3−3y(−3)=1− (−3) 2 =−8−2−2
y(−2)=1− (−2) 2 =−3−1−1y(−1)=1− (−1) 2 =000y(0)=1−0=111y(1)=1− (1) 2 =022y(2)=1− (2) 2 =−3
33y(3)=1− (3) 2 =−8 y=1− t 2 [−3, 3], t. t. x=0, x x= y 3 −2y.x(t)= t 3 −2t y(t)=t (−5, 3) (3, −1) −5
8 m 4 s , 2 m/s. x(t)=2t−5. y=mx+b,2t=mx −5=b. −1, −4 m 4 s , −1m/s. y(t)=−t+3. x y t

x(t)=2t−5 y(t)=−t+3

t,x, y t t tx(t)=2t−5y(t)=−t+30x=2(0)−5=−5y=−(0)+3=31x=2(1)−5=−3y=−(1)+3=22x=2(2)−5=−1
y=−(2)+3=13x=2(3)−5=1y=−(3)+3=04x=2(4)−5=3y=−(4)+3=−1 x t, y t, y x, t. x y. t t. t x y.
x(t)= t 2 +1 y(t)=2+t, y t.

y=2+t y−2=t

y−2 t x(t).
x= t 2 +1 x= (y−2) 2 +1 Substitute the expression for t into x. x= y 2 −4y+4+1 x= y 2 −4y+5 x= y 2
−4y+5

x= y 2 −4y+5. x y. (1,2), x(t) y(t), t x y y x. y x.

x(t)=2 t 2 +6 y(t)=5−t

y=5− 1 2 x−3 x(t)= e −t y(t)=3 e t , t>0. e t .

x= e −t e t = 1 x

y(t).

y=3 e t y=3(1 x) y= 3 x

y= 3 x . t>0. y= 3 x x≠0. x(t)= t +2 y(t)=log(t). t.

x= t +2 x−2= t (x−2) 2 =t Square both sides.

ty

y=log(t) y=log (x−2) 2

y=log (x−2) 2 . x= t +2 t>0; x x>2. y=log(t) t>0; y=log (x−2) 2 x>2.

x(t)= t 2 y(t)=ln t t>0

y=ln x

x(t)=acos t y(t)=bsin t

cos t sin t,

x a =cos t y b =sin t

cos 2 t+ sin 2 t=1

cos 2 t+ sin 2 t= (x a) 2 + (y b) 2 =1

0≤t≤2π

x(t)=4cos t y(t)=3sin t

cos t sin t,

x=4cos t x 4 =cos t y=3sin t y 3 =sin t

cos 2 t+ sin 2 t=1 (x 4) 2 + (y 3) 2 =1 x 2 16 + y 2 9 =1

x 2 16 + y 2 9 =1 (0,0). t=0 (4,0), t= π 2 (0,3). t. x(t)=2cos t y(t)=3sin t.x 2 4 + y 2 9 =1 x(t)=t.
y(t)

$$x(t)=t\ y(t)=t^2-3$$

$x\ t.\ y=x^2-3.$

$$x(t)=3t-2\ y(t)=t+1$$

$x\ t.\ y$

$$x=3t-2\ \ x+2=3t\ x+2\ 3=t$$

$t\ y$

$$y=t+1\ y=(x+2\ 3)+1\ y=x\ 3+2\ 3+1\ y=1\ 3\ x+5\ 3$$

$y\ t\ x$

$$y=t+1\ y-1=t$$

$y.$

$$x=3(y-1)-2 \qquad x=3y-3-2 \qquad x=3y-5\ \ x+5=3y\ x+5\ 3=y \qquad y=1\ 3\ x+5\ 3$$

$x(t)=t\ 3\ \ y(t)=t\ 6\ .y=x\ 2\ x,\ y\ \ x\ \ y=(x+3)\ 2+1.\ x(t)=t.\ \ y(t)=(t+3)\ 2+1.\ x=t+3?$

$$y=(x+3)\ 2+1\ y=((t+3)+3)\ 2+1\ y=(t+6)\ 2+1$$

$$x(t)=t+3\ y(t)=(t+6)\ 2+1$$

$x\ y,\ t,\ x\ \ y\ \ t,\ t,\ t\ \ x\ \ x=f(t)\ \ y=f(t).\ t,\ t\{\ x(t)=5-t\ y(t)=8-2ty=-2+2x\{\ x(t)=6-3t\ y(t)=10-t$
$\{\ x(t)=2t+1\ y(t)=3\ ty=3\ x-1\ 2\{\ x(t)=3t-1\ y(t)=2\ t\ 2\{\ x(t)=2\ e\ t\ y(t)=1-5tx=2\ e\ 1-y\ 5$
$y=1-5\ln(x\ 2)\{\ x(t)=e^{-2t}\ y(t)=2\ e^{-t}\{\ x(t)=4\log(t)\ y(t)=3+2tx=4\log(y-3\ 2)\{\ x(t)=\log(2t)\ y(t)=t-1$
$\{\ x(t)=t\ 3-t\ y(t)=2tx=(y\ 2)\ 3-y\ 2\{\ x(t)=t-t\ 4\ y(t)=t+2\{\ x(t)=e\ 2t\ y(t)=e\ 6ty=x\ 3$
$\{\ x(t)=t\ 5\ y(t)=t\ 10\{\ x(t)=4\cos t\ y(t)=5\sin t\ (x\ 4)\ 2+(y\ 5)\ 2=1\{\ x(t)=3\sin t\ y(t)=6\cos t$
$\{\ x(t)=2\cos 2t\ y(t)=-\sin t\ y\ 2=1-1\ 2\ x\{\ x(t)=\cos t+4\ y(t)=2\sin 2t\{\ x(t)=t-1\ y(t)=t\ 2y=x\ 2+2x+1$
$\{\ x(t)=-t\ y(t)=t\ 3+1\{\ x(t)=2t-1\ y(t)=t\ 3-2y=(x+1\ 2)\ 3-2x-y\{\ x(t)=2t-1\ y(t)=t+4\{\ x(t)=4-t\ y(t)=3t+2$
$y=-3x+14\{\ x(t)=2t-1\ y(t)=5t\{\ x(t)=4t-1\ y(t)=4t+2y=x+3x(t)=t\ y(t)=t.y(x)=3\ x\ 2+3y(x)=2\sin x+1$
$\{\ x(t)=t\ y(t)=2\sin t+1x(y)=3\log(y)+yx(y)=y+2y\{\ x(t)=t+2t\ y(t)=tx(t)=a\cos t\ y(t)=b\sin t.$
$x\ 2\ 4+y\ 2\ 9=1x\ 2\ 16+y\ 2\ 36=1\{\ x(t)=4\cos t\ y(t)=6\sin t\ ;\ x^2+y^2=16x^2+y^2=10$
$\{\ x(t)=10\ \text{cost}\ y(t)=10\ \text{sint}\ ;\ (3,0)\ (-2,-5)\ (3,0)\ t=0,\ (-2,-5)\ t=1.\ (-1,0)\ (3,-2)\ (-1,0)\ t=0,\ (3,-2)$
$t=1.\{\ x(t)=-1+4t\ y(t)=-2t\ (-1,5)\ (2,3)\ (-1,5)\ t=0,\ (2,3)\ t=1.\ (4,1)\ (6,-2)\ (4,1)\ t=0,\ (6,-2)\ t=1.$
$\{\ x(t)=4+2t\ y(t)=1-3t\{\ x\ 1\ (t)=3t\ y\ 1\ (t)=2t-1\ \text{and}\ \{\ x\ 2\ (t)=t+3\ y\ 2\ (t)=4t-4$
$\{\ x\ 1\ (t)=t\ 2\ y\ 1\ (t)=2t-1\ \text{and}\ \{\ x\ 2\ (t)=-t+6\ y\ 2\ (t)=t+1t=2\{\ x\ 1\ (t)=3\ t\ 2-3t+7\ y\ 1\ (t)=2t+3txy$
$\{\ x\ 1\ (t)=t\ 2-4\ y\ 1\ (t)=2\ t\ 2-1txytxy\{\ x\ 1\ (t)=t\ 4\ y\ 1\ (t)=t\ 3+4txy\ y=(x+1)\ 2.$
$\{\ x(t)=t-1\ y(t)=t\ 2\ \text{and}\ \{\ x(t)=t+1\ y(t)=(t+2)\ 2\ y=3x-2.\ y=x\ 2-4x+4.$
$\{\ x(t)=t\ y(t)=t\ 2-4t+4\ \text{and}\ \{\ x(t)=t+2\ y(t)=t\ 2\ x\ \ y\ 45°\ \ t,x(t),\text{and}\ \ y(t).xyt\ (x,y).$
$x(t)=t\ 2+1,\ y(t)=2+t.\ t,x(t),\ y(t),\ tx(t)=t\ 2+1y(t)=2+t-526-3-417-2-310-1-250-121012123254310$
$541765267\ (1,2),\ t\ \ t\ \ t.\ t,t.\ x=t\ ,\ y=2t+3,\ \ 0\le t\le 3.$

$$x=2\cos t\ y=4\sin t$$

$t,\ x\ \ y.\ \ t\ tx=2\cos ty=4\sin tx=2\cos(0)=2y=4\sin(0)=0\pi\ 6x=2\cos(\pi\ 6)=3y=4\sin(\pi\ 6)=2\pi\ 3$
$x=2\cos(\pi\ 3)=1y=4\sin(\pi\ 3)=2\ 3\pi\ 2x=2\cos(\pi\ 2)=0y=4\sin(\pi\ 2)=42\pi\ 3x=2\cos(2\pi\ 3)=-1$
$y=4\sin(2\pi\ 3)=2\ 35\pi\ 6x=2\cos(5\pi\ 6)=-\ 3y=4\sin(5\pi\ 6)=2\pi x=2\cos(\pi)=-2y=4\sin(\pi)=07\pi\ 6$
$x=2\cos(7\pi\ 6)=-\ 3y=4\sin(7\pi\ 6)=-24\pi\ 3x=2\cos(4\pi\ 3)=-1y=4\sin(4\pi\ 3)=-2\ 33\pi\ 2x=2\cos(3\pi\ 2)=0$
$y=4\sin(3\pi\ 2)=-45\pi\ 3x=2\cos(5\pi\ 3)=1y=4\sin(5\pi\ 3)=-2\ 311\pi\ 6x=2\cos(11\pi\ 6)=3y=4\sin(11\pi\ 6)=-2$
$2\pi x=2\cos(2\pi)=2y=4\sin(2\pi)=0x\ y,\ t\ \ Y=$

$$X\ 1T=\qquad Y\ 1T=$$

$Y\ 1=.\ x=5\cos t,\ y=3\sin t.\ x=5\cos t\ \ y=2\sin t.\ tx=5\cos ty=2\sin t0x=5\cos(0)=5y=2\sin(0)=01x=5\cos(1)\approx 2.7$
$y=2\sin(1)\approx 1.72x=5\cos(2)\approx -2.1y=2\sin(2)\approx 1.83x=5\cos(3)\approx -4.95y=2\sin(3)\approx 0.284x=5\cos(4)\approx -3.3$
$y=2\sin(4)\approx -1.55x=5\cos(5)\approx 1.4y=2\sin(5)\approx -1.9-1x=5\cos(-1)\approx 2.7y=2\sin(-1)\approx -1.7-2x=5\cos(-2)\approx -2.1$
$y=2\sin(-2)\approx -1.8-3x=5\cos(-3)\approx -4.95y=2\sin(-3)\approx -0.28-4x=5\cos(-4)\approx -3.3y=2\sin(-4)\approx 1.5-5$
$x=5\cos(-5)\approx 1.4y=2\sin(-5)\approx 1.9\ (x,y)\ \ t\ \ x(t)\ \ y(t),\ t\ y(x)\ t\ \ x,\ \ x(y)\ t\ \ y.$

$$x=5\cos t\ x\ 5=\cos t\ \text{Solve for cos t.}\ \ y=2\sin t\ \ \text{Solve for sin t.}\ y\ 2=\sin t$$

$$\cos 2t+\sin 2t=1\ (x\ 5)\ 2+(y\ 2)\ 2=1\ x\ 2\ 25+y\ 2\ 4=1$$

$x=t+1\ \ y=t\ ,\ t\ge 0,\ y=x-1\ \ \ y=t\ ,\ t\ge 0\ \ t\ \ t.\ \ x=2\cos\theta\ \ \text{and}\ \ y=4\sin\theta,\theta v\ 0\ ,h\theta v\ 0\ ,h$

$$x=(v 0 \cos\theta)t \quad y=- 1 2 g t 2 +(v 0 \sin\theta)t+h$$

g h g=32 ft / s 2 g=9.8 m / s 2 . x y x=(v 0 cos θ)t. v 0 . cos θ cos θ. y=− 1 2 g t 2 +(v 0 sin θ)t+h. − 1 2 g t 2 g=32 ft/s 2 g=9.8 m/s 2 . v 0 , h. t. 45° x.

$$x=(v 0 \cos\theta)t \; x=(140\cos(45°))t$$

y.

$$y=-16 t 2 +(v 0 \sin\theta)t+h \; y=-16 t 2 +(140\sin(45°))t+3$$

$$x=(140\cos(45°))(2) \; x=198 \text{ feet } y=-16 (2) 2 +(140\sin(45°))(2)+3 \; y=137 \text{ feet}$$

y=0.

$$y=-16 t 2 +(140\sin(45 ◦))t+3 \; y=0$$ Set y(t)=0 and solve the quadratic. t=6.2173

t=6.2173 t, t y.

$$x=(140\cos(45°))t \; 400=(140\cos(45°))t \quad t=4.04 \quad y=-16 (4.04) 2 +(140\sin(45°))(4.04)+3$$

$$y=141.8$$

x y t,x(t), y(t). t x y. x=(v 0 cos θ)t y=−16 t 2 +(v 0 sin θ)t+h. v 0 . θ h t.{ x(t)=t y(t)= t 2 −1txy −3−2−10123

$$\{ x(t)=t-1 \; y(t)= t 2$$

t−3−2−1012xy{ x(t)=2+t y(t)=3−2tt−2−10123xy{ x(t)=−2−2t y(t)=3+tt−3−2−101xy { x(t)= t 3 y(t)=t+2t−2−1012xy{ x(t)= t 2 y(t)=t+3t−2−1012xy{ x(t)=t y(t)= t{ x(t)=− t y(t)=t { x(t)=5−| t | y(t)=t+2{ x(t)=−t+2 y(t)=5−| t |{ x(t)=4sin t y(t)=2cos t{ x(t)=2sin t y(t)=4cos t { x(t)=3 cos 2 t y(t)=−3sin t{ x(t)=3 cos 2 t y(t)=−3 sin 2 t{ x(t)=sec t y(t)=tan t{ x(t)=sec t y(t)= tan 2 t { x(t)= 1 e 2t y(t)= e − t{ x(t)=t−1 y(t)=− t 2{ x(t)= t 3 y(t)=t+3{ x(t)=2cos t y(t)=−sin t { x(t)=7cos t y(t)=7sin t{ x(t)= e 2t y(t)=− e tx= t 2 , y = 3t, 0≤t≤5x=2t, y = t 2 , −5≤t≤5 x=t, y= 25− t 2 , 0<t≤5x(t)=−t,y(t)= t , t≥0x=−2cos t, y=6 sin t, 0≤t≤πx=−sec t, y=tan t, − π 2 <t< π 2

$$x(t)=a\cos((a+b)t) \; y(t)=a\cos((a-b)t)$$

[−π,0], a=2 b=1, [−π,0], a=3 b=2 [−π,0], a=4 b=3 [−π,0], a=5 b=4 a b, a b a=100 b=99. b a? x(t)= t 2 y(t)=6−3t x(t) x(t)=− t 2 y(t) y(t)= t 2 x(t) y(t)=− t 2 x(t) (0,0), { x(t)=5cost y(t)=5sint (0,0), [−3,3] [−3,3] [0,2π) a b

$$\{ x(t)=\sin(at) \; y(t)=\sin(bt)$$

a=1,b=2a=2,b=1a=3,b=3a=5,b=5a=2,b=5a=5,b=2 { x(t)=acos(bt) y(t)=csin(dt) . a,b,c,da=4, b=3, c=6, d=1 a=4, b=2, c=3, d=3{ x(t)=cost−1 y(t)=sint+t{ x(t)=cost+t y(t)=sint−1{ x(t)=t−sint y(t)=cost−1 [0, 2π]. [0,4π]. [−4π,6π]. sin t cos t y y(t)=−16 t 2 +20ty(x)=−16 (x 15) 2 +20(x 15) y(t)=−16 t 2 +10t+5. t. { x(t)=64tcos(52°) y(t)=−16 t 2 +64tsin(52°) x { x(t)=14cos t−cos(14t) y(t)=14sin t+sin(14t) [0,2π] { x(t)=6sin t+2sin(6t) y(t)=6cos t−2cos(6t) [0,2π]{ x(t)=2sin t+5cos(6t) y(t)=5cos t−2sin(6t) [0,2π] { x(t)=5sin(2t)sint y(t)=5sin(2t)cost [0,2π]vijv, u, w, v → , u → , w → . P Q, PQ → . (0,0) (a,b), ⟨ a,b ⟩.⟨ a,b ⟩(0,0) ⟨ a,b ⟩. CD → C(x 1 , y 1) D(x 2 , y 2),

$$AB \rightarrow = \langle x 2 - x 1 , y 2 - y 1 \rangle = \langle a,b \rangle$$

CD → AB → . (0,0) ⟨ a,b ⟩. P(2,3) Q(6,4).

$$v=\langle 6-2,4-3 \rangle =\langle 4,1 \rangle$$

(0,0) (4,1). ⟨ 4,1 ⟩. v (−3,2) (4,5),

$$v=\langle 4-(-3),5-2 \rangle =\langle 7,3 \rangle$$

(0,0) (7,3). v (3,5). v=⟨ a,b ⟩,| v |= a 2 + b 2 . tan θ=(b a)⇒θ= tan −1 (b a), P(−8,1) Q(−2,−5).

$$u=\langle -2,-(-8),-5-1 \rangle =\langle 6,-6 \rangle$$

$$|u|= (6) 2 + (-6) 2 = 72 = 6 2$$

$$\tan\theta= -6 6 =-1 \Rightarrow \theta= \tan -1 (-1) =-45°$$

−45°+360°=315°. (5,−3) (−1,2) (−1,−3) (−7,2). v (5,−3) (−1,2). u (−1,−3) (−7,2).

$$v=\langle -1-5,2-(-3) \rangle =\langle -6,5 \rangle \; u=\langle -7-(-1),2-(-3) \rangle =\langle -6,5 \rangle$$

$$|v|= (-1-5) 2 + (2-(-3)) 2 = (-6) 2 + (5) 2 = 36+25 = 61 \; |u|= (-7-(-1)) 2 + (2-(-3)) 2 = (-6) 2 + (5) 2 = 36+25 = 61$$

$$\tan\theta = -5\,6 \Rightarrow \theta = \tan^{-1}(-5\,6) = -39.8°$$

$180°. -39.8° + 180° = 140.2°.u = \langle x,y \rangle (x,y), u = \langle 3,-2 \rangle v = \langle -1,4 \rangle,$

$$u+v = \langle 3,-2 \rangle + \langle -1,4 \rangle = \langle 3+(-1),-2+4 \rangle = \langle 2,2 \rangle$$

$v\ u.$

$$u+(-v) = \langle 3,-2 \rangle + \langle 1,-4 \rangle = \langle 3+1,-2+(-4) \rangle = \langle 4,-6 \rangle$$

$v = \langle a,b \rangle k$

$$kv = \langle ka,kb \rangle$$

$k\ v = \langle 3,1 \rangle, 1\ 2v,\ v = \langle 3,1 \rangle,$

$$3v = \langle 3 \cdot 3, 3 \cdot 1 \rangle = \langle 9,3 \rangle \quad 1\ 2\ v = \langle 1\ 2 \cdot 3,\ 1\ 2 \cdot 1 \rangle = \langle 3\ 2,\ 1\ 2 \rangle - v = \langle -3,-1 \rangle$$

$1\ 2v\ uu = \langle 5,4 \rangle .3u = \langle 15,12 \rangle u = \langle 3,-2 \rangle v = \langle -1,4 \rangle,$

$$3u = 3\langle 3,-2 \rangle = \langle 9,-6 \rangle\ 2v = 2\langle -1,4 \rangle = \langle -2,8 \rangle$$

$$w = 3u+2v = \langle 9,-6 \rangle + \langle -2,8 \rangle = \langle 9-2,-6+8 \rangle = \langle 7,2 \rangle$$

$w = \langle 7,2 \rangle .\ x\ \ y\ \langle 2,3 \rangle\ (0,0)\ (2,0).$

$$v\ 1 = \langle 2-0,0-0 \rangle = \langle 2,0 \rangle$$

$(0,0)\ (0,3).$

$$v\ 2 = \langle 0-0,3-0 \rangle = \langle 0,3 \rangle$$

$$v = \langle 2+0,3+0 \rangle = \langle 2,3 \rangle$$

$$|v| = |\ v\ 1\ |\ 2 + |\ v\ 2\ |\ 2 = 2\ 2 + 3\ 2 = 13$$

$13\ .\ \tan\theta = y\ x\ .$

$$\tan\theta = v\ 2\ v\ 1\ \tan\theta = 3\ 2 \quad \theta = \tan^{-1}(3\ 2) = 56.3°$$

$v\ \ 13\ \ 56.3 \circ v\ (3,2)\ (7,4).$

$$v = \langle 7-3,4-2 \rangle = \langle 4,2 \rangle$$

$v\ 1 = \langle 4,0 \rangle\ \ v\ 2 = \langle 0,2 \rangle .i = \langle 1,0 \rangle j = \langle 0,1 \rangle\ v\ \ v\ |\ v\ |\ \ v.\ v = \langle -5,12 \rangle.$

$$|v| = (-5)\ 2 + (12)\ 2 = 25+144 = 169 = 13$$

$|\ v\ |,$

$$v\ |\ v\ | = -5\ 13\ i + 12\ 13\ j$$

$$v\ |\ v\ | = \langle -5\ 13,\ 12\ 13 \rangle$$

$-5\ 13\ i + 12\ 13\ j$

$$(-5\ 13)\ 2 + (12\ 13)\ 2 = 25\ 169 + 144\ 169 = 169\ 169 = 1$$

$= 5\ 13 + 12\ 13 = \langle -5,12 \rangle .\ v\ P = (x\ 1, y\ 1)\ Q = (x\ 2, y\ 2),$

$$v = (x\ 2 - x\ 1)i + (y\ 1 - y\ 2)j$$

$(0,0)\ (a,b), (x\ 2 - x\ 1) = a\ (y\ 2 - y\ 1) = b, |\ v\ | = a\ 2 + b\ 2\ .\ v\ P = (2,-6)\ Q = (-6,6),\ i\ j.$

$$v = (x\ 2 - x\ 1)i + (y\ 2 - y\ 1)j = (-6-2)i + (6-(-6))j = -8i + 12j$$

$P\ 1 = (-1,3)\ \ P\ 2 = (2,7),\ v\ i\ j.$

$$v = (x\ 2 - x\ 1)i + (y\ 2 - y\ 1)j\ v = (2-(-1))i + (7-3)j = 3i + 4j$$

$u\ P = (-1,6)\ Q = (7,-5)\ i\ j.u = 8i - 11j\ i\ j,$

$$v+u = (a+c)i + (b+d)j\ v-u = (a-c)i + (b-d)j$$

$v\ 1 = 2i - 3j\ \ v\ 2 = 4i + 5j.$

$$v\ 1 + v\ 2 = (2+4)i + (-3+5)j = 6i + 2j$$

i and j. $|\ v\ |$ r. $v = \langle x,y \rangle\ \theta,$

$$\cos\theta = x\ |v| \text{ and } \sin\theta = y\ |v| \quad x = |v|\cos\theta \quad y = |v|\sin\theta$$

$v = xi + yj = |\ v\ |\cos\theta i + |\ v\ |\sin\theta j,\ |\ v\ | = x\ 2 + y\ 2\ .\ x = |\ v\ |\cos\theta i\ \ y = |\ v\ |\sin\theta j,$

$$x = 7\cos(135°)i = -7\ 2\ 2\ \ y = 7\sin(135°)j = 7\ 2\ 2$$

$v = 7\cos(135°)i + 7\sin(135°)j$

$$v = -7\ 2\ 2\ i + 7\ 2\ 2\ j$$

$(3,5).\ v = 34\cos(59°)i + 34\sin(59°)j\ \ 34\theta = \tan^{-1}(5\ 3) = 59.04°\ v = \langle a,b \rangle\ u = \langle c,d \rangle$

$$v \cdot u = ac + bd$$

$$\cos\theta = \frac{v}{|v|} \cdot \frac{u}{|u|}$$

$v = \langle 5,12 \rangle$ $u = \langle -3,4 \rangle$.

$$v \cdot u = \langle 5,12 \rangle \cdot \langle -3,4 \rangle = 5 \cdot (-3) + 12 \cdot 4 = -15 + 48 = 33$$

$$v_1 \cdot v_2 = \langle 5,2 \rangle \cdot \langle 3,7 \rangle = 5 \cdot 3 + 2 \cdot 7 = 15 + 14 = 29$$

$\cos\theta = \frac{v}{|v|} \cdot \frac{u}{|u|}$.

$$\frac{v}{|v|} \cdot \frac{u}{|u|} = \left\langle \frac{5}{\sqrt{29}} + \frac{2}{\sqrt{29}} \right\rangle \cdot \left\langle \frac{3}{\sqrt{58}} + \frac{7}{\sqrt{58}} \right\rangle = \frac{5}{\sqrt{29} \cdot \sqrt{3}} \sqrt{58} + \frac{2}{\sqrt{29} \cdot \sqrt{7}} \sqrt{58}$$

$$= \frac{15}{\sqrt{1682}} + \frac{14}{\sqrt{1682}} = \frac{29}{\sqrt{1682}} = 0.707107 \cos^{-1}$$

$$(0.707107) = 45°$$

$u = \langle -3,4 \rangle$ $v = \langle 5,12 \rangle$.

$$\theta = \cos^{-1}\left(\frac{u}{|u|} \cdot \frac{v}{|v|}\right)\left(\frac{u}{|u|} \cdot \frac{v}{|v|}\right) = \frac{-3i+4j}{5} \cdot \frac{5i+12j}{13} = \left(-\frac{3}{5} \cdot \frac{5}{13}\right) + \left(\frac{4}{5} \cdot \frac{12}{13}\right) = -\frac{15}{65} + \frac{48}{65} = \frac{33}{65}$$

$$\theta = \cos^{-1}\left(\frac{33}{65}\right) = 59.5^\circ$$

x α $140° + \alpha$. BCO AOC BCO x

$$x^2 = (16.2)^2 + (200)^2 - 2(16.2)(200)\cos(140°) \quad x^2 = 45{,}226.41 \quad x = \sqrt{45{,}226.41} \quad x = 212.7$$

$$\frac{\sin\alpha}{16.2} = \frac{\sin(140°)}{212.7} \quad \sin\alpha = \frac{16.2\sin(140°)}{212.7}$$

$$= 0.04896 \sin^{-1}(0.04896) = 2.8°$$

$|v| = \sqrt{a^2 + b^2}$. ij i j v a and b. $v = |v|\cos\theta i + |v|\sin\theta j$. i j u,v,w i j, $\langle a,b \rangle$, i j? i, j. $(5,2)$ $(-1,-3)$, $(0,0)$. $\langle a,b \rangle$. $(-4,2)$ $(3,-3)$, $(0,0)$. $\langle a,b \rangle$. $\langle 7,-5 \rangle$ $(7,-1)$ $(-1,-7)$, $(0,0)$. $\langle a,b \rangle$. u v u P 1 P 2 v P 3 P 4 P 1 = $(5,1)$, P 2 = $(3,-2)$, P 3 = $(-1,3)$, P 4 = $(9,-4)$ P 1 = $(2,-3)$, P 2 = $(5,1)$, P 3 = $(6,-1)$, P 4 = $(9,3)$ P 1 = $(-1,-1)$, P 2 = $(-4,5)$, P 3 = $(-10,6)$, P 4 = $(-13,12)$ P 1 = $(3,7)$, P 2 = $(2,1)$, P 3 = $(1,2)$, P 4 = $(-1,-4)$ P 1 = $(8,3)$, P 2 = $(6,5)$, P 3 = $(11,8)$, P 4 = $(9,10)$ P 1 = $(-3,1)$ P 2 = $(5,2)$, v i j. P 1 = $(6,0)$ P 2 = $(-1,-3)$, v i j. $7i - 3j - 6i - 2j$ u = $\langle 2,-3 \rangle$, v = $\langle 1,5 \rangle$ u = $\langle -3,4 \rangle$, v = $\langle -2,1 \rangle$ u+v = $\langle -5,5 \rangle$, u−v = $\langle -1,3 \rangle$, $2u - 3v = \langle 0,5 \rangle$ v. v. $-10i - 4j - \frac{2}{\sqrt{29}} i + \frac{5}{\sqrt{29}} j$

$d = -\frac{1}{3} i + \frac{5}{2} j - \frac{2}{229\sqrt{229}} i + \frac{15}{229\sqrt{229}} j - \frac{7}{2} 10 i + \frac{2}{10} j$ $0 \le \theta < 2\pi$. $\langle 0,4 \rangle \langle 6,5 \rangle$ $|v| = 7.810, \theta = 39.806°$ $\langle 2,-5 \rangle \langle -4,-6 \rangle$ $|v| = 7.211, \theta = 236.310°$ u·v. u·v. −6 u = $\langle -2,4 \rangle$ v = $\langle -3,1 \rangle$, u·v. = $\langle -1,6 \rangle = \langle 6,-1 \rangle$, u·v. −12 v, v, 1 2 v. $\langle 2,-1 \rangle \langle -1,4 \rangle \langle -3,-2 \rangle \langle 4,1 \rangle$ P 1 = $(2,1)$ P 2 = $(-1,2)$, v i j, P 1 = $(4,-1)$ P 2 = $(-3,2)$, v i j. v = $-7i + 3j$ P 1 = $(3,3)$ P 2 = $(-3,3)$, v i j. $|v| = 6, \theta = 45°$ $3\sqrt{2} i + 3\sqrt{2} j$ $|v| = 8, \theta = 220°$ $|v| = 2, \theta = 300°$ $i - \sqrt{3} j$ $|v| = 5, \theta = 135°$ $x = 7.13$ $y = 3.63$ $x = 2.87$ $y = 4.10$ $(5,7)$ $(0.081,8.602)$ $(7,3)$ α a, β b, γ c. $\beta = 50°, a = 105, b = 45$ $\alpha = 43.1°, a = 184.2, b = 242.8$ $C = 120°, a = 23.1, c = 34.1$ A A: α a, β b, γ c: a=4, b=6, c=8. $B = 71.0°, C = 55.0°, a = 12.8$ $(3, \pi 6)$. $(5, -2\pi 3)$ $(6, -3\pi 4)$ $(-2, 3\pi 2)$ $(0,2)$ $(7,-2)$ $(-9,-4)$ $(9.8489, 203.96°)$ $x = -2x^2 + y^2 = 64 r = 8x^2 + y^2 = -2yr = 7\cos\theta x^2 + y^2 = 7x r = \frac{-2}{4\cos\theta + \sin\theta} \theta = \frac{3\pi}{4}$ $y = -x r = 5\sec\theta r = \frac{4}{4\sin\theta} \theta = \frac{\pi}{2} r = 7$ $r = 1 - 5\sin\theta$. $r = 5\sin(7\theta)$. $r = 3 - 3\cos\theta - 2 + 6i - 4 - 3i5 + 9i$ $1\sqrt{2} - 3\sqrt{2} i$ $\text{cis}(-\pi 3)$ $z = 5\text{cis}(5\pi 6)$ $z = 3\text{cis}(40°)$ $2.3 + 1.9i$ $z_1 z_2$ $z_1 = 2\text{cis}(89°)$ $z_2 = 5\text{cis}(23°)$ $z_1 = 10\text{cis}(\pi 6)$ $z_2 = 6\text{cis}(\pi 3)$ $60\text{cis}(\pi 2)$ $z_1 z_2$ $z_1 = 12\text{cis}(55°)$ $z_2 = 3\text{cis}(18°)$ $z_1 = 27\text{cis}(5\pi 3)$ $z_2 = 9\text{cis}(\pi 3)$ $3\text{cis}(4\pi 3)$ z^4 $z = 2\text{cis}(70°)$ z^2 $z = 5\text{cis}(3\pi 4)$ $25\text{cis}(3\pi 2)$ z $z = 64\text{cis}(210°)$. z $z = 25\text{cis}(3\pi 2)$. $5\text{cis}(3\pi 4)$, $5\text{cis}(7\pi 4)$ $6 - 2i - 1 + 3i$ t { x(t)=3t−1 y(t)= t{ x(t)=−cos t y(t)=2 sin 2 t $\frac{x^2}{2} + \frac{1}{2} y = 1$ x(t)=acos t y(t)=bsin t $\frac{x^2}{25} + \frac{y^2}{16} = 1$. $(-2,3)$ $(4,7)$ $(-2,3)$ t=0 $(4,7)$ t=1.{ x(t)=−2+6t y(t)=3+4t { x(t)=3 t 2 y(t)=2t−1{ x(t)= e t y(t)=−2 e 5 ty=−2 x 5{ x(t)=3cos t y(t)=2sin t { x(t)=(80cos(40°))t y(t)=−16 t 2 +(80sin(40°))t+4 u v, u P 1 P 2 , v P 3 P 4 . P 1 = $(-1,4)$, P 2 = $(3,1)$, P 3 = $(5,5)$ P 4 = $(9,2)$ P 1 = $(6,11)$, P 2 = $(-2,8)$, P 3 = $(0,-1)$ P 4 = $(-8,2)$ u = $2i - j$, v = $4i - 3j$, w = $-2i + 5j$ $-\frac{3}{10}\sqrt{10} - \frac{10}{10}\sqrt{10} \langle 6,-2 \rangle \langle -3,-3 \rangle \frac{3}{2}\sqrt{2}$, 225° u·v. 16 = $\langle -3,4 \rangle$ $\frac{1}{2}$ P 1 = $(3,2)$ P 2 = $(-5,-1)$, v i j. α a, β b, γ c. $\beta = 68°, b = 21, c = 16.$ $\alpha = 67.1°, \gamma = 44.9°, a = 20.91712$ miles $(2,2)$ $(2, \pi 3)$ $(1, 3)$ $x^2 + y^2 = 5y$. r=−3csc θ. y=−3 r=−4sin(2θ). r=3+3cos θ. r=3−5sin θ. 5−9i. 106 4+i. z=5cis(2π 3).−5 2 +i 5 3 2 $z_1 = 8\text{cis}(36°)$ $z_2 = 2\text{cis}(15°)$, $z_1 z_2$ $z_1 z_2$ 4cis(21°)(z 2) 3z 1 2 2 cis(18°),2 2 cis(198°) −5−i t { x(t)=t+1 y(t)=2 t 2 .y=2 (x−1) 2 x(t)=acos t y(t)=bsin t: $\frac{x^2}{36} + \frac{y^2}{100} = 1$. { x(t)=−2sin t y(t)=5cos t . u·v. v. 2 13 13 i + 3 13 13 j v P 1 = $(2,2)$ P 2 = $(-1,0)$, v

i j. v, −v. (0,0) (a,b), v 1 =⟨ 1,0 ⟩ v 2 =⟨ 0,1 ⟩.

2x+y= 15 3x−y= 5

2(4)+(7)=15 True 3(4)−(7)=5 True

(x,y). (5,1)

x+3y=8 2x−9=y

(5,1)

(5)+3(1)=8 8=8 True 2(5)−9=(1) 1=1 True

(5,1) (8,5)

5x−4y=20 2x+1=3y

2x+y=−8 x−y=−1

y.

2x+y=−8 y=−2x−8

y.

x−y=−1 y=x+1

(−3,−2).

2(−3)+(−2)=−8 −8=−8 True (−3)−(−2)=−1 −1=−1 True

(−3,−2),

2x−5y=−25 −4x+5y=35

(−5,3).

−x+y=−5 2x−5y=1

y.

−x+y=−5 y=x−5

x−5 y

2x−5y=1 2x−5(x−5)=1 2x−5x+25=1 −3x=−24 x=8

x=8 y.

−(8)+y=−5 y=3

(8,3). (8,3)

−x+y=−5 −(8)+(3)=−5 True 2x−5y=1 2(8)−5(3)=1 True

x=y+3 4=3x−2y

(−2,−5)

x+2y=−1 −x+y=3

x x x

x+2y=−1 −x+y=3 3y=2

x, y.

3y=2 y= 2 3

y x.

−x+y=3 −x+ 2 3 =3 −x=3− 2 3 −x= 7 3 x=− 7 3

(− 7 3 , 2 3).

x+2y=−1 (− 7 3)+2(2 3)= − 7 3 + 4 3 = − 3 3 = −1=−1 True

3x+5y=−11 x−2y=11

3x x. −3,

x−2y=11 −3(x−2y)=−3(11) Multiply both sides by −3. −3x+6y=−33 Use the distributive property.

3x+5y=−11 −3x+6y=−33 _____ 11y=−44 y=−4

y=−4 x.

3x+5y=−11 3x+5(−4)=−11 3x−20=−11 3x=9 x=3

(3,−4).

x−2y=11 (3)−2(−4)=3+8 =11 True

2x−7y=2 3x+y=−20

(−6,−2)

$$2x+3y=-16 \quad 5x-10y=30$$

2x 5x. 10x x −5 2.

$$-5(2x+3y)=-5(-16) \quad -10x-15y=80 \qquad 2(5x-10y)=2(30) \quad 10x-20y=60$$

$$-10x-15y=80 \quad 10x-20y=60 \_\_\_\_\_ \quad -35y=140 \quad y=-4$$

y=−4

$$2x+3(-4)=-16 \quad 2x-12=-16 \quad 2x=-4 \quad x=-2$$

(−2,−4).

$$5x-10y=30 \quad 5(-2)-10(-4)=30 \quad -10+40=30 \quad 30=30$$

$$x\ 3 + y\ 6 = 3 \quad x\ 2 - y\ 4 = 1$$

$$6(x\ 3 + y\ 6) = 6(3) \quad 2x+y=18 \quad 4(x\ 2 - y\ 4) = 4(1) \quad 2x-y=4$$

−1

$$-1(2x-y)=-1(4) \quad -2x+y=-4$$

$$2x+y=18 \quad -2x+y=-4 \_\_\_\_\_ \quad 2y=14 \quad y=7$$

y=7

$$2x+(7)=18 \quad 2x=11 \quad x=11\ 2 \quad =7.5$$

(11 2 ,7).

$$x\ 2 - y\ 4 = 1 \quad 11\ 2\ 2 - 7\ 4 = 1 \quad 11\ 4 - 7\ 4 = 1 \quad 4\ 4 = 1$$

$$2x+3y=8 \quad 3x+5y=10$$

(10,−4)

y 12=0.

$$x=9-2y \quad x+2y=13$$

x,

$$x+2y=13 \quad (9-2y)+2y=13 \quad 9+0y=13 \quad 9=13$$

9≠13.

$$x=9-2y \quad 2y=-x+9 \quad y=-\ 1\ 2\ x+\ 9\ 2$$

$$x+2y=13 \quad 2y=-x+13 \quad y=-\ 1\ 2\ x+\ 13\ 2$$

$$y=-\ 1\ 2\ x+\ 9\ 2 \quad y=-\ 1\ 2\ x+\ 13\ 2$$

$$2y-2x=2 \quad 2y-2x=6$$

0=0.

$$x+3y=2 \quad 3x+9y=6$$

x. −3, x

$$x+3y=2 \quad (-3)(x+3y)=(-3)(2) \quad -3x-9y=-6$$

$$-3x-9y=-6 + \quad 3x+9y=6 \_\_\_\_\_ \quad 0=0$$

x+3y=2 3y=−x+2 y=− 1 3 x+ 2 3 3x+9y=6 9y=−3x+6 y=− 3 9 x+ 6 9 y=− 1 3 x+ 2 3

(x, − 1 3 x+ 2 3).

$$y-2x=5 \quad -3y+6x=-15$$

(x,2x+5). R=xp, x= p= x P(x)=R(x)−C(x). C(x)=0.85x+35,000 R(x)=1.55x, y

$$y=0.85x+35,000 \quad y=1.55x$$

0.85x+35,000 x.

$$0.85x+35,000=1.55x \quad 35,000=0.7x \quad 50,000=x$$

x=50,000

$$1.55(50,000)=77,500$$

(50,000,77,500). P(x)=R(x)−C(x).

$$P(x)=1.55x-(0.85x+35,000) \quad =0.7x-35,000$$

P(x)=0.7x−35,000. x=50,000, $25.00 $50.00 2,000 $70,000. 2,000.

$$c+a=2,000$$

$25.00 25c. $50.00 50a. $70,000.

$$25c+50a=70,000$$

$$c+a=2{,}000 \quad 25c+50a=70{,}000$$

c a. a.

$$c+a=2{,}000 \qquad a=2{,}000-c$$

2,000−c a c.

$$25c+50(2{,}000-c)=70{,}000 \quad 25c+100{,}000-50c=70{,}000 \qquad -25c=-30{,}000$$

$$c=1{,}200$$

c=1,200 a.

$$1{,}200+a=2{,}000 \qquad a=800$$

1,200 800 $4.00 $12.00 1,650 $14,200, x y 5x−y=4 x+6y=2 (4,0)−3x−5y=13 −x+4y=10(−6,1) 3x+7y=1 2x+4y=0 (2,3)−2x+5y=7 2x+9y=7(−1,1)x+8y=43 3x−2y=−1 (3,5) x+3y=5 2x+3y=4(−1,2) 3x−2y=18 5x+10y=−104x+2y=−10 3x+9y=0(−3,1)2x+4y=−3.8 9x−5y=1.3−2x+3y=1.2 −3x−6y=1.8 (− 3 5 ,0) x−0.2y=1 −10x+2y=5 3x+5y=9 30x+50y=−90 −3x+y=2 12x−4y=−8 1 2 x+ 1 3 y=16 1 6 x+ 1 4 y=9(72 5 , 132 5)− 1 4 x+ 3 2 y=11 − 1 8 x+ 1 3 y=3 −2x+5y=−42 7x+2y=30(6,−6)6x−5y=−34 2x+6y=4 5x−y=−2.6 −4x−6y=1.4(− 1 2 , 1 10) 7x−2y=3 4x+5y=3.25 −x+2y=−1 5x−10y=6 7x+6y=2 −28x−24y=−8 5 6 x+ 1 4 y=0 1 8 x− 1 2 y=− 43 120(− 1 5 , 2 3) 1 3 x+ 1 9 y= 2 9 − 1 2 x+ 4 5 y=− 1 3 −0.2x+0.4y=0.6 x−2y=−3(x, x+3 2)−0.1x+0.2y=0.6 5x−10y=15x+9y=16 x+2y=4(−4,4) 6x−8y=−0.6 3x+2y=0.95x−2y=2.25 7x−4y=3(1 2 , 1 8) x− 5 12 y=− 55 12 −6x+ 5 2 y= 55 2 7x−4y= 7 6 2x+4y= 1 3(1 6 ,0)3x+6y=11 2x+4y=9 7 3 x− 1 6 y=2 − 21 6 x+ 3 12 y=−3(x,2(7x−6)) 1 2 x+ 1 3 y= 1 3 3 2 x+ 1 4 y=− 1 82.2x+1.3y=−0.1 4.2x+4.2y=2.1(− 5 6 , 4 3) 0.1x+0.2y=2 0.35x−0.3y=03x−y=0.6 x−2y=1.3−x+2y=4 2x−4y=1 x+2y=7 2x+6y=12 3x−5y=7 x−2y=3 3x−2y=5 −9x+6y=−15 0.1x+0.2y=0.3 −0.3x+0.5y=1 −0.01x+0.12y=0.62 0.15x+0.20y=0.52(−3.08,4.91) 0.5x+0.3y=4 0.25x−0.9y=0.46 0.15x+0.27y=0.39 −0.34x+0.56y=1.8(−1.52,2.29)−0.71x+0.92y=0.13 0.83x+0.05y=2.1 A,B,C,D,E, F A−F A≠B AE≠BD.x+y=A x−y=B(A+B 2 , A−B 2)x+Ay=1 x+By=1Ax+y=0 Bx+y=1 (−1 A−B , A A−B)Ax+By=C x+y=1Ax+By=C Dx+Ey=F(CE−BF BD−AE , AF−CD BD−AE) C=12x+30 R=20x. C(x)=11x+120 R(x)=5x. C(x)=150x+10,000 R(x)=200x. C(x)=64x+20,000, x (1,250,100,000) C(x)=75x+50,000. 2 7 8 % (x,y) P(x)=R(x)−C(x), R=xp, x= p= (x,y,z),

$$Ax+By+Cz=D \qquad Ey+Fz=G \qquad Hz=K$$

z, y x. { (x,y,z) }. { (x,y,z) }. 0=0. 3=0. (3,−2,1)

$$x+y+z=2 \quad 6x-4y+5z=31 \quad 5x+2y+2z=13$$

x,y, z.

x+y+z=2 (3)+(−2)+(1)=2 True 6x−4y+5z=31 6(3)−4(−2)+5(1)=31 18+8+5=31 True 5x+2y+2z=13 5(3)+2(−2)+2(1)=13 15−4+2=13 True

(3,−2,1)

$$x-2y+3z=9 \ (1) \quad -x+3y-z=-6 \ (2) \quad 2x-5y+5z=17 \ (3)$$

x

$$x-2y+3z=9 \ (1) \quad -x+3y-z=-6 \ (2) \qquad y+2z=3 \quad (3)$$

−2 x.

$$-2x+4y-6z=-18 \ (1) \text{ multiplied by } -2 \quad 2x-5y+5z=17 \ (3)$$

$$-y-z=-1 \ (5)$$

z

$$y+2z=3 \quad (4) \ -y-z=-1 \ (5) \qquad z=2 \quad (6)$$

$$x-2y+3z=9 \ (1) \qquad y+2z=3 \ (4) \qquad z=2 \ (6)$$

z=2 y.

$$y+2(2)=3 \qquad y+4=3 \qquad y=-1$$

z=2 y=−1 x.

$$x-2(-1)+3(2)=9 \qquad x+2+6=9 \qquad x=1$$

(1,−1,2).

x=amount invested in money-market fund y=amount invested in municipal bonds
z=amount invested in mutual funds

$$x+y+z=12{,}000$$

$$z=y+4{,}000$$

$$0.03x+0.04y+0.07z=670$$

$x+y+z=12{,}000$ $-y+z=4{,}000$ $0.03x+0.04y+0.07z=670$

$x+\ y+z=12{,}000$ (1) $-y+z=4{,}000$ (2) $3x+4y+7z=67{,}000$ (3)

$x+\ y+\ z=12{,}000$ $3x+4y+7z=67{,}000$ $-y\ +\ z=4{,}000$

-3

$x+y+z=12{,}000$ $y+4z=31{,}000$ $-y+z=4{,}000$

$x+y+\ z=12{,}000$ $y+4z=31{,}000$ $5z=35{,}000$

z y. z y x.

$5z=35{,}000$ $z=7{,}000$ $y+4(7{,}000)=31{,}000$ $y=3{,}000$

$x+3{,}000+7{,}000=12{,}000$ $x=2{,}000$

$2x+y-2z=-1$ $3x-3y-z=5$ $x-2y+3z=6$

(1,–1,1) 3=7

$x-3y+z=4$ (1) $-x+2y-5z=3$ (2) $5x-13y+13z=8$ (3)

x, x

$x-3y+z=4$ (1) $-x+2y-5z=3$ (2) $-y-4z=7$ (4)

-5

$-5x+15y-5z=-20$ (1) multiplied by -5 $5x-13y+13z=8$ (3)

$2y+8z=-12$ (5)

$-2y-8z=14$ (4) multiplied by 2 $2y+8z=-12$ (5)

$0=2$

0=2

$x+y+z=2$ $y-3z=1$ $2x+y+5z=0$

$2x+y-3z=0$ (1) $4x+2y-6z=0$ (2) $x-y+z=0$ (3)

-2

$-4x-2y+6z=0$ equation (1) multiplied by -2 $4x+2y-6z=0$ (2)

$0=0$

0=0, –2, 0=0.

$2x+y-3z=0$ $x-y+z=0$ $3x-2z=0$

z.

$3x-2z=0$ $z=\ 3\ 2\ x$

z y.

$2x+y-3(\ 3\ 2\ x\ )=0$ $2x+y-\ 9\ 2\ x=0$ $y=\ 9\ 2\ x-2x$ $y=\ 5\ 2\ x$

(x, 5 2 x, 3 2 x). x y z x. x? x y.

$x+y+z=7$ $3x-2y-z=4$ $x+6y+5z=24$

(x,4x–11,–5x+18). { (x,y,z) } (0,0,0) $2x+3y-6z=1$ $-4x-6y+12z=-2$ $x+2y+5z=10$

$2x-6y+6z=-12$ $x+4y+5z=-1$ $-x+2y+3z=-1$ (0,1,–1) $6x-y+3z=6$ $3x+5y+2z=0$ $x+y=0$(3,–3,–5)

$6x-7y+z=2$ $-x-y+3z=4$ $2x+y-z=1$ (4,2,–6) $x-y=0$ $x-z=5$ $x-y+z=-1$ (4,4,–1)

$-x-y+2z=3$ $5x+8y-3z=4$ $-x+3y-5z=-5$ (4,1,–7)$3x-4y+2z=-15$ $2x+4y+z=16$ $2x+3y+5z=20$(–1,4,2)

$5x-2y+3z=20$ $2x-4y-3z=-9$ $x+6y-8z=21$ $5x+2y+4z=9$ $-3x+2y+z=10$ $4x-3y+5z=-3$

(– 85 107 , 312 107 , 191 107)$4x-3y+5z=31$ $-x+2y+4z=20$ $x+5y-2z=-29$

$5x-2y+3z=4$ $-4x+6y-7z=-1$ $3x+2y-z=4$(1, 1 2 ,0) $4x+6y+9z=0$ $-5x+2y-6z=3$ $7x-4y+3z=-3$

$2x-y+3z=17$ $-5x+4y-2z=-46$ $2y+5z=-7$(4,–6,1)$5x-6y+3z=50$ $-x+4y=10$ $2x-z=10$

$2x+3y-6z=1$ $-4x-6y+12z=-2$ $x+2y+5z=10$(x, 1 27 (65–16x), x+28 27)

$4x+6y-2z=8$ $6x+9y-3z=12$ $-2x-3y+z=-4$ $2x+3y-4z=5$ $-3x+2y+z=11$ $-x+5y+3z=4$

(– 45 13 , 17 13 ,–2)$10x+2y-14z=8$ $-x-2y-4z=-1$ $-12x-6y+6z=-12$

x+y+z=14 2y+3z=−14 −16y−24z=−112 5x−3y+4z=−1 −4x+2y−3z=0 −x+5y+7z=−11
x+y+z=0 2x−y+3z=0 x−z=0(0,0,0)3x+2y−5z=6 5x−4y+3z=−12 4x+5y−2z=15
x+y+z=0 2x−y+3z=0 x−z=1(4 7 ,− 1 7 ,− 3 7)
3x− 1 2 y−z=− 1 2 4x+z=3 −x+ 3 2 y= 5 2
6x−5y+6z=38 1 5 x− 1 2 y+ 3 5 z=1 −4x− 3 2 y−z=−74(7,20,16)
1 2 x− 1 5 y+ 2 5 z=− 13 10 1 4 x− 2 5 y− 1 5 z=− 7 20 − 1 2 x− 3 4 y− 1 2 z=− 5 4
− 1 3 x− 1 2 y− 1 4 z= 3 4 − 1 2 x− 1 4 y− 1 2 z=2 − 1 4 x− 3 4 y− 1 2 z=− 1 2(−6,2,1)
1 2 x− 1 4 y+ 3 4 z=0 1 4 x− 1 10 y+ 2 5 z=−2 1 8 x+ 1 5 y− 1 8 z=2
4 5 x− 7 8 y+ 1 2 z=1 − 4 5 x− 3 4 y+ 1 3 z=−8 − 2 5 x− 7 8 y+ 1 2 z=−5(5,12,15)
− 1 3 x− 1 8 y+ 1 6 z=− 4 3 − 2 3 x− 7 8 y+ 1 3 z=− 23 3 − 1 3 x− 5 8 y+ 5 6 z=0
− 1 4 x− 5 4 y+ 5 2 z=−5 − 1 2 x− 5 3 y+ 5 4 z= 55 12 − 1 3 x− 1 3 y+ 1 3 z= 5 3(−5,−5,−5)
1 40 x+ 1 60 y+ 1 80 z= 1 100 − 1 2 x− 1 3 y− 1 4 z=− 1 5 3 8 x+ 3 12 y+ 3 16 z= 3 20
0.1x−0.2y+0.3z=2 0.5x−0.1y+0.4z=8 0.7x−0.2y+0.3z=8(10,10,10)
0.2x+0.1y−0.3z=0.2 0.8x+0.4y−1.2z=0.1 1.6x+0.8y−2.4z=0.2
1.1x+0.7y−3.1z=−1.79 2.1x+0.5y−1.6z=−0.13 0.5x+0.4y−0.5z=−0.07(1 2 , 1 5 , 4 5)
0.5x−0.5y+0.5z=10 0.2x−0.2y+0.2z=4 0.1x−0.1y+0.1z=2
0.1x+0.2y+0.3z=0.37 0.1x−0.2y−0.3z=−0.27 0.5x−0.1y−0.3z=−0.03(1 2 , 2 5 , 4 5)
0.5x−0.5y−0.3z=0.13 0.4x−0.1y−0.3z=0.11 0.2x−0.8y−0.9z=−0.32
0.5x+0.2y−0.3z=1 0.4x−0.6y+0.7z=0.8 0.3x−0.1y−0.9z=0.6(2,0,0)
0.3x+0.3y+0.5z=0.6 0.4x+0.4y+0.4z=1.8 0.4x+0.2y+0.1z=1.6
0.8x+0.8y+0.8z=2.4 0.3x−0.5y+0.2z=0 0.1x+0.2y+0.3z=0.6(1,1,1) x,y, z.
x+y+z=3 x−1 2 + y−3 2 + z+1 2 =0 x−2 3 + y+4 3 + z−3 3 = 2 3
5x−3y− z+1 2 = 1 2 6x+ y−9 2 +2z=−3 x+8 2 −4y+z=4(128 557 , 23 557 , 28 557)
x+4 7 − y−1 6 + z+2 3 =1 x−2 4 + y+1 8 − z+8 12 =0 x+6 3 − y+2 3 + z+4 2 =3
x−3 6 + y+2 2 − z−3 3 =2 x+2 4 + y−5 2 + z+4 2 =1 x+6 2 − y−3 2 +z+1=9(6,−1,0)
x−1 3 + y+3 4 + z+2 6 =1 4x+3y−2z=11 0.02x+0.015y−0.01z=0.065 3 4 3 1 8 % 2 1 2 %
Ax+By+C=0.

x−y=−1 y= x 2 +1

x

x−y=−1 x=y−1 Solve for x. y= x 2 +1 y= (y−1) 2 +1 Substitute expression for x.
y= (y−1) 2 =(y 2 −2y+1)+1 = y 2 −2y+2 0= y 2 −3y+2 =(y−2)(y−1)
y y=2 y=1. y x.

x−y=−1 x−(2)=−1 x=1 x−(1)=−1 x=0

(1,2) (0,1), (x,y) y x x x. y=1

y= x 2 +1 y= x 2 +1 x 2 =0 x=± 0 =0

y=2

y= x 2 +1 2= x 2 +1 x 2 =1 x=± 1 =±1

−1

3x−y=−2 2 x 2 −y=0

(− 1 2 , 1 2) (2,8)

x 2 + y 2 =5 y=3x−5

y. y=3x−5

x 2 + (3x−5) 2 =5 x 2 +9 x 2 −30x+25=5 10 x 2 −30x+20=0

x.

10(x 2 −3x+2)=0 10(x−2)(x−1)=0 x=2 x=1

y.

y=3(2)−5 =1 y=3(1)−5 =−2

(2,1) (1,−2), (x,y)

x 2 + y 2 =10 x−3y=−10

$(-1,3)$

$$x^2+y^2=26 \quad (1)\ 3x^2+25y^2=100\ (2)$$

-3,

$$(-3)(x^2+y^2)=(-3)(26) \qquad -3x^2-3y^2=-78 \qquad 3x^2+25y^2=100 \qquad 22y^2=22$$

y.

$$y^2=1 \quad y=\pm\sqrt{1}=\pm 1$$

$y=\pm 1$ x.

$x^2+(1)^2=26$ $x^2+1=26$ $x^2=25$ $x=\pm\sqrt{25}=\pm 5$ $x^2+(-1)^2=26$ $x^2+1=26$ $x^2=25=\pm 5$

$(5,1),(-5,1),(5,-1)$,and $(-5,-1)$.

$$4x^2+y^2=13 \quad x^2+y^2=10$$

$\{(1,3),(1,-3),(-1,3),(-1,-3)\}$ $y>a$, $y<a$, $y\geq a$, $y\leq a$, $y>a$; $y\geq a$; $y<a$; $y\leq a$ $\leq$ $\geq$ $y>x^2+1$. $y=x^2+1$. $y>x^2+1$ $(0,2)$ $(2,0)$.

$$y>x^2+1 \quad 2>(0)^2+1 \quad 2>1 \text{ True} \quad 0>(2)^2+1 \quad 0>5 \text{ False}$$

$$x^2-y\leq 0 \quad 2x^2+y\leq 12$$

y x.

$x^2-y=0$ $2x^2+y=12$ _____ $3x^2=12$ $x^2=4$ $x=\pm 2$

y.

$$x^2-y=0 \quad (2)^2-y=0 \quad 4-y=0 \quad y=4 \quad (-2)^2-y=0 \quad 4-y=0 \quad y=4$$

$(2,4)$ $(-2,4)$.

$x^2-y\leq 0$ $x^2\leq y$ $y\geq x^2$ $2x^2+y\leq 12$ $y\leq -2x^2+12$

$2x^2+y\leq 12$ $x^2-y\leq 0$

$$y\geq x^2-1 \quad x-y\geq -1$$

$C(x)$ $R(x)$. $C(x)<R(x)$, $x+y=4$ $x^2+y^2=9$ $y=x-3$ $x^2+y^2=9(0,-3),(3,0)$

$y=x$ $x^2+y^2=9$ $y=-x$ $x^2+y^2=9(-\frac{3\sqrt{2}}{2},\frac{3\sqrt{2}}{2}),(\frac{3\sqrt{2}}{2},-\frac{3\sqrt{2}}{2})$ $x=2$ $x^2-y^2=9$

$4x^2-9y^2=36$ $4x^2+9y^2=36(-3,0),(3,0)$ $x^2+y^2=25$ $x^2-y^2=1$

$2x^2+4y^2=4$ $2x^2-4y^2=25$ $x-10(\frac{1}{4},-\frac{\sqrt{62}}{8}),(\frac{1}{4},\frac{\sqrt{62}}{8})$ $y^2-x^2=9$ $3x^2+2y^2=8$

$x^2+y^2+\frac{1}{16}=2500$ $y=2x^2(-\frac{\sqrt{398}}{4},\frac{199}{4}),(\frac{\sqrt{398}}{4},\frac{199}{4})-2x^2+y=-5$ $6x-y=9$

$-x^2+y=2$ $-x+y=2(0,2),(1,3)$ $x^2+y^2=1$ $y=20x^2-1$ $x^2+y^2=1$ $y=-x^2$

$(-\frac{1}{2}(\sqrt{5}-1),\frac{1}{2}(1-\sqrt{5})),(\frac{1}{2}(\sqrt{5}-1),\frac{1}{2}(1-\sqrt{5}))$ $2x^3-x^2=y$ $y=\frac{1}{2}-x$

$9x^2+25y^2=225$ $(x-6)^2+y^2=1(5,0)$ $x^4-x^2=y$ $x^2+y=0$ $2x^3-x^2=y$ $x^2+y=0(0,0)$

$x^2+y^2=9$ $y=3-x$ $2x^2-y^2=9$ $x=3(3,0)$ $x^2-y^2=9$ $y=3x^2-y^2=9$ $x-y=0$

$-x^2+y=2$ $-4x+y=-1-x^2+y=2$ $2y=-x$ $x^2+y^2=25$ $x^2-y^2=36$ $x^2+y^2=1$ $y^2=x^2$

$(-2\sqrt{2},-2\sqrt{2}),(-2\sqrt{2},2\sqrt{2}),(2\sqrt{2},-2\sqrt{2}),(2\sqrt{2},2\sqrt{2})$ $16x^2-9y^2+144=0$ $y^2+x^2=16$

$3x^2-y^2=12$ $(x-1)^2+y^2=1(2,0)$ $3x^2-y^2=12$ $(x-1)^2+y^2=4$

$3x^2-y^2=12$ $x^2+y^2=16(-7,-3),(-7,3),(7,-3),(7,3)$

$x^2-y^2-6x-4y-11=0$ $-x^2+y^2=5$ $x^2+y^2-6y=7$ $x^2+y=1$

$(-\frac{1}{2}(\sqrt{73}-5),\frac{1}{2}(7-\sqrt{73})),(\frac{1}{2}(\sqrt{73}-5),\frac{1}{2}(7-\sqrt{73}))$ $x^2+y^2=6$ $xy=1$ $x^2+y<9$

$x^2+y^2<4$ $x^2+y<1$ $y>2x$ $x^2+y<-5$ $y>5x+10$ $x^2+y^2<25$ $3x^2-y^2>12$ $x^2-y^2>-4$ $x^2+y^2<12$

$x^2+3y^2>16$ $3x^2-y^2<1$ $y\geq e^x$ $y\leq \ln(x)+5$ $y\leq -\log(x)$ $y\leq e^x$ $4x^2+1y^2=24$ $5x^2-2y^2+4=0$

$6x^2-1y^2=8$ $1x^2-6y^2=1$ 8

$(-2\,70\,383,-2\,35\,29),(-2\,70\,383,2\,35\,29),(2\,70\,383,-2\,35\,29),(2\,70\,383,2\,35\,29)$

$x^2-xy+y^2-2=0$ $x+3y=4$ $x^2-xy-2y^2-6=0$ $x^2+y^2=1$

$x^2+4xy-2y^2-6=0$ $x=y+2$ $xy<1$ $y>x$ $xx=0, y>0$ $0<x<1$, $x<y<1$ $xx^2+y<3$ $y>2x$

$C(x)=3x^2-10x+200$ $R(x)=-2x^2+100x+50$. $C(x)=8x^2-600x+21{,}500$ $R(x)=-3x^2+480x$.

$P(x)$ $Q(x)$

$$\frac{2}{x-3}+\frac{-1}{x+2}$$

$(x+2)(x-3)$.

$\frac{2}{x-3}\left(\frac{x+2}{x+2}\right)+\frac{-1}{x+2}\left(\frac{x-3}{x-3}\right)=$ $\frac{2x+4-x+3}{(x+2)(x-3)}=\frac{x+7}{x^2-x-6}$

x+7 x 2 –x–6 Simplified sum = 2 x–3 + –1 x+2 Partial fraction decomposition

x 2 –x–6 (x–3)(x+2), P(x) Q(x) :Q(x) P(x) Q(x) Q(x) P(x) Q(x)

P(x) Q(x) = A 1 (a 1 x+ b 1) + A 2 (a 2 x+ b 2) + A 3 (a 3 x+ b 3) +···+ A n (a n x+ b n) .

A,B, C, A n

$$P(x)\,Q(x) = A_1(a_1x+b_1) + A_2(a_2x+b_2) + \cdots + A_n(a_nx+b_n)$$

$$3x\,(x+2)(x-1)$$

A,B ,C.

$$3x\,(x+2)(x-1) = A\,(x+2) + B\,(x-1)$$

$$(x+2)(x-1)[\,3x\,(x+2)(x-1)\,] = (x+2)\,(x-1)[\,A\,(x+2)\,] + (x+2)\,(x-1)\,[\,B\,(x-1)\,]$$

$$3x = A(x-1) + B(x+2)$$

$$3x = Ax - A + Bx + 2B \qquad 3x = (A+B)x - A + 2B$$

$$3 = A+B \qquad 0 = -A+2B$$

B.

$$3 = A+B \qquad 0 = -A+2B \quad -3 = 0 + 3B \qquad 1 = B$$

B=1

$$3 = A+1 \qquad 2 = A$$

$$3x\,(x+2)(x-1) = 2\,(x+2) + 1\,(x-1)$$

A B x x=1, A- B.

$$3x = A(x-1) + B(x+2) \qquad 3(1) = A[(1)-1] + B[(1)+2] \qquad 3 = 0 + 3B \qquad 1 = B$$

B=1 A, x=–2

$$3x = A(x-1) + B(x+2) \qquad 3(-2) = A[(-2)-1] + B[(-2)+2] \qquad -6 = -3A + 0 \qquad -6 - 3 = A \qquad 2 = A$$

A B

$$3x\,(x+2)(x-1) = 2\,(x+2) + 1\,(x-1)$$

$$x\,(x-3)(x-2)$$

3 x–3 – 2 x–2 P(x) Q(x) P(x) Q(x) :Q(x) P(x) Q(x) , Q(x) n P(x) Q(x),

P(x) Q(x) = A 1 (ax+b) + A 2 (ax+b) 2 + A 3 (ax+b) 3 +···+ A n (ax+b) n

A,B, C

$$P(x)\,Q(x) = A_1(ax+b) + A_2(ax+b)^2 + \ldots + A_n(ax+b)^n$$

$$-x^2+2x+4 \qquad x^3-4x^2+4x$$

x (x–2) 2 . (x–2), x,(x–2), (x–2) 2 .

$$-x^2+2x+4 \qquad x^3-4x^2+4x = A\,x + B\,(x-2) + C\,(x-2)^2$$

$$x\,(x-2)^2\,[\,-x^2+2x+4 \qquad x\,(x-2)^2\,] = [\,A\,x + B\,(x-2) + C\,(x-2)^2\,]\,x\,(x-2)^2 \qquad -x^2+2x+4 = A\,(x-2)^2 + Bx(x-2) + Cx$$

$$-x^2+2x+4 = A(x^2-4x+4) + B(x^2-2x) + Cx \qquad = Ax^2-4Ax+4A+Bx^2-2Bx+Cx$$

$$= (A+B)x^2 + (-4A-2B+C)x + 4A$$

$$-x^2+2x+4 = (A+B)x^2 + (-4A-2B+C)x + 4A$$

A+B=–1 (1) –4A–2B+C=2 (2) 4A=4 (3)

A

$$4A=4 \qquad A=1$$

A=1

$$A+B=-1 \qquad (1)+B=-1 \qquad B=-2$$

C, A B

$$-4A-2B+C=2 \qquad -4(1)-2(-2)+C=2 \qquad -4+4+C=2 \qquad C=2$$

$$-x^2+2x+4 \qquad x^3-4x^2+4x = 1\,x - 2\,(x-2) + 2\,(x-2)^2$$

$$6x-11\,(x-1)^2$$

6 x–1 – 5 (x–1) 2 P(x) Q(x) , A,B, C Ax+B,Bx+C, P(x) Q(x) :Q(x) P(x) Q(x) Q(x) P(x) Q(x)

P(x) Q(x) = A 1 x+ B 1 (a 1 x 2 + b 1 x+ c 1) + A 2 x+ B 2 (a 2 x 2 + b 2 x+ c 2) +···+ A n x+ B n (a n x 2 + b n x+ c n)

A,B,C, A,B, C A 1 x+ B 1 , A 2 x+ B 2 ,

P(x) Q(x) = A ax+b + A 1 x+ B 1 (a 1 x 2 + b 1 x+ c 1) + A 2 x+ B 2 (a 2 x 2 + b 2 x+ c 2) +⋯+ A n x+ B n (a n x 2 + b n x+ c n)

P(x) Q(x)

$$8x^2+12x-20\ (x+3)(x^2+x+2)$$

$$8x^2+12x-20\ (x+3)(x^2+x+2) = A\ (x+3) + Bx+C\ (x^2+x+2)$$

$$(x+3)(x^2+x+2)[\ 8x^2+12x-20\ (x+3)(x^2+x+2)\ ]=[\ A\ (x+3) + Bx+C\ (x^2+x+2)\ ](x+3)(x^2+x+2)$$

$$8x^2+12x-20=A(x^2+x+2)+(Bx+C)(x+3)$$

A x Bx+C x=−3

$$8x^2+12x-20=A(x^2+x+2)+(Bx+C)(x+3) \quad 8(-3)^2+12(-3)-20=A((-3)^2+(-3)+2)+(B(-3)+C)((-3)+3) \quad 16=8A \quad A=2$$

A,

$$8x^2+12x-20=2(x^2+x+2)+(Bx+C)(x+3)\ 8x^2+12x-20=2x^2+2x+4+Bx^2+3B+Cx+3C\ 8x^2+12x-20=(2+B)x^2+(2+3B+C)x+(4+3C)$$

$$2+B=8\ (1)\ 2+3B+C=12\ (2) \qquad 4+3C=-20\ (3)$$

B C

$$2+B=8\ (1) \qquad B=6\ 4+3C=-20\ (3) \qquad 3C=-24 \qquad C=-8$$

$$8x^2+12x-20\ (x+3)(x^2+x+2) = 2\ (x+3) + 6x-8\ (x^2+x+2)$$

A

$$8x^2+12x-20=Ax^2+Ax+2A+Bx^2+3B+Cx+3C\ 8x^2+12x-20=(A+B)x^2+(A+3B+C)x+(2A+3C)$$

$$A+B=8\ A+3B+C=12 \qquad 2A+3C=-20$$

$$5x^2-6x+7\ (x-1)(x^2+1)$$

3 x−1 + 2x−4 x 2 +1 P(x) Q(x) P(x) Q(x) P(x) Q(x) , Q(x) P(x) Q(x),

P(x) (a x 2 +bx+c) n = A 1 x+ B 1 (a x 2 +bx+c) + A 2 x+ B 2 (a x 2 +bx+c) 2 + A 3 x+ B 3 (a x 2 +bx+c) 3 +⋯+ A n x+ B n (a x 2 +bx+c) n

A,B, C A 1 x+ B 1 , A 2 x+ B 2 ,

P(x) Q(x) = A ax+b + A 1 x+ B 1 (a x 2 +bx+c) + A 2 x+ B 2 (a x 2 +bx+c) 2 +⋯+ A n + B n (a x 2 +bx+c) n

$$x^4+x^3+x^2-x+1\ x(x^2+1)^2$$

x,(x 2 +1), (x 2 +1) 2 . Ax+B.

$$x^4+x^3+x^2-x+1\ x(x^2+1)^2 = A\ x + Bx+C\ (x^2+1) + Dx+E\ (x^2+1)^2$$

x (x 2 +1) 2 .

$$x^4+x^3+x^2-x+1=A(x^2+1)^2+(Bx+C)(x)(x^2+1)+(Dx+E)(x)$$

$$x^4+x^3+x^2-x+1=A(x^4+2x^2+1)+Bx^4+Bx^2+Cx^3+Cx+Dx^2+Ex$$

$$=Ax^4+2Ax^2+A+Bx^4+Bx^2+Cx^3+Cx+Dx^2+Ex$$

$$x^4+x^3+x^2-x+1=(A+B)x^4+(C)x^3+(2A+B+D)x^2+(C+E)x+A$$

$$A+B=1 \qquad C=1\ 2A+B+D=1 \qquad C+E=-1 \qquad A=1$$

A=1

$$1+B=1 \qquad B=0$$

A=1 B=0

$$2(1)+0+D=1 \qquad D=-1$$

C=1

$$1+E=-1 \qquad E=-2$$

A=1, B=0, C=1, D=−1, E=−2.

$$x^4+x^3+x^2-x+1\ x(x^2+1)^2 = 1\ x + 1\ (x^2+1) - x+2\ (x^2+1)^2$$

$$x^3-4x^2+9x-5\ (x^2-2x+3)^2$$

x−2 x 2 −2x+3 + 2x+1 (x 2 −2x+3) 2 P(x) Q(x) A a 1 x+ b 1 + B a 2 x+ b 2 . P(x) Q(x)

P(x) Q(x) A x + Bx+C (a x 2 +bx+c) . P(x) Q(x) , Q(x)

Ax+B (a x 2 +bx+c) + A 2 x+ B 2 (a x 2 +bx+c) 2 +⋯+ A n x+ B n (a x 2 +bx+c) n .

1 x 2 +1 7x+13 3 x 2 +8x+15 = A x+1 + B 3x+5 , 7x+13=A(3x+5)+B(x+1). x A B A B. x=−1, A=3.
x=− 5 3 , −2.5x+16 x 2 +10x+243x−79 x 2 −5x−248 x+3 − 5 x−8−x−24 x 2 −2x−2410x+47 x 2 +7x+10
1 x+5 + 9 x+2x 6 x 2 +25x+2532x−11 20 x 2 −13x+23 5x−2 + 4 4x−1x+1 x 2 +7x+105x x 2 −9
5 2(x+3) + 5 2(x−3)10x x 2 −256x x 2 −43 x+2 + 3 x−22x−3 x 2 −6x+54x−1 x 2 −x−6
9 5(x+2) + 11 5(x−3)4x+3 x 2 +8x+153x−1 x 2 −5x+68 x−3 − 5 x−2−5x−19 (x+4) 2x (x−2) 2
1 x−2 + 2 (x−2) 27x+14 (x+3) 2−24x−27 (4x+5) 2− 6 4x+5 + 3 (4x+5) 2−24x−27 (6x−7) 2
5−x (x−7) 2− 1 x−7 − 2 (x−7) 25x+14 2 x 2 +12x+185 x 2 +20x+8 2x (x+1) 2
4 x − 3 2(x+1) + 7 2 (x+1) 24 x 2 +55x+25 5x (3x+5) 254 x 3 +127 x 2 +80x+16 2 x 2 (3x+2) 2
4 x + 2 x 2 − 3 3x+2 + 7 2 (3x+2) 2x 3 −5 x 2 +12x+144 x 2 (x 2 +12x+36)
4 x 2 +6x+11 (x+2)(x 2 +x+3)x+1 x 2 +x+3 + 3 x+24 x 2 +9x+23 (x−1)(x 2 +6x+11)
−2 x 2 +10x+4 (x−1)(x 2 +3x+8)4−3x x 2 +3x+8 + 1 x−1x 2 +3x+1 (x+1)(x 2 +5x−2)
4 x 2 +17x−1 (x+3)(x 2 +6x+1)2x−1 x 2 +6x+1 + 2 x+34 x 2 (x+5)(x 2 +7x−5)4 x 2 +5x+3 x 3 −1
1 x 2 +x+1 + 4 x−1−5 x 2 +18x−4 x 3 +83 x 2 −7x+33 x 3 +272 x 2 −3x+9 + 3 x+3x 2 +2x+40 x 3 −125
4 x 2 +4x+12 8 x 3 −27− 1 4 x 2 +6x+9 + 1 2x−3−50 x 2 +5x−3 125 x 3 −1
−2 x 3 −30 x 2 +36x+216 x 4 +216x1 x + 1 x+6 − 4x x 2 −6x+363 x 3 +2 x 2 +14x+15 (x 2 +4) 2
x 3 +6 x 2 +5x+9 (x 2 +1) 2x+6 x 2 +1 + 4x+3 (x 2 +1) 2x 3 − x 2 +x−1 (x 2 −3) 2x 2 +5x+5 (x+2) 2
x+1 x+2 + 2x+3 (x+2) 2x 3 +2 x 2 +4x (x 2 +2x+9) 2x 2 +25 (x 2 +3x+25) 2
1 x 2 +3x+25 − 3x (x 2 +3x+25) 22 x 3 +11x+7x+70 (2 x 2 +x+14) 25x+2 x (x 2 +4) 2
1 8x − x 8(x 2 +4) + 10−x 2 (x 2 +4) 2x 4 + x 3 +8 x 2 +6x+36 x (x 2 +6) 22x−9 (x 2 −x) 2
− 16 x − 9 x 2 + 16 x−1 − 7 (x−1) 25 x 3 −2x+1 (x 2 +2x) 2x 2 +4 (x+1) 3
1 x+1 − 2 (x+1) 2 + 5 (x+1) 3x 3 −4 x 2 +5x+4 (x−2) 37 x+8 + 5 x−2 − x−1 x 2 −6x−16
5 x−2 − 3 10(x+2) + 7 x+8 − 7 10(x−8)1 x−4 − 3 x+6 − 2x+7 x 2 +2x−24
2x x 2 −16 − 1−2x x 2 +6x+8 − x−5 x 2 −4x− 5 4x − 5 2(x+2) + 11 2(x+4) + 5 4(x+4) A,B, C

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}, B=\begin{bmatrix} 1 & 2 & 7 & 0 \\ -5 & 6 & 7 & 8 \\ 2 & \end{bmatrix}, C=\begin{bmatrix} -1 & 0 & 3 & 3 \\ 2 & 1 & \end{bmatrix}$$

m × n m n A a ij , i, j. A, a 23 .

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

n × n, 3×3 1 × n.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}$$

m × 1.

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$$

A,B,C, a ij , i j m × n m n A: A? a 31 a 22 ?

$$A=\begin{bmatrix} 2 & 1 & 0 \\ 2 & 4 & 7 \\ 3 & 1 & -2 \end{bmatrix}$$

3 × 3 a 31 a 22 3 × 3 3 × 3 2 × 3 3 × 3 A B A B C D

$$A+B=C \text{ such that } a_{ij} + b_{ij} = c_{ij}$$

$$A-B=D \text{ such that } a_{ij} - b_{ij} = d_{ij}$$

$$A+B=B+A$$

$$(A+B)+C=A+(B+C)$$

A B,

$$A=\begin{bmatrix} a & b & c & d \end{bmatrix} \text{ and } B=\begin{bmatrix} e & f & g & h \end{bmatrix}$$

$$A+B=\begin{bmatrix} a & b & c & d \end{bmatrix}+\begin{bmatrix} e & f & g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f & c+g & d+h \end{bmatrix}$$

A B.

$$A=\begin{bmatrix} 4 & 1 & 3 & 2 \end{bmatrix} \text{ and } B=\begin{bmatrix} 5 & 9 & 0 & 7 \end{bmatrix}$$

a 11 , A b 11 , B.

$$A+B=\begin{bmatrix} 4 & 1 & 3 & 2 \end{bmatrix}+\begin{bmatrix} 5 & 9 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 4+5 & 1+9 & 3+0 & 2+7 \end{bmatrix} = \begin{bmatrix} 9 & 10 & 3 & 9 \end{bmatrix}$$

A B.

$$A=\begin{bmatrix} -2 & 3 & 0 & 1 \end{bmatrix} \text{ and } B=\begin{bmatrix} 8 & 1 & 5 & 4 \end{bmatrix}$$

$$A-B=\begin{bmatrix} -2 & 3 & 0 & 1 \end{bmatrix}-\begin{bmatrix} 8 & 1 & 5 & 4 \end{bmatrix} = \begin{bmatrix} -2-8 & 3-1 & 0-5 & 1-4 \end{bmatrix} = \begin{bmatrix} -10 & 2 & -5 & -3 \end{bmatrix}$$

A B:

$$A=\begin{bmatrix} 2 & -10 & -2 & 14 \\ 12 & 10 & 4 & -2 \\ 2 & \end{bmatrix} \text{ and } B=\begin{bmatrix} 6 & 10 & -2 & 0 \\ -12 & -4 & -5 & 2 \\ -2 & \end{bmatrix}$$

A+B=[2 −10 −2 14 12 10 4 −2 2]+[6 10 −2 0 −12 −4 −5 2 −2] =[2+6 −10+10 −2−2 14+0 12−12 10−4 4−5 −2+2 2−2] =[8 0 −4 14 0 6 −1 0 0]

A−B=[2 −10 −2 14 12 10 4 −2 2]−[6 10 −2 0 −12 −4 −5 2 −2] =[2−6 −10−10 −2+2 14−0 12+12 10+4 4+5 −2−2 2+2] =[−4 −20 0 14 24 14 9 −4 4]

A B.

A=[2 6 1 0 1 −3] and B=[3 −2 1 5 −4 3]

A+B=[2 1 1 6 0 −3]+[3 1 −4 −2 5 3]=[2 + 3 1 + 1 1+(−4) 6+(−2) 0 + 5 −3 + 3]=[5 2 −3 4 5 0]

C 2013 =[15 16 16 27 34 34]

C

(0.15) C 2013 =[(0.15)15 (0.15)16 (0.15)16 (0.15)27 (0.15)34 (0.15)34]=[2.25 2.4 2.4 4.05 5.1 5.1]

[3 3 3 5 6 6]

[15 16 16 27 34 34]+[3 3 3 5 6 6]=[18 19 19 32 40 40]

C 2014 =[18 19 19 32 40 40]

A=[a 11 a 12 a 21 a 22]

cA

cA=c[a 11 a 12 a 21 a 22] =[c a 11 c a 12 c a 21 c a 22]

A,B,C a b,

a(A+B)=aA+aB (a+b)A=aA+bA

A

A=[8 1 5 4]

A

3A=3[8 1 5 4] = [3·8 3·1 3·5 3·4] = [24 3 15 12]

B, −2B

B=[4 1 3 2]

−2B=[−8 −2 −6 −4] 3A+2B.

A=[1 −2 0 0 −1 2 4 3 −6] and B=[−1 2 1 0 −3 2 0 1 −4]

3A, 2B.

3A=[3·1 3(−2) 3·0 3·0 3(−1) 3·2 3·4 3·3 3(−6)] =[3 −6 0 0 −3 6 12 9 −18]

2B=[2(−1) 2·2 2·1 2·0 2(−3) 2·2 2·0 2·1 2(−4)] =[−2 4 2 0 −6 4 0 2 −8]

3A+2B.

3A+2B=[3 −6 0 0 −3 6 12 9 −18]+[−2 4 2 0 −6 4 0 2 −8] =[3−2 −6+4 0+2 0+0 −3−6 6+4 12+0 9+2 −18−8] =[1 −2 2 0 −9 10 12 11 −26]

A m × r B r × n AB m × n AB A B. A B i AB, i A j B A B, A 2 × 3 B 3 × 3, AB 2 × 3

A=[a 11 a 12 a 13 a 21 a 22 a 23] and B=[b 11 b 12 b 13 b 21 b 22 b 23 b 31 b 32 b 33]

AB. AB, A B,

[a 11 a 12 a 13]·[b 11 b 21 b 31]= a 11 · b 11 + a 12 · b 21 + a 13 · b 31

AB, A B,

[a 11 a 12 a 13]·[b 12 b 22 b 32]= a 11 · b 12 + a 12 · b 22 + a 13 · b 32

AB, A B,

[a 11 a 12 a 13]·[b 13 b 23 b 33]= a 11 · b 13 + a 12 · b 23 + a 13 · b 33

AB. A B; A B; A B.

AB=[a 11 · b 11 + a 12 · b 21 + a 13 · b 31 a 21 · b 11 + a 22 · b 21 + a 23 · b 31 a 11 · b 12 + a 12 · b 22 + a 13 · b 32 a 21 · b 12 + a 22 · b 22 + a 23 · b 32 a 11 · b 13 + a 12 · b 23 + a 13 · b 33 a 21 · b 13 + a 22 · b 23 + a 23 · b 33]

A,B,C (AB)C=A(BC). C(A+B)=CA+CB, (A+B)C=AC+BC. A B.

A=[1 2 3 4] and B=[5 6 7 8]

A 2×2 B 2×2. 2×2. A B: AB. BA.

$$A=\begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix} \text{ and } B=\begin{bmatrix} 5 & -4 \\ 2 & -1 \\ 0 & 3 \end{bmatrix}$$

A 2×3 B 3×2, A B. 2×2 A B.

$$AB=\begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{bmatrix}=\begin{bmatrix} -1(5)+2(-4)+3(2) & -1(-1)+2(0)+3(3) \\ 4(5)+0(-4)+5(2) & 4(-1)+0(0)+5(3) \end{bmatrix}=\begin{bmatrix} -7 & 10 \\ 30 & 11 \end{bmatrix}$$

B 3×2 A 2×3. 3×3

$$BA=\begin{bmatrix} 5 & -1 \\ -4 & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix}=\begin{bmatrix} 5(-1)+-1(4) & 5(2)+-1(0) & 5(3)+-1(5) \\ -4(-1)+0(4) & -4(2)+0(0) & -4(3)+0(5) \\ 2(-1)+3(4) & 2(2)+3(0) & 2(3)+3(5) \end{bmatrix}=\begin{bmatrix} -9 & 10 & 10 \\ 4 & -8 & -12 \\ 10 & 4 & 21 \end{bmatrix}$$

AB BA

$$AB=\begin{bmatrix} -7 & 10 \\ 30 & 11 \end{bmatrix}\neq\begin{bmatrix} -9 & 10 & 10 \\ 4 & -8 & -12 \\ 10 & 4 & 21 \end{bmatrix}=BA$$

3×4 4×2.

$$E=\begin{bmatrix} 6 & 30 \\ 14 & 10 \\ 24 & 20 \end{bmatrix}$$

$$C=\begin{bmatrix} 300 & 10 & 30 \end{bmatrix}$$

$$CE=\begin{bmatrix} 300 & 10 & 30 \end{bmatrix}\cdot\begin{bmatrix} 6 & 10 \\ 30 & 24 \\ 14 & 20 \end{bmatrix}=\begin{bmatrix} 300(6)+10(30)+30(14) & 300(10)+10(24)+30(20) \end{bmatrix}=\begin{bmatrix} 2{,}520 & 3{,}840 \end{bmatrix}$$

[A],[B],[C],... AB−C

$$A=\begin{bmatrix} -15 & 25 & 32 \\ 41 & -7 & -28 \\ 10 & 34 & -2 \end{bmatrix},B=\begin{bmatrix} 45 & 21 & -37 \\ -24 & 52 & 19 \\ 6 & -48 & -31 \end{bmatrix},\text{and } C=\begin{bmatrix} -100 & -89 & -98 \\ 25 & -56 & 74 \\ -67 & 42 & -75 \end{bmatrix}.$$

A [A], B [B], C [C].

$$[\ A\ ]\times[\ B\ ]-[\ C\ ]$$

$$\begin{bmatrix} -983 & -462 & 136 \\ 1{,}820 & 1{,}897 & -856 \\ -311 & 2{,}032 & 413 \end{bmatrix}$$

3×2 A B, A B; A B, 2×2 2×3 $\begin{bmatrix} 1 & 2 & 3 \\ 4 & \end{bmatrix}+\begin{bmatrix} 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ AB BA A m×n B n×m, AB=BA? AB, A B AB. BA, B A BA.

$$A=\begin{bmatrix} 1 & 3 \\ 0 & 7 \end{bmatrix},B=\begin{bmatrix} 2 & 14 \\ 22 & 6 \end{bmatrix},C=\begin{bmatrix} 1 & 5 & 8 \\ 92 & 12 & 6 \end{bmatrix},D=\begin{bmatrix} 10 & 14 \\ 7 & 2 \\ 5 & 61 \end{bmatrix},E=\begin{bmatrix} 6 & 12 \\ 14 & 5 \end{bmatrix},F=\begin{bmatrix} 0 & 9 \\ 78 & 17 \\ 15 & 4 \end{bmatrix}$$

A+BC+D $\begin{bmatrix} 11 & 19 \\ 15 & 94 \\ 17 & 67 \end{bmatrix}$ A+CB−E $\begin{bmatrix} -4 & 2 \\ 8 & 1 \end{bmatrix}$ C+FD−B

$$A=\begin{bmatrix} 4 & 6 \\ 13 & 12 \end{bmatrix},B=\begin{bmatrix} 3 & 9 \\ 21 & 12 \\ 0 & 64 \end{bmatrix},C=\begin{bmatrix} 16 & 3 & 7 \\ 18 & 90 & 5 \\ 3 & 29 & \end{bmatrix},D=\begin{bmatrix} 18 & 12 & 13 & 8 \\ 14 & 6 & 7 & 4 \\ 21 & & & \end{bmatrix}$$

5A3B $\begin{bmatrix} 9 & 27 \\ 63 & 36 \\ 0 & 192 \end{bmatrix}$ −2B−4C $\begin{bmatrix} -64 & -12 \\ -28 & -72 \\ -360 & -20 \\ -12 & -116 \end{bmatrix}$ 1 2 C100D

$$\begin{bmatrix} 1{,}800 & 1{,}200 \\ 1{,}300 & 800 \\ 1{,}400 & 600 \\ 700 & 400 \\ 2{,}100 & \end{bmatrix}$$

$$A=\begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix},B=\begin{bmatrix} 3 & 6 \\ 4 & -8 \\ 0 & 12 \end{bmatrix},C=\begin{bmatrix} 4 & 10 & -2 \\ 6 & 5 & 9 \end{bmatrix},D=\begin{bmatrix} 2 & -3 & 12 \\ 9 & 3 & 1 \\ 0 & 8 & -10 \end{bmatrix}$$

ABBC $\begin{bmatrix} 20 & 102 \\ 28 & 28 \end{bmatrix}$ CABD $\begin{bmatrix} 60 & 41 & 2 \\ -16 & 120 & -216 \end{bmatrix}$ DCCB $\begin{bmatrix} -68 & 24 & 136 \\ -54 & -12 & 64 \\ -57 & 30 & 128 \end{bmatrix}$

$$A=\begin{bmatrix} 2 & -5 \\ 6 & 7 \end{bmatrix},B=\begin{bmatrix} -9 & 6 \\ -4 & 2 \end{bmatrix},C=\begin{bmatrix} 0 & 9 \\ 7 & 1 \end{bmatrix},D=\begin{bmatrix} -8 & 7 & -5 \\ 4 & 3 & 2 \\ 0 & 9 & 2 \end{bmatrix},E=\begin{bmatrix} 4 & 5 & 3 \\ 7 & -6 & -5 \\ 1 & 0 & 9 \end{bmatrix}$$

A+B−C4A+5D2C+B3D+4E $\begin{bmatrix} -8 & 41 \\ -3 & 40 \\ -15 & -14 \\ 4 & 27 \\ 42 & \end{bmatrix}$ C−0.5D100D−10E

$$\begin{bmatrix} -840 & 650 \\ -530 & 330 \\ 360 & 250 \\ -10 & 900 \\ 110 & \end{bmatrix}$$

A 2 =A·A

$$A=\begin{bmatrix} -10 & 20 \\ 5 & 25 \end{bmatrix},B=\begin{bmatrix} 40 & 10 \\ -20 & 30 \end{bmatrix},C=\begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

ABBA $\begin{bmatrix} -350 & 1{,}050 \\ 350 & 350 \end{bmatrix}$ CABCA 2B 2 $\begin{bmatrix} 1{,}400 & 700 \\ -1{,}400 & 700 \end{bmatrix}$ C 2B 2 A 2

$$\begin{bmatrix} 332{,}500 & 927{,}500 \\ -227{,}500 & 87{,}500 \end{bmatrix}$$

A 2 B 2(AB) 2 $\begin{bmatrix} 490{,}000 & 0 & 0 \\ 490{,}000 & & \end{bmatrix}$ (BA) 2 A 2 =A·A

$$A=\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix},B=\begin{bmatrix} -2 & 3 & 4 \\ -1 & 1 & -5 \end{bmatrix},C=\begin{bmatrix} 0.5 & 0.1 & 1 \\ 0.2 & -0.5 & 0.3 \end{bmatrix},D=\begin{bmatrix} 1 & 0 \\ -1 & -6 \\ 7 & 5 \\ 4 & 2 \\ 1 & \end{bmatrix}$$

AB $\begin{bmatrix} -2 & 3 & 4 \\ -7 & 9 & -7 \end{bmatrix}$ BABD $\begin{bmatrix} -4 & 29 \\ 21 & -27 \\ -3 & 1 \end{bmatrix}$ DCD 2 $\begin{bmatrix} -3 & -2 & -2 \\ -28 & 59 & 46 \\ -4 & 16 & 7 \end{bmatrix}$ A 2D 3

$$\begin{bmatrix} 1 & -18 \\ -9 & -198 \\ 505 & 369 \\ -72 & 126 \\ 91 & \end{bmatrix}$$

(AB)CA(BC)

$$\begin{bmatrix} 0 & 1.6 \\ 9 & -1 \end{bmatrix}$$

$$A=\begin{bmatrix} -2 & 0 & 9 \\ 1 & 8 & -3 \\ 0.5 & 4 & 5 \end{bmatrix},B=\begin{bmatrix} 0.5 & 3 \\ 0 & -4 \\ 1 & 6 \\ 8 & 7 \\ 2 & \end{bmatrix},C=\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

ABBA $\begin{bmatrix} 2 & 24 \\ -4.5 & 12 \\ 32 & -9 \\ -8 & 64 \\ 61 & \end{bmatrix}$ CABC $\begin{bmatrix} 0.5 & 3 & 0.5 \\ 2 & 1 & 2 \\ 10 & 7 & 10 \end{bmatrix}$ ABC

$$B=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

B 2 $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ B 3B 4 $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ B 5 B n . B 201 B 202 ,

$$B\ n=\begin{cases} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}, & n \text{ even}, \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}, & n \text{ odd}. \end{cases}$$

2×2

$$3x+4y=7$$

$$4x-2y=5$$

$$\begin{bmatrix} 3 & 4 \\ 4 & -2 \end{bmatrix}\ |\ \begin{matrix} 7 \\ 5 \end{matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 4 & -2 \end{bmatrix}$$

3x−y−z=0 x+y=5 2x−3z=2

[3 −1 −1 1 1 0 2 0 −3]

[3 −1 −1 1 1 0 2 0 −3 | 0 5 2]

ax+by+cz=d

x+2y−z=3 2x−y+2z=6 x−3y+3z=4

[1 2 −1 2 −1 2 1 −3 3 | 3 6 4]

4x−3y=11 3x+2y=4

[4 −3 3 2 | 11 4]

[1 −3 −5 2 −5 −4 −3 5 4 | −2 5 6]

x, y, z,

[1 −3 −5 2 −5 −4 −3 5 4 | −2 5 6]→ x−3y−5z=−2 2x−5y−4z=5 −3x+5y+4z=6

[1 −1 1 2 −1 3 0 1 1 | 5 1 −9]

x − y + z=5 2x − y + 3z=1 y + z=−9

Row-echelon form [1 a b 0 1 d 0 0 1]

R i ↔ R j c R i R i +c R j) A

A=[a 11 a 12 a 13 a 21 a 22 a 23 a 31 a 32 a 33] → After Gaussian elimination A=[1 b 12 b 13 0 1 b 23 0 0 1]

2×2

2x+3y=6 x−y= 1 2

[2 3 1 −1 | 6 1 2]

R 1 ↔ R 2 →[1 −1 2 3 | 1 2 6]

−2,

−2 R 1 + R 2 = R 2 →[1 −1 0 5 | 1 2 5]

1 5 .

1 5 R 2 = R 2 →[1 −1 0 1 | 1 2 1]

y=1. y=1

x−(1)= 1 2 x= 3 2

(3 2 ,1).

4x+3y=11 x−3y=−1

(2, 1) 2 × 2

2x+y=1 4x+2y=6

[2 1 4 2 | 1 6]

1 2 .

1 2 R 1 = R 1 →[1 1 2 4 2 | 1 2 6]

−4

−4 R 1 + R 2 = R 2 →[1 1 2 0 0 | 1 2 4]

0=4.

3x+4y=12 6x+8y=24

A=[3 4 6 8 | 12 24]

− 1 2 R 2 + R 1 = R 1 →[0 0 6 8 | 0 24] R 1 ↔ R 2 →[6 8 0 0 | 24 0]

0y=0. y.

3x+4y=12 4y=12−3x y=3− 3 4 x

(x,3− 3 4 x).

[1 −3 4 2 −5 6 −3 3 4 | 3 6 6]

−2

−2 R 1 + R 2 = R 2 →[1 −3 4 0 1 −2 −3 3 4 | 3 0 6]

3 R 1 + R 3 = R 3 →[1 −3 4 0 1 −2 0 −6 16 | 3 0 15]

6 R 2 + R 3 = R 3 →[1 −3 4 0 1 −2 0 0 4 | 3 0 15]

1 2 R 3 = R 3 →[1 −3 4 0 1 −2 0 0 1 | 3 −6 21 2]

x−2y+3z=9 −x+3y=−4 2x−5y+5z=17

[1 −5 2 5 2 0 1 5 0 0 1 | 17 2 9 2]

x − y + z= 8 2x + 3y − z=−2 3x − 2y −9z= 9

[1 −1 1 2 3 −1 3 −2 −9 | 8 −2 9]

−2 R 1 + R 2 = R 2 →[1 −1 1 0 5 −3 3 −2 −9 | 8 −18 9] −3 R 1 + R 3 = R 3 →[1 −1 1 0 5 −3 0 1 −12 | 8 −18 −15]

R 2 R 3 .

Interchange R 2 and R 3 →[1 −1 1 8 0 1 −12 −15 0 5 −3 −18]

−5 R 2 + R 3 = R 3 →[1 −1 1 0 1 −12 0 0 57 | 8 −15 57] − 1 57 R 3 = R 3 →[1 −1 1 0 1 −12 0 0 1 | 8 −15 1]

x−y+z=8 y−12z=−15 z=1

(4,−3,1).

−x−2y+z=−1 2x+3y=2 y−2z=0

[−1 −2 1 2 3 0 0 1 −2 | −1 2 0]

−1

− R 1 →[1 2 −1 1 2 3 0 2 0 1 −2 0]

R 2 ↔ R 3 →[1 2 −1 0 1 −2 2 3 0 | 1 0 2]

−2 R 1 + R 3 = R 3 →[1 2 −1 0 1 −2 0 −1 2 | 1 0 0]

R 2 + R 3 = R 3 →[1 2 −1 0 1 −2 0 0 0 | 2 1 0]

x+2y−z=1 y−2z=0 0=0

0=0 y z x.

x+2y−z=1 y=2z x+2(2z)−z=1 x+3z=1 z= 1−x 3

z y x.

y−2z=0 z= 1−x 3 y−2(1−x 3)=0 y= 2−2x 3

(x, 2−2x 3 , 1−x 3).

x+4y−z=4 2x+5y+8z=15 x+3y−3z=1

(1, 1, 1) [A], [B], [C], ….

5x+3y+9z=−1 −2x+3y−z=−2 −x−4y+5z=1

[5 3 9 −2 3 −1 −1 −4 5 | 5 −2 −1]

[A].

[A]=[5 3 9 −1 −2 3 −1 −2 −1 −4 5 1]

[A].

ref([A])

[1 3 5 9 5 1 5 0 1 13 21 − 4 7 0 0 1 − 24 187]→ x+ 3 5 y+ 9 5 z=− 1 5 y+ 13 21 z=− 4 7 z=− 24 187

(61 187 ,− 92 187 ,− 24 187). x= y=

x+y=12,000 0.105x+0.12y=1,335

[1 1 0.105 0.12 | 12,000 1,335]

−0.105

[1 1 0 0.015 | 12,000 75]

0.015y=75 y=5,000

12,000−5,000=7,000. x y z

x+y+z=10,000 0.05x+0.08y+0.09z=770 2x−z=0

[1 1 1 0.05 0.08 0.09 2 0 −1 | 10,000 770 0]

−0.05 R 1 + R 2 = R 2 →[1 1 1 0 0.03 0.04 2 0 −1 | 10,000 270 0] −2 R 1 + R 3 = R 3 →[1 1 1 0 0.03 0.04 0 −2 −3 | 10,000 270 −20,000] 1 0.03 R 2 = R 2 →[0 1 1 0 1 4 3 0 −2 −3 | 10,000 9,000 −20,000] 2 R 2 + R 3 = R 3 →[1 1 1 0 1 4 3 0 0 − 1 3 | 10,000 9,000 −2,000]

− 1 3 z=−2,000; z=6,000. y+ 4 3 z=9,000. z=6,000,

y+ 4 3 (6,000)=9,000 y+8,000=9,000 y=1,000

x+y+z=10,000. y=1,000 z=6,000,

x+1,000+6,000=10,000 x=3,000

[9 3 1 −2 | 0 6]. R 2 = R 2 −9 R 1 . R 2 = R 1 −9 R 2 . 8x−37y=8 2x+12y=3 16y=4 9x−y=2

[0 16 9 −1 | 4 2] 3x+2y+10z=3 −6x+2y+5z=13 4x+z=18

x+5y+8z=19 12x+3y=4 3x+4y+9z=−7[1 5 8 12 3 0 3 4 9 | 16 4 −7]

6x+12y+16z=4 19x−5y+3z=−9 x+2y=−8[−2 5 6 −18 | 5 26]−2x+5y=5 6x−18y=26

[3 4 10 17 | 10 439][3 2 0 −1 −9 4 8 5 7 | 3 −1 8]3x+2y=13 −x−9y+4z=53 8x+5y+7z=80

[8 29 1 −1 7 5 0 0 3 | 43 38 10][4 5 −2 0 1 58 8 7 −3 | 12 2 −5]

4x+5y−2z=12 y+58z=2 8x+7y−3z=−5[1 0 0 0 | 3 0][1 0 1 0 | 1 2][1 2 4 5 | 3 6]

[−1 2 4 −5 | −3 6](−1,−2)[−2 0 0 2 | 1 −1] 2x−3y=−9 5x+4y=58(6,7)6x+2y=−4 3x+4y=−17

2x+3y=12 4x+y=14(3,2)−4x−3y=−2 3x−5y=−13−5x+8y=3 10x+6y=5(1 5 , 1 2)

3x+4y=12 −6x−8y=−24−60x+45y=12 20x−15y=−4(x, 4 15 (5x+1))11x+10y=43 15x+20y=65

2x−y=2 3x+2y=17(3,4)−1.06x−2.25y=5.51 −5.03x−1.08y=5.403 4 x− 3 5 y=4 1 4 x+ 2 3 y=1

(196 39 ,− 5 13)1 4 x− 2 3 y=−1 1 2 x+ 1 3 y=3[1 0 0 0 1 1 0 0 1 | 31 45 87](31,−42,87)

[1 0 1 1 1 0 0 1 1 | 50 20 −90][1 2 3 0 5 6 0 0 8 | 4 7 9](21 40 , 1 20 , 9 8)

[−0.1 0.3 −0.1 −0.4 0.2 0.1 0.6 0.1 0.7 | 0.2 0.8 −0.8] −2x+3y−2z=3 4x+2y−z=9 4x−8y+2z=−6

(18 13 , 15 13 ,− 15 13) x+y−4z=−4 5x−3y−2z=0 2x+6y+7z=30

2x+3y+2z=1 −4x−6y−4z=−2 10x+15y+10z=5(x,y, 1 2 (1−2x−3y))

x+2y−z=1 −x−2y+2z=−2 3x+6y−3z=5 x+2y−z=1 −x−2y+2z=−2 3x+6y−3z=3(x,− x 2 ,−1)

x+y=2 x+z=1 −y−z=−3x+y+z=100 x+2z=125 −y+2z=25(125,−25,0)

1 4 x− 2 3 z=− 1 2 1 5 x+ 1 3 y= 4 7 1 5 y− 1 3 z= 2 9

− 1 2 x+ 1 2 y+ 1 7 z=− 53 14 1 2 x− 1 2 y+ 1 4 z=3 1 4 x+ 1 5 y+ 1 3 z= 23 15(8,1,−2)

− 1 2 x− 1 3 y+ 1 4 z=− 29 6 1 5 x+ 1 6 y− 1 7 z= 431 210 − 1 8 x+ 1 9 y+ 1 10 z=− 49 45

x−1 7 + y−2 8 + z−3 4 =0 x+y+z=6 x+2 3 +2y+ z−3 3 =5(1,2,3)

x−1 4 − y+1 4 +3z=−1 x+5 2 + y+7 4 −z=4 x+y− z−2 2 =1

x−3 4 − y−1 3 +2z=−1 x+5 2 + y+5 2 + z+5 2 =8 x+y+z=1

(x, 31 28 − 3x 4 , 1 28 (−7x−3))x−3 10 + y+3 2 −2z=3 x+5 4 − y−1 8 +z= 3 2 x−1 4 + y+4 2 +3z= 3 2

x−3 4 − y−1 3 +2z=−1 x+5 2 + y+5 2 + z+5 2 =7 x+y+z=1 A B a a −1 ,

a a −1 = a −1 a=(1 a)a=1. 2 −1 = 1 2 (1 2)2=1. A A −1 I n n 2×2 3×3

I 2 =[1 0 0 1]

I 3 =[1 0 0 0 1 0 0 0 1]

AI=IA=A.

A A −1 =I A −1 A=I

A A −1 = A −1 A=I, A A −1 2 × 2 2 × 2 3 × 3 I n ,

I 2 =[1 0 0 1] I 3 =[1 0 0 0 1 0 0 0 1] 2 × 2 3 × 3

A n×n B n×n AB=BA= I n , B= A −1 , A. AI=IA=A.

A=[3 4 −2 5]

A

AI=[3 4 −2 5] [1 0 0 1]=[3·1+4·0 3·0+4·1 −2·1+5·0 −2·0+5·1]=[3 4 −2 5]

AI=[1 0 0 1] [3 4 −2 5]=[1·3+0·(−2) 1·4+0·5 0·3+1·(−2) 0·4+1·5]=[3 4 −2 5]

A n×n B n×n AB. AB=I, BA. BA=I, B= A −1 A= B −1 .

A=[1 5 −2 −9],B=[−9 −5 2 1]

AB BA.

AB=[1 5 −2 −9]·[−9 −5 2 1] =[1(−9)+5(2) 1(−5)+5(1) −2(−9)−9(2) −2(−5)−9(1)] =[1 0 0 1]

BA=[−9 −5 2 1]·[1 5 −2 −9] =[−9(1)−5(−2) −9(5)−5(−9) 2(1)+1(−2) 2(−5)+1(−9)] =[1 0 0 1]

A B

A=[1 4 −1 −3],B=[−3 −4 1 1]

AB=[1 4 −1 −3] [−3 −4 1 1]=[1(−3)+4(1) 1(−4)+4(1) −1(−3)+−3(1) −1(−4)+−3(1)]=[1 0 0 1] BA=[−3 −4 1 1] [1 4 −1 −3]=[−3(1)+−4(−1) −3(4)+−4(−3) 1(1)+1(−1) 1(4)+1(−3)]=[1 0 0 1]

$$A=[\ 1\ -2\ 2\ -3\ ]$$

A

$$[\ 1\ -2\ 2\ -3\ ]\ \ [\ a\ b\ c\ d\ ]=[\ 1\ 0\ 0\ 1\ ]$$

$$[\ 1\ -2\ 2\ -3\ ]\ \ [\ a\ b\ c\ d\ ]=[\ 1a-2c\ 1b-2d\ 2a-3c\ 2b-3d\ ]$$

$$1a-2c=1 \quad R\ 1 \qquad 2a-3c=0 \quad R\ 2$$

$(-2)\ R\ 1 + R\ 2 \rightarrow R\ 2$. c.

$$1a-2c=1 \quad 0+1c=-2 \quad c=-2$$

a.

$$a-2(-2)=1 \quad a+4=1 \quad a=-3$$

$$1b-2d=0\ R\ 1 \qquad 2b-3d=1\ R\ 2$$

$(-2)\ R\ 1 + R\ 2 = R\ 2$. d.

$$1b-2d=0 \qquad 0+1d=1 \qquad d=1$$

b.

$$b-2(1)=0 \quad b-2=0 \quad b=2$$

$$A^{-1}=[\ -3\ 2\ -2\ 1\ ]$$

A I, I A^{-1} .

$$A=[\ 2\ 1\ 5\ 3\ ]$$

A

$$[\ 2\ 1\ 5\ 3\ \ |\ 1\ 0\ 0\ 1\ ]$$

A

$$[\ 5\ 3\ 2\ 1\ \ |\ 0\ 1\ 1\ 0\ ]$$

−2

$$[\ 1\ 1\ 2\ 1\ \ |\ -2\ 1\ 1\ 0\ ]$$

−2

$$[\ 1\ 1\ 0\ -1\ \ |\ -2\ 1\ 5\ -2\ ]$$

$$[\ 1\ 0\ 0\ -1\ \ |\ 3\ -1\ 5\ -2\ ]$$

−1.

$$[\ 1\ 0\ 0\ 1\ \ |\ 3\ -1\ -5\ 2\ ]$$

A^{-1} .

$$A^{-1}=[\ 3\ -1\ -5\ 2\ ]$$

2×2 A 2×2

$$A=[\ a\ b\ c\ d\ ]$$

A

$$A^{-1} = 1\ ad-bc\ [\ d\ -b\ -c\ a\ ]$$

$ad-bc \neq 0$. $ad-bc=0$, A

$$A=[\ 1\ -2\ 2\ -3\ ]$$

$$A^{-1} = 1\ (1)(-3)-(-2)(2)\ [\ -3\ 2\ -2\ 1\ ] \qquad = 1\ -3+4\ [\ -3\ 2\ -2\ 1\ ] \qquad =[\ -3\ 2\ -2\ 1\ ]$$

A

$$[\ 1\ -2\ 2\ -3\ \ |\ \ 1\ 0\ 0\ 1\ ]$$

A −2

$$[\ 1\ -2\ 0\ 1\ \ |\ \ 1\ 0\ -2\ 1\ ]$$

$$[\ 1\ 0\ 0\ 1\ \ |\ \ -3\ 2\ -2\ 1\ ]$$

$$A^{-1}=[\ -3\ 2\ -2\ 1\ ]$$

A.

$$A=[\ 1\ -1\ 2\ \ 3\ ]$$

$A^{-1}=[\ 3\ 5\ 1\ 5\ -\ 2\ 5\ 1\ 5\ ]$

$$A=[\ 3\ 6\ 1\ 2\ ]$$

$$[\ 3\ 6\ 1\ 3\ \ |\ \ 1\ 0\ 0\ 1\ ]$$

$$[\ 1\ 3\ 3\ 6\ \ |\ \ 0\ 1\ 1\ 0\ ]$$

[1 2 0 0 | 1 0 −3 1]

2×2 3×3 3 × 3

A=[2 3 1 3 3 1 2 4 1]

A

A|I=[2 3 1 3 3 1 2 4 1 | 1 0 0 0 1 0 0 0 1]

A 3 × 3 A A −1 =I A −1 A=I. 3 × 3 A,

A=[2 3 1 3 3 1 2 4 1]

A A. A.

[2 3 1 3 3 1 2 4 1 | 1 0 0 0 1 0 0 0 1] → Interchange R 2 and R 1 [3 3 1 2 3 1 2 4 1 | 0 1 0 1 0 0 0 0 1]

− R 2 + R 1 = R 1 →[1 0 0 2 3 1 2 4 1 | −1 1 0 1 0 0 0 0 1]

− R 2 + R 3 = R 3 →[1 0 0 2 3 1 0 1 0 | −1 1 0 1 0 0 −1 0 1]

R 3 ↔ R 2 →[1 0 0 0 1 0 2 3 1 | −1 1 0 −1 0 1 1 0 0]

−2 R 1 + R 3 = R 3 →[1 0 0 0 1 0 0 3 1 | −1 1 0 −1 0 1 3 −2 0]

−3 R 2 + R 3 = R 3 →[1 0 0 0 1 0 0 0 1 | −1 1 0 −1 0 1 6 −2 −3]

A −1 =B=[−1 1 0 −1 0 1 6 −2 −3]

B= A −1 , A A −1 =I A −1 A=I.

A A −1 =[2 3 1 3 3 1 2 4 1] [−1 1 0 −1 0 1 6 −2 −3] =[2(−1)+3(−1)+1(6) 2(1)+3(0)+1(−2) 2(0)+3(1)+1(−3) 3(−1)+3(−1)+1(6) 3(1)+3(0)+1(−2) 3(0)+3(1)+1(−3) 2(−1)+4(−1)+1(6) 2(1)+4(0)+1(−2) 2(0)+4(1)+1(−3)] =[1 0 0 0 1 0 0 0 1]

A −1 A=[−1 1 0 −1 0 1 6 −2 −3] [2 3 1 3 3 1 2 4 1] =[−1(2)+1(3)+0(2) −1(3)+1(3)+0(4) −1(1)+1(1)+0(1) −1(2)+0(3)+1(2) −1(3)+0(3)+1(4) −1(1)+0(1)+1(1) 6(2)+−2(3)+−3(2) 6(3)+−2(3)+−3(4) 6(1)+−2(1)+−3(1)] =[1 0 0 0 1 0 0 0 1]

3×3

A=[2 −17 11 −1 11 −7 0 3 −2]

A −1 =[1 1 2 2 4 −3 3 6 −5] X B

AX=B

A X B AX=B.

a 1 x+ b 1 y= c 1 a 2 x+ b 2 y= c 2

A=[a 1 b 1 a 2 b 2]

X=[x y]

B=[c 1 c 2]

AX=B

[a 1 b 1 a 2 b 2] [x y]=[c 1 c 2]

(2 −1) 2=(1 2) 2=1. ax=b x, a.

ax=b (1 a)ax=(1 a)b (a −1)ax=(a −1)b [(a −1)a]x=(a −1)b 1x=(a −1)b x=(a −1)b

2 × 2 3 × 3 A, X, B.

AX=B

A

(A −1)AX=(A −1)B [(A −1)A]X=(A −1)B IX=(A −1)B X=(A −1)B

3x+8y=5 4x+11y=7

A=[3 8 4 11],X=[x y],B=[5 7]

[3 8 4 11] [x y]=[5 7]

A −1 .

A −1 = 1 ad−bc [d −b −c a] = 1 3(11)−8(4) [11 −8 −4 3] = 1 1 [11 −8 −4 3]

A −1 =[11 −8 −4 3]

A −1 .

(A −1)AX=(A −1)B [11 −8 −4 3] [3 8 4 11] [x y]=[11 −8 −4 3] [5 7]
[1 0 0 1] [x y]=[11(5)+(−8)7 −4(5)+3(7)] [x y]=[−1 1]
(−1,1). X B A −1 ? A −1 B≠B A −1 .
(A −1)AX=(A −1)B [(A −1)A]X=(A −1)B IX=(A −1)B X=(A −1)B
A −1 , A −1 A B
5x+15y+56z=35 −4x−11y−41z=−26 −x−3y−11z=−7
AX=B.
[5 15 56 −4 −11 −41 −1 −3 −11] [x y z]=[35 −26 −7]
A
[5 15 56 −4 −11 −41 −1 −3 −11 | 1 0 0 0 1 0 0 0 1]
1 5 .
[1 3 56 5 −4 −11 −41 −1 −3 −11 | 1 5 0 0 0 1 0 0 0 1]
[1 3 56 5 0 1 19 5 −1 −3 −11 | 1 5 0 0 4 5 1 0 0 0 1]
[1 3 56 5 0 1 19 5 0 0 1 5 | 1 5 0 0 4 5 1 0 1 5 0 1]
[1 0 − 1 5 0 1 19 5 0 0 1 5 | − 11 5 −3 0 4 5 1 0 1 5 0 1]
[1 0 − 1 5 0 1 19 5 0 0 1 | − 11 5 −3 0 4 5 1 0 1 0 5]
1 5
[1 0 0 0 1 19 5 0 0 1 | −2 −3 1 4 5 1 0 1 0 5]
− 19 5
[1 0 0 0 1 0 0 0 1 | −2 −3 1 −3 1 −19 1 0 5]
A −1 =[−2 −3 1 −3 1 −19 1 0 5]
A −1 . A −1 AX= A −1 B:
[−2 −3 1 −3 1 −19 1 0 5] [5 15 56 −4 −11 −41 −1 −3 −11] [x y z]=[−2 −3 1 −3 1 −19 1 0 5] [35 −26 −7]
A −1 B=[−70+78−7 −105−26+133 35+0−35]=[1 2 0]
(1,2,0).
2x−17y+11z=0 −x+11y−7z=8 3y−2z=−2
X=[4 38 58]
[A] [B].
2x+3y+z=32 3x+3y+z=−27 2x+4y+z=−2
[A], [B].
[A]=[2 3 1 3 3 1 2 4 1], [B]=[32 −27 −2]
X,
[A] −1 ×[B]
[−59 −34 252]
2 × 2I 2 =[1 0 0 1]3 × 3I 3 =[1 0 0 0 1 0 0 0 1]2 × 2A −1 = 1 ad−bc [d −b −c a], where ad−bc≠0
AI=IA=A. A A −1 = A −1 A=I. 2×2 3×3 AX=B, A: A −1 AX= A −1 B. AB≠BA A −1 A=A A −1 ?
A −1 A, A A −1 =I, A A −1 , A −1 A=I. 2×2 2×2 2×2 ad bc ad−bc=0, 2×2 [0 1 1 0].
A −1 = 1 0(0)−1(1) [0 −1 −1 0]=[0 1 1 0]. A B.A=[1 0 −1 1], B=[1 0 1 1]
A=[1 2 3 4], B=[−2 1 3 2 − 1 2]AB=BA=[1 0 0 1]=IA=[4 5 7 0], B=[0 1 7 1 5 − 4 35]
A=[−2 1 2 3 −1], B=[−2 −1 −6 −4]AB=BA=[1 0 0 1]=I
A=[1 0 1 0 1 −1 0 1 1], B= 1 2 [2 1 −1 0 1 1 0 −1 1]
A=[1 2 3 4 0 2 1 6 9], B= 1 4 [6 0 −2 17 −3 −5 −12 2 4]AB=BA=[1 0 0 0 1 0 0 0 1]=I
A=[3 8 2 1 1 1 5 6 12], B= 1 36 [−6 84 −6 7 −26 1 −1 −22 5][3 −2 1 9]1 29 [9 2 −1 3][−2 2 3 1]
[−3 7 9 2]1 69 [−2 7 9 3][−4 −3 −5 8][1 1 2 2][0 1 1 0][0.5 1.5 1 −0.5]4 7 [0.5 1.5 1 −0.5]
[1 0 6 −2 1 7 3 0 2][0 1 −3 4 1 0 1 0 5]1 17 [−5 5 −3 20 −3 12 1 −1 4][1 2 −1 −3 4 1 −2 −4 −5]
[1 9 −3 2 5 6 4 −2 7]1 209 [47 −57 69 10 19 −12 −24 38 −13][1 −2 3 −4 8 −12 1 4 2]
[1 2 1 2 1 2 1 3 1 4 1 5 1 6 1 7 1 8][18 60 −168 −56 −140 448 40 80 −280][1 2 3 4 5 6 7 8 9] 2 × 2
5x−6y=−61 4x+3y=−2(−5,6)8x+4y=−100 3x−4y=1 3x−2y=6 −x+5y=−2(2,0)5x−4y=−5 4x+y=2.3

–3x–4y=9 12x+4y=–6(1 3 ,– 5 2)–2x+3y= 3 10 –x+5y= 1 2 8 5 x– 4 5 y= 2 5 – 8 5 x+ 1 5 y= 7 10 (– 2 3 ,– 11 6)1 2 x+ 1 5 y=– 1 4 1 2 x– 3 5 y=– 9 4 3×3 3x–2y+5z=21 5x+4y=37 x–2y–5z=5 (7, 1 2 , 1 5) 4x+4y+4z=40 2x–3y+4z=–12 –x+3y+4z=9 6x–5y–z=31 –x+2y+z=–6 3x+3y+2z=13 (5,0,–1)6x–5y+2z=–4 2x+5y–z=12 2x+5y+z=124x–2y+3z=–12 2x+2y–9z=33 6y–4z=1 1 34 (–35,–97,–154)1 10 x– 1 5 y+4z= –41 2 1 5 x–20y+ 2 5 z=–101 3 10 x+4y– 3 10 z=23 1 2 x– 1 5 y+ 1 5 z= 31 100 – 3 4 x– 1 4 y+ 1 2 z= 7 40 – 4 5 x– 1 2 y+ 3 2 z= 1 4 1 690 (65,–1136,–229)0.1x+0.2y+0.3z=–1.4 0.1x–0.2y+0.3z=0.6 0.4y+0.9z=–2 2x–y=–3 –x+2y=2.3(– 37 30 , 8 15)– 1 2 x– 3 2 y=– 43 20 5 2 x+ 11 5 y= 31 4 12.3x–2y–2.5z=2 36.9x+7y–7.5z=–7 8y–5z=–10(10 123 ,–1, 2 5) 0.5x–3y+6z=–0.8 0.7x–2y=–0.06 0.5x+4y+5z=0[1 0 1 0 0 1 0 1 0 1 1 0 0 0 1 1] 1 2 [2 1 –1 –1 0 1 1 –1 0 –1 1 1 0 1 –1 1][–1 0 2 5 0 0 0 2 0 2 –1 0 1 –3 0 1] [1 –2 3 0 0 1 0 2 1 4 –2 3 –5 0 1 1]1 39 [3 2 1 –7 18 –53 32 10 24 –36 21 9 –9 46 –16 –5] [1 2 0 2 3 0 2 1 0 0 0 0 3 0 1 0 2 0 0 1 0 0 1 2 0] [1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 1 1 1 1 1 1] [1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 –1 –1 –1 –1 –1 1] 14 ft 2 . 3 ft 2 2 × 2

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

det(A) | A |.

$$A=\begin{bmatrix} 5 & 2 \\ -6 & 3 \end{bmatrix}$$

$$\det(A)=\begin{vmatrix} 5 & 2 \\ -6 & 3 \end{vmatrix} = 5(3)-(-6)(2) = 27$$

$$a_1x+b_1y=c_1 \quad (1)$$
$$a_2x+b_2y=c_2 \quad (2)$$

x. y y y

$b_2a_1x+b_2b_1y=b_2c_1$ Multiply R 1 by b_2 – $b_1a_2x-b_1b_2y=-b_1c_2$ Multiply R 2 by– b_1 _____ $b_2a_1x-b_1a_2x=b_2c_1-b_1c_2$

x.

$b_2a_1x-b_1a_2x=b_2c_1-b_1c_2$ $x(b_2a_1-b_1a_2)=b_2c_1-b_1c_2$ $x=\frac{b_2c_1-b_1c_2}{b_2a_1-b_1a_2}=\frac{\begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}}{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}}$

y, x.

$a_2a_1x+a_2b_1y=a_2c_1$ Multiply R 1 by a_2 – $a_1a_2x-a_1b_2y=-a_1c_2$ Multiply R 2 by– a_1 _____ $a_2b_1y-a_1b_2y=a_2c_1-a_1c_2$

y

$a_2b_1y-a_1b_2y=a_2c_1-a_1c_2$ $y(a_2b_1-a_1b_2)=a_2c_1-a_1c_2$ $y=\frac{a_2c_1-a_1c_2}{a_2b_1-a_1b_2}=\frac{\begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}}{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}}$

x y x y, D:D x :x

$$x=\frac{D_x}{D}$$

D y : y

$$y=\frac{D_y}{D}$$

x y

$$a_1x+b_1y=c_1$$
$$a_2x+b_2y=c_2$$

$x=\frac{D_x}{D}=\frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$, $D\neq0$; $y=\frac{D_y}{D}=\frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$, $D\neq0$.

x, x y, y 2 × 2

$$12x+3y=15 \quad 2x-3y=13$$

x.

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 15 & 3 \\ 13 & -3 \end{vmatrix}}{\begin{vmatrix} 12 & 3 \\ 2 & -3 \end{vmatrix}}=\frac{-45-39}{-36-6}=\frac{-84}{-42}=2$$

y.

$$y=\frac{D_y}{D}=\frac{\begin{vmatrix} 12 & 15 \\ 2 & 13 \end{vmatrix}}{\begin{vmatrix} 12 & 3 \\ 2 & -3 \end{vmatrix}}=\frac{156-30}{-36-6}=\frac{126}{-42}=-3$$

(2,–3).

$$x+2y=-11 \quad -2x+y=-13$$

$(3,-7)$

$$A=[\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}]$$

A

$$\det(A)=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}$$

$$|A|=a_1b_2c_3+b_1c_2a_3+c_1a_2b_3-a_3b_2c_1-b_3c_2a_1-c_3a_2b_1$$

$$A=[\begin{matrix} 0 & 2 & 1 \\ 3 & -1 & 1 \\ 4 & 0 & 1 \end{matrix}]$$

$$|A|=\begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 1 \\ 4 & 0 & 1 \end{vmatrix} \quad \begin{matrix} 0 & 3 \\ 4 & 2 \\ -1 & 0 \end{matrix} \quad =0(-1)(1)+2(1)(4)+1(3)(0)-4(-1)(1)-0(1)(0)$$

$$-1(3)(2) \quad =0+8+0+4-0-6 \quad =6$$

$$\det(A)=\begin{vmatrix} 1 & -3 & 7 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

-10 2×2 3×3

$$x=\frac{D_x}{D},y=\frac{D_y}{D},z=\frac{D_z}{D},D\neq 0$$

D_x, x D_y, y D_z, z

$$x+y-z=6 \quad 3x-2y+z=-5 \quad x+3y-2z=14$$

$$D=\begin{vmatrix} 1 & 1 & -1 \\ 3 & -2 & 1 \\ 1 & 3 & -2 \end{vmatrix}, D_x=\begin{vmatrix} 6 & 1 & -1 \\ -5 & -2 & 1 \\ 14 & 3 & -2 \end{vmatrix}, D_y=\begin{vmatrix} 1 & 6 & -1 \\ 3 & -5 & 1 \\ 1 & 14 & -2 \end{vmatrix}, D_z=\begin{vmatrix} 1 & 1 & 6 \\ 3 & -2 & -5 \\ 1 & 3 & 14 \end{vmatrix}$$

$$x=\frac{D_x}{D}=\frac{-3}{-3}=1 \quad y=\frac{D_y}{D}=\frac{-9}{-3}=3 \quad z=\frac{D_z}{D}=\frac{6}{-3}=-2$$

$(1,3,-2)$.

$$x-3y+7z=13 \quad x+y+z=1 \quad x-2y+3z=4$$

$\left(-2,\frac{3}{5},\frac{12}{5}\right)$

$$3x-2y=4 \quad (1) \quad 6x-4y=0 \quad (2)$$

D, D_x, and D_y.

$$D=\begin{vmatrix} 3 & -2 \\ 6 & -4 \end{vmatrix}=3(-4)-6(-2)=0$$

-2. (2).

$$-6x+4y=-8 \quad 6x-4y=0 \quad \_\_\_\_\_ \quad 0=-8$$

$0=-8$,

$$x-2y+3z=0 \ (1) \quad 3x+y-2z=0 \ (2) \quad 2x-4y+6z=0 \ (3)$$

$$\begin{vmatrix} 1 & -2 & 3 \\ 3 & 1 & -2 \\ 2 & -4 & 6 \end{vmatrix} \quad \begin{matrix} 1 & -2 \\ 3 & 1 \\ 2 & -4 \end{matrix}$$

$$1(1)(6)+(-2)(-2)(2)+3(3)(-4)-2(1)(3)-(-4)(-2)(1)-6(3)(-2)=0$$

-2

$$-2x+4y-6x=0 \quad 2x-4y+6z=0 \quad 0=0$$

$0=0$, A^{-1} A.

$$A=[\begin{matrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{matrix}]$$

A

$$A=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & \end{bmatrix} \quad \begin{matrix} 1 & 0 & 0 \\ 2 & 2 & 0 \end{matrix}$$

$$\det(A)=1(2)(-1)+2(1)(0)+3(0)(0)-0(2)(3)-0(1)(1)+1(0)(2) \quad =-2$$

$A=[\begin{matrix} -1 & 5 \\ 4 & -3 \end{matrix}]$, $\det(A)=(-1)(-3)-(4)(5)=3-20=-17$ $B=[\begin{matrix} 4 & -3 \\ -1 & 5 \end{matrix}]$, $\det(B)=(4)(5)-(-1)(-3)=20-3=17$

$$A=\begin{bmatrix} 1 & 2 & 2 & 2 & 2 & 2 \\ -1 & 2 & 2 & \end{bmatrix} \quad \begin{matrix} 1 & 2 & -1 \\ 2 & 2 & 2 \end{matrix}$$ $\det(A)=1(2)(2)+2(2)(-1)+2(2)(2)+1(2)(2)-2(2)(1)-2(2)(2) \quad =4-4+8+4-4-8=0$

$$A=[\begin{matrix} 1 & 2 \\ 0 & 0 \end{matrix}], \quad \det(A)=1(0)-2(0)=0$$

A^{-1} A.

$A=[\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}]$, $\det(A)=1(4)-3(2)=-2$ $A^{-1}=[\begin{matrix} -2 & 1 \\ 3 & 2 \\ -1 & 2 \end{matrix}]$, $\det(A^{-1})=-2(-1\ 2)-(3\ 2)(1)=-1\ 2$

$A=[\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}]$, $\det(A)=1(4)-2(3)=-2$ $B=[\begin{matrix} 2(1) & 2(2) \\ 3 & 4 \end{matrix}]$, $\det(B)=2(4)-3(4)=-4$

$$2x+4y+4z=2 \ (1) \quad 3x+7y+7z=-5 \ (2) \quad x+2y+2z=4 \ (3)$$

$$D=\begin{vmatrix} 2 & 4 & 4 \\ 3 & 7 & 7 \\ 1 & 2 & 2 \end{vmatrix}$$

$$-2x-4y-4x=-8 \quad 2x+4y+4z=2 \quad 0=-6$$

$[\begin{matrix} a & b \\ c & d \end{matrix}]$ $ad-bc$. $x=\frac{D_x}{D},y=\frac{D_y}{D}$. $x=\frac{D_x}{D},y=\frac{D_y}{D},z=\frac{D_z}{D}$. A^{-1} A. 2×2 2×2 A

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}-2\begin{vmatrix} -1 & 2 \\ 3 & -4 \end{vmatrix}$$

| 2 −5 −1 6 |
7| −8 4 −1 5 || 1 0 3 −4 |−4| 10 20 0 −10 || 10 0.2 5 0.1 |0| 6 −3 8 4 || −2 −3 3.1 4,000 |−7,990.7
| −1.1 0.6 7.2 −0.5 || −1 0 0 0 1 0 0 0 −3 |3| −1 4 0 0 2 3 0 0 −3 || 1 0 1 0 1 0 1 0 0 |−1
| 2 −3 1 3 −4 1 −5 6 1 || −2 1 4 −4 2 −8 2 −8 −3 |224| 6 −1 2 −4 −3 5 1 9 −1 || 5 1 −1 2 3 1 3 −6 −3 |15
| 1.1 2 −1 −4 0 0 4.1 −0.4 2.5 || 2 −1.6 3.1 1.1 3 −8 −9.3 0 2 |−17.03| − 1 2 1 3 1 4 1 5 − 1 6 1 7 0 0 1 8 |
2x−3y=−1 4x+5y=9(1,1)5x−4y=2 −4x+7y=6 6x−3y=2 −8x+9y=−1(1 2 , 1 3)2x+6y=12 5x−2y=13
4x+3y=23 2x−y=−1(2,5)10x−6y=2 −5x+8y=−14x−3y=−3 2x+6y=−4(−1,− 1 3)
4x−5y=7 −3x+9y=04x+10y=180 −3x−5y=−105(15,12) 8x−2y=−3 −4x+6y=4
x+2y−4z=−1 7x+3y+5z=26 −2x−6y+7z=−6(1,3,2)
−5x+2y−4z=−47 4x−3y−z=−94 3x−3y+2z=94 4x+5y−z=−7 −2x−9y+2z=8 5y+7z=21
(−1,0,3)4x−3y+4z=10 5x−2z=−2 3x+2y−5z=−94x−2y+3z=6 −6x+y=−2 2x+7y+8z=24(1 2 ,1,2)
5x+2y−z=1 −7x−8y+3z=1.5 6x−12y+z=7
13x−17y+16z=73 −11x+15y+17z=61 46x+10y−30z=−18(2,1,4)
−4x−3y−8z=−7 2x−9y+5z=0.5 5x−6y−5z=−2 4x−6y+8z=10 −2x+3y−4z=−5 x+y+z=1
4x−6y+8z=10 −2x+3y−4z=−5 12x+18y−24z=−30| 1 0 8 9 0 2 1 0 1 0 3 0 0 2 4 3 |24
| 1 0 2 1 0 −9 1 3 3 0 −2 −1 0 1 1 −2 || 1 2 1 7 4 0 1 2 100 5 0 0 2 2,000 0 0 0 2 |1
| 1 0 0 0 2 3 0 0 4 5 6 0 7 8 9 0 | 3 4 3x−y=4 x+4y=−3 (−1,1)6x−2y=24 −3x+3y=18 (9,15)
10x+5y=−5 3x−2y=−12(−2,3)4 7 x+ 1 5 y= 43 70 5 6 x− 1 3 y=− 2 35x+6y=14 4x+8y=8(4,−1)
3x+2y=−7 2x+4y=63x+4y=2 9x+12y=38x+4y=2 6x−5y=0.7 C(x)=150x+15,000 R(x)=200x.
(300,60,000) C(x)=50x+10,000, x (400,30,000) 0.5x−0.5y=10 −0.2y+0.2x=4 0.1x+0.1z=2
(10,−10,10)5x+3y−z=5 3x−2y+4z=13 4x+3y+5z=22x+y+z=1 2x+2y+2z=1 3x+3y=2
2x−3y+z=−1 x+y+z=−4 4x+2y−3z=33 3x+2y−z=−10 x−y+2z=7 −x+3y+z=−2(−1,−2,3)
3x+4z=−11 x−2y=5 4y−z=−102x−3y+z=0 2x+4y−3z=0 6x−2y−z=0(x, 8x 5 , 14x 5)
6x−4y−2z=2 3x+2y−5z=4 6y−7z=5y= x 2 −7 y=5x−13(2,−3),(3,2)y= x 2 −4 y=5x+10
x 2 + y 2 =16 y=x−8x 2 + y 2 =25 y= x 2 +5x 2 + y 2 =4 y− x 2 =3y> x 2 −1
1 4 x 2 + y 2 <4x 2 + y 2 +2x<3 y>− x 2 −3
x 2 −2x+ y 2 −4x<4 y<−x+4x 2 + y 2 <1 y 2 <x−2x+6 x 2 +3x+2
2 x+2 , −4 x+110x+2 4 x 2 +4x+17x+20 x 2 +10x+257 x+5 , −15 (x+5) 2x−18 x 2 −12x+36
− x 2 +36x+70 x 3 −1253 x−5 , −4x+1 x 2 +5x+25−5 x 2 +6x−2 x 3 +27x 3 −4 x 2 +3x+11 (x 2 −2) 2
x−4 (x 2 −2) , 5x+3 (x 2 −2) 24 x 4 −2 x 3 +22 x 2 −6x+48 x (x 2 +4) 2
A=[4 −2 1 3],B=[6 7 −3 11 −2 4],C=[6 7 11 −2 14 0],D=[1 −4 9 10 5 −7 2 8 5],E=[7 −14 3 2 −1 3 0
1 9]
−4A[−16 8 −4 −12]10D−6EB+CABBABCCB[113 28 10 44 81 −41 84 98 −42]DEED
[−127 −74 176 −2 11 40 28 77 38]ECCEA 3[1 0 −3 0 1 2 0 0 0 | 7 −5 0]x−3z=7 y+2z=−5
[1 0 5 0 1 −2 0 0 0 | −9 4 3]−2x+2y+z=7 2x−8y+5z=0 19x−10y+22z=3
[−2 2 1 2 −8 5 19 −10 22 | 7 0 3] 4x+2y−3z=14 −12x+3y+z=100 9x−6y+2z=31
x+3z=12 −x+4y=0 y+2z=−7[1 0 3 −1 4 0 0 1 2 | 12 0 −7]3x−4y=−7 −6x+8y=14
3x−4y=1 −6x+8y=6−1.1x−2.3y=6.2 −5.2x−4.1y=4.32x+3y+2z=1 −4x−6y−4z=−2 10x+15y+10z=0
−x+2y−4z=8 3y+8z=−4 −7x+y+2z=1 [−0.2 1.4 1.2 −0.4]1 8 [2 7 6 1][1 2 − 1 2 − 1 4 3 4]
[12 9 −6 −1 3 2 −4 −3 2][2 1 3 1 2 3 3 2 1] 0.3x−0.1y=−10 −0.1x+0.3y=14(−20,40)
0.4x−0.2y=−0.6 −0.1x+0.05y=0.34x+3y−3z=−4.3 5x−4y−z=−6.1 x+z=−0.7(−1,0.2,0.3)
−2x−3y+2z=3 −x+2y+4z=−5 −2y+5z=−3| 100 0 0 0 || 0.2 −0.6 0.7 −1.1 || −1 4 3 0 2 3 0 0 −3 |
| 2 0 0 0 2 0 0 0 2 |4x−2y=23 −5x−10y=−35(6, 1 2)0.2x−0.1y=0 −0.3x+0.3y=2.5
−0.5x+0.1y=0.3 −0.25x+0.05y=0.15x+6y+3z=4 2x+y+2z=3 3x−2y+z=0
4x−3y+5z=− 5 2 7x−9y−3z= 3 2 x−5y−5z= 5 2 (0,0,− 1 2)
3 10 x− 1 5 y− 3 10 z=− 1 50 1 10 x− 1 10 y− 1 2 z=− 9 50 2 5 x− 1 2 y− 3 5 z=− 1 5−5x−y=12 x+4y=9
(−3,3)1 2 x− 1 3 y=4 3 2 x−y=0− 1 2 x−4y=4 2x+16y=25x−y=1 −10x+2y=−2
4x−6y−2z= 1 10 x−7y+5z=− 1 4 3x+6y−9z= 6 51 20 (10,5,4)x+z=20 x+y+z=20 x+2y+z=10
5x−4y−3z=0 2x+y+2z=0 x−6y−7z=0(x, 16x 5 − 13x 5)y= x 2 +2x−3 y=x−1y 2 + x 2 =25 y 2 −2 x 2 =1

(–2 2 ,– 17),(–2 2 , 17),(2 2 ,– 17),(2 2 , 17)y< x 2 +9x 2 + y 2 >4 y< x 2 +1–8x–30 x 2 +10x+25 13x+2 (3x+1) 25 3x+1 – 2x+3 (3x+1) 2x 4 – x 3 +2x–1 x (x 2 +1) 25[4 9 –2 3]+ 1 2 [–6 12 4 –8] [17 51 –8 11][1 4 –7 –2 9 5 12 0 –4] [3 –4 1 3 5 10][1 2 1 3 1 4 1 5] –1[12 –20 –15 30] detl 0 0 400 4,000 ldetl 1 2 – 1 2 0 – 1 2 0 1 2 0 1 2 0 l– 1 8 det(A)=–6,

14x–2y+13z=140 –2x+3y–6z=–1 x–5y+12z=11

[14 –2 13 –2 3 –6 1 –5 12 | 140 –1 11]

[1 0 3 –2 4 9 –6 1 2 l 12 –5 8]

x–6y=4 2x–12y=0

2x+y+z=–3 x–2y+3z=6 x–y–z=6

4x–5y=–50 –x+2y=80 (100,90)

1 100 x– 3 100 y+ 1 20 z=–49 3 100 x– 7 100 y– 1 100 z=13 9 100 x– 9 100 y– 9 100 z=99

200x–300y=2 400x+715y=4(1 100 ,0)

0.1x+0.1y–0.1z=–1.2 0.1x–0.2y+0.4z=–1.2 0.5x–0.3y+0.8z=–5.9 C(x)= x 2 +75x+2,688

R(x)= x 2 +160x. (x,y) (–c,0) (c,0). (x,y) (x,y) (a,0) (–c,0) (a,0) a–(–c)=a+c. (c,0) (a,0) a–c

(a+c)+(a–c)=2a

(x,y)

d 1 =the distance from (–c,0) to (x,y) d 2 =the distance from (c,0) to (x,y)

d 1 + d 2 (x,y) 2a (a,0). d 1 + d 2 =2a

d 1 + d 2 = (x–(–c)) 2 + (y–0) 2 + (x–c) 2 + (y–0) 2 =2a Distance formula (x+c) 2 + y 2 + (x–c) 2 + y 2 =2a Simplify expressions. (x+c) 2 + y 2 =2a– (x–c) 2 + y 2 Move radical to opposite side. (x+c) 2 + y 2 = [2a– (x–c) 2 + y 2] 2 Square both sides. x 2 +2cx+ c 2 + y 2 =4 a 2 –4a (x–c) 2 + y 2 + (x–c) 2 + y 2 Expand the squares. x 2 +2cx+ c 2 + y 2 =4 a 2 –4a (x–c) 2 + y 2 + x 2 –2cx+ c 2 + y 2 Expand remaining squares. 2cx=4 a 2 –4a (x–c) 2 + y 2 –2cx Combine like terms. 4cx–4 a 2 =–4a (x–c) 2 + y 2 Isolate the radical. cx– a 2 =–a (x–c) 2 + y 2 Divide by 4. [cx– a 2] 2 = a 2 [(x–c) 2 + y 2] 2 Square both sides. c 2 x 2 –2 a 2 cx+ a 4 = a 2 (x 2 –2cx+ c 2 + y 2) Expand the squares. c 2 x 2 –2 a 2 cx+ a 4 = a 2 x 2 –2 a 2 cx+ a 2 c 2 + a 2 y 2 Distribute a 2 . a 2 x 2 – c 2 x 2 + a 2 y 2 = a 4 – a 2 c 2 Rewrite. x 2 (a 2 – c 2)+ a 2 y 2 = a 2 (a 2 – c 2) Factor common terms. x 2 b 2 + a 2 y 2 = a 2 b 2 Set b 2 = a 2 – c 2 .

x 2 b 2 a 2 b 2 + a 2 y 2 a 2 b 2 = a 2 b 2 a 2 b 2 Divide both sides by a 2 b 2 .

x 2 a 2 + y 2 b 2 =1 Simplify.

x 2 a 2 + y 2 b 2 =1. a>b, b>a, (0,0)

x 2 a 2 + y 2 b 2 =1

a>b 2a (±a,0) 2b (0,±b) (±c,0) c 2 = a 2 – b 2 . (0,0)

x 2 b 2 + y 2 a 2 =1

a>b 2a (0,±a) 2b (±b,0) (0,±c) c 2 = a 2 – b 2 . c 2 = a 2 – b 2 . (0,0) (0,0) (±a,0) (±c,0) x 2 a 2 + y 2 b 2 =1. (0,±a) (±c,0), x 2 b 2 + y 2 a 2 =1. c 2 = a 2 – b 2 , b 2 . a 2 b 2 (±8,0) (±5,0)?

x 2 a 2 + y 2 b 2 =1

(±8,0), a=8 a 2 =64. (±5,0), c=5 c 2 =25. c 2 = a 2 – b 2 . b 2 ,

c 2 = a 2 – b 2 25=64– b 2 Substitute for c 2 and a 2 . b 2 =39 Solve for b 2 .

a 2 =64 b 2 =39 x 2 64 + y 2 39 =1. (0,±4) (0,± 15)?x 2 + y 2 16 =1 (±a,0) (0, ±a). (±c,0) (0, ±c). a c c 2 = a 2 – b 2 , b 2 . h k (h,k). x (x–h) (y–k). (h, k)

(x–h) 2 a 2 + (y–k) 2 b 2 =1

a>b 2a (h±a,k) 2b (h,k±b) (h±c,k), c 2 = a 2 – b 2 . (h,k)

(x–h) 2 b 2 + (y–k) 2 a 2 =1

a>b 2a (h,k±a) 2b (h±b,k) (h,k±c), c 2 = a 2 – b 2 . (h,k) c 2 = a 2 – b 2 . (h,k) (h,k) (x–h) 2 a 2 + (y–k) 2 b 2 =1. (x–h) 2 b 2 + (y–k) 2 a 2 =1. (h,k) a 2 2a, c 2 h k, b 2

$c^2 = a^2 - b^2$. h, k, a^2 , b^2 (−2,−8) (−2,2) (−2,−7) (−2,1)?

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

(h,k). (−2,−8) (−2,2).

$$(h,k) = (-2+(-2)^2 , -8+2^2) = (-2,-3)$$

a^2 . 2a, a

$$2a = 2-(-8) \quad 2a = 10 \quad a = 5$$

$a^2 = 25$. c^2 . (h,k±c). (h,k−c)=(−2,−7) (h,k+c)=(−2,1). k=−3 c.

$$k+c = 1 \quad -3+c = 1 \quad c = 4$$

$c^2 = 16$. b^2 $c^2 = a^2 - b^2$.

$$c^2 = a^2 - b^2 \quad 16 = 25 - b^2 \quad b^2 = 9$$

h,k, a^2 , b^2

$$\frac{(x+2)^2}{9} + \frac{(y+3)^2}{25} = 1$$

(−3,3) (5,3) ($1-2\sqrt{3}$,3) ($1+2\sqrt{3}$,3)?$\frac{(x-1)^2}{16} + \frac{(y-3)^2}{4} = 1$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a>b$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a>b$ (0, 0), $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a>b, (±a,0) (0,±b) (±c,0)

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a>b$, (0,±a) (±b,0) (0,±c) c $c^2 = a^2 - b^2$. $\frac{x^2}{9} + \frac{y^2}{25} = 1$. 25>9,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $b^2 = 9$ $a^2 = 25$. (0,0) (0,±a)=(0,± 25)=(0,±5) (±b,0)=(± 9 ,0)=(±3,0)

(0,±c), $c^2 = a^2 - b^2$ c,

$$c = \pm\sqrt{a^2 - b^2} = \pm\sqrt{25-9} = \pm\sqrt{16} = \pm 4$$

(0,±4). $\frac{x^2}{36} + \frac{y^2}{4} = 1$. (0,0); (±6,0); (0,±2); (±4 $\sqrt{2}$,0) $4x^2 + 25y^2 = 100$.

$4x^2 + 25y^2 = 100$ $4x^2\,100 + 25y^2\,100 = 100\,100$ $\frac{x^2}{25} + \frac{y^2}{4} = 1$

25>4, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a^2 = 25$ $b^2 = 4$. (0,0) (±a,0)=(± 25 ,0)=(±5,0)

(0,±b)=(0,± 4)=(0,±2) (±c,0), $c^2 = a^2 - b^2$. c,

$$c = \pm\sqrt{a^2 - b^2} = \pm\sqrt{25-4} = \pm\sqrt{21}$$

(± 21 ,0). $49x^2 + 16y^2 = 784$. $\frac{x^2}{16} + \frac{y^2}{49} = 1$; (0,0); (0,±7); (±4,0); (0,± 33) (h,k),

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, a>b$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, a>b$ (h, k),

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, a>b, (h,k) (h±a,k) (h,k±b) (h±c,k) $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$,

a>b, (h,k) (h,k±a) (h±b,k) (h,k±c) c $c^2 = a^2 - b^2$. $\frac{(x+2)^2}{4} + \frac{(y-5)^2}{9} = 1$. 9>4,

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$, $b^2 = 4$ $a^2 = 9$. (h,k)=(−2,5) (h,k±a)=(−2,5± 9)=(−2,5±3), (−2,2)

(−2,8) (h±b,k)=(−2± 4 ,5)=(−2±2,5), (−4,5) (0,5) (h,k±c), $c^2 = a^2 - b^2$. c,

$$c = \pm\sqrt{a^2 - b^2} = \pm\sqrt{9-4} = \pm\sqrt{5}$$

(−2,5− 5) (−2,5+ 5). $\frac{(x-4)^2}{36} + \frac{(y-2)^2}{20} = 1$. (4,2); (−2,2) (10,2); (4,2−2 5) (4,2+2 5);

(0,2) (8,2) $ax^2 + by^2 + cx + dy + e = 0$ x^2 y^2 $m_1(x-h)^2 + m_2(y-k)^2 = m_3$, m_1 , m_2 ,

m_3 $4x^2 + 9y^2 - 40x + 36y + 100 = 0$.

$$4x^2 + 9y^2 - 40x + 36y + 100 = 0$$

$$(4x^2 - 40x) + (9y^2 + 36y) = -100$$

$$4(x^2 - 10x) + 9(y^2 + 4y) = -100$$

$$4(x^2 - 10x + 25) + 9(y^2 + 4y + 4) = -100 + 100 + 36$$

$$4(x-5)^2 + 9(y+2)^2 = 36$$

$$\frac{(x-5)^2}{9} + \frac{(y+2)^2}{4} = 1$$

9>4, $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, $a^2 = 9$ $b^2 = 4$. (h,k)=(5,−2) (h±a,k)=(5± 9 ,−2)=(5±3,−2),

(2,−2) (8,−2) (h,k±b)=(5,−2± 4)=(5,−2±2), (5,−4) (5,0) (h±c,k), $c^2 = a^2 - b^2$. c,

$$c = \pm\sqrt{a^2 - b^2} = \pm\sqrt{9-4} = \pm\sqrt{5}$$

(5− 5 ,−2) (5+ 5 ,−2).

$$4x^2 + y^2 - 24x + 2y + 21 = 0$$

$\frac{(x-3)^2}{4} + \frac{(y+1)^2}{16} = 1$; (3,−1); (3,−5) (3,3); (1,−1) (5,−1); (3,−1−2 3) (3,−1+2 3) (0,0).

(0,0), $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a>b. 2a, 2b. a, 2a=96, a=48, $a^2 = 2304$. b, 2b=46, b=23, $b^2 = 529$.

$\frac{x^2}{2304} + \frac{y^2}{529} = 1$. (±c,0), $c^2 = a^2 - b^2$. c,

c 2 = a 2 – b 2 c 2 =2304–529 Substitute using the values found in part (a). c=± 2304–529
Take the square root of both sides. c=± 1775 Subtract. c≈±42 Round to the nearest foot.
(±42,0) 2(42)=84 (0,0).x 2 57,600 + y 2 25,600 =1x 2 a 2 + y 2 b 2 =1, a>bx 2 b 2 + y 2 a 2 =1, a>b
(h,k)(x–h) 2 a 2 + (y–k) 2 b 2 =1, a>b (h,k)(x–h) 2 b 2 + (y–k) 2 a 2 =1, a>b (x,y) 2 x 2 +y=4
4 x 2 +9 y 2 =36 x 2 3 2 + y 2 2 2 =14 x 2 – y 2 =44 x 2 +9 y 2 =1x 2 (1 2) 2 + y 2 (1 3) 2 =1
4 x 2 –8x+9 y 2 –72y+112=0x 2 4 + y 2 49 =1x 2 2 2 + y 2 7 2 =1; (0,7) (0,–7). (2,0) (–2,0).
(0,3 5),(0,–3 5).x 2 100 + y 2 64 =1x 2 +9 y 2 =1x 2 (1) 2 + y 2 (1 3) 2 =1; (1,0) (–1,0).
(0, 1 3),(0,– 1 3). (2 2 3 ,0),(– 2 2 3 ,0).4 x 2 +16 y 2 =1(x–2) 2 49 + (y–4) 2 25 =1
(x–2) 2 7 2 + (y–4) 2 5 2 =1; (9,4),(–5,4). (2,9),(2,–1). (2+2 6 ,4),(2–2 6 ,4).
(x–2) 2 81 + (y+1) 2 16 =1(x+5) 2 4 + (y–7) 2 9 =1(x+5) 2 2 2 + (y–7) 2 3 2 =1;
(–5,10),(–5,4). (–3,7),(–7,7). (–5,7+ 5),(–5,7– 5).(x–7) 2 49 + (y–7) 2 49 =1
4 x 2 –8x+9 y 2 –72y+112=0(x–1) 2 3 2 + (y–4) 2 2 2 =1; (4,4),(–2,4). (1,6),(1,2).
(1+ 5 ,4),(1– 5 ,4).9 x 2 –54x+9 y 2 –54y+81=04 x 2 –24x+36 y 2 –360y+864=0
(x–3) 2 (3 2) 2 + (y–5) 2 (2) 2 =1; (3+3 2 ,5),(3–3 2 ,5). (3,5+ 2),(3,5– 2). (7,5),(–1,5).
4 x 2 +24x+16 y 2 –128y+228=04 x 2 +40x+25 y 2 –100y+100=0(x+5) 2 (5) 2 + (y–2) 2 (2) 2 =1;
(0,2),(–10,2). (–5,4),(–5,0). (–5+ 21 ,2),(–5– 21 ,2).x 2 +2x+100 y 2 –1000y+2401=0
4 x 2 +24x+25 y 2 +200y+336=0(x+3) 2 (5) 2 + (y+4) 2 (2) 2 =1; (2,–4),(–8,–4).
(–3,–2),(–3,–6). (–3+ 21 ,–4),(–3– 21 ,–4).9 x 2 +72x+16 y 2 +16y+4=0
(x+3) 2 25 + (y+1) 2 36 =1 (–3,–1+ 11),(–3,–1– 11)(x+1) 2 100 + (y–2) 2 4 =1x 2 + y 2 =1 (0,0)
x 2 +4 y 2 +4x+8y=110 x 2 + y 2 +200x=0 (–10,30),(–10,–30)x 2 25 + y 2 36 =1x 2 16 + y 2 9 =1
(0,0), (4,0),(–4,0),(0,3),(0,–3), (7 ,0),(– 7 ,0)4 x 2 +9 y 2 =181 x 2 +49 y 2 =1 (0,0),
(1 9 ,0),(– 1 9 ,0),(0, 1 7),(0,– 1 7), (0, 4 2 63),(0,– 4 2 63)(x–2) 2 64 + (y–4) 2 16 =1
(x+3) 2 9 + (y–3) 2 9 =1 (–3,3), (0,3),(–6,3),(–3,0),(–3,6), (–3,3) x 2 2 + (y+1) 2 5 =1
4 x 2 –8x+16 y 2 –32y–44=0 (1,1), (5,1),(–3,1),(1,3),(1,–1), (1,1+4 3),(1,1–4 3)
x 2 –8x+25 y 2 –100y+91=0x 2 +8x+4 y 2 –40y+112=0 (–4,5), (–2,5),(–6,4),(–4,6),(–4,4),
(–4+ 3 ,5),(–4– 3 ,5)64 x 2 +128x+9 y 2 –72y–368=016 x 2 +64x+4 y 2 –8y+4=0 (–2,1),
(0,1),(–4,1),(–2,5),(–2,–3), (–2,1+2 3),(–2,1–2 3)100 x 2 +1000x+ y 2 –10y+2425=0
4 x 2 +16x+4 y 2 +16y+16=0 (–2,–2), (0,–2),(–4,–2),(–2,0),(–2,–4), (–2,–2) (4,0), (0,3). (0,–2),
(5,0).x 2 25 + y 2 29 =1 (3,0), (4,2) (9,2) (4+2 6 ,2)(x–4) 2 25 + (y–2) 2 1 =1 (3,5) (3,11)
(3, 5+4 2) (–3,4) (1,4) (–3+2 3 ,4)(x+3) 2 16 + (y–4) 2 4 =1x 2 81 + y 2 9 =1
(x+2) 2 4 + (y–2) 2 9 =1 Area=a·b·π.(x–3) 2 9 + (y–3) 2 16 =1Area = 12π square units
(x+6) 2 16 + (y–6) 2 36 =1(x+1) 2 4 + (y–2) 2 5 =1Area = 2 5 π square units
4 x 2 –8x+9 y 2 –72y+112=09 x 2 –54x+9 y 2 –54y+81=0Area = 9π square units h,
x 2 4 h 2 + y 2 1 4 h 2 =1x 2 400 + y 2 144 =1 (x,y) (x,y) (x,y) (x,y) (–c,0) (c,0) (x,y) (x,y)
(a,0) (–c,0) (a,0) a–(–c)=a+c. (c,0) (a,0) c–a.
(a+c)–(c–a)=2a
(x,y)
d 2 =the distance from (–c,0) to (x,y) d 1 =the distance from (c,0) to (x,y)
d 2 – d 1 (x,y) 2a (a,0). d 2 – d 1 =2a

d 2 – d 1 = (x–(–c)) 2 + (y–0) 2 – (x–c) 2 + (y–0) 2 =2a Distance Formula (x+c) 2 + y 2 – (x–c) 2 + y 2 =2a Simplify expressions. (x+c) 2 + y 2 =2a+ (x–c) 2 + y 2 Move radical to opposite side. (x+c) 2 + y 2 = (2a+ (x–c) 2 + y 2) 2 Square both sides. x 2 +2cx+ c 2 + y 2 =4 a 2 +4a (x–c) 2 + y 2 + (x–c) 2 + y 2 Expand the squares. x 2 +2cx+ c 2 + y 2 =4 a 2 +4a (x–c) 2 + y 2 + x 2 –2cx+ c 2 + y 2 Expand remaining square. 2cx=4 a 2 +4a (x–c) 2 + y 2 –2cx Combine like terms. 4cx–4 a 2 =4a (x–c) 2 + y 2 Isolate the radical. cx– a 2 =a (x–c) 2 + y 2 Divide by 4. (cx– a 2) 2 = a 2 [(x –c) 2 + y 2] 2 Square both sides. c 2 x 2 –2 a 2 cx+ a 4 = a 2 (x 2 –2cx+ c 2 + y 2) Expand the squares. c 2 x 2 –2 a 2 cx+ a 4 = a 2 x 2 –2 a 2 cx+ a 2 c 2 + a 2 y 2 Distribute a 2 . a 4 + c 2 x 2 = a 2 x 2 + a 2 c 2 + a 2 y 2 Combine like terms. c 2 x 2 – a 2 x 2 – a 2 y 2 = a 2 c 2 – a 4 Rearrange terms. x 2 (c 2 – a 2)– a 2 y 2 = a 2 (c 2 – a 2) Factor common terms. x 2 b 2 – a 2 y 2 = a 2 b 2 Set b 2 = c 2 – a 2 .

x 2 b 2 a 2 b 2 – a 2 y 2 a 2 b 2 = a 2 b 2 a 2 b 2 Divide both sides by a 2 b 2

x 2 a 2 – y 2 b 2 =1

(±a,0) (0±b). (0,0)

x 2 a 2 – y 2 b 2 =1

2a (±a,0) 2b (0,±b) 2c, c 2 = a 2 + b 2 (±c,0) y=± b a x (0,0)

y 2 a 2 – x 2 b 2 =1

2a (0,±a) 2b (±b,0) 2c, c 2 = a 2 + b 2 (0,±c) y=± a b x c 2 = a 2 + b 2 . (0,0) (0,0) a 2 x 2 a 2 – y 2 b 2 =1, (±a,0), (±c,0). y 2 a 2 – x 2 b 2 =1, (0,±a), (0,±c). a a= a 2 . c c= a 2 + b 2 . y 2 49 – x 2 32 =1. y 2 a 2 – x 2 b 2 =1, x=0, y.

1= y 2 49 – x 2 32 1= y 2 49 – 0 2 32 1= y 2 49 y 2 =49 y=± 49 =±7

(0,±c). c,

c= a 2 + b 2 = 49+32 = 81 =9

(0,±7), (0,9). x 2 9 – y 2 25 =1. (±3,0); (± 34 ,0) (0,0), c 2 = a 2 + b 2 . b 2 = c 2 – a 2 . (0,0), (±a,0) (±c,0), x 2 a 2 – y 2 b 2 =1. (0,±a) (0,±c), y 2 a 2 – x 2 b 2 =1. b 2 b 2 = c 2 – a 2 . a 2 b 2 (±6,0) (±2 10 ,0)? x 2 a 2 – y 2 b 2 =1. (±6,0), a=6 a 2 =36. (±2 10 ,0), c=2 10 c 2 =40. b 2 ,

b 2 = c 2 – a 2 b 2 =40–36 Substitute for c 2 and a 2 . b 2 =4 Subtract.

a 2 =36 b 2 =4 x 2 a 2 – y 2 b 2 =1. x 2 36 – y 2 4 =1, (0,±2) (0,±2 5)?y 2 4 – x 2 16 =1 h k (h,k). x (x–h) y (y–k). (h,k)

(x–h) 2 a 2 – (y–k) 2 b 2 =1

2a (h±a,k) 2b (h,k±b) 2c, c 2 = a 2 + b 2 (h±c,k) 2a 2b. ± b a , (h,k). y=± b a (x–h)+k. (h,k)

(y–k) 2 a 2 – (x–h) 2 b 2 =1

2a (h,k±a) 2b (h±b,k) 2c, c 2 = a 2 + b 2 (h,k±c) y=± a b (x–h)+k. (h,k) (h,k) (h,k) c 2 = a 2 + b 2 . (h,k), (x–h) 2 a 2 – (y–k) 2 b 2 =1. (y–k) 2 a 2 – (x–h) 2 b 2 =1. (h,k), a 2 2a c 2 h k b 2 b 2 = c 2 – a 2 . h, k, a 2 , b 2 (0,–2)(6,–2)(–2,–2)(8,–2)?

(x–h) 2 a 2 – (y–k) 2 b 2 =1

(h,k). (0,–2) (6,–2).

(h,k)=(0+6 2 , –2+(–2) 2)=(3,–2)

a 2 . 2a, a 2

2a=| 0–6 | 2a=6 a=3 a 2 =9

c 2 . (h±c,k). (h–c,k)=(–2,–2) (h+c,k)=(8,–2). c. (8,–2), h=3,

h+c=8 3+c=8 c=5 c 2 =25

b 2 b 2 = c 2 – a 2 :

b 2 = c 2 – a 2 =25–9 =16

h,k, a 2 , b 2

(x–3) 2 9 – (y+2) 2 16 =1

(1,–2) (1,8) (1,–10) (1,16)?(y–3) 2 25 + (x–1) 2 144 =1 x 2 a 2 – y 2 b 2 =1
y 2 a 2 – x 2 b 2 =1 (0,0), x 2 a 2 – y 2 b 2 =1, (±a,0) (0,±b) (±c,0) y=± b a x y 2 a 2 – x 2 b 2 =1,
(0,±a) (±b,0) (0,±c) y=± a b x c=± a 2 + b 2 . y 2 64 – x 2 36 =1. y 2 a 2 – x 2 b 2 =1.
(0,±a)=(0,± 64)=(0,±8) (±b,0)=(± 36 , 0)=(±6,0) (0,±c), c=± a 2 + b 2 . c,

$$c=\pm \sqrt{a^2+b^2}=\pm \sqrt{64+36}=\pm \sqrt{100}=\pm 10$$

(0,±10) y=± a b x=± 8 6 x=± 4 3 x x 2 144 – y 2 81 =1. (±12,0); (0,±9); (±15,0); y=± 3 4 x; (h,k)
(x–h) 2 a 2 – (y–k) 2 b 2 =1 (y–k) 2 a 2 – (x–h) 2 b 2 =1 (h, k), (x–h) 2 a 2 – (y–k) 2 b 2 =1,
(h,k) (h±a,k) (h,k±b) (h±c,k) y=± b a (x–h)+k (y–k) 2 a 2 – (x–h) 2 b 2 =1, (h,k) (h,k±a)
(h±b,k) (h,k±c) y=± a b (x–h)+k c=± a 2 + b 2 . 9 x 2 –4 y 2 –36x–40y–388=0.

$$(9x^2-36x)-(4y^2+40y)=388$$

$$9(x^2-4x)-4(y^2+10y)=388$$

$$9(x^2-4x+4)-4(y^2+10y+25)=388+36-100$$

$$9(x-2)^2-4(y+5)^2=324$$

$$\frac{(x-2)^2}{36}-\frac{(y+5)^2}{81}=1$$

(x–h) 2 a 2 – (y–k) 2 b 2 =1, a 2 =36 b 2 =81, a=6 b=9. (h,k)=(2,–5) (h±a,k)=(2±6,–5),
(–4,–5) (8,–5) (h,k±b)=(2,–5±9), (2,–14) (2,4) (h±c,k), c=± a 2 + b 2 . c,

$$c=\pm \sqrt{36+81}=\pm \sqrt{117}=\pm 3\sqrt{13}$$

(2–3 13 ,–5) (2+3 13 ,–5). y=± b a (x–h)+k=± 3 2 (x–2)–5. (y+4) 2 100 – (x–3) 2 64 =1.
(3,–4); (3,–14) (3,6); (–5,–4); (11,–4); (3,–4–2 41) (3,–4+2 41); y=± 5 4 (x–3)–4
x 2 a 2 – y 2 b 2 =1, a 2 b 2 a 2 . 2a. 65.3 2a=60. a=30 a 2 =900. b 2 , x y (x,y)
x 2 a 2 – y 2 b 2 =1 Standard form of horizontal hyperbola. b 2 = y 2 x 2 a 2 –1 Isolate b 2
= (79.6) 2 (36) 2 900 –1 Substitute for a 2 ,x, and y ≈14400.3636
Round to four decimal places

$$\frac{x^2}{900}-\frac{y^2}{14400.3636}=1,\text{or}\ \frac{x^2}{30^2}-\frac{y^2}{120.0015^2}=1$$

x 2 400 – y 2 3600 =1or x 2 20 2 – y 2 60 2 =1.x 2 a 2 – y 2 b 2 =1y 2 a 2 – x 2 b 2 =1 (h,k),
(x–h) 2 a 2 – (y–k) 2 b 2 =1 (h,k),(y–k) 2 a 2 – (x–h) 2 b 2 =1 (x,y) (x,y) 3 y 2 +2x=6
x 2 36 – y 2 9 =1 x 2 6 2 – y 2 3 2 =15 y 2 +4 x 2 =6x25 x 2 –16 y 2 =400 x 2 4 2 – y 2 5 2 =1
–9 x 2 +18x+ y 2 +4y–14=0x 2 25 – y 2 36 =1x 2 5 2 – y 2 6 2 =1; (5,0),(–5,0); (61 ,0),(– 61 ,0);
y= 6 5 x,y=– 6 5 x x 2 100 – y 2 9 =1y 2 4 – x 2 81 =1y 2 2 2 – x 2 9 2 =1; (0,2),(0,–2);
(0, 85),(0,– 85); y= 2 9 x,y=– 2 9 x9 y 2 –4 x 2 =1(x–1) 2 9 – (y–2) 2 16 =1
(x–1) 2 3 2 – (y–2) 2 4 2 =1; (4,2),(–2,2); (6,2),(–4,2); y= 4 3 (x–1)+2,y=– 4 3 (x–1)+2
(y–6) 2 36 – (x+1) 2 16 =1(x–2) 2 49 – (y+7) 2 49 =1(x–2) 2 7 2 – (y+7) 2 7 2 =1;
(9,–7),(–5,–7); (2+7 2 ,–7),(2–7 2 ,–7); y=x–9,y=–x–54 x 2 –8x–9 y 2 –72y+112=0
–9 x 2 –54x+9 y 2 –54y+81=0(x+3) 2 3 2 – (y–3) 2 3 2 =1; (0,3),(–6,3);
(–3+3 2 ,1),(–3–3 2 ,1); y=x+6,y=–x4 x 2 –24x–36 y 2 –360y+864=0
–4 x 2 +24x+16 y 2 –128y+156=0(y–4) 2 2 2 – (x–3) 2 4 2 =1; (3,6),(3,2); (3,4+2 5),(3,4–2 5);
y= 1 2 (x–3)+4,y=– 1 2 (x–3)+4–4 x 2 +40x+25 y 2 –100y+100=0x 2 +2x–100 y 2 –1000y+2401=0
(y+5) 2 7 2 – (x+1) 2 70 2 =1; (–1,2),(–1,–12); (–1,–5+7 101),(–1,–5–7 101);
y= 1 10 (x+1)–5,y=– 1 10 (x+1)–5–9 x 2 +72x+16 y 2 +16y+4=04 x 2 +24x–25 y 2 +200y–464=0
(x+3) 2 5 2 – (y–4) 2 2 2 =1; (2,4),(–8,4); (–3+ 29 ,4),(–3– 29 ,4);
y= 2 5 (x+3)+4,y=– 2 5 (x+3)+4y 2 3 2 – x 2 3 2 =1(x–3) 2 5 2 – (y+4) 2 2 2 =1
y= 2 5 (x–3)–4,y=– 2 5 (x–3)–4(y–3) 2 3 2 – (x+5) 2 6 2 =19 x 2 –18x–16 y 2 +32y–151=0
y= 3 4 (x–1)+1,y=– 3 4 (x–1)+116 y 2 +96y–4 x 2 +16x+112=0x 2 49 – y 2 16 =1x 2 64 – y 2 4 =1
y 2 9 – x 2 25 =181 x 2 –9 y 2 =1(y+5) 2 9 – (x–4) 2 25 =1(x–2) 2 8 – (y+3) 2 27 =1
(y–3) 2 9 – (x–3) 2 9 =1–4 x 2 –8x+16 y 2 –32y–52=0x 2 –8x–25 y 2 –100y–109=0
– x 2 +8x+4 y 2 –40y+88=064 x 2 +128x–9 y 2 –72y–656=016 x 2 +64x–4 y 2 –8y–4=0
–100 x 2 +1000x+ y 2 –10y–2575=04 x 2 +16x–4 y 2 +16y+16=0 (3,0) (–3,0) (5,0).
x 2 9 – y 2 16 =1 (0,6) (0,–6) (0,–8). (1,1) (11,1) (12,1).(x–6) 2 25 – (y–1) 2 11 =1 (0,0);
(0,–13); (0, 313). (4,2); (9,2); (4+ 26 ,2).(x–4) 2 25 – (y–2) 2 1 =1 (3,5); (3,11); (3,5+2 10).

y 2 16 – x 2 25 =1y 2 9 – (x+1) 2 9 =1(x+3) 2 25 – (y+3) 2 25 =1 y x. x 2 4 – y 2 9 =1
y 2 9 – x 2 1 =1y(x)=3 x 2 +1 ,y(x)=–3 x 2 +1(x–2) 2 16 – (y+3) 2 25 =1
–4 x 2 –16x+ y 2 –2y–19=0y(x)=1+2 x 2 +4x+5 ,y(x)=1–2 x 2 +4x+54 x 2 –24x– y 2 –4y+16=0
y=x and y=–x,x 2 25 – y 2 25 =1 y=2x and y=–2x, y= 1 2 x y=– 1 2 x,x 2 100 – y 2 25 =1 y= 2 3 x
y=– 2 3 x, y= 3 4 x and y=– 3 4 x,x 2 400 – y 2 225 =1 y=x–2 y=–x+2. y=2x–2 y=–2x+2.
(x–1) 2 0.25 – y 2 0.75 =1 y=0.5x+2 y=–0.5x–2. y= 1 3 x–1 y=– 1 3 x+1.(x–3) 2 4 – y 2 5 =1
y=3x–9 y=–3x+9. (x,y) (x,y) (x,y) d P P (x,y) (0,0), (0,p), y= –p d (x,y) (x,–p) d=y+p. (0,p)
(x,y) d

$$d= (x–0)^2 + (y–p)^2 = x^2 + (y–p)^2$$

d y (x,y) (0,p) (x,y) (x, –p).

$$x^2 + (y–p)^2 =y+p$$

$$x^2 + (y–p)^2 = (y+p)^2 \quad x^2 + y^2 –2py+ p^2 = y^2 +2py+ p^2 \quad x^2 –2py=2py \quad x^2 =4py$$

(0,0) y 2 =4px x 2 =4py y 2 =4px(p, 0)x=–p(p, ±2p)x 2 =4py(0, p)y=–p(±2p, p) p>0 p<0 p<0
p<0 y 2 =4px x 2 =4py. y 2 =4px, y=0 4p p. p>0, p<0, p (p,0) p x=–p p (p,±2p). x=p x 2 =4py,
x=0 4p p. p>0, p<0, p (0,p) p y=–p p (±2p,p) y 2 =24x. y 2 =4px. 24=4p, p=6. p>0, (p,0)=(6,0)
x=–p=–6 x=6 (6,±12) y 2 =–16x. (–4,0); x=4; (–4,±8) x 2 =–6y. x 2 =4py. –6=4p, p=– 3 2 . p<0,
(0,p)=(0,– 3 2) y=–p= 3 2 y= 3 2 (±3,– 3 2) x 2 =8y. (0,2); y=–2; (±4,2). (p,0), y 2 =4px.
(0,p), x 2 =4py. 4p. (– 1 2 ,0) x= 1 2 ? (p,0), y 2 =4px. 4p, 4p=4(– 1 2)=–2. 4p, y 2 =4px=–2x.
y 2 =–2x. (0, 7 2) y=– 7 2 ?x 2 =14y. h k (h,k). x (x–h) y (y–k). (h,k) (y–k) 2 =4p(x–h)
(x–h) 2 =4p(y–k) (h,k).y=k(y–k) 2 =4p(x–h)(h+p, k)x=h–p(h+p, k±2p)x=h(x–h) 2 =4p(y–k)
(h, k+p)y=k–p(h±2p, k+p) p>0, p<0, p>0, p<0, (y–k) 2 =4p(x–h) (x–h) 2 =4p(y–k).
(y–k) 2 =4p(x–h), h k (h,k) k y=k 4p (x–h) p. p>0, p<0, h,k, p (h+p, k) h p x=h–p h,k, p
(h+p,k±2p) (x–h) 2 =4p(y–k), h k (h,k) h x=h 4p (y–k) p. p>0, p<0, h,k, p (h, k+p) k p y=k–p
h, k, p (h±2p, k+p) (y–1) 2 =–16(x+3). (y–k) 2 =4p(x–h). (h,k)=(–3,1) y=k=1–16=4p, p=–4.
p<0, (h+p,k)=(–3+(–4),1)=(–7,1) x=h–p=–3–(–4)=1 (h+p,k±2p)=(–3+(–4),1±2(–4)),
(–7,–7) (–7,9) (y+1) 2 =4(x–8). (8,–1); y=–1; (9,–1); x=7; (9,–3) (9,1).
x 2 –8x–28y–208=0. (x–h) 2 =4p(y–k). x
x 2 –8x–28y–208=0 x 2 –8x=28y+208 x 2 –8x+16=28y+208+16 (x
–4) 2 =28y+224 (x–4) 2 =28(y+8) (x–4) 2 =4·7·(y+8)
(h,k)=(4,–8) x=h=4 p=7,p>0 (h,k+p)=(4,–8+7)=(4,–1) y=k–p=–8–7=–15
(h±2p,k+p)=(4±2(7),–8+7), (–10,–1) (18,–1) (x+2) 2 =–20(y–3). (–2,3); x=–2; (–2,–2);
y=8; (–12,–2) (8,–2). x 2 =4py, p>0. p=1.7.
x 2 =4py Standard form of upward-facing parabola with vertex (0,0) x 2 =4(1.7)y Substitute 1.7 for p. x 2
=6.8y Multiply.
4.5 2 =2.25 x
x 2 =6.8y Equation found in part (a). (2.25) 2 =6.8y Substitute 2.25 for x. y≈0.74 Solve for y.
y 2 =1280xy 2 =4pxx 2 =4py (h,k),(y–k) 2 =4p(x–h) (h,k),(x–h) 2 =4p(y–k) (x,y) (0,0) p>0, p<0,
(0,0) p>0, p<0, (h,k) p>0, p<0, (h,k) p>0, p<0, p p p?y 2 =4– x 2y=4 x 2 y=4(1) x 23 x 2 –6 y 2 =12
(y–3) 2 =8(x–2) (y–3) 2 =4(2)(x–2)y 2 +12x–6y–51=0 (V), (F), (d) x=8 y 2
y 2 = 1 8 x,V:(0,0);F:(1 32 ,0);d:x=– 1 32y= 1 4 x 2y=–4 x 2
x 2 =– 1 4 y,V:(0,0);F:(0,– 1 16);d:y= 1 16x= 1 8 y 2x=36 y 2
y 2 = 1 36 x,V:(0,0);F:(1 144 ,0);d:x=– 1 144x= 1 36 y 2(x–1) 2 =4(y–1)
(x–1) 2 =4(y–1),V:(1,1);F:(1,2);d:y=0(y–2) 2 = 4 5 (x+4)(y–4) 2 =2(x+3)
(y–4) 2 =2(x+3),V:(–3,4);F:(– 5 2 ,4);d:x=– 7 2(x+1) 2 =2(y+4)(x+4) 2 =24(y+1)
(x+4) 2 =24(y+1),V:(–4,–1);F:(–4,5);d:y=–7(y+4) 2 =16(x+4)y 2 +12x–6y+21=0
(y–3) 2 =–12(x+1),V:(–1,3);F:(–4,3);d:x=2x 2 –4x–24y+28=05 x 2 –50x–4y+113=0
(x–5) 2 = 4 5 (y+3),V:(5,–3);F:(5,– 14 5);d:y=– 16 5y 2 –24x+4y–68=0x 2 –4x+2y–6=0
(x–2) 2 =–2(y–5),V:(2,5);F:(2, 9 2);d:y= 11 2y 2 –6y+12x–3=03 y 2 –4x–6y+23=0
(y–1) 2 = 4 3 (x–5),V:(5,1);F:(16 3 ,1);d:x= 14 3x 2 +4x+8y–4=0x= 1 8 y 2y=36 x 2y= 1 36 x 2

y=−9 x 2(y−2) 2 =− 4 3 (x+2)−5 (x+5) 2 =4(y+5)−6 (y+5) 2 =4(x−4)y 2 −6y−8x+1=0
x 2 +8x+4y+20=03 x 2 +30x−4y+95=0y 2 −8x+10y+9=0x 2 +4x+2y+2=0y 2 +2y−12x+61=0
−2 x 2 +8x−4y−24=0 (0,0); y=4, (0,−4).x 2 =−16y (0,0); x=4, (−4,0). (2,2); x=2− 2 , (2+ 2 ,2).
(y−2) 2 =4 2 (x−2) (−2,3); x=− 7 2 , (− 1 2 ,3). (2 ,− 3); x=2 2 , (0,− 3).(y+ 3) 2 =−4 2 (x− 2)
(1,2); y= 11 3 , (1, 1 3).x 2 =y(y−2) 2 = 1 4 (x+2)(y− 3) 2 =4 5 (x+ 2)
V(0,0), Endpoints (2,1),(−2,1)V(0,0), Endpoints (−2,4),(−2,−4)y 2 =−8x
V(1,2), Endpoints (−5,5),(7,5)V(−3,−1),Endpoints (0,5),(0,−7)(y+1) 2 =12(x+3)
V(4,−3), Endpoints (5,− 7 2),(3,− 7 2) x 2 =4y. (0,1) (0,0.25),x 2 =−125(y−20), y=− x 2 +96x, x
y y=−0.5 x 2 +80x. (x,y)

$$A x^2 + Bxy + C y^2 + Dx + Ey + F = 0$$

A,B,C xy xy B 4 x 2 +9 y 2 =14 x 2 +4 y 2 =14 x 2 −9 y 2 =14 x 2 =9y or 4 y 2 =9x4x+9y=1
(x−4)(y+4)=0(x−4)(x−9)=04 x 2 +4 y 2 =04 x 2 +4 y 2 = − 1

$$A x^2 + Bxy + C y^2 + Dx + Ey + F = 0$$

A,B,C B=0, A C A x 2 +C y 2 +Dx+Ey+F=0, A≠C and AC>0A x 2 +C y 2 +Dx+Ey+F=0, A=C
A x 2 −C y 2 +Dx+Ey+F=0 or −A x 2 +C y 2 +Dx+Ey+F=0, A C
A x 2 +Dx+Ey+F=0 or C y 2 +Dx+Ey+F=0A x 2 +Bxy+C y 2 +Dx+Ey+F=0. A C A C A C A C A C
A x 2 +By2=0,A x 2 +By2=0,A x 2 +By2+1=0,4 x 2 −9 y 2 +36x+36y−125=09 y 2 +16x+36y−10=0
3 x 2 +3 y 2 −2x−6y−4=0−25 x 2 −4 y 2 +100x+16y+20=0A=4 C=−9, A C A=0 C=9. A A=3 C=3.
A=C,A=−25 C=−4. AC>0 A≠C,16 y 2 − x 2 +x−4y−9=016 x 2 +4 y 2 +16x+49y−81=0 xy xy θ, (x,y)
(x ′ , y ′) x 2 + y 2 −xy−15=0 x y x ′ y ′ θ. i j . i ′ j ′ . θ

$$i' = \cos\theta\, i + \sin\theta\, j \qquad j' = -\sin\theta\, i + \cos\theta\, j$$

u

$$u = x'i' + y'j' \qquad u = x'(i\cos\theta + j\sin\theta) + y'(-i\sin\theta + j\cos\theta)$$

Substitute. $u = ix'\cos\theta + jx'\sin\theta - iy'\sin\theta + jy'\cos\theta$ Distribute. $u = ix'\cos\theta - iy'\sin\theta + jx'\sin\theta + jy'\cos\theta$ Apply commutative property. $u = (x'\cos\theta - y'\sin\theta)i + (x'\sin\theta + y'\cos\theta)j$ Factor by grouping.

u= x ′ i ′ + y ′ j ′ , x y

$$x = x'\cos\theta - y'\sin\theta \text{ and } y = x'\sin\theta + y'\cos\theta$$

(x,y) θ (x ′ , y ′). (x,y) (x ′ , y ′):

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

x y x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ. x y x ′ y ′ 2 x 2 −xy+2 y 2 −30=0 θ=45°. x y,
x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ. θ=45°,

$$x = x'\cos(45°) - y'\sin(45°) \quad x = x'\left(\tfrac{1}{2}\right) - y'\left(\tfrac{1}{2}\right) \quad x = \frac{x' - y'}{2}$$
$$y = x'\sin(45°) + y'\cos(45°) \quad y = x'\left(\tfrac{1}{2}\right) + y'\left(\tfrac{1}{2}\right) \quad y = \frac{x' + y'}{2}$$

x= x ′ cosθ− y ′ sinθ y= x ′ sin θ+ y ′ cos θ 2 x 2 −xy+2 y 2 −30=0.

$$2\left(\frac{x' - y'}{2}\right)^2 - \left(\frac{x' - y'}{2}\right)\left(\frac{x' + y'}{2}\right) + 2\left(\frac{x' + y'}{2}\right)^2 - 30 = 0$$

$$2\frac{(x' - y')(x' - y')}{2} - \frac{(x' - y')(x' + y')}{2} + 2\frac{(x' + y')(x' + y')}{2} - 30 = 0$$ FOIL method

$$x'^2 - 2x'y' + y'^2 - \frac{(x'^2 - y'^2)}{2} + x'^2 + 2x'y' + y'^2 - 30 = 0$$ Combine like terms.

$$2x'^2 + 2y'^2 - \frac{(x'^2 - y'^2)}{2} = 30$$ Combine like terms.

$$2\left(2x'^2 + 2y'^2 - \frac{(x'^2 - y'^2)}{2}\right) = 2(30)$$ Multiply both sides by 2.

$$4x'^2 + 4y'^2 - (x'^2 - y'^2) = 60$$ Simplify.

$$4x'^2 + 4y'^2 - x'^2 + y'^2 = 60$$ Distribute.

$$\frac{3x'^2}{60} + \frac{5y'^2}{60} = \frac{60}{60}$$ Set equal to 1.

x ′ y ′

$$\frac{x'^2}{20} + \frac{y'^2}{12} = 1$$

A x 2 +Bxy+C y 2 +Dx+Ey+F=0 x ′ y ′ x ′ y ′ θ

$$\cot(2\theta) = \frac{A - C}{B}$$

$$A x^2 + Bxy + C y^2 + Dx + Ey + F = 0$$

A,B,C B≠0, xy θ cot(2θ)= A−C B . cot(2θ)>0, 2θ θ (0°,45°). cot(2θ)<0, 2θ θ (45°,90°). A=C, θ=45°. x ′ y ′ x ′ y ′ x ′ y ′ , x ′ y ′ θ cot(2θ). sin θ cos θ. sin θ cos θ x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ. x y x ′ y ′ 8 x 2 −12xy+17 y 2 =20 x ′ y ′ x ′ y ′ cot(2θ).

8 x 2 −12xy+17 y 2 =20⇒A=8, B=−12 and C=17 cot(2θ)= A−C B = 8−17 −12

cot(2θ)= −9 −12 = 3 4

cot(2θ)= 3 4 = adjacent opposite

3 2 + 4 2 = h 2 9+16= h 2 25= h 2 h=5

sin θcos θ.

sin θ= 1−cos(2θ) 2 = 1− 3 5 2 = 5 5 − 3 5 2 = 5−3 5 · 1 2 = 2 10 = 1 5 sin θ= 1 5 cos θ= 1+cos(2θ) 2 = 1+ 3 5 2 = 5 5 + 3 5 2 = 5+3 5 · 1 2 = 8 10 = 4 5 cos θ= 2 5

sin θ cos θ x= x ′ cos θ− y ′ sin θ y= x ′ sin θ+ y ′ cos θ.

x= x ′ cos θ− y ′ sin θ x= x ′ (2 5)− y ′ (1 5) x= 2 x ′ − y ′ 5

y= x ′ sin θ+ y ′ cos θ y= x ′ (1 5)+ y ′ (2 5) y= x ′ +2 y ′ 5

x y

8 (2 x ′ − y ′ 5) 2 −12(2 x ′ − y ′ 5)(x ′ +2 y ′ 5)+17 (x ′ +2 y ′ 5) 2 =20 8((2 x ′ − y ′)(2 x ′ − y ′) 5)−12((2 x ′ − y ′)(x ′ +2 y ′) 5)+17((x ′ +2 y ′)(x ′ +2 y ′) 5)=20 8(4 x ′ 2 −4 x ′ y ′ + y ′ 2)−12(2 x ′ 2 +3 x ′ y ′ −2 y ′ 2)+17(x ′ 2 +4 x ′ y ′ +4 y ′ 2)=100 32 x ′ 2 −32 x ′ y ′ +8 y ′ 2 −24 x ′ 2 −36 x ′ y ′ +24 y ′ 2 +17 x ′ 2 +68 x ′ y ′ +68 y ′ 2 =100

25 x ′ 2 +100 y ′ 2 =100

25 100 x ′ 2 + 100 100 y ′ 2 = 100 100

x ′ y ′

x ′ 2 4 + y ′ 2 1 =1

13 x 2 −6 3 xy+7 y 2 =16 x ′ y ′ x ′ y ′ x ′ 2 4 + y ′ 2 1 =1 x ′ y ′

x 2 +12xy−4 y 2 =30

cot(2θ).

x 2 +12xy−4 y 2 =20⇒A=1, B=12,and C=−4

cot(2θ)= A−C B cot(2θ)= 1−(−4) 12 cot(2θ)= 5 12

cot(2θ)= 5 12 ,

cot(2θ)= 5 12 = adjacent opposite

5 2 + 12 2 = h 2 25+144= h 2 169= h 2 h=13

sin θ cos θ.

sin θ= 1−cos(2θ) 2 = 1− 5 13 2 = 13 13 − 5 13 2 = 8 13 · 1 2 = 2 13 cos θ= 1+cos(2θ) 2 = 1+ 5 13 2 = 13 13 + 5 13 2 = 18 13 · 1 2 = 3 13

x y.

x= x ′ cos θ− y ′ sin θ x= x ′ (3 13)− y ′ (2 13) x= 3 x ′ −2 y ′ 13

y= x ′ sin θ+ y ′ cos θ y= x ′ (2 13)+ y ′ (3 13) y= 2 x ′ +3 y ′ 13

x= 3 x ′ −2 y ′ 13 y= 2 x ′ +3 y ′ 13 x 2 +12xy−4 y 2 =30.

(3 x ′ −2 y ′ 13) 2 +12(3 x ′ −2 y ′ 13)(2 x ′ +3 y ′ 13)−4 (2 x ′ +3 y ′ 13) 2 =30 (1 13)[(3 x ′ −2 y ′) 2 +12(3 x ′ −2 y ′)(2 x ′ +3 y ′)−4 (2 x ′ +3 y ′) 2]=30

Factor. (1 13)[9 x ′ 2 −12 x ′ y ′ +4 y ′ 2 +12(6 x ′ 2 +5 x ′ y ′ −6 y ′ 2)−4(4 x ′ 2 +12 x ′ y ′ +9 y ′ 2)]=30 Multiply. (1 13)[9 x ′ 2 −12 x ′ y ′ +4 y ′ 2 +72 x ′ 2 +60 x ′ y ′ −72 y ′ 2 −16 x ′ 2 −48 x ′ y ′ −36 y ′ 2]=30 Distribute. (1 13)[65 x ′ 2 −104 y ′ 2]=30 Combine like terms. 65 x ′ 2

−104 y ′ 2 =390 Multiply.

x ′ 2 6 − 4 y ′ 2 15 =1

Divide by 390.

x ′ 2 6 − 4 y ′ 2 15 =1.

A x 2 +Bxy+C y 2 +Dx+Ey+F=0

$$A′x′2+B′x′y′+C′y′2+D′x′+E′y′+F′=0$$

$B2–4AC=B′2–4A′C′$. $B2–4AC$, $Ax2+Bxy+Cy2+Dx+Ey+F=0$

$A′x′2+B′x′y′+C′y′2+D′x′+E′y′+F′=0$, $B2–4AC=B′2–4A′C′$.

$Ax2+Bxy+Cy2+Dx+Ey+F=0$ $B2–4AC$, <0, $=0$, >0, $5x2+23xy+2y2–5=0$

$5x2+23xy+12y2–5=0$ A, B, C.

$$5‾Ax2+23‾Bxy+2‾Cy2–5=0$$

$$B2–4AC=(23)2–4(5)(2)=4(3)–40=12–40=–28<0$$

$5x2+23xy+2y2–5=0$ A, B, C.

$$5‾Ax2+23‾Bxy+12‾Cy2–5=0$$

$$B2–4AC=(23)2–4(5)(12)=4(3)–240=12–240=–228<0$$

$5x2+23xy+12y2–5=0$ $x2–9xy+3y2–12=0$ $10x2–9xy+4y2–4=0$

$Ax2+Bxy+Cy2+Dx+Ey+F=0$

$$x=x′\cos\theta–y′\sin\theta \quad y=x′\sin\theta+y′\cos\theta$$

θ, where $\cot(2\theta)=\frac{A–C}{B}$ A $x2+Bxy+Cy2+Dx+Ey+F=0$ A, B C A, B, C xy $x′$ $y′$ $x′y′$ xy xy

$Ax2+By2+Cx+Dy+E=0$ $AB=0$, $Ax2+Bxy+Cy2+Dx+Ey+F=0$, $B2–4AC>0$,

$ax2+4x+3y2–12=0$, $a>0$? $Ax2+Bxy+Cy2+Dx+Ey+F=0$, θ $\cot(2\theta)=\frac{A–C}{B}$ xy

$9x2+4y2+72x+36y–500=0$ $x2–10x+4y–10=0$ $AB=0$, $2x2–2y2+4x–6y–2=0$ $4x2–y2+8x–1=0$

$AB=–4<0$, $4y2–5x+9y+1=0$ $2x2+3y2–8x–12y+2=0$ $AB=6>0$, $4x2+9xy+4y2–36y–125=0$

$3x2+6xy+3y2–36y–125=0$ $B2–4AC=0$, $–3x2+33xy–4y2+9=0$ $2x2+43xy+6y2–6x–3=0$

$B2–4AC=0$, $–x2+42xy+2y2–2y+1=0$ $8x2+42xy+4y2–10x+1=0$ $B2–4AC=–96<0$,

$3x2+xy+3y2–5=0$, $\theta=45°$ $4x2–xy+4y2–2=0$, $\theta=45°$ $7x′2+9y′2–4=0$ $2x2+8xy–1=0$, $\theta=30°$

$–2x2+8xy+1=0$, $\theta=45°$ $3x′2+2x′y′–5y′2+1=0$ $4x2+2xy+4y2+y+2=0$, $\theta=45°$ θ xy xy

$x2+33xy+4y2+y–2=0$ $\theta=60°$, $11x′2–y′2+3x′+y′–4=0$ $4x2+23xy+6y2+y–2=0$

$9x2–33xy+6y2+4y–3=0$ $\theta=150°$, $21x′2+9y′2+4x′–43y′–6=0$ $–3x2–3xy–2y2–x=0$

$16x2+24xy+9y2+6x–6y+2=0$ $\theta\approx 36.9°$, $125x′2+6x′–42y′+10=0$ $x2+4xy+4y2+3x–2=0$

$x2+4xy+y2–2x+1=0$ $\theta=45°$, $3x′2–y′2–2x′+2y′+1=0$ $4x2–23xy+6y2–1=0$

$y=–x2$, $\theta=–45°$ $22(x′+y′)=12(x′–y′)$ $2x=y2$, $\theta=45°$ $x242+y212=1$, $\theta=45°$

$\frac{(x′–y′)2}{8}+\frac{(x′+y′)2}{2}=1$ $\frac{y2}{16}+\frac{x2}{9}=1$, $\theta=45°$ $y2–x2=1$, $\theta=45°$

$\frac{(x′+y′)2}{2}–\frac{(x′–y′)2}{2}=1$ $y=\frac{x2}{2}$, $\theta=30°$ $x=\frac{(y–1)2}{2}$, $\theta=30°$

$\frac{3}{2}x′–\frac{1}{2}y′=(\frac{1}{2}x′+\frac{3}{2}y′–1)$ $2x29+y24=1$, $\theta=30°$ $x′y′$ $x′y′$ $xy=9$

$x2+10xy+y2–6=0$ $x2–10xy+y2–24=0$ $4x2–33xy+y2–22=0$ $6x2+23xy+4y2–21=0$

$11x2+103xy+y2–64=0$ $21x2+23xy+19y2–18=0$ $16x2+24xy+9y2–130x+90y=0$

$16x2+24xy+9y2–60x+80y=0$ $13x2–63xy+7y2–16=0$ $4x2–4xy+y2–85x–165y=0$ xy

$6x2–53xy+y2+10x–12y=0$ $6x2–5xy+6y2+20x–y=0$ $\theta=45°$ $6x2–83xy+14y2+10x–3y=0$

$4x2+63xy+10y2+20x–40y=0$ $\theta=60°$ $8x2+3xy+4y2+2x–4=0$ $16x2+24xy+9y2+20x–44y=0$

$\theta\approx 36.9°$ k $4x2+kxy+16y2+8x+24y–48=0$, k $2x2+kxy+12y2+10x–16y+28=0$, $k–4\,6<k<4\,6$

$3x2+kxy+4y2–6x+20y+128=0$, k k $x2+8xy+8y2–12x+16y+18=0$, k $k=2$

$6x2+12xy+ky2+16x+10y+4=0$, k $\cot(2\theta)>0$, θ $(0°,45°)$; $\cot(2\theta)<0$, θ $(45°,90°)$; $\cot(2\theta)=0$, $\theta=45°$

$x=2+y2$ P(r,θ) F D e P $e=\frac{PF}{PD}$ P P F P D e. e, $0\le e<1$, $e=1$, $e>1$, $x=\pm p$, e, θ. r θ. $x=\pm p$, p e,

$$r=\frac{ep}{1\pm e\cos\theta}$$

$y=\pm p$, p e,

$$r=\frac{ep}{1\pm e\sin\theta}$$

e e $x=p$ $y=p$ ep x y. $r=\frac{6}{3+2\sin\theta}$ $r=\frac{12}{4+5\cos\theta}$ $r=\frac{7}{2–2\sin\theta}$ 1 c, c 1 3.

$$r=\frac{6}{3+2\sin\theta}\cdot\frac{(13)}{(13)}=\frac{6(13)}{3(13)+2(13)\sin\theta}=\frac{21}{1+23\sin\theta}$$

$\sin\theta$ $y=p$. $e=23$.

$$2=ep \quad 2=23p \quad (32)2=(32)23p \quad 3=p$$

$e<1$, $e=23$ $y=3$. 1 4.

$$r=\frac{12}{4+5\cos\theta}\cdot\frac{(14)}{(14)} \quad r=\frac{12(14)}{4(14)+5(14)\cos\theta} \quad r=\frac{3}{1+\frac{5}{4}\cos\theta}$$

cos θ x=p. e= 5 4 .

3=ep 3= 5 4 p (4 5)3=(4 5) 5 4 p 12 5 =p

e>1, e= 5 4 x= 12 5 =2.4. 1 2 .

r= 7 2−2 sin θ · (1 2) (1 2) r= 7(1 2) 2(1 2)−2(1 2) sin θ r= 7 2 1−sin θ

y=−p. e=1.

7 2 =ep 7 2 =(1)p 7 2 =p

e=1, e=1 y=− 7 2 =−3.5. r= 2 3−cos θ . e= 1 3 ; x=−2 e θ r θ 0, π 2 ,π, 3π 2 r= 5 3+3 cos θ . 1 3 .

r= 5 3+3 cos θ = 5(1 3) 3(1 3)+3(1 3)cos θ r= 5 3 1+cos θ

e=1, cos θ, x=p.

5 3 =ep 5 3 =(1)p 5 3 =p

x= 5 3 .θ0π 2π3π 2r= 5 3+3 cos θ5 6 ≈0.835 3 ≈1.675 3 ≈1.67 r= 8 2−3 sin θ . 1 2 .

r= 8 2−3sin θ = 8(1 2) 2(1 2)−3(1 2)sin θ r= 4 1− 3 2 sin θ

e= 3 2 ,e>1, sin θ y=−p.

4=ep 4=(3 2)p 4(2 3)=p 8 3 =p

y=− 8 3 .θ0π 2π3π 2

r= 8 2−3sin θ

4−848 5 =1.6 r= 10 5−4 cos θ . 1 5 .

r= 10 5−4cos θ = 10(1 5) 5(1 5)−4(1 5)cos θ r= 2 1− 4 5 cos θ

e= 4 5 ,e<1, cos θ, x=−p.

2=ep 2=(4 5)p 2(5 4)=p 5 2 =p

x=− 5 2 .θ0π 2π3π 2r= 10 5−4 cos θ10210 9 ≈1.12r= 10 5−4 cos θ [−3,12,1] [−4,4,1],θ min = 0

θ max = 2π. r= 2 4−cos θ . y, x, p<0, p>0, p e=3 y=−2. y=−p, y=−2,−2<0,

r= ep 1−e sin θ

e=3 | −2 |=2=p.

r= (3)(2) 1−3 sin θ r= 6 1−3 sin θ

e= 3 5 , x=4. x=p, x=4,4>0,

r= ep 1+e cos θ

e= 3 5 | 4 |=4=p.

r= (3 5)(4) 1+ 3 5 cos θ r= 12 5 1+ 3 5 cos θ r= 12 5 1(5 5)+ 3 5 cos θ r= 12 5 5 5 + 3 5 cos θ r= 12 5 · 5 5+3 cos θ r= 12 5+3 cos θ

e=1, x=−1.r= 1 1−cosθ r= 1 5−5sin θ r= x 2 + y 2 ,x=r cos θ,and y=r sin θ.

r= 1 5−5 sin θ r·(5−5 sin θ)= 1 5−5 sin θ ·(5−5 sin θ) Eliminate the fraction. 5r −5r sin θ=1 Distribute. 5r=1+5r sin θ Isolate 5r. 25 r 2 = (1+5r sin θ) 2 Square both sides. 25(x 2 + y 2)= (1+5y) 2 Substitute r= x 2 + y 2 and y=r sin θ. 25 x 2 +25 y 2 =1+10y+25 y 2 Distribute and use FOIL. 25 x 2 −10y=1 Rearrange terms and set equal to 1.

r= 2 1+2 cos θ 4−8x+3 x 2 − y 2 =0 P(r,θ) e= PF PD , e r= x 2 + y 2 ,x=r cos θ, y=r sin θ sin θ,

r= 6 1−2 cos θr= 3 4−4 sin θ e=1 3 4 r= 8 4−3 cos θr= 5 1+2 sin θ e=2 5 2 r= 16 4+3 cos θ

r= 3 10+10 cos θ e=1 3 10 r= 2 1−cos θr= 4 7+2 cos θ e= 2 7 2 r(1−cos θ)=3r(3+5sin θ)=11 e= 5 3

11 5 r(4−5sin θ)=1r(7+8cos θ)=7 e= 8 7 7 8 r= 4 1+3 sin θr= 2 5−3 sin θ25 x 2 +16 y 2 −12y−4=0

r= 8 3−2 cos θr= 3 2+5 cos θ21 x 2 −4 y 2 −30x+9=0r= 4 2+2 sin θr= 3 8−8 cos θ64 y 2 =48x+9

r= 2 6+7 cos θr= 5 5−11 sin θ96 y 2 −25 x 2 +110y+25=0r(5+2 cos θ)=6r(2−cos θ)=1

3 x 2 +4 y 2 −2x−1=0r(2.5−2.5 sin θ)=5r= 6sec θ −2+3 sec θ5 x 2 +9 y 2 −24x−36=0r= 6csc θ 3+2 csc θ

r= 5 2+cos θr= 2 3+3 sin θr= 10 5−4 sin θr= 3 1+2 cos θr= 8 4−5 cos θr= 3 4−4 cos θr= 2 1−sin θ

r= 6 3+2 sin θr(1+cos θ)=5r(3−4sin θ)=9r(3−2sin θ)=6r(6−4cos θ)=5x=4; e= 1 5r= 4 5+cosθx=−4; e=5

y=2; e=2r= 4 1+2sinθy=−2; e= 1 2x=1; e=1r= 1 1+cosθx=−1; e=1x=− 1 4 ; e= 7 2r= 7 8−28cosθ

y= 2 5 ; e= 7 2y=4; e= 3 2r= 12 2+3sinθx=−2; e= 8 3x=−5; e= 3 4r= 15 4−3cosθy=2; e=2.5x=−3; e= 1 3

r= 3 3−3cosθ xy r θ.xy=2x 2 +xy+ y 2 =4r=± 2 1+sinθcosθ2 x 2 +4xy+2 y 2 =916 x 2 +24xy+9 y 2 =4

r=± 2 4cosθ+3sinθ2xy+y=1x 2 25 + y 2 64 =1x 2 5 2 + y 2 8 2 =1; (0,0); (5,0),(−5,0),(0,8),(0,−8);

(0, 39),(0,− 39)(x−2) 2 100 + (y+3) 2 36 =19 x 2 + y 2 +54x−4y+76=0

(x+3) 2 1 2 + (y−2) 2 3 2 =1 (−3,2); (−2,2),(−4,2),(−3,5),(−3,−1); (−3,2+2 2),(−3,2−2 2)
9 x 2 +36 y 2 −36x+72y+36=0x 2 36 + y 2 9 =1 (0,0); (6,0),(−6,0),(0,3),(0,−3); (3 3 ,0),(−3 3 ,0)
(x−4) 2 25 + (y+3) 2 49 =14 x 2 + y 2 +16x+4y−44=0 (−2,−2); (2,−2),(−6,−2),(−2,6),(−2,−10);
(−2,−2+4 3 ,),(−2,−2−4 3) 2 x 2 +3 y 2 −20x+12y+38=0 (0,0), (3,0), (−5,0)x 2 25 + y 2 16 =1
(2,−2), (7,−2), (4,−2)x 2 81 − y 2 9 =1(y+1) 2 16 − (x−4) 2 36 =1(y+1) 2 4 2 − (x−4) 2 6 2 =1;
(4,−1); (4,3),(4,−5); (4,−1+2 13),(4,−1−2 13)9 y 2 −4 x 2 +54y−16x+29=0
3 x 2 − y 2 −12x−6y−9=0(x−2) 2 2 2 − (y+3) 2 (2 3) 2 =1; (2,−3); (4,−3),(0,−3);
(6,−3),(−2,−3)x 2 9 − y 2 16 =1(y−1) 2 49 − (x+1) 2 4 =1x 2 −4 y 2 +6x+32y−91=0
2 y 2 − x 2 −12y−6=0 (0,0), (0,4), (0,−6) (3,7) (7,7), (6,7)(x−5) 2 1 − (y−7) 2 3 =1y 2 =12x
(x+2) 2 = 1 2 (y−1)(x+2) 2 = 1 2 (y−1); (−2,1); (−2, 9 8); y= 7 8y 2 −6y−6x−3=0
x 2 +10x−y+23=0(x+5) 2 =(y+2); (−5,−2); (−5,− 7 4); y=− 9 4x 2 +4y=0(y−1) 2 = 1 2 (x+3)
x 2 −8x−10y+46=02 y 2 +12y+6x+15=0 (−4,0); x=4 (2, 9 8); y= 7 8(x−2) 2 =(1 2)(y−1)
16 x 2 +24xy+9 y 2 +24x−60y−60=0B 2 −4AC=0,4 x 2 +14xy+5 y 2 +18x−6y+30=0
4 x 2 +xy+2 y 2 +8x−26y+9=0B 2 −4AC=−31<0, θ xy xy x 2 +4xy−2 y 2 −6=0x 2 −xy+ y 2 −6=0
θ= 45 ◦ , x ′ 2 +3 y ′ 2 −12=0 x ′ y ′ x ′ y ′ 9 x 2 −24xy+16 y 2 −80x−60y+100=0x 2 −xy+ y 2 −2=0
θ= 45 ◦6 x 2 +24xy− y 2 −12x+26y+11=0r= 10 1−5 cos θ e=5 2 r= 6 3+2 cos θr= 1 4+3 sin θ e= 3 4
1 3 r= 3 5−5 sin θr= 3 1−sin θr= 8 4+3 sin θr= 10 4+5 cos θr= 9 3−6 cos θ x=3 e=1r= 3 1+cos θ y=−2
e=4x 2 9 + y 2 4 =1x 2 3 2 + y 2 2 2 =1; (0,0); (3,0),(−3,0),(0,2),(0,−2); (5 ,0),(− 5 ,0)
9 y 2 +16 x 2 −36y+32x−92=0(x−3) 2 64 + (y−2) 2 36 =1 (3,2); (11,2),(−5,2),(3,8),(3,−4);
(3+2 7 ,2),(3−2 7 ,2)2 x 2 + y 2 +8x−6y−7=0 (1,2), (7,2), (4,2).(x−1) 2 36 + (y−2) 2 27 =1
x 2 49 − y 2 81 =1x 2 7 2 − y 2 9 2 =1; (0,0); (7,0),(−7,0); (130 ,0),(− 130 ,0); y=± 9 7 x
16 y 2 −9 x 2 +128y+112=0(x−3) 2 25 − (y+3) 2 1 =1 (3,−3); (8,−3),(−2,−3);
(3+ 26 ,−3),(3− 26 ,−3); y=± 1 5 (x−3)−3y 2 − x 2 +4y−4x−18=0 (1,0) (1,6), (1,2).
(y−3) 2 1 − (x−1) 2 8 =1y 2 +10x=03 x 2 −12x−y+11=0(x−2) 2 = 1 3 (y+1); (2,−1); (2,− 11 12);
y=− 13 12(x−1) 2 =−4(y+3)y 2 +8x−8y+40=0 (2,3) y=−1. 8.49 θ xy 3 x 2 −2xy+3 y 2 =4
x 2 +4xy+4 y 2 +6x−8y=0 θ≈ 63.4 ◦ x ′ y ′ x ′ y ′ 11 x 2 +10 3 xy+ y 2 =416 x 2 +24xy+9 y 2 −125x=0
x ′ 2 −4 x ′ +3 y ′ =0r= 3 2−sin θr= 5 4+6 cos θ e= 3 2 , 5 6 r= 12 4−8 sin θr= 2 4+4 sin θ e=2, x=3. P F
D e= PF PD , e r θ

{2,4,8,16,32,…}.

nthnnth 2 1 =2, 2 2 =4, 2 3 =8, nthntha,b,c...n.

a n = 2 n .

nthn

a 31 = 2 31 =2,147,483,648

2,147,483,648nthnnntha n2 n

{2,4,8,16,32,…, 2 n ,…}.

{2,4,8,16,32,…, 2 n ,…,1024}.

na

a 1 , a 2 , a 3 ,…, a n ,…

a 1a 2a 3a nnthnth a 1 ?a 0 a 1 . n nn=1a 1 .a 2 ,n=2.na n =−3n+8.n=1n.
n=1 a 1 =−3(1)+8=5 n=2 a 2 =−3(2)+8=2 n=3 a 3 =−3(3)+8=−1 n=4 a 4 =−3(4)+8=−4 n=5 a 5 =
−3(5)+8=−7

{5, 2, −1, −4, −7}.na n t n =5n−4.{ 1,6, 11, 16, 21 }.n

{2,−4,6,−8}

nnn=1a 1 .(−1) n a 2 , n=2. n

a n = (−1) n n 2 n+1

n=1,n=2,
n=1 a 1 = (−1) 1 2 2 1+1 =− 1 2 n=2 a 2 = (−1) 2 2 2 2+1 = 4 3 n=3 a 3 = (−1) 3 3 2 3+1 =− 9 4 n=4 a 4 =
(−1) 4 4 2 4+1 = 16 5 n=5 a 5 = (−1) 5 5 2 5+1 =− 25 6

{ – 1 2 , 4 3 ,– 9 4 , 16 5 ,– 25 6 }.nn,n+1,n–1,

a n = 4n (–2) n

{ –2, 2, – 3 2 , 1, – 5 8 }. n n=1 a 1 , n=1 n=2 a 2 , n=2 n

a n ={ n 2 if n is not divisible by 3 n 3 if n is divisible by 3

n=1,n=2, n 2nn 3n

a 1 = 1 2 =1 1 is not a multiple of 3. Use n 2 . a 2 = 2 2 =4 2 is not a multiple of 3. Use n 2 . a 3 = 3 3 =1 3 is a multiple of 3. Use n 3 . a 4 = 4 2 =16 4 is not a multiple of 3. Use n 2 . a 5 = 5 2 =25 5 is not a multiple of 3. Use n 2 . a 6 = 6 3 =2 6 is a multiple of 3. Use n 3 .

{ 1, 4, 1, 16, 25, 2 }.

a n ={ 2 n 3 if n is odd 5n 2 if n is even

{ 2, 5, 54, 10, 250, 15 }. nth a nn.n=1, n=2,n=3.nth{ – 2 11 , 3 13 ,– 4 15 , 5 17 ,– 6 19 ,… }

{– 2 25 ,– 2 125 ,– 2 625 ,– 2 3,125 ,– 2 15,625 ,…}{ e 4 , e 5 , e 6 , e 7 , e 8 ,…} (–1) n n+1.2n+9.

a n = (–1) n (n+1) 2n+95 n+1 .

a n =– 2 5 n+1

e.n=1,e 4n+3.

a n = e n+3

nth

{9, –81, 729, –6,561, 59,049, …}

a n = (–1) n+1 9 n nth

{ – 3 4 ,– 9 8 ,– 27 12 ,– 81 16 ,– 243 20 ,... }

a n =– 3 n 4n nth

{ 1 e 2 , 1 e , 1, e, e 2 ,... }

a n = e n–3a n

a 1 =3 a n =2 a n–1 –1, for n≥2

a 1 =3 a 2 =2 a 1 –1=2(3)–1=5 a 3 =2 a 2 –1=2(5)–1=9 a 4 =2 a 3 –1=2(9)–1=17

{ 3, 5, 9, 17 }

a 1 =1 a 2 =1 a n = a n–1 + a n–2 , for n≥3

a 10 = a 9 + a 8 =34+21=55

n a 1 , a 2 ,a n–1 . a 3 ,nth

a 1 =9 a n =3 a n–1 –20, for n≥2

a n–1

n=1 a 1 =9 n=2 a 2 =3 a 1 –20=3(9)–20=27–20=7 n=3 a 3 =3 a 2 –20=3(7)–20=21–20=1 n=4 a 4 =3 a 3 –20=3(1)–20=3–20=–17 n=5 a 5 =3 a 4 –20=3(–17)–20=–51–20=–71

{ 9, 7, 1, –17, –71 }.

a 1 =2 a n =2 a n–1 +1, for n≥2

{ 2, 5, 11, 23, 47 }n a 1 , a 2 , nth

a 1 =1 a 2 =2 a n =3 a n–1 +4 a n–2 , for n≥3

a n–1a n–2

n=3 a 3 =3 a 2 +4 a 1 =3(2)+4(1)=10 n=4 a 4 =3 a 3 +4 a 2 =3(10)+4(2)=38 n=5 a 5 =3 a 4 +4 a 3 =3(38)+4(10)=154 n=6 a 6 =3 a 5 +4 a 4 =3(154)+4(38)=614

{1,2,10,38,154,614}.

a 1 =0 a 2 =1 a 3 =1 a n = a n–1 a n–2 + a n–3 , for n≥4

{ 0, 1, 1, 1, 2, 3, 5 2 , 17 6 }.nn!,n.

4!=4·3·2·1=24 5!=5·4·3·2·1=120

a n =(n+1)!.n.

a 6 =(6+1)!=7!=7·6·5·4·3·2·1=5040

nn(n–1)!5! 5·4!. n, n!, n

0!=1 1!=1 n!=n(n–1)(n–2)···(2)(1), for n≥2

0!0!=1.a n = 5n (n+2)! . n=1,n=2,

n=1 a 1 = 5(1) (1+2)! = 5 3! = 5 3·2·1 = 5 6 n=2 a 2 = 5(2) (2+2)! = 10 4! = 10 4·3·2·1 = 5 12 n=3 a 3 = 5(3) (3+2)! = 15 5! = 15 5·4·3·2·1 = 1 8 n=4 a 4 = 5(4) (4+2)! = 20 6! = 20 6·5·4·3·2·1 = 1 36 n=5 a 5 = 5(5) (5+2)! = 25 7! = 25 7·6·5·4·3·2·1 = 5 1,008

{ 5 6 , 5 12 , 1 8 , 1 36 , 5 1,008 }.na n = (n+1)! 2n .{ 1, 3 2 , 4, 15, 72 }.

0!=1 1!=1 n!=n(n−1)(n−2)⋯(2)(1), for n≥2

nth n n a nn?13·12·11·10·9·8·7·6·5·4·3·2·1.a n = 2 n −2a n =− 16 n+1−8, − 16 3 , −4, − 16 5 a n =− (−5) n−1a n = 2 n n 32, 1 2 , 8 27 , 1 4a n = 2n+1 n 3a n =1.25· (−4) n−11.25, −5, 20, −80 a n =−4· (−6) n−1a n = n 2 2n+11 3 , 4 5 , 9 7 , 16 9a n = (−10) n +1a n =−(4· (−5) n−1 5) − 4 5 , 4, −20, 100a n ={ (−2) n −2 if n is even (3) n−1 if n is odda n ={ n 2 2n+1 if n ≤5 n 2 −5 if n >5 1 3 , 4 5 , 9 7 , 16 9 , 25 11 , 31, 44, 59a n ={ (2n+1) 2 if n is divisible by 4 2 n if n is not divisible by 4 a n ={ −0.6· 5 n−1 if n is prime or 1 2.5· (−2) n−1 if n is composite −0.6,−3,−15,−20,−375,−80,−9375,−320a n ={ 4(n 2 −2) if n ≤3 or n > 6 n 2 −2 4 if 3<n≤6 4, 7, 12, 19, 28,…a n = n 2 +3−4,2,−10,14,−34,…1,1, 4 3 ,2, 16 5 ,…a n = 2 n 2n or 2 n−1 n 0, 1− e 1 1+ e 2 , 1− e 2 1+ e 3 , 1− e 3 1+ e 4 , 1− e 4 1+ e 5 ,…1,− 1 2 , 1 4 ,− 1 8 , 1 16 ,… a n = (− 1 2) n−1a 1 =9, a n = a n−1 +na 1 =3, a n =(−3) a n−13, −9, 27, −81, 243 a 1 =−4, a n = a n−1 +2n a n−1 −1a 1 =−1, a n = (−3) n−1 a n−1 −2−1, 1, −9, 27 11 , 891 5 a 1 =−30, a n =(2+ a n−1) (1 2) na 1 = 1 24 , a 2 =1, a n =(2 a n−2)(3 a n−1) 1 24 , 1, 1 4 , 3 2 , 9 4 , 81 4 , 2187 8 , 531,441 16a 1 =−1, a 2 =5, a n = a n−2 (3− a n−1) a 1 =2, a 2 =10, a n = 2(a n−1 +2) a n−22, 10, 12, 14 5 , 4 5 , 2, 10, 12−2.5,−5,−10,−20,−40,… −8,−6,−3,1,6,…a 1 =−8, a n = a n−1 +n2, 4, 12, 48, 240, …35, 38, 41, 44, 47, …a 1 =35, a n = a n−1 +3 15,3, 3 5 , 3 25 , 3 125 ,⋯6!720(12 6)!12! 6!665,280100! 99!a n = n! n 21, 1 2 , 2 3 , 3 2a n = 3·n! 4·n! a n = n! n 2 −n−1−1,2, 6 5 , 24 11a n = 100·n n(n−1)!a n = (−1) n n +n a n ={ 4+n 2n if n is even 3+n if n is odda 1 =2, a n = (− a n−1 +1) 2a n =1, a n = a n−1 +8 a n = (n+1)! (n−1)!a n = 2 n−2a 1 =6, a n =2 a n−1 −5 a 1 a n−1 . a 1 = 87 111 , a n = 4 3 a n−1 + 12 37 .29 37 , 152 111 , 716 333 , 3188 999 , 13724 2997 a 1 =625, a n =0.8 a n−1 +18. a 1 =2, a n = 2 [(a n −1)−1] +1.2,3,5,17,65537 a 1 =8, a n = (a n−1 +1)! a n−1 ! .a 1 =2, a n =n a n−1a 10 =7,257,600 [X,T,θ,n] n n n a n =− 28 9 n+ 5 3 . a n = n 3 −3.5 n 2 + 4.1n−1.5 2.4n .0.042,0.146,0.875,2.385,4.708 a n = 15n· (−2) n−1 47a n = 5.7 n +0.275(n−1)!5.975,32.765,185.743,1057.25,6023.521a n = n! n . a n =−6−8n.a n =−421 a n =−421 −421=−6−8nn −421=−6−8n, n=51.875a n =−421 a n = n 2 +4n+4 2(n+2)41?1, 0, −1, −1, 0, 1, 1, 0, −1, −1, 0, 1, 1, ... a n a 1 =1, a 2 =0, a n = a n−1 − a n−2a n = (n+2)! (n−1)!b n = n 3 +3 n 2 +2n, (n+2)! (n−1)! = (n+2)·(n+1)·(n)·(n−1)·...·3·2·1 (n−1)·...·3·2·1 =n(n+1)(n+2)= n 3 +3 n 2 +2n {1,2,…n} n n a 1d

{ a n }={ a 1 , a 1 +d, a 1 +2d, a 1 +3d,...}

{1,2,4,8,16,...}{−3,1,5,9,13,...}ab

{18, 16, 14, 12, 10,…}

−2.

{1, 3, 6, 10, 15,…}

3−1≠6−3.nd

a n = a 1 +(n−1)d

a 1 =17d=−3 −3 {17, 14, 11, 8, 5}a 1 =1d=5{ 1 , 6 , 11 , 16 , 21 }a 1 , a n ,n a n = a 1 +(n−1)d d. a 1 ,n, d a n = a 1 +(n−1)d.a 1 =8a 4 =14a 5d

{ 8,8+d,8+2d,8+3d }

a 1 +3d=8+3dd

a n = a 1 +(n−1)d a 4 = a 1 +3d a 4 =8+3d Write the fourth term of the sequence in terms of a_1 and d.

14=8+3d Substitute 14 for a_4. d=2 Solve for the common difference.

a 5 = a 4 +2=16

a n = a 1 +(n−1)d.a 3 =7a 5 =17a 2a 2 = 2

$$a_n = a_{n-1} + d \quad n \geq 2$$

d

$$a_n = a_{n-1} + d \quad n \geq 2$$

$$\{-18, -7, 4, 15, 26, \ldots\}$$

−18

$$d = -7 - (-18) = 11$$

$$a_1 = -18 \quad a_n = a_{n-1} + 11, \text{ for } n \geq 2$$

$$\{25,\ 37,\ 49,\ 61,\ \ldots\}$$

$a_1 = 25 \quad a_n = a_{n-1} + 12, \text{ for } n \geq 2$

$$a_n = a_1 + d(n-1)$$

−50−50y−50200: 200−(−50)=200+50=250y y=mx+b. a nynx. mb −50 250

$$a_n = -50n + 250$$

$a_n = 200 - 50(n-1)$ $a_n = -50n + 250$.nth

$$a_n = a_1 + d(n-1)$$

$a_2 - a_1$.$a_n = a_1 + d(n-1)$.

$$\{2, 12, 22, 32, 42, \ldots\}$$

$$d = a_2 - a_1 = 12 - 2 = 10$$

$$a_n = 2 + 10(n-1) \quad a_n = 10n - 8$$

−8

$$\{50, 47, 44, 41, \ldots\}$$

$a_n = 53 - 3n$d.$a_n = a_1 + d(n-1)$.a nn.

$$\{8, 1, -6, \ldots, -41\}$$

$$1 - 8 = -7$$

−7nth

$$a_n = a_1 + d(n-1) \quad a_n = 8 + -7(n-1) \quad a_n = 15 - 7n$$

−41a nn

$$-41 = 15 - 7n \qquad 8 = n$$

$$\{6, 11, 16, \ldots, 56\}$$

a 0a 1 .

$$a_n = a_0 + dn$$

An

$$A_n = 1 + 2n$$

$$16 - 5 = 11$$

$$A_{11} = 1 + 2(11) = 23$$

$T_n = 10 + 4n$, $a_n = a_{n-1} + d \quad n \geq 2$$a_n = a_1 + d(n-1)$da $n = a_{n-1} + d, n \geq 2$.da $n = a_1 + d(n-1)$.$a_n = a_0 + dn$.
{ 5 , 11 , 17 , 23 , 29 , ... }{ 0 , 1 2 , 1 , 3 2 , 2 , ... }1 2{ 11.4 , 9.3 , 7.2 , 5.1 , 3 , ... }
{ 4 , 16 , 64 , 256 , 1024 , ... }16−4≠64−16.$a_1 = -25 \quad d = -9$a $1 = 0$d$= 2\ 3$

$$0,\ 2\ 3,\ 4\ 3,\ 2,\ 8\ 3$$

$a_1 = 17,\ a_7 = -31$a $13 = -60,\ a_{33} = -1600$, − 5 , − 10 , − 15 , − 20a 4 =19a 6 =41a 1a 6 =12a 14 =28.
$a_1 = 2$a 1a 7 =21a 15 =42. a 1a 8 =40a 23 =115.a 1 =5a 1a 9 =54a 17 =102.a 1a 11 =11a 21 =16.a 1 =6
$a_1 = 33$ $a_7 = -15$. a 4 .a 3 =−17.1 a 10 =−15.7.a 21 .a 21 =−13.5a 1 =39; $a_n = a_{n-1} - 3$
$a_1 = -19$; $a_n = a_{n-1} - 1.4$−19,−20.4,−21.8,−23.2,−24.6a n ={ 40,60,80,... }a n ={17,26,35,...}
$a_1 = 17$; $a_n = a_{n-1} + 9 \quad n \geq 2$a n ={−1,2,5,...}a n ={12,17,22,...}a $1 = 12$; $a_n = a_{n-1} + 5 \quad n \geq 2$
a n ={−15,−7,1,...}a n ={8.9,10.3,11.7,...}a $1 = 8.9$; $a_n = a_{n-1} + 1.4 \quad n \geq 2$a n ={−0.52,−1.02,−1.52,...}
a n ={ 1 5 , 9 20 , 7 10 ,... }a $1 = 1\ 5$; $a_n = a_{n-1} + 1\ 4 \quad n \geq 2$a n ={ − 1 2 ,− 5 4 ,−2,... }
a n ={ 1 6 ,− 11 12 ,−2,... }$1 = 1\ 6$; $a_n = a_{n-1} - 13\ 12 \quad n \geq 2$a n ={7, 4, 1, ...}; a n ={4, 11, 18, ...};
$a_1 = 4$; $a_n = a_{n-1} + 7$; a 14 =95a n ={2, 6, 10, ...}; a n =24−4n20,16,12,8,4.a n = 1 2 n− 1 2
a n ={3,5,7,...}a n =1+2na n ={32,24,16,...}a n ={−5, 95, 195, ...}a n =−105+100n
a n ={−17, −217, −417,...}a n ={1.8, 3.6, 5.4, ...}a n =1.8na n ={−18.1,−16.2,−14.3,...}
a n ={15.8,18.5,21.2,...}a n =13.1+2.7na n ={ 1 3 ,− 4 3 ,−3, ... }a n ={ 0, 1 3 , 2 3 ,... }a n = 1 3 n− 1 3

a n ={ −5,− 10 3 ,− 5 3 ,… }a n ={3,−4,−11, ...,−60}a n ={1.2,1.4,1.6,...,3.8}a n ={ 1 2 ,2, 7 2 ,...,8 }
a 1 =0,d=4a 1 =9; a n = a n−1 −10a n =−12+5na n =3n−2nMin nMin=1u(n) u(n)=3n−2u(nMin)
u(nMin)=1 TblStart=1 ΔTbl=1u(n)?1,4,7,10,13,16,19n=50.u(n)?
nMin=1,nMax=5,xMin=0,xMax=6,yMin=−1, yMax=14. a n = 1 2 n+5 u(n) 9.206.a n =20.6n
a n =2+20.4n.{9b,5b,b,…}.{3a−2b,a+2b,−a+6b…}.a 11 =−17a+38b{5.4,14.5,23.6,...}
{ 17 3 , 31 6 , 14 3 ,... }a 13 =− 1 3{ 5 2 , 19 8 , 9 4 ,..., 1 8 }a 1 =3, a n = a n−1 −3.3,0,−3,−6 a 31 =−87
a 1r

$$\{ a 1 , \ a 1 r, a 1 r 2 , a 1 r 3 , ...\}.$$

1, 2, 4, 8, 16, ...48, 12, 4, 2, ...2 1 =2 4 2 =2 8 4 =2 16 8 =212 48 = 1 4 4 12 = 1 3 2 4 = 1 2

$$5,10,15,20,...$$

10 5 ≠ 15 10

$$100,20,4, 4 5 ,...$$

1 5a 1 =−2r=4,−2·4−8−8·4−32

$$a 1 =−2 a 2 =(−2·4)=−8 a 3 =(−8·4)=−32 a 4 =(−32·4)−128$$

{−2, −8, −32, −128}.a 1 ,a 2 .a n = a 2a 3a 3a 4,a 1 =5r=−2.a 1−2a 2 .a 2a 3 ,

$$a 1 =5 a 2 =−2 a 1 =−10 a 3 =−2 a 2 =20 a 4 =−2 a 3 =−40$$

{ 5,−10,20,−40 }.a 1 =18r= 1 3 .

$$\{ 18,6,2, 2 3 , 2 9 \}$$

ra 1

$$a n =r a n−1 ,n≥2$$

$$\{6, 9, 13.5, 20.25, ...\}$$

$$r= 9 6 =1.5$$

a 1 .

$$a n =r a n−1 \ a n =1.5 a n−1 \ \text{for } n≥2 \ a 1 =6$$

$$\{2, \ 4 3 , \ 8 9 , \ 16 27 , ...\}$$

$$a 1 =2 \ a n = 2 3 a n−1 \ \text{for } n≥2$$

$$a n = a 1 r n−1$$

{18, 36, 72, 144, 288, ...}.

$$a n =18· 2 n−1$$

$$a n = a 1 r n−1$$

a 1 =3 a 4 =24, a 2 . r.

$$3,3r,3 r 2 ,3 r 3 ,...$$

$a n = a 1 r n−1$ $a 4 =3 r 3$ Write the fourth term of sequence in terms of $\alpha 1$ and $r 24=3 r 3$

Substitute 24 for $a 4$ $8= r 3$ Divide $r=2$ Solve for the common ratio

$$a 2 =2 a 1 =2(3) =6$$

a 2 =4a 3 =32a 6 .a 6 =16,384nth

$$\{2, 10, 50, 250, ...\}$$

$$10 2 =5$$

$$a n = a 1 r (n−1) \ a n =2· 5 n−1$$

$$\{−1, 3, −9, 27, ...\}$$

a n =− (−3) n−1a 0a 1 .

$$a n = a 0 r n$$

Pn

$$P n =284· 1.04 n$$

$$2020−2013=7$$

n

$$P 7 =284· 1.04 7 ≈374$$

P n = 293·1.026 a nntha n =r a n−1 ,n≥ 2 nth

$$a n = a 1 r n−1$$

r a n =r a n−1 n≥2r a n = a 1 r n−1 . a n = a 0 r n . 1,3,9,27,81,...−0.125,0.25,−0.5,1,−2,...−2

–2,– 1 2 ,– 1 8 ,– 1 32 ,– 1 128 ,...–6,–12,–24,–48,–96,...5,5.2,5.4,5.6,5.8,...–1, 1 2 ,– 1 4 , 1 8 ,– 1 16 ,... – 1 2 .6,8,11,15,20,...0.8,4,20,100,500,...5.a 1 =8, r=0.3a 1 =5, r= 1 55,1, 1 5 , 1 25 , 1 125
a 7 =64, a 10 =512a 6 =25, a 8 =6.25800,400,200,100,502,3.– 1 3 .a 4 =– 16 27a n ={ –1,2,–4,8,... }.
a 12 .a n ={ –2, 2 3 ,– 2 9 , 2 27 ,... }.a 7 .a 7 =– 2 729a 1 =–486, a n =– 1 3 a n–1a 1 =7, a n =0.2 a n–1
7,1.4,0.28,0.056,0.0112a n ={ –1,5,–25,125,... }a n ={ –32,–16,–8,–4,... }a = 1 –32, a n = 1 2 a n–1
a n ={ 14,56,224,896,... }a n ={ 10,–3,0.9,–0.27,... }a 1 =10, a n =–0.3 a n–1
a n ={ 0.61,1.83,5.49,16.47,... }a n ={ 3 5 , 1 10 , 1 60 , 1 360 ,... }a 1 = 3 5 , a n = 1 6 a n–1
a n ={ –2, 4 3 ,– 8 9 , 16 27 ,... }a n ={ 1 512 ,– 1 128 , 1 32 ,– 1 8 ,... }a 1 = 1 512 , a n =–4 a n–1
a n =–4· 5 n–1a n =12· (– 1 2) n–112,–6,3,– 3 2 , 3 4a n ={ –2,–4,–8,–16,... }a n ={ 1,3,9,27,... }
a n = 3 n–1a n ={ –4,–12,–36,–108,... }a n ={ 0.8,–4,20,–100,... }a n =0.8· (–5) n–1
a n ={–1.25,–5,–20,–80,...}a n ={ –1,– 4 5 ,– 16 25 ,– 64 125 ,... }a n =– (4 5) n–1
a n ={ 2, 1 3 , 1 18 , 1 108 ,... }a n ={ 3,–1, 1 3 ,– 1 9 ,... }a n =3· (– 1 3) n–1a 1 =4,a n =–3 a n–1 .a 8 .
a n =– (– 1 3) n–1 .a 12 .a 12 = 1 177,147a n ={ –1,3,–9,...,2187 }a n ={ 2,1, 1 2 ,..., 1 1024 }12
a 1 =1, r= 1 2a 1 =3, a n =2 a n–1a n =27· 0.3 n–1 200.a 1 =800, a n =0.5a n–1a 1 =12.5, a n =4a n–1
1024.{b,4b,16b,...}.a 5 =256b{64a(–b),32a(–3b),16a(–9b),...}.{10,12,14.4,17.28, ...}100?100a 14 ≈107.
{ 1 2187 , 1 729 , 1 243 , 1 81 ... }a n =–36 (2 3) n–1a 4 =– 32 3 a n =400· 0.5 n–1 ;
400,200,100,50; a 8 =3.125

$$3+7+11+15+19+...$$

nth S_n

$$S_1=3\ S_2=3+7=10\ S_3=3+7+11=21\ S_4=3+7+11+15=36$$

$\sum, a_k=2k_{k=1}^{k=5}. k$

$$a_1=2(1)=2\ a_2=2(2)=4\ a_3=2(3)=6\ a_4=2(4)=8\ a_5=2(5)=10$$

$$\sum_{k=1}^{5} 2k=2+4+6+8+10=30$$

n

$$\sum_{k=1}^{n} a_k$$

$a_{k_{k=1}^{k=n}}.k\ nk\sum_{k=3}^{7} k^2 .k\ 2k=3k=7.k=3,4,5,6,7k^2 .$

$$\sum_{k=3}^{7} k^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2 =9+16+25+36+49 =135$$

$\sum_{k=2}^{5} (3k–1) .d.n$

$$S_n = a_1 +(a_1 +d)+(a_1 +2d)+...+(a_n –d)+ a_n .$$
$$S_n = a_n +(a_n –d)+(a_n –2d)+...+(a_1 +d)+ a_1 .$$

nn

$$S_n = a_1 +(a_1 +d)+(a_1 +2d)+...+(a_n –d)+ a_n + \quad S_n = a_n +(a_n –d)+(a_n –2d)+...+(a_1 +d)+ a_1$$
$$2 S_n =(a_1 + a_n)+(a_1 + a_n)+...+(a_1 + a_n)$$

n

$$2 S_n =n(a_1 + a_n).$$

n

$$S_n = n(a_1 + a_n) 2$$

n

$$S_n = n(a_1 + a_n) 2$$

$na_1a_n .n.a_1 , a_n , n\ S_n = n(a_1 + a_n) 2 .S_n .5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32$
20 + 15 + 10 +…+ $–50_{\sum k=1}^{12} 3k–8a_1 =5 \ a_n =32.n=10. \ a_1 , a_n ,n$

$$S_n = n(a_1 + a_n) 2\ S_{10} = 10(5+32) 2 =185$$

$a_1 =20a_n =–50.n.$

$$a_n = a_1 +(n–1)d \quad –50=20+(n–1)(–5) \quad –70=(n–1)(–5) \quad 14=n–1 \quad 15=n$$

a_1 , a_n , n

$$S_n = n(a_1 + a_n) 2\ S_{15} = 15(20–50) 2 =–225$$

$a_1 , k=1$

$$a_k =3k–8 \quad a_1 =3(1)–8=–5$$

$n=12.a_{12} , k=12$

$$a_k =3k–8\ a_{12} =3(12)–8=28$$

a 1 , a n ,n

$$S_n = n(a_1 + a_n)\ 2 \quad S_{12} = 12(-5+28)\ 2 = 138$$

1.4 + 1.6 + 1.8 + 2.0 + 2.2 + 2.4 + 2.6 + 2.8 + 3.0 + 3.2 + 3.426.413 + 21 + 29 + …+ 69328

$\sum_{k=1}^{10} 5 - 6k - 280$ a 1 = 1 2 d= 1 4 . n=8, S 8 . a 8 ,

$$a_n = a_1 + d(n-1) \quad a_8 = 1\ 2 + 1\ 4\ (8-1) = 9\ 4$$

$$S_n = n(a_1 + a_n)\ 2 \quad S_8 = 8(1\ 2 + 9\ 4)\ 2 = 11$$

r. n

$$S_n = a_1 + r\,a_1 + r^2 a_1 + \ldots + r^{n-1} a_1 .$$

n r.

$$r\,S_n = r\,a_1 + r^2 a_1 + r^3 a_1 + \ldots + r^n a_1$$

$$S_n = a_1 + r\,a_1 + r^2 a_1 + \ldots + r^{n-1} a_1 - r\,S_n = -(r\,a_1 + r^2 a_1 + r^3 a_1 + \ldots + r^n a_1)\ (1-r)\ S_n = a_1 - r^n a_1$$

S n ,(1–r).

$$S_n = a_1 (1 - r^n)\ 1 - r \quad r \neq 1$$

n

$$S_n = a_1 (1 - r^n)\ 1 - r \quad r \neq 1$$

a 1 , r, and n. a 1 , r,nS n = a 1 (1– r n) 1–r .S n .S 11 8 + -4 + 2 + …$\sum_{k=1}^{6} 3 \cdot 2^k a_1$ =8,n=11.r

r= –4 8 =– 1 2

a 1 , r, and n

$$S_n = a_1 (1 - r^n)\ 1 - r \quad S_{11} = 8(1 - (-1\ 2)^{11})\ 1 - (-1\ 2) \approx 5.336$$

$a_{1k=1}$

$$a_1 = 3 \cdot 2\ 1 = 6$$

r=2.n=6.a 1 , r,n

$$S_n = a_1 (1 - r^n)\ 1 - r \quad S_6 = 6(1 - 2^6)\ 1 - 2 = 378$$

S 20 1,000 + 500 + 250 + …≈2,000.00$\sum_{k=1}^{8} 3\,k a_1$ =26,750; n=5; r=1.016. a 1 , r,n

$$S_n = a_1 (1 - r^n)\ 1 - r \quad S_5 = 26{,}750(1 - 1.016^5)\ 1 - 1.016 \approx 138{,}099.03$$

n2+4+6+8+...$\sum_{k=1}^{\infty} 2k$,

1+0.2+0.04+0.008+0.0016+...

r = 0.2. nr n–1<r<1–1<r<1.–1<r<1.r,–1<r<112 + 8 + 4 + …3 4 + 1 2 + 1 3 +...$\sum_{k=1}^{\infty} 27 \cdot (1\ 3)^k$

$\sum_{k=1}^{\infty} 5k2\ 3$,1 2 .2 3 .1 3 ;1 3 .1 3 + 1 2 + 3 4 + 9 8 +...24+(–12)+6+(–3)+...$\sum_{k=1}^{\infty} 15 \cdot (-0.3)^{kn}$

$$S_n = a_1 (1 - r^n)\ 1 - r$$

r= 1 2 .r nn

$$(1\ 2)^2 = 1\ 4 \quad (1\ 2)^3 = 1\ 8 \quad (1\ 2)^4 = 1\ 16$$

r n n?

$$(1\ 2)^{10} = 1\ 1{,}024 \quad (1\ 2)^{20} = 1\ 1{,}048{,}576 \quad (1\ 2)^{30} = 1\ 1{,}073{,}741{,}824$$

nr nnr nr n1– r n a 1 .–1<r<1

$$S = a_1\ 1 - r$$

a 1r.–1<r<1.a 1rS= a 1 1–r . S.10+9+8+7+…248.6+99.44+39.776+ …$\sum_{k=1}^{\infty} 4{,}374 \cdot (-1\ 3)^{k-1}$

$\sum_{k=1}^{\infty} 1\ 9 \cdot (4\ 3)^k a_1$ =248.6r= 99.44 248.6 =0.4,a 1 =248.6r=0.4

$$S = a_1\ 1 - r \quad S = 248.6\ 1 - 0.4 = 414.\ 3^{-}$$

r=– 1 3 .a 1k=1

$$a_1 = 4{,}374 \cdot (-1\ 3)\ 1 - 1 = 4{,}374$$

a 1 =4,374r=– 1 3

$$S = a_1\ 1 - r \quad S = 4{,}374\ 1 - (-1\ 3) = 3{,}280.5$$

r>1. 0.3¯0.3¯=0.333...

$$0.3^{-} = 0.3 + 0.03 + 0.003 + \ldots$$

$$S_n = a_1\ 1 - r = 0.3\ 1 - 0.1 = 0.3\ 0.9 = 1\ 3\ .$$

2+ 2 3 + 2 9 +...$\sum_{k=1}^{\infty} 0.76k+1$ $\sum_{k=1}^{\infty} (-3\ 8)^k - 3\ 11 a_1$ =50r=100.5%=1.005.nnn=72.

a 1 =50, r=1.005, and n=72

$$S_{72} = 50(1 - 1.005^{72})\ 1 - 1.005 \approx 4{,}320.44$$

72(50) = $3,600. a 1 , n, r.r. a 1 , r, and n nS n = a 1 (1− r n) 1−r .S n ,n a 1 =100. n=120.r,

$$r=1+ 0.09 12 =1.0075$$

a 1 =100, r=1.0075, and n=120 n

$$S 120 = 100(1− 1.0075 120) 1−1.0075 ≈19,351.43$$

n S n = n(a 1 + a n) 2 n

$$S n = a 1 (1− r n) 1−r ·r≠1$$

−1<r< 1

$$S n = a 1 1−r ·r≠1$$

nn−1<r<1.nthnthnnthm 2 +3mm=1m=5n=0n=45n∑ n=0 4 5n6k−5k=−2k=1∑ k=1 5 4 5+10+15+20+25+30+35+40+45+5010+18+26+…+162∑ k=1 20 8k+21 2 +1+ 3 2 +2+…+4n 3 2 +2+ 5 2 +3+ 7 2S 5 = 5(3 2 + 7 2) 219+25+31+…+733.2+3.4+3.6+…+5.6S 13 = 13(3.2+5.6) 2 1+3+9+27+81+243+729+21878+4+2+…+0.125∑ k=1 7 8· 0.5 k−1− 1 6 + 1 12 − 1 24 +…+ 1 768n 9+3+1+ 1 3 + 1 9S 5 = 9(1− (1 3) 5) 1− 1 3 = 121 9 ≈13.44∑ n=1 9 5· 2 n−1∑ a=1 11 64· 0.2 a−1 S 11 = 64(1− 0.2 11) 1−0.2 = 781,249,984 9,765,625 ≈8012+18+24+30+...2+1.6+1.28+1.024+... S= 2 1−0.8∑ m=1 ∞ 4 m−1∑ ∞ k=1 − (− 1 2) k−1S= −1 1−(− 1 2)∑ k=1 ∞ (1 2) k .S nS n ∑ k=1 ∞ (1 2) kS= 1 2 1−(1 2) =1.∑ a=1 14 a∑ n=1 6 n(n−2)∑ k=1 17 k 2∑ k=1 7 2 kn −1.7+−0.4+0.9+2.2+3.5+4.86+ 15 2 +9+ 21 2 +12+ 27 2 +15S 7 = 147 2−1+3+7+...+31 ∑ k=1 11 (k 2 − 1 2)S 11 = 55 2nS 6−2−10−50−250...S 70.4−2+10−50...S 7 =5208.4∑ k=1 9 2 k−1 ∑ n=1 10 −2· (1 2) n−1S 10 =− 1023 2564+2+1+ 1 2 ...−1− 1 4 − 1 16 − 1 64 ...S=− 4 3 ∑ ∞ k=1 3· (1 4) k−1∑ n=1 ∞ 4.6· 0.5 n−1S=9.2$50;60;5%,$150;24;3%,$450;60;4.5%,$100;120;10%, 50− k 2k=x7115.a k∑ k=0 6 a k =189.a k =30−k∑ k=1 n (3k−5)>100.−1−3−5−7.... −75?0. 65 ¯0. 65 ¯ r= 4 5$10,000 1 30 3 4nmnm+n

# of appetizer options × # of entree options × # of dessert options 2 × 3 × 2 =12

mnm×n n n n!. n r P(n,r). n P r , P(n,r), n!, n (n−r)! (n−r) 6×5×4=120.

$$6! 3! = 6·5·4·3! 3! =6·5·4=120$$

$$P(n,r)= n! (n−r)!$$

n (n−n)! 0!, n n n! 1 n!. n r

$$P(n,r)= n! (n−r)!$$

n r n r n=12 r=9

$$P(n,r)= n! (n−r)! \quad P(12,9)= 12! (12−9)! = 12! 3! =79,833,600$$

n P r n P r 15·14·13·12·11·10·9·8·7·6·5·4 P(7,7)=5,040 P(7,5)=2,520 r n C(n,r). C(n,r) n C r .

$$C(n,r)= n! r!(n−r)!$$

3!=3·2·1=624 6 , n r

$$C(n,r)= n! r!(n−r)!$$

n r n r

$$C(5,2)= 5! 2!(5−2)! =10$$

$$C(5,3)= 5! 3!(5−3)! =10$$

n C r , n C r , (n−r) C(n,r)=C(n,n−r). C(10,3)=120 rC(5,0)=1C(5,1)=5

$$C(5,0)+C(5,1)+C(5,2)+C(5,3)+C(5,4)+C(5,5)=32$$

2 5 . nn 2·2·2·…·22 nn=4

$$2 n = 2 4 =16$$

12!12!

$$n! r 1 ! r 2 !… r k !$$

4!3!

$$12! 4!3! =3,326,400$$

nr 1 r 2 r 3 r k ,

$$n! r 1 ! r 2 !… r k !$$

n=8, r 1 =2, r 2 =2

$$8!\ 2!2!=10{,}080$$

$n\ r$

$$P(n,r)=n!\ (n-r)!$$

$n\ r$

$$C(n,r)=n!\ r!(n-r)!$$

n

$$n!\ r\ 1\ !\ r\ 2\ !\ldots\ r\ k\ !$$

mnm+nmnm×nnnrP(n,r).P(n,r).nrC(n,r)n2 nnAmBA and BA or B m+n A B A and B nrnrnr

$C(n,r)=n!\ (n-r)!r!$ A={−5,−3,−1,2,3,4,5,6}.A?B={−23,−16,−7,−2,20,36,48,72}.A? 4+2=6 5+4+7=16 2×6=12 A B— A={b,c,d} B={a,e,i,o,u}? 10 3 =1000 P(5,2) P(5,2)=20 P(8,4) P(3,3) P(3,3)=6 P(9,6) P(11,5) P(11,5)=55,440 C(8,5) C(12,4) C(12,4)=495 C(26,3) C(7,6) C(7,6)=7 C(10,3) {1,2,3,4,5,6,7,8,9,10} 2 10 =1024 {a,b,c,…,z} 2 12 =4096 2 9 =512 8! 3! =6720 12! 3!2!3!4! S 900,000,000 S? n n? C(n,r)P(n,r)?r=0r=1.r=0,C(n,r)=P(n,r)=1. r=1,r=1,C(n,r)=P(n,r)=n.AA? 6! 2! ×4!=8640 6−3+8−3=8 4×2×5=40 4×12×3=144 8 11 P(15,9)=1,816,214,400 C(10,3)×C(6,5)×C(5,2)=7,200 2 11 =2048 20! 6!6!8! =116,396,280 mnm+nmnm×nmnm×n (x+y) n n (x+y) n , (n r) C(n,r),

$$(\ n\ r\ )=C(n,r)=n!\ r!(n-r)!$$

(n r) (5 2)=C(5,2)=10. nrn≥r,

$$(\ n\ r\ )=C(n,r)=n!\ r!(n-r)!$$

(5 3)(9 2)(9 7)n C r

$$(\ n\ r\ )=C(n,r)=n!\ r!(n-r)!$$

$(\ 5\ 3\ )=5!\ 3!(5-3)!=5\cdot4\cdot3!\ 3!2!=10$ $(\ 9\ 2\ )=9!\ 2!(9-2)!=9\cdot8\cdot7!\ 2!7!=36$
$(\ 9\ 7\ )=9!\ 7!(9-7)!=9\cdot8\cdot7!\ 7!2!=36$

$$(\ n\ r\ )=(\ n\ n-r\ )$$

(7 3) (11 4) (x+y) n(x+y) 52 ,(x+y)

$(x+y)^2=x^2+2xy+y^2$ $(x+y)^3=x^3+3x^2y+3xy^2+y^3$ $(x+y)^4=x^4+4x^3y+6x^2y^2+4xy^3+y^4$

xyn

$$(\ n\ 0\ ),(\ n\ 1\ ),(\ n\ 2\ ),...,(\ n\ n\ ).$$

$(x+y)^n=\sum_{k=0}^{n}(\ n\ k\ )x^{n-k}y^k=x^n+(\ n\ 1\ )x^{n-1}y+(\ n\ 2\ )x^{n-2}y^2+...+(\ n\ n-1\ )xy^{n-1}+y^n$ x+y,

$(x+y)^1=x+y$ $(x+y)^2=x^2+2xy+y^2$ $(x+y)^3=x^3+3x^2y+3xy^2+y^3$ $(x+y)^4=x^4+4x^3y+6x^2y^2+4xy^3+y^4$

$(x+y)^5$? n+1$(x+y)^n$ .n.xnyn.$(x+y)^5$,n=5,x,x^5 ,xx 0 =1y 0 =1,yy 5

$$x^5,\ x^4y,\ x^3y^2,\ x^2y^3,\ xy^4,\ y^5$$

$$(x+y)^5=x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5.$$

1+1nth

$(x+y)^n=\sum_{k=0}^{n}(\ n\ k\ )x^{n-k}y^k=x^n+(\ n\ 1\ )x^{n-1}y+(\ n\ 2\ )x^{n-2}y^2+...+(\ n\ n-1\ )xy^{n-1}+y^n$ nk=0k=n $(x+y)^5$ $(\ 3x-y\ )^4$ n=5k=0k=5

$$(x+y)^5=(\ 5\ 0\ )x^5y^0+(\ 5\ 1\ )x^4y^1+(\ 5\ 2\ )x^3y^2+(\ 5\ 3\ )x^2y^3+(\ 5\ 4\ )x^1y^4+(\ 5\ 5\ )x^0y^5$$

$$(x+y)^5=x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$$

n=4k=0k=43xx−yy.

$$(3x-y)^4=(\ 4\ 0\ )(3x)^4(-y)^0+(\ 4\ 1\ )(3x)^3(-y)^1+(\ 4\ 2\ )(3x)^2(-y)^2+(\ 4\ 3\ )(3x)^1(-y)^3+(\ 4\ 4\ )(3x)^0(-y)^4$$

$$(3x-y)^4=81x^4-108x^3y+54x^2y^2-12xy^3+y^4$$

(−y) (−y) (x−y) 5(2x+5y) $3x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$
$8x^3+60x^2y+150xy^2+125y^3$ (x+2y) 16 $(x+y)^5$.

$$(x+y)^5=x^5+(\ 5\ 1\ )x^4y+(\ 5\ 2\ )x^3y^2+(\ 5\ 3\ )x^2y^3+(\ 5\ 4\ )xy^4+y^5$$

$(\ 5\ 1\ )x^4y.$ $(\ 5\ 2\ )x^3y^2.$

$$(\ n\ r\ )\ x^{n-r}\ y^r$$

(r+1)th (x+y) n

$$\binom{n}{r} x^{n-r} y^{r}$$

n(r+1).r.r(r+1)th (x+2y) 16 r+1=10, r=9

$$\binom{n}{r} x^{n-r} y^{r}$$

$$\binom{16}{9} x^{16-9} (2y)^{9} = 5{,}857{,}280 x^{7} y^{9}$$

(3x−y) 9 −10,206 x 4 y 5(x+y) n = $\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^{k}$(r+1)th

$$\binom{n}{r} x^{n-r} y^{r}$$

$\binom{n}{r}$ C(n,r). C(n,r). $\binom{n}{r}$= C(n,r) = n! r!(n−r)! . $(x+y)^{n} = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^{k}$ $\binom{6}{2}$ $\binom{5}{3}$ $\binom{7}{4}$ $\binom{9}{7}$ $\binom{10}{9}$ $\binom{25}{11}$ $\binom{17}{6}$ $\binom{200}{199}$ (4a−b) 364 a 3 −48 a 2 b+12a b 2 − b 3(5a+2) 3(3a+2b) 3 27 a 3 +54 a 2 b+36a b 2 +8 b 3(2x+3y) 4(4x+2y) 5 1024 x 5 +2560 x 4 y+2560 x 3 y 2 +1280 x 2 y 3 +320x y 4 +32 y 5(3x−2y) 4(4x−3y) 5 1024 x 5 −3840 x 4 y+5760 x 3 y 2 −4320 x 2 y 3 +1620x y 4 −243 y 5(1 x +3y) 5(x −1 +2 y −1) 4 1 x 4 + 8 x 3 y + 24 x 2 y 2 + 32 x y 3 + 16 y 4(x − y) 5(a+b) 17a 17 +17 a 16 b+136 a 15 b 2(x−1) 18 (a−2b) 15a 15 −30 a 14 b+420 a 13 b 2(x−2y) 8(3a+b) 20 3,486,784,401 a 20 +23,245,229,340 a 19 b+73,609,892,910 a 18 b 2(2a+4b) 7(x 3 − y) 8 x 24 −8 x 21 y +28 x 18 y (2x−3y) 4 (3x−2y) 5−720 x 2 y 3 (6x−3y) 7 (7+5y) 14 220,812,466,875,000 y 7 (a+b) 11 (x−y) 735 x 3 y 4 (x−1) 12 (a−3 b 2) 111,082,565 a 3 b 16 (x 3 − 1 2) 10 (y 2 + 2 x) 91152 y 2 x 7f(x)= (x+3) 4 . f 1 (x), f 1 (x) f 2 (x), f 2 (x) f 2 (x)= x 4 +12 x 3 f 3 (x), f 3 (x) f 4 (x), f 4 (x) f 4 (x)= x 4 +12 x 3 +54 x 2 +108x f 5 (x), f 5 (x) (5x+3y) n , $\binom{n}{k} a^{n-k} b^{k}$, where k 0,1,2, ..., n. $\binom{n}{k}$=(7 2), 590,625 x 5 y 2 (a+b) n , $a^{n-k} b^{k}$ (x+b) 40 . bkthk−1 $\binom{n}{k-1}$+$\binom{n}{k}$ $\binom{n}{k}$. p, p≥1, p!=p(p−1)!.$\binom{n}{k-1}$+$\binom{n}{k}$=(n+1 k);

$\binom{n}{k-1}$+$\binom{n}{k}$ = n! k!(n−k)! + n! (k−1)!(n−(k−1))! = n! k!(n−k)! + n! (k−1)!(n−k+1)! = (n −k+1)n! (n−k+1)k!(n−k)! + kn! k(k−1)!(n−k+1)! = (n−k+1)n!+kn! k!(n−k+1)! = (n+1)n! k!((n+1)−k)! = (n+1)! k!((n+1)−k)! =(n+1 k)

(x 2 −2x+1)(a +4 a −5) 8(x 3 +2 y 2 −z) 5(3 x 2 − 2 y 3) 12 (x 3 +2 y 2 −z) 5 C(n,r), $\binom{n}{r}$ $(x+y)^{n}$ {1,2,3,4,5,6 }. p0≤p≤1, 1 6 .1 61 61 61 61 61 6 S S. S. S, 4 6 = 2 3 . ES

P(E)= number of elements in E number of elements in S = n(E) n(S)

ES,0≤P(E)≤1.

P(E)= 3 6 = 1 2

2 3 E and F,written E∪F,

P(E∪F)=P(E)+P(F)−P(E∩F)

b. 3 6 = 1 2 . b. b2 6 = 1 3 .bb,b.

1 2 + 1 3 − 1 6 = 2 3

b 2 3 .EF E∪FEFEF(EFE∩F

P(E∪F)=P(E)+P(F)−P(E∩F)

1 4 . 1 13 . 1 52 . P(H)= 1 4 , P(7)= 1 13 , and P(H∩7)= 1 52

P(E ∪ F)=P(E)+P(F)−P(E ∩ F) = 1 4 + 1 13 − 1 52 = 4 13

4 13 . 7 13 d. d,

P(E∪F)=P(E)+P(F)

E F 3 6 = 1 2 d 1 6 . d

P(E ∪ F)=P(E)+P(F) = 1 2 + 1 6 = 2 3

d 2 3 . E and F

P(E∪F)=P(E)+P(F)

1 4 , 1 4 ,

1 4 + 1 4 = 1 2

2 13 E, E ′ , E. W W

P(E ′)=1−P(E)

1 9 ,

1– 1 9 = 8 9

P(E ′)=1–P(E)

6×6, 36 1-11-21-31-41-51-62-12-22-32-42-52-63-13-23-33-43-53-64-14-24-34-44-54-65-15-25-35-45-55-66-16-26-36-46-56-6

3 36 = 1 12

P(E ′)=1–P(E) =1– 1 12 = 11 12

5 6 C(5,2) C(8,2)

ways to select 2 phones that are not defective ways to select 2 phones = C(5,2) C(8,2)

= 10 28 = 5 14

C(6,5) C(14,5)

C(6,5) C(14,5) = 6 2,002 = 3 1,001

C(6,2) C(5,3) C(6,2)·C(5,3)

C(6,2)C(5,3) C(14,5) = 15·10 2,002 = 75 1,001

C(9,5) C(14,5)

C(9,5) C(14,5) = 63 1,001

C(5,1)·C(9,4)

C(5,1)C(9,4) C(14,5) = 5·126 2,002 = 315 1,001

63 1,001 + 315 1,001 = 378 1,001

1– 378 1,001 = 623 1,001

a. 1 91 ; b. 5 91 ; c. 86 91

P(E)= n(E) n(S)

P(E∪F)=P(E)+P(F)–P(E∩F)

P(E∪F)=P(E)+P(F)

P(E')=1–P(E)

0 1, 0 1. A and B A and B, A or B 0 1. 1 2 . 5 8 . 1 2 . 3 8 . 1 4 . 3 4 . 3 8 . 1 8 . 15 16 . 5 8 . 52 1 13 . 1 26 . 12 13 . 3. 8. 5 12 . 9. 15. 0. 15. 6 9. 4 9 . 6 9, 5 6. 1 4 . 5 6. 3 4 21 26 12 6 10 8 8 4 C(12,5) C(48,5) = 1 2162 4 3 C(12,3)C(36,2) C(48,5) = 175 2162 20 1 80. 20 1 80. 3,4, 5 20 C(20,3)C(60,17) C(80,20) ≈12.49% C(20,5)C(60,15) C(80,20) ≈23.33% 20.50+23.33–12.49=31.34% C(40000000,1)C(277000000,4) C(317000000,5) =36.78% C(40000000,4)C(277000000,1) C(317000000,5) =0.11% a 1 =2, a n = a n–1 +n.2,4,7,11 6! (5–3)!3! . a n = 10 n +3.13,103,1003,10003 a n = n! n(n+1) . 4 7 , 47 21 , 82 21 , 39 7 , ... d= 5 3 . 2,4,8,16, ... a 1 =18 d=–8. 18,10,2,–6,–14a 3 =11.7a 8 =–14.6.–20,–10,0,10,…a 1 =–20, a n = a n–1 +10 0, – 1 2 , –1, – 3 2 ,…,7 8 , 29 24 , 37 24 , 15 8 ,…a n = 1 3 n+ 13 24 12,20,28,…,172?2.5, 5, 10, 20,… r=24, 16, 28, 40,… a 7 =16,384 a 9 =262,144 .4, 16, 64, 256, 1024 a 1 =–3 r= 1 2 . a 1 =3, a n =4· a n–1 ?3, 12, 48, 192, 7681, 1 3 , 1 9 , 1 27 ,…– 1 5 , – 1 15 , – 1 45 , – 1 135 ,… a n =– 1 5 · (1 3) n–1–5, – 5 3 , – 5 9 ,…, – 5 59,049 ?1 2 m+5m=0m=5.∑ m=0 5 (1 2 m+5).13n S 11 =11015S 912, 6, 3, 3 2 ,…S 9 ≈23.95∑ k=1 ∞ 45· (– 1 3) k–1 .S= 135 43 555 {–10,–6, 4, 10, 12, 18, 24, 32} 46?20127210 4 =10,000P(18,4).P(18,4)=73,44051032C(15,6). C(15,6)=5005{ 1, 3, 5, …, 99 }2 50 =1.13× 10 158! 3!2! =3360 (23 8).490,314(3x+ 1 2 y) 6 . (2a+b) 17 .131,072 a 17 +1,114,112 a 16 b+4,456,448 a 15 b 2(3 a 2 –2b) 112?1 65 94 9 1– C(350,8) C(500,8) ≈94.4%C(150,3)C(350,5) C(500,8) ≈25.6%a=–14, a n = 2+ a n–1 2 . –14,–6,–2,0a n = n 2 –n–1 n! .0.3, 1.2, 2.1, 3,…d=0.9.a 1 =–4d=– 4 3 .–2, – 7 2 , –5, – 13 2 ,… a 1 =–2, a n = a n–1 – 3 2 ; a 22 =– 67 215.6, 15, 14.4, 13.8,… –2,–1,– 1 2 ,– 1 4 ,…r= 1 2 . –1.5,–3,–6,–12,…? 1, – 1 2 , 1 4 , – 1 8 ,…a 1 =1, a n =– 1 2 · a n –14, – 4 3 , 4 9 , – 4 27 ,… 3 k 2 – 5 6 k k=–3 k=15.∑ k=–3 15 (3 k 2 – 5 6 k)n∑ k=1 7 –0.2· (–5) k–1 .S 7 =–2604.2 ∑ k=1 ∞ 1 3 · (– 1 5) k–1 .$140,355.75;$14,355.755×3×2×3×2=180C(15,3)=45510! 2!3!2! =151,200 (3 2 x– 1 2 y) 5 .(x 2 – 1 2) 13429 x 14 164 75 7C(14,3)C(26,4) C(40,7) ≈29.2% E

1, 1 2 , 1 4 , 1 8 ...

f(x)=L, x a, L. L. x a, f(x) L.L a.

$$\lim x\to a\ f(x)=L.$$

x a x=a x=a, L.

$$f(x)=\ x\ 2\ -6x-7\ x-7\ .$$

f(x)= (x−7) (x+1) x−7 Cancel like factors in numerator and denominator. f(x)=x+1,x≠7 Simplify.

x x≠7, x=7. x=7 x=7

$$\lim x\to\ 7\ f(x)=8$$

x x=7? x=7,

$$f(7)\ \text{does not exist.}$$

$$f(x)=x+1,\ x\neq 7.$$

f(x) x=a. x a, x=a L f(x) x a x a a), f(x) L. f(x) a. a L

$$\lim x\to a\ f(x)=L$$

a,f(x), L.

$$\lim x\to 2\ \ (\ 3x+5\ )=11$$

x a,

$$\lim x\to a\ \ f(x)=L.$$

$$\lim x\to 2\ (\ 3x+5\ )=11.$$

a=2,f(x)=3x+5, and L=11. y=3x+5 lim x→2 (3x+5)=11 , x f(x)=3x+5 3(2)+5, L, x x=2. a,f(x), L.

$$\lim x\to 5\ (\ 2\ x\ 2\ -4\ )=46$$

a=5, f(x)=2 x 2 −4, L=46.

$$f(x)=x+1,x\neq 7$$

6.9, 6.99, 6.999. 7.9,7.99, 7.999. f(x) x f(x)=x+1,x≠7 x 7.1, 7.01, 7.001. 8.1, 8.01, 8.001.
f(x) x f(x)=x+1,x≠7 x x 6.9<x<7.1 f(x) 7.9<f(x)<8.1. f(x) f(x), L=8 x x

$$\lim x\to\ 7\ -\ f(x)=8.$$

$$\lim x\to\ 7\ +\ f(x)=8.$$

f(x) x a L,

$$\lim x\to\ a\ -\ f(x)=L.$$

f(x) L x a x<a x≠a. f(x), x a L,

$$\lim x\to\ a\ +\ f(x)=L.$$

f(x) L x a a. a L x a f(x) x a. f(x), x a, L,

$$\lim x\to a\ \ f(x)=L$$

$$\lim x\to\ a\ -\ f(x)=\lim x\to\ a\ +\ f(x).$$

f(x) x a x a. x a, x x=a. a. x=a, x=a. x<a. L x=a, x=a. x>a. L, f(x) x a. (a,f(a)).
f(a) f(a) f(x), x a. x=a, f(a) f lim x→ 2 − f(x)lim x→ 2 + f(x)lim x→2 f(x)f(2) f
lim x→ 2 − f(x)lim x→ 2 + f(x)lim x→2 f(x)f(2)lim x→ 2 − f(x)=8; x<2, y=8.lim x→ 2 + f(x)=3;
x>2, y=3.lim x→ 2 f(x) lim x→ 2 − f(x)≠ lim x→ 2 + f(x); f(2)=3 f (2,f(2)) (2,3).
lim x→ 2 − f(x)=8; x<2 y=8.lim x→ 2 + f(x)=8; x>2 y=8.lim x→ 2 f(x)=8
lim x→ 2 − f(x)= lim x→ 2 + f(x)=8; f(2)=4 f (2,f(2)) (2,4). y=f(x) −2; x a L.

$$\lim x\to\ 5\ (\ x\ 3\ -125\ x-5\ )$$

x=5. x>5 x>5

$$\lim x\to\ 5\ -\ f(x)=75=\lim x\to\ 5\ +\ f(x),$$

$$\lim x\to 5\ f(x)=75.$$

f(5) f, x a f(a), a x a f(a).

$$\lim x\to 0\ (\ 5\sin(x)\ 3x\ )$$

x=0. x=0

$$f(x)=\ 5\sin(x)\ 3x$$

x=0, x=0 x=0. x<0 5 3 . x>0 5 3 .

$$\lim x\to\ 0\ -\ f(x)=\ 5\ 3\ =\lim x\to\ 0\ +\ f(x),$$

$$\lim x\to 0\ f(x)= 5\ 3\ .$$

f(x)= x 3 −125 x−5 x x x=5 75.

$$\lim x\to 0\ (\ 20\sin(x)\ 4x\ )$$

$\lim x\to 0\ (\ 20\sin(x)\ 4x\ )=5$ x x

$$f(x)=3\sin(\ \pi\ x\ )$$

x=0. x=0, x=0 [−2,2] [−3,3]. [−0.1,0.1] [−3,3]. f(x) x

$\lim x\to 0 -\ (\ 3\sin(\ \pi\ x\ )\ )$ does not exist.

$\lim x\to 0 +\ (\ 3\sin(\ \pi\ x\ )\ )$ does not exist.

$\lim x\to 0\ (\ 3\sin(\ \pi\ x\ )\ )$ does not exist.

$\lim x\to 0\ (\ \sin(\ 2\ x\ )\ )$. L a. $\lim x\to a\ f(x)=L$, x a, x=a x=a, L. f(x) L x a x<a. f(x) L x a x>a. x a, y- x=a a x=a x a. x=a f(a). $\lim x\to a\ f(\ x\ )$ x a a. x a x a x a, f $\lim x\to -\ 2\ -\ f(x)$ $\lim x\to -\ 2\ +\ f(x)$ $\lim x\to -2\ f(x)$f(−2) $\lim x\to -\ 1\ -\ f(x)$ $\lim x\to 1\ +\ f(x)$ $\lim x\to 1\ f(x)$f(1) $\lim x\to 4\ -\ f(x)$ $\lim x\to 4\ +\ f(x)$ $\lim x\to 4\ f(x)$f(4) $\lim x\to 0\ -\ f(x)=2$, $\lim x\to 0\ +\ f(x)=-3$, $\lim x\to 2\ f(x)=2$, f(0)=4, f(2)=−1, f(−3) does not exist. $\lim x\to 2\ -\ f(x)=0$, $\lim x\to 2\ +=-2$, $\lim x\to 0\ f(x)=3$, f(2)=5, f(0) $\lim x\to 2\ -\ f(x)=2$, $\lim x\to 2\ +\ f(x)=-3$, $\lim x\to 0\ f(x)=5$, f(0)=1, f(1)=0 $\lim x\to 3\ -\ f(x)=0$, $\lim x\to 3\ +\ f(x)=5$, $\lim x\to 5\ f(x)=0$, f(5)=4, f(3) does not exist. $\lim x\to 4\ f(x)=6$, $\lim x\to 6\ +\ f(x)=-1$, $\lim x\to 0\ f(x)=5$, f(4)=6, f(2)=6 $\lim x\to -3\ f(x)=2$, $\lim x\to 1\ +\ f(x)=-2$, $\lim x\to 3\ f(x)=-4$, f(−3)=0, f(0)=0 $\lim x\to \pi\ f(x)=\ \pi\ 2$, $\lim x\to -\pi\ f(x)=\ \pi\ 2$, $\lim x\to 1\ -\ f(x)=0$, $f(\pi)=\ 2$, f(0) does not exist. x f(x)= (1+x) 1 xg(x)= (1+x) 2 xh(x)= (1+x) 3 xi(x)= (1+x) 4 xj(x)= (1+x) 5 x f(x)= (1+x) 6 x , g(x)= (1+x) 7 x , and h(x)= (1+x) n x .e 6 ≈403.428794, e 7 ≈1096.633158, e n x a. x a, (x)={ | x |−1, if x≠1 x 3 , if x=1 a=1(x)={ 1 x+1 , if x=−2 (x+1) 2 , if x≠−2 a=−2$\lim x\to -2\ f(x)=1$ x=a. x=a. f(x)= x 2 −4x 16− x 2 ;a=4f(x)= x 2 −x−6 x 2 −9 ;a=3$\lim x\to 3\ (\ x\ 2\ -x-6\ x\ 2\ -9\ )=\ 5\ 6\ \approx 0.83$ f(x)= x 2 −6x−7 x 2 − 7x ;a=7f(x)= x 2 −1 x 2 −3x+2 ;a=1$\lim x\to 1\ (\ x\ 2\ -1\ x\ 2\ -3x+2\ )=-2.00$ f(x)= 1− x 2 x 2 −3x+2 ;a=1f(x)= 10−10 x 2 x 2 −3x+2 ;a=1$\lim x\to 1\ (\ 10-10\ x\ 2\ x\ 2\ -3x+2\ )=20.00$ f(x)= x 6 x 2 −5x−6 ;a= 3 2f(x)= x 4 x 2 +4x+1 ;a=− 1 2$\lim x\to -1\ 2\ (\ x\ 4\ x\ 2\ +4x+1\ )$ x f(x)= 2 x−4 ; a=4 x $\lim x\to 0\ 7\tan x\ 3x$ $\lim x\to 0\ 7\tan x\ 3x = 7\ 3$ $\lim x\to 4\ x\ 2\ x-4$ $\lim x\to 0\ 2\sin x\ 4\tan x$ $\lim x\to 0\ 2\sin x\ 4\tan x = 1\ 2$ x a. x a, $\lim x\to 0\ e\ e\ 1\ x$ $\lim x\to 0\ e\ e -\ 1\ x\ 2$ $\lim x\to 0\ e\ e -\ 1\ x\ 2 =1.0$ $\lim x\to 0\ |\ x\ |\ x$ $\lim x\to -1\ |\ x+1\ |\ x+1$ $\lim x\to -\ 1\ -\ |\ x+1\ |\ x+1 = -(x+1)\ (x+1) =-1$ $\lim x\to -\ 1\ +\ |\ x+1\ |\ x+1 = (x+1)\ (x+1) =1$; $\lim x\to -1\ |\ x+1\ |\ x+1$ $\lim x\to 5\ |\ x-5\ |\ 5-x$ $\lim x\to -1\ 1\ (\ x+1\ )\ 2$ $\lim x\to -1\ 1\ (\ x+1\ )\ 2$ x −1 $\lim x\to 1\ 1\ (\ x-1\ )\ 3$ $\lim x\to 0\ 5\ 1-\ e\ 2\ x$ $\lim x\to 0\ 5\ 1-\ e\ 2\ x$ f(x)=| 1−x x | g(x)=| 1+x x | x f(x) g(x) x x m v

$$m=\ m\ o\ 1-(\ v\ 2\ /\ c\ 2\ )$$

m o c m, v c − . v→c, m→∞.

$$\lim v\to\ c\ -\ m=\lim v\to\ c\ -\ m\ o\ 1-(\ v\ 2\ /\ c\ 2\ )\ =\infty$$

c, m, m v→c? v vm

$$f(\ x\ )$$

x a

$$\lim x\to\ a\ -\ f(x)=L.$$

$$f(x)$$

L x a

$$x<a$$

$$x\neq a.$$

a L L, f(x) x a a.

$$f(x),$$

L x x=a, x=a. a L L

lim x→a f(x)=L.

f(x)

x

a

lim x→ a + f(x)=L.

f(x)

L

x a

x>a,

x≠a.

a

L

f(x),

x a, L,

lim x→a f(x)=L

lim x→ a – f(x)= lim x→ a + f(x).

f(x)= x 2 –6x–7 x–7

f(x)= (x–7) (x+1) x–7 , which gives us f(x)=x+1,x≠7.

f g(x)=x+1? f x=7 g f(x) x=7, g(x), f x=7 x=7. g x x a, x=a. x a. x

lim x→7 f(x)= lim x→7 g(x).

x=a, x=a. a, k, A, B f g lim x→a f(x)=A lim x→a g(x)=B. lim x→a k=k

lim x→a [k·f(x)]=k lim x→a f(x)=kAlim x→a [f(x)+g(x)]= lim x→a f(x)+ lim x→a g(x)=A+B

lim x→a [f(x)–g(x)]= lim x→a f(x)– lim x→a g(x)=A–B

lim x→a [f(x)·g(x)]= lim x→a f(x)· lim x→a g(x)=A·B

lim x→a f(x) g(x) = lim x→a f(x) lim x→a g(x) = A B ,B≠0

lim x→a [f(x)] n = [lim x→∞ f(x)] n = A n , n lim x→a f(x) n = lim x→a [f(x)] n = A n

lim x→a p(x)=p(a) lim x→3 (2x+5).

lim x→3 (2x+5)= lim x→3 (2x)+ lim x→3 (5) Sum of functions property = 2lim x→3 (x)+ lim x→3 (5) Constant times a function property =2(3)+5 Evaluate =11

lim x→–12 (–2x+2). x a a a lim x→3 (5 x 2).

lim x→3 (5 x 2)=5 lim x→3 (x 2) Constant times a function property =5(3 2) Function raised to an exponent property =45

lim x→4 (x 3 –5). lim x→5 (2 x 3 –3x+1).

lim x→5 (2 x 3 –3x+1)= lim x→5 (2 x 3)– lim x→5 (3x)+ lim x→5 (1) Sum of functions = 2lim x→5 (x 3)– 3lim x→5 (x)+ lim x→5 (1) Constant times a function =2(5 3)–3(5)+1 Function raised to an exponent =236 Evaluate

lim x→–1 (x 4 –4 x 3 +5). lim x→2 (3x+1) 5 . x

lim x→2 (3x+1) 5 = (lim x→2 (3x+1)) 5 = (3(2)+1) 5 = 7 5 =16,807

lim x→–4 (10x+36) 3 .–64lim x→2 (x 2 +6x+8 x–2)xa lim x→2 (x 2 –6x+8 x–2).

lim x→2 (x 2 –6x+8 x–2)= lim x→2 ((x–2)(x–4) x–2) Factor the numerator. = lim x→2 ((x–2) (x–4) x–2) Cancel the common factors. = lim x→2 (x–4) Evaluate. =2–4=–2

f(x)= x 2 –6x+8 x–2

f(x)=x–4,x≠2.

x = 2. lim x→7 (x 2 –11x+28 7–x).–3 lim x→5 (1 x – 1 5 x–5). lim x→–5 (1 5 + 1 x 10+2x). – 1 50 (0 0) a± b lim x→0 (25–x –5 x).

lim x→0 (25−x −5 x)= lim x→0 ((25−x −5) x · (25−x +5) (25−x +5)) Multiply numerator and denominator by the conjugate. = lim x→0 ((25−x)−25 x(25−x +5)) Multiply: (25−x −5)·(25−x +5)=(25−x)−25. = lim x→0 (−x x(25−x +5)) Combine like terms. = lim x→0 (− x x (25−x +5)) Simplify −x x =−1. = −1 25−0 +5 Evaluate. = −1 5+5 =− 1 10

lim h→0 (16−h −4 h).− 1 8 lim x→4 (4−x x −2).

lim x→4 (4−x x −2) = lim x→4 ((2+ x)(2− x) x −2) Factor. = lim x→4 ((2+ x) (2− x) − (2− x)) Factor −1 out of the denominator. Simplify. = lim x→4 −(2+ x) Evaluate. =−(2+ 4) =−4

a 2 − b 2

(a+b)(a−b).

lim x→3 (x−3 x − 3).2 3 lim x→7 | x−7 | x−7 . x=7 ,

| 6.9−7 | 6.9−7 = | 6.99−7 | 6.99−7 = | 6.999−7 | 6.999−7 =−1

| 7.1−7 | 7.1−7 = | 7.01−7 | 7.01−7 = | 7.001−7 | 7.001−7 =1 lim x→ 6 + 6−x | x−6 | .−1 f x a, f(a). f x a f(a). x a f(a)= 0 0 , f f(x) , x c, x c,lim x→0 (3)lim x→2 (−5x x 2 −1)−10 3

lim x→2 (x 2 −5x+6 x+2)lim x→3 (x 2 −9 x−3)lim x→−1 (x 2 −2x−3 x+1)

lim x→ 3 2 (6 x 2 −17x+12 2x−3)1 2lim x→− 7 2 (8 x 2 +18x−35 2x+7)lim x→3 (x 2 −9 x−5x+6)

lim x→−3 (−7 x 4 −21 x 3 −12 x 4 +108 x 2)lim x→3 (x 2 +2x−3 x−3)lim h→0 ((3+h) 3 −27 h)

lim h→0 ((2−h) 3 −8 h)−12lim h→0 ((h+3) 2 −9 h)lim h→0 (5−h − 5 h)− 5 10

lim x→0 (3−x − 3 x)lim x→9 (x 2 −81 3− x)−108lim x→1 (x − x 2 1− x)lim x→0 (x 1+2x −1)

lim x→ 1 2 (x 2 − 1 4 2x−1)lim x→4 (x 3 −64 x 2 −16)lim x→ 2 − (|x−2| x−2)

lim x→ 2 + (| x−2 | x−2)lim x→2 (| x−2 | x−2)lim x→ 4 − (| x−4 | 4−x)lim x→ 4 + (| x−4 | 4−x)

lim x→4 (| x−4 | 4−x)lim x→2 (−8+6x− x 2 x−2) lim x→c f(x)=3, lim x→c g(x)=5

lim x→c [2f(x)+ g(x)]6+ 5lim x→c [3f(x)+ g(x)]lim x→c f(x) g(x)3 5lim x→2 cos(πx)

lim x→2 sin(πx)lim x→2 sin(π x)f(x)={ 2 x 2 +2x+1, x≤0 x−3, x>0 ; lim x→ 0 + f(x)−3

f(x)={ 2 x 2 +2x+1, x≤0 x−3, x>0 ; lim x→ 0 − f(x)f(x)={ 2 x 2 +2x+1, x≤0 x−3, x>0 ; lim x→0 f(x)

lim x→4 x+5 −3 x−4lim x→ 2 + (2x− ⟦x⟧)lim x→2 x+7 −3 x 2 −x−2lim x→ 3 + x 2 x 2 −9

f(x+h)−f(x) h .f(x)=x+1f(x)=2 x 2 −14x+2hf(x)= x 2 +3x+4f(x)= x 2 +4x−1002x+h+4f(x)=3 x 2 +1

f(x)=cos(x)cos(x+h)−cos(x) hf(x)=2 x 3 −4xf(x)= 1 x−1 x(x+h)f(x)= 1 x 2f(x)= x−1 x+h + x

f(x)= x 2 +5x+6 x+3 x x s(t)=−16 t 2 +144t [1,2] s(t)=−64 t 2 +192t t=1 t=1.5 t

A= A 0 e 0.0425t , A 0 t=1 t=2 118 ◦ F 95 ◦ F. T, T(x) x 96 ◦ F 116 ◦ F, 118 ◦ F.

110.5 ◦ F. 96 ◦ F 118 ◦ F 96 ◦ F 118 ◦ F D, D(x) x y=f(x) f(a) x=a x=a x=a

lim x→a f(x) , x=a f x=a x=a y x=a f. lim x→a f(x)=f(a). x=a. x=a x=a f(x) x=a f(a)

lim x→a f(x) x=a lim x→a f(x)=f(a) f(x) x=a , x=a x=a y=f(x) x a f(x) x=a

lim x→ a − f(x)≠ lim x→ a + f(x) y=f(x) x=a x=a x=a f x=a f(x) x=a lim x→a f(x) ,f(a)

f(a), x=a f(a)≠ lim x→a f(x).f(x)= x 2 −2x−15 x−5g(x)={ x+1, x<2 −x, x≥2 x=5. f(5) x x=5.

g(2)=−2. x lim x→ 2 − (x+1)=2+1=3. lim x→ 2 + (−x)=−2.

lim x→ 2 − f(x)≠ lim x→ 2 + f(x).

lim x→2 f(x) x=2.f(x)= x 2 −6x x−6g(x)={ x , 0≤x<4 2x, x≥4 x=6; x=4 f(x) x a f(x) x=a.

lim x→a f(x)=f(a). f(x)= x 4 −9 x 2 f(x)= 4 x+2 −5 f(x)=sin(2x)−4 f(x)=−cos(x+ π 3) f(x)=2ln(x)

x>0 f(x)=tan(x)+2, x≠ π 2 +kπ, k f(x)= x 2 −25 x−7 , x≠7 f(x), x=a. f(a) lim x→a f(x) x=a.

lim x→a f(x)=f(a). x=a. x=a. f(x)={ 4x, x≤3 8+x, x>3 x=3x= 8 3 f x=a, x=a f(a)

f(3)=4(3)=12 ⇒Condition 1 is satisfied.

lim x→3 f(x) x=3, f(x)=4x; x=3, f(x)=8+x. x lim x→ 3 − f(x)= lim x→ 3 − 4(3)=12

lim x→ 3 + f(x)= lim x→ 3 + (8+x)=8+3=11 lim x→ 1 − f(x)≠ lim x→ 1 + f(x), lim x→1 f(x)

⇒ Condition 2 fails.

x=3. x=3, f(x) x=3.x= 8 3 f(8 3)

f(8 3)=4(8 3)= 32 3 ⇒Condition 1 is satisfied.

lim x→ 8 3 f(x) x= 8 3 , f(x)=4x; x= 8 3 , f(x)=8+x. x 8 3 .

lim x→ 8 3 – f(x)= lim x→ 8 3 – 4(8 3)= 32 3 lim x→ 8 3 + f(x)= lim x→ 8 3 + (8+x)=8+ 8 3 = 32 3 lim x→ 8 3 f(x)

⇒Condition 2 is satisfied.

f(8 3)= lim x→ 8 3 f(x)?

f(32 3)= 32 3 = lim x→ 8 3 f(x) ⇒Condition 3 is satisfied.

x= 8 3 , f(x) x= 8 3 . f(x)={ 1 x , x≤2 9x−11.5, x>2 x=2. f(x)= x 2 −25 x−5 x=5. f x=5, x=5.

f(5) does not exist. ⇒Condition 1 fails.

x=5. x=5, f x=5.

lim x→5 x 2 −25 x−5 = lim x→3 (x−5) (x+5) x−5 = lim x→5 (x+5) =5+5=10

⇒Condition 2 is satisfied.

x=5, f(x)= 9− x 2 x 2 −3x x=3. x=3. x=3. a x a, a. f(a) lim x→a f(x) lim x→a f(x)=f(a). x=a. x=a. f

fx={ x+1, x<2 3, 2≤x<4 x 2 −11, x≥4

f(x)=x+1 x<2, f(x)=3 2≤x<4, f(x)= x 2 −5 x≥4. x=2 x=4. x=2,

f(2)=3 ⇒Condition 1 is satisfied.

x=2, lim x→ 2 – f(x)= lim x→ 2 + f(x)? lim x→ 2 – f(x)= lim x→ 2 – (x+1)=2+1=3

lim x→ 2 + f(x)= lim x→ 2 + 3=3 3=3 lim x→ 2 – f(x)= lim x→ 2 + f(x)

⇒Condition 2 is satisfied.

lim x→2 f(x)=3=f(2) ⇒Condition 3 is satisfied.

x=2, f(x) x=2. x=4, x=4, lim x→ 4 – f(x)= lim x→ 4 + f(x)? lim x→ 4 – f(x)= lim x→ 4 – 3=3

lim x→ 4 + f(x)= lim x→ 4 + (x 2 −11)= 4 2 −11=5 3≠5 lim x→ 4 – f(x)≠ lim x→ 4 + f(x) , lim x→4 f(x)

⇒Condition 2 fails.

x=4, f(x) x=4. x=4, x=2. x=2 x=4. f(x)={ πx 4 , x<2 π x , 2≤x≤6 2πx, x>6 x=6 x=a

fx={ sin(x), x<0 x 3 , x>0

f(x)=sin(x) x<0 f(x)= x 3 x>0. x=0,

f(0) does not exist. ⇒Condition 1 fails.

x=0, f(x) x=0. x=0; lim x→0 f(x)=0, f(a) f(a) lim x→a f(x) x=a. lim x→a f(x)=f(a). f x=c. x=c , f(c), f(c) (a,b). f a

f(x)=ln | x+3 |,a=−3

a=−3 f(−3)

f(x)=ln | 5x−2 |,a= 2 5

f(x)= x 2 −16 x+4 ,a=−4 a=−4 f(−4) f(x)= x 2 −16x x ,a=0f(x)={ x, x≠3 2x,x=3 a=3 a=3 lim x→3 f(x)=3 , f(3)=6 ,f(x)={ 5, x≠0 3, x=0 a=0f(x)={ 1 2−x , x≠2 3, x=2 a=2lim x→2 f(x) f(x)={ 1 x+6 , x=−6 x 2 , x≠−6 a=−6f(x)={ 3+x, x<1 x, x=1 x 2 , x>1 a=1 lim x→ 1 – f(x)=4; lim x→ 1 + f(x)=1 lim x→1 f(x) f(x)={ 3−x, x<1 x, x=1 2 x 2 , x>1 a=1 f(x)={ 3+2x, x<1 x, x=1 – x 2 , x>1 a=1lim x→ 1 – f(x)=5≠ lim x→ 1 + f(x)=−1 lim x→1 f(x) f(x)={ x 2 , x<−2 2x+1, x=−2 x 3 , x>−2 a=−2 f(x)={ x 2 −9 x+3 , x<−3 x−9, x=−3 1 x , x>−3 a=−3lim x→− 3 – f(x)=−6 lim x→− 3 + f(x)=− 1 3 lim x→−3 f(x) f(x)={ x 2 −9 x+3 , x<−3 x−9, x=−3 −6, x>−3 a=3f(x)= x 2 −4 x−2 , a=2f(2) f(x)= 25− x 2 x 2 −10x+25 , a=5f(x)= x 3 −9x x 2 +11x+24 , a=−3f(−3) f(x)= x 3 −27 x 2 −3x , a=3f(x)= x |x| , a=0f(0) f(x)= 2| x+2 | x+2 , a=−2 f f(x)= x 3 −2x−15 (−∞,∞)f(x)= x 2 −2x−15 x−5f(x)=2· 3 x+4 (−∞,∞)f(x)=−sin(3x)f(x)= | x−2 | x 2 −2x x=0 x=2 f(x)=tan(x)+2f(x)=2x+ 5 x x=0f(x)= log 2 (x)f(x)=ln x 2 (0,∞)f(x)= e 2xf(x)= x−4 [4,∞) f(x)=sec(x)−3f(x)= x 2 +sin(x) (−∞,∞) b c

f(x)= { x+1, 1<x<3 x 2 +bx+c, | x−2 |≥1 }

a, x=a x=−3x=2x=4 f(x)=sin(12π x) f(0).f(0) x f(x)=0. f(x)?(−∞,0)∪(0,∞) x=−1, x=1, f(1) x=2, x=−7 x=1.x 3 +6 x 2 −7x (x+7)(x−1) f(x)= x 3 −1 x−1 [−3,3], x lim x→1 f(x) x=1:

fx={ x 2 +4 x≠1 2 x=1

x=1 x f(1)=2. f(x)= sin(2x) x f(x) x=0? x=a f(x) x=a lim x→ a − f(x)≠ lim x→ a + f(x) f(x) f(x)= x 3 −4x. x=−2 x=−1 x=2 f x=a (a,f(a)). x=a, h. (a+h,f(a+h)) h. a x=a. (a,f(a)) (a+h,f(a+h)),

slope = change in y change in x

slope = change in y change in x

m sec

m sec = f(a+h)−f(a) (a+h)−(a) = f(a+h)−f(a) a +h− a

m sec (a,f(a)) (a+h,f(a+h)).

m sec = f(a+h)−f(a) h

(a,f(a)) (a+h,f(a+h)) f

AROC= f(a+h)−f(a) h

(2,−6) (−1,5).

AROC= f(a+h)−f(a) h .

(2,−6), (2,f(2)), f(2)=−6. h 2 −1, −1−2=−3. f(a+h) a+h , 2+(−3) −1, f(a+h)=f(−1)=5.

AROC= f(a+h)−f(a) h = 5−(−6) −3 = 11 −3 =− 11 3

(−5,1.5) (−2.5,9). h a+h a h (a+h,f(a+h)) (a,f(a)). x=a, x=a. x=a. x=a, f x=a. x=a f x=a f'(a),

f ′ (a)= lim h→0 f(a+h)−f(a) h

f x=a ,

f'(a)= lim h→0 f(a+h)−f(a) h

f(a+h)−f(a) h h f(x) x=a f(x) x=a. f(x) x=a f ′ (a). f ′ (a) (a,f(a)). f ′ (a) f(x) x=a. t=a. f(x) y ′ = f ′ (x), y=f(x). f ′ (x) f prime of x.

f ′ (x)= y ′ = dy dx = df dx = d dx f(x)=Df(x)

f ′ (x) x y=f(x) x. f(x) x=a f ′ (a). f, f(a+h). f(a). f(a+h)−f(a) h .

f ′ (a)= lim h→0 f(a+h)−f(a) h . f(x)= x 2 −3x+5 x=a.

f ′ (a)= lim h→0 f(a+h)−f(a) h Definition of a derivative

f(a+h)= (a+h) 2 −3(a+h)+5 f(a)= a 2 −3a+5.

f ′ (a)= lim h→0 (a+h)(a+h)−3(a+h)+5−(a 2 −3a+5) h = lim h→0 a 2 +2ah+ h 2 −3a−3h+5− a 2 +3a −5 h Evaluate to remove parentheses. = lim h→0 a 2 +2ah+ h 2 −3a −3h +5 − a 2 +3a −5 h Simplify. = lim h→0 2ah+ h 2 −3h h = lim h→0 h (2a+h−3) h Factor out an h. =2a+0−3 Evaluate the limit. =2a−3

f(x)=3 x 2 +7x x=a.f ′ (a)=6a+7 f(x)= 3+x 2−x x=a.

f ′ (a)= lim h→0 f(a+h)−f(a) h = lim h→0 3+(a+h) 2−(a+h) −(3+a 2−a) h Substitute f(a+h) and f(a) = lim h→0 (2−(a+h))(2−a)[3+(a+h) 2−(a+h) −(3+a 2−a)] (2−(a+h))(2−a)(h) Multiply numerator and denominator by (2−(a+h))(2−a) = lim h→0 (2−(a+h)) (2−a)(3+(a+h) (2−(a+h)))−(2−(a+h)) (2−a) (3+a 2−a) (2−(a+h))(2−a)(h) Distribute = lim h→0 6−3a+2a− a 2 +2h−ah−6+3a+3h−2a+ a 2 +ah (2−(a+h))(2−a)(h) Multiply = lim h→0 5 h (2−(a+h))(2−a)(h) Combine like terms = lim h→0 5 (2−(a+h))(2−a) Cancel like factors = 5 (2−(a+0))(2−a) = 5 (2−a)(2−a) = 5 (2−a) 2 Evaluate the limit

f(x)= 10x+11 5x+4 x=a.f ′ (a)= −15 (5a+4) 2 f(x)=4 x x=36.

f ′ (a)= lim h→0 f(a+h)−f(a) h = lim h→0 4 a+h −4 a h Substitute f(a+h) and f(a)

4 a+h +4 a 4 a+h +4 a .

f ′ (a)= lim h→0 (4 a+h −4 a h)·(4 a+h +4 a 4 a+h +4 a) = lim h→0 (16(a+h)−16a h4(a+h +4 a)) Multiply. = lim h→0 (16a +16h− 16a h4(a+h +4 a)) Distribute and combine like terms. = lim h→0 (16 h h (4 a+h +4 a)) Simplify. = lim h→0 (16 4 a+h +4 a) Evaluate the limit by letting h=0. = 16 8 a = 2 a f ′ (36)= 2 36 Evaluate the derivative at x=36. = 2 6 = 1 3

f(x)=9 x x=9.3 2 s(t)=−16 t 2 +64t+6, t s(t) s(t)=−16 t 2 +64t+6, t=1 t=3 t=1 t=3:

f ′ (a)= lim h → 0 f(a+h)−f(a) h = lim h → 0 −16 (t+h) 2 +64(t+h)+6−(−16 t 2 +64t+6) h Substitute s(t+h) and s(t). = lim h → 0 −16 t 2 −32ht− h 2 +64t+64h+6+16 t 2 −64t−6 h Distribute. = lim h → 0 −32ht− h 2 +64h h Simplify. = lim h → 0 h (−32t−h+64) h Factor the numerator. = lim h → 0 −32t−h+64 Cancel out the common factor h. s ′ (t)=−32t+64 Evaluate the limit by letting h=0.

t s ′ (t) t. t=1 t=3.

s ′ (1)=−32(1)+64=32 s ′ (3)=−32(3)+64=−32

−32 s(t)=−16 t 2 +64t+6. x=a f(x) x=a. x=a f(x), x=a. x=a f(x). x=a a change in y change in x .

y=f(x) f(0)f(2)f'(0)f'(2) f(a), x=a. x=a, f ′ (a), x=a,f(0) x=0. (0,1), f(0)=1.f(2) x=2. (2,1), f(2)=1.f ′ (0) x=0. x=0 f ′ (0)=0.f ′ (2) x=2. x=2. x y f ′ (2)=4. f(x)= x 3 −3x f(1), f ′ (1), f(0), f ′ (0).−2 , −3 x=a f(x)

output units input unit

x C(x),x C ′ (x) x C ′ (11) C ′ (11)=2.50 x f(x)= x 2 −100x. f(x)= x 2 −100x x f ′ (x) f(a+b)= (x+h) 2 −100(x+h) f(a)= a 2 −100a

f ′ (x)= f(a+h)−f(a) h Formula for a derivative = (x+h) 2 −100(x+h)−(x 2 −100x) h Substitute f(a+h) and f(a). = x 2 +2xh+ h 2 −100x−100h− x 2 +100x h Multiply polynomials, distribute. = 2xh+ h 2 −100h h Collect like terms. = h (2x+h−100) h Factor and cancel like terms. =2x+h−100 Simplify. =2x−100 Evaluate when h=0. f ′ (x)=2x−100 Formula for marginal cost f ′ (200)=2(200)−100=300 Evaluate for 200 units.

f(t), t f(0)=0f ′ (1)=60f(1)=70f(2.5)=150 f(t) f ′ (t), f(t), t f ′ (t), t f(0)=0 f ′ (1)=60 f(1)=70 f(2.5)=150 f(t) tf(0)=0f(10)=150f ′ (10)=15f ′ (20)=−10f(40)=−100 f(x) x=a x=a f(x) f(x)=| x |, x=0, x=0. f(x)=| x |, x=0. f(x)=| x |, f(x) x=a.

f(x)={ x 2 , x≤2 8−x, x>2 .

x x x=2. f(x) x=2. f(x)=| x |. x=0. f(x)=| x | x=0 f(x)= x 2 3 x=0. x f(x)= x 2 3 x=0. f(x)= x 1 3 x=0. f(x)= x 1 3 x=0. f(x) x=a x=a, f ′ (a) f(x) x=a. x=a. x=a. x=a. x=a.f(x) (−∞,−2)∪(−2, 1)∪(1,∞).f(x)x=−2x=1.(−∞,−2)∪(−2,−1)∪(−1,1)∪(1,2)∪(2,∞).f(x)x=−2x=−1 x=1x=2 y=f(x) f (−∞,1)∪(1,3)∪(3,∞). f x=1 x=3. f (−∞,1)∪(1,3)∪(3,∞). f x=1 x=3. f(x) x=a y=m(x− x 1)+ y 1 . x=a f ′ (a), f(x) x=a. x=a (a,f(a)).

y=f'(a)(x−a)+f(a)

f x=a

y=f'(a)(x−a)+f(a)

f, x=a. f(x) x=a f ′ (a)= lim h→0 f(a+h)−f(a) h . x=a. f(a). (a,f(a)) f ′ (a) y=f'(a)(x−a)+f(a). y=mx+b. f(x)= x 2 −4x x=3.

f'(a)= lim h→0 f(a+h)−f(a) h

f(a+h)= (a+h) 2 −4(a+h) f(a)= a 2 −4a.

f ′ (a)= lim h→0 (a+h)(a+h)−4(a+h)−(a 2 −4a) h = lim h→0 a 2 +2ah+ h 2 −4a−4h− a 2 +4a h Remove parentheses. = lim h→0 a 2 +2ah+ h 2 −4a −4h − a 2 +4a h Combine like terms. = lim h→0 2ah+ h 2 −4h h = lim h→0 h (2a+h−4) h Factor out h. =2a+0−4 f ′ (a)=2a−4 Evaluate the limit. f ′ (3)=2(3)−4=2

x=3:

y=f'(a)(x−a)+f(a) y=f'(3)(x−3)+f(3) y=2(x−3)+(−3) y=2x−9

x=3 x=3. f(x)=5 x 2 −x+4 x=2.y=19x−16 s(t) t. t=a

$$s'(a)=\lim_{h\to 0}\frac{s(a+h)-s(a)}{h}$$

t s(t)=−16 t 2 +36t+200, t=2. s ′ (t) t=2, s(a+h)=−16 (a+h) 2 +36(a+h)+200 s(a)=−16 a 2 +36a+200.

$$s'(a)=\lim_{h\to 0}\frac{s(a+h)-s(a)}{h}=\lim_{h\to 0}\frac{-16(a+h)^2+36(a+h)+200-(-16a^2+36a+200)}{h}=\lim_{h\to 0}\frac{-16(a^2+2ah+h^2)+36(a+h)+200-(-16a^2+36a+200)}{h}=\lim_{h\to 0}\frac{-16a^2-32ah-16h^2+36a+36h+200+16a^2-36a-200}{h}=\lim_{h\to 0}\frac{-16a^2-32ah-16h^2+36a+36h+200+16a^2-36a-200}{h}=\lim_{h\to 0}\frac{-32ah-16h^2+36h}{h}=\lim_{h\to 0}\frac{h(-32a-16h+36)}{h}=\lim_{h\to 0}(-32a-16h+36)=-32a-16\cdot 0+36$$

s ′ (a)=−32a+36 s ′ (2)=−32(2)+36 =−28 t=2 t s=−16 t 2 +60t−12. AROC= f(a+h)−f(a) h f ′ (a)= lim h→0 f(a+h)−f(a) hf(a+h)−f(a) h x. [x,x+h] x? x. f(x) t, f(x) t=a t=b f(a)+45(b−a). lim h→0 f(x+h)−f(x) h f(x)=3x−4 f(x)=−2x+1f ′ (x)=−2f(x)= x 2 −2x+1f(x)=2 x 2 +x−3f ′ (x)=4x+1f(x)=2 x 2 +5f(x)= −1 x−2 f ′ (x)= 1 (x−2) 2f(x)= 2+x 1−xf(x)= 5−2x 3+2x−16 (3+2x) 2f(x)= 1+3xf(x)=3 x 3 − x 2 +2x+5 f ′ (x)=9 x 2 −2x+2f(x)=5f(x)=5πf ′ (x)=0(−2,0) (−4,5)(4,−3) (−2,−1)− 1 3(0,5) (6,5)(7,−2) (7,10)f(x)= x 3 +1f(x)=−3 x 2 −7x=6f ′ (x)=−6x−7f(x)=7 x 2f(x)=3 x 3 +2 x 2 +x−26 f ′ (x)=9 x 2 +4x+1 x f(x)=2 x 2 −3x x=3f(x)= x 3 +1 x=2y=12x−15f(x)= x x=9 kf(x)= x 2 −kx, y=4x−9 k=−10 k=2 f x=−2 x=0. x=5. x x,f(−1)f(0)f(0)=−2f(1)f(2)f(2)=−6f(3)f ′ (−1)f ′ (−1)=9 f ′ (0)f ′ (1)f ′ (1)=−3f ′ (2)f ′ (3)f ′ (3)=9f ′ (x)=2x f(2)=4 f(x)= x 2 x=1 [0.9,1.1] [0.99,1.01] [0.999,1.001] , [0.9999, 1.0001] x=1, x=1. x=1 f(t) t f(0)=600f'(30)=−20f(30)=0 f'(200)=30f(240)=500 s, t s(t)=−16 t 2 +80t.s(2)=96s'(2)=16s(3)=96 t=3 s'(3)=−16s(0)=0,s(5)=0. t=0 t=5. V r V= 4 3 π r 3 .V r V r=3 cm.36π x R(x)=2 x 2 +10x. x x=10 x=20. R'(10) R'(15) R'(15) R'(10), x C(x)= x 2 −4x+1000. x x=10 to x=15. x=a, lim x→a f(x)−f(a) x−a ,f(x)= 1 x 2 f(x)=5 x 2 −x+4f'(x)=10a−1f(x)=− x 2 +4x+7f(x)= −4 3− x 24 (3−x) 2lim x→ −1 + f(x) lim x→ −1 − f(x)lim x→−1 f(x)lim x→3 f(x) x

Discontinuous at x=−1(lim x→ a f(x) does not exist),x=3 (jump discontinuity), and x=7 (lim x→ a f(x) does not exist).

lim x→0 f(x).xF(x) x a. x a, f(x)={ | x |−1, if x≠1 x 3 , if x=1 a=1 f(x)={ 1 x+1 , if x=−2 (x+1) 2 , if x≠−2 a=−2lim x→−2 f(x)=1f(x)={ x+3 , if x<1 − x 3 , if x>1 a=1 lim x→c f(x)=−3 lim x→c g(x)=5.lim x→c (f(x)+g(x))lim x→c f(x) g(x)lim x→c (f(x)·g(x))−15 lim x→ 0 + f(x),f(x)={ 3 x 2 +2x+1 5x+3 x>0 x<0lim x→ 0 − f(x),f(x)={ 3 x 2 +2x+1 5x+3 x>0 x<0 lim x→ 3 + (3x− ⟦x⟧)lim h→0 ((h+6) 2 −36 h)lim x→25 (x 2 −625 x −5)lim x→1 (− x 2 −9x x) −10lim x→4 7− 12x+1 x−4lim x→−3 (1 3 + 1 x 3+x)− 1 9 x=a. x=a.f(x)= −2 x−4 ; a=4 f(x)= −2 (x−4) 2 ; a=4 x=4,f(x)= −x x 2 −x−6 ; a=3f(x)= 6 x 2 +23x+20 4 x 2 −25 ; a=− 5 2 a=− 5 2 f(x)= x −3 9−x ; a=9 f(x) f(x)= x 2 −2x−15 (−∞,∞)f(x)= x 2 −2x−15 x−5f(x)= x 2 −2x x 2 −4x+4 x=2. f(2) f(x)= x 3 −125 2 x 2 −12x+10f(x)= x 2 − 1 x 2−x x=0 x=2. f(0) f(2) f(x)= x+2 x 2 −3x−10 f(x)= x+2 x 3 +8 x=−2. f(−2) f(x+h)−f(x) h .f(x)=3x+2f(x)=5f(x)= 1 x+1f(x)=ln(x)ln(x+h)−ln(x) h f(x)= e 2xf(x)=4x−6=4f(x)=5 x 2 −3x f(x) x f(x)=− x 3 +4x x=2.y=−8x+16f(x)= x | x | V= 1 3 π r 2 h π12π f f(1)lim x→ −1 + f(x)lim x→ −1 − f(x)lim x→−1 f(x)lim x→−2 f(x)−1 x f x a. x a, f(x)={ 1 x −3, if x≤2 x 3 +1,if x>2 a=2lim x→ 2 − f(x)=− 5 2 a lim x→ 2 + f(x)=9 x f(x)={ x 3 +1, if x<1 3 x 2 −1, if x=1 − x+3 +4, if x>1 a=1lim x→−5 (1 5 + 1 x 10+2x)− 1 50 lim h→0 (h 2 +25 −5 h 2)lim h→0 (1 h − 1 h 2 +h) f f(x)= x 2 −4 f(x)= x 3 −4 x 2 −9x+36 x 3 −3 x 2 +2x−6 x=3 x=a.f(x)= 3 5+2xf(x)= 3 xf'(x)=− 3 2 a 3 2f(x)=2 x 2 +9x f(x)=| x−2 |−| x+2 |f(x)= 2 1+ e 2 x x=0 s, t s(t).s(0)s(2) t=2 s'(2)s(2)−s(1) 2−1 t=1 to t=2s(t)=0 lim x→0 sin(x) 3x1 3lim x→0 tan 2 (x) 2xlim x→0 sin(x)(1−cos(x)) 2 x 2 lim x→1 f(x), where f(x)={ 4x−7 x≠1 x 2 −4 x=1 x f(x)={ 4x−7 x≠1 x 2 −4 x=1 x=1 x=1 to x=3. x f'(x)=0. x=1 x f'(x) f f(x)=3 x 2 −2x−6, x=−2y=−14x−18 f(x)=x (1−x) 2 5 f(x)=x (1−x) 2 5 f(x)=x ((1−x) 2) 1 5 f(x)=x ((1−x) 1 5) 2 f(x) x=1 x=1. x=1 lim h→0 f(x+h)−f(x) h

f(x)=2x−8f(x)=4 x 2 −7f ' (x)=8xf(x)=x− 1 2 x 2f(x)= 1 x+2f ' (x)=− 1 (2+x) 2f(x)= 3 x−1f(x)=− x 3 +1
f ' (x)=−3 x 2f(x)= x 2 + x 3f(x)= x−1f'(x)= 1 2 x−1 (a,f(a)) (a+h,f(a+h)) f(x);

$$AROC = \frac{f(a+h)-f(a)}{h}.$$

f ′ (a), x=a f ′ (a)= lim h→0 f(a+h)−f(a) h , f(x) x=a. f ′ (a) x=a
f ′ (a)= lim h→0 f(a+h)−f(a) h . s(t) t, t=a s ′ (a)= lim h→0 s(a+h)−s(a) h .

$\cos^2 t+\sin^2 t=1$ $1+\tan^2 t=\sec^2 t$ $1+\cot^2 t=\csc^2 t$

$\cos(-t)=\cos t$ $\sec(-t)=\sec t$ $\sin(-t)=-\sin t$ $\tan(-t)=-\tan t$ $\csc(-t)=-\csc t$ $\cot(-t)=-\cot t$

$\cos t=\sin(\frac{\pi}{2}-t)$ $\sin t=\cos(\frac{\pi}{2}-t)$ $\tan t=\cot(\frac{\pi}{2}-t)$ $\cot t=\tan(\frac{\pi}{2}-t)$ $\sec t=\csc(\frac{\pi}{2}-t)$ $\csc t=\sec(\frac{\pi}{2}-t)$

$\tan t=\frac{\sin t}{\cos t}$ $\sec t=\frac{1}{\cos t}$ $\csc t=\frac{1}{\sin t}$ $\cot t=\frac{1}{\tan t}=\frac{\cos t}{\sin t}$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$\sin(2\theta)=2\sin\theta\cos\theta$ $\cos(2\theta)=\cos^2\theta-\sin^2\theta$ $\cos(2\theta)=1-2\sin^2\theta$ $\cos(2\theta)=2\cos^2\theta-1$ $\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$

$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$ $\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$ $\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}$ $\tan\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}$

$\sin^2\theta=\frac{1-\cos(2\theta)}{2}$ $\cos^2\theta=\frac{1+\cos(2\theta)}{2}$ $\tan^2\theta=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

$\cos\alpha\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$ $\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$ $\sin\alpha\sin\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$ $\cos\alpha\sin\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$

$\sin\alpha+\sin\beta=2\sin(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$ $\sin\alpha-\sin\beta=2\sin(\frac{\alpha-\beta}{2})\cos(\frac{\alpha+\beta}{2})$ $\cos\alpha-\cos\beta=-2\sin(\frac{\alpha+\beta}{2})\sin(\frac{\alpha-\beta}{2})$ $\cos\alpha+\cos\beta=2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$

$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$ $a\sin\alpha=b\sin\beta=c\sin\gamma$

$a^2=b^2+c^2-2bc\cos\alpha$ $b^2=a^2+c^2-2ac\cos\beta$ $c^2=a^2+b^2-2ab\cos\gamma$